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Committee Design in the Presence of Communication*

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Abstract

The goal of this paper is to introduce communication in a collective choice environment with information acquisition. We concentrate on decision panels that are comprised of agents sharing a common goal and having a joint task. Members of the panel decide whether to acquire costly information or not, preceding the communication stage. We take a mechanism design approach and consider a designer who can choose the size of the decision panel, the procedure by which it selects the collective choice, and the communication protocol by which its members abide prior to casting their individual action choices. We characterize the solution of this extended design problem. We find that the optimal communication protocol in such an environment balances a tradeoff between inducing players to acquire information and extracting the maximal amount of information from them. In particular, the optimal device may lead to suboptimal aggregation of information from a statistical point of view. Furthermore, groups producing the optimal collective decisions are bounded in size. Comparative statics results shed light on the regularities the design solution exhibits. For example, the expected utility of all agents decreases with the cost of private information and increases with its accuracy, but the optimal panel size is not monotonic in the signals' accuracy.

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I delight in talking politics. I talk them all day long. But I can't bear listening to them.

- Oscar Wilde, *An Ideal Husband.*

1 Introduction

Many real world group decisions incorporate a communication phase as part of the decision making procedure. For example, trial jurors are made to deliberate before casting their votes, hiring committees meet before making their final hiring recommendations, and top management teams convene before determining their firm's investment strategies.

The focus of this paper is the introduction of communication into a mechanism design problem of collective choice with information acquisition.

We analyze a mechanism design problem involving the choice of one out of two alternatives. We consider a group of homogeneous agents who are capable of investing in information acquisition. The designer chooses the size, communication system, and voting rule in order to maximize the (common) expected utility of the collective decision. Our goal is to characterize the solution of this mechanism design problem.

The characterization of the optimal mechanism yields a few interesting insights: 1. In order to provide strong incentives for information acquisition, the optimal device does not necessarily utilize all the information that is reported; 2. The optimal size of the decision panel is bounded and does not necessarily coincide with the maximal number of agents who can be induced to purchase information in equilibrium; and 3. The comparative statics of the optimal mechanism exhibit some regularities and irregularities, e.g., the expected social value is monotonic in the cost of information and accuracy of private information, but the optimal panel size is not monotonic in the signals' accuracy.

Formally, we consider the standard voting setup (see Feddersen and Pesendorfer [1998] or Persico [2002]). There are two possible states of the world and two alternatives that need to be matched to the states. All agents have the same utility function and a common prior on the state of the world. At the outset of the game, each agent in the chosen panel can pay a positive cost and receive, in return, a signal of fixed accuracy. Players can then communicate with one another after which they cast their simultaneous votes. The model can serve as a parable to the decision making process of a jury (each juror decides whether to attend the testimonies or not, the jurors then meet to discuss the case, after which they simultaneously cast their votes), an advisory committee (each member invests in information and gives advice following conversation with the other committee members), etc.

We consider a designer with identical preferences to those of the agents in the population. The designer chooses the size of the decision panel, the voting rule by which the collective choice is made, and the protocol according to which panel members communicate.

For any fixed size n of the decision panel, there are many voting rules available for the designer to choose from. For example, one class of well-known procedures is that of threshold voting rules, parametrized by $r = 1, \ldots, n$. Under voting rule r, the first alternative is chosen if and only if at least r agents vote in favor of it. The results in Gerardi and Yariv [2003] simplify the designer's problem tremendously by illustrating that communication renders a large class of voting rules equivalent with respect to the sequential equilibrium outcomes they yield (see Section 3 for a complete description of the results). For example, the intermediate threshold voting rules $2, ..., n-1$ are non-dictatorial and hence yield the same set of sequential equilibrium outcomes. Furthermore, the set of sequential equilibrium outcomes corresponding to any other voting rule (such as the threshold voting rules $r = 1$ and $r = n$) is included in this set. In our current framework, Gerardi and Yariv [2003] implies that the designer's choice of voting rule does not affect the collective outcomes and that she can concentrate on communication devices that give only unanimous recommendations.

For any fixed number of agents, the optimal scenario, which we term the first best, entails all agents purchasing information and reporting truthfully. This information is then utilized in a statistically efficient way. That is, for every profile of signals, the optimal device calculates, using Bayes' rule, the probability that each state of the world has been realized and chooses the optimal alternative.

Unfortunately, for large numbers of agents, the marginal contribution of each signal becomes quite low and, since the cost of information is positive, a free rider problem disables the first best scenario from constituting an equilibrium.

The benevolent designer faces two options. The mechanism can simply induce a small number of agents to purchase information and make the best statistical use of it. Alternatively, the designer can alleviate the free rider problem by using an aggregation rule which is not statistically efficient, thus increasing the incentives to acquire information. This approach involves a compromise between achieving more information in the population, but creating intentional distortions in the interpretation of the collective signals.

Our theoretical results indicate that the optimal design employs both approaches. One can achieve the optimal expected social value by using distortionary mechanisms. These are devices that induce more players to acquire information than would be possible if the mechanism were using statistically efficient rules in creating recommendations. Nonetheless, the optimal mechanism does not always exhaust the number of agents that can be induced to purchase information. That is, the optimal size of a panel of decision makers may be smaller than the maximal number of players who can be made to invest in information in equilibrium.

The mechanism design problem described so far depends on essentially three parameters: a preference parameter indicating the relative preference of matching each alternative to each state (in the terminology of the jury literature, this is the weight each juror puts on convicting the innocent relative to acquitting the guilty), the accuracy of the private information, and the cost of private information.

There are a few interesting insights regarding the comparative statics of the mechanism design problem. As was already mentioned, the expected (common) utility of the optimal mechanism is decreasing in the cost of information and increasing in the accuracy of available signals. It also appears that as the cost of information increases, the optimal decision panel decreases in size. Finally, the optimal committee size is not monotonic in the signals' accuracy.

The paper is structured as follows. Section 2 overviews some of the related literature. Section 3 presents the design problem analyzed in the paper. Section 4 illustrates the important features of the optimal mechanism, regarding the size of the committee, and the way the information agents report is aggregated. Section 5 provides some comparative statics pertaining to the optimal design solution. Section 6 concludes. Most technical proofs are relegated to the Appendix. Throughout the paper we label the group of decision makers a decision panel or committee interchangeably.

2 Related Literature

The current paper is linked to a few strands of literature. First, the paper contributes to the literature on mechanism design with endogenous information. While most of this literature deals with auction and public good models (see, e.g., Auriol and Gary-Bobo [1999], Bergemann and Välimäki [2002], and references therein), there are a few exceptions focusing on collective decision-making.

Persico [2002] is possibly the closest paper to ours. He considers jury decisions and allows the jurors to acquire information before voting. In contrast to our model, the jurors are not allowed to communicate. Persico [2002] analyzes the problem of the designer who can choose the size of the jury and the voting rule. While the tension between giving incentives to acquire information and aggregating information efficiently comes through in his framework, the optimal mechanism is very different from ours. In particular, the distinction between different voting rules plays a crucial role in Persico [2002] but becomes irrelevant in our context once we allow for communication.

Li [2001] considers a committee of a fixed size and allows each player to invest in the precision of her private signal. Information is a public good and, thus, there is an insufficient effort to gather information. To alleviate this problem, it is optimal to introduce statistical distortions to the decision rule. In Li [2001] investments as well as signals are publicly observed and thereby verifiable. As we show below, in our setup verifiability assures that a non-distortionary rule is optimal when the committee is large enough. It is in environments in which investment and acquired signals are not transparent (such as in the case of jury decisions, hiring committee decisions, etc.) that distortionary devices end up being optimal.

Mukhopadhaya [1998] restricts attention to majority rule elections and compares committees of different sizes. Players decide whether to acquire information or not before voting (and communication is prohibited). Mukhopadhaya [1998] shows that the quality of the decision may worsen when the size of the committee increases. Cai [2003] looks at a continuous framework in which the policy preferences and information structures are captured by normal random variables. Members exert non-verifiable efforts in gathering information, report these preferences to the principal, who then uses the mean decision rule to determine the collective policy. Cai characterizes the optimal committee size in this setting and shows that it is finite. Furthermore, Cai illustrates that the optimal size is non-monotonic in the variation of preferences of the committee members.

Our paper is also connected with a few recent attempts to model strategic voting with communication. Coughlan [2000] adds a straw poll preceding the voting stage. He shows that voters reveal their information truthfully if and only if their preferences are sufficiently close. Doraszelski, Gerardi and Squintani [2002] study a two-player model with communication and voting. Preferences are heterogenous (not necessarily aligned) and private information. They show that some, but not all, information is transmitted in equilibrium, and that communication is beneficial. Austen-Smith and Feddersen [2002] analyze a model in which a deliberative committee of three agents needs to choose one of two alternatives. Each player has private information on two dimensions: perfect information concerning her preferences and noisy information concerning the state of the world. Austen-Smith and Feddersen model deliberations as a one-round process in which all players simultaneously send public messages. They show that when such deliberations precede the voting stage, majority rule induces more information sharing and fewer decision-making errors than unanimity.

As mentioned, Gerardi and Yariv [2003] show that communication renders a wide range of voting rules equivalent with respect to the sequential equilibrium outcomes they produce. These results are used in our current analysis. They imply that the designer's choice of voting rule does not affect the collective outcomes and that she can concentrate on communication devices that give only unanimous recommendations.

In the political science literature, Habermas [1976] was one of the first to lay foundations for the deliberative democracy school. He put forward a universal theory of pragmatism and directed attention to the importance of communication as foundations for social action. His theory served as a trigger for an entire body of work focusing on the effects of communication on how institutions function and, consequentially, should be designed (see Elster [1998] for a good review of the state of the art of the field). The research presented here provides an initial formal framework to study some of these issues.

3 Mechanism Design with Information Acquisition

In this section we introduce our general setup and some preliminary observations. We concentrate on the case replicating the standard committee voting problem (see, e.g., Feddersen and Pesendorfer [1998] or Persico [2002]). While our setup is germane to many collective decision environments, the reader may find it useful to trace our modeling choices with a jury metaphor in mind.

