Econometric Modeling of Differentiated Durable Goods Markets:
An Application to Telephone

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SUMMARY

As the structure of consumer preferences plays a crucial role in the analysis of differentiated product markets, estimation of demand systems is a sensitive task. This paper contributes to this project in two ways. First, we develop a method to deal with the simultaneous choice of an equipment and a level of usage. This question is crucial for durable goods. Second, our method is suited for surveys of households, i.e., micro data. The main feature of our method is to specify a direct utility function, which provides correct substitution patterns among products at the aggregate level, i.e., compatible with the intuition. Applied to the market of telephone equipment for households, our approach is also able to deal with two classical problems encountered in the empirical IO literature on differentiated product markets, i.e., price endogeneity and dimensionality of product sets. A distinguished feature of our study is that we provide an estimate of product shares in terms of the whole stock of telephones rather than in terms of total sales in a given period. From a marketing point of view, this is a useful information since stock shares measure the effective penetration of a brand over its lifetime. Finally, we also provide markups assuming that firms follow Nash strategies.

RESUME

Cet article est une contribution à l'analyse des marchés de produits durables et différenciés. Le modèle économétrique traite simultanément les décisions d'usage et de choix d'équipement. Dans ce cadre, est donnée une réponse aux difficultés généralement rencontrées en économie industrielle appliquée : l'endogénéité des prix, la dimension de l'ensemble de choix et la structure du sentier de substitution entre produits. A partir de données individuelles sur le marché du téléphone, le modèle fournit une estimation du stock et des ventes de téléphones. Enfin une hypothèse de comportement stratégique permet de calculer des taux de marge par modèle de téléphone.

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1. INTRODUCTION

Product differentiation is a common feature of competition in most consumer goods markets. Not surprisingly it is an important topic in theoretical industrial organization since Hotelling (1929), Chamberlin (1933), and more recently Spence (1976) and Dixit and Stiglitz (1977). Two issues of competition mainly concern the theoretical research: The strategic pricing under differentiated products and the choice of products in oligopoly. (See Tirole, 1989.) These questions motivate a growing empirical research. Besides the question of selecting the relevant concept of conduct that is an essential step toward a correct measure of market power, a major focus of the recent empirical literature is the estimation of unrestrictive demand systems. Indeed, it is a quite sensitive task as the structure of consumer preferences plays a crucial role for defining the equilibrium type that could be reached in models of product differentiation. It is also a challenging problem as it involves dealing with the dimension of the commodity set which can be very large.

This paper contributes to this project in two ways. First, we develop a method to estimate a demand model for consumer durable goods, namely telephones. The distinguished aspect of our approach is to deal with the simultaneous choice of a product and a level of usage, a question that has not been often addressed in previous econometric studies on differentiated product markets. In our view, this question is crucial for dealing with durable goods. Second, our method is suited for surveys of households, i.e., for micro-data, and provides adequate and practicable solutions to the main issues raised by the econometric analysis of this type of markets: Price endogeneity, dimensionality of product sets, and correctness of substitution patterns. The following review of the literature is aimed at clarifying these issues.

In applied industrial organization literature, most studies use aggregate data for which two approaches are applied: Estimation of demand systems and estimation of discrete choice models. As proponents of the first approach, Hausman, Leonard and Zona (1994) estimate a multitiered system of demand, justified on the assumption of multistage budgeting and the theory of price indices. This method considerably reduces the number of parameters while obtaining a good approximation of the working of the market. In direct relationship with marketing practice, which identifies step by step the different tiers on a market, the method is adequate for transactional data recorded at the store level from scanners. In this approach, the endogeneity of prices could matter. The supply side tells us that prices are correlated with the unobservable product characteristics as part of the disturbance terms of demand equations. This correlation may be not negligible since, even at the lowest tier, data are aggregated. The authors take into account this situation at the estimation stage by using instrumental-variable methods. Note that here the supply side plays no structural role.

Alternatively, Berry, Levinshon and Pakes (1995) favor the theory of random utility and develop a full method based on the estimation of market shares defined from a discrete choice model. Already the theoretical literature had recognized the advantage of discrete choice models to study differentiated product markets. (See Anderson, De Palma and Thisse (1992) for a review of these models.) Well suited to deal with the dimensionality problem, this approach requires some care when specifying stochastic assumptions in order to avoid unduly restrictive patterns of substitution among goods. In particular, modeling interactions between product characteristics and non-observable individual characteristics is particularly crucial. However, this is not sufficient to obtain good estimators as stressed by these authors. The supply side and the market equilibrium cannot be neglected for estimating aggregate industry models.

Goldberg (1995) notices that the use of micro data for fitting a demand model alleviates the question of the endogeneity of prices. First, an individual consumer cannot alter prices. Second, micro data (i.e., data collected at the individual level) should permit to envision a high degree of product differentiation and to seize the heterogeneity of consumer behaviors. Hence, supply side considerations can be postponed until one turns to the study of the industry equilibrium. Precisely, an aggregate demand can be consistently recovered from the estimated individual demands as micro-data allow us to obtain the distribution of individual characteristics, and so to monitor the aggregation process. The only critical task left to the econometrician is to achieve a correct estimation of individual demands.

In the case of durable goods, the analysis can be performed by means of discrete choice models, for which the nested multinomial logit model is an adequate tool. It is simple to estimate and it escapes from the effect of the assumption of independence of irrelevant alternatives (IIA). Indeed, if this assumption is maintained within each nest, it is relaxed in between. However, this approach can be criticized on different grounds. First, there is some *adhockery* in choosing the different nests, even if they correspond to common sense or expert assessment. Second, this logit-type model remains generally based on linear indirect utility, which is restrictive. Third, in consumer surveys, the observed discrete choice is often completed by an information on the usage of the durable good. For instance, one observes both the type of housing ownership and the size of dwelling, the type of car and the average annual mileage, the choice of an heating system and the power consumption, etc. Separating the two choices, the discrete choice on the durable product and the continuous choice on its usage, lies on some separability assumptions that are sometimes unrealistic. Dubbin and McFadden (1984) for the case of electric appliance, and Goldberg (1998) for the case of automobile, specify a nonlinear indirect utility function that allows recovering discrete and continuous choices. However these authors focus mainly on the continuous choice.

Consider now the telecommunications demand. It involves access to the network and telephone usage that are clearly interdependent and cannot be separated. Even if a consumer does not make calls, he or she may be willing to have access just for the option value of a call in case of emergency.[1] Note that access to the network requires a device or an equipment, here a telephone which is a durable good. Now, many examples show that traffic and equipment allowing for network access are interrelated in the telecommunications industry. For instance, because of the overwhelming success of Internet, producers of microcomputers are introducing simpler machines just equipped with the function of accessing and searching on the web. The emerging information economy requires network equipment capable to offer access to a large range of services and to transfer various types of information like voice, data, or images. In part, habit formation and new usage may be the spring of technical changes on equipment. More generally, the consumption pattern of a consumer informs us on his private valuation for the equipment. Understanding the relationship between product choice and usage is at the heart of business and marketing strategies, like product introduction, positioning or pricing strategies.

By several specific features, the household telephone equipment (HTE) market is a very stimulating field of investigation. This market is characterized by a high degree of both horizontal and vertical differentiation of products. Marketing studies usually identify four distinct segments on the French market: one-block phone, two-block phones, phones with

[1] See Wolak (1993) on modeling telecommunications demand.

answering devices, and cordless phones. Within a segment, products are likely to be horizontally differentiated. In this context, providing realistic elasticities of substitution is a sensitive task. Moreover, as in other European countries, the French HTE market presents two further dispositions which raise interesting economic questions. First, during the eighties, this market has evolved from a monopoly, the historic telecommunications operator France Telecom, renting few telephone sets to a free-entry market where few firms (the majors like Philips, Matra, Alcatel, and some smaller companies) sell a large variety of telephone models. Note that the incumbent not only keeps renting but also sells many different types of equipment. The convergence to a new equilibrium as well as the effect of ownership type (renting or buying) must receive some attention, which we do in the empirical part of our study.

A survey made in 1992 on a representative sample of French households allows us to observe the choices of telephone models, the telephone bill and several socio-demographic variables on the individuals. Another marketing database provides the technical characteristics and the price ranges of different telephone models. The number of models of telephone we consider is about 160. However, since a household can enjoy more than one phone at home, the number of combinations of models that it faces, is huge (more than 12500 if we restrict attention to symmetric tuples). To cope with the dimension of the choice set, we apply a procedure introduced by McFadden (1978) which consists of drawing a subset of alternatives and performing the maximum likelihood on this subset after having corrected the choice probabilities accordingly.

Finally, fitting a structural model, i.e., a model derived from a direct utility function, to micro data for analyzing a market of differentiated durable goods turns out to be a fruitful method. First, we are able to deal with the classical problems of the empirical IO literature on differentiated product markets, i.e., price endogeneity, dimensionality of product sets, and correctness of substitution patterns. Second, we provide an estimate of product shares in the whole stock of telephones as the survey bears on the equipment of households and not on recent acquisitions. From a marketing point of view, this is a helpful information since stock shares measure the effective penetration of a brand over its lifetime. Third, considering that the market solution is approximated as a Nash equilibrium, we derive markups at the product level.

The next section is devoted to the specification of a demand model for HTE, taking into account the specific aspects of the HTE market. The econometric model and its estimation are presented in section 3. Empirical results are discussed in section 4 and the analysis of the market equilibrium is performed in section 5. Section 6 concludes and proposes a research agenda.

2. A CONTINUOUS-DISCRETE CHOICE MODEL OF TELEPHONE DEMAND

2.1. Notations and definitions

Consider a household (or a consumer) who simultaneously chooses a telephone equipment in a set E of alternatives which are precisely defined below, and a level of telecommunications usage which is a continuous variable. This consumer is assumed to be connected to the network and to own at least one phone.[2]

[2] As about ninety nine percent of French households are connected to the network, there is no need to consider an outside alternative composed with no telephone. In a standard discrete choice model, omitting the outside alternative implies the unrealistic feature that aggregate demand for all brands or products remain unchanged due to a general increase in price.

An alternative or a choice is *an equipment* composed with one or two brands of telephone.[3] An equipment is denoted by (j,k), j or k being a model of telephone. Note that, without loss of generality, an equipment composed of one phone is treated as a two-phones bundle, with one of the two telephones being the *nil* phone. A symmetry condition on the structure of equipment is needed as we do not observe which component of the equipment (i.e., j or k) has been bought first. Hence, the model does not differentiate between alternative (j,k) and alternative (k,j). (See however in Section 4 how the price of an alternative is defined.) Finally, the set E contains all mutually exclusive alternatives (j,k) comprising one or two phones.

