Ecological discounting

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Abstract

Which rates should we use to discount costs and benefits of different natures at different time horizons? We answer this question by considering a representative agent consuming two goods whose availability evolves over time in a stochastic way. We extend the Ramsey rule by taking into account the degree of substitutability between the two goods and of the uncertainty surrounding the economic and environmental growths. The rate at which environmental impacts should be discounted is in general different from the one at which monetary benefits should be discounted. We provide arguments in favor of an ecological discount rate smaller than the economic discount rate. In particular, we show that, under certainty and Cobb-Douglas preferences, the difference between the economic and the ecological discount rates equals the difference between the economic and the ecological growth rates. Using data about the link between biodiversity and economic development, I estimate that the rate at which changes in biodiversity should be discounted is 1.5%, whereas changes in consumption should be discounted at 3.2%.

Keywords: Discounting, Ramsey rule, bivariate utility function, prudence, sustainable development.

JEL Classification: G12, E43, Q51

1 Introduction

The current debate on the intensity of the global effort to fight climate change has focused much on the choice of the discount rate, which is crucial to answer this question because of the long term nature of the impacts of the reduction of emissions of most greenhouse gases. Since Ramsey (1928), we know that the main economic justification of discounting is based on a wealth effect. If one believes that future generations will be wealthier than us, one more unit of consumption is more valuable to us than to them, under decreasing marginal utility of consumption. However, a large fraction of the impacts of climate change affects the quality of the environment (increased temperature, reduced biodiversity, or destruction of environmental assets for example) rather than consumption. In this paper, we address the question of how one should discount future changes in the quality of the environment. If we believe that the environment is deteriorating over time, and if we assume that the marginal utility of the quality of the environment is decreasing, then increasing the environmental quality is more valuable to future generations than to us. This argument, which is symmetric to the Ramsey's wealth effect, is in favour of using a smaller discount rate for changes in the environment than for changes in consumption. The full characterization of this "ecological" discount rate should also take into account of the substitutability between environmental assets and consumption, and of the uncertainty that affects the dynamics of consumption and of the environment. This paper provides a full description of the determinants of the ecological discount rate.

There are two possible methods to evaluate the present monetary value of a sure future environmental impact. The classical one consists in first measuring the future monetary value of the impact, and second discounting this monetary equivalent impact to the present. This involves a pricing formula to value future changes in environmental quality, and an economic discount rate to discount these monetarized impacts. As first suggested by Malinvaud (1953), the second approach consists in first discounting the future environmental impact to transform it into an immediate equivalent environmental impact, and then measuring the monetary value of this immediate impact. This involves an ecological discount rate, to discount environmental impacts. Of course, these two methods are strictly equivalent. As shown by Guesnerie (2004), Weikard and Zhu (2005) and Hoel and Sterner (2007) in the case of certainty, the two discount rates differ if the monetary value of environmental assets evolves over time.

The classical method is not well adapted to the case of uncertainty. Indeed, the value of environmental assets in the future depends upon their relative scarcity, which is unknown. This is a problem because the economic discount rate is useful to discount sure future monetary benefits. Because the monetary value of environmental impacts is uncertain, one needs to compute its certainty equivalent. This requires the use of a stochastic discount factor, which determines at the same time the risk premium and the economic discount rate. Standard pricing formulas exist that can be borrowed from the theory of finance, but they are seldom used in cost-benefit analyses of environmental projects because of their complexity. In this paper, we follow the alternative methods based on the ecological discount rate. The ecological discount factor associated to date t is the immediate sure environmental impact that has the same impact on intergenerational welfare than a unit environmental impact at date t. The (shadow) price of an immediate environmental impact can then be used to value environmental projects. This alternative method is simpler because one does not need to compute certainty equivalent future values.

The efficient economic (resp. ecological) discount rate equals the marginal rate of substitution between future and present consumption (resp. environmental qualities). If the quality of the environment improves with time, and if the marginal utility of the quality of the environment is decreasing, this environmental growth effect justifies a positive ecological discount rate. On the contrary, if one believes that the quality of the environment will deteriorate over time, a negative ecological discount rate may be socially efficient. However, assuming that consumption is a substitute to the quality of the environment, economic growth has a positive impact on the ecological discount rate, thereby potentially counterbalancing the effect of the deterioration of the environment. As observed for example by Traeger (2007), the possibility to substitute the deteriorating environment quality by other goods is at the core of the notion of sustainable development. If the substitutability is limited, the environmental deterioration effect dominates the economic growth effect, and the ecological discount rate should be small or negative, thereby inducing us to preserve environmental assets.

Following Weitzman (2007) and Gollier (2002, 2007), we consider a consumption-based theory of discount rates under uncertainty. Uncertainty adds new elements into the picture. Besides the growth effect and the substitution effect, there is a precautionary effect. The uncertainty associated to the future quality of the environment reduces the ecological rate if the marginal utility of the environment is convex in it. As explained in the paper, other third derivatives of the bivariate utility function also play a role in the determination of the ecological discount rate if the economic growth is uncertain.

Our analysis exhibits two arguments in favor of using an ecological discount rate smaller than the economic discount rate. Under certainty, we show that the difference between the economic and the ecological discount rates equals the difference between the economic and the ecological growth rates. A first argument is thus derived from the hypothesis that the growth of environmental quality is smaller than the economic growth. A second argument is based on the hypothesis that there is more uncertainty about the evolution of the environmental quality than on the evolution of the economy. The precautionary argument, which tends to reduce the discount rate, is thus stronger for the ecological discount rate.

An important question is to determine whether the ecological and the economic discount rates should be sensitive to the time horizon. Weitzman (2007) and Gollier (2007) have justified a decreasing term structure of the economic rate based on a learning effect in a model in which there is some parametric uncertainty affecting the growth process. We show that a similar result holds for the ecological discount rate in a model with a multi-attribute utility function when the sensitiveness of the environmental quality to changes in GDP per capita is uncertain. We believe that this argument is particularly relevant for the ecological discount rate, because of the considerable parametric uncertainty underlying the evolution of the quality of the environment.

Section 2 describes the intuitive assumptions that one needs to consider on preferences to sign the various determinants of the ecological discount rate: risk aversion, correlation aversion, prudence and cross-prudence. In section 3, we derive the pricing formulas for the economic and ecological discount rates. The 5 determinants of the ecological discount rate are described in Section 4, whereas Section 5 is devoted to the special case of the CES utility function under certainty. In Section 6, we derive an analytical solution to the ecological discount rate when the utility function is Cobb-Douglas, and the uncertainty is described by a bivariate brownian motion. From this benchmark, we explore various extensions, in particular the case of the more general CES utility function under uncertainty, or the case of parametric uncertainty.

2 The basic preference concepts

We consider a simple aggregate model with two goods. The first one is an aggregate consumption good, whereas the second one is an aggregate environmental good. The latter can be seen as a quality index of the environment, which includes the comfort generated from the climate, the services extracted from the biodiversity, the morbidity due to various pollutions, or the life expectancy for example. Discounting future costs and benefits in the context of two goods is driven by specific preference traits of the representative agent which are described in this section.

