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(Working Papers)

Sorting, reputation and entry in a market for experts

by Enrico Sette

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# **SORTING, REPUTATION AND ENTRY IN A MARKET FOR EXPERTS**

by Enrico Sette\*

## **Abstract**

This paper analyses the market for professional (expert) services where the experts are motivated by reputational concerns. A key feature of such markets, which is often overlooked, is that clients can have specific characteristics that affect their evaluation of the service, and (or) the likelihood the service can be provided successfully. These different characteristics can induce clients to choose between experts with different reputations. The paper shows that clients choices have an important impact on the incentives of experts to provide a high quality service. In particular, sorting of clients affects incentives through three channels: changes in the types of client who are indifferent between getting the service from experts of different reputation, changes in the information on good performance as a signal of an expert's talent, and changes in the average complexity of the service the expert provides which impacts on the marginal efficiency of effort. The paper also investigates under what conditions increased entry of experts increases their incentives to exert effort. The results of the model can be applied to examine the effects of entry into the markets for doctors, lawyers, professional consultancies.

**JEL Classification:** D82, L15, L84.

**Keywords:** reputation, competition, sorting, experts, entry.

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# 1 Introduction and Motivation<sup>1</sup>

In markets for professional services such as healthcare, legal practice, professional consultancy, experts (doctors, lawyers, etc.) with different reputations compete in providing a service to clients: patients can be treated by luminaries or young general practitioners; legal assistance can be provided by a “Perry Mason” as well as by unknown members of the Bar; intermediaries of different standing compete in the market for financial services. An importance aspect of such markets is that clients can be widely heterogenous, too: patients can require sophisticated operations or more standard treatments, defendants can go to court facing an involved murder accusation or a trivial quarrel with neighbours, merging firms can be of different size, and a merger between two large multinationals is typically much more complex than a merger between two small firms. Heterogenous clients typically derive a different expected utility from hiring experts of different reputation: a patient with a rare illness might benefit from the expertise of a luminary more than a patient needing to fix a broken arm; similarly, a firm with a very promising project might value the services of a famed investment bank in the going public process more than firms with less promising investment plans. Thus, heterogenous clients may find it optimal to sort into experts with different reputation.

The goal of this paper is to investigate the effect of clients’ sorting on the incentives of experts, motivated by reputational concerns, to provide their service at a high standard. Then, the paper analyzes how increased entry of experts in the market modifies clients’ sorting and, in this way, incentives of experts. The paper shows under what conditions increased entry boosts incentives to exert effort and under what conditions increased entry depresses such incentives. The analysis of the effects of sorting on reputational incentives help to shed new light on the debate about the impact of increasing the entry of experts in a market on the average quality of the service provided. This is a hot topic in the market for healthcare services as there is a debate about the extent to which countries should constrain the supply of doctors by limiting the number of student who can access medical colleges, or by tightly controlling the immigration of foreign doctors.

This paper builds a theoretical model to study the interaction between experts motivated by career concerns and clients that can have specific characteristics affecting their valuation for the service, and (or) the likelihood the service can be provided successfully. In equilibrium, clients find it optimal to sort into experts of different reputation for being talented. Then, the model identifies three channels through which sorting affects the incentives of experts to exert effort. The first is based on the fact that the way clients sort into experts determines which type of client is indifferent

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<sup>1</sup>This paper is a revised version of chapter 3 of my PhD dissertation, completed at the London School of Economics. My supervisor Antoine Faure - Grimaud provided great advice, suggestions and support. I also thank Heski Bar Isaac, Rocco Macchiavello, Giuliana Palumbo, Andrea Prat, Giacomo Rodano, Lucy White, seminar participants at the RES 2008 and NASM 2008 conferences, and colleagues at LSE and Bank of Italy for helpful comments. All remaining errors are my own. The views expressed in this paper do not necessarily coincide with those of the Bank of Italy. Corresponding address: enrico.sette@bancaditalia.it

between getting the service from experts of different reputation. The valuation these types have for the service impacts on the premium clients pay in equilibrium to be served by more reputable experts. This, in turn, affects the value to build a reputation and thus incentives to exert effort. I dub this the “*direct channel*” as it is directly linked to the clients’ valuation for the service. The second channel stems from the fact that sorting of clients may influence the informativeness of a success as a signal of talent. If the type of a client affects the difficulty of providing the service, an expert who serves successfully clients who are on average “more difficult”, will build a reputation more quickly. This channel can be quite relevant in practice: the reputation of a lawyer winning a complex trial, or of a doctor successfully performing a difficult operation will get an especially large boost. I dub this the “*signalling channel*”. The third channel is closely related to the previous one: if the type of a client affects not only the learning process about the expert’s talent but also the likelihood the expert provides the service successfully, then the way clients sort affects the effectiveness of effort in rising the probability the expert succeeds, and thus the incentives to exert effort. Succeeding in a complex operation can provide a strong boost to a doctor’s reputation, but a more complex operation is more likely to fail: thus, the premium for being successful may increase, but the likelihood of being successful may be reduced. I dub the latter the “*complexity channel*”.

The focus of this paper is on markets where reputational (career) concerns are the only incentives motivating the behaviour of experts. This is a suitable modelling strategy for many markets for professional services where the quality of the service provided cannot be easily verified by third parties.

The paper develops as follows: section 2 reviews the literature, section 3 contains the description of the model set up, section 4 derives the equilibrium and investigates the three different channels through which changes in the sorting of clients, induced by increased entry of experts, can affect incentives to exert effort. Section 5 performs a welfare analysis, section 6 discusses results and section 7 concludes.

## 2 Related Literature

This paper contributes to the existing literature in two ways: it investigates how sorting of clients into experts affects the value of building a reputation and in this way the incentives for effort exertion; it analyzes the effects of entry on incentives, by examining the impact of entry of experts on clients’ sorting.

There is a large literature on career concerns and on expert advice. The former has been firstly modelled in the seminal paper of Holmstrom (1982) which spurred a large literature emphasizing different aspects and applications. Another important paper which clarifies the nature and the



mechanics of reputation is Mailath and Samuelson (2001)<sup>2</sup>. They underline the notion of reputation as an asset and show the importance of maintaining uncertainty on a player's type in order for reputation to play an incentivizing role.

Two recent papers investigated the interaction of sorting and reputational incentives. Almeida - Costa and Vasconcelos (2008) investigate the dynamics of firms' reputation when firms implement joint projects. In this case, the reputation of the partner affects the updating process about a firm's reputation conditional on the outcome of the project. Importantly, they show that firms may prefer to start a joint project with a low reputation than with a high reputation partner, as the equilibrium matching influences the dynamics of reputation. Anderson and Smith (2009) develop a dynamic matching model which the characteristics of agents (reputations) evolve depending on their chosen match partner. They show conditions in which positive assortative matching does not occur despite the assumption of productive complementarity. They also investigate the effect of the information rents generated by certain matches on wage profiles. This paper shares the idea that the learning on an agent's type is affected by the type of the partner an agent is matched with. However, here the focus is on the impact of sorting (matching) on the incentives of agents to exert effort. Moreover, experts have relatively little control of their matching with clients as this depends upon equilibrium fees.

Sorting of clients into experts has been analyzed by two interesting papers on investment banking. The first is Chemmanur and Fulghieri (1994). The authors derive a model showing how reputational concerns affect investment banks' standards for Initial Public Offerings (IPOs) valuation. They also sketch the effects of sorting of clients, but this has not a direct impact on experts incentives. The second is Fernando, Gatchev and Spindt (2005) who investigate the matching between issuers and underwriters of IPOs and Secondary Equity Offerings (SEOs). They show that the equilibrium features positive sorting as better firms hire more talented underwriters.

A few papers deal with career concerns and heterogenous principals. Casas-Arce (2009) is very related as he analyzes a situation in which employers differ both in their productivity and in the visibility of the agents working for them. The latter influences the extent to which career concerns provide incentives to exert effort and the possibility of agents to move into different jobs. This paper is different in that the type of the client (the principal) can affect also the learning process about the ability of the agent<sup>3</sup>, and can affect her valuation for the service, which impacts on equilibrium sorting, and in this way, on incentives. Moreover, the setting and the focus are different as Casas-Arce does not investigate the effect of increased entry of agents (more competition) on their incentives. Kovrijnykh (2007) and Martinez (2009) are also related as they both study the

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<sup>2</sup>See also the review paper by Bar-Isaac and Tadelis (2008).

<sup>3</sup>Casas-Arce considers an extension of Holmstrom (1982) in which the type of the principal (her productivity) and the effort of the agent are additive.

a job assignment problem in a career concerns model. They consider different types of jobs or principals, but do not allow for heterogeneity in visibility.

Most of the work on reputational incentives for expert advice (or firm behaviour) focus essentially on one expert only and do not really deal with the effects of competition and entry. An important exception is Horner (2002) who shows that competition among firms acts as a strong disciplinary device and allows to sustain an equilibrium with repeated play of high effort, even when reputational concerns fade out due to learning about firms' type. Kranton (2003) and Bar-Isaac (2005) suggest that competition can have harmful effects on reputational incentives.

There is a large literature investigating the role of competition in markets for expert advice and credence goods<sup>4</sup>. Dulleck and Kerschbamer (2006) provide a thorough survey of the literature and of the most relevant issues. They underline the critical assumptions that sustain the different results proposed by the literature. Using their terminology, this paper does not impose either the verifiability assumption (as I assume the type or quality of service is not verifiable) or the liability assumption (as I assume that clients cannot fine experts for malpractice). However, I assume the quality of the service is observed by the market, that experts are characterized by a different ability in providing a good service and that they live more than one period, so that reputational concerns generate incentives for the provision of a good quality service (or for refraining from fraudulent behaviour).

The analysis of this paper is also related to the literature on competition and incentives. A key contribution in this area is Raith (2003) who showed that tougher competition raises incentives to exert high effort. Raith derives his results in a context where firms provide explicit incentives to managers in order to induce them to exert effort in reducing production costs. The mechanism at work here is different: firstly there are no explicit incentives to motivate experts to exert effort to provide a high quality service and secondly tougher competition affects incentives through the sorting behaviour of clients.

Finally, this paper is also related to the literature on the role of middlemen, in that it underlines the importance of the distribution of clients for the incentives of the intermediary to report her information fully and/or correctly. This literature showed that middlemen help reducing time to find a suitable partner for exchange in a search and matching framework (Rubinstein and Wolinsky, 1987), study the impact of adverse selection on the incentives to collect and reveal information (Biglaiser 1993, Lizzeri 1999), and the role of competition in shaping such incentives (Faure-Grimaud et al., 2008).

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<sup>4</sup>Following the early work by Pitchik and Schotter (1987), a large literature developed. Wolinsky (1993), Emons (1997), Pesendorfer and Wolinsky (2003), Park (2005) and Fong (2006) are related contributions. None of these papers investigates the role of clients' characteristics in shaping the incentives of experts.

### 3 The model

The model is set to describe a market for expert services, such as the market for doctors, lawyers, professional consultancies. The market for doctors is going to be used as the main running example throughout the paper, but the set-up aims at modelling a market for professional services more generally.

**Players:** the economy is populated by a continuum of experts and clients and has an overlapping generation structure. Experts can be of different type (talented / untalented), and both experts and clients only know the probability that experts are talented conditional on the history of the game. Experts do not know their type: they share the same information about their talent as the other market participants<sup>5</sup>. Moreover, experts are of a different vintage: experts live for two periods, and in each period there are young and old experts. In every period  $t$ , there is measure  $Q_t$  of new entrants and measure  $Q_{t-1}$  of old experts (new entrants of the previous period). Overall, in period  $t$  there is measure  $Q_t + Q_{t-1}$  of experts. Therefore, in each period there are young experts (new entrants) whose probability of being talented is given by the prior probability an expert is talented, old experts who were successful in the previous period and old experts who were unsuccessful in the previous period. The probability that old experts, either successful or unsuccessful, are talented is given by the posterior probability they are talented conditional on the outcome of the service they provided in the previous period. The probability of being talented represents an expert's reputation. When providing the service, experts can exert unobservable and costly effort which increases the likelihood the service is of high quality.

Clients live one period only<sup>6</sup> and there is measure  $M > Q_t + Q_{t-1}$  of them. As the total measure of experts is smaller than the measure of clients, then some clients are rationed<sup>7</sup>. Depending on their valuation for the service and the equilibrium fees, clients sort into experts, who provide the service. I assume the service is indivisible<sup>8</sup>.

**Technology:** the service provided by experts can be of high or low quality<sup>9</sup> depending upon the talent and effort choice of the expert, and, in some cases, also upon the intrinsic type of the client. Talented experts generate a high quality service with probability  $\gamma < 1$ , while untalented

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<sup>5</sup>This point is further explained when discussing the information structure below. The same assumption is, among others, in Holmstrom 1982.

<sup>6</sup>Alternatively, clients can live forever, but their type changes. The main idea is that the characteristics of the problem of a client, which are summarized in a client's type, are not constant, but change period by period.

<sup>7</sup>In equilibrium, fees determine which clients are better off not purchasing the service. This is with little loss of generality as results hold as long as there is excess demand for the most reputable experts because this is necessary to provide some incentives to build a reputation.

<sup>8</sup>Experts cannot agree to provide "half" service.

<sup>9</sup>In some applications, it can be said that the service can have a positive outcome (e.g. the successful treatment of an illness) or a negative outcome. This would just be a relabeling. Thus, in the paper "high quality", "positive outcome" and "successful provision" are used interchangeably.

experts need to exert effort  $e \in [0, 1]$ , in order to generate a high quality service with probability  $\gamma e^{10}$ . The cost of effort is a continuous function  $c(e)$  where  $\frac{\partial c}{\partial e} > 0$ ,  $\frac{\partial c(0)}{\partial e} = 0$ ,  $\frac{\partial c(1)}{\partial e} = \infty$ ,  $\frac{\partial^2 c}{\partial e^2} > 0$ , so that the cost of effort is strictly convex.

**Players payoff:** experts get a fee  $F$  for the service they offer and sustain an unobservable cost of effort  $c(e)$ . The period payoff for an expert is then given by  $F_t - c(e_t)$ . However, experts live for more than one period and they also take into account the continuation payoff when choosing effort optimally. Therefore, the full payoff for an expert is  $F_t - c(e_t) + \delta EW(\lambda_{t+1})$  where  $\delta \leq 1$  is a discount factor and  $EW(\lambda)$  is the expected continuation payoff, function of future beliefs about the expert's talent. In order to ease notation, and without loss of generality,  $\delta = 1$ .

Clients are characterized by a type  $\theta$ . This can represent either their valuation for the service, or the “complexity” of their case (or both). The latter can be, for example, the gravity of an illness, the complexity of a judicial case, the quality of a firm, etc. Three cases are analyzed:

1. The type  $\theta$  is clients' valuation for the service. In such a case denote as  $U_H$  and  $U_L$  the payoffs in case of high (successful) and low (unsuccessful) quality (provision) of the service.  $U_L(\theta)$  is normalized to zero and the subscript  $H$  is dropped<sup>11</sup>. The function  $U$  is assumed to be continuous in  $\theta$ . The function  $p$  represents the probability the service provided is of high quality. In this case, the type of the client does not affect the probability of high quality provision of the service and  $p$  is linear in  $\lambda$  and  $e$  so that,  $p = \lambda\gamma + (1 - \lambda)\gamma e$ , and the expected utility of getting the service from an expert of expected talent (reputation)  $\lambda$  exerting effort  $e$  is  $[\lambda\gamma + (1 - \lambda)\gamma e]U(\theta)$ . In order to ease notation, the expected value of the service is defined as  $V(\lambda, \theta, e) = [\lambda\gamma + (1 - \lambda)\gamma e]U(\theta)$ , and the following holds:

$$\frac{\partial U(\theta)}{\partial \theta} > 0, \frac{\partial V(\lambda, \theta, e)}{\partial \lambda} > 0, \frac{\partial V(\lambda, \theta, e)}{\partial e} \geq 0$$

so that higher types value the service more; the expected value of the service is increasing in the expert's talent, it is not decreasing in effort, and the inequality is strict when the expert is untalented, otherwise, effort has no effect. When  $\theta$  represents clients' valuation for the service, the effect of sorting operates only through the the *direct channel*.

2. The type  $\theta$  represents the difficulty of the case which affects the likelihood of successful provision of the service. Apart from this, all clients have the same valuation for a successful service,  $U_H$  which is normalized to 1, while  $U_L$  is still normalized to zero. The probability of a successful

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<sup>10</sup>This formulation borrows from Tadelis (2002) and it is useful as it ensures the existence of a unique equilibrium effort level  $e$  for given beliefs about future period effort levels. This ensures comparative statics on the effect of entry on effort are well defined. However, results hold if it was assumed instead that effort is effective only for talented experts, as long as the equilibrium effort level is unique. This formulation was used in a previous version of the paper.

<sup>11</sup>Notice that it is sufficient that at least the payoff from receiving a high quality service depends upon  $\theta$ . The normalization is with little loss of generality.

provision of the service is now  $p(\lambda, e, \theta)$  and it is modelled as follows

$$\Pr(\text{success} \mid \text{talented}, \theta) = \gamma k$$

$$\text{while } \Pr(\text{success} \mid \text{not talented}, \theta) = \gamma ez(\theta)$$

where  $k \in (0, 1)$ ,  $z(\theta) \in (0, 1)$  for all  $\theta$ ,  $\frac{\partial k}{\partial \theta} = 0$ ,  $\frac{\partial z}{\partial \theta} < 0$ ,  $|\frac{\partial k}{\partial \theta}| < |\frac{\partial z}{\partial \theta}|$  and  $z$  is continuous. In words, the probability of providing the service successfully decreases as  $\theta$  gets larger if the expert is untalented, while it is constant if the expert is talented<sup>12</sup>. I also assume that  $k(\underline{\theta}) \geq z(\underline{\theta})$ , so that the probability of a success is always larger for a talented than for an untalented expert. Thus, the expected utility of a client of type  $\theta$  obtaining the service from a client with expected talent (reputation)  $\lambda$  is  $V(\lambda, \theta, e) = \lambda\gamma k + (1 - \lambda)\gamma ez(\theta)$ . In this case the signalling and the complexity channels are at work (but not the direct channel). If clients' sorting changes, experts face clients with a different  $\theta$  and the conditional probability of a success changes. This implies that the informativeness of a success as a signal of talent is affected by sorting. In fact, a success on a more complex case is more likely to come from a talented expert (for example, the successful treatment of a rare disease is a stronger signal that the doctor is reputable). In this case, sorting affects incentives through the *signalling channel*. However, as  $z(\theta)$  changes, also does the total probability of a success: experts are more likely to provide the service successfully if they face clients with lower types (simpler cases). Thus, sorting affects incentives also through the *complexity channel*, together with the *signalling channel*.

3. The type  $\theta$  represents both a client's valuation for the service and the "difficulty" of providing the service at a high standard. This formulation aims at capturing the fact that more difficult cases can be harder to solve and that a high quality service can be more valuable for more difficult cases. This is a good modelling strategy for some markets. Among these, stands the market for doctors. For example, a patient needing a complex liver operation may survive only if the surgeon performs the operation successfully. The benefit for a high quality service of a patient that needs to fix a broken arm is likely much lower. In such a case, there is a positive correlation between clients' valuation for the service, and the probability the service is provided successfully. Then, the expected value for a client of type  $\theta$  of the service provided by an expert of expected talent  $\lambda$ , exerting effort  $e$  is given by

$$p(\lambda, e, \theta)U_H(\theta) = [\lambda\gamma k + (1 - \lambda)\gamma ez(\theta)]U_H(\theta) \equiv V(\lambda, \theta, e)$$

where the notation is the same as above. This case combines the *direct*, the *signalling* and the *complexity channels*.

In all the three cases, in any period  $t$ , clients obtain expected value  $V(\lambda_t, \theta, e_t)$  from the service

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<sup>12</sup>This assumption is not strictly necessary for results but it is useful to simplify the analysis. As it will become clear in section 4.2, it ensures that the premium for being successful does not depend on next period equilibrium effort level.

and pay the fee  $F_t$ . The latter is set competitively according to clients' demand and experts' supply. If a client does not get the service she obtains an outside option normalized to zero. I assume that experts cannot offer screening contracts. This is not a very restrictive assumption in this setting both because experts have only one instrument, the fee, to screen clients, and because there is a continuum of experts with the same reputation, so that it may not be possible, in equilibrium, for experts to provide rents to different types of clients in order to induce them to reveal their type.