Consumer n's preferences are translated into a conditional direct utility function, which provides the utility level of using a level x of telecommunications and a level z of *numéraire*, conditional to holding the alternative (j,k). A modified version of the Blackburn utility function, discussed by Hanemann (1984) and applied by Hobson and Spady (1988) for analyzing the demand of telephone usage, serves to specify this conditional utility function as:

$$U_{njk} = U\left(x, z, y_{jk}, q_n, z_{njk}, e_{njk}\right) = \frac{x}{b}\left(1 + \ln q_n - \ln x\right) + x y_{jk} + h_n z + z_{njk} + e_{njk}, \quad (1)$$

where b is a non-negative parameter of scale to be estimated, q_n is a heterogeneity index (supposed to be strictly positive) which can be measured through observable characteristics of household n, h_n is the marginal utility of *numéraire* specific to household n, y_{jk} is a quality index of equipment (j,k) that depends on observable attributes of each component of alternative (j,k), z_{njk} is a term aimed at measuring the individual valuation for quality, i.e., relating the taste of individual n for equipment (j,k) to observable variates, and e_{njk} is a random term which is not observed by the econometrician. Note that the quasi-linearity is a fairly admissible assumption as telephone call patterns should not depend on the consumption structure of other goods.

The household faces the budget constraint

$$px + z = y_n - F_{jk}, \quad (2)$$

where p is the price of a call unit,[4] F_{jk} is the cost of equipment (j,k), and y_n is the household income over the period under study.

Here, the possibility of combining brands of telephone to set up an equipment avoids a similar drawback. In response to a general price increase, the consumer could shift to an equipment composed of only one phone, when he/she used to own a two-pieces equipment.

[3] Restricting attention to equipment that has less than two telephones is not a strong assumption as the percent of households having more than two telephones is very low.

[4] Because of the tariff structure practised by the French operator, the cost of each call depends on the calling area and the time-of-use. In fact, this tariff structure provides exchange rates between calls of different types. All calls performed by a customer are aggregated in terms of a measurement unit of duration associated with a particular call, i.e., a specific duration for a call on a particular calling area at a particular time of the day. The price p is the price for one unit of this type of call. Note that, because of this tariff structure, the price per minute is an endogenous variable. In other terms, the average expenditure on telephone calls is specific to the individual and depends on his calling pattern. In this context, the demand model we present could be understood as the second step of the a full consumer program or as the result of a two-stage budgeting analysis. In a first stage, the consumer decides for the total expenditure, px, on telephone usage and on the composite good. The other stage is devoted to the allocation of this telephone budget to different types of calls and, to the

The utility function (1) is strictly quasi-concave, continuous and smooth with respect to x and z. On the one hand, it ensures that the conditional optimal level of telephone usage, x^*_{njk}, exists and is unique for the given discrete choice (j,k). On the other hand, the optimal demand of the composite commodity, z^*_{njk}, is positive provided that income is large enough compared to the value of consumption. Assuming that household n selects an alternative (j,k), the optimal conditional level of telephone consumption is given by

$$x^*_{njk} = q_n \exp\left[b\left(y_{jk} - h_n p\right)\right]. \tag{3}$$

Note that the telephone demand is always positive as long as q_n is positive.

Individual heterogeneity enters the demand in two ways. First, due to the non-monotonicity of preferences, the telephone consumption can reach a saturation level, which is specific to the consumer, a realistic hypothesis in the case of telephone. This saturation level, i.e., the total usage of telephone services that an individual is able to consume when the price of telephone calls is zero, is given by $q_n \exp\left(b\, y_{jk}\right)$, a function of the heterogeneity index. Second, another source of heterogeneity is introduced through the marginal utility of *numéraire*, namely h_n (also called marginal utility of income herein). It is realistic that individuals differ in their valuations of the exchange rate between the utility levels provided by the telephone usage and the *numéraire* (i.e., French Francs). It allows us to discriminate among the different consumption levels in terms of income levels, which turns out to be empirically adapted to our context. (See section 4 paragraph 3.)

The optimal discrete choice can now be defined. Inserting equations (2) and (3) into equation (1) yields the conditional indirect utility

$$V_{njk} = U\left(x^*_{njk}, z^*_{njk}, y_{jk}, q_n, z_{njk}, e_{njk}\right) = \frac{q_n}{b}\exp\left[b\left(y_{jk} - h_n p\right)\right] + h_n\left(y_n - F_{jk}\right) + z_{njk} + e_{njk}. \tag{4}$$

Alternative (j,k) is chosen when

$$V_{njk} \geq V_{nj'k'}, \quad \forall (j',k') \neq (j,k), \tag{5}$$

which is equivalent to

$$\exp\left(b\, y_{jk}\right) - \exp\left(b\, y_{j'k'}\right) \geq \frac{b\, \exp\left(b\, h_n p\right)}{q_n}\left[h_n\left(F_{jk} - F_{j'k'}\right) + z_{nj'k'} - z_{njk} + e_{nj'k'} - e_{njk}\right]. \tag{6}$$

In other terms, equation (6) shows how the choice of an equipment results from an arbitrage between quality and price. The higher (resp. lower) the saturation level, q_n, and/or the less (resp. more) the marginal utility of income, h_n, the more (resp. less) is the weight on quality instead of the price.

allocation of the budget on the composite good. The result of the second stage can be aggregated to provide the quantity x of call units purchased by the consumer.

2.2. Stochastic assumptions and substitution patterns

The theoretical structure is completed with stochastic assumptions, which mainly drive the properties of the econometric model. Assume that the econometrician does not observe all variates explaining the consumer choices. The stochastic error, e_{njk}, results from the interaction among unobservable individual characteristics and unobservable attributes of equipment alternatives. The first stochastic assumption is stated as:

ASSUMPTION 1: *The random terms $e_{njk}, \forall(j,k) \in E, \forall n$ are independent and distributed according to a Gumbel density function with scale parameter m (the location parameter being normalised at zero).*

This assumption yields the multinomial logit (MNL, herein) model. (See McFadden, 1973.) The probability of selecting alternative (j,k) is obtained as

$$P_{njk} \equiv P_{jk}(q_n, h_n, p, y, F, z_n) = \frac{\exp(mW_{njk})}{\sum_{(j',k')\in E} \exp(mW_{nj'k'})}, \tag{7}$$

where W_{njk} is the deterministic part in $V_{njk} \equiv W_{njk} + h_n y_n + e_{njk}$, and where $y \equiv (y_{jk})_{(j,k)\in E}$, $F \equiv (F_{jk})_{(j,k)\in E}$ and $z_n = (z_{njk})_{(j,k)\in E}$.

As it is well known, these individual choice probabilities exhibit the property of independence of irrelevant alternatives (IIA), which simplifies estimation but imposes strong restrictions on the substitution path among alternatives. Indeed, under IIA, the cross-elasticity of the choice probability for alternative (j,k) with respect to any characteristic of an alternative (j',k') is the same for all alternatives (j,k) such that $(j,k) \neq (j',k')$. In particular, if the price of an alternative increases, then the choice probabilities of all other alternatives increase in the same proportion. This result is restrictive in the sense that the choice probabilities of similar alternatives (i.e., alternatives that provide similar level of utility) should rise proportionally more than the probabilities of dissimilar alternatives.[5]

The counterintuitive result associated with MNL motivates several approaches, as in Berry, Levinshon and Pakes (1995) or in Goldberg (1995). It also impels our method founded on the specification of a direct utility function, which provides a reasonable substitution path between products at the aggregate level, i.e., compatible with the intuition. Thanks to the structure of the selected utility function, the following result obtains:

PROPOSITION 1: *The elasticity of product j's market share with respect to price of any product q is not the same for all $j \neq q$.*

PROOF: See Appendix 1.

Three remarks shed light on this proposition. First, although the analysis bears on equipment, the result applies to products, i.e., to brands, which are our primary interest. For sake of completeness, Appendix 1 provides the computation of product market shares from market shares of alternatives. Second, as in the usual logit model, the main reason for this

[5] The intuition dictates that the higher the change of the choice probability of product B due to a price increase of product A, "the closer" B to A.

result comes from the interaction between the individual heterogeneity index, q_n, and the quality index, y_{jk}, on the one side, and between the marginal utility of *numéraire*, h_n, and the equipment price, F_{jk}, on the other side. The higher the relationship between individual characteristics and quality of a particular product, the higher the desire to select products of similar characteristics. For instance, households living in a large house are keener to look for cordless telephones. Third, this result just allows us to get rid of the harmful effect of the IIA property. However, that estimated cross-elasticities are in conformity with the above intuition remains an empirical issue.

The crucial assumption here is the independence of stochastic terms e_{njk}. Relaxing this hypothesis as in the nested multinomial logit (NMNL) or multinomial probit (MNP) models indeed provides more flexibility in the substitution patterns among alternatives. However, on the one hand, estimation of MNP models is computationally cumbersome and often not practicable (even using simulation methods) when it involves integrals of very high order as soon as the number of alternatives gets large. On the other hand, although the NMNL model is much easier to implement, it heavily relies on the correctness of the selected nests (i.e., different segments of market). This question is particularly critical in our case where the grouping of alternatives in a reasonable number of nests is not obvious. In addition, deriving a nested logit from a general non-linear utility function and estimating it, does not seem to be a simple task.

Berry, Levinshon and Pakes (1995) propose an alternative approach. Assuming that the quality index is a linear combination of interactions between product attributes and private valuations (which are random variables), these authors obtain a correlation among alternatives. One advantage of this procedure is to derive individual choice probabilities, which do not satisfy the IIA hypothesis. For estimating their model, Berry *et alii* integrates the logit probabilities of choice over the vector of private values which, in their case, is of a relatively small dimension.

A similar approach can be attempted in our case. However, introducing an unobservable individual heterogeneity conflicts with our estimation method, which requires the IIA to hold at the individual level. We leave this technical difficulty to further research. Nonetheless, given that we deal with a rich set of micro data and observable individual variables, adding an unobservable private valuation for quality has a limited interest and increases the estimation cost (in terms of computation time, in particular).