Variables x_{1t} and x_{2t} denote respectively the quantity of the consumption good consumed by the representative agent at date t, and the quality of the environment at that date. Let $l_i > 0$ and ε_i denote respectively any sure loss in x_i and any zero-mean risk in x_i, $i = 1, 2$. I assume that the representative agent is averse to consumption risks and to environmental risks, which means that for all (x_1, x_2) and all zero-mean risks $(\varepsilon_1, \varepsilon_2)$:

$$\begin{array}{rcl}(x_1, x_2) & \succsim & (x_1 + \varepsilon_1, x_2) \\ (x_1, x_2) & \succsim & (x_1, x_2 + \varepsilon_2).\end{array}$$

In various parts of the paper, we consider the following additional atemporal definitions on collective preferences:

- The representative agent is correlation-averse if he always prefers a 50-50 gamble of a loss in consumption or a loss in environmental quality over another 50-50 gamble offering a loss in neither dimension or a loss in both:

$$((x_1-l_1, x_2), 1/2; (x_1, x_2-l_2), 1/2) \succsim ((x_1, x_2), 1/2; (x_1-l_1, x_2-l_2), 1/2).$$

- The representative agent is prudent in consumption (environment) if risk in consumption (environment) can be tempered by a sure increase in consumption (environment). More precisely, prudence in consumption means that one always prefers a 50-50 gamble of a zero-mean risk in consumption or of a sure loss in consumption over a 50-50 gamble with nothing in one state, or the zero-mean risk in consumption combined with the sure loss in consumption in the other state:

$$((x_1+\varepsilon_1, x_2), 1/2; (x_1-l_1, x_2), 1/2) \succsim ((x_1, x_2), 1/2; (x_1+\varepsilon_1-l_1, x_2), 1/2).$$

Prudence in environmental quality is easily defined by symmetry:

$$((x_1, x_2+\varepsilon_2), 1/2; (x_1, x_2-l_2), 1/2) \succsim ((x_1, x_2), 1/2; (x_1, x_2+\varepsilon_2-l_2), 1/2).$$

- The representative agent is cross-prudent if risk in one dimension can be tempered by a sure increase in the other dimension. Namely, cross-prudence in consumption means that one always prefers a 50-50 gamble of a zero-mean risk in consumption or a sure loss in environmental quality over a 50-50 gamble with nothing in one state, or the zero-mean risk in consumption combined with the sure loss in environmental quality in the other state:

$$((x_1+\varepsilon_1, x_2), 1/2; (x_1, x_2-l_2), 1/2) \succsim ((x_1, x_2), 1/2; (x_1+\varepsilon_1, x_2-l_2), 1/2).$$

Symmetrically, cross-prudence in environmental quality is defined as follows:

$$((x_1, x_2+\varepsilon_2), 1/2; (x_1-l_1, x_2), 1/2) \succsim ((x_1, x_2), 1/2; (x_1-l_1, x_2+\varepsilon_2), 1/2).$$

Because a sure loss and a zero-mean risk are two "harms" for risk-averse agents, one can summarize the above definitions by saying that *the representative agent always prefer to incur one of the two harms for certain, with the only uncertainty being about which one will be received, as opposed to a 50-50 gamble of receiving the two harms simultaneously, or receiving neither.* Following a terminology introduced by Kimball (1993) and Eeckhoudt and Schlesinger (2006), pairs of harms are "mutually aggravating".

We hereafter assume that the representative agent is an expected-utility maximizer. His preferences are represented by a three times differentiable and increasing von Neumann-Morgenstern utility function $U : R^2 \to R$. In that framework, risk aversion means that U is concave in both dimensions. Eeckhoudt, Rey and Schlesinger (2007) have shown that the concepts of correlation aversion, prudence and cross-prudence are easy to characterize in this framework. We summarize their findings in the following proposition.

Proposition 1 *(Eeckhoudt, Rey and Schlesinger) In the expected utility framework with a bivariate utility function U, the different concepts of mutually aggravating harms are characterized by the sign of different derivatives of U :*

1. *U is correlation-averse if and only if U_{12} is non-positive;*
2. *U is prudent in consumption (environmental quality) if and only if U_{111} (U_{222}) is non-negative;*
3. *U is cross-prudent in consumption (environmental quality) if and only if U_{112} (U_{122}) is non-negative.*

We hereafter assume that this conditions are satisfied by the representative agent's preferences.

3 A model for efficient discount rates

At date $t = 0$, the representative agent evaluates actions by using the following utilitarian social welfare function:

$$V = E\left[\int_0^\infty e^{-\delta t} U(x_{1t}, x_{2t})dt\right], \tag{1}$$

where δ is an ethical parameter valuing future utils relative to current ones, and where E is the expectation operator that takes into account the fact that the pair (x_{1t}, x_{2t}) is uncertain at date $t = 0$. The representative agent contemplates the possibility to sacrifice some current utility either to increase consumption at date t or to improve environmental quality at that date. The first problem refers to the choice of the economic discount rate, which discounts future consumption. The second problem refers to the choice of the ecological discount rate, which discounts future changes in the environmental quality.

We first examine the economic discount rate. Let us consider a simple marginal project that would increase consumption by a sure amount ε in period $[t, t + \Delta t]$, and that would reduce consumption by $\varepsilon e^{-r(t)t}$ in period $[0, \Delta t]$, leaving the environment unaffected by the action. We assume that ε and Δt tend to zero. Observe that this simple project has a sure internal rate of return $r(t)$. Implementing this marginal project would increase social welfare if

$$\left[-e^{-r(t)t}U_1(x_{10},x_{20})+e^{-\delta t}EU_1(x_{1t},x_{2t})\right]\varepsilon\Delta t\geq 0,$$

or equivalently, if

$$r(t)\geq r_1(t)=\delta-\frac{1}{t}\ln\frac{EU_1(x_{1t},x_{2t})}{U_1(x_{10},x_{20})}. \tag{2}$$

In other words, the internal rate of return of the project must exceed a minimum threshold, $r_1(t)$, to be socially efficient. Thus, $r_1(t)$ defined by equation 2 is the socially efficient economic discount rate associated to time horizon t. It allows for the comparison of the value of different consumption increments at different dates.

Consider alternatively an investment project that increases the environmental quality by ε in period $[t, t+\Delta t]$. The standard way to include this environmental impact in the cost-benefit analysis would be to first express this impact in future monetary terms. The instantaneous value v_t of the environment at date t is measured by the marginal rate of substitution between consumption and the environment:

$$v_t=-\left.\frac{dx_{1t}}{dx_{2t}}\right|_U=\frac{U_2(x_{1t},x_{2t})}{U_1(x_{1t},x_{2t})}. \tag{3}$$

If the quality of the environment would be traded, v_t would be its equilibrium price, taking the aggregate consumption good as the numeraire. More generally, v_t is the instantaneous willingness to pay for improving environmental quality. Its evolution over time is uncertain, i.e., v_t is a random variable seen from $t=0$. So is the future monetary benefit εv_t of the sure improvement of the environmental quality. Its certainty equivalent is

$$CE_t=\varepsilon\frac{Ev_tU_1(x_{1t},x_{2t})}{EU_1(x_{1t},x_{2t})}.$$

CE_t is the sure increase in consumption at date t that has the same effect on welfare as an ε increase in environmental quality at date t, seen from date 0. It would be the equilibrium future price P^f of an asset traded at date 0 that delivers one unit of the environmental good with certainty at date t

against the payment of P^f at that date. This certainty equivalent must then be discounted at the economic discount rate $r_1(t)$ to measure the net present monetary value of a sure future improvement of the environment.