**Information structure:** The set of clients' types is  $\Theta = [\underline{\theta}, \bar{\theta}]$  where  $0 < \underline{\theta} < \bar{\theta}$  and clients are distributed uniformly<sup>13</sup> so that  $\bar{\theta} - \underline{\theta} = M$ . I assume a client's type is private information, while the distribution of types is common knowledge<sup>14</sup>. Experts do not know the type of clients when choosing effort, but hold correct beliefs about the average type they are providing the service to<sup>15</sup>. Experts can be talented or not talented, and they do not know their type. Both experts and the market share the same information about the probability the expert is talented. All players active in a given period  $t$  know the current reputation of an expert which is denoted by  $\lambda_t$ , indicating the probability, conditional on the information available at time  $t$ , that the expert is talented. The outcome (for example, the quality) of the service is observed and the market is able to update beliefs about expert's type; however, the outcome is not verifiable to court, preventing the possibility to offer experts contracts contingent on the quality of output<sup>16</sup>. To summarize, clients observe the reputation level (probability of being talented) of each expert, the distribution of experts of each reputation level, the fees and the outcome of the service.

Notice that the market does not observe exactly the type of each client, but knows, in equilibrium, the distribution of types being attended by a given expert. Thus beliefs about experts' reputation are updated over the average type attended by experts of a given reputation. This helps keeping the reputation levels of experts limited to three. If the type of each client was perfectly observed ex-post together with the realization of output, there would be a continuum of reputation levels for successful and unsuccessful experts, parameterized by  $\theta$ . Allowing for this possibility would

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<sup>13</sup>This is with little loss of generality.

<sup>14</sup>The fact that clients know their need may seem in contrast with some of the proposed applications: often patients visit doctors in order to discover what is their problem. However, the type  $\theta$  may represent the physical status of the person such as a strong pain in the chest or just a strong cold. In this case, patients would probably not be able to fully understand the true  $\theta$ , but all result would hold also if patients (clients) had imperfect, but private, information on  $\theta$ . This point is further discussed in section 6.

<sup>15</sup>Experts can learn the type of the clients after exerting effort. This does not change results as long as this information is soft and cannot be credibly transmitted to the market. The assumption that experts do not know the type of clients before exerting effort plays a role only when the type of the client affects the likelihood of a success, or using the terminology of the paper, only when the *complexity channel* of sorting is at work. Even in that case, if experts knew the type of clients before exerting effort, result would hold: each expert will exert effort as a function of the type of client she faces. The effect of entry, in such a case, is on the aggregate effort level. See section 6 for more on this point.

<sup>16</sup>If the outcome was verifiable, it could be possible to offer fees contingent on realized quality of the service. Then, these explicit incentives would coexist with reputational incentives. The effect of entry on the latter, through sorting would still be the same as outlined in this model, although equilibrium sorting may be different.

add little to the economic intuition, while rendering the model more cumbersome. This point is further discussed in section 6.

**Strategies and beliefs:** clients compete for the services of experts of different reputation. Equilibrium fees are determined so as to balance the demand and the supply of services provided by experts of each reputation level<sup>17</sup>. Given equilibrium fees, clients choose the experts from whom to purchase the service and have no incentive to deviate. Experts choose optimally their effort level, as a function of beliefs about their talent and the expected fees in the future period, denoted as  $F_{t+1}$ . Formally,  $e_t : \{F_{t+1}, \lambda\} \rightarrow [0, 1]$ , when the type of the clients does not affect the probability of success, and  $e_t : \{F_{t+1}, \lambda, E[(\theta) | \Theta(\lambda)]\} \rightarrow [0, 1]$  when it does ( $\Theta(\lambda)$  is the set of clients' types getting the service from experts with reputation  $\lambda$ ). Beliefs about experts' talent are updated according to Bayes rule. Notice that current period effort depends upon expected fees in the future period. In general, these are affected by future period effort. Therefore, effort in period  $t$  is going to depend on beliefs about effort in period  $t + 1$  as these affect fees in period  $t + 1$ . This point is illustrated and discussed at some length in the rest of the analysis.

**Timing:** each period a cohort of experts and clients drawn from the same respective distribution replaces those who exit the market, so that the distributions of clients and experts are stationary. Clients demand the service of experts and fees are determined. Then, clients choose the expert giving them the highest expected payoff (difference between expected value and fee charged). Clients get served until there are idle experts. If some clients cannot be served, they get an outside option normalized to zero. Then, experts exert effort and provide the service, whose outcome is observed by the market which updates beliefs about experts' reputation. Finally, clients exit the market and the period ends. The fee can be paid up-front or after the service has been provided, this has no effect on results.

## 4 Equilibrium

The equilibrium concept is Perfect Bayesian Equilibrium. Clients choose optimally which expert to get the service from, and experts choose optimally their effort level. Beliefs are confirmed in equilibrium, and updated according to Bayes Rule. I start assuming that the measure of new entrants is constant across periods, so that  $Q_{t-1} = Q_t = Q_{t+1} = Q$ . Then, when focussing on the effect of entry, comparative statics on  $Q$  are performed.

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<sup>17</sup>The actual process through which fees are determined can take different forms. In particular, clients may bid for the service in an ascending auction. When the supply of experts of a given reputation is filled, the fee is determined. Alternatively, experts can post fees. However, experts are atomistic (there is a continuum of them), and they would be competed away if they posted a price which is higher than that of competitors (with the same reputation). Results are robust to different choices and the paper does not explicitly model the fee formation process in order to streamline the exposition. It just assumes that fees clear the market and players have no incentives to deviate.

The first step is the analysis the behaviour of experts with different “vintage”, then I will turn to derive equilibrium fees and analyse the application policy and sorting behaviour of clients.

**Lemma 1** *Experts exert zero effort in their last period, while they may exert positive effort in their first period.*

**Proof.** See Appendix. ■

This result is common to all cases, and it is not new.

The novel part of the analysis lies in the strategic behaviour of clients and in its consequences. I firstly study the simplest model, case 1, where the type of clients does not affect either the learning process about experts’ talent, or the likelihood that a talented expert succeeds in providing a high quality service. Then, I will focus on case 2, where the type of the client represents the complexity of providing the service successfully. Finally, I will deal with case 3, where the type of the client both represents her valuation for the service and the complexity of providing the service successfully. Studying cases 1 and 2 separately is interesting in itself, but it is especially useful to gain the intuition towards case 3, which combines them. Moreover, analyzing cases 1 and 2 in isolation helps to underline that the signalling and the complexity channels can operate independently of the direct channel.

#### 4.1 Case 1 - Types as valuation for the service

The first step is deriving the preferences of clients over experts with different reputation. This allows to determine equilibrium fees and the set of types faced by experts of different reputation in equilibrium. The former depend upon the value clients attach to the service. Therefore they are function the reputation of the expert, and, implicitly, of the expected effort exerted ( $e^*$ ) and of the expected type of client applying to experts of that reputation.

The reputation of experts following a success is given by:

$$\Pr(Talented | Success, e^*) = \lambda_{t+1}^+ = \frac{\lambda\gamma}{\lambda\gamma + (1-\lambda)\gamma e_t^*}$$

as the probability of a success conditional on being talented is  $\gamma$ , while the probability of a success conditional on the expert being untalented and exerting effort  $e^*$  is  $\gamma e^*$ <sup>18</sup>. If the service provided turns out to be poor, the reputation of the expert is lowered to

$$\Pr(Talented | Failure, e^*) = \lambda_{t+1}^- = \frac{\lambda(1-\gamma)}{\lambda(1-\gamma) + (1-\lambda)(1-\gamma e_t^*)}$$

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<sup>18</sup>The effort level  $e^*$  is the market expectation of the effort exerted by the expert.



In each period  $t + 1$  there is measure  $Q_{t+1}$  of newly born experts with reputation  $\lambda$ , measure  $\gamma[\lambda + (1 - \lambda)e_t]Q_t$  of old experts that were successful in their first period and thus have reputation  $\lambda_{t+1}^+$ , and measure  $[(1 - \gamma)\lambda + (1 - \lambda)(1 - \gamma e_t)]Q_t$  of old experts who were unsuccessful in their first period and thus have reputation  $\lambda_{t+1}^-$ . For the moment,  $Q$  is assumed to be constant over time, so that there is no change in the measure of new entrants and in each period there is measure  $2Q$  of experts ( $Q$  old experts and  $Q$  new entrants).

The expected value from the service,  $V(\lambda, \theta, e^*)$ ,  $V(\lambda^+, \theta, 0)$ ,  $V(\lambda^-, \theta, 0)$ <sup>19</sup>, is increasing in clients' types, thus higher types are willing to pay more for experts that are more likely to provide the service at a high quality. The fact that experts do not exert effort in the last period has implications for the preferences, and thus the sorting behaviour, of clients. It is obvious that all clients prefer old successful to old unsuccessful experts, and new entrants to old unsuccessful experts, as the latter both have a lower probability of being talented, and exert no effort. In fact, the assumptions of the model imply that

$$\begin{aligned} V(\lambda^+, \theta, 0) &> V(\lambda^-, \theta, 0), \forall \theta \\ V(\lambda, \theta, e) &> V(\lambda^-, \theta, 0), \forall \theta \end{aligned}$$

as  $\lambda^+ > \lambda^-$ ,  $\lambda > \lambda^-$  and  $e \geq 0$ . It is less obvious how do clients rate successful experts relative to new entrants. Experts who are successful in the first period have a larger value for clients, as they are more likely to be talented and thus to produce a high quality service. On the other hand, they do not exert effort, while new entrants do, and therefore the latter might provide a service whose expected value is larger than that offered by more reputable experts. In fact, it is not possible, ex-ante, to tell whether

$$V(\lambda^+, \theta, 0) > V(\lambda, \theta, e) \text{ or } V(\lambda^+, \theta, 0) < V(\lambda, \theta, e)$$

because the increase in value from applying to a more reputable expert can be more than compensated by the decrease in value due to the lower effort level exerted by an expert with no career concerns. Notice that this effect is always present, even in a model with infinitely lived experts, as incentives to exert effort fade out as learning about an expert's type becomes more precise.

As old experts do not exert effort, whether the highest valuation types prefer to get the service from the most reputable experts (even if they do not exert effort), or from new entrants depends upon whether  $\gamma\lambda^+ \geq \gamma\lambda + (1 - \lambda)\gamma e^*$ . When  $\gamma\lambda^+ > \gamma\lambda + (1 - \lambda)\gamma e^*$ , old successful experts are valued more than new entrants, even if the former do not exert effort. As talent is somehow valued more than effort, I denote this situation as the *talent intensive* case. This condition depends upon

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<sup>19</sup>Lemma 1 proved that old experts exerts no effort in equilibrium.

clients' beliefs about equilibrium effort. For this equilibrium to exist it is necessary that the true equilibrium value of effort chosen by experts as a function of the application strategy of clients, which depends upon beliefs, actually satisfy the condition. Rewriting the condition one gets, for the talent intensive case

$$\gamma \frac{\lambda}{\lambda + (1 - \lambda)e_{t-1}^*} > \gamma[\lambda + (1 - \lambda)e_t^*]$$

as the updating about current old successful experts is made on the basis of beliefs about their previous period effort level, while the right hand side contains beliefs about current period effort level by young (new entrant) experts in period  $t$ . In principle  $e_{t-1}^*$  can be different from  $e_t^*$ . This point will be discussed further in the proof of Proposition 2.

When, instead,  $\gamma\lambda^+ < \gamma\lambda + (1 - \lambda)\gamma e^*$  new entrants are preferred to old successful experts, as the fact that they exert effort in equilibrium compensates for the lower probability of being talented. For this reason I denote this case as the *effort intensive* case. The paper deals with both cases<sup>20</sup>. The talent intensive case is analyzed first. Then, attention will be placed on the effort intensive case.

**Case A - Talent intensive case:**  $\gamma\lambda^+ > \gamma\lambda + (1 - \lambda)\gamma e^*$ , and  $\gamma\lambda_{+1}^+ > \gamma\lambda + (1 - \lambda)\gamma e_{+1}^*$  so that all clients prefer to be served by old successful experts both in the current and in the next period. The equilibrium fee for the services of experts of reputation  $\lambda^+$ , in any period  $t$ , must be such that all experts are busy and therefore demand for the services of experts of that reputation equals supply, and clients have no incentive to deviate. Those clients that are not served by the most reputable experts, get served by new entrant experts with reputation  $\lambda$ . Finally, part of the remaining clients are served by the least reputable experts, and some clients get no service at all, as  $M$ , the total measure of clients, is larger than 1, the total measure of experts. For ease of notation, I drop the dependence on  $t$ , and instead denote current period variables without subscript, previous period with subscript  $-1$  and future period with subscript  $+1$ <sup>21</sup>. The discussion above is formalized in the following:

**Proposition 2** *In equilibrium, clients sort as follows: clients of type  $\theta \in [\theta^*, \bar{\theta}]$  are served by experts of reputation  $\lambda^+$ , clients of type  $\theta \in [\theta^{**}, \theta^*]$  are served by experts of reputation  $\lambda$ , clients of type  $\theta \in [\theta^{***}, \theta^{**}]$  are served by experts of reputation  $\lambda^-$ . Finally clients of type  $\theta \in [\underline{\theta}, \theta^{***}]$  do*

<sup>20</sup>Notice that both conditions concern beliefs about equilibrium values of a choice variable. The following analysis studies the conditions ensuring the existence of equilibrium in both the talent and the effort intensive cases.

<sup>21</sup>The incentives to exert effort of a young expert in period  $t$ , depend upon the fees she will get in period  $t + 1$ . These depend, among other things, upon the beliefs about the effort level exerted by young agents in period  $t + 1$ . The model has stationary equilibria, in which the effort level of young agents is constant in each period (as long as no change in the measure of entrants occur). However, the model may also have non stationary equilibria in which effort level of young agents oscillates from one period to the other. In the latter case, dropping the dependence on  $t$  entails loss of information. However, for the purpose of this analysis, such loss is very limited. In fact, results on the effect of entry hold taking any equilibrium, and the beliefs about the effort exerted in the next period that support it, as given.

not get the service. The thresholds separating the three subsets satisfy  $\theta^{***} < \theta^{**} < \theta^*$  and are such that supply equals demand for experts:

$$\begin{aligned} [\lambda\gamma + (1-\lambda)\gamma e_{-1}]Q &= \bar{\theta} - \theta^* \\ Q &= \theta^* - \theta^{**} \\ [\lambda(1-\gamma) + (1-\lambda)(1-\gamma e_{-1})]Q &= \theta^{**} - \theta^{***} \\ M - 1 &= \theta^{***} - \underline{\theta} \end{aligned}$$

equilibrium fees satisfy the following equalities

$$\begin{aligned} V(\lambda^+, \theta^*, 0) - F(\lambda^+) &= V(\lambda, \theta^*, e^*) - F(\lambda) \\ V(\lambda, \theta^{**}, e^*) - F(\lambda) &= V(\lambda^-, \theta^{**}, 0) - F(\lambda^-) \\ V(\lambda^-, \theta^{***}, 0) - F(\lambda^-) &= 0 \end{aligned}$$

The equilibrium effort level satisfies the following conditions

$$\begin{aligned} \gamma(1-\lambda)[F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)] - \frac{\partial c(e)}{\partial e} &= 0 \\ \gamma\lambda_{+1}^+ &> \gamma\lambda_{+1} + (1-\lambda_{+1})\gamma e_{+1}^* \\ e^* &= e \end{aligned}$$

If a talent intensive equilibrium exists, then the equilibrium effort level is unique, for given beliefs  $e_{+1}^*$ , about next period effort level. The difference  $F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)$  is increasing in threshold types.

**Proof.** See Appendix ■

Figure 1 illustrates the equilibrium.

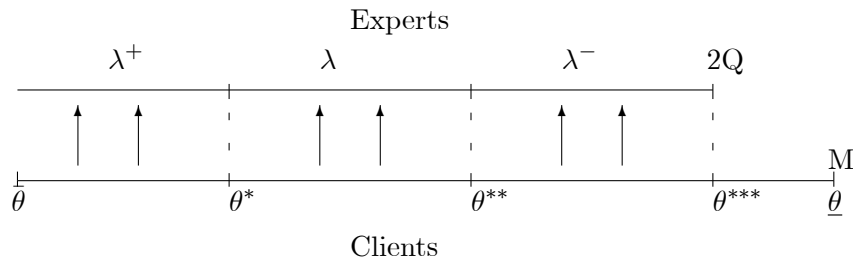


Figure 1

This proposition shows that clients sort into experts, so that highest types purchase the service from the most reputable experts, intermediate types purchase the service from experts with intermediate reputation, lower types purchase the service from the experts with the lowest reputation, and finally very low types do not get served at all, as they value the service too little. Equilibrium fees ensure that demand equals supply for the services of experts of different reputation and clients have no incentive to deviate. For this equilibrium to exist, it must be that parameters are such that the equilibrium level of effort actually satisfies the conditions,  $\gamma\lambda^+ > \gamma\lambda + (1 - \lambda)\gamma e^*$  and  $\gamma\lambda_{+1}^+ > \gamma\lambda + (1 - \lambda)\gamma e_{+1}^*$ , so that clients in equilibrium truly prefer to get the service from old successful experts than from new entrants. Notice that current period effort level affects  $\lambda_{+1}^+$ . Equilibrium effort level also depends upon beliefs about next period effort level  $e_{+1}^*$ , which is the effort that is going to be exerted by new entrants next period. In principle, this can differ from the current effort level, just because in period +1 beliefs about period +2 effort level are further different. However, there can also exist an equilibrium where beliefs are “stationary”, meaning that beliefs about effort are the same as long as the measure of experts and the other parameters of the model are the same<sup>22</sup>.

A further result of the proposition is that the difference in fees charged by successful and unsuccessful experts  $F(\lambda^+) - F(\lambda^-)$  is increasing in threshold types. This follows from the conditions ensuring that clients sort. Thus, if there is sorting of clients in equilibrium, the difference  $F(\lambda^+) - F(\lambda^-)$  increases in threshold types.

**Case B:** I now consider the case  $\lambda^+\gamma < \lambda\gamma + (1 - \lambda)\gamma e^*$  and  $\lambda_{+1}^+\gamma < \lambda\gamma + (1 - \lambda)\gamma e_{+1}^*$ . I label this the “effort intensive” case, as all clients prefer to be served by new entrants because these exert effort and the probability of a success is larger than for old successful experts, even if the latter are more likely to be talented. In such a case, there is little change in results. From the same reasoning as in Proposition 2, it is possible to get

**Proposition 3** *In equilibrium, clients sort as follows: clients of type  $\theta \in [\theta^*, \bar{\theta}]$  are served by experts of reputation  $\lambda$  (new entrants), clients of type  $\theta \in [\theta^{**}, \theta^*]$  are served by experts of reputation  $\lambda^+$ , clients of type  $\theta \in [\theta^{***}, \theta^{**}]$  are served by experts of reputation  $\lambda^-$ . Finally clients of type  $\theta \in [\underline{\theta}, \theta^{***}]$  do not get served. The thresholds separating the three subsets satisfy  $\theta^{***} < \theta^{**} < \theta^*$  and are such*

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<sup>22</sup> Assuming beliefs are stationary would simplify the analysis, with little loss of generality for results. This point is further discussed in section 4.3.

that supply equals demand for experts:

$$\begin{aligned}
Q &= \bar{\theta} - \theta^* \\
[\lambda\gamma + (1 - \lambda)\gamma e_{-1}]Q &= \theta^* - \theta^{**} \\
[\lambda(1 - \gamma) + (1 - \lambda)(1 - \gamma e_{-1})]Q &= \theta^{**} - \theta^{***} \\
M - 1 &= \theta^{***} - \underline{\theta}
\end{aligned}$$

where  $\theta^{***} < \theta^{**} < \theta^*$ . Equilibrium fees satisfy

$$\begin{aligned}
V(\lambda, \theta^*, e^*) - F(\lambda) &= V(\lambda^+, \theta^*, 0) - F(\lambda^+) \\
V(\lambda^+, \theta^{**}, 0) - F(\lambda^+) &= V(\lambda^-, \theta^{**}, 0) - F(\lambda^-) \\
V(\lambda^-, \theta^{***}, 0) - F(\lambda^-) &= 0
\end{aligned}$$

The first order condition for optimal effort is given by

$$\gamma(1 - \lambda)[F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)] - \frac{\partial c(e)}{\partial e} = 0$$

which is

$$\gamma(1 - \lambda)(\gamma\lambda_{+1}^+ - \gamma\lambda_{+1}^-)U(\theta_{+1}^{**}) - \frac{\partial c(e)}{\partial e} = 0$$

The difference  $F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)$  is increasing in threshold types  $\theta^{**}$ . If an effort intensive equilibrium exists, then the equilibrium effort level is unique, for given beliefs  $e_{+1}^*$  about next period effort level.

**Proof.** See Appendix. ■

Propositions 2 and 3 show that equilibrium in both the talent and the effort intensive case feature sorting of clients. Changes in sorting can have important impacts on incentives to exert effort. In particular, entry of experts can modify the way clients sort in equilibrium and thus impact on incentives to exert effort. However, before turning to the analysis of the effect of entry, it is important to prove some results about the equilibrium, which are useful for comparative statics.

**Lemma 4** *The equilibrium function is continuous. The equilibrium is either a talent intensive equilibrium, or an effort intensive equilibrium and equilibrium effort is unique.*

**Proof.** See Appendix. ■

These results ensures that at least one equilibrium exists, that such equilibrium is unique and it is either a talent intensive, or an effort intensive equilibrium. This result is especially important for the analysis of the effect of entry, as it allows to conduct comparative statics.