It remains to specify the stochastic assumption on the continuous choice. As there is no interaction between the level of telephone usage, *x*, and the unobservable attributes of the chosen equipment in the specification of our random utility function, the optimal conditional usage x^*_{njk} defined in equation (3) does not depend on e_{njk}. In our case, as the quality of each equipment is well identified by a precise list of observable characteristics, unobserved attributes should only be related to subjective elements, such as the perceived design of a phone by the consumers. They could play a role in the decision to buy a particular model, not really on the usage. In addition, when, as in Hanemann (1984), one admits an interaction between the level of telephone usage and the unobservable attributes of the chosen equipment, the distribution of usage (i.e., the conditional continuous choice) should depend on the attributes of all alternatives which is not particularly realistic.[6]

[6] There is a critical difference between Hanemann's approach and our model. In his paper, the continuous and discrete choices bear on the same commodity. For instance, he has clearly in mind the case of housing where the question is to

However, the continuous decision is not deterministic for the econometrician. As a matter of fact, the household may report its telephone bill incorrectly.[7] Let $\tilde{x}_{njk}$ be the reported level of telephone usage if alternative (j,k) is chosen. It is related to the true level x^*_{njk} according to $\tilde{x}_{njk} = x^*_{njk}\exp(u_n)$ with the following hypothesis:

ASSUMPTION 2: *The random term $u_n, \forall n$, has a normal density function with mean 0 and variance s^2.*

Denote by

$$f_{njk}(x) = \frac{1}{s\sqrt{2p}}\exp\left(-\frac{1}{2}\left(\frac{\ln x - \ln q_n - b\left(y_{jk} - h_n p\right)}{s}\right)^2\right) \tag{8}$$

the density of the logarithm of observed usage conditional to the choice of alternative (j,k) .by individual n characterized by the vector (q_n, h_n)

Summing up, equations (7) and (8), together with Assumptions 1 and 2, will constitute the basis of our econometric model.

3. ESTIMATION PROCEDURE

Implementing the maximum likelihood (ML herein) method in order to estimate the model here involves two practical difficulties that arise from the features of the data set.[8] First, the number of alternatives is very large which makes the estimation procedure computationally burdensome.[9] The solution is to randomly draw a set of alternatives for each individual according to some selection probabilities to be defined. Clearly, in order to reduce the dimension of the choice set, the intuition is to select alternatives that are close to the observed choice, i.e., we restrict the individual choice set to the most probable choices. Second, the database does not allow the econometrician to always identify the chosen alternative. For some individuals, we just observe a sub-group of alternatives in which the choice has been made. This technical problem is addressed by "integrating" over the possible alternatives in the sub-group. The solution we propose in order to deal with these two practical questions accounts for the IIA property and allows us to predict the choice probabilities for each elementary alternative of the choice set for any individual.

Consider a sample of N individuals, independently drawn from the French population having access to the network. Each individual is labelled by n = 1,....,N. Denote by x_n his

decide the size together with the type of a dwelling. Here the joint choices refer to two different commodities: The act of communicating with somebody else and the ownership of a phone.

[7] The surveyed household may or may not provide its telephone bill to the interviewer. It may just answer with a rough estimate, which is a first source of measurement error. Moreover, a second source of error can be introduced as we need to approximate the annual telephone bill while the survey asks for the telephone expenditure during two-months.

[8] The reader, who is not concerned by statistical techniques, could skip this section without altering the understanding of the sequel. Nonetheless, this methodological section shows the practicability of our structural approach.

[9] The large number of alternatives should not afraid a big computer! Nonetheless, the procedure we propose has the advantage of producing efficient estimates while allowing the estimation of our model on standard PCs in a reasonable time.

observed level of telephone usage over the given period (measured in call unit with a measurement error), by (j_n, k_n) his chosen alternative, and by Γ the vector of the parameters to be estimated.

Conditionally to the exogenous variables (such as socio-demographic variables, classes of income, attributes and prices of telephones), the likelihood for observation *n* is

$$l[j_n, k_n, x_n; \Gamma] = P_{nj_nk_n} f_{nj_nk_n}(x_n), \tag{9}$$

i.e., given the stochastic structure of the model, the likelihood function is the product of the marginal probability $P_{nj_nk_n}$ associated with the choice of equipment as in equation (7), and the conditional probability of telephone usage $f_{nj_nk_n}(x_n)$ as in equation (8).

To cope with the large number of alternatives that an individual faces, we apply a procedure initially introduced by McFadden (1978) and developed by Ben-Akiva and Lerman (1985). Define for each individual n a selection probability $\Pi(n, j, k)$ for each alternative $(j,k) \in E$. Then alternatives can be drawn according to an importance random sampling. To alternatives, which an individual would choose with very low (resp. large) probabilities, should be imputed very low (resp. large) selection probabilities. In terms of efficiency, these selection probabilities must be such that the ratio $P_{njk} / \Pi(n, j, k)$ varies as little as possible. An efficient solution is to choose them such that $\Pi(n, j, k) = \hat{P}_{njk}$, $\forall\, n = 1,\ldots, N$, $\forall (j,k) \in E$, where $\hat{P}_{njk}$ is a consistent estimator of the choice probability provided that the model is well specified.[10] Note that this procedure is valid only when there is no alternative-specific parameter in the model and the IIA property is verified. (See McFadden, 1978 and 1984).

Denote by D_n the reduced set of alternatives obtained from applying a sampling procedure with replacement. Note that the chosen alternative is added to this set systematically. The probability of obtaining the set D_n conditionally to the chosen alternative is denoted by $\Pr(D_n | j_n, k_n)$ and is computed according to the type of random sampling and the selection probabilities. (See Foncel, 1997).

Now, an assumption is required for proving consistency of ML estimates when the preceding procedure is applied to compute the choice probabilities. (See McFadden, 1978.)

ASSUMPTION 3: *i) The sampling protocol satisfies the positive conditioning property, i.e.,* $\Pr(D_n | j,k) > 0$, $\forall (j,k) \in D_n$. *ii) It is fully specified with the property that* $\Pr(D_n | j,k) = 0$ $\forall (j,k) \notin D_n$.

In other terms, an alternative in D_n has a positive probability of being an observed choice effectively, and could be assigned the set D_n by the sampling mechanism. Behind this assumption is the idea that the knowledge and the relevance of selection probabilities allow us to correct for the bias involved in the estimation of choice probabilities on a reduced set of alternatives for each individual.

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[10] In order to obtain consistent estimates of the choice probabilities, a first run consists of applying the whole estimation method using any selection probabilities. For instance a set of selection probabilities is easily obtained by allocating to each alternative, a probability equal to the inverse of the total number of alternatives.

Before modifying the likelihood function to account for this sampling process, one must address a second problem. As mentioned above, in a few cases, we do not precisely observe which alternative has been chosen. Instead, one just knows a subset of E in which the selected alternative lies. The definition of these subsets is imposed by the database. There are M subsets, $\left(E_m\right)_{m=1,\ldots,M}$, such that $\bigcup_{m=1}^{M} E_m = E$ and $E_m \cap E_{m'} = \{\varnothing\}$, $\forall\, m \neq m'$. We observe E_{mn}, the class in which individual n has selected an equipment.[11]

This problem of incomplete observability of alternatives affects the selection process of alternatives. Indeed, the selection set D_n should include all alternatives in E_{mn}, instead of just the observed chosen alternative. Each alternative $(j,k) \in E_{mn}$ must belong to the same selection set, D_n, in order to achieve consistency of estimators. Note that the probability of drawing D_n has to be modified accordingly. (See Foncel, 1997).

We can now derive the final expression of the likelihood function conditional to the reduced set of alternatives.

PROPOSITION 2:
i) The modified log-likelihood of the sample is

$$L_N(\Gamma) = \sum_{n=1}^{N} \ln \left[\sum_{(j,k)\in E_{mn}} \frac{\exp\left(mW_{njk}\right) \Pr\left(D_n \,|\, j,k\right) \; f_{njk}\left(x_n\right)}{\sum_{(j',k')\in D_n} \exp\left(mW_{nj'k'}\right) \Pr\left(D_n \,|\, j',k'\right)} \right] \tag{10}$$

ii) Under usual regularity conditions, $\hat{\Gamma} = \arg\max_{\Gamma} L_N(\Gamma)$ *is a consistent estimator of* Γ^0*, the true values of the parameters.*

PROOF: See Appendix 2.

Given regularity conditions, asymptotic normality is achieved. Note that the variance-covariance matrix of parameters is the White matrix.

4. EMPIRICAL RESULTS

This section is devoted to the estimation of the model, which requires a preliminary step consisting of building a bridge between the theoretical model and our data sources.

4.1. The data

A first database[12] contains the responses to a survey realized in 1992 on a sample of 1500 households representative of the population connected to the French telecommunications network. The survey provides a description of the household's telephone equipment and the

[11] To avoid the problem of not observing systematically the chosen alternative, one may aggregate alternatives. Then, we would have to define the level of utility procured by these aggregate alternatives. This is not so obvious to do when the indirect utility function is not linear in product attributes. Moreover, one can show that one is not able to recover the estimated market shares of products, which is one of our main objectives.

[12] The marketing institute DEMOSCOPIE, Paris, has provided this database.

level of its telephone bill over two successive months of 1992.[13] It also reports a lot of socio-demographic variables, such as the number of individuals in the household, the job position of the family head, the population density of the area where the household lives or the ownership of a teletext terminal, and the income class.

The second database[14] describes all the 134 models of telephone marketed in the survey year, including their prices, their attributes and in particular their type. In 1992, marketing analysts considered four market segments corresponding to four different types of telephone: One-block telephone (referenced by the acronym OB) for which dialling buttons are located on the listening part; two-blocks telephone (TB); cordless telephone (CD); answering telephone (AW). The telephone type is used as an attribute in the empirical model. Other attributes are for instance, the number of memories, the presence or not of a speaker or a screen to show the numbers of incoming or outgoing calls. Further informations about sales by brand and type of telephone from 1988 (after deregulation) to 1992 are also provided. For each of these years, the set of models and their attributes (including price) are reported.

Variables used in the empirical model are listed in Appendix 3 while descriptive statistics on individual and product variables are gathered in Appendix 4.

4.2. Specification

Now we specify the heterogeneity index q_n, the marginal utility of *numéraire* h_n, the quality index y_{jk}, the cost of equipment F_{jk}, and the private valuation of equipment z_{njk}, in terms of the observable variables.

The heterogeneity index

As this index is individual-specific and must be positive always, a classical specification is to set

$$q_n = \exp\left(c_0 + \sum_{q=1}^{Q} c_q v_{nq} \right), \tag{11}$$

where $\left(c_q\right)_{q=0,..,Q}$ are parameters to be estimated and v_{nq} is the vector of observable individual characteristics q of household n. The selection of relevant socio-demographic variables is an empirical issue.

The marginal utility of income

In order to differentiate the marginal utility of *numéraire* among individuals, we set

$$h_n = \sum_{i=1}^{I} m_i y_{in}, \tag{12}$$

where $i = 1, \dots, I$ indexes income classes (increasing with i), y_{in} is a dummy variable which takes value equal to one if the household's income belongs to class i or value zero otherwise,

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[13] If d_n denotes the telephone bill (expressed in French francs) of household n during two months, the annual level of usage x_n is approximated by computing $6d_n/p$ with p=0.73 FF. We assume that any seasonal variation is included in the measurement error of the equation of telephone consumption.