There are two states of the world, I (innocent) and G (guilty), with prior distribution $(P(I), P(G))$. The alternatives (or decisions) are A (acquittal), and C (conviction). There is an infinite pool of identical agents. All the agents as well as the mechanism designer share the same preferences which depend on the state of the world and the final decision.

Let q be a number in $(0, 1)$. The common utility is given by:

$$u(d, \omega) = \begin{cases} -q & \text{if } d = C \text{ and } \omega = I \\ -(1-q) & \text{if } d = A \text{ and } \omega = G \\ 0 & \text{otherwise.} \end{cases}$$

where d and ω denote the collective decision and the state of the world, respectively.

To intuit this utility specification, consider a jury decision scenario. Jurors prefer to make the correct decision, i.e., acquitting the innocent and convicting the guilty (in this case we normalize the utility to zero). The ratio $\frac{q}{1-q}$ can be thought of as the jurors' perceived cost of convicting the innocent relative to that of acquitting the guilty.

Each agent can purchase a signal of accuracy $p > \frac{1}{2}$. That is, upon paying the cost $c > 0$, the agent receives a signal $s \in \{i, g\}$ satisfying $\Pr(s = i|I) = \Pr(s = g|G) = p$. Using the jury metaphor, each juror has to decide whether to pay attention or not to the testimonies presented during the trial. These testimonies provide a noisy signal concerning the guilt of the defendant.

If more than one agent purchases information, we assume their signals are conditionally independent. Moreover, we only attend to the case in which an agent can buy at most one signal. While these assumptions may not always be completely realistic, they serve as a first approximation and make our benchmark model tractable.

Committees make joint decisions by voting. A voting rule specifies a set of actions for each player and an alternative (A or C) for every profile of actions. A well-known class of voting procedures consists of *threshold* voting rules. Consider a committee of size n. Under the threshold voting rule $r = 1, \ldots, n$, each member can vote for either alternative, A or C, and the final decision is C if and only if r or more members vote in favor of it. Later on we will be using the notion of non-dictatorial voting rules. A voting rule is *non-dictatorial* if for every alternative, A or C, there exists an action profile that yields that alternative, and is robust to unilateral deviations (i.e., the alternative is chosen even if a single player changes her action). For example, in a committee of size n, any threshold voting rule

$r = 2, \ldots, n-1$ is non-dictatorial, while the voting rules $r = 1$ and $r = n$ (the unanimity rule) are dictatorial.

We allow the members of a committee to communicate before casting their votes. We therefore add a cheap talk phase to the voting game and consider cheap talk extensions. A cheap talk extension is a game that consists of one or more communication stages and a voting stage. In every communication stage, the members exchange messages among themselves (we assume that an impartial mediator is not available). In the last stage (the voting stage), the players choose their votes simultaneously. Payoffs depend on the players' signals and votes, but not on their messages (see Myerson [1991] for a general definition of cheap talk extensions to arbitrary games).

In our environment there are infinitely many ways to make a collective decision. First, we can have committees of different sizes. Second, for a committee of a given size we can choose different voting rules. Finally, we can select different protocols according to which the members of a committee communicate before casting their votes. Of course, these variables will affect the agents' decisions (whether they acquire information or not, as well as how they communicate and vote) and, therefore, the quality of the final decision. We now analyze the problem of designing the optimal mechanism. To accomplish this, we study the following game.

Stage 1 The designer chooses an *extended mechanism*, i.e., the size of the committee n, the voting rule, and a cheap talk extension (i.e., how the players can exchange messages before voting).

Stage 2 All agents observe the designer's mechanism. Each agent $j = 1, ..., n$ decides whether to purchase a signal. These choices are made simultaneously, and each member of the committee does not observe whether other members have acquired information.[1]

[1] Our analysis would, in fact, be tremendously simplified if investments were overt (see Footnote 5).

Stage 3 All the agents in the committee exchange messages (as specified by the cheap talk extension) and vote.

Note that the chosen extended mechanism is comprised partly of the size of the decision panel. This is where we use the assumption that there exists an infinite pool of identical players free to be selected by the designer as participants in the collective decision making process.

Stages 2 and 3 constitute an extensive-form game played by the agents $1, ..., n$. We restrict attention to sequential equilibria in which the players use pure (behavioral) strategies in Stage 2, and are allowed to randomize in Stage 3. A strategy profile of this game determines an outcome (i.e., the probabilities that the correct decision is made in state I and in state G) and therefore, the expected common utility of the decision. The designer chooses the mechanism to maximize her utility (from the decision). In particular, the designer does not take into account the cost incurred by the informed agents. There are different situations in which this assumption is appropriate. The designer may be a principal who delegates the final decision to a committee. Alternatively, the decision may affect the welfare of every individual in a large society and the designer can be a benevolent planner (Persico [2002]). In this case, any increase in the utility from the decision can compensate for the information costs paid by the agents.

The assumption that players can invest in information, thereby endogenizing their types, which does appear in Persico [2002], is a deviation from some of the prevalent models in the literature (e.g., Austen-Smith and Banks [1996] and Feddersen and Pesendorfer [1996, 1998]). We find this model particularly appealing both on realistic and theoretical grounds. Indeed, when thinking about real-life decision panels, players need to decide whether (and sometimes to what extent) to invest in information acquisition: jurors choose whether to

However, in many situations in which agents engage in information acquisition, investment in information is indeed covert and signals are non-verifiable. For example, jurors would have a hard time proving they had attended testimonies, committee members do not check whether their colleagues have gone over the relevant background information before convening, etc.

attend or not the testimonies presented to them, hiring committee members decide whether to carefully go over the candidate's portfolio or not, etc. Theoretically, this framework allows us to study mechanism design in situations where there are two forces at play. On the one hand, the mechanism should use the information available as efficiently as possible. On the other hand, the mechanism needs to provide agents with incentives to invest in information.

We denote each agent j's type in Stage 3 by $t_j \in T_j \equiv \{\phi, i, g\}$, where ϕ stands for an agent who does not purchase information, and i or g stand for an agent who purchases information and receives i or g as the realized signals, respectively.

The analysis presented in Gerardi and Yariv [2003] simplifies enormously the designer's problem. Indeed, once players decide whether to acquire information or not, and the appropriate signals are realized, we are in the setup of that paper. Suppose the jury has at least three members. Consider a voting rule and a communication protocol. A strategy profile of the corresponding cheap talk extension induces a mapping from the set of types' profiles into $[0, 1]$, the set of probability distributions over the alternatives A and C. For any voting rule ψ, Gerardi and Yariv characterize Γ_ψ, the set of mappings induced by sequential equilibria of arbitrary cheap talk extensions. We remind the reader of the notion of non-dictatorial voting rules, introduced earlier, which identifies rules such that for every alternative, A or C, there exists an action profile that yields that alternative, and is robust to unilateral deviations. Gerardi and Yariv show that all non-dictatorial voting rules are equivalent, in the sense that they all implement the same set of mappings ($\Gamma_\psi = \Gamma_{\psi'}$ for any pair (ψ, ψ') of non-dictatorial voting rules). Gerardi and Yariv also show that with a dictatorial voting rule it is possible to implement only a subset of mappings (if ψ is non-dictatorial and ψ' is dictatorial, then $\Gamma_{\psi'} \subseteq \Gamma_\psi$). Finally, they demonstrate that any element of Γ_ψ can be implemented with a sequential equilibrium in which there is always unanimous consensus at the voting stage.

These results imply that when the size of the committee is $n > 2$, we can ignore the choice of the voting rule. Moreover, we only need to consider mappings from types into final decisions that give all players an incentive to reveal their private information. To state formally this simplified problem, we need to introduce some notation. A communication device in a committee of size n is a mapping $\gamma : T_1 \times ... \times T_n \rightarrow [0,1]$ (recall that $T_j = \{\phi, i, g\}$). Given a committee with n members and a communication device γ, consider the following game. All players simultaneously decide whether to purchase a signal. Then each player $j = 1, ..., n$ reports her type to the device, and $\gamma(t)$ denotes the probability that the defendant is convicted when the vector of reports is t.[2] We refer to this game as the game induced by γ. Let σ_j define player j's choice at the information acquisition stage, i.e., σ_j specifies whether player j acquires information or not (as already mentioned, we do not allow for mixed strategies in this stage of the game). Given σ_j, we let $(\sigma_j, *)$ define the strategy under which player j chooses σ_j at the information acquisition stage and reports truthfully her type to the device.

It follows from Gerardi and Yariv [2003] that when the designer chooses a committee of size $n > 2$, we need only to consider pairs of decision profiles $(\sigma_1, \ldots, \sigma_n)$ and communication devices γ such that the strategy profile $((\sigma_1, *), \ldots, (\sigma_n, *))$ constitutes a (sequential) equilibrium of the game induced by γ.[3] This means that the acquisition strategies $(\sigma_1, \ldots, \sigma_n)$ and the communication device γ have to satisfy the following two constraints. First, given $(\sigma_1, \ldots, \sigma_n)$, the device γ is incentive compatible (i.e., each player has an incentive to reveal her type truthfully). Second, if all players report their true types

[2] Although we restrict attention to direct communication, the reader may find it useful to interpret the device as an exogenous mediator who helps the members of the committee communicate (in Gerardi and Yariv [2003] we show that there is no difference between mediated and unmediated communication). If the players report vector t to the mediator, then with probability $\gamma(t)$ she recommends to every player to vote in favor of conviction (and with probability $1 - \gamma(t)$ the mediator invites all player to vote to acquit). If the voting rule is non-dictatorial, each player has no incentive to disobey the mediator's recommendation provided that all her opponents behave likewise.

[3] In this paper, we do not assume that the designer is able to commit to a function that maps the players' information into final decisions (perfect commitment). One would think that perfect commitment is beneficial for the designer. Our analysis shows that this is not the case. With communication and non-dictatorial voting rules it is possible to implement the same outcomes that can be implemented with perfect commitment.

to the device γ and player j's opponents use strategies $(\sigma_1, \ldots, \sigma_{j-1}, \sigma_{j+1}, \ldots, \sigma_n)$ then σ_j is optimal for player j (where $j = 1, \ldots, n$).