I now assume that the measure  $Q$  of new entrants is not constant anymore. In particular, I model entry as a permanent increase in the measure of young experts, and I also assume that entry is not anticipated by agents in the previous period<sup>23</sup>. Increased entry of new experts raises the supply of experts with intermediate reputation. Suppose, then, that after entry occurs in period 0 there is measure  $\widehat{Q}$  of new entrants with reputation  $\lambda$ , where and  $M > Q_{-1} + \widehat{Q}$ <sup>24</sup>. In period +1 the total measure of experts is going to be  $2\widehat{Q}$ . Then, the equilibrium thresholds for clients' sorting are modified and this impacts on the types who are indifferent in equilibrium, and, in this way, on the incentives to exert effort.

I firstly analyze the talent intensive case. The effect of entry on incentives through its impact on sorting is formalized in the following

**Proposition 5** *Entry modifies the sorting behaviour of clients both in the current and in the future period. The change in the sorting of clients in the future period impacts on the premium that clients are willing to pay to be served by the most reputable experts in a way that unambiguously reduces the equilibrium effort.*

**Proof.** See Appendix ■

This proposition shows that increased entry of experts reduces equilibrium effort. This follows because increased entry in the current period raises both the measure of successful and of unsuccessful experts in the future period. This reduces the threshold types indifferent between being served by experts of different reputation, and thus lowers the premium for being a successful expert, which is increasing in threshold types. Thus, entry modifies the sorting behaviour of clients and this impacts on the incentives of experts to exert effort.

Figures 2 and 3 show the effect of entry in the current and in the future period on threshold types. Threshold types  $\widehat{\theta}_{+1}^*$  and  $\widehat{\theta}_{+1}^{**}$  have to be lower than the corresponding threshold types had entry not occurred, as shown in the proof of proposition 5. Finally, the fact that  $\widehat{\theta}_{+1}^{**} < \theta_{+1}^{**}$  also implies that  $\widehat{\theta}_{+1}^{***} < \theta_{+1}^{***}$ .

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<sup>23</sup>This is only to facilitate the description of the effect of entry, as it means that effort level in period  $-1$ , i.e. in the period before entry occurs, is not affected by entry. This assumption has no significant impact on any result.

<sup>24</sup>Assuming the measure of new entrants is  $Q > 1$ , or that  $M < 2Q$  would change little.

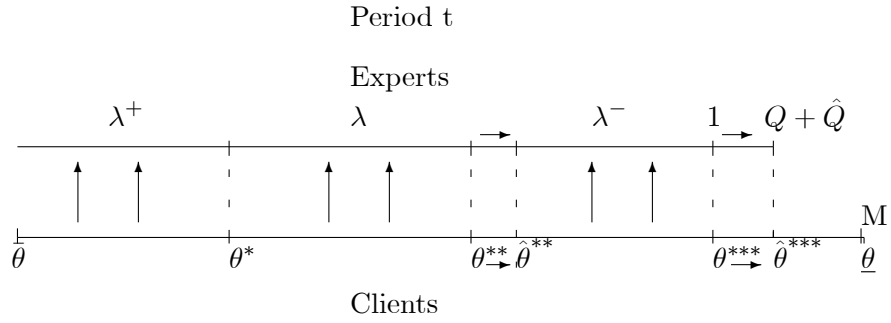


Figure 2

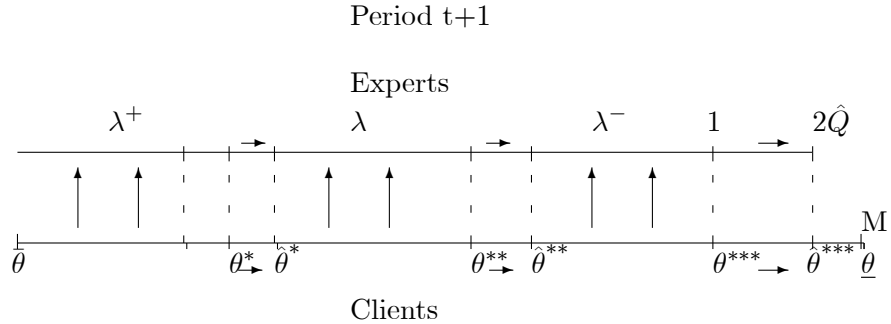


Figure 3

Entry also affects equilibrium fees both in the period when the increase in entry occurs, and in future periods.

**Proposition 6** *Fees charged by new entrants and unsuccessful experts are reduced in the period entry occurs, while the effect on the fees for the service of the most reputable experts is ambiguous. On the contrary, all fees are reduced in future periods, when, moreover, the premium to be paid to get the service from the most reputable experts must be lower.*

**Proof.** See Appendix ■

This proposition shows that after entry occurs, fees charged by new entrants and by less reputable experts are lower. This is due to the downward movement in the thresholds  $\theta^{**}$  and  $\theta^{***}$ . On the contrary, the effect on the fees paid to most reputable experts is ambiguous in the period entry occurs. On the one hand, the decrease in the thresholds  $\theta^{**}$  and  $\theta^{***}$  induces all fees to decrease. On the other hand, the decrease in equilibrium effort raises the difference between the value of being

served by the most reputable expert and the value of being served by a new entrant. However, fees in the period after entry occurs are all lower.

I now analyze the effort intensive case. Assume now that an effort intensive equilibrium exists before entry occurs ( $\gamma\lambda_{+1}^+ < \gamma\lambda_{+1} + (1 - \lambda)\gamma e_{+1}^*$ ). Then, the effect of entry can be immediately verified

**Proposition 7** *Entry reduces equilibrium effort.*

**Proof.** See Appendix ■

The reasoning is the same as for the talent intensive case. Again entry changes equilibrium sorting. This changes the types who are indifferent between getting the service from new entrants or from old successful experts and from old successful or from old unsuccessful experts. This in turn impacts on equilibrium fees and thus on the premium for being successful, and in this way on equilibrium effort.

The main difference with the “talent intensive” case lies in the behaviour of equilibrium fees in the period in which entry occurs. In fact, now all fees must be lower, both in period 0 and in period +1. This follows as  $\theta^* = \bar{\theta} - Q > \hat{\theta}^* = \bar{\theta} - \hat{Q}$ , so that  $\theta^* > \hat{\theta}^*$  as  $Q > \frac{1}{2}$ . Then, this necessarily implies that  $\hat{\theta}^{**} < \theta^{**}$  and  $\hat{\theta}^{***} < \theta^{***}$ . On the contrary, the behaviour of fees in the future period is analogous to that in the talent intensive case.

## 4.2 Case 2 - Clients’ type represents the complexity of the service

This section analyzes the case in which the type of clients affects the probability of successful provision of the service by experts, so that untalented experts are less likely to provide the service at a high standard to certain types. This situation is likely to occur in practice. A surgeon performing successfully a liver transplant with an innovative technique increases her reputation more than if she repairs a knee joint (similarly, winning a class action suit against a large corporation is a strong signal of talent for a lawyer as typically these are difficult cases). As discussed in section 3 describing the model, the exact type of the client (thus the exact difficulty of the operation, or of the legal case) is not observed. The market infers the average type faced by experts of reputation  $\lambda^+, \lambda, \lambda^-$ , as equilibrium sorting behaviour of types is known in equilibrium.

In order to isolate the effect of modelling type as the difficulty of providing the service successfully, clients are assumed to have the same valuation for the service. The probability of a success of an expert facing clients of expected quality  $E(\theta \mid \Theta(\lambda))$ <sup>25</sup>, is  $\lambda\gamma k + (1 - \lambda)\gamma e^* E(z(\theta) \mid \Theta(\lambda))$ , where  $E$  is the expectation operator. The impact of the complexity of the service on the probability

<sup>25</sup>The expectation is conditional on the set of clients applying to experts of reputation  $\lambda$ .



of success is illustrated in section 3. It is rewritten here for convenience:

$$\frac{\partial k(\theta)}{\partial \theta} = 0, \quad \frac{\partial z(\theta)}{\partial \theta} < 0 \quad ; \quad \frac{\partial k(\theta)}{\partial \theta} = 0 > \frac{\partial z(\theta)}{\partial \theta} \quad ; \quad k(\underline{\theta}) \geq z(\underline{\theta})$$

in other words, providing the service to a higher type reduces the probability of success for an untalented expert, while the probability a talented expert succeeds is constant, and does not decrease as types are higher and cases become tougher<sup>26</sup>. The probability of a success when providing the service to the lowest type is weakly higher for talented experts, so that the probability of a success for a talented expert is greater than for an untalented expert for all type of clients  $\theta > \underline{\theta}$ .

Then, beliefs about the talent of the expert evolve as follows:

$$\Pr(\textit{Talented} \mid \textit{Success}, \theta, e^*) = \lambda_{+1}^+ = \frac{\gamma \lambda k}{\gamma \lambda k + \gamma(1 - \lambda)e^*E[z(\theta) \mid \Theta(\lambda)]}$$

as the probability of a success for an untalented expert depends upon  $\theta$ . If the service provided turns out to be poor, the reputation of the expert is lowered to

$$\Pr(\textit{Talented} \mid \textit{Failure}, \theta, e^*) = \lambda_{+1}^- = \frac{\lambda(1 - \gamma k)}{\lambda(1 - \gamma k) + (1 - \lambda)(1 - \gamma e^*E[z(\theta) \mid \Theta(\lambda)])}$$

and the measure of successful experts in period +1 is  $\{\lambda \gamma k + (1 - \lambda) \gamma e^* E[z(\theta) \mid \Theta(\lambda)]\}Q$ , while the measure of unsuccessful experts is  $\{\lambda(1 - \gamma k) + (1 - \lambda)(1 - \gamma e^* E[z(\theta) \mid \Theta(\lambda)])\}Q$ .

It is important to notice that, if threshold types move upwards,  $E[z(\theta) \mid \Theta(\lambda)]$  is reduced as  $z(\theta)$  is decreasing in  $\theta$ . Then, providing the service successfully to a higher type raises the probability the expert is talented as

$$\frac{\partial \lambda_{+1}^+}{\partial \theta} = \frac{\gamma^2 \lambda (1 - \lambda) e^* [-\frac{\partial E[z(\theta) \mid \Theta(\lambda)]}{\partial \theta} k]}{[\gamma \lambda k + \gamma(1 - \lambda) e^* E[z(\theta) \mid \Theta(\lambda)]]^2} > 0$$

from the assumptions about  $k$  and  $z$ . However, providing the service to a higher type reduces the probability an expert is talented following a failure, as untalented experts are less likely to succeed on tougher cases than talented experts. In fact,

$$\frac{\partial \lambda_{+1}^-}{\partial \theta} = \frac{\gamma \lambda (1 - \lambda) [+ \frac{\partial E[z(\theta) \mid \Theta(\lambda)]}{\partial \theta} e^* (1 - \gamma k)]}{[\lambda(1 - \gamma k) + (1 - \lambda)(1 - \gamma e^* E[z(\theta) \mid \Theta(\lambda)])]^2} < 0$$

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<sup>26</sup>This assumption is not strictly necessary for results but it is useful to simplify the analysis. In fact, it ensures that the value of a talented expert is always decreasing in the type of the client  $\theta$  for all levels of effort. If  $k$  was dependent on  $\theta$ , the total probability of a success would be  $\lambda \gamma k(\theta) + (1 - \lambda) \gamma e z(\theta)$ . Its cross derivative with respect to  $\theta$  and  $\lambda$ , is  $\gamma [\frac{\partial E[k(\theta) \mid \Theta(\lambda)]}{\partial \theta} - e \frac{\partial E[z(\theta) \mid \Theta(\lambda)]}{\partial \theta}]$ . This can be either positive or negative, and it would be necessary to impose conditions ensuring sorting, either positive or negative, occurs in equilibrium for result to hold.

The total probability of success decreases if the client has a higher type. This has important implications for equilibrium sorting. In fact, the valuation for the service provided by new entrants is decreasing in the complexity of the case. Figure 4 represents possible configurations of preferences.

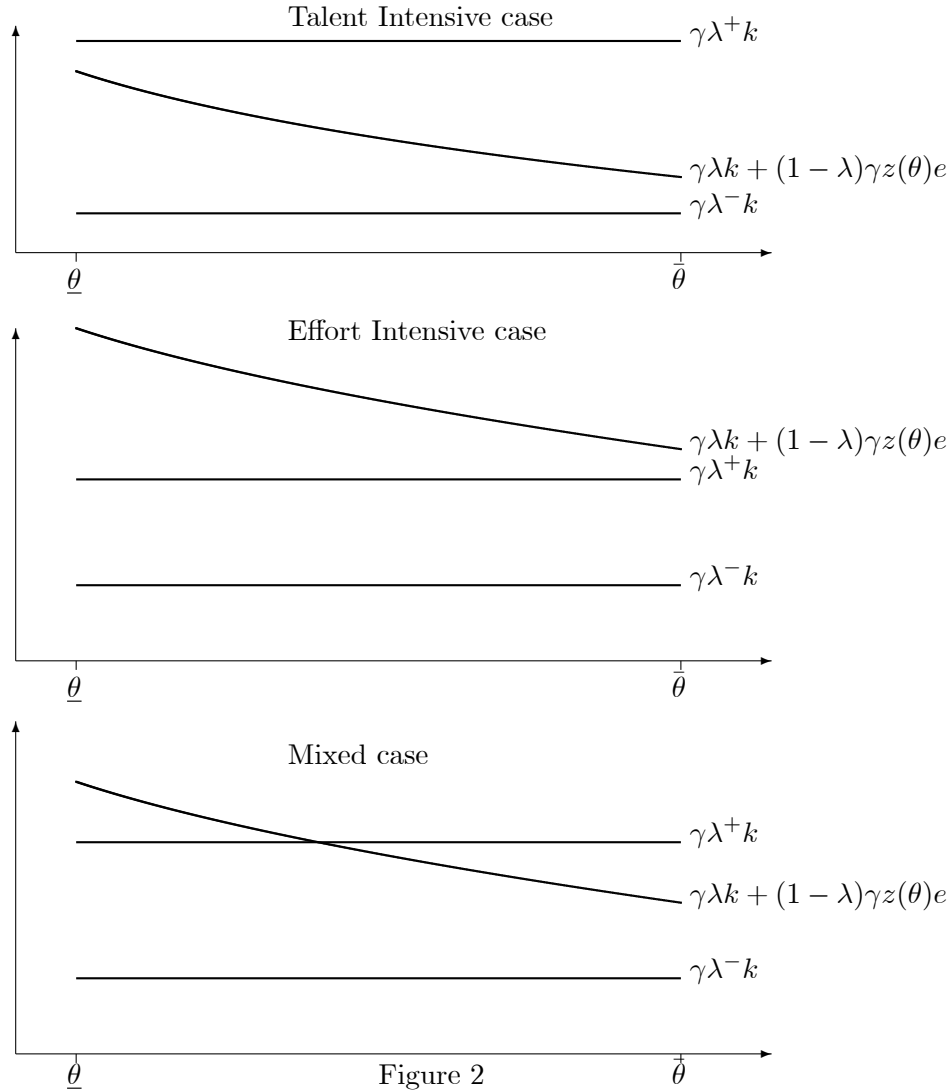


Figure 2

The expected value of being served by a new entrant is decreasing in clients' type. The following proposition characterizes the equilibrium independently of whether preferences are talent intensive, effort intensive or "mixed".

**Proposition 8** *In equilibrium, clients with type  $\theta \leq \theta^*$  get the service from new entrants. Clients with types  $\theta > \theta^*$  randomize between getting the service from old successful, old unsuccessful, or no*

service. Equilibrium fees satisfy the equality

$$\lambda\gamma k + (1 - \lambda)\gamma e^* z(\theta^*) - F(\lambda) = \lambda^+\gamma k - F(\lambda^+) = \lambda^-\gamma k - F(\lambda^-) = 0$$

and  $\theta^* - \underline{\theta} = Q$ .

**Proof.** See Appendix ■

This proposition shows that in equilibrium lower types get the service from new entrants (young experts). A key point is that, given equilibrium fees, higher types derive a lower utility from being served by new entrants than lower types, because the probability of success decreases with  $\theta$ . The value of being served by old experts does not depend upon  $\theta$  because it was assumed that  $k$  is constant<sup>27</sup>. Thus, at the going equilibrium fees, higher types prefer old (successful or unsuccessful) experts than new entrants.

It can be seen that if threshold type changes, incentives to exert effort change. In fact, as  $\theta^*$  moves, so do  $\lambda^+$ ,  $\lambda^-$  and  $E[z(\theta) | \theta \leq \theta^*]$ . In particular, it was shown above that  $\frac{\partial \lambda_{+1}^+}{\partial \theta} > 0$ ,  $\frac{\partial \lambda_{+1}^-}{\partial \theta} < 0$ . This represents the signalling channel. Moreover,  $\frac{\partial E[z(\theta) | \theta \leq \theta^*]}{\partial \theta^*} < 0$  as the average type faced by new entrants increases when the threshold type  $\theta^*$  is higher. Then, the probability of success must be lower. This is the complexity channel.

Entry of experts modifies the equilibrium threshold  $\theta^*$ . This impacts on equilibrium effort.

**Proposition 9** *Entry induces conflicting forces on equilibrium effort. Entry increases equilibrium effort as long as the elasticity of the premium for being a successful experts ( $F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)$ ) is larger than the elasticity of the expected probability of a success, both computed with respect to the threshold type  $\theta^*$ .*

This proposition shows that entry has an ambiguous effect on equilibrium effort. This follows because the change in threshold types induced by increased entry of experts impacts both the informativeness of a success as a signal of talent and the total probability of a success, thus the marginal efficiency of effort. These two effects always go in opposite directions, so that the total effect on equilibrium effort is ambiguous. The proof shows that the net effect depends on the relative size of the elasticity of the premium for being a successful expert,  $F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)$ , and the

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<sup>27</sup>If  $k$  depended on  $\theta$ , the equilibrium would feature negative sorting. Depending upon whether the equilibrium is talent intensive or effort intensive, lower types would get the service from old successful experts or new entrants, intermediate types from new entrants or old successful experts, higher types from old unsuccessful experts and very high type will not get the service. This is due to the fact that all clients have the same valuation for the service, but the probability they will be served successfully decreases with the type  $\theta$ . Thus, higher types have a low chance of getting the service at a high standard, and thus have a lower willingness to pay than clients with a lower  $\theta$ . Results for the model with  $k$  dependent on  $\theta$  are available upon request. The next section will deal with the empirically more interesting case where the type  $\theta$  affects both the complexity of the case and the valuation of a successful provision of the service.

elasticity of the expected probability of a success, both computed with respect to the threshold type  $\theta^*$ . These quantities may be observable in some markets. This point is further discussed in the comment of Proposition 11.

Equilibrium fees are also affected by the increase in the entry of experts, due to the change in the threshold type. However, in this case only fees for new entrants change in the period entry occurs. In the following period all fees change, and they may either increase or decrease.

**Proposition 10** *Entry changes both current period fees for new entrants  $F(\lambda)$  and all future periods fees  $F_{+1}(\lambda^+)$ ,  $F_{+1}(\lambda)$ ,  $F_{+1}(\lambda^-)$ . Fees for new entrants may either increase or decrease. In the period after the change in entry fees for old successful experts,  $F_{+1}(\lambda^+)$ , are higher, fees for old unsuccessful experts,  $F_{+1}(\lambda^-)$  are lower.*

**Proof.** See Appendix ■

This proposition shows that entry has an ambiguous effect on equilibrium fees charged by new entrants, while it increases the premium for being successful as  $F_{+1}(\lambda^+)$  is higher and  $F_{+1}(\lambda^-)$  lower. The latter effect is due to the signalling channel: as threshold types are higher in equilibrium, both a success and a failure are more informative about the type of the client. This in turn impacts on clients' willingness to pay for the service provided by old experts. The impact of entry on fees of new entrants depends upon whether equilibrium effort raises after the increase in entry takes place. When effort is higher, then fees may be higher, as there is a higher probability new entrants provide the service successfully, and this raises clients' valuation for the service of new entrants. On the other hand, the higher threshold type reduces the probability of success, and this depresses clients' valuation for the service of new entrants.

### 4.3 Case 3 - Types represent both valuation and complexity of the service

This section combines the two previous cases. Now, a clients' type represents both her valuation for the service, and the complexity of providing the service successfully. This is an interesting case for certain markets, but possibly not for others. In particular, the positive correlation between valuation for a positive outcome and complexity of the case seems to be a suitable situation for healthcare, for legal services, possibly for professional consultancy.