[14] The marketing institute GFK, Paris, has provided this database.

and $(m_i)_{i=1,...,I}$ are unknown parameters to estimate. With a lower (higher) income class should be associated a higher (lower) value of the marginal utility of income.

The quality index

This index is defined as $y_{jk} = y_j + y_k$, where y_j and y_k are the quality indexes of each equipment component. We define

$$y_{jk} = a'b_j + a'b_k = \sum_{l=1}^{L} a_l \left(b_{jl} + b_{kl}\right), \tag{13}$$

where $(a_l)_{l=1,...,L}$ are parameters to be estimated, and b_{jl} (b_{kl}) is the attribute l of telephone j (respectively, k).[15] In other terms, when a consumer owns two phones, one adds their quality indexes to obtain the overall quality of his/her equipment. This linear specification could be deemed too restrictive. However, our attempts to use non-linear expressions (quadratic form, for instance) have not been successful so far. Note that all parameters associated with different attributes should be positive since, *ceteris paribus,* quality should rise with a higher availability of technical characteristics.

The cost of an equipment

As the model is cast in a static setting, the customer behaves as if he had to renew his equipment decision at each period. However, since each element of a telephone equipment has a different acquisition date, discounted prices apply. Two pieces of information are helpful in this situation. On the one hand, the customer may buy or rent his telephones on the French HTE market, renting being only practised by the historic company. On the other hand, we know from the survey whether the household's equipment is recent (i.e., roughly less than one year) or old.

Denote by f_j the sale price or the annual renting fee of telephone j. This variable is an average price provided by the marketing institute, GFK.[16] An actualization issue prevents the direct use of this price in the model.

Define the indicator variable l_j that is equal to one ($l_j = 1$) if telephone model j is proposed for renting and is equal to zero ($l_j = 0$) if it is proposed for sale. First, consider the case of recent equipment. If f_j is the sale price or the annual renting fee of telephone j, then its cost is $\left(1 + l_j r_1\right) f_j$ where r_1, a parameter to be estimated, is introduced so that one can compare renting or sale of this telephone on an equal footing. In particular, this parameter accounts for the net effect between the expected costs of pursuing the renting contract and the ones of maintaining the telephone (when it has been bought). Hence, for new equipment (j,k), the total cost is

$$F_{jk}^{N} = \left(1 + l_j r_1\right) f_j + \left(1 + l_k r_1\right) f_k, \tag{14}$$

[15] The quality and price of a *nil* phone are normalised to zero.

[16] One should differentiate the transaction price from the average price f_j, like for instance the special discount that the consumer has obtained from his/her shopkeeper. We account for this second measurement through the measurement error e_{njk}.

where *N* stands for "New".

Now, consider the case of an "Old" equipment. The discounted cost today of a telephone *j* bought at some point in the past, is approximated by $(1+r_2)f_j$ where f_j is its present sale price. It is approximated by $(1+r_3)f_j$ when it is rented and f_j is its annual rental cost. Then, the total cost of an old equipment is

$$F_{jk}^{O} = \left[(1-l_j)(1+r_2)+l_j(1+r_3)\right]f_j + \left[(1-l_k)(1+r_2)+l_k(1+r_3)\right]f_k \,. \tag{15}$$

Note that equation (14) is equivalent to equation (15) if $r_2 = 0$ and $r_3 = r_1$. In other words, there is a normalization which allows us to designate a new equipment.

Parameters r_1, r_2, and r_3 are function of the equipment duration, the interest rate, or the individual discount rate. One expects r_1 and r_3 to be positive since they are basically rates of change between buying or renting. While r_2 should be negative in general, it may turn out to be positive if prices of telephones decreases more drastically than the consumer price index.

Summing up, the total cost of any equipment (j,k) is given by

$$F_{njk} = g_n F_{jk}^{N} + (1-g_n)F_{jk}^{O}, \tag{16}$$

where the variable g_n takes value 0 if the equipment is old, i.e., if the household owns it since a long time, and value 1 if it is recent. The cost F_{njk} of an equipment (j,k) hold by the household *n*, is indexed by *n* as it is function of exogenous decisions of the household.[17]

The individual valuation for equipment

Assume that the individual valuation of the intrinsic quality of an equipment results from the personal characteristics of its owner and the product attributes. Given the preceding notations, we propose to specify this individual valuation according to the function

$$z_{njk} \equiv (1+\ln q_n)(y_{jk} + x_{jk}), \tag{17}$$

where the term x_{jk} encompasses all product or market attributes that could affect the quality of the product (and so its demand) without affecting its usage. In particular, it is aimed at taking into account market characteristics in the demand model. Assume that $x_{jk} = x_j + x_k$ and specify

$$x_j = \sum_{g=1}^{G} d_g t_{jg} + d_{G+1} l_j + d_{G+2} R_j \,, \tag{18}$$

where *G* is the number of firms present on the market, $(d_g)_{g=1,\dots,G}$, d_{G+1} and d_{G+2} are parameters to estimate, t_{jg} equals 1 if telephone *j* is proposed for sale by firm *g* et 0 otherwise, l_j equals

[17] The variable g_n must be viewed as an exogenous variable in our static model even if it should result from a household choice in a dynamic framework.

1 if telephone *j* is proposed for renting and 0 otherwise, and R_j equals 1 if telephone *j* is new on the market and 0 otherwise.

Note that the component x_j has no direct link with the telephone usage and should not be confused with the quality index, which enters the consumption function. The first term in equation (18) is a classic way to introduce the reputation of firms in the demand model. The term $d_{G+1}\ l_j$ is used to account for two potential disequilibrium effects. First, at the survey date, all households may not be equally and perfectly informed on the new rules of the HTE market after deregulation started. Second, there could be some inertia in the household behavior, in part because of the customer loyalty vis-à-vis the historic company. Recall that the renting contract, only proposed by this firm, offers a free maintenance service, which could be perceived as a sign of quality. Finally, the last term on the RHS of equation (18) measures the effect of launching new brands on the market.

Neglecting the function given in equation (17) may result in implausible values for model parameters or counterintuitive effects. It has a very sensitive role for the quality of estimates. Note that it is a way to control for the endogeneity of prices, which could occur when one omits to account for the fact that firms determines their marketing strategies by observing consumer tastes on the quality of their products. Knowing that the analyst does not have the same information set than the firms, the introduction of product-specific dummies in the model could care the pain. This solution requires many observations per product, which is not our case. The alternative is to use observable control variables as here or as in Goldberg (1995).

The set of marketed phones

The set *E* of all equipment that consumers hold must be precisely defined. It must take into account all models of telephones sold or rented in 1992, and also those models that are not sold anymore (but still hold by consumers). For this purpose, we include the most representative old models (in terms of market shares) in order to avoid an overestimation of the stock shares of the most recent telephone models. Hence, our final set *E* is built from symmetric combinations of the 158 different models of telephone (including the *nil* phone).

4.3. Estimates

Parameter estimates are gathered in Table 1. After several experiments comprising different sets of exogenous variables, we report an estimation for which all parameters (except one) are strongly significant. Since the mean log-likelihood is equal to –12.0626 and the mean log-likelihood when all parameters are set to 0 (except s set to 1) is equal to -40.3631. The pseudo R^2 is then equal to 0.701.

Hausman and McFadden (1984) provide a test of IIA in the case of multinomial logit models. It is not directly applicable in our case for two reasons. First, we must account for the continuous part in our likelihood function. Second, the test is based on the comparison to zero of the difference between the estimator obtained with the full choice set and another one based on an arbitrarily reduced set of alternatives. Our estimation strategy is precisely built in order to avoid using the full choice set. The appropriate test for our case is left to further research. Here, we just check that two estimators based on two different reduced choice sets randomly drawn give similar results, i.e., provide similar predictions for the product shares.

Table 1: Estimation of model parameters

Model parameters	Variable acronyms	Estimates	Standard errors	t-ratios
Heterogeneity index, q	*Constant*	6.9106	0.2341	29.520
c_q*'s*	*URBA1*	0.1946	0.0437	4.452
	URBA2	0.1540	0.0355	4.335
	URBA3	0.0821	0.0424	1.937
	NUMB	0.0259	0.0102	2.525
	SES1	0.2095	0.0535	3.918
	SES2	0.1828	0.0439	4.168
	SES3	0.0974	0.0418	2.328
	MINIT	0.2436	0.0352	6.913
Marginal utility of income, h	*INC1*	0.0025	0.0007	3.710
m_i*'s*	*INC2*	0.0021	0.0006	3.329
	INC3	0.0021	0.0006	3.308
	INC4	0.0017	0.0006	2.812
Quality index, y	*OB*	0.0017	0.0004	4.341
a_l*'s*	*TB*	0.0017	0.0004	4.459
	CD	0.0021	0.0004	4.769
	AW	0.0019	0.0004	4.220
	MEM	0.0000	0.0000	1.784
	AMPL	0.0001	0.0000	2.270
	AFFIC	0.0002	0.0001	2.006
	NMD	0.0001	0.0001	0.957
	VOL	0.0001	0.0001	2.236
Equipment cost, F r_1		2.2947	0.9467	2.424
r_2		0.2546	0.1107	2.298
r_3		3.1976	1.1682	2.737
Equipment valuation, z	*France Telecom*	0.2568	0.0320	8.022
d_g*'s*	*Philips*	0.1410	0.0292	4.835
	Alcatel	0.0592	0.0314	1.886
	Matra	0.1811	0.0245	7.400
d_5		0.6387	0.0552	11.570
d_6		-0.3570	0.0880	-4.068
Other parameters m		0.8150	0.0827	9.856
s		0.5390	0.0091	59.080
b		470.9400	107.5300	4.379

Note: Standard errors are obtained from the consistent-heteroskedastic matrix of White

Comparing our joint model with a simpler model omitting the continuous choice would also be of interest. However, the discrete part of our model is not identifiable from the data. Note that, as the estimated value of s is relatively small, the continuous part of the model is meaningful. Moreover, as the estimated value of m is large, the variance of the e's tends to become small, so that the discrete choice model is informative.

What do the other results teach us on the microeconomic behavior of households? First, the effects of socio-demographic variables on telephone consumption agree with the common intuition. The higher is the population density in the area where the household lives, the lengthier is the duration of telephone usage. This result should be interpreted as an indicator of the presence of a network effect. Similarly, the larger the size of household, the higher the telephone usage. Note that white collars have a higher consumption than blue collars. Finally, the ownership of a teletex terminal also increases consumption.