A much simpler approach is obtained by defining an ecological discount rate. Consider a marginal project that would increase the environmental quality by a sure amount ε in period $[t, t + \Delta t]$, and that would reduce the environmental quality by $\varepsilon e^{-r(t)t}$ in period $[0, \Delta t]$. Implementing this project would be socially efficient if

$$r(t) \geq r_2(t) = \delta - \frac{1}{t} \ln \frac{EU_2(x_{1t}, x_{2t})}{U_2(x_{10}, x_{20})}. \tag{4}$$

This equation defines the ecological discount rate $r_2(t)$ associated to time horizon t. It allows us to compare sure changes in the environment quality at different dates. Namely, an increase in environmental quality by ε at date t has an effect on intertemporal welfare that is equivalent to an increase in current environmental quality by $\varepsilon e^{-r_2(t)t}$. In monetary terms, this is equal to $v_0 \varepsilon e^{-r_2(t)t}$.

The two methods value the environmental impact in the same way, since

$$e^{-r_1(t)t} CE_t = e^{-\delta t} \frac{EU_2(x_{1t}, x_{2t})}{U_1(x_{10}, x_{20})} = v_0 e^{-r_2(t)t}.$$

To sum up, the benefit of a unit increment in environmental quality at date t should be accounted for in the evaluation of a project as equivalent to an immediate increase in consumption by $v_0 e^{-r_2(t)t}$. This really means that environmental costs and benefits should be discounted at the ecological rate $r_2(t)$, which needs not to be the same as the economic discount rate $r_1(t)$. The potential discrepancy between the economic discount rate and the ecological discount rate takes into account the stochastic changes in the relative social valuation of the environment.

4 The determinants of the ecological discount rate

In this section, we describe the five determinants of the ecological discount rate. We compare two economies, $j = a$ or b, having the same representative agent characterized by (δ, U), and the same initial economic and ecological environment (x_{10}, x_{20}). They differ by their expectations on economic and ecological growths. Let F_1^j and F_2^j denote respectively the marginal distributions of x_{1t}^j and x_{2t}^j. In order to guarantee the existence of expectations, we hereafter suppose that the supports of x_{1t} and x_{2t} are bounded.

We first examine the role of the expectations on the growth of environmental quality. In the following definition and proposition, we assume that the prospects of economic growth are the same in the two economies. The following definition is based on the classical two stochastic dominance orders applied to the conditional distributions of x_{2t}.[1]

Definition 1 *(Ecological Dominance) Consider two economies, $j = a$ or b, with the same the marginal distributions of x_{1t}: $F_1^a \equiv F_1^b$.*

1. *We say that economy b is ecologically dominated by economy a in the sense of first-degree stochastic dominance (FSD) if $x_{2t}^b \mid x_{1t}^b = y_1$ is FSD-dominated by $x_{2t}^a \mid x_{1t}^a = y_1$ for all y_1.*

2. *We say that economy b is ecologically riskier than economy a if $x_{2t}^b \mid x_{1t}^b = y_1$ is riskier than $x_{2t}^a \mid x_{1t}^a = y_1$ in the sense of Rothschild-Stiglitz (1970) for all y_1.*

A first-degree ecologically dominated shift is obtained when all conditional distributions of environmental quality are shifted downwards, conditional to all possible economic outcomes $x_{1t} = y_1$. An example of an increase in ecological riskiness is when each conditional distribution of environmental quality undergoes a mean-preserving spread, as defined by Rothschild and Stiglitz (1970). The following proposition characterizes the conditions under which such changes in beliefs reduce the ecological discount rate.

Proposition 2 *Consider any pair of economies with the same marginal distributions of economic outcome x_{1t}: $F_1^a \equiv F_1^b$.*

[1] Let $F_{2|1}^j$ be the distribution of $x_{2t}^j \mid x_{1t}^j$. As is well-known, $x_{2t}^b \mid x_{1t}^b = y_1$ is FSD-dominated by $x_{2t}^a \mid x_{1t}^a = y_1$ if $F_{2|1}^a(y_1, y_2) \leq F_{2|1}^b(y_1, y_2)$ for all y_2. Similarly, $x_{2t}^b \mid x_{1t}^b = y_1$ is riskier than $x_{2t}^a \mid x_{1t}^a = y_1$ in the sense of Rothschild-Stiglitz if $\int^{y_2} F_{2|1}^a(y_1, z)dz \leq \int^{y_2} F_{2|1}^b(y_1, z)dz$ for all y_2, with an equality for y_2 being the supremum of the support of x_{2t}^b.

1. **(ecological growth effect)** *Suppose that economy b is ecologically dominated by economy a in the sense of first-degree stochastic dominance. The ecological discount rate is smaller in economy b than in economy a if and only if U_2 is non-increasing in x_2.*

2. **(ecological prudence effect)** *Suppose that economy b is ecologically riskier than economy a. The ecological discount rate is smaller in economy b than in economy a if and only if U_2 is convex in x_2.*

Proof: We start with the proof of claim 1. Because economy b is ecologically dominated by economy a in the sense of FSD, and because U_2 is decreasing in its second argument, we have that $EU_2(y_1, x_{2t}^b \mid x_{1t}^b = y_1)$ is larger $EU_2(y_1, x_{2t}^a \mid x_{1t}^a = y_1)$ for all y_1. It implies that

$$\begin{aligned} EU_2(x_1^b, x_2^b) &= \int EU_2(y_1, x_{2t}^b \mid x_{1t}^b = y_1) dF_1^b(y_1) \\ &\geq \int EU_2(y_1, x_{2t}^a \mid x_{1t}^a = y_1) dF_1^a(y_1) = EU_2(x_1^a, x_2^a). \end{aligned}$$

It implies that

$$r_2^b(t) = \delta - \frac{1}{t} \ln \frac{EU_2(x_{1t}^b, x_{2t}^b)}{U_2(x_{10}, x_{20})} \leq \delta - \frac{1}{t} \ln \frac{EU_2(x_{1t}^a, x_{2t}^a)}{U_2(x_{10}, x_{20})} = r_2^a(t).$$

This proves the sufficiency of $U_{22} \leq 0$. To prove necessity, suppose by contradiction that U_{22} is not uniformly negative. By continuity, there exists $(y_1, y_2) \in R^2$ and a neighborhood $N \in R^2$ such that $(y_1, y_2) \in N$ and U_{22} is uniformly positive in N. Consider two economies a and b with the same marginal distribution of x_{1t} and with a support of (x_{1t}^j, x_{2t}^j) entirely contained in N. Because economy b is ecologically dominated by economy a in the sense of FSD, and because U_2 is increasing in x_2 in the support of (x_{1t}^j, x_{2t}^j), $EU_2(x_1^b, x_2^b)$ is smaller than $EU_2(x_1^a, x_2^a)$, which implies that $r_2^b(t)$ is larger than $r_2^a(t)$. This is a contradiction. The proof of claim 2 is perfectly parallel, and is therefore skipped. ■

A FSD-deterioration of the ecological growth reduces the ecological discount rate under the standard assumption that U is concave in x_2. The willingness to improve the environmental quality in the future is negatively related to the expected future environmental quality. This ecological growth effect is parallel to the wealth effect that is apparent in the Ramsey rule. The ecological prudence effect states that an increase in the uncertainty on the future environmental quality reduces the ecological discount rate if U_{222} is non-negative, i.e., if the representative agent is prudent in the environment quality.