Results from the previous sections show that whether the expected value of being served by an expert of a given reputation increases in  $\theta$  or not, crucially shape the equilibrium sorting. For the sake of conciseness, I focus on the case in which the expected value  $V(\lambda, \theta, e) = \{\lambda\gamma + (1 - \lambda)\gamma ez(\theta)\}U(\theta)$  is increasing in  $\theta$ . When this occurs, the equilibrium sorting will resemble that of section 4.1. This is the most interesting case, empirically, as we observe in practice that clients with more difficult cases tend to choose experts with relatively high reputation (old successful, or

new entrants in the model). Notice, however, that results in the other case (negative sorting) are quite similar, as it will become clear that the signalling and the complexity channels go in opposite directions, and the effect of sorting on incentives is going to be ambiguous, even if the direct channel is at work, too. Thus, I assume that

$$z(\theta) \frac{\partial U(\theta)}{\partial \theta} > -\frac{\partial z(\theta)}{\partial \theta} U(\theta) \text{ for all } \theta$$

This is a sufficient, although not necessary, condition to ensure that

$$\begin{aligned} \frac{\partial V(\lambda, \theta, e)}{\partial \theta} &= (1 - \lambda)\gamma e \frac{\partial z(\theta)}{\partial \theta} U(\theta) + \{\lambda\gamma + (1 - \lambda)\gamma e z(\theta)\} \frac{\partial U(\theta)}{\partial \theta} = \\ &\lambda\gamma \frac{\partial U(\theta)}{\partial \theta} + (1 - \lambda)\gamma e \left[ \frac{\partial z(\theta)}{\partial \theta} U(\theta) + z(\theta) \frac{\partial U(\theta)}{\partial \theta} \right] > 0 \end{aligned}$$

for all  $e, \lambda, \gamma$ , so that higher types are more willing to pay for the service. The fact that  $\frac{\partial V(\lambda, \theta, e)}{\partial \theta}$  is monotonic ensures that there is sorting in equilibrium.

I first analyse the talent intensive case, which now requires that  $\lambda^+ \gamma > \lambda\gamma + (1 - \lambda)\gamma e^* E[z(\theta) | \Theta(\lambda)]$ . In this case, equilibrium sorting is the same as in case 1 (derived in Proposition 2). In particular, types  $\theta^{**} < \theta < \theta^*$  get the service from new entrants<sup>28</sup>, so that  $E[z(\theta) | \lambda] = E[z(\theta) | \theta^{**} < \theta < \theta^*]$ .

The effect of sorting on incentives to exert effort can be understood investigating the derivative of the first order conditions for optimal effort,  $\gamma(1 - \lambda)E[z(\theta) | \theta^{**} < \theta < \theta^*][F(\lambda^+) - F(\lambda^-)] - \frac{\partial c(e)}{\partial e}$ , with respect to the expected type buying the service from new entrants  $E(\theta | \theta^{**} < \theta < \theta^*)$ . Changes in either  $\theta^*$  or  $\theta^{**}$  (or both) induce changes in  $E(\theta | \theta^{**} < \theta < \theta^*)$ . In order to be consistent with the analysis of the effect of entry, performed in section 4.1, I assume that  $\theta^{**}$ ,  $\theta_{+1}^*$  and  $\theta_{+1}^{**}$  decrease. The change in the current period threshold type  $\theta^{**}$  impacts on  $E[z(\theta) | \theta^{**} < \theta < \theta^*]$ , and thus also on  $\lambda_{+1}^+$  and  $\lambda_{+1}^-$ , while the change in next period threshold types  $\theta_{+1}^*$  and  $\theta_{+1}^{**}$ , affects the valuation for the service, both directly through the utility function of clients, and through the impact on

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<sup>28</sup>A formal proof characterizing the equilibrium is omitted but it is available upon request. In general the proof is along the lines of that of Proposition 2. The part about the sorting behaviour of clients makes use of the assumption that  $z(\theta) \frac{\partial U(\theta)}{\partial \theta} > -\frac{\partial z(\theta)}{\partial \theta} U(\theta)$  for all  $\theta$ , which ensures that clients sort in equilibrium.

$E_{+1}[z(\theta) \mid \theta_{+1}^{**} < \theta < \theta_{+1}^*]$ . Then, differentiating the first order condition for effort exertion yields:

$$\gamma(1 - \lambda) \frac{\partial E[z(\theta) \mid \theta_{+1}^{**} < \theta < \theta^*]}{\partial \theta^{**}} [F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)] + \quad (1)$$

$$\gamma(1 - \lambda) E[z(\theta) \mid \theta_{+1}^{**} < \theta < \theta^*] \left\{ \gamma k \frac{\partial \lambda_{+1}^+}{\partial \theta^{**}} U(\theta_{+1}^*) - \gamma k \frac{\partial \lambda_{+1}^-}{\partial \theta^{**}} U(\theta_{+1}^{**}) \right\} \quad (2)$$

$$-\gamma(1 - \lambda) e_{+1}^* \frac{\partial E_{+1}[z(\theta) \mid \theta_{+1}^{**} < \theta < \theta_{+1}^*]}{\partial E_{+1}(\theta \mid \theta_{+1}^{**} < \theta < \theta_{+1}^*)} [U(\theta_{+1}^*) - U(\theta_{+1}^{**})] + \quad (3)$$

$$\begin{aligned} & \gamma(1 - \lambda) [\lambda_{+1}^+ k \gamma - \lambda k \gamma - (1 - \lambda) \gamma e_{+1}^* E_{+1}[z(\theta) \mid \theta_{+1}^{**} < \theta < \theta_{+1}^*]] \frac{\partial U(\theta_{+1}^*)}{\partial \theta_{+1}^*} + \\ & \gamma(1 - \lambda) [\lambda k \gamma + (1 - \lambda) \gamma e_{+1}^* E_{+1}[z(\theta) \mid \theta_{+1}^{**} < \theta < \theta_{+1}^*]] - \lambda_{+1}^- k \gamma \frac{\partial U(\theta_{+1}^{**})}{\partial \theta_{+1}^{**}} \end{aligned} \quad (4)$$

where  $\partial E_{+1}(\theta \mid \theta_{+1}^{**} < \theta < \theta_{+1}^*)$  captures, with some abuse of notation, a change of both  $\theta_{+1}^*$  and  $\theta_{+1}^{**}$  in the same direction (i.e. either both increase or both decreased). The term

$$\gamma(1 - \lambda) \frac{\partial E[z(\theta) \mid \theta_{+1}^{**} < \theta < \theta^*]}{\partial \theta^{**}} [F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]$$

represents the effect of sorting on the marginal efficiency of effort. As  $\theta^{**}$  decreases,  $E[z(\theta) \mid \theta^{**} < \theta < \theta^*]$  raises, as untalented experts are more likely to succeed on easier cases. Thus, effort is more effective in rising the probability the expert succeeds. This is the *complexity channel*.

The term

$$\begin{aligned} & \gamma(1 - \lambda) E[z(\theta) \mid \theta_{+1}^{**} < \theta < \theta^*] \left\{ \gamma k \frac{\partial \lambda_{+1}^+}{\partial E[z(\theta) \mid \theta_{+1}^{**} < \theta < \theta^*]} \frac{dE[z(\theta) \mid \theta_{+1}^{**} < \theta < \theta^*]}{d\theta^{**}} U(\theta^*) - \right. \\ & \left. \gamma k \frac{\partial \lambda_{+1}^-}{\partial E[z(\theta) \mid \theta_{+1}^{**} < \theta < \theta^*]} \frac{dE[z(\theta) \mid \theta_{+1}^{**} < \theta < \theta^*]}{d\theta^{**}} U(\theta^{**}) \right\} \end{aligned}$$

represents the effect of sorting on the informativeness of a success as a signal of talent. As  $\frac{\partial \lambda_{+1}^+}{\partial E[z(\theta) \mid \lambda]} \frac{dE[z(\theta) \mid \lambda]}{d\theta^{**}} > 0$  and  $\frac{\partial \lambda_{+1}^-}{\partial E[z(\theta) \mid \lambda]} \frac{dE[z(\theta) \mid \lambda]}{d\theta^{**}} < 0$  this term decreases if  $\theta^{**}$  is lower. This follows because facing easier cases makes Bayesian updating less precise, and this reduces the premium for being successful as the probability the expert is talented conditional on a success is lower, and the probability the expert is talented following a failure is higher, than before threshold type  $\theta^{**}$  decreased. This is the *signalling channel*.

The term

$$\gamma(1 - \lambda) e_{+1}^* \frac{\partial E_{+1}[z(\theta \mid \theta_{+1}^{**} < \theta < \theta^*)]}{\partial E_{+1}(\theta \mid \theta_{+1}^{**} < \theta < \theta^*)} [U(\theta_{+1}^*) - U(\theta_{+1}^{**})]$$

represents the clients' "valuation" of the different (higher) probability of a success. As  $E_{+1}(\theta \mid \theta_{+1}^{**} < \theta < \theta_{+1}^*)$  is reduced, due to the reduction in both  $\theta_{+1}^*$  and  $\theta_{+1}^{**}$ ,  $\frac{\partial E_{+1}[z(\theta \mid \theta_{+1}^{**} < \theta < \theta_{+1}^*)]}{\partial E_{+1}(\theta \mid \theta_{+1}^{**} < \theta < \theta_{+1}^*)} > 0$

and this term goes up. This represents the larger willingness to pay of clients for the fact that the probability of receiving the service successfully is higher. This raises the value of being served by a new entrant, as only this class of experts exert effort, and it reduces the relative value of being served by an old successful expert. For this reason it enters with a negative sign and reduces the premium for being successful. This is a new part of the *direct channel*. Finally, the term

$$\begin{aligned} & \gamma(1 - \lambda)\{\lambda_{+1}^+ k\gamma - \lambda k\gamma - (1 - \lambda)\gamma e_{+1}^* E_{+1}[z(\theta) \mid \theta_{+1}^{**} < \theta < \theta_{+1}^*]\} \frac{\partial U(\theta_{+1}^*)}{\partial \theta_{+1}^*} + \\ & \gamma(1 - \lambda)\{\lambda k\gamma + (1 - \lambda)\gamma e_{+1}^* E_{+1}[z(\theta) \mid \theta_{+1}^{**} < \theta < \theta_{+1}^*] - \lambda_{+1}^- k\gamma\} \frac{\partial U(\theta_{+1}^{**})}{\partial \theta_{+1}^{**}} \end{aligned}$$

represents the *direct channel*: the same valuation effect due to higher types deriving a larger utility from a successful outcome, as described in section 4.1. As both  $\theta_{+1}^*$  and  $\theta_{+1}^{**}$ , decrease, the term is negative.

This analysis illustrated the effect of sorting on incentives to exert effort<sup>29</sup>. In order to investigate the full effect of changes in sorting on equilibrium effort it is necessary to perform comparative statics on the equilibrium function. This is done in the following

**Proposition 11** *When the type of clients both represents their valuation for the service and affects the informativeness of success and failure, entry induces conflicting forces on the equilibrium effort. The valuation for the service of threshold types is reduced, as threshold types are lower. The informativeness of a success and of a failure as signals of talent is reduced, decreasing incentives to exert effort. Finally, the total probability of a success increases, raising incentives to exert effort. The net effect depends upon the relative size of the elasticity of the expected probability of success (in absolute value) and that of the premium for being successful. When the former is larger entry increases equilibrium effort.*

**Proof.** See Appendix ■

This proposition shows that the sorting behaviour of clients affects equilibrium fees in three ways: 1) through the increase in the supply of expert services, which changes the types that are indifferent between getting the service from experts with different reputation. Different threshold types have a different valuation for the service and this affects equilibrium fees (*direct channel*); 2) through the change in the informativeness of good performance as a signal of the talent of the expert: in

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<sup>29</sup>Notice that the complexity and the signalling channels depend upon parameters in the current period. In particular, there is measure  $Q$  of old experts, and measure  $\hat{Q}$  if new entrants. Thus, threshold type  $\theta^{**}$  is going to be lower, while threshold type  $\theta^*$  is unaffected. From period +1 onwards, there is going to be measure  $\hat{Q}$  of old experts and measure  $\hat{Q}$  of new entrants. Thus, both threshold type  $\theta_{+1}^*$  and  $\theta_{+1}^{**}$  are affected. This implies that the complexity and the signalling channels will not have the same impact on incentives in the current period as in period +1. This did not happen in the basic version of the model, because the direct channel only operates through period +1 parameters. In period +1 there are going to be measure  $\hat{Q}$  of new entrants and measure  $\hat{Q}$  of old experts.

particular, if the average type being served by new entrants is lower, then learning about an expert type occurs more slowly and less information is released after the market observes the outcome of the service (*signalling channel*); 3) through the change in the probability of success, which modifies the marginal efficiency of effort: facing lower types leads more easily to a success (*complexity channel*). When the latter happens to be particularly strong, entry can increase incentives to exert effort. This occurs if the elasticity of the expected probability of success is larger than the elasticity of the premium for being a successful expert with respect to threshold types. In principle, both these quantities can be estimated, at least for some markets.

A notable example is the market for doctors. The elasticity of the expected probability of success can be recovered by historical data on outcomes of treatments of different operations from very complex, such as liver transplants, to simpler ones. These data are typically available, and actually doctors inform patients about the probability of success when discussing the opportunity of performing a treatment or an operation. The elasticity of the premium paid to successful experts is more complex to observe, although not impossible. As a first step, an econometrician can estimate how the difference between the fees charged by more reputable and less reputable doctors changes for patients with different illnesses. To do this, it is necessary to get a measure of doctor's reputation and data on the fees charged by different doctors for performing different treatments. This amounts to estimating the desired elasticity and it is relatively easy if such data are available. The more difficult step is to ascertain which cases are "threshold" cases. This can be done by identifying which patients (actually, patients with which illness) split about equally between more reputable doctors and younger, new entrant doctors, and between new entrants and older less reputable doctors<sup>30</sup>. These are the threshold cases. Then, one should compute the elasticity of the fees for more reputable and less reputable doctors, with respect to the relevant threshold cases. Such information can guide policy makers in deciding whether increasing entry of doctors is a desirable policy. In fact, by comparing the values of the elasticities at the threshold cases, it would be possible to understand whether entry increases or decreases treatment "quality". A full analysis of the whole impact on welfare of entry of experts (doctors) is contained in the next section.

Finally, it is also interesting to investigate how fees are affected by increased entry.

**Proposition 12** *Fees are lower in the period after entry occurs, both when entry leads to stronger incentives to exert effort, and when entry weakens incentives to exert effort.*

**Proof.** See Appendix ■

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<sup>30</sup>The model says literally that one should observe that cases of similar gravity ( $\theta$  that are close) are treated by doctors of different reputation. Those are the threshold types. In practice, it will probably be observed that each treatment, independent of gravity is performed by doctors of different reputation. However, the proportion of treatments performed by very reputable doctors will be very high for higher  $\theta$ , low for low  $\theta$  and about the same as that performed by younger doctors for threshold types.



Even if incentives to exert effort are stronger, fees are lower. This follows because the signalling and the direct channels impact the premium for being successful, while the complexity channel impacts on the marginal efficiency of effort in the current period, which is not priced in the difference  $F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)$ .

The effort intensive case is essentially analogous. Entry modifies the sorting behaviour of clients, and this affects incentives to exert effort through the same 3 channels identified above. For this reason, a full treatment of the effort intensive case is omitted.

## 5 Welfare Analysis

The model shows that, in many instances, entry can reduce equilibrium effort and thus the average “quality” in the market. However, when the type of the client affects the likelihood of success, increased entry may lead to an equilibrium where effort is higher. This occurs as long as the elasticity of the probability of success is larger (in absolute value) than the elasticity of the premium for reputable experts.

However, entry has a beneficial effect, even when it decreases equilibrium effort. This occurs because when more experts are active in the market a larger fraction of clients get served. In fact the increase in the supply of experts modifies threshold types, impacting on fees, and some clients who preferred to get no service at the ongoing fees, decide to purchase the service. Therefore, entry may generate a higher level of social welfare even if effort is reduced in equilibrium. This can be seen by examining social welfare in the talent intensive case<sup>31</sup>. I consider social welfare in the second period following the increase in entry, for two reasons: firstly, this takes into account the full effect of having a larger size of experts in the market, as there will be a larger measure of both young and old experts; secondly, the increase in entry impacts on threshold types and equilibrium effort and in this way on learning about expert’s talent ( $\lambda^+$  and  $\lambda^-$ ), but this affects clients from the period after the increase in entry occurred. Thus, assuming increased entry occurred in period  $-2$ , social welfare in period 0 is:

$$\int_{\theta^{***}}^{\bar{\theta}} V(\lambda, \theta, e) d\theta - c(e) = \int_{\theta^*}^{\bar{\theta}} V(\lambda^+, \theta, 0) d\theta + \int_{\theta^{**}}^{\theta^*} V(\lambda, \theta, e) d\theta + \int_{\theta^{***}}^{\theta^{**}} V(\lambda^-, \theta, 0) d\theta - c(e)$$

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<sup>31</sup>The effort intensive case is analogous.

Differentiating social welfare with respect to  $Q$ , the size of new entrants, yields the following

$$\begin{aligned}
& \int_{\theta^*}^{\bar{\theta}} \gamma k \left[ \frac{\partial \lambda^+}{\partial e_{-1}} \frac{\partial e_{-1}}{\partial Q} + \right. \\
& \left. \frac{\partial \lambda^+}{\partial E[z(\theta) | \theta_{-1}^* < \theta < \theta_{-1}^{**}]} \left( \frac{\partial E[z(\theta) | \theta_{-1}^* < \theta < \theta_{-1}^{**}]}{\partial \theta_{-1}^*} \frac{\partial \theta_{-1}^*}{\partial Q} + \frac{\partial E[z(\theta) | \theta_{-1}^* < \theta < \theta_{-1}^{**}]}{\partial \theta_{-1}^{**}} \frac{\partial \theta_{-1}^{**}}{\partial Q} \right) \right] U(\theta) d\theta - \\
& - \gamma k \lambda^+ U(\theta^*) \frac{\partial \theta^*}{\partial Q} + \int_{\theta^{**}}^{\theta^*} \gamma [k\lambda + (1-\lambda)z(\theta)] \frac{\partial e}{\partial Q} U(\theta) d\theta + \\
& + \gamma [k\lambda + (1-\lambda)z(\theta^*)e] U(\theta^*) \frac{\partial \theta^*}{\partial Q} - \gamma [k\lambda + (1-\lambda)z(\theta^{**})e] U(\theta^{**}) \frac{\partial \theta^{**}}{\partial Q} + \\
& + \gamma k \lambda^- U(\theta^{**}) \frac{\partial \theta^{**}}{\partial Q} - \gamma k \lambda^- U(\theta^{***}) \frac{\partial \theta^{***}}{\partial Q} \\
& \int_{\theta^{***}}^{\theta^{**}} \gamma k \left[ \frac{\partial \lambda^-}{\partial e_{-1}} \frac{\partial e_{-1}}{\partial Q} + \right. \\
& \left. \frac{\partial \lambda^-}{\partial E[z(\theta) | \theta_{-1}^{**} < \theta < \theta_{-1}^{***}]} \left( \frac{\partial E[z(\theta) | \theta_{-1}^{**} < \theta < \theta_{-1}^{***}]}{\partial \theta_{-1}^{**}} \frac{\partial \theta_{-1}^{**}}{\partial Q} + \frac{\partial E[z(\theta) | \theta_{-1}^{**} < \theta < \theta_{-1}^{***}]}{\partial \theta_{-1}^{***}} \frac{\partial \theta_{-1}^{***}}{\partial Q} \right) \right] U(\theta) d\theta \\
& - \frac{\partial c(e)}{\partial e} \frac{\partial e}{\partial Q}
\end{aligned}$$

this represents the trade-off associated with increased entry.