Although this problem is much simpler to analyze than the original one, in order to characterize the optimal mechanism we still have to consider all possible sizes and, for each size, all pairs $((\sigma_1, \ldots, \sigma_n), \gamma)$ that satisfy the requirements described above. We now present a number of steps that further simplify the designer's problem.

Given the committee size n, there are many pairs $((\sigma_1, \ldots, \sigma_n), \gamma)$ for which the strategy profile $((\sigma_1, *), \ldots, (\sigma_n, *))$ is an equilibrium of the game induced by γ. In particular, there are profiles $(\sigma_1, \ldots, \sigma_n)$ in which all players acquire information, and profiles $(\sigma_1, \ldots, \sigma_n)$ in which one or more players do not pay the cost c to observe a signal. However, without loss of generality, we can restrict attention to profiles $(\sigma_1, \ldots, \sigma_n)$ in which every member of the committee buys the signal. Consider an outcome implemented by a pair $(((\sigma_1, \ldots, \sigma_n), \gamma))$ in which only players $1, ..., n'$ acquire the signal, where $n' < n$. It is easy to show that the same outcome can be implemented with a committee of size n' and a communication device γ' that gives incentives to all members to acquire information and report it truthfully. For every vector of reports $t_1, ..., t_{n'}$, let $\gamma'(t_1, ..., t_{n'}) = \gamma(t_1, ..., t_{n'}, \phi, ..., \phi)$. Under the original pair $(((\sigma_1, \ldots, \sigma_n), \gamma))$, the first n' players know that players $n'+1, ..., n$ do not purchase the signal and report message ϕ to the device γ (remember that γ induces truthful revelation). If players $1, ..., n'$ decide to acquire information and be sincere under γ then they have an incentive to do the same under γ'. Therefore, in the remainder of the section we focus on communication devices that induce all players to acquire information and reveal it sincerely (provided that her opponents behave likewise). We call these devices *admissible.* It is important to note that admissible devices are characterized by two classes of incentive compatibility constraints. The first is the already introduced truthful revelation constraint. The second guarantees that each player best responds by acquiring information.

The next step of our analysis is to show that we can ignore what a communication device specifies when one or more players report the message ϕ. Consider an admissible device

$\gamma : \{\phi, i, g\}^n \to [0,1]$. Let $U_j\left(t_j, t'_j\right)$ denote the expected utility (from the decision) of player j when her type is t_j, she reports message t'_j and all her opponents acquire information and are sincere. *Truthful revelation* holds if and only if:

$$U_j\left(t_j, t_j\right) \geqslant U_j\left(t_j, t'_j\right) \quad \forall j = 1, ..., n, \quad \forall \left(t_j, t'_j\right) \in \{\phi, i, g\}^2 . \tag{1}$$

The expected utility of player j when she does not acquire information can be expressed as:

$$U_j(\phi, \phi) = \Pr(i)\, U_j(i, \phi) + \Pr(g)\, U_j(g, \phi),$$

where $\Pr(s)$ denotes the probability that agent j will observe signal $s = i, g$ if she acquires information. It follows that agent j will purchase the signal if and only if the following *information acquisition* constraint is satisfied:

$$\Pr(i)\, U_j(i, i) + \Pr(g)\, U_j(g, g) - c \geqslant \Pr(i)\, U_j(i, \phi) + \Pr(g)\, U_j(g, \phi). \tag{2}$$

Constraints (1) and (2) imply that a necessary condition for a communication device to be admissible is that for every player $j = 1, ..., n$:

$$\Pr(g)\left(U_j(g, g) - U_j(g, i)\right) \geqslant c, \tag{3}$$

$$\Pr(i)\left(U_j(i, i) - U_j(i, g)\right) \geqslant c. \tag{4}$$

These inequalities guarantee that agent j prefers to buy the signal and be sincere rather than not buy the signal and always report one of $s = i, g$ (i in the first inequality, g in the second one).

We now explain in which sense inequalities (3) and (4) are also a sufficient condition for a device to be admissible. Consider a device $\gamma : \{\phi, i, g\}^n \to [0,1]$, and suppose inequalities (3) and (4) hold. This device may not be incentive compatible. In particular, a player may have an incentive to lie when her type is ϕ. Consider, however, the outcome induced by γ when all players acquire information and are sincere. This outcome can be implemented with the following admissible device γ'. Given γ, consider player $j = 1, ..., n$ and assume that all her opponents acquire information and are sincere. Suppose that player j does not observe a signal and has to choose between message i and message g. Denote by s_j the message that agent j prefers to send. That is, $s_j \in \{i, g\}$ and is such that

$$\Pr(i) U_j(i, s_j) + \Pr(g) U_j(g, s_j) \geqslant \Pr(i) U_j\left(i, s'_j\right) + \Pr(g) U_j\left(g, s'_j\right),$$

where $s'_j = i, g$.

Given the device γ, we construct γ' as follows:

$$\gamma'(t) = \begin{cases} \gamma(t) & \text{if } t \in \{i, g\}^n \\ \gamma(s_j, t_{-j}) & \text{if } t = (\phi, t_{-j}) \text{ and } t_{-j} \in \{i, g\}^{n-1} \end{cases},$$

and we assign an arbitrary value to $\gamma'(t)$ when two or more players report message ϕ. Intuitively, when each player different from j sends either i or g, the device γ' interprets message ϕ of player j as message s_j.

Notice that the expressions in inequalities (3) and (4) do not depend on what the device specifies when some players report ϕ. We can, therefore, think of an admissible device as a mapping $\gamma : \{i, g\}^n \to [0, 1]$ which satisfies conditions (3) and (4).

An admissible device γ is *symmetric* if for every vector $(t_1, ..., t_n)$ in $\{i, g\}^n$ and every permutation φ on $\{1, ..., n\}$, $\gamma(t_1, ..., t_n) = \gamma\left(t_{\varphi(1)}, ..., t_{\varphi(n)}\right)$. In a symmetric device, the probability that the defendant is convicted depends only on the number of messages g (or i) but not on the identity of the players who send g. We now argue that there is no loss of generality in considering only symmetric devices. In fact, suppose that γ is an admissible device and consider a permutation φ on $\{1, ..., n\}$ (let Λ_n denote the set of such permutations). Consider the device γ_φ, where $\gamma_\varphi(t_1, ..., t_n) = \gamma\left(t_{\varphi(1)}, ..., t_{\varphi(n)}\right)$ for every $\{t_1, ..., t_n\}$ in $\{i, g\}^n$. Since all players are identical and γ is admissible, the device γ_φ is also admissible and outcome equivalent to γ. Of course, convex combinations of admissible devices are admissible. It follows that the symmetric device $\tilde{\gamma} = \left(\frac{1}{|\Lambda_n|}\right) \sum_{\varphi \in \Lambda_n} \gamma_\varphi = \frac{1}{n!} \sum_{\varphi \in \Lambda_n} \gamma_\varphi$ is admissible and outcome equivalent to the original device γ.

In order to emphasize the points illustrated so far, we recap our discussion as follows:

Summary Without loss of generality, the designer can restrict attention to mechanisms in which the following two qualifications pertain:

1. Admissibility - all committee members acquire information and report their signals truthfully;

2. Symmetry - committee members are anonymous.

A symmetric device can be represented as a mapping $\gamma : \{0, 1, ..., n\} \to [0, 1]$, where $\gamma(k)$ denotes the probability that the defendant is convicted (alternative C is chosen) when k players report the guilty signal g (each player can report either i or g). For a symmetric device $\gamma : \{0, 1, ..., n\} \to [0, 1]$ conditions (3) and (4) can be expressed as:

$$\sum_{k=0}^{n-1} \binom{n-1}{k} f(k+1; n) (\gamma(k+1) - \gamma(k)) \geqslant c, \tag{ICi}$$

$$\sum_{k=0}^{n-1} \binom{n-1}{k} f(k; n) (\gamma(k) - \gamma(k+1)) \geqslant c, \tag{ICg}$$

where $f(\cdot; n) : \mathbb{R} \to \mathbb{R}$ is defined by:

$$f(x; n) = -qP(I)(1-p)^x p^{n-x} + (1-q) P(G) p^x (1-p)^{n-x}.$$

For each $n > 2$, we look for the optimal admissible device, i.e., the admissible device that maximizes the expected utility of the decision. This amounts to solving the following linear programming problem P_n :

$$\max_{\gamma:\{0,...,n\}\to[0,1]} -(1-q) P(G) + \sum_{k=0}^{n} \binom{n}{k} f(k; n) \gamma(k)$$

$$\text{s.t. } (ICi), (ICg).$$

We denote by $\bar{\gamma}_n$ the solution to problem P_n (if it exists), and by $V(n)$ the expected utility of the optimal device. If problem P_n does not have any feasible solution, we set $V(n) = -1$.

To complete the description of all mechanisms, we need to consider committees with less than three players. We let $V(0) = \max\{-qP(I), -(1-q)P(G)\}$ denote the expected utility of the optimal decision when no information is available.

Suppose $n = 1$, i.e., the designer delegates the final decision to a single agent. Let $\hat{V}(1)$ be the expected utility of the optimal decision of the agent when she acquires information. Formally,

$$\hat{V}(1) = \max_{\gamma:\{0,1\}\to[0,1]} -(1-q)P(G) + \sum_{k=0}^{1} f(k;1)\gamma(k).$$

If the benefit of buying the signal, $\hat{V}(1) - V(0)$, is greater than or equal to the cost c, the agent will acquire information, and we set $V(1) = \hat{V}(1)$. Otherwise, we set $V(1) = -1$.

Finally, suppose that the designer chooses a committee with two agents. We define problem P_2 in the same way as P_n for $n > 2$. We let $V(2)$ denote the value of the objective function at the solution (if it exists). Notice, however, that problem P_2 does not guarantee that $V(2)$ can be achieved by the extended mechanism designer. In fact, when $n = 2$, a player can be pivotal at the voting stage and we need to take into account her incentives to follow the mediator's recommendation (constrains (ICi) and (ICg) do not capture these incentives). In other words, $V(2)$ represents an upper bound of the expected utility that can be achieved with two players. If the problem P_2 does not admit any feasible solution, we set $V(2) = -1$.