Second, as expected, for each individual, the marginal utility of *numéraire* is positive and decreasing with the class of income level. Moreover, individual price elasticities of telephone consumption (i.e., $\P \ln x_n / \P \ln p = -bh_n p$) are negative and increasing in absolute value with the class of income level.[18] Our estimates of these elasticities of telephone usage are higher than ones published in previous empirical studies. It is probably because most of these studies use aggregate panel data and thus provides short-run elasticities. On the opposite, our static model should reflect long run changes in consumption.[19]

Third, the results provide evidence on the usual issues of differentiation, namely, on how customers value the different products and are able to substitute among telephone models. Parameters of product attributes are all positive. Thus, for any individual *n*, the marginal utility of any attribute *l* (i.e., $a_l q_n \exp\left(b\left(y_{jk} - h_n p\right)\right) + a_l\left(1 + \ln q_n\right) > 0$) is positive when computed at the estimated value, which is an indirect test of a right specification. Given the estimated parameters, the quality of a telephone (or the contribution of its attributes to the utility) is essentially measured by its type, the effect of each other characteristic being much lower. So, vertical differentiation is mainly driven by the telephone type, which distinguishes market segments, while the other characteristics of telephone models play a role within each market segment, introducing by this way a sense of horizontal differentiation.

Fourth, concerning the equipment cost, parameters r's are ordered as $0 < r_2 < r_1 < r_3$. From the definition of the cost of equipment provided above, the parameters r_1, r_2 and r_3 are associated with three cases: A telephone recently rented, bought or rented a long time ago, respectively. (Recall that the parameter associated with a recent acquisition is normalized to zero). That r_2 is positive is probably due to the large decrease of the average price level of telephone compared to the evolution of the cost of living. That r_3 is greater than r_1 is trivial in the sense that a longer period of renting corresponds to a higher cost.

Fifth, several remarks can be made on the determinants of the individual valuation of equipment. As expected, customers assign a positive value to telephones bought from the biggest firms (Alcatel, France Telecom, Philips, and Matra) compared to other competitors on the market whose effect is normalized at zero. It is the result of combined effects: Reputation of firms or loyalty of consumers. Note that France Telecom has the highest grade followed by Matra, Philips and Alcatel in a decreasing order. (See values of parameters d_1 to d_4.) Moreover, French customers would associate a higher valuation to renting than to buying a telephone. (See value of parameter d_5.) As there is no objective reason for this fact, this effect is either due to the lack of information of households on the new rules after deregulation, either due to the strong support for renting France Telecom's telephones, which provides free replacement for out-of-order telephones. Finally, a new telephone model has a lower valuation than an old brand, because all customers at the survey date do not know it. (See value of parameter d_6.)

The preceding remarks are heuristic proofs of the robustness of our econometric model that can be now used to analyze the conduct on the market.

[18] For confidentiality reasons, these elasticities are not reported.

[19] See Blundell, Pashardes and Weber (1993) on the reliability of estimates of price coefficients in demand models when one uses different levels of aggregation.

5. MARKET ANALYSIS

All ingredients are now available to derive market shares and to compute price elasticities and markups.

5.1. Stock and market shares

Recall that, as the survey bears on the household equipment, the model provides the shares of each telephone brand in the whole stock. If $\hat{P}_{njk}^{t}$ is the estimate at the survey period *t* (namely, 1992) of the choice probability P_{njk} defined in equation (7), then the stock share of equipment (j,k) in this period is

$$\hat{s}_{jk}^{t} = \frac{1}{N}\sum_{n=1}^{N}\hat{P}_{njk}^{t} , \tag{19}$$

and the stock of product *j* is obtained as

$$\hat{S}_{j}^{t} = \aleph\left(\hat{s}_{jj}^{t} + \sum_{k\in C_t^*}\hat{s}_{jk}^{t}\right), \tag{20}$$

where C_t^* is the set of all telephones in 1992 including the *nil* phone and the old models still owned by consumers. The total number of households in the French population $\aleph$ is equal to 23.92 millions, which is assumed to be constant in the sequel.

We estimate the size of the total stock to be equal to 37.19 millions of telephones hold by French households in 1992, while France Telecom officially estimates it at 34.93 millions. Rented telephones represents 47.43 percent of this estimated stock, namely, 17.64 millions of telephones. France Telecom estimates this proportion of rented telephones to be 52 percent in 1992. According to our model, each household owns and/or rents 1.55 telephone. For France Telecom, this average level of equipment is evaluated at 1.46 telephone per household while the marketing institute, Demoscopie, estimates it at 1.65. The fit of our estimated model is fairly satisfactory.

Table 2 provides stock shares of telephones in 1992. Since there are too many telephone models, only aggregate shares at the segment level are reported. Note that the shares are computed on the estimated total stock after having excluded the stock of rented telephones. As the estimated stock shares well mimic the observed shares given in Table 3, the model is also able to provide a good approximation of the variability in the market.

Now, as the set of prices in 1991 (called period *t-1*) is available, we can estimate the stocks in 1991. For this purpose, define C_{t-1}^* the set of marketed models in 1991 (where products appeared in 1992 are excluded). At the estimated value of parameters, the choice probabilities, namely $\hat{P}_{njk}^{t-1}$, are computed in applying equation (7) to the set C_{t-1}^* and the 1991 prices. Then, the stock $\hat{S}_{j}^{t-1}$ of product $j \in C_{t-1}^*$ is derived in using the estimated probabilities $\hat{P}_{njk}^{t-1}$ in equations (19) and (20). Finally, an estimate of sales $\hat{D}_j$ of telephone $j \in C_t^*$ during period *t* can be readily obtained by computing

$$\hat{D}_j = \hat{S}_j^{t} - \hat{S}_j^{t-1} . \tag{21}$$

Table 2: Telephone equipment of French households: Estimated stock shares in percent

Market segment / Firms	One-block OB	Two-block TB	Cordless CD	Answering AW	All segments
France Telecom	0.62	28.60	2.90	1.27	**33.39**
Matra	6.66	14.70	3.02	0.95	**25.33**
Philips	6.34	3.01	1.37	1.44	**12.15**
Alcatel	4.03	5.72	0.85	0.53	**11.13**
Modulophone	2.05	5.39	0.18	0.26	**7.87**
Comoc	0.62	4.16			**4.78**
HPF	0.98	1.88			**2.86**
Téfal	1.01	0.75			**1.77**
Radialva	0.00	0.62			**0.62**
Dialatron	0.10	0.00	0.00		**0.10**
All firms	**22.41**	**64.83**	**8.30**	**4.46**	**100.00**

Notes: *- An empty cell means that the firm is not present on the corresponding market segment.*
- The stock does include rented telephones

Table 3: Telephone equipment of French households: Observed stock shares in percent

Market segment / Firms	One-block OB	Two-block TB	Cordless CD	Answering AW	All segments
France Telecom	2.72	27.81	4.00	1.16	**35.69**
Matra	8.34	10.90	4.13	1.33	**24.70**
Philips	4.58	3.35	3.09	3.06	**14.08**
Alcatel	4.00	4.31	1.80	1.19	**11.30**
Modulophone	1.24	2.56	0.21	0.47	**4.49**
Comoc	2.54	2.20			**4.74**
HPF	0.72	0.97			**1.69**
Téfal	1.21	0.59			**1.80**
Radialva	0.00	0.40			**0.40**
Dialatron	0.71	0.24	0.16		**1.11**
All firms	**26.07**	**53.32**	**13.40**	**7.21**	**100.00**

Note: All figures are obtained from the marketing institute GFK.

Table 4: Telephone equipment of French households: Estimated market shares in percent

Market segment / Firms	One-block OB	Two-block TB	Cordless CD	Answering AW	All segments
France Telecom	4.20	25.58	7.11	1.99	**38.88**
Matra	6.21	8.27	8.54	1.77	**24.79**
Philips	5.97	2.81	2.45	2.57	**13.80**
Alcatel	4.44	1.91	2.14	0.91	**9.40**
Modulophone	0.89	2.76	0.22	0.70	**4.57**
Comoc	0.40	2.57			**2.98**
HPF	0.71	2.77			**3.47**
Téfal	0.85	0.42			**1.27**
Radialva		0.16			**0.16**
Dialatron	0.67	0.00			**0.67**
All firms	**24.35**	**47.26**	**20.46**	**7.94**	**100.00**

Table 5: Telephone equipment of French households: Observed market shares in percent

Market segment / Firms	One-block OB	Two-block TB	Cordless CD	Answering AW	All segments
France Telecom	7.92	23.73	5.73	3.25	**40.63**
Matra	7.50	10.29	4.79	2.26	**24.84**
Philips	4.22	2.39	4.24	2.65	**13.50**
Alcatel	3.18	4.02	2.46	1.08	**10.74**
Modulophone	0.21	0.82	0.55	0.08	**1.66**
Comoc	2.14	1.71			**3.85**
HPF	0.21	0.29			**0.50**
Téfal	0.88	0.64			**1.52**
Radialva		0.20			**0.20**
Dialatron	1.90	0.66			**2.56**
All firms	**28.16**	**44.75**	**17.77**	**9.32**	**100.00**

Note: All figures are obtained from the marketing institute GFK.

If product j is new, $\hat{D}_j = \hat{S}_j^t$. Of course equation (21) applies only to products that are sold in period t. Note that for all products sold in 1992, our estimated demands turn out to be positive.

The model predicts a volume of total sales up to 2.88 millions of telephones. This number should be compared to the estimation of 3.06 millions telephones sold in 1992, according to GFK. Table 4 provides the 1992 estimated market shares as percent of total sales by market segment and firm, while observed shares are given in Table 5. Again we observe that the model behaves quite well. These simulations permit also to analyze the renting side of the market. In 1992, the stock of rented telephones has increased slightly by 2.3 percent from a stock of 17.25 millions in 1991. However, as the estimated proportion of rented telephones decreases from 50.86 percent in 1991 to 47.43 in 1992, our estimate agrees with the facts as reported by marketing experts.

5.2. Price elasticities

As explained in Section 2, the structure of the utility function does not impose unrealistic substitution patterns of aggregate demands. This can be easily checked by a glance at Table 6 that displays the own and cross-price elasticities of demand for some models of telephone. The elasticity of product j's demand with respect to the 1992 price of product q is computed according to $\partial \ln \hat{D}_j \big/ \partial \ln f_q = \left(f_q \Big/ \left(\hat{S}_j^t - \hat{S}_j^{t-1} \right) \right) \left(\partial \hat{S}_j^t \big/ \partial f_q \right)$. On average, individuals perform substitution among products that provide them with comparable level of utility. As the type of a product plays an important role in the discrete choice, products are generally more substitutable within segments. For instance, the cross-elasticities on the two-block segment are between 3 and 5 percent while they are lower with telephones of other segments. Note also that the own price elasticities increase (resp. decrease) with the quality (price) of telephones.