We now examine the role of the expectations on economic growth. The ceteris paribus condition is hereafter that the expectations on the future environmental quality are unchanged.

Definition 2 *(Economic Dominance) Consider two economies, $j = a$ or b, with the same the marginal distributions of x_{2t}: $F_2^a \equiv F_2^b$.*

1. *We say that economy b is economically dominated by economy a in the sense of first-degree stochastic dominance (FSD) if $x_{1t}^b \mid x_{2t}^b = y_2$ is FSD-dominated by $x_{1t}^a \mid x_{2t}^a = y_2$ for all y_2.*

2. *We say that economy b is economically riskier than economy a if $x_{1t}^b \mid x_{2t}^b = y_2$ is riskier than $x_{1t}^a \mid x_{2t}^a = y_2$ in the sense of Rothschild-Stiglitz (1970) for all y_2.*

Proposition 3 *Consider any pair of economies with the same marginal distributions of environmental quality x_{2t}: $F_2^a \equiv F_2^b$.*

1. **(substitution effect)** *Suppose that economy b is economically dominated by economy a in the sense of first-degree stochastic dominance. The ecological discount rate is smaller in economy b than in economy a if and only if U_2 is non-increasing in x_1.*

2. **(cross-prudence effect)** *Suppose that economy b is economically riskier than economy a. The ecological discount rate is smaller in economy b than in economy a if and only if U_2 is convex in x_1.*

Proof: The proof of this proposition is parallel to the proof of Proposition 2, and is therefore skipped. ■

If the representative agent is correlation-averse, i.e., if U_{12} is non-positive, a FSD-dominated shift in the expectations of economic growth reduces the ecological discount rate. This substitution effect shows that the willingness to invest in the environment is decreasing in the rate of economic growth under this condition. Similarly, under the assumption that the representative agent is cross-prudent in consumption ($U_{211} \geq 0$), an increase in the uncertainty surrounding the economic growth raises the willingness to invest in the environment. It thus reduces the ecological discount rate.

Finally, we examine the effect of the correlation between ecological and economic growth. We consider the following notion of statistical dependence.

Definition 3 *(Correlation) Consider a pair of random variables $(\widetilde{x}_{1t}, \widetilde{x}_{2t})$. We say that there is positive FSD dependence between $\widetilde{x}_{1t}$ and $\widetilde{x}_{2t}$ if any increase in y_1 yields a FSD-dominant shift in $x_{2t} \mid x_{1t} = y_1$.*

In other words, an increase in economic growth generates a first-order stochastic dominant shift in the conditional distribution of the environmental quality. In the statistical literature (see for example Joe (1997)), this notion is referred to as the "stochastic increasing positive dependence", because x_{2t} is more likely to take on larger value when x_{1t} increases. Milgrom (1981) uses this concept to define the notion of a good news.

Let us compare two economies a and b with the same marginal distributions: $F_1^a \equiv F_1^b$ and $F_2^a \equiv F_2^b$. In words, when considered in isolation, both the economic risk and the ecological risk look the same in the two economies. In the benchmark economy a, we assume that the two risks are independent, whereas, in economy b, there is positive FSD dependence between them. How does this positive statistical dependence affect the ecological discount rate? Because it tends to raise the global risk, it should intuitively reduce the discount rates. This is the case if $EU_2(x_{1t}, x_{2t})$ is increased by the positive FSD dependence of x_{1t} and x_{2t}. As shown in the following proposition, this is the case if and only if U_2 is supermodular, i.e., if U_{122} is positive.

Proposition 4 (Correlation effect) *Consider two economies with the same marginal distributions for x_{1t} and x_{2t}: $F_1^a \equiv F_1^b$ and $F_2^a \equiv F_2^b$. In economy a, the two random variables are independent, whereas they are positive FSD-dependent in economy b. Then, the ecological discount rate is smaller in economy b than in economy a if and only if the representative agent is cross-prudent in environmental quality (U_{122} positive).*

Proof: Tchen (1980) and Gollier (2007) proved that $Eh(x_{1t}, x_{2t})$ is increased by positive FSD-dependence if and only if function h is supermodular. Applying this to function $h = U_2$ implies that EU_2 is increased by the positive statistical dependence under scrutinty if and only if U_{122} is positive. ■

The three propositions presented in this section characterize the 5 determinants of the ecological discount rate. A symmetric analysis can be made about the economic discount rate $r_1(t)$.

5 CES utility in the certainty case

Guesnerie (2004), Hoel and Sterner (2007), Sterner and Persson (2008) and Traeger (2007) considered the case of certainty, which implies that the only determinants at play for the ecological discount rate are the ecological growth effect and the substitution effect. They examined the following set of CES functions:

$$U(x_1, x_2) = \frac{1}{1-\alpha} y^{1-\alpha} \text{ with } y = \left[(1-\gamma) x_1^{\frac{\sigma-1}{\sigma}} + \gamma x_2^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{5}$$

where $\sigma > 0$ is the constant elasticity of substitution, $\alpha > 0$ is relative aversion towards the risk on "aggregate good" y, and $\gamma \in [0, 1]$ is a preference weight in favor of the environment. Parameter σ is the rate at which the demand for x_2 declines when the relative price of x_2 is increased by 1%. This function is defined on R^2_+. We have that

$$U_2(x_1, x_2) = \gamma y^{\frac{1}{\sigma}-\alpha} x_2^{-\frac{1}{\sigma}}.$$

From this expression, we see that the representative agent is correlation-averse ($U_{12} \leq 0$) if and only if $\alpha\sigma - 1$ is nonnegative. Thus, from Proposition 3.1, we know that the ecological discount rate is increasing with the economic growth rate if $\alpha\sigma - 1$ is nonnegative. We also observe that this condition is sufficient for aversion to ecological risk ($U_{22} \leq 0$). Thus, from Property 1 of Proposition 2., this condition is sufficient for the ecological discount rate to be increasing with the growth rate of environmental quality. To make this more explicit, suppose that growth rates are constant, which means that $x_{1t} = e^{g_1 t}$ and $x_{2t} = e^{g_2 t}$. The following equation is a direct rewriting of equation (4) under this specification:

$$r_2(t) = \delta + \frac{g_2}{\sigma} + \left(\alpha - \frac{1}{\sigma} \right) G(t) \tag{6}$$

with

$$G(t) = \frac{\sigma}{\sigma - 1}\frac{1}{t}\ln\left[(1-\gamma)e^{g_1\frac{\sigma-1}{\sigma}t} + \gamma e^{g_2\frac{\sigma-1}{\sigma}t}\right].$$

We directly obtain the following proposition. Some of these results are in Guesnerie (2004) and Hoel and Sterner (2007).

