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Unbiased Estimation of the Half-Life to Price Index Convergence among US Cities

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Abstract

Cecchetti et al. (2002) estimate the half-life to price index convergence among U.S. cities to be approximately nine years. Although they correct for the small-sample bias in their panel estimate of the half-life, they do not adjust for biases that may potentially arise due to heterogeneity in the dynamic behavior of prices across cities, and time aggregation of price indices. This paper finds no evidence of significant heterogeneity in the dynamics of prices in different cities. However, corrected for the combined small-sample and time aggregation bias, the panel estimate of the half-life is found to be about seven years – two years shorter than the previous estimate.

Keywords: Price index convergence; Half-life; Nickell Bias; Time aggregation bias

JEL classifications: C33; E31

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1. Introduction

Using consumer price index (CPI) data for 19 major cities in the U.S. from 1918 to 1995, Cecchetti et al. (2002) find evidence of convergence in price indices.[2] Applying panel unit root procedures suggested by Levin and Lin (1993) and Im et al. (1997), they estimate the half-life to convergence to be approximately nine years.[3] Use of the panel techniques to test for convergence and to estimate half-life is appealing because such methods combine cross-section with time series to increase the number of observations, potentially increasing the power of the tests and the precision of half-life estimates. However, estimating half-life from panel data may introduce three potential sources of bias which have recently been emphasized by Choi et al. (2006).[4] Because the magnitude of half-life is very sensitive to the value of autoregressive coefficients, failure to correct for all these biases in the estimation of those coefficients can lead to inaccurate estimate of the half-life.

Although Cecchetti et al. (2002) correct for the downward bias introduced by small size of the sample – also known as Nickell bias – they fail to recognize and correct for two other sources of potential bias.[5] If the dynamic behavior of price indices across cities

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[2] There are other studies that examine the price behavior across the U.S. cities. Most notably, Parsley and Wei (1996) and Engel and Rogers (2001) examine violations of the law of one price within the U.S. However, to the best of our knowledge, Cecchetti et al (2002) is the only published study that applies panel technique to aggregate price indices data that span a relatively longer sample period, to study dynamic price behavior across cities in the United Sates.

[3] Half-life is a measure of the speed at which an autoregressive process converges, and is the time required for any divergence to dissipate by one half. In the case of an autoregressive process of order 1 (AR(1)), half-life is calculated as: $\frac{ln(0.5)}{ln(\hat{\rho})}$ where $\hat{\rho}$ is the estimated AR(1) coefficient.

[4] Although the literature has discussed these biases separately in the context of international purchasing power parity (PPP), Choi et al (2006) is the first to combine all these three biases and to estimate an unbiased half-life.

[5] The small-sample bias in the panel context was studied by Nickell (1981) who showed that in the estimation of a dynamic panel with fixed effects, because the errors would be correlated with current and future values of the dependent variable, the pooled estimates are biased downward and this bias does not disappear even asymptotically as the number of cross-section increases.

exhibits sufficient heterogeneity (that is, the autoregressive coefficients are significantly different across cities), then panel estimation of a common autoregressive coefficient will be biased upward, and so will be the implied half-life.[6] Furthermore, the annual CPI data are averages of goods and services prices recorded monthly, rather than point-in-time sampled prices. This time-averaging (also referred to as time aggregation) process introduces a moving average structure into the regression error when city prices are modeled as autoregressive processes. Failure to account for this imparts an additional upward bias in the estimation of the autoregressive coefficient and the implied half-life.

The objective of this paper is to take cognizance of, and correct for these biases in the estimation of the half-life to price index convergence among U. S. cities. We extend the dataset used by Cecchetti et al. (2002) to include more recent observations and apply a panel estimation technique that accounts for possible cross-sectional dependence in both original and extended datasets. We show that the estimated half-life to CPI convergence with no bias correction could be longer than the previous estimate. We do not find in the data any support of significant heterogeneity in the dynamic behavior of price indices across cities, and therefore no evidence of any upward bias stemming from such heterogeneity. However, we do find that the half life – after correcting for the combined Nickell and time aggregation bias – is about seven years, two years shorter than that estimated by Cechetti et al. (2002).

The rest of the paper is organized as follows. Section 2 describes the data and methodology. The results are presented in section 3. Section 4 includes our concluding remarks.