Own and cross-price elasticities on stocks (not reported here) can also be computed. They are less sensitive to prices while they follow a similar path of substitution as the own and cross-price elasticities of market shares.

Table 6: Own and cross-price elasticities

	AW1	AW1	CD1	CD2	CD3	OB1	OB2	OB3	OB4	OB5	OB6	TB1	TB2	TB3	TB4	TB5	TB6
AW1	-38.5	1.99	1.02	0.96	1.23	0.19	0.2	0.27	0.18	0.15	0.18	0.27	0.16	0.14	0.21	0.18	0.2
AW2	1.99	-38.67	1.01	0.95	1.22	0.18	0.2	0.29	0.18	0.14	0.17	0.26	0.15	0.14	0.2	0.17	0.19
CD1	1.38	1.36	-34.59	1.64	1.75	0.1	0.12	0.15	0.1	0.08	0.1	0.16	0.08	0.07	0.12	0.09	0.11
CD2	1.23	1.22	1.71	-33.94	1.68	0.08	0.1	0.13	0.09	0.06	0.09	0.13	0.05	0.04	0.09	0.06	0.08
CD3	1.02	1.05	1.48	1.48	-52.73	0.13	0.15	0.33	0.14	0.11	0.13	0.22	0.15	0.13	0.18	0.16	0.18
OB1	0.22	0.24	0.026	0.08	0.1	-153.1	8.59	8.51	8.39	7.99	7.28	2.48	1.79	1.69	2.06	1.93	1.99
OB2	0.21	0.31	0.024	0.06	0.09	7.31	-258.4	6.37	9.25	6.85	9.14	2.37	1.67	1.58	1.95	1.82	1.87
OB3	0.23	0.13	0.026	0.12	0.21	8.54	8.68	-142.4	8.48	8.08	5.37	3.55	1.86	1.76	3.13	1.89	2.06
OB4	0.2	0.2	0.024	0.11	0.19	8.2	8.75	4.26	-219.5	7.74	8.03	2.28	1.59	1.49	1.86	1.73	1.79
OB5	0.17	0.08	0.022	0.051	0.18	7.68	8.12	7.03	7.91	-148.8	7.8	2.6	1.41	1.31	2.68	1.55	1.61
OB6	0.09	0.19	0.022	0.1	0.19	8.05	10.19	7.11	7.99	7.58	-233.8	2.16	1.46	1.37	1.74	1.61	1.66
TB1	0.26	0.27	0.034	0.03	0.14	2.6	2.57	2.63	1.52	1.41	1.46	-119.1	3.63	3.58	3.77	3.7	3.73
TB2	0.19	0.18	0.052	0.12	0.13	2.25	2.22	1.28	1.17	1.06	1.11	3.93	-87.2	3.23	2.42	3.35	3.39
TB3	0.17	0.16	0.07	0.11	0.29	1.2	1.18	1.24	1.12	1.02	1.07	3.59	3.23	-77.58	2.38	3.3	3.34
TB4	0.22	0.25	0.04	0.08	0.14	2.69	2.36	2.42	1.31	1.2	1.25	3.77	3.42	3.36	-116.5	3.49	3.52
TB5	0.2	0.33	0.09	0.14	0.32	1.13	1.04	1.36	0.95	1.14	1.19	3.71	3.36	3.72	3.5	-72.87	3.46
TB6	0.2	0.21	0.038	0.015	0.05	3.34	2.32	3.58	2.26	1.16	1.21	4.73	2.38	2.32	4.63	3.44	-132.3
Price	1413	1386	1450	1363	894	282	157	309	185	273	171	369	459	586	353	576	304

Note: Cell entries (*i.j*). where *i* indexes row and *j* column, give the percentage change in market share of *i* with a change in the price of *j* by an amount of FF 50. The last row gives the 1992 prices in French Francs of the products.

5.3. Market equilibrium

Provided that we adopt assumptions on marginal cost structure and on the conduct of firms, we are able to compute markups from the estimated demand system. Assume that, for a given commodity produced by a given firm, the marginal cost is constant and independent of marginal costs of other products proposed by this firm. Moreover, assume that each firm plays a Nash strategy in prices. These assumptions are relaxed in Foncel (1998).

Consider a firm *g* that provides the set of telephones Ω_g in 1992. Its strategy consists of choosing prices $(f_j)_{j\in\Omega_g}$ of its products given the prices of competitors.[20] Note that the set of all products and their attributes are exogenous. Assume that the marginal cost of product *j* can be written $\overline{mc}_j = mc_j + w_j$ where mc_j is the deterministic part of marginal cost (which is a function of product attributes and parameters) and where the component w_j is not observable for the econometrician and probably correlated with unobserved quality. Firm *g* maximizes expected profits, i.e., solves

$$\underset{(f_j)_{j\in\Omega_g}}{Max}\ E\left[\sum_{j\in\Omega_g}\left(f_j - mc_j + w_j\right)\ \left(\hat{D}_j(f) + c_j\right)\right], \tag{22}$$

where $\hat{D}_j(f)$ is the estimated demand for product *j*, which depends on the vector *f* of product prices (and on all other exogenous variables that are omitted for convenience), and c_j is a random term relative to product *j*, related to the non observable attributes of this product, in particular the unobservable perceived quality. This last term is introduced to represent what it is not explained by the model. Recall that, when prices are correlated with unobservable attributes of products (i.e., with a term like c_j) on the demand side, there is a potential problem of endogeneity, which may prevent to find a correct solution to the first order conditions associated with the program defined by equation (22). Neglecting this endogeneity problem has statistical implications when one estimates a system of demand and supply with aggregate data. As already noticed by Goldberg (1995), this problem is alleviated when demand and supply are estimated sequentially on micro data as here. Indeed, because of our specification, our estimated demands account correctly for the perceived quality of products by the consumers. Hence it is reasonable to assume that c_j is not correlated with the unobservable perceived quality, and hence not correlated with w_j (or that the correlation between c_j and w_j is negligible). Then, a good approximation of the program solved by firm *g* is

$$\underset{(f_j)_{j\in\Omega_g}}{Max}\ \sum_{j\in\Omega_g}\left(f_j - mc_j\right)\ \hat{D}_j(f). \tag{23}$$

[20] The operator France Telecom has an asymmetric role in this game. We assume here that its choice concerning the price of telecommunications is not related to the competition on the differentiated product market. Its strategy can be analyzed conditionally to the given tariff of telecommunications.

This leads to the first order condition[21]

$$\hat{D}_j(f) + \sum_{r\in\Omega_g}\left(f_r - mc_r\right)\frac{\partial \hat{D}_r(f)}{\partial f_j} = 0\,,\ \ \forall\ \ j \in \Omega_g\,. \tag{24}$$

Define by Δ a $J \times J$ matrix whose element (*i.j*) is such that $\Delta_{ij} = \partial \hat{D}_i(f) / \partial f_j$, with J the number of products. Let E be the $J \times J$ matrix with generic element (*i.j*) such that $\mathrm{E}_{ij} = 1$ if products *i* and *j* are produced by the same firm and 0 otherwise. In matrix notations, equations (24) is written

$$\hat{D}(f) + (\Delta'.*\mathrm{E})(f - mc) = 0\,. \tag{25}$$

where the operator $.*$ defines the element-by-element matrix multiplication and *mc* is the vector of marginal costs.

Assuming that $\Delta'.*\mathrm{E}$ is invertible, the vector of markups evaluated at the observed prices is obtained as

$$MK = -(\Delta'.*\mathrm{E})^{-1}\hat{D}(f)\,. \tag{26}$$

For some models of telephone, Table 7 provides the associated markup, this markup as a percent of the price and the profit (expressed in millions of French Francs) which is simply the markup times the estimated sales.

At the segment level, the average ratio of markups to prices are equal to 31.46 percent for answering telephones, 34.09 percent for cordless telephones, 19.03 percent for two-blocks type and 15.27 percent for one-block type. These numbers seems high although the pattern is plausible and agrees with a common feature of differentiated products markets: The higher the ratios, the higher the quality.

Concerning profits now (see last column of Table 7), note that products in segments with an intensive competition in terms of number of products (like OB and TB) are generally associated with lower level of profits than the other segments (CD and AW). While it is often observed in market studies that high markups come with small market shares, here they are combined with relatively large market shares. In our case, demand for high-quality products is rising while competition is not so fierce since the market is just experiencing deregulation and the number of sophisticated telephone models remains limited. After 1992, many new high-quality models appear (like telephones with both answering and cordless functions) and prices decreased significantly.

The most striking result is the very large profit generated by the two-blocks RONDO which was a very popular telephone and which was also proposed for renting from 1993. Table 8 below confirms the leadership of France Telecom, Matra and Philips in terms of profits as in terms of shares. Note that the high quality segments are very profitable (especially the cordless one).

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[21] Checking that second order conditions hold ensures that estimated parameters are consistent with the existence of the equilibrium even if unicity is not guaranteed (see also Berry, Levinshon and Pakes, 1998, Feenstra and Levinshon, 1995, Petrin, 1999).

Table 7: Estimated markups and profits by model of telephone

Brand	Model	Type	Price	% Markup	Markup	Profits
MATRA	RIP30	AW	1413.84	31.34	443.10	6.73
PHILIPS	TD9460C	AW	1386.26	34.90	483.80	6.95
MATRA	AMPLITE	CD	1450.43	36.55	530.13	5.88
ALCATEL	DAYTONR	CD	1363.80	31.20	425.51	2.39
PHILIPS	TD9230	CD	894.93	28.20	252.37	4.45
ALCATEL	SURFMEM	OB	282.38	16.88	47.67	1.19
COMOC	DAND101	OB	157.01	13.11	20.58	0.24
MATRA	VOILE10	OB	309.64	16.13	49.94	2.33
MODULOPHONE	MP2020T	OB	185.19	12.82	23.74	0.68
TEFAL	COMPAC2	OB	273.96	12.84	35.18	0.49
DIALATON	SCANDAS	OB	171.45	13.09	22.44	0.24
FRANCE TELECOM	DUO	TB	369.39	22.73	83.96	7.68
FRANCE TELECOM	RONDO	TB	459.00	24.36	111.81	8.70
MATRA	ADVENT1	TB	586.77	26.71	156.73	2.35
MATRA	CONTACP	TB	353.37	21.86	77.25	1.38
MATRA	TM1	TB	576.30	26.83	154.62	3.51
MODULOPHONE	DYNASTM	TB	304.55	18.53	56.43	0.17

Table 8: Estimated profits by segment and by firm

Market segment Firms	One-block OB	Two-block TB	Cordless CD	Answering AW	All segments
France Telecom	9.81	79.93	44.83	22.26	156.83
Matra	8.23	28.37	62.17	22.23	121.00
Philips	8.82	12.34	40.42	43.62	105.20
Alcatel	5.93	10.48	13.70	8.44	38.55
Modulophone	1.48	6.11	1.29	2.39	11.27
Comoc	0.24	6.06			6.30
HPF	0.82	3.83			4.65
Téfal	0.84	0.66			1.50
Radialva		0.40			0.40
Dialatron	0.57				0.57
All firms	36.74	148.18	162.41	98.94	446.27

Note: Unit is million of French Francs.