The optimal mechanism consists of the optimal size of the committee n^*, and the optimal admissible device $\bar{\gamma}_{n^*}$. n^* is such that $V(n^*) \geqslant V(n)$, for every nonnegative integer n.[4] In what follows, we will demonstrate that the optimal size of the committee is always finite. Furthermore, when the cost of acquiring information is sufficiently low, the optimal size n^* is greater than 2 (see below) and, therefore, the expected utility $V(n^*)$ can be achieved.

[4]Notice that $V(0) > -1$, and, thus, $n > 1$ can be the optimal size only if problem P_n admits a feasible solution. Similarly, $n = 1$ can be optimal only if the agent who has to make the final decision acquires information.

4 Features of The Optimal Extended Mechanism

The extended mechanism the designer chooses is comprised of the size of the decision panel as well as the incentive scheme it will operate under. In Subsection 4.1 we tackle the first aspect of this design problem. Namely, we illustrate that the optimal committee is of bounded size. In Subsection 4.2 we illustrate some traits of the optimal device the designer would choose. In particular, we show that imperfectly aggregating the available information may induce more players to acquire information, thereby yielding a higher overall expected utility level.

4.1 The Scope of The Committee

Our first result, Proposition 1, shows that the solution to our design problem always exists. In fact, we show that when the size of the committee is very large it is impossible to give the incentive to all the members to acquire information. That is, committees with too many members do not have any admissible device. Intuitively, when there are many agents in the committee, the marginal contribution of an additional signal is relatively small. Therefore, each agent has an incentive to save the cost c and benefit from the information acquired by the other participants. In other words, in large committees there is a severe free rider problem.

Proposition 1 *Fix $P(I)$, q, p and c. There exists $\bar{n}$ such that for every $n \geqslant \bar{n}$, problem P_n does not have any feasible solution.*

Proof. See Appendix.

Clearly, it follows from Proposition 1 that the optimal size of the committee n^* is finite and smaller than $\bar{n}$. This, in turn, implies that when information is costly, the probability of making the wrong decision is bounded away from zero. This observation stands in contrast to the underlying message of the information aggregation literature (see, e.g., Feddersen and Pesendorfer [1996, 1997]) in which a large pool of agents yields complete aggregation of all of the available information.

It is interesting to note that the fact that $P_{\tilde{n}}$ does not have any feasible solution does not imply that P_n does not have any feasible solution for all $n > \tilde{n}$. For example, for $P(I) = P(G) = \frac{1}{2}$, $p = 0.8$, $q = 0.7$, and $c = 0.32$, P_6 and P_7 have feasible solutions, while P_5 does not. In the proof of Proposition 1, we provide a formula for an upper bound beyond which no feasible solution exists. This is potentially useful for computational reasons. Indeed, in order to find the optimal n^*, one needs to check for the values produced by the feasible solutions of P_n. As illustrated, calculating consecutively the solutions for P_1, P_2, P_3, etc. and arriving at an n for which no feasible solution to P_n exists is no indication that committees bigger than n are not optimal. Thus, an upper bound is necessary for the designer to limit her search.

4.2 Optimal Distortionary Devices

The next question is how the optimal device uses the information of the agents. To analyze this problem, let us first consider the case in which the designer makes the final decision after observing n free signals. This will give us an upper bound on what the designer can achieve when she chooses a committee of size n and information is costly.

To find the optimal decision rule, we simply need to maximize the objective function of problem P_n (without the constraints). We let γ_n^B denote the solution to this maximization problem. γ_n^B, which we call a *Bayesian* device, is of the form:

$$\gamma_n^B(k) = \begin{cases} 0 & \text{if } f(k;n) \leqslant 0 \\ 1 & \text{if } f(k;n) > 0 \end{cases},$$

(in fact, when $f(k;n) = 0$, $\gamma_n^B(k)$ can be any number in the unit interval).

To interpret this result, notice that $f(k;n)$ is positive (negative) if and only if the cost of convicting the innocent q is smaller (greater) than the probability that the defendant is guilty given that k of n signals are g.

The function $f(\cdot;n)$ is increasing and we let $z(n)$ be defined by $f(z(n);n)=0$. We have:

$$z(n)=\frac{1}{2}\left(n+\frac{\ln\left(\frac{qP(I)}{(1-q)P(G)}\right)}{\ln\left(\frac{p}{1-p}\right)}\right).$$

Another way to express the Bayesian device γ_n^B is:

$$\gamma_n^B(k)=\left\{\begin{array}{ll}0 & \text{if } k\leqslant z(n)\\ 1 & \text{if } k>z(n)\end{array}\right. .$$

For small values of n, $z(n)$ can be negative or greater than n. In the first case, the optimal decision is always to convict the defendant. In the latter case, the optimal decision is always to acquit. These cases arise when the designer is very concerned with a particular mistake (acquitting the guilty or convicting the innocent), and the signal is not very accurate, i.e., p is close to $1/2$. In both cases the n signals are of no value. For large values of n, however, $z(n)$ is positive and smaller than n ($z(n)/n$ converges to $1/2$ as n goes to infinity), and the defendant will be convicted if and only if the designer observes sufficiently many guilty signals.

We let $\hat{V}(n)$ denote the expected utility of the Bayesian device:

$$\hat{V}(n)=-(1-q)P(G)+\sum_{k\in\{0,..,n\},\ k>z(n)}\binom{n}{k}f(k;n),$$

The utility $\hat{V}(n)$ is nondecreasing in the number of signals n. Moreover, $\hat{V}(n)$ is strictly greater than $V(0)$, the expected utility of the optimal uninformed decision, if and only if $z(n)$ belongs to $(0,n)$. If $z(n)$ is not in $(0,n)$, then $V(0)=\hat{V}(1)=...=\hat{V}(n)$.

When n becomes unboundedly large, the Bayesian device uses an infinitely increasing number of i.i.d. signals. The law of large numbers ensures that all uncertainty vanishes asymptotically. In particular, $\hat{V}(n)$ converges to zero, the no uncertainty value, when n goes to infinity.[5]

[5] Note that if information acquisition is overt and $c<1$, then $\hat{V}(n)$ is implementable (in Nash equilibrium) for sufficiently large n. Indeed, consider the following scenario. The designer selects the Bayesian

We now return to the original design problem. Clearly, the expected utility of the optimal admissible device $V(n)$ cannot be greater than $\hat{V}(n)$, since the Bayesian device γ_n^B is the solution to the unconstrained problem. On the other hand, when the Bayesian device γ_n^B is admissible, we have $V(n) = \hat{V}(n)$. In this case the designer is able to give the incentive to the n agents to acquire the signal and, at the same time, to make the best use of the available information. Proposition 2 shows that this can happen if and only if the contribution of the last signal to the utility of a single decision maker is greater than or equal to its cost.

Proposition 2 *For every* $n \geqslant 2$, $V(n) = \hat{V}(n)$ *if and only if* $\hat{V}(n) - \hat{V}(n-1) \geqslant c$.

Proof. See Appendix.

The following example provides an intuition for Proposition 2. The Bayesian device γ_9^B of a committee of size 9 selects conviction if at least 5 players report a guilty signal. Consider the Bayesian device γ_8^B for the committee of size 8. There are two cases, depending on p, q and $P(I)$: (a) the device selects conviction when there are at least 4 guilty signals; (b) it selects conviction when there are at least 5 guilty signals. Consider the committee with 9 members and suppose that each opponent of player 1 acquires information and is sincere. Player 1's expected utility if she also acquires information and is sincere is equal to $\hat{V}(9) - c$. However, if player 1 does not purchase the signal and reports message g in case (a) and message i in case (b), she gets $\hat{V}(8)$. The proof of Proposition 2 formalizes this argument.

We assume that there exists at least one integer for which the Bayesian device is admissible. Let n^B denote the greatest such integer. That is, $\hat{V}\left(n^B\right) - \hat{V}\left(n^B - 1\right) \geqslant c$,

device γ_n^B as long as everyone purchases information, and a device γ that makes a choice contrary to the Bayesian prescription if any agent does not purchase information (i.e., for all k, $\gamma(k) = 1 - \gamma_n^B(k)$). The strategy profile under which all players acquire information and are always sincere constitutes a Nash equilibrium. Under this profile, the expected utility of the decision approaches 0. If one player deviates and does not acquire information, she drives the common utility to a level that approaches -1. Finally, no agent has an incentive to lie upon acquiring information.

and $\hat{V}(n) - \hat{V}(n-1) < c$ for every $n > n^B$. The existence of n^B is guaranteed by the fact that the sequence $\left\{\hat{V}(1), ..., \hat{V}(n), ...\right\}$ converges (to zero). The designer can induce more than n^B players to acquire information only if she selects a device that aggregates the available information suboptimally. On the other hand, more information will be available in larger committees. How should the designer solve this trade-off? Is the optimal size of the committee equal to or larger than n^B? Before answering these questions, let us explain why we believe they are important.

Suppose n is such that the Bayesian device is admissible. We now show that there is a very simple mechanism that does not require communication and allows the designer to obtain utility $\hat{V}(n)$. Let k_n be the smallest integer greater than $z(n)$. Notice that k_n belongs to $\{1, ..., n\}$ since $\hat{V}(n)$ can be greater than $\hat{V}(n-1)$ only if $z(n)$ is in $(0, n)$. Consider the following game. Each agent decides whether to buy a signal or not. Then the players vote, and the defendant is convicted if and only if at least k_n agents vote to convict. This game admits an equilibrium in which each player acquires the signal and votes sincerely (i.e., she votes to convict if and only if she observes signal g). The expected utility of the decision when the agents play this equilibrium is, of course, $\hat{V}(n)$.

Consider our design problem. If the optimal size of the committee is n^B, then communication is unnecessary and the designer simply needs to select the threshold voting rule k_{n^B} (this is the optimal mechanism in Persico [2002] where communication is not allowed). On the other hand, if the optimal size is larger than n^B, then communication may play a very important role in the solution to the designer's problem.[6] Proposition 3 shows that this is indeed the case (at least when the cost is sufficiently low).