[6] In this case, pooling is not appropriate.

2. Data and methodology

To make our results comparable with those of Cecchetti et al. (2002), we use their dataset that includes annual CPI data for 19 U.S. cities from 1918 to 1995.[7] We also use an updated version of the above dataset in which more recent observations are included. We construct the latter by extending the sample period until 2006 using additional data obtained from the Bureau of Labor Statistics (BLS). The BLS, however, discontinued publishing separate CPI series for Baltimore and Washington, D.C. – two cities included in the original dataset, since 1997.[8] Therefore, the extended dataset includes data only for the remaining 17 cities. We construct relative price series for each city by using the following equation:

$$r_{i,t} = 100 \times \left(\ln P_{i,t} - \frac{1}{n} \sum_{j=1}^{n} \ln P_{j,t} \right) \tag{1}$$

where $r_{i,t}$ is the relative price and $P_{i,t}$ is the CPI in city i in year t, and n is the number of cities. Note that this relative price represents the percentage deviation of CPI in a city from the average across cities, and is equivalent to modeling price indices in different cities having a common time effect component.[9, 10]

We closely follow the methodology described in Choi et al. (2006). In order to determine if there is any possibility of an upward bias in the panel estimate of the half-

[7] These data are available on the Bureau of Labor Statistics (BLS) website: www.bls.gov. The dataset can also be downloaded from Professor Nelson C. Mark's homepage: http://www.nd.edu/~nmark/

[8] The cities in our sample are: Atlanta, Boston, Chicago, Cincinnati, Cleveland, Detroit, Houston, Kansas City, Los Angeles, Minneapolis, New York, Philadelphia, Pittsburg, Portland, San Francisco, Seattle, and St. Louis.

[9] In the international PPP literature, this relative price would be equivalent to the real exchange rate. Although a numeraire currency is chosen for calculating real exchange rate, we use the average city CPI, an approach previously adopted by Cecchetti et al. (2002) and Chen and Devereux (2003).

[10] We conduct panel unit root tests to determine the stochastic trending properties of relative prices. In addition to Levin-Lin and Im-Pesaran-Shin test procedures as used by Cecchetti et al. (2002), we also conduct the Phillips and Sul (2004) panel unit root test. Results indicate that relative prices are stationary, and thus support previous findings. Since our objective is to obtain unbiased estimates of the half-life we do not report the unit root test results.

life due to heterogeneous dynamic behavior of prices across cities, we first conduct a test of heterogeneity of estimated autoregressive coefficients of relative prices in different cities. This test procedure – discussed in detail by Choi et al. (2004) – involves obtaining recursive mean adjusted seemingly unrelated regression (SUR) estimates of the autoregressive coefficients of relative prices, and subsequently constructing a Wald test statistic with homogeneity restrictions under the null hypothesis. Note that this procedure has the desirable property of mitigating size distortion due to the small-sample bias. Once we determine that there is little evidence of cross-sectional heterogeneity (which is the case in our study as we will see in the next section), it is appropriate to apply panel estimation techniques to pooled data. This leaves us with two potential biases in the panel estimates of autoregressive coefficient and half-life: a downward bias due to small sample size and an upward bias due to the moving average error term introduced by time aggregation of data.

We implement a fixed effects panel generalized least squares (GLS) estimation technique as described in Phillips and Sul (2004). In contrast to the least squares dummy variable (LSDV) method, this technique increases the efficiency of the estimates by controlling for cross-sectional dependence. To sketch an outline of the procedure, suppose relative price in city *i* follows an AR(1) process:

$$r_{i,t} = \alpha_i + \rho_i r_{i,t-1} + u_{i,t} \tag{2}$$

where α_i is a city-specific constant; $i = 1, 2, \ldots, n$; and $t = 1, 2, \ldots, T$. In the absence of time aggregation, the errors are generated by a single factor structure,

$$u_{i,t} = \delta_i \theta_t + \varepsilon_{i,t} \tag{3}$$

where δ_is are factor loadings, θ_t is the common shock, and $\varepsilon_{i,t}$s are serially and mutually independent. The factor loadings and the error covariance matrix are estimated by iterative method of moments, and the estimated covariance matrix is then used to obtain the feasible GLS estimate of ρ.