The terms  $\frac{\partial \lambda^+}{\partial e_{-1}} \frac{\partial e_{-1}}{\partial Q}$  and  $\frac{\partial \lambda^+}{\partial E[z(\theta) | \theta_{-1}^* < \theta < \theta_{-1}^{**}]} \left( \frac{\partial E[z(\theta) | \theta_{-1}^* < \theta < \theta_{-1}^{**}]}{\partial \theta_{-1}^*} \frac{\partial \theta_{-1}^*}{\partial Q} + \frac{\partial E[z(\theta) | \theta_{-1}^* < \theta < \theta_{-1}^{**}]}{\partial \theta_{-1}^{**}} \frac{\partial \theta_{-1}^{**}}{\partial Q} \right)$  represent the effect on learning about old successful types. These can have opposite sign when entry reduces equilibrium effort, as this raises  $\lambda^+$  (lower effort means that a success is more likely to come from talented experts), while entry also reduces threshold types and this raises the probability of success. A similar reasoning applies to the terms  $\frac{\partial \lambda^-}{\partial e_{-1}} \frac{\partial e_{-1}}{\partial Q}$  and  $\frac{\partial \lambda^-}{\partial E[z(\theta) | \theta_{-1}^{**} < \theta < \theta_{-1}^{***}]} \left( \frac{\partial E[z(\theta) | \theta_{-1}^{**} < \theta < \theta_{-1}^{***}]}{\partial \theta_{-1}^{**}} \frac{\partial \theta_{-1}^{**}}{\partial Q} + \frac{\partial E[z(\theta) | \theta_{-1}^{**} < \theta < \theta_{-1}^{***}]}{\partial \theta_{-1}^{***}} \frac{\partial \theta_{-1}^{***}}{\partial Q} \right)$  which capture learning about old unsuccessful types. Then, the reduction in threshold types changes the fraction of clients served by experts of different reputation. This is captured by the terms  $-\gamma k \lambda^+ U(\theta^*) \frac{\partial \theta^*}{\partial Q}$ ,  $\gamma [k\lambda + (1-\lambda)z(\theta^*)e] U(\theta^*) \frac{\partial \theta^*}{\partial Q} - \gamma [k\lambda + (1-\lambda)z(\theta^{**})e] U(\theta^{**}) \frac{\partial \theta^{**}}{\partial Q}$ ,  $\gamma k \lambda^- U(\theta^{**}) \frac{\partial \theta^{**}}{\partial Q} - \gamma k \lambda^- U(\theta^{***}) \frac{\partial \theta^{***}}{\partial Q}$ . The overall effect is

$$\begin{aligned}
& \{-\gamma k \lambda^+ U(\theta^*) + \gamma [k\lambda + (1-\lambda)z(\theta^*)e]\} U(\theta^*) \frac{\partial \theta^*}{\partial Q} \\
& + \{-\gamma [k\lambda + (1-\lambda)z(\theta^{**})e] + \gamma k \lambda^-\} U(\theta^{**}) \frac{\partial \theta^{**}}{\partial Q} \\
& - \gamma k \lambda^- U(\theta^{***}) \frac{\partial \theta^{***}}{\partial Q}
\end{aligned}$$

the first two lines are positive as  $\frac{\partial \theta^*}{\partial Q}$  and  $\frac{\partial \theta^{**}}{\partial Q}$  are negative and  $\gamma k \lambda^+ > k\lambda + (1-\lambda)z(\theta^*)e$  in the talent intensive case, and capture the fact that a fraction of clients who were getting the service

from new entrants are now getting it from old successful experts, and a fraction of clients who were getting the service from old unsuccessful experts are now getting it from new entrants. The last line captures the fact that some clients who were not getting any service, are now getting it. Finally, the term  $\int_{\theta^{**}}^{\theta^*} \gamma[k\lambda + (1 - \lambda)z(\theta)\frac{\partial e}{\partial Q}]U(\theta)d\theta$  captures the effect of increased entry on equilibrium effort, and thus on the probability that clients served by new entrants obtain the service at a high standard. This term has the same sign as  $\frac{\partial e}{\partial Q}$ . Thus, increased entry has an ambiguous effect on social welfare<sup>32</sup> as it may lead to lower equilibrium effort and to slower learning about experts' talent which could more than compensate the positive effect stemming from the increase in the measure of clients getting the service.

This result provides support for policies that attempt to ensure that the supply of experts in certain markets is somehow constrained. The market for doctors represents an important example. Some countries limit the number of places for students in medical schools and put bounds on the number of foreign doctors that can work in the country. Putting limits to the entry of doctors may help to attain an optimal trade-off between increasing the access of the population to medical services and keeping quality of the service high.

This analysis also suggests that a possible way to avoid the potential adverse effect of entry on the incentives to exert effort is to introduce a test, or a certification system, for successful experts. The certificate, to be useful, must be correlated with talent. Then, if only the top end of the distribution of experts got the certificate, it would be possible to restore rents from getting a good reputation. In fact, even if entry occurs, those who succeed have a chance to get into the top league and the premium to be served by top league successful experts would be independent of entry. Somehow, this kind of institutions seem to arise in practice: league tables for investment banks or financial analysts are published every year, and that could be a way to preserve the rents from building a reputation even if competition is fierce<sup>33</sup>.

## 6 Discussion

This section reviews some of the most important aspects of the modelling strategy.

The intertemporal dimension is very important, as the incentives to exert effort depend on beliefs about current period effort (which in a Perfect Bayesian Equilibrium reflect correctly the

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<sup>32</sup>In principle, it may be possible to identify the optimal size of new entrants, and in general of experts in the market. This would solve the optimal trade off between the potential negative impact of a larger number of experts on effort and the positive impact of a larger number of experts in terms of more clients obtaining the service. The solution to this problem depends upon whether the welfare function is concave or convex in  $Q$ . This is difficult to establish in a general case, without making more specific assumptions on utility functions, cost functions and parameters.

<sup>33</sup>Entry could still have some adverse effects as the premium for successful experts also depends upon the whole structure of fees in the market, which may still decrease due to increased entry. However, "rationing" the supply of the most talented experts could help to boost the benefit from increasing entry in a market for expert services.

equilibrium effort level chosen by experts), and on beliefs about future period effort levels, as the two determine the equilibrium fees. In principle, beliefs about future period effort level could be set rather freely, although they should be correct ex-post in a Perfect Bayesian Equilibrium. Then, there could exist equilibria in which the equilibrium effort level oscillates between a high and a low value, even if there is no change in entry, nor in other parameters. However, even in such a case, entry modifies incentives to exert effort, and all results would hold with respect to the equilibrium effort had entry not occurred. Thus, for example, effort could be larger after entry occurs, just because it oscillates in equilibrium between a low and a large value, but it would have been larger had entry not occurred.

The model assumes the exact type of clients is not observable, not even after the output of the service is realized and observed. However, the market knows the average type of client being served by experts of a given reputation level. This is not unreasonable in practice, as often evaluators lack the skills to fully judge the complexity of the task performed by an expert, and the model assumes evaluators infer an average degree of complexity. While this is fine for some applications, it may be less appealing in other cases. For example, the assumption would imply that the market cannot observe the exact complexity of a case treated by a doctor, but knows that doctors with a given reputation level treat cases of a certain degree of complexity. This is fine for what concerns evaluations performed by patients, but in other situations, in which experts are evaluated by their peers, the true complexity of the task performed may be fully observed. Thus, it may seem that this assumption limits the applicability of the model. However, this is not the case, as the same results would hold assuming that the type of the client, and thus, for example, the complexity of the operation, is perfectly observed after the outcome is realized. In such a case, old experts would have each a different reputation, depending on the exact  $\theta$  they are matched with in their first period on the market. Then, clients' sorting would still have similar effects as those highlighted in the paper: when choosing effort levels, in their first period on the market, young experts take an expectation about the type they are going to face in that first period, exactly as in the model<sup>34</sup>. Thus, the true role of the assumption is to streamline the model, rendering its presentation easier, as reputation levels will be constrained to three: prior, posterior following a success, posterior following a failure.

Clients are assumed to know perfectly their type. Literally, this means that they know how much they value the service and (or) how difficult it is for the expert to provide the service successfully. Moreover, at least until the expert has exerted effort, the client knows more about her type than

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<sup>34</sup>If experts could observe the true type of clients before choosing effort in the current period, their choice of effort would depend on the observed  $\theta$ . Thus, the effort level will be different for each new entrant expert as a function of the actual type  $\theta$  the expert is matched with. In such a case, results would hold for the average effort exerted by experts in the market. As, due to increased entry, new entrants face a different pool of clients, the average effort chosen by such experts will be different, as predicted by the model. A version of the model allowing for the observability of  $\theta$  by the market, by the expert, or by both is available upon request and may be added as an extension of the base model.

the expert does. In principle, this can limit the applicability of the model to some markets. For example, patients typically do not know the treatment they need, and part of the service provided by doctors is the discovery of the disease, i.e. of the type  $\theta$ , through a diagnosis. Still, the model can be applicable to the market for health services if the type of the client is interpreted as follows: the patient has a problem, which can be an headache, or a difficulty to breathe. This is a noisy, but private, signal of the true disease  $\theta$ . Given how bad this problem is, the client decides whether a more or a less reputable doctor is more likely to provide a successful treatment. The impact of the problem on the patient's welfare also represents her valuation for a successful solution: an headache is more bearable than a difficulty to breathe. The doctor may learn something from the patient, but does not know exactly the true disease  $\theta$  when exerting effort. Then, effort in this context can be interpreted as the provision of both a correct diagnosis and of a successful treatment. The modelling of clients' type may be enriched, but what is important for results to hold is that there is some sorting in equilibrium. When this happens, the impact of sorting on reputational incentives will operate through the three channels identified by this paper.

The model assumes that fees cannot be made contingent on the outcome of the service, so that the only incentives are reputational. This is fine for some applications, such as in the case of doctors<sup>35</sup>, but it is probably less so for other applications. For example, in the United States, lawyers can get fees contingent on whether they win the lawsuit. When fees can be made contingent on observed outcomes, the model is not fully applicable, as experts are motivated both by reputational concerns and by the explicit incentives provided by contingent fees. However, the effect of sorting on reputational concerns would still be the same, although explicit incentives, in equilibrium, would be set taking into account also the impact of sorting on reputational incentives.

Finally, experts charge a fee  $F$  for the service, and the service is not divisible, so that this rules out the possibility that experts offer screening contracts in order to attract different types<sup>36</sup>. Moreover, the fact that experts compete for types makes it difficult to offer screening contracts, as there may not be enough rents to induce clients to separate (revealing their types) while providing experts with adequate incentives to exert effort. This is a reasonable assumption for some applications, but possibly less for others. In the case of doctors, treatments are typically not divisible and doctors have few instruments to screen types.

## 7 Conclusion

This paper investigates how the sorting behaviour of clients affects the incentives of experts motivated by reputational concerns. The model shows that sorting can affect incentives in three ways:

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<sup>35</sup>Doctors may be sued for malpractice, although they are often insured. Then, the "loss" for doctors providing a poor treatment to patients is often mostly reputational.

<sup>36</sup>Experts have only one instrument, the fee, to screen types.

firstly through a change in the type of client who is indifferent between getting the service from experts of different reputation (*direct channel*); secondly by affecting the informativeness of a successful provision of the service by the expert: if successful provision of the service for certain types of clients is more likely to be delivered by talented experts, then, the pool of clients applying to an expert affects the updating of beliefs about an expert's talent (*signalling channel*); finally, if the type of the client affects the likelihood an expert succeeds, facing "tougher" types could reduce the total probability of success (*complexity channel*), affecting the marginal efficiency of effort.

The paper investigates the effect of entry in this framework. Entry affects the sorting behaviour of clients in a way that reduces equilibrium effort through the direct channel. The signalling and the complexity channels have instead conflicting effects on equilibrium effort. When the type of clients affects both their valuation for the service, and the informativeness of a success as a signal of experts' talent, entry increases incentives as long as the elasticity of the probability of success is larger (in absolute value) than the elasticity of the premium for successful experts. However, even if entry reduces equilibrium effort, it can lead to higher welfare as more clients get access to the service. The model also suggests that the use of league tables, or of other certification mechanisms aimed at "constraining" the supply of most reputable experts can be beneficial as they may help preserving the rents from building a reputation.

## References

- [1] Almeida-Costa, L. and Vasconcelos, L. (2008). “Share the Fame or Share the Blame? The Reputational Implications of Partnerships”, *RAND Journal of Economics*, Forthcoming.
- [2] Anderson, A. and Smith, L. (2009), “Dynamic Matching and Evolving Reputations”, *Review of Economic Studies*, Forthcoming.
- [3] Bar-Isaac, H. (2005). “Imperfect Competition and Reputational Commitment”, *Economic Letters*, 89, 167 - 173.
- [4] Bar-Isaac, H. and Tadelis, S. (2008). “Seller Reputation”, *Foundations and Trends in Microeconomics*, 4, 273 - 351.
- [5] Biglaiser, G. (1993). “Middlemen as Experts”. *RAND Journal of Economics*, 24, 212-223.
- [6] Casas-Arce, (2009). “Career Tournaments”, mimeo, Universitat Pompeu Fabra.
- [7] Chemmanur, T. and Fulghieri, P. (1994). “Investment Bank Reputation, Information Production, and Financial Intermediation”, *Journal of Finance*, 49, 57-79.
- [8] Dulleck, U. and Kerschbamer, R. (2006). “On doctors, Mechanics, and Computer Specialists: the Economics of Credence Goods”, *Journal of Economic literature*, XLIV, 5 - 42.
- [9] Emons, W. (1997). “Credence Goods and Fraudulent Experts”, *RAND Journal of Economics*, 28, 107 - 119.
- [10] Faure Grimaud, A., Peyrache, E. and Quesada, L. (2008). “The Ownership of Ratings”, *RAND Journal of Economics*, Forthcoming.
- [11] Fernando, C. S., Gatchev, V. A. and Spindt, P. A. (2005). “Wanna Dance? How Firms and Underwriters Choose Each Other”, *Journal of Finance*, 60, 2437 - 2469.
- [12] Fong, Y. F. (2006). “When Do Experts cheat and Whom Do They Target?”, *Rand Journal of Economics*, 36, 113 - 130.
- [13] Holmstrom, B. (1982). “Managerial Incentive Schemes. A Dynamic Perspective”, in *Essays in Economics and Management in Honour of Lars Wahlbeck*, Swedish School of Economics.
- [14] Horner, J. (2002). “Reputation and Competition”, *American Economic Review*, 92, 644 - 663.
- [15] Kovrijnykh, A. (2007). “Career Uncertainty and Dynamic Incentives”, mimeo, University of Chicago.

- [16] Kranton, R. (2003). “Competition and the Incentive to Produce High - Quality”, *Economica*, 70, 385 - 404.
- [17] Lizzeri, A. (1999). “Information Revelation and Certification intermediaries”, *RAND Journal of Economics*, 30, 214 - 231.
- [18] Mailath, G. and Samuelson, P. (2001). “Who Wants a Good Reputation?”, *Review of Economic Studies*, 68, 415 - 441.
- [19] Martinez, L. (2009). “Reputation, Career Concerns, and Job Assignments”. *B.E. Journal of Theoretical Economics*, 9, Art. 15.
- [20] Park, I.U. (2005). “Cheap-talk referrals of differentiated experts in repeated relationships”, *RAND Journal of Economics*, 36, 391 - 411.
- [21] Pesendorfer, W. and Wolinsky, A. (2003). “Second Opinions and Price Competition: Inefficiency in the Market for Expert Advice”, *Review of Economic Studies*, 70, 417 - 437.
- [22] Pitchik, C. and Schotter, A. (1987). “Honesty in a Model of Strategic Information Transmission”, *American Economic Review*, 77, 1032 - 1036.
- [23] Raith, M. (2003). “Competition, Risk, and Managerial Incentives”, *American Economic Review*, 93, 1425 - 1436.
- [24] Rubinstein, A. and Wolinsky, A. (1987). “Middlemen”, *Quarterly Journal of Economics*, 102, 581 - 594.
- [25] Tadelis, S. (2002). “The Market for Reputations as an Incentive Mechanism”, *Journal of Political Economy*, 92, 854 - 882 .
- [26] Wolinsky, A. (1993). “Competition in a Market for Informed Expert’s Services”, *RAND Journal of Economics*, 24, 380 - 398.

## Appendix

**Proof of Lemma 1.** The payoff of any expert in the last period is given by  $F_t - c(e_t)$ , where  $F_t$  cannot depend upon the effort level  $e_t$ , nor on realized performance, as both are not verifiable. Thus, current period effort  $e_t$  does not affect the revenues from providing the services, while it costs  $c(e_t)$ , therefore  $e_t = 0$  is the unique optimal effort choice for old experts. The behaviour of experts in their first (penultimate) period is more interesting, although the logic is still standard in models



with career concerns. In case 1, in which the type of clients only affects the valuation for the service, payoff for experts in their first period is

$$F_t + [\lambda\gamma + (1 - \lambda)\gamma e_t]F_{t+1}(\lambda_{t+1}^+) + \\ [\lambda(1 - \gamma) + (1 - \lambda)(1 - \gamma e_t)]F_{t+1}(\lambda_{t+1}^-)$$

subject to the constraint that  $0 \leq e \leq 1$ , and the first order condition for effort exertion is<sup>37</sup>

$$\frac{\partial c(e_t)}{\partial e_t} = \gamma(1 - \lambda)[F_{t+1}(\lambda_{t+1}^+) - F_{t+1}(\lambda_{t+1}^-)]$$

as now exerting effort in period  $t$ , raises the chances of obtaining in period  $t + 1$  the fee conditional on being successful in the period  $t$ . Incentives to exert effort increase in the premium for being served by an expert who has been successful in the first period (and thus improved her reputation). In cases 2 and 3, the type of clients also affects the probability of success and payoff for experts in their first period is

$$F_t + [\lambda k\gamma + (1 - \lambda)\gamma E(z(\theta) | \Theta(\lambda))e_t]F_{t+1}(\lambda_{t+1}^+) + \\ [\lambda(1 - k\gamma) + (1 - \lambda)(1 - \gamma E(z(\theta) | \Theta(\lambda))e_t)]F_{t+1}(\lambda_{t+1}^-)$$

where  $\Theta(\lambda)$  is the set of clients' types getting the service from experts with reputation  $\lambda$ . The first order condition for effort exertion is

$$\frac{\partial c(e_t)}{\partial e_t} = \gamma(1 - \lambda)E(z(\theta) | \Theta(\lambda))[F_{t+1}(\lambda_{t+1}^+) - F_{t+1}(\lambda_{t+1}^-)]$$

and again exerting effort in period  $t$  raises the chances of obtaining a higher payoff in period  $t + 1$ .

■

**Proof of Proposition 2.** Under the assumption of case A,  $\gamma\lambda^+ > \gamma\lambda + (1 - \lambda)\gamma e^*$ , and experts with reputation  $\lambda^+$  are preferred by all clients so that there is excess demand for them. Then, fees raise so that demand equals supply and there is no incentive to deviate. As higher types are more willing to pay for the service, when fees go up they are still willing to pay for the services of most reputable experts. As there are  $[\lambda\gamma + (1 - \lambda)\gamma e_{-1}]Q$  experts with reputation  $\lambda^+$  this represents supply, while  $\bar{\theta} - \theta^*$  is demand. Fees must make the marginal client indifferent between getting the service from experts of reputation  $\lambda^+$  and from experts of reputation  $\lambda$ . Then, it can be seen that all clients with  $\theta > \theta^*$  do not want to deviate. In fact, by getting the service from an expert with reputation  $\lambda^+$  clients' payoff is  $V(\lambda^+, \theta, 0) - F(\lambda^+)$  while by getting the service from an expert with

<sup>37</sup>Ignoring the constraint that  $0 \leq e \leq 1$ . On the one hand, as  $\frac{\partial c(1)}{\partial e} = +\infty$  ensures that  $e = 1$  cannot be a solution. For the possibility that  $e = 0$ , it will be shown that in equilibrium effort is strictly positive.

reputation  $\lambda$  payoff is  $V(\lambda, \theta, e^*) - F(\lambda)$ . Then, it must be true that:

$$V(\lambda^+, \theta, 0) - F(\lambda^+) > V(\lambda, \theta, e^*) - F(\lambda)$$

or

$$V(\lambda^+, \theta, 0) - V(\lambda, \theta, e^*) > V(\lambda^+, \theta^*, 0) - V(\lambda, \theta^*, e^*)$$

which is equal to

$$V(\lambda^+, \theta, 0) - V(\lambda^+, \theta^*, 0) > V(\lambda, \theta, e^*) - V(\lambda, \theta^*, e^*)$$

As  $V(\lambda^+, \theta, 0) - V(\lambda^+, \theta^*, 0) = \gamma\lambda^+[U(\theta) - U(\theta^*)]$  and  $V(\lambda, \theta, e^*) - V(\lambda, \theta^*, e^*) = [\lambda\gamma + (1 - \lambda)\gamma e^*][U(\theta) - U(\theta^*)]$  the inequality is verified as  $\theta > \theta^*$  and as in the talent intensive case  $\gamma\lambda^+ > \lambda\gamma + (1 - \lambda)\gamma e^*$ .