Before formally stating the result, we need to introduce one technical assumption. We

[6] As noted by Nicola Persico, our results have an alternative interpretation. Namely, the optimal device γ described in our analysis could be implemented without communication, but with a probabilistic voting rule (essentially, a voting rule that determines the final choice to be C with probability $\gamma(k)$ when k players vote for that alternative). Since such rules do not seem to be very prevalent (consequentially, they have received little, if no, attention in the literature), we find our interpretation of communication as the channel by which aggregate choices are randomized to be more natural.

say the environment is *regular* if $\frac{\ln\left(\frac{qP(I)}{(1-q)P(G)}\right)}{\ln\left(\frac{p}{1-p}\right)}$ is not an integer. This implies that for all n, $z(n)$, the Bayesian threshold value, is not an integer. In a regular environment, if n is such that $\hat{V}(n) > V(0)$, then for all $n' \geqslant n$, $\hat{V}(n'+1) > \hat{V}(n')$. Note that assuming the environment is regular is not restrictive since this is, generically, the case.

Proposition 3 *Fix $P(I)$, q and p and assume the environment is regular. Let $n^*(c)$ denote the optimal size of the committee when the cost of acquiring information is c. There exists $\bar{c} > 0$ such that for every $c < \bar{c}$, $V(n^*(c)) < \hat{V}(n^*(c))$.*

Proof. See Appendix.

In the proof of Proposition 3, we show that if n^B is sufficiently large (or equivalently, if c is sufficiently low), then there exists a non Bayesian device for a committee of size n^B+1 that is better than the Bayesian device with n^B players, i.e., $V\left(n^B+1\right) > \hat{V}\left(n^B\right)$. Although we do not have a necessary condition for the optimal size to be greater than n^B, notice that the result in Proposition 3 cannot be extended to all values of c. Consider the environment $P(I) = P(G)$, $p = 0.85$, $q = 0.52$, and $c = 0.035$ (the environment is regular). In this case the optimal size n^* and n^B coincide and are equal to three.

4.3 Example

In this section, we provide a simple example illustrating the forces that are at play in constructing the optimal mechanism. We choose one configuration of parameters that allows us to focus on a small committee (five members) and show how distorting the Bayesian device can be beneficial.

Specifically, consider the environment characterized by equally likely states, i.e., $P(I) = P(G) = \frac{1}{2}$, preference parameter of $q = 0.72$, and signals of accuracy $p = 0.65$ available for the cost $c = 0.014$.

Without communication, we can concentrate on $\hat{V}(n)$. A simple calculation yields that $\hat{V}(1) = -0.14$, $\hat{V}(2) = -0.12495$, $\hat{V}(3) = -0.1169875$, and $\hat{V}(n) - \hat{V}(n-1) < c$ for any

$n > 2$, thereby leading to $n^B = 2$. Moreover, the corresponding Bayesian device, γ_2^B, is given by:

$$\gamma_2^B(k) = \begin{cases} 0 & \text{if } k < 2 \\ 1 & \text{if } k = 2 \end{cases}.$$

In particular, any one of the two agents is pivotal only when the other receives the g signal, which occurs with probability $\frac{1}{2}$.

Consider now a group of size $n = 5$. The corresponding Bayesian device, γ_5^B, is given by:

$$\gamma_5^B(k) = \begin{cases} 0 & \text{if } k \leqslant 3 \\ 1 & \text{if } k > 3 \end{cases}$$

(using the above notation $z(5) = 3.2628$). To illustrate how distorting the Bayesian device can help satisfy the incentive constraints, we now introduce a device γ^α of the following form:

$$\gamma^\alpha(k) = \begin{cases} 0 & k < 3 \\ \alpha & k = 3 \\ 1 & k > 3 \end{cases}, \text{ where } \alpha \in [0,1].$$

Note that γ^0 coincides with the Bayesian device γ_5^B. Given a device γ^α, let us consider the incentives that a player faces. Without loss of generality, we focus on player 1. Suppose that players $2, \ldots, 5$ acquire information and report it truthfully. We let $y = 0, \ldots, 4$ denote the number of guilty signals observed by player 1's opponents. We let $W_g(\alpha)$ denote the difference between player 1's expected utility from the decision when she acquires information and behaves sincerely and her utility when she remains uninformed and reports message g ($W_g(\alpha)$ does not take into account the cost of information). A necessary condition for player 1 to acquire information is $W_g(\alpha) \geqslant c$ (the other condition is $W_i(\alpha) \geqslant c$, where $W_i(\alpha)$ is defined in the obvious way with respect to the blind report of i).

In order to compute $W_g(\alpha)$ it will be helpful to consider Table 1. The first column describes the events in which player 1's information is relevant, that is, situations in which she is pivotal. The second column contains the probabilities of these events. In the third column, we report the change in the probability of conviction between the case in which player 1 sends message i and the case in which she sends message g (the analysis above allows us to ignore the incentives to report ϕ rather than i or g). Finally, the last column describes the difference in player 1's utility between acquittal and conviction.

Relevant events	$\Pr(\omega)\Pr(y\|\omega)\Pr(i\|\omega)$	$\|\gamma^{\alpha}(y)-\gamma^{\alpha}(y+1)\|$	Change in utility
$\omega=I, y=2, s_1=i$	$\frac{1}{2}(0.3105375)(0.65)$	α	0.72
$\omega=I, y=3, s_1=i$	$\frac{1}{2}(0.111475)(0.65)$	$1-\alpha$	0.72
$\omega=G, y=2, s_1=i$	$\frac{1}{2}(0.3105375)(0.35)$	α	-0.28
$\omega=G, y=3, s_1=i$	$\frac{1}{2}(0.384475)(0.35)$	$1-\alpha$	-0.28

Table 1: Expected Benefits of Information Acquisition

Since we are comparing the case in which player 1 invests in information and is sincere with the case in which she is uninformed and reports message g, it is obvious that player 1's utility is affected only when she observes an innocent signal. There are four relevant cases to consider, corresponding to the rows of Table 1:

1. **(First Row)** Suppose that the state of the world is $\omega=I$, $y=2$ and player 1 observes i. The probability of this event is $\frac{1}{2}(0.3105375)(0.65)$. Since the defendant is innocent, player 1 will benefit from a decrease in the probability of conviction (the benefit is equal to $q=0.72$). Compare now the case in which player 1 sends message i with the case in which she sends message g. Since $y=2$, the decrease in the probability of conviction is $\gamma^{\alpha}(3)-\gamma^{\alpha}(2)=\alpha$.

2. **(Second Row)** The second row considers the analogous case in which the state is I, and player 1 and one of her opponents observe an innocent signal. The probability of this event is $\frac{1}{2}(0.111475)(0.65)$. Again, the state is I and player 1 will benefit from a decrease in the probability of conviction. By reporting message i instead of message g, player 1 decreases the probability of conviction by $1-\alpha$. Of course, we do not need

to consider $y = 0, 1, 4$ because in these cases the final decision does not depend on player 1's message (player 1 is not pivotal).

3. **(Third Row)** Suppose now that the state is $\omega = G$, $y = 2$ and player 1 observes i (third row). The probability of this event is $\frac{1}{2}(0.3105375)(0.35)$. The state of the world is G and, thus, player 1 will suffer from a decrease in the probability of conviction (the net benefit is $-(1-q) = -0.28$). By reporting message i instead of message g the probability of conviction decreases by α.

4. **(Fourth Row)** Finally, the last case to consider is $\omega = G$ and player 1 and one of her opponents observe an innocent signal (last row). This happens with probability $\frac{1}{2}(0.384475)(0.35)$. A decrease in the probability of conviction will decrease player 1's utility (since the defendant is guilty). A change from message g to message i decreases the probability of conviction by $1-\alpha$.

Simple calculations yield $W_g(\alpha) = 0.0072459 + 0.0502036\alpha$. With the Bayesian device γ^0 we have $W_g(0) = 0.0072459$. When the cost is $c = 0.014$ the player prefers to remain uninformed and report message i. From Table 1, when $\alpha = 0$, information acquisition is more profitable than the uninformative message g only if the state is I, player 1 observes i and only one of her opponents observes an innocent signal. Player 1 does not have an incentive to acquire information since the probability of this event is low (equal to 0.036229). As α increases, it will become easier to satisfy the incentive constraint. In fact, when α is greater than 0, information acquisition is also beneficial in the event in which $\omega = I$ and player 1 and two of her opponents observe an innocent signal. The probability of this event is 0.10092 (greater than 0.036229).

It is easy to check that $W_i(\alpha) = 0.0209412 - 0.0318102\alpha$. Thus, by increasing α it becomes more difficult to satisfy the second constraint. However, notice that for $\alpha = 0$ we have $W_i(0) > c$ (i.e., the constraint is not binding). Therefore, we can choose levels of α greater than 0 such that both constraints are still satisfied.

Of course, this increase in incentives comes at the cost of information aggregation. Indeed, ex-post efficiency would require $\alpha = 0$. We are thus interested in finding the minimal α such that the incentives to purchase information by all five agents would be satisfied. A simple calculation yields a solution of $\alpha = 0.1345$. Moreover, the resulting expected common value is $V(5) = -0.1019 > \hat{V}(2)$.

As it turns out, an admissible device exists in a committee of size no greater than six. However, the cost of inducing a sixth player to acquire information is greater than the benefit of having more information available. That is, $\gamma^{0.1345}$ is the solution of the extended mechanism design and the optimal committee size in this environment is indeed $n^* = 5 > n^B$. A graphical illustration of $\gamma^{0.1345}$ appears in the next section.

The possibility of communication has significant effects on the probabilities of the two type of errors the committee can make. Table 2 summarizes the probabilities arising when choosing optimal committees with and without communication.

	$\text{Prob}(d = C \mid \omega = I)$ (Convicting the Innocent)	$\text{Prob}(d = A \mid \omega = G)$ (Acquitting the Guilty)
Without Communication	0.1225	0.5775
With Communication	0.0783868	0.5263371

Table 2: Probabilities of Errors With and Without Communication

Note that in this example both probabilities of error decrease by a significant amount. We should note that numerous other examples illustrate that this is not a global property. Introducing communication always decreases at least one of the probabilities of error. The parameters of the environment determine which of these probabilities decreases.