Proposition 5 *Suppose that x_i grows at a constant rate g_i, $i = 1, 2$, and that the utility function satisfies (5). Then, the ecological discount rate $r_2(t)$*

1. *is increasing in the economic growth rate g_1 if and only if $\alpha\sigma - 1$ is nonnegative;*
2. *is increasing in the ecological growth rate g_2 if $\alpha\sigma - 1$ is nonnegative;*
3. *is decreasing with the time horizon t if and only if $(\alpha\sigma - 1)(1 - \sigma)$ is positive;*
4. *tends to $r_{20} = \delta + \frac{g_2}{\sigma} + (\alpha - \frac{1}{\sigma})((1-\gamma)g_1 + \gamma g_2)$ when t tends to zero;*
5. *tends to*

$$r_{2\infty} = \begin{cases} \delta + \frac{g_2}{\sigma} + (\alpha - \frac{1}{\sigma})\min(g_1, g_2) & \text{if } \sigma < 1 \\ \delta + \frac{g_2}{\sigma} + (\alpha - \frac{1}{\sigma})\max(g_1, g_2) & \text{if } \sigma > 1 \end{cases}$$

when t tends to infinity.

Proof: Properties 1 and 2 have already been proved above. Property 3 comes from the observation that $e^{G(t)}$ is the certainty equivalent of $(e^{g_1}, 1 - \gamma; e^{g_2}, \gamma)$ under utility function $v_t(z) = \frac{\sigma-1}{\sigma}z^{\frac{\sigma-1}{\sigma}t}$, which is increasing, and whose Arrow-Pratt coefficient of risk aversion is increasing (decreasing) in t when σ is smaller (larger) than unity. By the main theorem in Pratt (1964), this implies that the certainty equivalent $G(t)$ is decreasing (increasing) in t when σ is smaller (larger) than unity. Equation (6) concludes the proof of property 3. Properties 4 and 5 are direct consequences of applying L'Hospital's rule to the limits of $r_2(t)$ when t tends to zero and infinity. ■

The most interesting result described in the above proposition is property 3, which states that the term structure of the ecological discount rate should be decreasing if $(\alpha\sigma - 1)(1 - \sigma)$ is positive. The intuition of this result is easiest to understand by assuming that the environment remains stable

($g_2 = 0$), as in Guesnerie (2004). In that case, only the substitution effect is at play, and it is easy to check that equations (4) simplifies to

$$r_2(t) = \delta + g_1 \frac{\int_0^t R_{21}(e^{g_1\tau}, 1)d\tau}{t},$$

where $R_{21}(x_1, x_2) = -x_1 U_{21}(x_1, x_2)/U_2(x_1, x_2)$ is the relative correlation aversion, i.e., the elasticity of the marginal utility of the environment with respect to consumption. Intuitively, if this elasticity is decreasing with time, the substitution effect is decreasing with the time horizon. If g_1 is positive, this is the case if R_{21} is decreasing in its first argument. Under specification (5), it is easy to check that

$$R_{21}(x_1, x_2) = \left(\alpha - \frac{1}{\sigma}\right)(1 - \gamma^*(x_1/x_2)). \tag{7}$$

with

$$\gamma^*(x) = \frac{\gamma}{(1-\gamma)x^{\frac{\sigma-1}{\sigma}} + \gamma}.$$

Therefore, R_{21} are decreasing with x_1 if $(\alpha\sigma - 1)(1 - \sigma)$ is positive. The substitution effect is diminishing over time in that case. This provides an intuition for property 3. This is reminiscent of Gollier (2002) who linked decreasing relative risk aversion to the decreasing nature of the term structure of the economic discount rate.[2]

6 Lognormal distributions

Introducing uncertainty into this model under the CES specification (5) is difficult because there is in general no analytical expression to the expectation of $U_2(x_{1t}, x_{2t})$, except when σ tends to unity. When σ tends to unity, y tends to $x_1^{1-\gamma}x_2^{\gamma}$, and U tends to a Cobb-Douglas specification:

$$U(x_1, x_2) = kx_1^{1-\gamma_1}x_2^{1-\gamma_2}, \tag{8}$$

[2]Under the same conditions, we have that $r_1(t) = \delta + g_1 t^{-1} \int_0^t R_{11}(e^{g_1\tau}, 1)d\tau$. This implies that r_1 is decreasing if and only if relative risk aversion is decreasing (Gollier (2002)). The main result in Guesnerie (2004) is a special case of this observation.

with $1-\gamma_1=(1-\gamma)(1-\alpha)$ and $1-\gamma_2=\gamma(1-\alpha)$. The monotonicity of U with respect to x_1 and x_2 requires that

$$sgn(1-\gamma_1)=sgn(1-\gamma_2)=sgn(k).$$

The concavity of U with respect to x_1 and x_2 implies that γ_1 and γ_2 must be positive. If we assume that γ_1 and γ_2 are both larger than unity, it is easy to check that the representative agent considers pairs of harms as mutually aggravating, implying correlation aversion and (cross-)prudence.

We consider four different specifications for the dynamics of (x_{1t},x_{2t}).

6.1 A bivariate brownian motion with Cobb-Douglas preferences

In the first one, we suppose that this pair follows a bivariate geometric brownian motion. It implies that for all t, $(\ln x_{1t},\ln x_{2t})$ is jointly normally distributed with mean $(\ln x_{10}+\mu_1 t,\ln x_{20}+\mu_2 t)$ and variance-covariance matrix $\Sigma=(\sigma_{ij}t)_{i,j=1,2}$. The proof of the following propositions are relegated to the Appendix.

Proposition 6 *Suppose that $U(x_1,x_2)=kx_1^{1-\gamma_1}x_2^{1-\gamma_2}$ and that (x_{1t},x_{2t}) follows a bivariate geometric Brownian motion. It implies that the ecological discount rate equals*

$$r_2(t)=\delta+\gamma_2\left[g_2-\frac{1}{2}(\gamma_2+1)\sigma_{22}\right]+(\gamma_1-1)\left[g_1-\frac{1}{2}\gamma_1\sigma_{11}\right]-(\gamma_1-1)\gamma_2\sigma_{12}, \quad (9)$$

where $\sigma_{ij}=t^{-1}cov(x_{it},x_{jt})$ and $g_i=t^{-1}\ln Ex_{it}/x_{i0}=\mu_i+0.5\sigma_{ii}$.

Symmetrically, the economic discount rate equals

$$r_1(t)=\delta+\gamma_1\left[g_1-\frac{1}{2}(\gamma_1+1)\sigma_{11}\right]+(\gamma_2-1)\left[g_2-\frac{1}{2}\gamma_2\sigma_{22}\right]-(\gamma_2-1)\gamma_1\sigma_{12}. \quad (10)$$

These formulas extend the Ramsey rule to an ecological economy. The different terms in the right-hand side of equation (9) are easily linked to the five determinants of the ecological discount rate that we obtained in the previous section:

- $\gamma_2 g_2$ is the ecological growth effect;
- $-\frac{1}{2}\gamma_2(\gamma_2+1)\sigma_{22}$ is the ecological prudence effect;
- $(\gamma_1-1)g_1$ is the substitution effect;
- $-\frac{1}{2}(\gamma_1-1)\gamma_2\sigma_{22}$ is the cross-prudence effect;
- $-(\gamma_1-1)\gamma_2\sigma_{12}$ is correlation effect.