Similarly, clients of type  $\theta > \theta^*$  prefer to get the service from experts of reputation  $\lambda^+$  than from experts of reputation  $\lambda^-$ . In fact,  $V(\lambda^+, \theta, 0) - F(\lambda^+) > V(\lambda^-, \theta) - F(\lambda^-)$ . This inequality can be rewritten as

$$V(\lambda^+, \theta, 0) - V(\lambda^-, \theta, 0) > F(\lambda^+) - F(\lambda^-)$$

and by adding and subtracting  $\gamma\lambda U(\theta)$ :

$$V(\lambda^+, \theta, 0) - V(\lambda^-, \theta, 0) = \gamma(\lambda^+ - \lambda^-)U(\theta) = \gamma(\lambda^+ - \lambda)U(\theta) + \gamma(\lambda - \lambda^-)U(\theta)$$

so that:

$$\begin{aligned} F(\lambda^+) - F(\lambda^-) &= V(\lambda^+, \theta^*, 0) - V(\lambda, \theta^*, e^*) + V(\lambda, \theta^{**}, e^*) - V(\lambda^-, \theta^{**}, 0) = \\ &\quad \gamma(\lambda^+ - \lambda)U(\theta^*) + \gamma(\lambda - \lambda^-)U(\theta^{**}) - \lambda\gamma e^*[U(\theta^*) - U(\theta^{**})] \end{aligned}$$

Then,

$$\begin{aligned} V(\lambda^+, \theta, 0) - V(\lambda^-, \theta, 0) &= \gamma(\lambda^+ - \lambda)U(\theta) + \gamma(\lambda - \lambda^-)U(\theta) > \\ F(\lambda^+) - F(\lambda^-) &= \gamma(\lambda^+ - \lambda)U(\theta^*) + \gamma(\lambda - \lambda^-)U(\theta^{**}) - \lambda\gamma e^*[U(\theta^*) - U(\theta^{**})] \end{aligned}$$

because  $\gamma(\lambda^+ - \lambda)U(\theta) > \gamma(\lambda^+ - \lambda)U(\theta^*)$  due to  $\theta > \theta^*$ ,  $\gamma(\lambda - \lambda^-)U(\theta) > \gamma(\lambda - \lambda^-)U(\theta^{**})$  due to  $\theta > \theta^{**}$ ,  $-\lambda\gamma e^*[U(\theta^*) - U(\theta^{**})] < 0$  as  $\theta^* > \theta^{**}$ . Therefore, types  $\theta > \theta^*$  have no incentive to deviate and cannot do better than pay  $F(\lambda^+)$  and being matched to an expert with type  $\lambda^+$ .

Clients with types  $\theta^* > \theta > \theta^{**}$  prefer to be served by experts with reputation  $\lambda^+$ , but are rationed by equilibrium fees, so they switch to experts of reputation  $\lambda$ . By applying to an expert

with higher reputation they would get  $V(\lambda^+, \theta, 0) - F(\lambda^+)$ . It can be seen that:

$$V(\lambda^+, \theta, 0) - F(\lambda^+) < V(\lambda, \theta, e^*) - F(\lambda)$$

as

$$V(\lambda^+, \theta, 0) - V(\lambda, \theta, e^*) < V(\lambda^+, \theta^*, 0) - V(\lambda, \theta^*, e^*)$$

by the same reasoning as above and noting that now  $\theta < \theta^*$ . In the same fashion it is possible to show that also the other types of clients have no incentive to deviate. To complete the proof, it can be shown that experts cannot do better. Experts are atomistic, if they tried to charge a higher price, they would be competed away from other experts with the same reputation.

Finally, equilibrium effort is unique. Effort depends upon expected next period premium for being successful which is:

$$F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-) = V(\lambda_{+1}^+, \theta_{+1}^*, 0) - V(\lambda, \theta_{+1}^*, e_{+1}^*) + V(\lambda, \theta_{+1}^{**}, e_{+1}^*) - V(\lambda_{+1}^-, \theta_{+1}^{**}, 0)$$

this expression can be rewritten as

$$\gamma(\lambda_{+1}^+ - \lambda)U(\theta_{+1}^*) + \gamma(\lambda - \lambda_{+1}^-)U(\theta_{+1}^{**}) - \lambda\gamma e_{+1}^*[U(\theta_{+1}^*) - U(\theta_{+1}^{**})]$$

This depends both upon beliefs about the current period effort level  $e^*$ , which determines the value of  $\lambda_{+1}^+$  and  $\lambda_{+1}^-$ , and upon beliefs about next period effort level, which will be exerted by new entrants in the next period. The left hand side is decreasing in  $e^*$ . In fact,  $\frac{\partial \lambda^+}{\partial e^*} < 0$ ,  $\frac{\partial \lambda^-}{\partial e^*} > 0$  but it enters with a negative sign, and  $\theta_{+1}^*$  and  $\theta_{+1}^{**}$  are decreasing in  $e^*$ . Then the first order condition is

$$\gamma(1 - \lambda)\{\gamma(\lambda_{+1}^+ - \lambda)U(\theta_{+1}^*) + \gamma(\lambda - \lambda_{+1}^-)U(\theta_{+1}^{**}) - \lambda\gamma e_{+1}^*[U(\theta_{+1}^*) - U(\theta_{+1}^{**})]\} = \frac{\partial c(e)}{\partial e} \quad (5)$$

the right hand side is increasing in  $e$ , while the left hand side is decreasing in  $e^* = e$  in a perfect Bayesian equilibrium. Moreover,  $\frac{\partial c(0)}{\partial e} = 0$ , while the left hand side is strictly positive when  $e = 0$ , and the left hand side is equal to zero when  $e = 1$ , while the right hand side is positive (actually it is  $+\infty$ ). Both the left and the right hand side of equation 5 are continuous in  $e$  by continuity of  $c$ ,  $U$ , and of  $\lambda^+$  and  $\lambda^-$ . Thus, there is only one value of  $e$  that satisfies equation 5 and the equilibrium level of effort is thus unique, for a given level of expected next period effort  $e_{+1}^*$ . This equilibrium exists as long as the equilibrium value of  $e$  satisfies  $\gamma\lambda_{+1}^+ > \gamma\lambda + (1 - \lambda)\gamma e_{+1}^*$ , where  $\lambda_{+1}^+ = \frac{\lambda}{\lambda + (1 - \lambda)e^*}$ , after imposing the condition that beliefs are correct in equilibrium so that  $e^* = e$ . If one further assumes that beliefs about effort are stationary, meaning that if the market has the same measure of experts and technology is still the same, beliefs about effort level across generations

are constant, then  $e_{+1}^* = e^* = e$ , then equilibrium effort level is still unique (in fact, the left hand side of equation 5 is still decreasing in  $e$ , the right hand side is increasing, the left hand side is positive when  $e = 0$  and negative when  $e = 1$ ). Then, the condition  $\gamma\lambda_{+1}^+ > \gamma\lambda + (1 - \lambda)\gamma e_{+1}^*$  yields a quadratic inequality in  $e$ , which is satisfied as long as  $e < \frac{\sqrt{\lambda - \lambda}}{1 - \lambda}$ . Finally,

$$F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-) = \gamma(\lambda^+ - \lambda)U(\theta^*) + \gamma(\lambda - \lambda^-)U(\theta^{**}) - \lambda\gamma e^*[U(\theta^*) - U(\theta^{**})] = \\ \gamma[(\lambda^+ - \lambda - (1 - \lambda)e^*)U(\theta^*) + \gamma[\lambda + (1 - \lambda)e^* - \lambda^-]U(\theta^{**})]$$

from the conditions that ensure sorting in a talent intensive equilibrium,  $\lambda^+ > \lambda + (1 - \lambda)e^*$ , and in any equilibrium  $\lambda + (1 - \lambda)e^* > \lambda^-$ , so that the difference  $F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)$  is increasing in threshold types  $\theta^*, \theta^{**}$ . ■

**Proof of Proposition 3.** It follows the same reasoning used in the proof of Proposition 2. ■

**Proof of Lemma 4.** Consider a Perfect Bayesian equilibrium where  $e = e^*$ . In such an equilibrium, effort must satisfy the following:

$$\gamma(1 - \lambda)[F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)] - \frac{\partial c(e)}{\partial e} = 0 \\ \text{where } \lambda_{+1}^+ = \frac{\lambda\gamma}{\lambda\gamma + (1 - \lambda)\gamma e^*}, \\ \lambda_{+1}^- = \frac{\lambda(1 - \gamma)}{\lambda(1 - \gamma) + (1 - \lambda)(1 - \gamma e)}$$

The equilibrium function is

$$\gamma(1 - \lambda)[\gamma\lambda_{+1}^+ - \gamma\lambda - (1 - \lambda)\gamma e_{+1}^*]U(\theta_{+1}^*) + \\ [\gamma\lambda + (1 - \lambda)\gamma e_{+1}^* - \gamma\lambda_{+1}^-]U(\theta_{+1}^{**}) - \frac{\partial c(e)}{\partial e} \text{ if } e \leq \frac{\lambda(1 - e_{+1}^*)}{\lambda + (1 - \lambda)e_{+1}^*} \\ \gamma(1 - \lambda)(\gamma\lambda_{+1}^+ - \gamma\lambda_{+1}^-)U(\theta_{+1}^{**}) - \frac{\partial c(e)}{\partial e} \text{ if } e > \frac{\lambda(1 - e_{+1}^*)}{\lambda + (1 - \lambda)e_{+1}^*}$$

the function is continuous in  $e$  because  $U$  and  $c$  are continuous (notice that  $e_{+1}^*$  is taken as a parameter). Moreover, the function is decreasing in  $e$ . It is positive for  $e = 0$  as proved in the proof of Proposition 2 (notice that also  $\gamma(1 - \lambda)(\gamma\lambda_{+1}^+ - \gamma\lambda_{+1}^-)U(\theta_{+1}^{**}) > 0$ , when  $e = 0$ ), and negative for  $e = 1$ , in fact

$$\gamma(1 - \lambda)(\gamma\lambda_{+1}^+ - \gamma\lambda_{+1}^-)U(\theta_{+1}^{**}) - \frac{\partial c(e_t)}{\partial e_t} < 0$$

as  $\lambda_{+1}^+ = \lambda_{+1}^- = \lambda$  (notice that also  $\gamma(1 - \lambda)[\gamma\lambda_{+1}^+ - \gamma\lambda - (1 - \lambda)\gamma e_{+1}^*]U(\theta_{+1}^*) + [\gamma\lambda + (1 - \lambda)\gamma e_{+1}^* - \gamma\lambda_{+1}^-]U(\theta_{+1}^{**}) - \frac{\partial c(e)}{\partial e} < 0$  when  $e = 1$ ).

It can also be shown that when  $e = \frac{\lambda(1 - e_{+1}^*)}{\lambda + (1 - \lambda)e_{+1}^*}$ , the equilibrium function in the talent and in

the effort intensive case takes the same value,

$$\begin{aligned} & \gamma(1-\lambda)[\gamma\lambda_{+1}^+ - \gamma\lambda - (1-\lambda)\gamma e_{+1}^*]U(\theta_{+1}^*) + \\ & [\gamma\lambda + (1-\lambda)\gamma e_{+1}^* - \gamma\lambda_{+1}^-]U(\theta_{+1}^{**}) - \frac{\partial c(e)}{\partial e} = \\ & \gamma(1-\lambda)(\gamma\lambda_{+1}^+ - \gamma\lambda_{+1}^-)U(\theta_{+1}^{**}) - \frac{\partial c(e)}{\partial e} \end{aligned}$$

and the threshold  $\theta_{+1}^{**}$  is the same both in the talent and in the effort intensive case (in the talent intensive case,  $\bar{\theta} - \theta_{+1}^* = [\gamma\lambda + (1-\lambda)\gamma e]Q$  and  $\theta_{+1}^* - \theta_{+1}^{**} = Q_{t+1}$ , so that  $\theta_{+1}^{**} = \bar{\theta} - [\gamma\lambda + (1-\lambda)\gamma e]Q - Q_{+1}$ ; in the effort intensive case,  $\bar{\theta} - \theta_{+1}^* = Q_{t+1}$  and  $\theta_{+1}^* - \theta_{+1}^{**} = [\gamma\lambda + (1-\lambda)\gamma e]Q$ , so that  $\theta_{+1}^{**} = \bar{\theta} - Q_{+1} - [\gamma\lambda + (1-\lambda)\gamma e]Q$ ). Therefore the equilibrium function is continuous, is decreasing in  $e$ , is positive for the lowest value of  $e$  and negative for the highest value. Thus, there exists one value of  $e$  such that the equilibrium function is equal to zero. In other words, there exists one equilibrium level of effort. This is unique by monotonicity and continuity of the equilibrium function. Thus the equilibrium is going to be either a talent intensive equilibrium, if the equilibrium function computed at  $e = \frac{\lambda(1-e_{+1}^*)}{\lambda+(1-\lambda)e_{+1}^*}$  is negative, or an effort intensive equilibrium if the equilibrium function computed at  $e = \frac{\lambda(1-e_{+1}^*)}{\lambda+(1-\lambda)e_{+1}^*}$  is positive. This holds for given beliefs about next period effort level  $e_{+1}^*$ . Assuming that beliefs are stationary, i.e., that  $e^* = e_{+1}^*$  so that, in a Perfect Bayesian Equilibrium  $e = e^* = e_{+1}^*$ , changes little. Also in this case, the equilibrium function is continuous, the equilibrium is unique and it is either a talent or an effort intensive equilibrium. This can be verified by setting  $e_{+1}^* = e$  in the equilibrium function. ■

**Proof of Proposition 5.** I denote with a hat all variables after entry occurs. Therefore  $\hat{\theta}^*$  is the threshold  $\theta^*$  after entry occurred. I firstly illustrate how the equilibrium changes in period 0, even though this has no effect on the incentives to exert effort. In equilibrium, in period 0

$$\begin{aligned} [\lambda\gamma + (1-\lambda)\gamma e_{-1}]Q_{-1} &= \bar{\theta} - \hat{\theta}^* \\ \hat{Q} &= \hat{\theta}^* - \hat{\theta}^{**} \\ [\lambda(1-\gamma) + (1-\lambda)(1-\gamma e_{-1})]Q_{-1} &= \hat{\theta}^{**} - \hat{\theta}^{***} \\ M-1 &= \hat{\theta}^{***} - \underline{\theta} \end{aligned}$$

It can be seen that  $\hat{\theta}^* = \theta^*$ , so that there is no change after entry, in the current period. However, this implies that  $\hat{\theta}^{**}$  is now lower. In fact  $\hat{\theta}^*$  is unchanged, while there is now measure  $\hat{Q} > Q$  entrants, and  $\hat{\theta}^{**} = \hat{\theta}^* - \hat{Q}$  implies that  $\hat{\theta}^{**}$  is lower. This also implies that  $\hat{\theta}^{***}$  is reduced. This, however, has no effect on the incentives to exert effort because the latter depend upon the equilibrium fees for old experts in period +1. Entry in period  $t$  implies that there will be a different

measure of experts in period +1. Successful experts will be

$$[\lambda\gamma + (1 - \lambda)\gamma\widehat{e}]\widehat{Q}$$

and unsuccessful experts

$$[\lambda(1 - \gamma) + (1 - \lambda)(1 - \gamma\widehat{e})]\widehat{Q}$$

and  $\widehat{e}$  is equilibrium effort level in period 0, when entry occurs. The measure of successful and unsuccessful experts depends both upon the measure of entrants,  $\widehat{Q}$ , and endogenously on the new equilibrium effort level  $\widehat{e}$ . The measure of entrants, successful and unsuccessful experts in period +1, determine the thresholds for indifference  $\widehat{\theta}_{+1}^*$ ,  $\widehat{\theta}_{+1}^{**}$ ,  $\widehat{\theta}_{+1}^{***}$  and the equilibrium fees in that period. The latter are critical for effort exertion in the previous period. In period +1 thresholds are given by

$$\begin{aligned} [\lambda\gamma + (1 - \lambda)\gamma\widehat{e}]\widehat{Q} &= \bar{\theta} - \widehat{\theta}_{+1}^* \\ \widehat{Q} &= \widehat{\theta}_{+1}^* - \widehat{\theta}_{+1}^{**} \\ [\lambda(1 - \gamma) + (1 - \lambda)(1 - \gamma\widehat{e})]\widehat{Q} &= \widehat{\theta}_{+1}^{**} - \widehat{\theta}_{+1}^{***} \\ M - 1 &= \widehat{\theta}_{+1}^{***} - \underline{\theta} \end{aligned}$$

The first order condition for effort exertion is

$$\frac{\partial c(e_t)}{\partial e_t} = \gamma(1 - \lambda)[F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)] \quad (6)$$

where

$$\begin{aligned} F_{+1}(\lambda_{+1}^+) &= [V(\lambda_{+1}^+; \theta_{+1}^*, 0) - V(\lambda_{+1}, \theta_{+1}^*, e_{+1}^*)] + \\ &[V(\lambda_{+1}, \theta_{+1}^{**}, e_{+1}^*) - V(\lambda_{+1}^-, \theta_{+1}^{**}, 0)] + V(\lambda_{+1}^-, \theta_{+1}^{***}, 0) \end{aligned}$$

and

$$F_{+1}(\lambda_{+1}^-) = V(\lambda_{+1}^-, \theta_{+1}^{***}, 0)$$

so that

$$\begin{aligned} F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-) &= \\ [V(\lambda_{+1}^+; \theta_{+1}^*, 0) - V(\lambda_{+1}, \theta_{+1}^*, e_{+1}^*)] + [V(\lambda_{+1}, \theta_{+1}^{**}, e_{+1}^*) - V(\lambda_{+1}^-, \theta_{+1}^{**}, 0)] &= \\ [\gamma\lambda_{+1}^+ - \gamma\lambda - (1 - \lambda)\gamma e_{+1}^*]U(\theta_{+1}^*) + [\gamma\lambda + (1 - \lambda)\gamma e_{+1}^* - \gamma\lambda_{+1}^-]U(\theta_{+1}^{**}) \end{aligned}$$

Moreover,  $\widehat{\theta}_{+1}^* = \bar{\theta} - [\lambda\gamma + (1 - \lambda)\gamma\widehat{e}]\widehat{Q}$  and  $\widehat{\theta}_{+1}^{**} = \widehat{\theta}_{+1}^* - \widehat{Q} = \bar{\theta} - [\lambda\gamma + (1 - \lambda)\gamma\widehat{e}]\widehat{Q} - \widehat{Q}$ .

In a Perfect Bayesian Equilibrium,  $e = e^*$ . Thus, the equilibrium effort level satisfies the following equation:

$$\gamma(1-\lambda)[\gamma\lambda_{+1}^+ - \gamma\lambda - (1-\lambda)\gamma e_{+1}^*]U(\theta_{+1}^*) + [\gamma\lambda + (1-\lambda)\gamma e_{+1}^* - \gamma\lambda_{+1}^-]U(\theta_{+1}^{**}) - \frac{\partial c(e_t)}{\partial e_t} = 0$$

or

$$\gamma(1-\lambda)\left\{\gamma\frac{\lambda\gamma}{\lambda\gamma + (1-\lambda)\gamma e} - \gamma\lambda - (1-\lambda)\gamma e_{+1}^*\right\}U(\theta_{+1}^*(e, Q)) + [\gamma\lambda + (1-\lambda)\gamma e_{+1}^* - \gamma\frac{\lambda(1-\gamma)}{\lambda(1-\gamma) + (1-\lambda)(1-\gamma e)}]U(\theta_{+1}^{**}(e, Q)) - \frac{\partial c(e_t)}{\partial e_t} = 0 \quad (7)$$

where the notation underlines the fact that  $\theta_{+1}^*$  and  $\theta_{+1}^{**}$  are functions of  $e$  and  $Q$ . As long as the condition for the existence of a talent intensive equilibrium,  $\gamma\lambda_{+1}^+ > \gamma\lambda_{+1} + (1-\lambda)\gamma e_{+1}^*$  is satisfied, the implicit function (7) is continuously differentiable from the results proved in Lemma 4 (it will be shown below that if a talent intensive equilibrium exists before entry occurs, it is going to exist also after entry occurs). Thus, applying the implicit function theorem to equation (7) yields the effect of entry on equilibrium effort:

$$\frac{de}{dQ} = \frac{\partial \frac{\gamma(1-\lambda)[F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]}{\partial \theta} \frac{d\theta}{dQ}}{-\partial \frac{\gamma(1-\lambda)[F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]}{\partial e} + \frac{\partial^2 c(e)}{\partial e^2}}$$

where

$$\begin{aligned} & \partial \frac{\gamma(1-\lambda)[F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]}{\partial e} = \\ & \gamma(1-\lambda)\left\{\gamma\frac{\partial \lambda_{+1}^+}{\partial e} - \gamma\frac{\partial \lambda_{+1}^-}{\partial e} + \right. \\ & \left. [\gamma\lambda_{+1}^+ - \gamma\lambda - (1-\lambda)\gamma e_{+1}^*]\frac{\partial U(\theta_{+1}^*(e, Q))}{\partial \theta_{+1}^*} \frac{d\theta_{+1}^*}{de} + [\gamma\lambda + (1-\lambda)\gamma e_{+1}^* - \gamma\lambda_{+1}^-]\frac{\partial U(\theta_{+1}^{**}(e, Q))}{\partial \theta_{+1}^{**}} \frac{d\theta_{+1}^{**}}{de}\right\} \end{aligned}$$

which is always negative as  $\frac{\partial \lambda_{+1}^+}{\partial e} < 0$ ,  $\frac{\partial \lambda_{+1}^-}{\partial e} > 0$ ,  $\frac{d\theta_{+1}^*}{de} < 0$ ,  $\frac{d\theta_{+1}^{**}}{de} < 0$ . As  $\frac{\partial^2 c(e)}{\partial e^2} > 0$  by convexity of the cost function, then the denominator is always positive.