5 Comparative Statics

In this section we analyze how the optimal extended mechanism and the quality of the decision depend on the primitives of the model. We first look at the impact that changes in the information cost c and in the accuracy of the signal p have on the expected utility of the designer and on the optimal size of the committee. We then perform the comparative statics for the agents' preferences. Finally, we focus on the optimal admissible device.

The Cost of Information

The first, obvious, result is that the expected utility of the optimal mechanism is decreasing in the cost of information acquisition. This follows from the fact that for any given size of the committee, the utility of the optimal devices increases (weakly) when the cost decreases. In fact, if a device is admissible when the cost is c, then the device is also admissible when the cost is lower than c.

We now consider how the optimal size is affected by a change in the information cost. Clearly, given any cost c with optimal size $n^*(c)$, we can always find another cost c', sufficiently lower than c, such that $n^*(c')$ is greater than $n^*(c)$. In fact, it is enough to choose c' such that the Bayesian device γ_n^B is admissible for some n greater than $n^*(c)$. Unfortunately, it is less straightforward to perform the comparative statics for small changes of the information cost. In all the examples we have constructed, the optimal size decreases when the information cost increases. However, we have not been able to prove that this is a general result. To illustrate what constitutes a problem, we consider two committees of size n and $n+1$. For any cost c, let us consider the difference between the utility of the optimal device at $n+1$ and the utility of the optimal device at n. It is possible to construct examples such that this difference is positive for low and high values of the cost, but is negative for intermediate values (in a sense, the utility does not exhibit a single crossing property).[7] In other words, suppose we start with a level of the cost such that size $n+1$

[7] A possible example is the following: $P(I) = \frac{1}{2}$, $q = 0.82$, $p = 0.55$ and $n = 7$.

is better than size n. In general, we cannot conclude that this relation holds when the cost becomes smaller. Of course, this discussion does not show that the optimal size may increase with the cost. It only explains why it could be difficult to obtain analytical results. But it remains an open question whether the optimal size is indeed always decreasing in the cost or not.

The Signals' Accuracy

As one would expect, the utility of optimal extended mechanism increases when the signal becomes more accurate, i.e. when p increases. In fact, a stronger result holds. For a committee of a given size n, the utility of the optimal device increases when the quality of the signal improves. Intuitively, when the signal is more accurate, the device can ignore some information and replicate an environment with less information. Formally, let γ be an admissible device when the accuracy of the signal is p. Suppose now the accuracy is $\hat{p} > p$ and consider the following device $\hat{\gamma}$. The simplest way to describe $\hat{\gamma}$ is to imagine that each player j reports her signal $s_j = i, g$ to a mediator. The mediator then generates a new variable $s'_j = i, g$ at random according to the distribution:

$$\Pr\left(s'_j = i | s_j = i\right) = \Pr\left(s'_j = g | s_j = g\right) = \frac{p + \hat{p} - 1}{2\hat{p} - 1}.$$

The variables $s'_1, ..., s'_n$ are independent of each other. Finally, the mediator applies the original device γ to the vector $(s'_1, ..., s'_n)$.[8] Notice that the mediator's distribution is appropriately chosen so that $\Pr\left(s'_j = i | I\right) = \Pr\left(s'_j = g | G\right) = p$. Thus, the expected utility of the device $\hat{\gamma}$ when the accuracy is $\hat{p}$ coincides with the utility of γ when the accuracy is p. It is also easy to show that $\hat{\gamma}$ is admissible (when the accuracy is $\hat{p}$). This implies that for any committee size, the utility of the optimal device is increasing in p.

[8] That is, for every vector $(s_1, ..., s_n)$ in $\{i, g\}^n$,

$$\hat{\gamma}(s_1, ..., s_n) = \sum_{(s'_1, ..., s'_n) \in \{i,g\}^n} \Pr\left((s'_1, ..., s'_n) \mid (s_1, ..., s_n)\right) \gamma(s'_1, ..., s'_n),$$

where $\Pr\left((s'_1, ..., s'_n) \mid (s_1, ..., s_n)\right)$ denotes the probability that the mediator generates the vector $(s'_1, ..., s'_n)$ when the vector of reports is $(s_1, ..., s_n)$.

While in our model the designer always benefits from a more informative signal, this is not necessarily the case when communication is not possible. For example, in Persico [2002] the utility of the optimal mechanism can decrease when p increases. When players are not allowed to communicate, the designer can induce n agents to acquire information if and only if $\hat{V}(n) - \hat{V}(n-1)$ is greater than the information cost. When the signal is very informative, $\hat{V}(n)$ is close to zero for n relatively small, and therefore it is impossible to induce many players to acquire information. In contrast, when the signal is less accurate, the difference $\hat{V}(n) - \hat{V}(n-1)$ can be larger than the cost for large values of n. It is possible for the designer to prefer having many uninformative signals over a few very accurate ones.[9]

As far as the optimal size of the committee is concerned, several examples indicate that it is not monotonic in p. Consider, for instance, the case $P(I) = \frac{1}{2}$, $q = 0.62$, and $c = 0.004$. The optimal size is 13 for $p = 0.55$, 24 for $p = 0.65$, and 15 for $p = 0.75$.

The Preference Parameter

The agents and the designer's preferences are characterized by the parameter q, the cost of convicting the innocent. We do not have a formal result for the relation between q and the optimal size of the committee. However, all the examples that we have constructed suggest that it depends on the comparison between q and $P(G)$, the probability that the defendant is guilty. If q is greater than $P(G)$, the agents (and the designer) are more concerned with the error of convicting the innocent, and an uninformed agent would acquit the defendant. In this case, the optimal size of the committee decreases when q increases. Conversely, if q is smaller than $P(G)$, the optimal uninformed decision is to convict the defendant. In this case, the optimal size is increasing in q. Our examples also show that the utility of the optimal mechanism is not monotonic in q (even if we restrict attention to values of q above or below $P(G)$). Finally, notice that similar results hold for the prior distribution, since q and $P(I)$ play an interchangeable role throughout all of our analysis.

[9] To give a concrete example, let us assume that $P(I) = \frac{1}{2}$, $q = 0.82$, and $c = 0.0013$. When $p = 0.85$, the optimal size is 10 and the utility is -0.001455 (the voting rule is $r = 6$). However, when $p = 0.95$, the optimal size is 4 and the utility is -0.001459 (the voting rule is $r = 3$).

The Optimal Device

As was illustrated in Proposition 3, for certain parameters, the optimal device does not coincide with the corresponding Bayesian device. It is then natural to compare $\bar{\gamma}_{n^*}$ with the Bayesian $\gamma^B_{n^*}$, which is graphically represented as a step function achieving the value of 0 for any $k \leqslant z(n^*)$ and the value of 1 for any $k > z(n^*)$. There are three main observations that may prove significant for future theoretical investigations of the optimal extended mechanism.

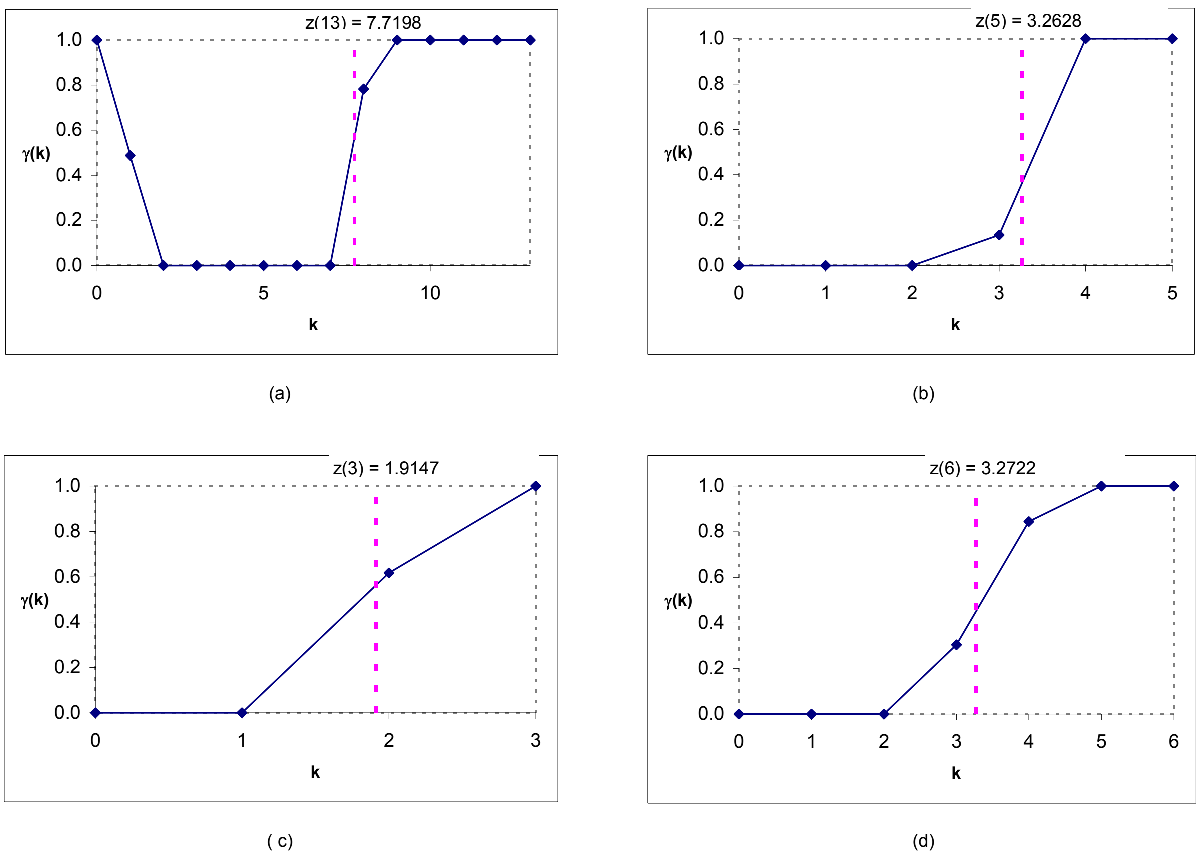

Figure 1: The Optimal Devices

First, unlike its Bayesian counterpart, $\bar{\gamma}_{n^*}$ may be non-monotonic in the number of guilty signals k. For the sake of illustration, consider the example in which $P(I) = \frac{1}{2}$,

$c = 0.004$, $p = 0.55$, and $q = 0.62$. In this case, the optimal size of the team is $n^* = 13$. Figure 1a plots $\bar{\gamma}_{n^*}$ and the Bayesian threshold $z(n^*)$ and illustrates that $\bar{\gamma}_{n^*}$ may indeed be non-monotonic.