An important implication of this proposition is that the term structures of the economic discount rates and of the ecological discount rates are flat. In such an economy, the random evolution of aggregate consumption and of the environmental quality does not justify to use a smaller rate to discount benefits occurring in a more distant future.

Another immediate consequence of Proposition 6 is that

$$r_2 - r_1 = (g_2 - g_1) + (\gamma_1\sigma_{11} - \gamma_2\sigma_{22}) + (\gamma_2 - \gamma_1)\sigma_{12}. \tag{11}$$

Interestingly enough, under certainty, the difference between the two discount rates is independent of the parameters of the Cobb-Douglas utility function. This equation provides two arguments in favor of using an ecological discount rate smaller than the economic discount rate. First, it is often suggested that the growth rate of environmental quality is smaller than the economic growth rate ($g_2 \leq g_1$), the first being potentially negative. Second, it seems that there is much more uncertainty surrounding the evolution of the environmental quality than the evolution of the economy itself ($\sigma_{22} \geq \sigma_{11}$). If the degrees aversion to risk on x_1 and on x_2 are not too heterogeneous, this would imply that $\gamma_2\sigma_{22} - \gamma_1\sigma_{11}$ be positive. The last term of the right-hand side of equation (11) is more difficult to sign.

6.2 A single brownian motion with Cobb-Douglas preferences

Because of the lack of time-series data about environmental quality, calibrating this specification is problematic. Various authors have argued in favor of a closer link between the environmental quality and economic growth than the one that we assumed in Proposition 6. Following this idea, let us alternatively assume that the environmental quality is a deterministic function of economic achievement: $x_2 = f(x_1)$. Common wisdom suggests that the environmental quality is a decreasing function of GDP per capita, but this is heavily debated in scientific circles. The environmental Kuznets curve hypothesizes that the relationship between per capita income and the environmental quality has an inverted U-shape, but there is no consensus about it (see for example Millimet, List and Stengos (2003)). We hereafter hypothesize a monotone relationship by assuming that there exists $\rho \in R$ such that $x_2 = x_1^\rho$, where ρ can be either positive or negative. If we assume that x_1 follows a geometric brownian motion, we obtain an analytical solution for r_1 and r_2.

Proposition 7 *Suppose that $U(x_1, x_2) = kx_1^{1-\gamma_1} x_2^{1-\gamma_2}$, that $x_2 = x_1^\rho$ and that x_{1t} follows a geometric brownian motion. It implies that the ecological discount rate equals*

$$r_2(t) = \delta + (\rho\gamma_2 + \gamma_1 - 1)\left[g_1 - 0.5(\rho\gamma_2 + \gamma_1)\sigma_{11}\right], \tag{12}$$

where $g_1 = t^{-1} \ln Ex_{1t}/x_{10}$ and $\sigma_{11} = t^{-1}Var(x_{1t})$.

Symmetrically, the economic discount rate equals

$$r_1(t) = \delta + (\gamma_1 + \rho(\gamma_2 - 1))\left[g_1 - 0.5(1 + \gamma_1 + \rho(\gamma_2 - 1))\sigma_{11}\right]. \tag{13}$$

In order to calibrate this model, let us assume that the rate of pure preference for the present δ is zero. We also assume that the relative aversion to risk on consumption is a constant $\gamma_1 = 2$, which is often considered as a reasonable estimation.[3] The parameter γ_2 of aversion to environmental risk is not easy to calibrate. Observe however that

$$\gamma^* = \frac{\gamma_2 - 1}{\gamma_1 + \gamma_2 - 2}$$

is the share of total consumption expenditures that the representative agent would use on environmental quality if environmental quality would be a tradable good.[4] Hoel and Sterner (2007) and Sterner and Persson (2008) suggested γ^* somewhere 10% and 50%, which implies that γ_2 should be

[3] See Drèze (1981) for example.

[4] Because the price elasticity equals -1 under this specification, this share remains constant over time.

somewhere between 1.1 and 2 under our specification. We hereafter assume $\gamma^* = 30\%$, which implies $\gamma_2 = 1.4$.

Kocherlakota (1996) estimated the parameters of the growth process of consumption in the United States with yearly data between 1889 and 1978. He obtained $g_1 = 1.8\%$ and $\sigma_{11}^{1/2} = 3.6\%$. The choice of ρ depends upon how we define the environmental quality. In order to estimate ρ, we considered the SYS_LAN indicator contained in the Environmental Sustainability Index (ESI2005, Yale Center for Environmental Law and Policy, (2005)), which measures for 146 countries in 2005 the percentage of total land area (including inland waters) having very high anthropogenic impact. The OLS estimation of the regression coefficients are as follows:

$$\ln x_2 = 1.93 - 0.10 \ln x_1 + \varepsilon$$

where x_1 is the country's GDP/cap[5] whereas x_2 is 3 plus the country's SYS_LAN indicator contained in ESI2005. The p-value for the slope-coefficient is -4.69, whereas the R2 coefficient equals 0.13. Plugging $\rho = -0.10$ in equations (12) and (13) yields $r_2 = 1.5\%$ and $r_1 = 3.2\%$. It is useful to provide a few comments on this result:

- The difference bteween the ecological rate and the economic rate comes mostly from the large expected economic growth rate ($g_1 = 1.8\%$) compared to the expected environmental growth rate ($g_2 = \rho g_1 = -0.18\%$).
- The elevel of the ecological rate is mostly determined by the substitution effect. Because ρ is small in absolute value, the (negative) ecological growth effect $\gamma_2 \rho g_1 = -0.25\%$ is indeed small. This needs to be compared to the substitution effect $(\gamma_1 - 1) g_1 = 1.8\%$.
- The effect of the uncertainty (prudence, cross-prudence and correlation effects) is marginal because of the low volatility of x_1 and x_2, and because we assume that shocks are not serially correlated.

6.3 A single brownian motion with CES preferences

In the numerical illustration presented above, we assumed that the elasticity of substitution σ equals unity. In Figure 1, we describe the term structure of

[5]We used data from the World Economic Outlook Database of IMF, April 2008.

the ecological discount rates when the constant elasticity of substitution σ is either 0.5, 1, or 1.5. We use the CES specification (5) with $\gamma = 2/7$, $\alpha = 2.4$. When σ tends to unity, this is equivalent to the Cobb-Douglas specification (8) with $\gamma_1 = 2$ and $\gamma_2 = 1.4$, as in the above numerical illustration. We also assume that $\ln x_{1t}$ follows an arithmetic brownian motion with the same trend and volatility, and that $x_{2t} = x_{1t}^{\rho}$ with the same $\rho = -0.1$. When $\sigma = 1$, we know from above that the ecological discount rate is a flat 1.5%, independent of the time horizon. As can be seen from (7), the relative correlation aversion $R_{21}(1,1) = -U_{21}(1,1)/U_2(1,1)$ measured at $t = 0$ (where we normalized $x_{10} = x_{20} = 1$) is increasing in the elasticity of substitution σ. It implies that the crucial substitution effect – which tends to raise the ecological discount rate – is made more powerful by an increase in σ. This explains the relative position of the three plain curves in Figure 1, at least for small t. When $\sigma = 1.5$, equation (7) tells us that the substitution effect is increasing with time, because of the increased scarcity of the environment. This explains the positive slope of r_2 in that case. The effect is reversed for $\sigma = 0.5$.