Finally, look at the profits of the historic operator on its rented products as given in Table 9. Clearly, renting produces a lot of profits for France Telecom. Total profits for the period based on the stock of five rented products amount to 682.29 millions of French Francs whereas the aggregate annual profits on sales (with 134 products) is around 446.27 millions. This may explain the toughness of competition after 1992.

Table 9: France Telecom's profits on renting

Models	Markup	Stock	Profits
S63	21.00	6.51	136.84
ALTO	34.88	3.71	129.43
CHORUS	82.50	1.65	136.43
FIDELIO	79.45	1.65	131.39
DIGITEL	36.12	4.10	148.20

Note: Unit is million of French Francs.

6- REMARKS AND EXTENSIONS

This study provides, using a direct utility approach, a structural analysis of the demand of telephone usage and equipment choice in a static setting. Based on survey data, our model explains how the stock of telephones results from consumer choices. It fits for the main features of durable goods. Different problems are addressed, from the dimensionality of the choice set, the definition of prices in a static approach and the characterization of vertical and horizontal differentiation, to the correctness of substitution patterns, and the question of the different distribution strategies (selling or renting). This is particularly useful in marketing studies that are aimed at defining product and price strategies according to the actual marketed products but also according to products yet in the stock. In some sense, our model accounts for the fact that durable goods create their own competition. It could be easily applied to similar instances like cellular phones. However, it is also relevant for various markets, like the software market where the installed base plays a crucial role.

Several by-products of this analysis can be developed. First, one can estimate marginal costs for each product as a function of observable attributes of products and one can avoid the assumption of constant marginal costs. Second, one can test for the conduct of firms. Most applications posit a Nash assumption except Gasmi, Laffont, Vuong (1992) and Feenstra and Levinshon (1995). Testing for different firm behavior is crucial in order to compute correct markups. (See Foncel, 1998.) Finding the relevant equilibrium concept should also be required in order to measure the effect of new products on welfare as in Petrin (1999) or to analyze mergers or anti-competitive practices from an antitrust point of view.

The main advantage of our approach lies in the use of micro data. This explains the richness and also the complexity of the econometric modeling. Methodological extensions of our mixed continuous-discrete choice model are on the research agenda. For instance, one could develop a framework that enables us to estimate simultaneously demand and supply along the line of Berry, Levinshon and Pakes (1998). One would like also to address the question of the joint choice of quality and price, while here quality is considered as given. Finally, this study also tells us that, although our static setting turns out to be quite fruitful, econometrics of differentiated durable goods markets should greatly benefit from a true dynamic approach.

APPENDIX 1: Proof of Proposition 1

The first task is to define the market share for any product. Denoting by s_{jk} the market share of equipment (j,k), we have that

$$s_{jk} = \frac{1}{N}\sum_{n=1}^{N} P_{njk} \ , \tag{A1.1}$$

where P_{njk} is defined in equation (7). If $\aleph$ is the number of households in the population, the total demand for equipment (j,k) is $S_{jk} = \aleph s_{jk}$. Now, taking into account the symmetry of alternatives, the total stock for product j is $S_j = S_{jj} + \sum_{k \in C^*} S_{jk}$, where C^* is the set C of all telephones to which one adds the *nil* telephone. The market share of product j is then $s_j = S_j \Big/ \sum_{j' \in C} S_j$ or

$$s_j = \frac{s_{jj} + \sum_{k\in C^*} s_{jk}}{\sum_{j'\in C}\left(s_{j'j'} + \sum_{k\in C^*} s_{j'k}\right)} = \frac{\sum_{n=1}^{N}\left(P_{njj} + \sum_{k\in C^*} P_{njk}\right)}{\sum_{n=1}^{N}\sum_{j'\in C}\left(P_{nj'j'} + \sum_{k\in C^*} P_{nj'k}\right)}. \tag{A1.2}$$

Now we compute the cross price-elasticity of product j with respect of product q. We have the following expressions (for a given n)

$$\frac{\partial P_{njj}}{\partial F_q} = \frac{-m\exp\left(mW_{njj}\right)\left(-2h\exp\left(mW_{nqq}\right) - h_n \sum_{k\in C^*\setminus\{q\}}\exp\left(mW_{nqk}\right)\right)}{\left(\sum_{(j'',k'')\in E}\exp\left(mW_{nj''k''}\right)\right)^2} = mh_n P_{njj}\left(P_{nqq} + \sum_{k\in C^*} P_{nqk}\right),$$

$$\frac{\partial\left(\sum_{k\in C^*} P_{njk}\right)}{\partial F_q} = mh_n\left[\left(P_{nqq} + \sum_{k\in C^*} P_{nqk}\right)\sum_{k\in C^*} P_{njk} - P_{njq}\right].$$

Hence

$$\frac{\partial}{\partial F_q}\left[\sum_{n=1}^{N}\left(P_{njj} + \sum_{k\in C^*} P_{njk}\right)\right] = m\sum_{n=1}^{N} h_n\left[\left(P_{njj} + \sum_{k\in C^*} P_{njk}\right)\left(P_{nqq} + \sum_{k\in C^*} P_{nqk}\right) - P_{njq}\right],$$

and

$$\frac{\partial}{\partial F_q}\left(\sum_{j'\in C} P_{nj'j'}\right) = mh_n\left[\left(P_{nqq} + \sum_{k\in C^*} P_{nqk}\right)\left(\sum_{j'\in C} P_{nj'j'}\right) - 2P_{nqq}\right],$$

$$\frac{\partial}{\partial F_q}\left(\sum_{j'\in C}\sum_{k\in C^*} P_{nj'k}\right) = mh_n\left\{-\sum_{j'\in C} P_{nj'q} + \left(P_{nqq} + \sum_{k\in C^*} P_{nqk}\right)\sum_{j'\in C}\sum_{k\in C^*} P_{nj'k} - \sum_{k\in C^*} P_{nqk}\right\}.$$

So we have

$$\frac{\partial s_j}{\partial F_q} = \frac{\sum_{n=1}^{N}\left[\frac{\partial P_{njj}}{\partial F_q} + \frac{\partial\left(\sum_{k\in C^*} P_{njk}\right)}{\partial F_q}\right]\sum_{n=1}^{N}\sum_{j'\in C}\left(P_{nj'j'} + \sum_{k\in C^*} P_{nj'k}\right)}{\left[\sum_{n=1}^{N}\sum_{j'\in C}\left(P_{nj'j'} + \sum_{k\in C^*} P_{nj'k}\right)\right]^2}$$

$$- \frac{\sum_{n=1}^{N}\left(P_{njj} + \sum_{k\in C^*} P_{njk}\right)\sum_{n=1}^{N}\left[\frac{\partial}{\partial F_q}\left(\sum_{j'\in C} P_{nj'j'}\right) + \frac{\partial}{\partial F_q}\left(\sum_{j'\in C}\sum_{k\in C^*} P_{nj'k}\right)\right]}{\left[\sum_{n=1}^{N}\sum_{j'\in C}\left(P_{nj'j'} + \sum_{k\in C^*} P_{nj'k}\right)\right]^2}.$$

This can be rewritten

$$\frac{\partial s_j}{\partial F_q} = \frac{m\sum_{n=1}^{N} h_n \left[\left(P_{njj} + \sum_{k\in C^*} P_{njk} \right)\left(P_{nqq} + \sum_{k\in C^*} P_{nqk} \right) - P_{njq} \right]}{\sum_{n=1}^{N} \sum_{j'\in C} \left(P_{nj'j'} + \sum_{k\in C^*} P_{nj'k} \right)}$$

$$- s_j \frac{m\sum_{n=1}^{N} h_n \left[-\sum_{j'\in C} P_{nj'q} + \left(P_{nqq} + \sum_{k\in C^*} P_{nqk} \right)\left(-1 + \sum_{j'\in C} P_{nj'j'} + \sum_{j'\in C}\sum_{k\in C^*} P_{nj'k} \right) - P_{nqq} \right]}{\sum_{n=1}^{N} \sum_{j'\in C} \left(P_{nj'j'} + \sum_{k\in C^*} P_{nj'k} \right)}.$$

Finally:

$$\frac{\partial s_j}{\partial F_q}\frac{F_q}{s_j} = F_q \frac{m\sum_{n=1}^{N} h_n \left[\left(P_{njj} + \sum_{k\in C^*} P_{njk} \right)\left(P_{nqq} + \sum_{k\in C^*} P_{nqk} \right) - P_{njq} \right]}{\sum_{n=1}^{N} \left(P_{njj} + \sum_{k\in C^*} P_{njk} \right)}$$

$$- F_q \frac{m\sum_{n=1}^{N} h_n \left[-\sum_{j'\in C} P_{nj'q} + \left(P_{nqq} + \sum_{k\in C^*} P_{nqk} \right)\left(-1 + \sum_{j'\in C} P_{nj'j'} + \sum_{j'\in C}\sum_{k\in C^*} P_{nj'k} \right) - P_{nqq} \right]}{\sum_{n=1}^{N} \sum_{j'\in C} \left(P_{nj'j'} + \sum_{k\in C^*} P_{nj'k} \right)}.$$

As this expression depends on j, the result is proved. By the same token, own-price elasticities can be computed. (See Foncel, 1997.)