Second, when $\bar{\gamma}_{n^*}$ is monotonic, there is no global regularity in its relation to the Bayesian incentive scheme. Specifically, it can be above $\gamma^B_{n^*}$ (thereby inducing a higher or equal probability of conviction for any profile of signals), as illustrated in Figure 1b for the case of $P(I) = \frac{1}{2}$, $c = 0.014$, $p = 0.65$, and $q = 0.72$. It can be below $\gamma^B_{n^*}$ (thereby inducing a lower or equal probability of conviction), as illustrated in Figure 1c for the case of $P(I) = \frac{1}{2}$, $c = 0.014, p = 0.95$, and $q = 0.92$ (which corresponds to the example of Section 4.3). It can also be neither below nor above $\gamma^B_{n^*}$, as illustrated in Figure 1d for the case of $P(I) = \frac{1}{2}$, $c = 0.014, p = 0.85$, and $q = 0.72$.

Third, in cases as described above, an agent is pivotal for more than one value of guilty signals. Intuitively, increasing an agent's probability of being pivotal (relative to the Bayesian device) increases her incentives to purchase information. In such situations, this effect is stronger than the statistical efficiency loss.

6 Conclusions

A group can be identified as a collection of agents satisfying one or more of the following three elements: sharing a common goal, having a joint task, or possessing the ability to communicate and exchange information at no costs. The current paper considers groups satisfying all three conditions and introduces a model of group decision making under uncertainty.

Our analysis yielded three key insights. First, the optimal incentive scheme in such an environment balances a trade-off between inducing players to acquire information and extracting the maximal amount of information from them. In particular, the optimal device may aggregate information suboptimally from a statistical point of view. Second, when members of the group decide whether to acquire costly information or not preceding the communication stage, groups producing the optimal collective decisions are bounded in size. Third, the comparative statics of the optimal mechanism exhibit some regularities and irregularities, e.g., the expected social value is monotonic in the cost of information and accuracy of private information, but the optimal panel size is not monotonic in the signals' accuracy.

In what follows we outline several avenues in which the current framework can be extended.

First, it would be interesting to investigate additional extended mechanisms. For instance, one could consider a setup in which the designer is able to subsidize agents' information. One example would be a department chair investing in the creation of a centralized web-site containing all information pertaining to job candidates. Such an investment would potentially reduce the cost of studying each candidate's portfolio for all of the hiring committee members. Formally, in the extended mechanism the designer has to choose the level of informational subsidies in addition to the size of the group and the communication protocol. Selecting a high level of subsidies creates a trade-off between inducing more agents to acquire information, and internalizing some of this information cost by the designer.

A second interesting extension concerns the homogeneity of the players. So far we have considered homogenous decision panels, in the sense that all players, including the mechanism designer, share the same preferences. Concretely, in our model, both the designer and all of the players share the same utility parameter q. However, in many situations it is conceivable that agents have heterogenous preferences. One could then study the extended mechanism design problem in which, at stage 1, the designer chooses the distribution of preference parameters of the decision panel members, in addition to choosing the panel's size and the device. An analysis of such a scenario would entail defining carefully the goal of the designer (maximizing her own preferences, as characterized by one given q, or implementing a point in the Pareto frontier of the equilibria set). We are especially interested in the optimal composition of the decision panel. In particular, would the designer choose a committee comprised of agents with preferences coinciding with her own or would she choose agents with diverging tastes (as observed, e.g., in the optimal choice of central banker - see, for example, Alesina and Gatti [1995] and references therein)?

Appendix

Proof of Proposition 1

We prove Proposition 1 by demonstrating that when the size of the committee is sufficiently large there is no device that satisfies constraint (ICi). The choice of the constraint is arbitrary since we could prove the same result for constraint (ICg).[10]

Given a device $\gamma : \{0, ..., n\} \rightarrow [0, 1]$, let $H_n(\gamma)$ denote the left hand side of constraint (ICi):

$$H_n(\gamma) = \sum_{k=0}^{n-1} \binom{n-1}{k} f(k+1; n) (\gamma(k+1) - \gamma(k)).$$

For ease of presentation, we will drop the argument n in f throughout this proof. $H_n(\gamma)$ can be expressed as:

$$H_n(\gamma) = -f(1)\gamma(0) + \sum_{k=1}^{n-1} \binom{n-1}{k-1} (1-p)^k p^{n-k-1} \frac{1}{k} h(k) \gamma(k) + f(n) \gamma(n),$$

where

$$h(k) = qP(I)(n(1-p) - k) + (1-q) P(G) \left(\frac{1-p}{p}\right)^{n-2k-1} (k - np).$$

For n sufficiently large, $-f(1) > 0$ and $f(n) > 0$. Moreover, $h(k) < 0$ for k in $[n(1-p), np]$.

Let $\lfloor n(1-p) \rfloor$ denote the greatest integer smaller than $n(1-p)$. For every $k = 1, ..., \lfloor n(1-p) \rfloor - 1$, we have:

$$h(k) \geqslant qP(I) - (1-q) P(G) \left(\frac{1-p}{p}\right)^{n(2p-1)+1} (np-1).$$

[10] In this proof, we show how to construct an upper bound on the size of the committees that have admissible devices. The fact that we do not consider both constraints at the same time implies that our bound is not tight. Admissible devices may not exist even when the size of the committee is smaller than our bound.

Notice that the right hand side of the above inequality is positive when n is sufficiently large. Similarly, let $\lceil np \rceil$ be the smallest integer greater than np. For $k = \lceil np \rceil + 1, ..., n-1$,

$$h(k) \geqslant -qP(I)(np-1) + (1-q)P(G)\left(\frac{p}{1-p}\right)^{n(2p-1)+1},$$

and the right hand side is positive for n sufficiently large.

Consider the following maximization problem:

$$\max_{\gamma:\{0,...,n\}\to[0,1]} H_n(\gamma),$$

and let γ^+ denote the solution. Define $\bar{H}_n = H_n(\gamma^+)$. It follows from the analysis above that when n is sufficiently large, the device γ^+ is of the form:

$$\gamma^+(k) = \begin{cases} 1 & \text{if } k = 0, ..., k', k'', ..., n \\ 0 & \text{if } k = k'+1, ..., k''-1 \end{cases}$$

where k' is either $\lfloor n(1-p) \rfloor - 1$ or $\lfloor n(1-p) \rfloor$, and k'' is either $\lceil np \rceil$ or $\lceil np \rceil + 1$. This, in turn, implies:

$$\bar{H}_n = -\binom{n-1}{k'} f(k'+1) + \binom{n-1}{k''-1} f(k'').$$

When n is sufficiently large, both $-f(k'+1)$ and $f(k'')$ are positive. Furthermore,

$$-f(k'+1) \leqslant qP(I)(1-p)^{n(1-p)-1}p^{np+1} - (1-q)P(G)p^{n(1-p)-1}(1-p)^{np+1}$$

since k' belongs to $[n(1-p)-2, n(1-p))$, and

$$f(k'') \leqslant -qP(I)(1-p)^{np+2}p^{n(1-p)-2} + (1-q)P(G)p^{np+2}(1-p)^{n(1-p)-2}$$

since k'' belongs to $(np, np+2]$.

To bound the binomial coefficients in the expression of $\bar{H}_n$, we now introduce Stirling's approximation (see Feller [1968]):

$$l_1(n) = \sqrt{2\pi}n^{n+\frac{1}{2}}e^{-n} < n! < \sqrt{2\pi}n^{n+\frac{1}{2}}e^{-n+\frac{1}{12n}} = l_2(n).$$

For every n, the function $\frac{l_2(n)}{l_1(x)l_1(n-x)}$ is increasing for $x < \frac{n}{2}$, and decreasing for $x > \frac{n}{2}$. This and the range of k' and k'' imply:

$$\binom{n-1}{k'} = \frac{n-k'}{n} \frac{n!}{(k')!\,(n-k')!} \leqslant \frac{np+2}{n} \frac{l_2\,(n)}{l_1\,(k')\,l_1\,(n-k')} \leqslant$$

$$\frac{np+2}{n} \frac{l_2\,(n)}{l_1\,(n\,(1-p))\,l_1\,(np)} = \frac{np+2}{n} \frac{e^{\frac{1}{12n}}}{\sqrt{2\pi n}\,(1-p)^{n(1-p)+\frac{1}{2}}\,p^{np+\frac{1}{2}}}.$$

Similarly,

$$\binom{n-1}{k''-1} = \frac{k''}{n} \frac{n!}{(k'')!\,(n-k'')!} \leqslant \frac{np+2}{n} \frac{l_2\,(n)}{l_1\,(k'')\,l_1\,(n-k'')} \leqslant \frac{np+2}{n} \frac{l_2\,(n)}{l_1\,(np)\,l_1\,(n\,(1-p))}.$$

After substituting the above inequalities in the expression of $\bar{H}_n$ and performing some algebraic manipulations, we get:

$$\bar{H}_n \leqslant \frac{np+2}{n} \frac{e^{\frac{1}{12n}}}{\sqrt{2\pi n}} \left(qP\,(I)\,(1-p) + (1-q)\,P\,(G)\,p\right) \frac{p^{\frac{1}{2}}}{(1-p)^{\frac{5}{2}}}.$$

The right hand side of the above inequality decreases in n and converges to zero as n goes to infinity. Thus, the claim of Proposition 1 follows. $\blacksquare$

Proof of Proposition 2

First, suppose that $\hat{V}\,(n) = V\,(0) = \hat{V}\,(n-1)$. It follows that either $\gamma_n^B\,(0) = ... = \gamma_n^B\,(n)$, or $\gamma_n^B\,(0) = 0$, $f\,(0;n) = 0$ and $\gamma_n^B\,(1) = ... = \gamma_n^B\,(n) = 1$. In both cases, the left hand side of constraint (ICg) is zero.