The dashed curves in Figure 1 are obtained by ignoring the three effects associated to uncertainty ($\sigma_{11} = 0$), in which case equation (6) may be applied. The comparison with the corresponding plain curves tells us that the effect of uncertainty is globally negative. This is because the third derivatives of U are negative under the specifications under scrutiny.

6.4 Parametric uncertainty

In the last specification for the dynamics of (x_{1t}, x_{2t}), we introduce some parametric uncertainty. Conditional to parameter θ, x_{1t} follows a geometric Brownian motion with drift $g_1(\theta)$ and volatility $\sigma_{11}^{1/2}(\theta)$, whereas $x_2 = x_1^{\rho(\theta)}$. In this case, we obtain the following proposition. The true value of θ is unknown, and the prior beliefs on it is described by the cumulative distribution function F.

Proposition 8 *Suppose that $U(x_1, x_2) = kx_1^{1-\gamma_1}x_2^{1-\gamma_2}$, that $x_2 = x_1^{\rho}$ and x_{1t} follows a geometric brownian motion. Suppose that the true value of triplet (g_1, σ_{11}, ρ) is uncertain at date 0 so that it depends upon some parameter θ whose cumulative distribution function is F. It implies that the ecological*

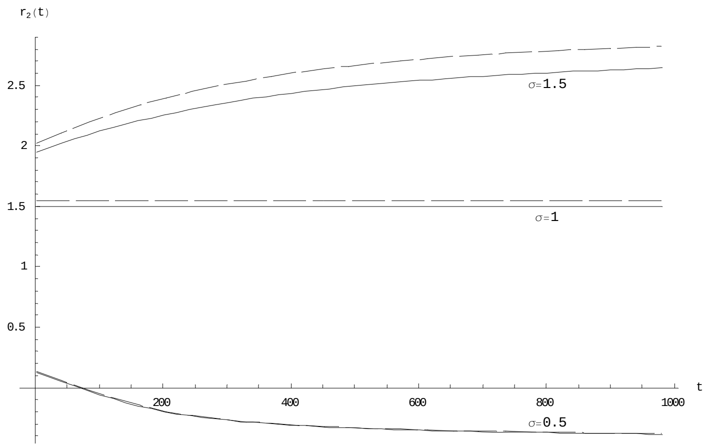

Figure 1: The term structure of the ecological discount rate (in %) under specification (5) with $\alpha = 2.4$ and $\gamma = 2/7$. We assume that $\ln x_{1t}$ follows a brownian motion with trend 1.8% and volatility 3.6% (plain curves) or 0% (dashed curves), and that $x_{2t} = x_{1t}^{-0.1}$.

discount rate equals

$$r_2(t) = \delta - \frac{1}{t} \ln \int \exp[-R_2(\theta)t]dF(\theta), \tag{14}$$

where $R_2(\theta) = (\rho\gamma_2 + \gamma_1 - 1)[g_1 - 0.5(\rho\gamma_2 + \gamma_1)\sigma_{11}]$.

Symmetrically, the economic discount rate equals

$$r_1(t) = \delta - \frac{1}{t} \ln \int \exp[-R_1(\theta)t]dF(\theta), \tag{15}$$

where $R_1(\theta) = (\gamma_1 + \rho(\gamma_2 - 1))\,[g_1 - 0.5(1 + \gamma_1 + \rho(\gamma_2 - 1))\sigma_{11}]$. By Jensen inequality, this immediately implies that the term structures of r_1 and r_2 are decreasing. The short-term discount rate $r_i(t)$ equals δ plus the mean of R_i when t tends to zero, and it tends to δ plus the smallest possible value of $R_i(\theta)$ when t tends to infinity. The reason for the decreasing nature of the term structure comes from the combination of two properties of this specification. First, the parametric uncertainty raises the long term uncertainty

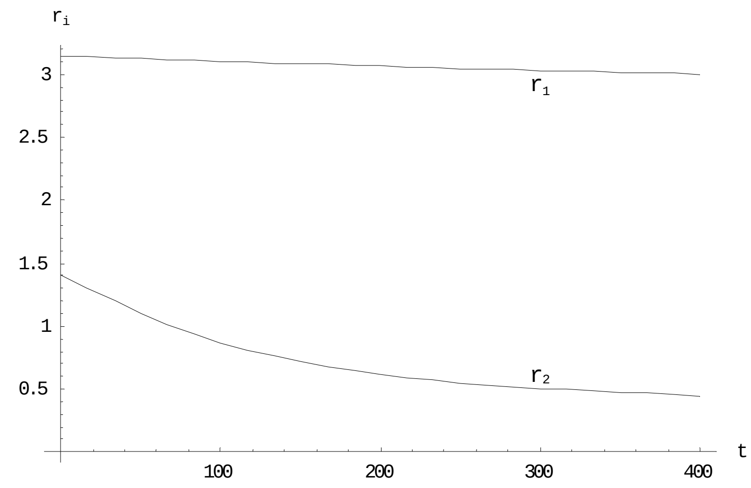

Figure 2: The term structures of the economic and ecological discount rates (in %), assuming $\delta = 0$, $\gamma = 2$, $\gamma_2 = 1.4$, $g_1 = 1.8\%$, $\sigma_{11}^{1/2} = 3.6\%$ and $\rho \sim (-0.6, 1/2; 0.4, 1/2)$.

surrounding (x_{1t}, x_{2t}) relatively more for the long term than for the short term. Second, because the representative agent is prudent, cross-prudent and correlation-averse, Propositions 2, 3 and 4 implies that these (negative) effects will be increasing (in absolute value) with time.

These results generalize those obtained by Weitzman (2007) and Gollier (2007) to multiattribute utility functions. They both assumed that the economic growth rate was affected by parametric uncertainty. Suppose alternatively that g_1 and σ_{11} are known, but the elasticity ρ of environmental quality to changes in GDP is not. Rather than assuming that $\rho = -0.1$ as above, let us suppose that ρ is either -0.6 or $+0.4$ with equal probabilities. All other parameters remain unchanged compared to section 6.2. We draw the term structure of r_1 and r_2 in Figure 2. Whereas the economic discount rate is almost independent of time horizon, the ecological discount rate goes from 1.4% to 0.3% when t goes from 0 to infinity. The high uncertainty affecting the long-term evolution of the environment in this specification explains why the term structure of the ecological discount rate is decreasing.