Thus:

$$\text{sign}\left\{\frac{de}{dQ}\right\} = \text{sign}\left\{\partial \frac{\gamma(1-\lambda)[F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]}{\partial \theta} \frac{d\theta}{dQ}\right\}$$

From the proof of proposition 2, the difference

$$F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-) = [\gamma\lambda_{+1}^+ - \gamma\lambda - (1-\lambda)\gamma e_{+1}^*]U(\theta_{+1}^*) + [\gamma\lambda + (1-\lambda)\gamma e_{+1}^* - \gamma\lambda_{+1}^-]U(\theta_{+1}^{**})$$

increases in  $\theta$ , due to the conditions on the sorting of clients,  $\gamma\lambda_{+1}^+ > \gamma\lambda + (1-\lambda)\gamma e_{+1}^*$ <sup>38</sup>. Then, the result can be proved by showing that entry reduces threshold types. First of all, consider  $\widehat{\theta}_{+1}^*$ . This is given by  $\bar{\theta} - [\lambda\gamma + (1-\lambda)\gamma\widehat{e}^*]\widehat{Q}$ , and it decreases as  $Q$  rises. In particular, as  $\widehat{Q} > Q$ , then  $\widehat{\theta}_{+1}^* < \theta_{+1}^*$ . The same applies to  $\widehat{\theta}_{+1}^{**}$  as  $\widehat{\theta}_{+1}^{**} = \widehat{\theta}_{+1}^* - Q = \bar{\theta} - [\lambda\gamma + (1-\lambda)\gamma\widehat{e}^*]\widehat{Q} - \widehat{Q}$ . Thus, the increase in the measure of entrants reduces threshold types. Then, this implies that  $\frac{de}{dQ} < 0$ , so that equilibrium effort decreases as entry of new experts is larger. A last point to notice is that the change in equilibrium effort exertion could in principle lead to a violation of the condition  $\gamma\lambda_{+1}^+ > \gamma\lambda_{+1} + (1-\lambda)\gamma e_{+1}^*$ . However,  $\lambda^+$  decreases in  $e^*$ . Therefore, if the initial equilibrium satisfied the condition for the existence of a talent intensive equilibrium, then the condition is surely satisfied after entry occurs, as after entry, equilibrium effort is lower.

If one further assumes that beliefs are stationary,  $e = e^* = e_{+1}^*$  (effort in period +1 depends upon market conditions in period +2)<sup>39</sup>, all results are the same. as it can be verified by setting  $e_{+1}^* = e$ . ■

**Proof of Proposition 6.** The previous proposition showed that, when entry occurs in the current period:

$$\widehat{\theta}^* = \theta^*, \widehat{\theta}^{**} < \theta^{**}, \widehat{\theta}^{***} < \theta^{***}$$

so that  $\theta^*$  is constant, while  $\theta^{**}$  and  $\theta^{***}$  are both reduced. Equilibrium fees are as follows

$$\widehat{F}(\lambda^-) = V(\lambda^-, \widehat{\theta}^{***}, 0) < F(\lambda^-) = V(\lambda^-, \theta^{***}, 0)$$

in fact,  $V(\lambda^-, \widehat{\theta}^{***}, 0)$  is lower than before entry, as  $\widehat{\theta}^{***} < \theta^{***}$ . Then,

$$\begin{aligned} \widehat{F}(\lambda) &= V(\lambda, \widehat{\theta}^{**}, \widehat{e}^*) - V(\lambda^-, \widehat{\theta}^{**}, 0) + V(\lambda^-, \widehat{\theta}^{***}, 0) < \\ F(\lambda) &= [V(\lambda, \theta^{**}, e^*) - V(\lambda^-, \theta^{**}, 0)] + V(\lambda^-, \theta^{***}, 0) \end{aligned}$$

as  $\widehat{F}(\lambda)$  is reduced because the difference  $V(\lambda, \theta^{**}, \widehat{e}^*) - V(\lambda^-, \theta^{**}, 0)$  increases in  $\theta^{**}$ , but after

<sup>38</sup>Notice that, in principle, as beliefs about  $e_{+1}^*$  are set freely, it could be that the equilibrium in the second period lies in the effort intensive case. In such a case, the premium for being successful is increasing in threshold types, too, as shown in Proposition 7 below.

<sup>39</sup>Notice that in period +2, which is the relevant for the incentives to exert effort in period +1, parameters of the problem (including the total measure of experts in the market which is  $2\widehat{Q}$ ) are the same as in period +1, which is the relevant for the incentives to exert effort in the current period, thus in a stationary equilibrium  $e_{+1}^* = e^* = e$ .



entry  $\widehat{\theta}^{**} < \theta^{**}$ , and both  $\widehat{e}^*$  and  $V(\lambda^-, \widehat{\theta}^{***}, 0)$  are lower. Finally,

$$\begin{aligned}\widehat{F}(\lambda^+) &= [V(\lambda^+, \widehat{\theta}^*, 0) - V(\lambda, \widehat{\theta}^*, \widehat{e}^*)] + [V(\lambda, \widehat{\theta}^{**}, \widehat{e}^*) - V(\lambda^-, \widehat{\theta}^{**}, 0)] + V(\lambda^-, \widehat{\theta}^{***}, 0) \leq \\ F(\lambda^+) &= [V(\lambda^+, \theta^*, 0) - V(\lambda, \theta^*, e^*)] + [V(\lambda, \theta^{**}, e^*) - V(\lambda^-, \theta^{**}, 0)] + V(\lambda^-, \theta^{***}, 0)\end{aligned}$$

as  $\widehat{F}(\lambda^+)$  moves in an ambiguous way. In fact, the terms  $[V(\lambda, \widehat{\theta}^{**}, \widehat{e}^*) - V(\lambda^-, \widehat{\theta}^{**}, 0)] + V(\lambda^-, \widehat{\theta}^{***}, 0)$  are reduced, while  $[V(\lambda^+, \widehat{\theta}^*, 0) - V(\lambda, \widehat{\theta}^*, \widehat{e}^*)]$  raises as  $\widehat{\theta}^*$  is unchanged, and  $\widehat{e}^*$  is lower, so that  $V(\lambda, \widehat{\theta}^*, \widehat{e}^*)$  is lower. Fees in the future period will all be lower. This follows from the fact that the premium for being successful,  $[V(\lambda^+, \widehat{\theta}_{+1}^*, 0) - V(\lambda, \widehat{\theta}_{+1}^*, \widehat{e}_{+1}^*)] + [V(\lambda, \widehat{\theta}_{+1}^{**}, \widehat{e}_{+1}^*) - V(\lambda^-, \widehat{\theta}_{+1}^{**}, 0)]$ , is lower in equilibrium as proved in proposition 5 (due to the lower threshold types for given equilibrium effort). If beliefs about effort are stationary, the premium for being successful is still lower, despite lower equilibrium effort in period +1. ■

**Proof of Proposition 7.** The proof is analogous to that of proposition 5. The premium for successful experts is given by

$$\begin{aligned}F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-) &= V(\lambda_{+1}^+, \theta_{+1}^{**}, 0) - V(\lambda_{+1}^-, \theta_{+1}^{**}, 0) = \\ &(\gamma\lambda_{+1}^+ - \gamma\lambda_{+1}^-)U(\theta_{+1}^{**})\end{aligned}$$

Increased entry reduces threshold types, and as valuation is increasing in  $\theta$ , the premium for being successful drops, exactly as in the talent intensive case analyzed in Proposition 5. Here, however, if equilibrium effort decreases too much, the condition ensuring the existence of an equilibrium in the effort intensive case could be violated. When this happens, the implicit function theorem cannot be applied as the equilibrium function is continuous but it is not differentiable at  $e = \frac{\lambda(1-e_{+1}^*)}{\lambda+(1-\lambda)e_{+1}^*}$ . However, in such a case, clients anticipate the new equilibrium effort and sort accordingly as in the talent intensive case. As showed in Lemma 4, either a talent intensive, or an effort intensive equilibrium exists. If entry reduces incentives so much that the equilibrium effort will be talent intensive, then clearly equilibrium effort will be lower after entry occurs. ■

**Proof of Proposition 8.** As a first step, it will be shown that the proposed equilibrium can be an equilibrium, indeed. The second step is proving that equilibria with different sorting of clients do not exist. The first point to notice is that the value of a new entrant is decreasing in clients' types. Thus, for types  $\theta < \theta^*$ ,  $\lambda\gamma k + (1-\lambda)\gamma e^* z(\theta) - F(\lambda) > \lambda\gamma k + (1-\lambda)\gamma e^* z(\theta^*) - F(\lambda)$  as  $\theta < \theta^*$  and  $z$  is decreasing in  $\theta$ . For the same reason, types  $\theta > \theta^*$  are worse off if they pay the fee  $F(\lambda)$ , as the expected value they get is lower. Then, all such clients with type  $\theta > \theta^*$  get the service from the other available experts. However, the value of other experts is the same for all clients, irrespective of type. Therefore, competition of clients raises fees to the point in which they are indifferent between getting the service from old successful, from old unsuccessful, or getting no

service. Therefore,  $\lambda^+ \gamma k - F(\lambda^+) = \lambda^- \gamma k - F(\lambda^-) = 0$ , as the outside option has been normalized to zero. Then, no client has incentives to deviate and it can be proved that other equilibria are not feasible. First of all, there cannot be full negative sorting (higher types getting the service from old unsuccessful experts) as all clients are willing to pay the same for higher types and competition among clients would drive fees up to the expected value of old successful experts. Lower types can instead obtain rents by getting the service from new entrants. There cannot exist equilibria where highest types get the service from old successful experts, lower types from new entrants and intermediate types either get the service from old unsuccessful, or prefer the outside option. In such a case, both higher and intermediate types have the same willingness to pay for the service of old successful experts and competition raises fees to the expected value of obtaining the service from old successful experts. Other equilibrium configurations can be ruled out following the same logic. This is true in any period  $t$ . Therefore, first order conditions for effort exertion in period 0 are:

$$\gamma(1 - \lambda)E(z(\theta | \theta \leq \theta^*)) [F(\lambda_{+1}^+) - F(\lambda_{+1}^-)] - \frac{\partial c(e)}{\partial e} = 0$$

or

$$\gamma(1 - \lambda)E(z(\theta | \theta \leq \theta^*)) \left[ \gamma k \left( \frac{\gamma \lambda k}{\gamma \lambda k + \gamma(1 - \lambda)e^* E[z(\theta) | \theta \leq \theta^*]} - \frac{\lambda(1 - \gamma k)}{\lambda(1 - \gamma k) + (1 - \lambda)(1 - \gamma e^* E[z(\theta) | \theta \leq \theta^*])} \right) \right] - \frac{\partial c(e)}{\partial e} = 0 \quad (8)$$

it can be seen that equilibrium effort is unique. In a Perfect Bayesian equilibrium,  $e = e^*$ , then substituting  $e^* = e$  in eq. (8), for  $e = 0$  the expression is strictly positive as  $\lambda^+ > \lambda^-$  and  $\frac{\partial c(e)}{\partial e} |_{e=0} = 0$ . The difference  $\lambda^+ - \lambda^-$  is strictly decreasing in  $e$ , and so is  $-\frac{\partial c(e)}{\partial e}$ , so the equilibrium function is monotonic. It is also continuous by continuity of  $\lambda^+, \lambda^-$  and  $c$ . Finally, when  $e = 1$ , the function is negative, as  $\frac{\partial c(e)}{\partial e} |_{e=1} = +\infty$ . Therefore, there exists a value of  $e$  which satisfies the first order condition, and such value is unique. Notice that next period effort does not show up in the first order condition, so that the analysis is much simpler as current period effort does not depend upon beliefs about next period effort. ■

**Proof of Proposition 9.** First of all, the proof of Proposition 8 showed that the equilibrium function is continuous in  $e$ , and that the equilibrium is unique. Thus, applying the implicit function theorem yields

$$\frac{de}{dQ} = \frac{\partial \frac{\gamma(1-\lambda)E(z(\theta|\theta \leq \theta^*)) [F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]}{\partial \theta} \frac{d\theta}{dQ}}{-\partial \frac{\gamma(1-\lambda)E(z(\theta|\theta \leq \theta^*)) [F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]}{\partial e} + \frac{\partial^2 c(e)}{\partial e^2}}$$

as  $\partial \frac{\gamma(1-\lambda)[F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]}{\partial e} < 0$ , and  $\frac{\partial^2 c(e)}{\partial e^2} > 0$ , the denominator is positive. Therefore

$$\text{sign}\left\{\frac{de}{dQ}\right\} = \text{sign}\left\{\partial \frac{\gamma(1-\lambda)E(z(\theta | \theta \leq \theta^*)[F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)])}{\partial \theta} \frac{d\theta}{dQ}\right\}$$

as  $Q$  raises, so does  $\theta^*$ . Then,

$$\begin{aligned} & \frac{\gamma(1-\lambda)E(z(\theta | \theta \leq \theta^*)[F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)])}{\partial \theta^*} = \\ & \gamma(1-\lambda) \frac{\partial E[z(\theta) | \theta \leq \theta^*]}{\partial \theta^*} [\gamma k(\lambda^+ - \lambda^-)] + \\ & \gamma(1-\lambda)E(z(\theta | \theta \leq \theta^*)[\gamma k(\frac{\partial \lambda_{+1}^+}{\partial \theta^*} - \frac{\partial \lambda_{+1}^-}{\partial \theta^*})]) = \\ & \gamma^2 k(1-\lambda) \left[ \frac{\partial E[z(\theta) | \theta \leq \theta^*]}{\partial \theta^*} (\lambda^+ - \lambda^-) + E(z(\theta | \theta \leq \theta^*)) (\frac{\partial \lambda_{+1}^+}{\partial \theta^*} - \frac{\partial \lambda_{+1}^-}{\partial \theta^*}) \right] \end{aligned}$$

where the first term

$$\frac{\partial E[z(\theta) | \theta \leq \theta^*]}{\partial \theta^*} (\lambda^+ - \lambda^-)$$

is negative, as the probability of a success is lower when the threshold type moves up (this is the *complexity channel*). The second term

$$E(z(\theta | \theta \leq \theta^*)) (\frac{\partial \lambda_{+1}^+}{\partial \theta^*} - \frac{\partial \lambda_{+1}^-}{\partial \theta^*})$$

is positive, as the signalling value of a success is higher when the threshold type is higher (this is the *signalling channel*). In particular, the effect of entry on equilibrium effort is positive as long as

$$\frac{\frac{\partial \lambda_{+1}^+}{\partial \theta^*} - \frac{\partial \lambda_{+1}^-}{\partial \theta^*}}{\lambda^+ - \lambda^-} > - \frac{\frac{\partial E[z(\theta) | \theta \leq \theta^*]}{\partial \theta^*}}{E(z(\theta | \theta \leq \theta^*))}$$

or, recalling that  $F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-) = \gamma k(\lambda_{+1}^+ - \lambda_{+1}^-)$ , as long as

$$\frac{\frac{\partial [F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]}{\partial \theta^*}}{F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)} > - \frac{\frac{\partial E[z(\theta) | \theta \leq \theta^*]}{\partial \theta^*}}{E(z(\theta | \theta \leq \theta^*))}$$

■

**Proof of Proposition 10.** Proposition 8 showed that in equilibrium,  $\lambda \gamma k + (1-\lambda) \gamma e^* z(\theta^*) - F(\lambda) = \lambda^+ \gamma k - F(\lambda^+) = \lambda^- \gamma k - F(\lambda^-) = 0$ . Then,  $F(\lambda) = \lambda \gamma k + (1-\lambda) \gamma e^* z(\theta^*)$ . As entry raises the threshold type,  $\hat{\theta}^* > \theta^*$ , this tends to decrease  $F(\lambda)$ , as the probability of success,  $z(\theta)$  is lower, and thus the expected value of the service provided by new entrants. On the other hand, equilibrium

effort may be higher, and this tends to increase equilibrium effort. The same reasoning holds for  $F_{+1}(\lambda)$ . The increase in threshold type  $\theta^*$  raises  $\lambda_{+1}^+$  and lowers  $\lambda_{+1}^-$ . This in turn is reflected in a higher  $F_{+1}(\lambda^+)$  and a lower  $F_{+1}(\lambda^-)$ . ■

**Proof of Proposition 11.** The proposition can be proved by investigating the sign of  $\frac{de}{dQ}$ , the effect of a change in the measure of new entrants  $Q$  on effort. The equilibrium function is continuous from the proof of lemma 4 and proposition 8 and from the fact that  $z$  is continuous and monotonic. The implicit function theorem applied to the first order condition for effort exertion yields:

$$\frac{de}{dQ} = \frac{\frac{\partial \gamma(1-\lambda)E[z(\theta)|\theta^{**} < \theta < \theta^*][F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]}{\partial \theta} \frac{d\theta}{dQ}}{-\frac{\partial \gamma(1-\lambda)E[z(\theta)|\theta^{**} < \theta < \theta^*][F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]}{\partial e} + \frac{\partial^2 c(e)}{\partial e^2}}$$

and as  $-\frac{\partial \gamma(1-\lambda)E[z(\theta)|\theta^{**} < \theta < \theta^*][F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]}{\partial e} + \frac{\partial^2 c(e)}{\partial e^2} > 0$ , and  $\frac{d\theta^{**}}{dQ} < 0$ ,  $\frac{d\theta_{+1}^*}{dQ} < 0$ ,  $\frac{d\theta_{+1}^{**}}{dQ} < 0$  as shown in the proof of proposition 5,  $sign\{\frac{de}{dQ}\} = sign\{\frac{\partial \gamma(1-\lambda)E[z(\theta)|\theta^{**} < \theta < \theta^*][F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]}{\partial \theta}\}$ . Then, the proposition follows from the analysis of the change in the left hand side of the first order condition for effort exertion as threshold types change, performed in the text. Term (1) raises as  $\theta^{**}$  decrease, while terms (2), (3), (4) decrease as  $\theta^{**}$ ,  $\theta_{+1}^*$  and  $\theta_{+1}^{**}$  drop. If term (1), the *complexity channel*, dominates the other terms, the *signalling* and the *direct channel*, there can exist an equilibrium where effort is higher after an increase in entry occurs; otherwise, equilibrium effort is lower. This condition can be rewritten in terms of elasticities as follows

$$sign\left\{\frac{\partial \gamma(1-\lambda)E[z(\theta) | \theta^{**} < \theta < \theta^*][F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]}{\partial \theta}\right\} = sign\left\{\frac{\frac{\partial E[z(\theta)|\theta^{**} < \theta < \theta^*]}{\partial \theta}}{E[z(\theta) | \theta^{**} < \theta < \theta^*]} + \frac{\frac{\partial [F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]}{\partial \theta}}{[F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]}\right\}$$

this is positive as long as

$$\left| \frac{\frac{\partial E[z(\theta)|\theta^{**} < \theta < \theta^*]}{\partial \theta}}{E[z(\theta) | \theta^{**} < \theta < \theta^*]} \right| > \left| \frac{\frac{\partial [F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]}{\partial \theta}}{[F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]} \right|$$

where the derivative  $\frac{\partial [F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]}{\partial \theta}$  is taken over the vector of threshold  $\theta^{**}, \theta_{+1}^*, \theta_{+1}^{**}$ , thus this represents an elasticity with respect to a vector of thresholds. Recall that  $\frac{\partial E[z(\theta)|\theta^{**} < \theta < \theta^*]}{\partial \theta} < 0$ , while  $\frac{\partial [F_{+1}(\lambda_{+1}^+) - F_{+1}(\lambda_{+1}^-)]}{\partial \theta} > 0$ , and that threshold types are lower in equilibrium.

To complete the proof, conditions to ensure the existence of a talent intensive equilibrium must be distinguished:

I) equilibrium effort can be higher after entry occurs. In the talent intensive case, this may

in principle destroy the condition for the existence of a talent intensive equilibrium. Then, if the new effort level is so large as to be above the thresholds ensuring the existence of a talent intensive equilibrium, then an effort intensive equilibrium exists, in which case effort is larger. This is analogous to the reasoning in the proof of proposition 7 (now the direction of the effect is different, though, as effort is increasing).