APPENDIX 2: Proof of Proposition 2

i) The joint probability $\Pr(E_{mn}, x_n, D_n)$ of drawing a level of usage x_n, a subset D_n and an alternative which is known to belong to the group E_{mn}, is given by

$$\Pr\left[\bigcup_{(j,k)\in E_{mn}} \left\{ V_{njk} + e_{njk} \geq V_{nj'k'} + e_{nj'k'}, \forall (j',k') \in E \text{ and } (j',k') \neq (j,k) \right\} \cap \{x_n\} \cap \{D_n\} \right].$$

Then,

$$\Pr(E_{mn}, x_n, D_n) = \sum_{(j,k)\in E_{mn}} \Pr(j,k,x_n,D_n).$$

However,

$$\Pr(j,k,x_n,D_n) = \Pr(D_n | j,k,x_n) \Pr(j,k,x_n).$$

Given the sampling procedure, the way alternatives are selected is independent of the consumption level. So we have $\Pr(D_n | j,k,x_n) = \Pr(D_n | j,k)$. By Bayes' theorem,

$$\Pr(j,k,x_n | D_n) = \frac{\Pr(D_n | j,k) \Pr(j,k,x_n)}{\Pr(D_n)}.$$

However,

$$\Pr(D_n) = \int_x \sum_{(j,k)\in E} \Pr(D_n | j,k) f_{njk}(x) P_{njk}\, dx,$$

where, based on our notations, we use that $\Pr(j,k,x) = f_{njk}(x) P_{njk}$. From Assumptions 2 and 3, we have

$$\Pr(D_n) = \sum_{(j,k)\in D_n} \Pr(D_n | j,k) P_{njk} \ .$$

Hence,

$$\Pr(j,k,x_n | D_n) = \frac{\Pr(D_n | j,k) P_{njk} f_{njk}(x_n)}{\sum_{(j',k')\in D_n} P_{nj'k'} \Pr(D_n | j',k')} \ .$$

Then,

$$\Pr(E_{mn}, x_n | D_n) = \sum_{(j,k)\in E_{mn}} \frac{\Pr(D_n | j,k) P_{njk} f_{njk}(x_n)}{\sum_{(j',k')\in D_n} P_{nj'k'} \Pr(D_n | j',k')} \ ,$$

which leads to the expression of the likelihood function given in equation (10).

ii) Now consider the limit problem of the log-likelihood function in equation (10)

$$L = \int_{\bar v}\int_x \sum_{m=1}^{M} \sum_{D\subset E} \Pr(E_m, x, D, \bar v; \Gamma^0) \ln\left[\sum_{(j,k)\in E_m} \frac{\exp(m W_{jk}(\bar v;\Gamma_1)) \Pr(D|\bar v, j,k) f_{jk}(x;\Gamma_2)}{\sum_{(j',k')\in D} \exp(m W_{j'k'}(\bar v;\Gamma_1)) \Pr(D|\bar v, j',k')}\right] dx\, d\bar v \ ,$$

where $\bar v$ denotes the vector of all exogenous variables of an individual and Γ_1 and Γ_2 are two subsets of parameters to be estimated such that $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \{m\}$. Define

$$B(\bar v, x, D, m; \Gamma) = \sum_{(j,k)\in E_m} \frac{\exp(m W_{jk}(\bar v;\Gamma_1)) \Pr(D|\bar v, j,k) f_{jk}(x;\Gamma_2)}{\sum_{(j',k')\in D} \exp(m W_{j'k'}(\bar v;\Gamma_1)) \Pr(D|\bar v, j',k')} \ .$$

The limit problem can be written

$$L = \int_{\bar v}\int_x \sum_{D\subset E} \sum_{m=1}^{M} \sum_{(j,k)\in E_m} \Pr(D|\bar v, j,k) \Pr(j,k|\bar v, \Gamma_1^0) f_{jk}(x|\bar v, \Gamma_2^0) \ln B(\bar v, x, D, m; \Gamma)\, dx dG(\bar v)$$

with G the distribution of $\bar v$. Define

$$A(\bar v, D; \Gamma^0) = \frac{\sum_{(j',k')\in D} \exp(m W_{jk}(\bar v;\Gamma_1^0)) \Pr(D|\bar v, j',k')}{\sum_{(j',k')\in E} \exp(m W_{j'k'}(\bar v;\Gamma_1^0))} \ .$$

Hence, we have

$$L = \int_{\bar v}\left[\int_x \sum_{D\subset E} \sum_{m=1}^{M} \sum_{(j,k)\in E_m} A(\bar v, D; \Gamma^0) \frac{\exp(m W_{jk}(\bar v;\Gamma_1^0)) \Pr(D|\bar v, j,k) f_{jk}(x;\Gamma_2^0)}{\sum_{(j',k')\in D} \exp(m W_{j'k'}(\bar v;\Gamma_1^0)) \Pr(D|\bar v, j',k')} \ln B(\bar v, x, D, m; \Gamma)\, dx\right] G(\bar v)\, d\bar v.$$

Then,

$$L = \int_{\bar v}\left[\int_x \sum_{D\subset E} A(\bar v, D; \Gamma^0) \sum_{m=1}^{M} B(\bar v, x, D, m; \Gamma^0) \ln B(\bar v, x, D, m; \Gamma)\, dx\right] G(\bar v)\, d\bar v \ ,$$

and

$$L = \int_{\bar v}\left[\sum_{D\subset E} A(\bar v, D; \Gamma^0) \sum_{m=1}^{M} \int_x B(\bar v, x, D, m; \Gamma^0) \ln B(\bar v, x, D, m; \Gamma)\, dx\right] G(\bar v)\, d\bar v \ .$$

LEMMA 1 (Manski and McFadden, 1981): *Let* $B(s,\Gamma)$ *be a real value function over a space* $S \times \Omega$ *such that B is integrable with respect to a measure* H *over S and* $B(s,\Gamma) \geq 0$*, all* $s \in S, \Gamma \in \Omega$*. Let* Γ^0 *be an element of* Ω *such that* $B(s,\Gamma^0) > 0$ *for almost every* $s \in S$ *and*

$$\int_s \left(B(s,\Gamma^0) - B(s,\Gamma)\right) dH \geq 0, \quad \textit{for all } \Gamma \in \Omega .$$

Then the expression $\int_s \left(B(s,\Gamma^0) - B(s,\Gamma)\right) dH$ *attains its maximum at* $\Gamma = \Gamma^0$*. The maximum is unique if, for every* $\Gamma \in \Omega$ *such that* $\Gamma \neq \Gamma^0$*, there exists* $S_\Gamma \subset S$ *such that* $\int_{S_\Gamma} B(s,\Gamma) dH \neq \int_{S_\Gamma} B(s,\Gamma^0) dH$ *.*

LEMMA 2: *Suppose that* $y^0 = \arg\max_y f(y,z)$ *and* $q(y^0,z) > 0$*, all* $z \in Z$*. Then the expression* $\int_z q(y^0,z) f(y,z) dH$ *attains its maximum at* $y = y^0$*.*

PROOF: Suppose $y^1 = \arg\max_y \int_z q(y^0,z) f(y,z)\, dH$. Then,

$$\int_z q(y^0,z) f(y^1,z) dH \geq \int_z q(y^0,z) f(y^0,z) dH \text{ , and } \int_z q(y^0,z)\left[f(y^1,z) - f(y^0,z)\right] dH \geq 0 .$$

This implies that there exists $z \in Z$ such that $f(y^1,z) \geq f(y^0,z)$ which contradicts the assumption.

From Lemma 1, the expression $\sum_{m=1}^{M} \int_x B(\bar{v},x,D,m;\Gamma^0) \ln B(\bar{v},x,D,m;\Gamma)\, dx$ attains its maximum at $\Gamma = \Gamma^0$, for all $(D,\bar{v})$. It is just required to take an appropriate measure composed by a Lebesgue measure over *x* and a discrete one over *m*. From Lemma 2, consistency is achieved by taking the appropriate measure over *D* and *v*.

APPENDIX 3: Variable definitions

A3.1. Individual variables

All these variables enter the heterogeneity index, q .

URBA1 : takes value 1 if the household lives in a city with a population size larger than 1,000,000 inhabitants and 0 otherwise.

URBA2 : takes value 1 if the household lives in a city with a population size between 100,000 and 1,000,000 inhabitants and 0 otherwise.

URBA3 : takes value 1 if the household lives in a city with a population size between 20,000 and 100,000 inhabitants and 0 otherwise.

URBA4 : takes the value 1 if the household lives in a city with a population size lower than 100,000 inhabitants or in a rural area (the parameter relative to this variable is normalised to 0).

NUMB : is the number of persons in the household.

SES1 : takes value 1 if the household head is an upper level white collar.

SES2 : takes value 1 if the household head is a lower level white collar.

SES3 : takes value 1 if the household head is a blue collar, a farmer or a craftsman.

SES4 : takes value 1 if the household head is unproductive or pensioned off (the parameter relative to this variable is normalised to 0).

MINIT : takes value 1 if the household owns a teletext terminal.

A3.2. Income variables

These variables correspond to vector $(y_i)_{i=1,...,I}$ in the model.

INC1 : takes value 1 if household's annual income *y* is lower than FF 84,000.
INC2 : takes value 1 if household's annual income is between FF 84,000 and FF 165,000.
INC3 : takes value 1 if household's annual income is between FF 165,000 and FF 270,000.
INC4 : takes value 1 if household's annual income is greater than FF 270,000.

3. Product variables

All these variables (i.e., the vector $(b_l)_{l=1,...,L}$) enter the quality index y .

OB : takes the value 1 if the telephone is made of one block and is not cordless.
TB : takes the value 1 if the telephone is made of two blocks and has not the answering function.
CD : takes the value 1 if the telephone is cordless.
AW : takes the value 1 if the telephone has the answering function.
MEM : number of available memories.
AMPL : takes the value 1 if possible to amplify the sound.
SCRE : takes the value 1 if possible to show the last telephone number dialed.
NMD : takes the value 1 if possible to dial directly.
VOL : takes the value 1 if possible to modulate listening.

APPENDIX 4: Descriptive statistics

Table A.1: Descriptive Statistics on Individual Variables

Variables	Mean	Standard Error
INC1	0.293	0.455
INC2	0.407	0.491
INC3	0.196	0.397
INC4	0.104	0.306
URBA1	0.169	0.374
URBA2	0.265	0.441
URBA3	0.333	0.472
URBA4	0.233	0.423
NUMB	2.978	1.455
SES1	0.116	0.320
SES2	0.179	0.383
SES3	0.397	0.489
SES4	0.308	0.462
MINIT	0.251	0.434
Number of rented phones by equipment (a)	1.034	0.625

Table A.2: Descriptive Statistics on Product Variables

Variables	Mean	Standard Error
OB	0.213	0.410
TB	0.529	0.499
CD	0.147	0.354
AW	0.110	0.313
MEM	0.669	2.022
AMPL	0.397	0.489
SCRE	0.037	0.188
NMD	0.199	0.399
VOL	0.067	0.083

Table A.3: Mean Prices in French Francs

Type of telephone	Mean	Standard Error
OB	265.40	28.55
TB	445.67	131.17
CD	1328.99	113.74
AW	1400.99	77.89
Whole market	642.50	469.69

Table A.4: Product Repartition by Brand and Type

	OB	TB	AW	CD
Alcatel	6	12	3	4
Comoc	1	12	0	0
Dialaton	2	0	0	0
FrTelecom	1	6	1	2
HPF	2	4	0	0
Matra	5	9	3	6
Modulophone	3	13	1	1
Philips	6	6	7	7
Radialva	0	2	0	0
Tefal	2	2	0	0

Note: Telephones rented by France Telecom are not included.

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