7 Conclusion

Environmentalists are often quite skeptical about using standard cost-benefit analysis to shape environmental policies because environmental damages incurred in the distant future are claimed to receive insufficient weights in the economic evaluation. This may be due either because future environmental assets are undervalued, or because the economic discount rate is too large. In this paper, we address these two questions altogether by defining an ecological discount rate compatible with social welfare when the representative agent cares about both the economic and ecological environment faced by future generations. This ecological rate at which future environmental damages are discounted may be much smaller than the economic rate at which economic damages are discounted, because of the integration of the potentially increasing willingness to pay for the environment into the ecological discount rate. We have also shown in this paper that the uncertainties surrounding the evolutions of the environment and the economy tend to reduce the discount rates, in particular if they are positively correlated.

REFERENCES

Drèze, J. H. (1981). "Inferring Risk Tolerance from Deductibles in Insurance Contracts", *The Geneva Papers* 6, 48-52.

Eeckhoudt, L., and H. Schlesinger, (2006), Putting risk in its proper place, *American Economic Review*, 96(1), 280-289.

Eeckhoudt, L., B. Rey, and H. Schlesinger, (2007), A good sign for multivariate risk taking, *Management Science*, Vol. 53 (1), 117-124.

Gollier, C., (2002), Time horizon and the discount rate, *Journal of Economic Theory*, 107, 463-473.

Gollier, C., (2007), The consumption-based determinants of the term structure of discount rates, *Mathematics and Financial Economics*, 1 (2), 81-102.

Groom, B., P. Koundouri, E. Panopoulou and T. Pantelidis, (2007), An Econometric Approach to Estimating Long-Run Discount Rates, *Journal of Applied Econometrics*, 22, 641-656.

Guesnerie, R., (2004), Calcul économique et développement durable, *Revue Economique*.

Hoel, M., and T. Sterner, (2007), Discounting and relative prices, *Climatic Change*, DOI 10.1007/s10584-007-9255-2, March 2007.

Joe, H., (1997), *Multivariate models and dependence concepts*, Chapman and Hall/CRC.

Kimball, M.S., (1990), Precautionary savings in the small and in the large, *Econometrica*, 58, 53-73.

Kimball, M.S., (1993), Standard risk aversion, *Econometrica*, 61, 589-611.

Kocherlakota, N.R., (1996), The Equity Premium: It's Still a Puzzle, *Journal of Economic Literature*, 34, 42-71.

Malinvaud, E., (1953), Capital accumulation and efficient allocation of resources, *Econometrica*, 21 (2), 233-268.

Milgrom, P., (1981), Good news and bad news: Representation theorems and applications, *Bell Journal of Economics*, 12, 380-91.

Millimet, D. L., J. A. List and T. Stengos (2003), "The environmental Kuznets curve: Real progress or misspecified models?", *The Review of Economics and Statistics*, 85, 1038-1047.

Pratt, J., (1964), Risk aversion in the small and in the large, *Econometrica*, 32, 122-136.

Ramsey, F.P., (1928), A mathematical theory of savings, *The Economic Journal*, 38, 543-59.

Rothschild, M. and J. Stiglitz, (1970), Increasing risk: I. A definition, *Journal of Economic Theory*, 2, 225-243.

Sterner, T. and M. Persson, (2008),"An Even Sterner Report": Introducing Relative Prices into the Discounting Debate, *Review of Environmental Economics and Policy*, vol 2, issue 1.

Tchen, A.H., (1980), Inequalities for distributions with given marginals, *Annals of Probability*, 8, 814-827.

Traeger, C.P., (2007), Sustainability, limited substituability and non-constant social discount rates, Dpt of Agricultural & Resource Economics DP 1045, Berkeley.

Weikard, H.-P., and X. Zhu, (2005), Discounting and environmental quality: When should dual rates be used?, *Economic Modelling*, 22, 868-878.

Weitzman, M.L., (2007), Subjective expectations and asset-return puzzles, *American Economic Review*, 97(4), pp. 1102-1130.

Yale Center for Environmental Law and Policy, (2005), 2005 Environmental Sustainability Index: Benchmarking national environmental stewardship, Yale University.

APPENDIX

Proof of Proposition 6

Under the specification of this proposition, we can rewrite $EU_2(x_{1t}, x_{2t})$ as

$$EU_2(x_{1t}, x_{2t}) = k(1-\gamma_2)E\left[\exp z_t\right],$$

where $z_t = (1-\gamma_1)\ln x_{1t} - \gamma_2 \ln x_{2t}$ is normally distributed with mean

$$Ez_t = (1-\gamma_1)(\ln x_{10} + \mu_1 t) - \gamma_2(\ln x_{20} + \mu_2 t)$$

and variance

$$Var(z_t) = \left((1-\gamma_1)^2\sigma_{11} + \gamma_2^2\sigma_{22} - 2(1-\gamma_1)\gamma_2\sigma_{12}\right)t.$$

As is well-known, the Arrow-Pratt approximation is exact for an exponential utility function with a normally distributed random variable. It implies that

$$EU_2(x_{1t}, x_{2t}) = k(1-\gamma_2)E\left[\exp z_t\right] = k(1-\gamma_2)\exp\left(Ez_t + 0.5Var(z_t)\right).$$

This implies in turn that

$$\frac{EU_2(x_{1t}, x_{2t})}{U_2(x_{10}, x_{20})} = \exp\left((1-\gamma_1)g_1 - \gamma_2 g_2 + 0.5\left(\gamma_1(\gamma_1-1)\sigma_{11} + \gamma_2(\gamma_2+1)\sigma_{22} - 2(1-\gamma_1)\gamma_2\sigma_{12}\right)\right)t,$$

where g_i is the expected growth rate of x_{it}: $Ex_{it} = x_{i0}e^{g_i t}$.[6] Applying (4) concludes this proof. ■

Proof of Proposition 7

We can rewrite $U_2(x_1, x_2) = k\eta^{-\gamma_2}(1-\gamma_2)x_1^{1-\gamma_1-\rho\gamma_2}$, which implies that $EU_2(x_{1t}, x_{2t})$ be proportional to $E\exp\left[(1-\gamma_1-\rho\gamma_2)\ln x_{1t}\right]$. Again, since the Arrow-Pratt approximation is exact for an exponential utility function with a normally distributed random variable, we have that

$$\frac{EU_2(x_{1t}, x_{2t})}{U_2(x_{10}, x_{20})} = \exp\left((1-\gamma_1-\rho\gamma_2)(\mu_1 t + 0.5(1-\gamma_1-\rho\gamma_2)\sigma_{11}t\right), \quad (16)$$

[6] Using Ito's Lemma or the property that the Arrow-Pratt approximation is exact in this framework yields that $g_i = \mu_i + 0.5\sigma_{ii}$.

with $\mu_1 = t^{-1}E\ln(x_{1t}/x_{10}) = g_1 - 0.5\sigma_{11}$. Applying (4) concludes this proof. A symmetric analysis can be made for $r_1(t)$, after noticing that $U_1(x_1, x_2) = k\eta^{1-\gamma_2}(1-\gamma_1)x_1^{-\gamma_1-\rho(\gamma_2-1)}$. ■

Proof of Proposition 8

We limit the proof to $r_2(t)$. Conditional to θ, equation (16) holds, which implies that

$$\frac{EU_2(x_{1t}, x_{2t})}{U_2(x_{10}, x_{20})} = \int \exp\left[-R_2(\theta)t\right] dF(\theta).$$

Applying (4) concludes this proof. ■