II) equilibrium equilibrium can be lower after entry occurs. In this case, if a talent intensive equilibrium existed before entry occurred, it will surely exist now as effort is reduced after entry occurs. ■

**Proof of Proposition 12.** Entry increases incentives to exert effort when the complexity channel dominates the direct and signalling channels. Threshold types go down and this depresses  $F_{+1}(\lambda_{+1}^-)$ . For the same reason  $F_{+1}(\lambda_{+1}^+)$  is lower. Incentives to exert effort may still be larger because by exerting effort there are larger chances of obtaining a (smaller) premium for being successful because sorting increases the likelihood of a success. When entry leads to weaker incentives to exert effort, the reasoning is the same. ■

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- W. Fuchs and Lippi F., *Monetary union with voluntary participation*, Review of Economic Studies, Vol. 73, pp. 437-457 **TD No. 512 (July 2004)**.
- E. Gaiotti and A. Secchi, *Is there a cost channel of monetary transmission? An investigation into the pricing behaviour of 2000 firms*, Journal of Money, Credit and Banking, Vol. 38, 8, pp. 2013-2038 **TD No. 525 (December 2004)**.
- A. Brandolini, P. Cipollone and E. Viviano, *Does the ILO definition capture all unemployment?*, Journal of the European Economic Association, Vol. 4, 1, pp. 153-179, **TD No. 529 (December 2004)**.
- A. Brandolini, L. Cannari, G. D'Alessio and I. Faiella, *Household wealth distribution in Italy in the 1990s*, in E. N. Wolff (ed.) *International Perspectives on Household Wealth*, Cheltenham, Edward Elgar, **TD No. 530 (December 2004)**.
- P. Del Giovane and R. Sabbatini, *Perceived and measured inflation after the launch of the Euro: Explaining the gap in Italy*, Giornale degli economisti e annali di economia, Vol. 65, 2, pp. 155-192, **TD No. 532 (December 2004)**.
- M. Caruso, *Monetary policy impulses, local output and the transmission mechanism*, Giornale degli economisti e annali di economia, Vol. 65, 1, pp. 1-30, **TD No. 537 (December 2004)**.
- L. Guiso and M. Paiella, *The role of risk aversion in predicting individual behavior*, In P. A. Chiappori e C. Gollier (eds.) *Competitive Failures in Insurance Markets: Theory and Policy Implications*, Monaco, CESifo, **TD No. 546 (February 2005)**.
- G. M. Tomat, *Prices product differentiation and quality measurement: A comparison between hedonic and matched model methods*, Research in Economics, Vol. 60, 1, pp. 54-68, **TD No. 547 (February 2005)**.
- L. Guiso, M. Paiella and I. Visco, *Do capital gains affect consumption? Estimates of wealth effects from Italian household's behavior*, in L. Klein (ed), *Long Run Growth and Short Run Stabilization: Essays in Memory of Albert Ando (1929-2002)*, Cheltenham, Elgar, **TD No. 555 (June 2005)**.
- F. Busetti, S. Fabiani and A. Harvey, *Convergence of prices and rates of inflation*, Oxford Bulletin of Economics and Statistics, Vol. 68, 1, pp. 863-878, **TD No. 575 (February 2006)**.
- M. Caruso, *Stock market fluctuations and money demand in Italy, 1913 - 2003*, Economic Notes, Vol. 35, 1, pp. 1-47, **TD No. 576 (February 2006)**.
- R. Bronzini and G. De Blasio, *Evaluating the impact of investment incentives: The case of Italy's Law 488/92*. Journal of Urban Economics, Vol. 60, 2, pp. 327-349, **TD No. 582 (March 2006)**.
- R. Bronzini and G. De Blasio, *Una valutazione degli incentivi pubblici agli investimenti*, Rivista Italiana degli Economisti, Vol. 11, 3, pp. 331-362, **TD No. 582 (March 2006)**.
- A. Di Cesare, *Do market-based indicators anticipate rating agencies? Evidence for international banks*, Economic Notes, Vol. 35, pp. 121-150, **TD No. 593 (May 2006)**.
- R. Golinelli and S. Momigliano, *Real-time determinants of fiscal policies in the euro area*, Journal of Policy Modeling, Vol. 28, 9, pp. 943-964, **TD No. 609 (December 2006)**.

- S. SIVIERO and D. TERLIZZESE, *Macroeconomic forecasting: Debunking a few old wives' tales*, Journal of Business Cycle Measurement and Analysis, v. 3, 3, pp. 287-316, **TD No. 395 (February 2001)**.
- S. MAGRI, *Italian households' debt: The participation to the debt market and the size of the loan*, Empirical Economics, v. 33, 3, pp. 401-426, **TD No. 454 (October 2002)**.
- L. CASOLARO and G. GOBBI, *Information technology and productivity changes in the banking industry*, Economic Notes, Vol. 36, 1, pp. 43-76, **TD No. 489 (March 2004)**.
- G. FERRERO, *Monetary policy, learning and the speed of convergence*, Journal of Economic Dynamics and Control, v. 31, 9, pp. 3006-3041, **TD No. 499 (June 2004)**.
- M. PAIELLA, *Does wealth affect consumption? Evidence for Italy*, Journal of Macroeconomics, Vol. 29, 1, pp. 189-205, **TD No. 510 (July 2004)**.
- F. LIPPI and S. NERI, *Information variables for monetary policy in a small structural model of the euro area*, Journal of Monetary Economics, Vol. 54, 4, pp. 1256-1270, **TD No. 511 (July 2004)**.
- A. ANZUINI and A. LEVY, *Monetary policy shocks in the new EU members: A VAR approach*, Applied Economics, Vol. 39, 9, pp. 1147-1161, **TD No. 514 (July 2004)**.
- D. JR. MARCHETTI and F. Nucci, *Pricing behavior and the response of hours to productivity shocks*, Journal of Money Credit and Banking, v. 39, 7, pp. 1587-1611, **TD No. 524 (December 2004)**.
- R. BRONZINI, *FDI Inflows, agglomeration and host country firms' size: Evidence from Italy*, Regional Studies, Vol. 41, 7, pp. 963-978, **TD No. 526 (December 2004)**.
- L. MONTEFORTE, *Aggregation bias in macro models: Does it matter for the euro area?*, Economic Modelling, 24, pp. 236-261, **TD No. 534 (December 2004)**.
- A. NOBILI, *Assessing the predictive power of financial spreads in the euro area: does parameters instability matter?*, Empirical Economics, Vol. 31, 1, pp. 177-195, **TD No. 544 (February 2005)**.
- A. DALMAZZO and G. DE BLASIO, *Production and consumption externalities of human capital: An empirical study for Italy*, Journal of Population Economics, Vol. 20, 2, pp. 359-382, **TD No. 554 (June 2005)**.
- M. BUGAMELLI and R. TEDESCHI, *Le strategie di prezzo delle imprese esportatrici italiane*, Politica Economica, v. 23, 3, pp. 321-350, **TD No. 563 (November 2005)**.
- L. GAMBACORTA and S. IANNOTTI, *Are there asymmetries in the response of bank interest rates to monetary shocks?*, Applied Economics, v. 39, 19, pp. 2503-2517, **TD No. 566 (November 2005)**.
- P. ANGELINI and F. LIPPI, *Did prices really soar after the euro cash changeover? Evidence from ATM withdrawals*, International Journal of Central Banking, Vol. 3, 4, pp. 1-22, **TD No. 581 (March 2006)**.
- A. LOCARNO, *Imperfect knowledge, adaptive learning and the bias against activist monetary policies*, International Journal of Central Banking, v. 3, 3, pp. 47-85, **TD No. 590 (May 2006)**.
- F. LOTTI and J. MARCUCCI, *Revisiting the empirical evidence on firms' money demand*, Journal of Economics and Business, Vol. 59, 1, pp. 51-73, **TD No. 595 (May 2006)**.
- P. CIPOLLONE and A. ROSOLIA, *Social interactions in high school: Lessons from an earthquake*, American Economic Review, Vol. 97, 3, pp. 948-965, **TD No. 596 (September 2006)**.
- L. DEDOLA and S. NERI, *What does a technology shock do? A VAR analysis with model-based sign restrictions*, Journal of Monetary Economics, Vol. 54, 2, pp. 512-549, **TD No. 607 (December 2006)**.
- F. VERGARA CAFFARELLI, *Merge and compete: strategic incentives for vertical integration*, Rivista di politica economica, v. 97, 9-10, serie 3, pp. 203-243, **TD No. 608 (December 2006)**.
- A. BRANDOLINI, *Measurement of income distribution in supranational entities: The case of the European Union*, in S. P. Jenkins e J. Micklewright (eds.), *Inequality and Poverty Re-examined*, Oxford, Oxford University Press, **TD No. 623 (April 2007)**.
- M. PAIELLA, *The foregone gains of incomplete portfolios*, Review of Financial Studies, Vol. 20, 5, pp. 1623-1646, **TD No. 625 (April 2007)**.
- K. BEHRENS, A. R. LAMORGESE, G.I.P. OTTAVIANO and T. TABUCHI, *Changes in transport and non transport costs: local vs. global impacts in a spatial network*, Regional Science and Urban Economics, Vol. 37, 6, pp. 625-648, **TD No. 628 (April 2007)**.
- M. BUGAMELLI, *Prezzi delle esportazioni, qualità dei prodotti e caratteristiche di impresa: analisi su un campione di imprese italiane*, v. 34, 3, pp. 71-103, *Economia e Politica Industriale*, **TD No. 634 (June 2007)**.
- G. ASCARI and T. ROPELE, *Optimal monetary policy under low trend inflation*, Journal of Monetary Economics, v. 54, 8, pp. 2568-2583, **TD No. 647 (November 2007)**.



- R. GIORDANO, S. MOMIGLIANO, S. NERI and R. PEROTTI, *The Effects of Fiscal Policy in Italy: Evidence from a VAR Model*, European Journal of Political Economy, Vol. 23, 3, pp. 707-733, **TD No. 656 (January 2008)**.
- B. ROFFIA and A. ZAGHINI, *Excess money growth and inflation dynamics*, International Finance, v. 10, 3, pp. 241-280, **TD No. 657 (January 2008)**.
- G. BARBIERI, P. CIPOLLONE and P. SESTITO, *Labour market for teachers: demographic characteristics and allocative mechanisms*, Giornale degli economisti e annali di economia, v. 66, 3, pp. 335-373, **TD No. 672 (June 2008)**.
- E. BREDA, R. CAPPARIELLO and R. ZIZZA, *Vertical specialisation in Europe: evidence from the import content of exports*, Rivista di politica economica, numero monografico, **TD No. 682 (August 2008)**.

2008

- P. ANGELINI, *Liquidity and announcement effects in the euro area*, Giornale degli Economisti e Annali di Economia, v. 67, 1, pp. 1-20, **TD No. 451 (October 2002)**.
- P. ANGELINI, P. DEL GIOVANE, S. SIVIERO and D. TERLIZZESE, *Monetary policy in a monetary union: What role for regional information?*, International Journal of Central Banking, v. 4, 3, pp. 1-28, **TD No. 457 (December 2002)**.
- F. SCHIVARDI and R. TORRINI, *Identifying the effects of firing restrictions through size-contingent Differences in regulation*, Labour Economics, v. 15, 3, pp. 482-511, **TD No. 504 (June 2004)**.
- L. GUIISO and M. PAIELLA, *Risk aversion, wealth and background risk*, Journal of the European Economic Association, v. 6, 6, pp. 1109-1150, **TD No. 483 (September 2003)**.
- C. BIANCOTTI, G. D'ALESSIO and A. NERI, *Measurement errors in the Bank of Italy's survey of household income and wealth*, Review of Income and Wealth, v. 54, 3, pp. 466-493, **TD No. 520 (October 2004)**.
- S. MOMIGLIANO, J. HENRY and P. HERNÁNDEZ DE COS, *The impact of government budget on prices: Evidence from macroeconomic models*, Journal of Policy Modelling, v. 30, 1, pp. 123-143 **TD No. 523 (October 2004)**.
- L. GAMBACORTA, *How do banks set interest rates?*, European Economic Review, v. 52, 5, pp. 792-819, **TD No. 542 (February 2005)**.
- P. ANGELINI and A. GENERALE, *On the evolution of firm size distributions*, American Economic Review, v. 98, 1, pp. 426-438, **TD No. 549 (June 2005)**.
- R. FELICI and M. PAGNINI, *Distance, bank heterogeneity and entry in local banking markets*, The Journal of Industrial Economics, v. 56, 3, pp. 500-534, **No. 557 (June 2005)**.
- S. DI ADDARIO and E. PATACCHINI, *Wages and the city. Evidence from Italy*, Labour Economics, v.15, 5, pp. 1040-1061, **TD No. 570 (January 2006)**.
- M. PERICOLI and M. TABOGA, *Canonical term-structure models with observable factors and the dynamics of bond risk premia*, Journal of Money, Credit and Banking, v. 40, 7, pp. 1471-88, **TD No. 580 (February 2006)**.
- E. VIVIANO, *Entry regulations and labour market outcomes. Evidence from the Italian retail trade sector*, Labour Economics, v. 15, 6, pp. 1200-1222, **TD No. 594 (May 2006)**.
- S. FEDERICO and G. A. MINERVA, *Outward FDI and local employment growth in Italy*, Review of World Economics, Journal of Money, Credit and Banking, v. 144, 2, pp. 295-324, **TD No. 613 (February 2007)**.
- F. Busetti and A. HARVEY, *Testing for trend*, Econometric Theory, v. 24, 1, pp. 72-87, **TD No. 614 (February 2007)**.
- V. CESTARI, P. DEL GIOVANE and C. ROSSI-ARNAUD, *Memory for prices and the Euro cash changeover: an analysis for cinema prices in Italy*, In P. Del Giovane e R. Sabbatini (eds.), The Euro Inflation and Consumers' Perceptions. Lessons from Italy, Berlin-Heidelberg, Springer, **TD No. 619 (February 2007)**.
- B. H. HALL, F. LOTTI and J. MAIRESSE, *Employment, innovation and productivity: evidence from Italian manufacturing microdata*, Industrial and Corporate Change, v. 17, 4, pp. 813-839, **TD No. 622 (April 2007)**.

- J. SOUSA and A. ZAGHINI, *Monetary policy shocks in the Euro Area and global liquidity spillovers*, International Journal of Finance and Economics, v.13, 3, pp. 205-218, **TD No. 629 (June 2007)**.
- M. DEL GATTO, GIANMARCO I. P. OTTAVIANO and M. PAGNINI, *Openness to trade and industry cost dispersion: Evidence from a panel of Italian firms*, Journal of Regional Science, v. 48, 1, pp. 97-129, **TD No. 635 (June 2007)**.
- P. DEL GIOVANE, S. FABIANI and R. SABBATINI, *What's behind "inflation perceptions"? A survey-based analysis of Italian consumers*, in P. Del Giovane e R. Sabbatini (eds.), *The Euro Inflation and Consumers' Perceptions. Lessons from Italy*, Berlin-Heidelberg, Springer, **TD No. 655 (January 2008)**.
- B. BORTOLOTTI, and P. PINOTTI, *Delayed privatization*, Public Choice, v. 136, 3-4, pp. 331-351, **TD No. 663 (April 2008)**.
- R. BONCI and F. COLUMBA, *Monetary policy effects: New evidence from the Italian flow of funds*, Applied Economics, v. 40, 21, pp. 2803-2818, **TD No. 678 (June 2008)**.
- M. CUCCULELLI, and G. MICUCCI, *Family Succession and firm performance: evidence from Italian family firms*, Journal of Corporate Finance, v. 14, 1, pp. 17-31, **TD No. 680 (June 2008)**.
- A. SILVESTRINI and D. VEREDAS, *Temporal aggregation of univariate and multivariate time series models: a survey*, Journal of Economic Surveys, v. 22, 3, pp. 458-497, **TD No. 685 (August 2008)**.

2009

- F. PANETTA, F. SCHIVARDI and M. SHUM, *Do mergers improve information? Evidence from the loan market*, Journal of Money, Credit, and Banking, v. 41, 4, pp. 673-709, **TD No. 521 (October 2004)**.
- P. PAGANO and M. PISANI, *Risk-adjusted forecasts of oil prices*, The B.E. Journal of Macroeconomics, v. 9, 1, Article 24, **TD No. 585 (March 2006)**.
- M. PERICOLI and M. SBRACIA, *The CAPM and the risk appetite index: theoretical differences, empirical similarities, and implementation problems*, International Finance, v. 12, 2, pp. 123-150, **TD No. 586 (March 2006)**.
- S. MAGRI, *The financing of small innovative firms: the Italian case*, Economics of Innovation and New Technology, v. 18, 2, pp. 181-204, **TD No. 640 (September 2007)**.
- S. MAGRI, *The financing of small entrepreneurs in Italy*, Annals of Finance, v. 5, 3-4, pp. 397-419, **TD No. 640 (September 2007)**.
- F. LORENZO, L. MONTEFORTE and L. SESSA, *The general equilibrium effects of fiscal policy: estimates for the euro area*, Journal of Public Economics, v. 93, 3-4, pp. 559-585, **TD No. 652 (November 2007)**.
- R. GOLINELLI and S. MOMIGLIANO, *The Cyclical Reaction of Fiscal Policies in the Euro Area. A Critical Survey of Empirical Research*, Fiscal Studies, v. 30, 1, pp. 39-72, **TD No. 654 (January 2008)**.
- P. DEL GIOVANE, S. FABIANI and R. SABBATINI, *What's behind "Inflation Perceptions"? A survey-based analysis of Italian consumers*, Giornale degli Economisti e Annali di Economia, v. 68, 1, pp. 25-52, **TD No. 655 (January 2008)**.
- F. MACCHERONI, M. MARINACCI, A. RUSTICHINI and M. TABOGA, *Portfolio selection with monotone mean-variance preferences*, Mathematical Finance, v. 19, 3, pp. 487-521, **TD No. 664 (April 2008)**.
- M. AFFINITO and M. PIAZZA, *What are borders made of? An analysis of barriers to European banking integration*, in P. Alessandrini, M. Fratianni and A. Zazzaro (eds.): *The Changing Geography of Banking and Finance*, Dordrecht Heidelberg London New York, Springer, **TD No. 666 (April 2008)**.
- L. ARCIERO, C. BIANCOTTI, L. D'AURIZIO and C. IMPENNA, *Exploring agent-based methods for the analysis of payment systems: A crisis model for StarLogo TNG*, Journal of Artificial Societies and Social Simulation, v. 12, 1, **TD No. 686 (August 2008)**.
- A. CALZA and A. ZAGHINI, *Nonlinearities in the dynamics of the euro area demand for M1*, Macroeconomic Dynamics, v. 13, 1, pp. 1-19, **TD No. 690 (September 2008)**.
- L. FRANCESCO and A. SECCHI, *Technological change and the households' demand for currency*, Journal of Monetary Economics, v. 56, 2, pp. 222-230, **TD No. 697 (December 2008)**.
- M. BUGAMELLI, F. SCHIVARDI and R. ZIZZA, *The euro and firm restructuring*, in A. Alesina e F. Giavazzi (eds): *Europe and the Euro*, Chicago, University of Chicago Press, **TD No. 716 (June 2009)**.
- B. HALL, F. LOTTI and J. MAIRESSE, *Innovation and productivity in SMEs: empirical evidence for Italy*, Small Business Economics, v. 33, 1, pp. 13-33, **TD No. 718 (June 2009)**.

*FORTHCOMING*

- L. MONTEFORTE and S. SIVIERO, *The Economic Consequences of Euro Area Modelling Shortcuts*, Applied Economics, **TD No. 458 (December 2002)**.
- M. BUGAMELLI and A. ROSOLIA, *Produttività e concorrenza estera*, Rivista di politica economica, **TD No. 578 (February 2006)**.
- G. DE BLASIO and G. NUZZO, *Historical traditions of civicness and local economic development*, Journal of Regional Science, **TD No. 591 (May 2006)**.
- R. BRONZINI and P. PISELLI, *Determinants of long-run regional productivity with geographical spillovers: the role of R&D, human capital and public infrastructure*, Regional Science and Urban Economics, **TD No. 597 (September 2006)**.
- E. IOSSA and G. PALUMBO, *Over-optimism and lender liability in the consumer credit market*, Oxford Economic Papers, **TD No. 598 (September 2006)**.
- U. ALBERTAZZI and L. GAMBACORTA, *Bank profitability and the business cycle*, Journal of Financial Stability, **TD No. 601 (September 2006)**.
- A. CIARLONE, P. PISELLI and G. TREBESCHI, *Emerging Markets' Spreads and Global Financial Conditions*, Journal of International Financial Markets, Institutions & Money, **TD No. 637 (June 2007)**.
- V. DI GIACINTO and G. MICUCCI, *The producer service sector in Italy: long-term growth and its local determinants*, Spatial Economic Analysis, **TD No. 643 (September 2007)**.
- Y. ALTUNBAS, L. GAMBACORTA and D. MARQUÉS, *Securitisation and the bank lending channel*, European Economic Review, **TD No. 653 (November 2007)**.
- F. BALASSONE, F. MAURA and S. ZOTTERI, *Cyclical asymmetry in fiscal variables in the EU*, Empirica, **TD No. 671 (June 2008)**.
- M. BUGAMELLI and F. PATERNÒ, *Output growth volatility and remittances*, Economica, **TD No. 673 (June 2008)**.
- M. IACOVIELLO and S. NERI, *Housing market spillovers: evidence from an estimated DSGE model*, American Economic Journal: Macroeconomics, **TD No. 659 (January 2008)**.
- A. ACCETTURO, *Agglomeration and growth: the effects of commuting costs*, Papers in Regional Science, **TD No. 688 (September 2008)**.
- L. FORNI, A. GERALI and M. PISANI, *Macroeconomic effects of greater competition in the service sector: the case of Italy*, Macroeconomic Dynamics, **TD No. 706 (March 2009)**.
- Y. ALTUNBAS, L. GAMBACORTA, and D. MARQUÉS-IBÁÑEZ, *Bank risk and monetary policy*, Journal of Financial Stability, **TD No. 712 (May 2009)**.