Thus, we now assume that $\hat{V}\,(n) > V\,(0)$. This implies that $z\,(n)$ is in $(0,n)$ and k_n, the smallest integer greater than $z\,(n)$, belongs to $\{1,...,n\}$. Depending on the distance between k_n and $z\,(n)$, there are two cases to consider.

Case (i): $0 < k_n - z\,(n) \leqslant \frac{1}{2}$. In this case, it is easy to check that:

$$\hat{V}\,(n) - \hat{V}\,(n-1) = \binom{n-1}{k_n-1} f\,(k_n;n)\,,$$

and

$$-f\,(k_n-1;n) \geqslant f\,(k_n;n)\,.$$

Case (ii): $\frac{1}{2} < k_n - z(n) \leqslant 1$. Then we have:

$$\hat{V}(n) - \hat{V}(n-1) = -\binom{n-1}{k_n - 1} f(k_n - 1; n),$$

and

$$-f(k_n - 1; n) < f(k_n; n).$$

The Bayesian device is admissible if it satisfies the following constraints:

$$\binom{n-1}{k_n - 1} f(k_n; n) \geqslant c, \tag{5}$$

$$-\binom{n-1}{k_n - 1} f(k_n - 1; n) \geqslant c. \tag{6}$$

Clearly, the two inequalities above hold if and only if $\hat{V}(n) - \hat{V}(n-1) \geqslant c$. ■

Proof of Proposition 3

For every c, let $n^B(c)$ denote the largest integer for which the Bayesian device is admissible. We show that if $n^B(c)$ is sufficiently large then $V\left(n^B(c) + 1\right) > \hat{V}\left(n^B(c)\right)$. This will complete the proof of Proposition 3 since $n^B(c)$ is decreasing in c.

We now fix c and write n for $n^B(c)$. We assume $0 < k_n - z(n) < \frac{1}{2}$ (the proof for the case $\frac{1}{2} < k_n - z(n) < 1$ is similar and is therefore omitted).[11] The Bayesian device γ_n^B is admissible, and so inequalities (5) and (6) hold.

Consider now a committee of size $n+1$. For every α in the unit interval, let the device $\gamma_\alpha : \{0, ..., n+1\}$ be defined by:

$$\gamma_\alpha(k) = \begin{cases} 0 & \text{if } k = 0, ..., k_n - 1 \\ \alpha & \text{if } k = k_n \\ 1 & \text{if } k = k_n + 1, ..., n+1 \end{cases}.$$

The constraints that the device γ_α has to satisfy to be admissible can be expressed as:

$$F(\alpha) = \binom{n}{k_n} f(k_n + 1; n+1) + \alpha \left[\binom{n}{k_n - 1} f(k_n; n+1) - \binom{n}{k_n} f(k_n + 1; n+1) \right] \geqslant c, \tag{7}$$

[11] Since the environment is regular, $k_n - z(n) \neq \frac{1}{2}, 1$.

$$L(\alpha) = -\binom{n}{k_n} f(k_n; n+1) + \alpha \left[\binom{n}{k_n} f(k_n; n+1) - \binom{n}{k_n - 1} f(k_n - 1; n+1) \right] \geqslant c. \tag{8}$$

The function F is decreasing in α. We now assume that n is sufficiently large, so that $k_n - 1 \geqslant n(1-p)$ and $k_n \leqslant np$. This implies:

$$F(0) = \binom{n}{k_n} f(k_n + 1; n+1) > \binom{n-1}{k_n - 1} f(k_n; n) \geqslant c,$$

and that L is an increasing function that satisfies:

$$L(1) = -\binom{n}{k_n - 1} f(k_n - 1; n+1) > -\binom{n-1}{k_n - 1} f(k_n - 1; n) \geqslant c.$$

We let $\hat{\alpha}_1$ denote the greatest value of α for which the device γ_α satisfies constraint (7). Similarly, we let $\hat{\alpha}_2$ denote the smallest value of α for which the device γ_α satisfies constraint (8). Notice that $-f(k_n - 1; n) \geqslant f(k_n; n)$ since we are assuming that $k_n - z(n)$ is in $\left(0, \frac{1}{2}\right)$ (see the proof of Proposition 2). Thus, the cost c can be at most $\binom{n-1}{k_n-1} f(k_n; n)$ since the Bayesian device γ_n^B is admissible. It follows that:

$$\hat{\alpha}_1 \geqslant \frac{\binom{n}{k_n} f(k_n + 1; n+1) - \binom{n-1}{k_n-1} f(k_n; n)}{\binom{n}{k_n} f(k_n + 1; n+1) - \binom{n}{k_n-1} f(k_n; n+1)} \equiv \alpha_1,$$

$$\hat{\alpha}_2 \leqslant \frac{\binom{n-1}{k_n-1} f(k_n; n) + \binom{n}{k_n} f(k_n; n+1)}{\binom{n}{k_n} f(k_n; n+1) - \binom{n}{k_n-1} f(k_n - 1; n+1)} \equiv \alpha_2.$$

With a slight abuse of notation we let $V(\alpha)$ denote the expected utility of the device γ_α :

$$V(\alpha) = -(1-q) P(G) + \alpha \binom{n+1}{k_n} f(k_n; n+1) + \sum_{k=k_n+1}^{n+1} \binom{n+1}{k} f(k; n+1).$$

The difference between $V(\alpha)$ and $\hat{V}(n)$ is equal to:

$$V(\alpha) - \hat{V}(n) = \binom{n}{k_n} f(k_n; n+1) \left(\frac{n+1}{n - k_n + 1} \alpha - 1 \right).$$

Let $\alpha^* = \frac{n-k_n+1}{n+1}$. Then $V(\alpha)$ is greater than $\hat{V}(n)$ if and only if $\alpha < \alpha^*$.

It remains to be shown that $\alpha_2 < \alpha^*$ and $\alpha_2 < \alpha_1$ for sufficiently large values of n. Let us start with the first inequality. We need to show:

$$(n - k_n + 1)\left[\binom{n}{k_n} f(k_n; n+1) - \binom{n}{k_n - 1} f(k_n - 1; n+1)\right] >$$

$$(n+1)\left[\binom{n-1}{k_n - 1} f(k_n; n) + \binom{n}{k_n} f(k_n; n+1)\right]$$

which can be rewritten as:

$$-\binom{n-1}{k_n - 1}\left(nf(k_n; n+1) + nf(k_n - 1; n+1) + (n+1) f(k_n; n)\right) > 0.$$

We divide the above the expression by $\binom{n-1}{k_n-1}$, and notice that

$$f(k_n; n+1) + f(k_n - 1; n+1) = f(k_n - 1; n).$$

We obtain:

$$-nf(k_n - 1; n) - (n+1) f(k_n; n) > 0.$$

After dividing both sides by $qP(I)(1-p)^{z(n)} p^{n-z(n)}$ and rearranging terms we have:

$$\left(\frac{p}{1-p}\right)^{1-2\lambda} > \frac{n+p}{n+1-p},$$

where $\lambda = k_n - z(n)$. The left hand side is greater than 1 since λ belongs to $\left(0, \frac{1}{2}\right)$, while the right hand side is decreasing in n, and converges to 1 as n goes to infinity.

We now compare α_1 and α_2. We divide both the numerator and the denominator of α_1 by $\binom{n-1}{k_n-1} qP(I)(1-p)^{z(n)} p^{n-z(n)}$, rearrange terms and obtain:

$$\alpha_1 = \frac{\left((1-p)^{\lambda} p^{-\lambda} - p^{\lambda}(1-p)^{-\lambda}\right) + \left(\frac{n}{k_n}\right)\left(-(1-p)^{1+\lambda} p^{-\lambda} + p^{1+\lambda}(1-p)^{-\lambda}\right)}{\left(\frac{n}{k_n}\right)\left(-(1-p)^{1+\lambda} p^{-\lambda} + p^{1+\lambda}(1-p)^{-\lambda}\right) + \left(\frac{n}{n-k_n+1}\right)\left((1-p)^{\lambda} p^{1-\lambda} - p^{\lambda}(1-p)^{1-\lambda}\right)}.$$

We now take the limit of α_1 as n goes to infinity. Both $\left(\frac{n}{k_n}\right)$ and $\left(\frac{n}{n-k_n+1}\right)$ converge to 2 as n grows large. Thus, we have:

$$\bar{\alpha}_1 = \lim_{n\to\infty} \alpha_1 = \frac{\frac{1}{2}\left((1-p)^{\lambda} p^{-\lambda} - p^{\lambda}(1-p)^{-\lambda}\right) - (1-p)^{1+\lambda} p^{-\lambda} + p^{1+\lambda}(1-p)^{-\lambda}}{-(1-p)^{1+\lambda} p^{-\lambda} + p^{1+\lambda}(1-p)^{-\lambda} + (1-p)^{\lambda} p^{1-\lambda} - p^{\lambda}(1-p)^{1-\lambda}}.$$

In a similar way we derive:

$$\bar{\alpha}_2 = \lim_{n\to\infty} \alpha_2 = \frac{\frac{1}{2}\left(-(1-p)^{\lambda} p^{-\lambda} + p^{\lambda}(1-p)^{-\lambda}\right) - (1-p)^{\lambda} p^{1-\lambda} + p^{\lambda}(1-p)^{1-\lambda}}{-(1-p)^{\lambda} p^{1-\lambda} + p^{\lambda}(1-p)^{1-\lambda} + (1-p)^{\lambda-1} p^{2-\lambda} - p^{\lambda-1}(1-p)^{2-\lambda}}.$$

It is tedious but simple to show that $\bar{\alpha}_2 < \bar{\alpha}_1$ for every p in $\left(\frac{1}{2}, 1\right)$ and every λ in $\left(0, \frac{1}{2}\right)$.

■

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