However, in the presence of time aggregation the regression error has a moving average (MA) structure. Suppose $u_{i,t}$ follows an MA(1) process:

$$u_{i,t} = \upsilon_{i,t} + \gamma\upsilon_{i,t-1} \text{ and } \upsilon_{i,t} = \delta_i\theta_t + \varepsilon_{i,t} \tag{4}$$

In this specification, the estimated covariance matrix – which is used to transform the variables to obtain the feasible GLS estimate of ρ – includes both the contemporaneous and the long-run covariance. This estimated autoregressive coefficient is then adjusted for the Nickell bias, the time aggregation bias, and the combined Nickell and time aggregation bias as discussed in Choi et al. (2006).[11] These bias-corrected estimates of autoregressive coefficient are used to obtain various unbiased estimates of the half-life to price index convergence among U.S. cities.

3. Results

Table 1 presents the results of our homogeneity test. In the first column we report the estimated Wald statistics for the original dataset used by Cecchetti et al. (2002) as well as for our extended dataset. The second column shows the corresponding critical chi-squared values at the 5 percent significance level. A comparison of column 1 and 2 indicates that we cannot reject the null of homogeneity. Thus, the dynamic behavior of

[11] Time aggregation of the data introduces an interaction between the Nickell bias and the time aggregation bias, which requires additional adjustment in the estimation of the autoregressive coefficient. For a discussion, see Choi et al. (2006). The combined Nickell and time aggregation bias correction incorporates this adjustment.

relative prices does not vary significantly across the U.S. cities, and the panel estimate of the autoregressive coefficient is not likely to be biased upward. It implies that pooling of data is appropriate.

Table 2 presents our estimates of the autoregressive coefficient along with the implied half-life with no bias correction, and with various types of bias corrections. The first two columns give us the estimated values of ρ and associated half-life with no bias correction for the original and the extended dataset. Note that the values of $\hat{\rho}$ and half-life are larger than those estimated by Cecchetti et al. (2002). This difference can be ascribed to our estimation method which exploits the cross-sectional covariance structure of the observations to control for cross-sectional dependence. In contrast, inclusion of a common time effect to account for cross-sectional dependence in Cecchetti et al. (2002) works only asymptotically as n increases.[12] Column 3 and 4 report the estimated ρ and corresponding half-life when only the Nickell bias is corrected. Our estimate shows that the Nickell bias corrected half-life is about 13 years when the original dataset is used. It is even longer when we use the updated dataset. The estimates of ρ and associated half-life when only the time aggregation bias is corrected are presented in column 5 and 6. The half-life is more than 5 years. Finally, when corrections are made for the combined Nickell and time aggregation bias, the half-life turns out to be about 7 years. Again with the updated dataset, it is slightly longer. In comparison, the half-life is about two years shorter than the result reported in Cecchetti et al. (2002)

[12] Like Cecchetti et al. (2002), we also include a time effect by taking the deviations of individual city prices from city average. In addition, we use estimated cross-sectional covariance to transform the variables. See Phillips and Sul (2004) for details of our method.

4. Conclusion

Cecchetti et al. (2002) find that the speed of convergence in price indices across U.S. cities is surprisingly slow. Although they explore several reasons, they do not find one single explanation responsible for this slow convergence. This paper shows that, if corrected for the time aggregation bias in combination with the small-sample bias, the estimated speed of convergence is faster than the previous estimate. The estimated half-life of 7 years – as reported in this study – still implies slow convergence by international PPP standards, the explanation for which is beyond the scope of this short article.

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Table 1

Homogeneity Test Results

	Estimated Test Statistics	5% Critical Value $\left(\chi^2_{n-1,0.05}\right)$
	(1)	(2)
Relative prices in 19 cities (Sample period: 1918-1995)	17.46	28.87
Relative prices in 17 cities Sample period: 1918-2006)	12.28	26.30

Note: The null hypothesis is H_0: $\rho_1 = \rho_2 = \ldots = \rho_n$ where ρ_i is the AR(1) coefficient of relative price in city *i*. Under the null, the estimated test statistic follows a chi-squared distribution with *n*-1 degrees of freedom. The Wald test is described in Choi et al. (2004).

Table 2

Panel Feasible GLS Estimation of ρ and Implied Half-life

	No bias corrections		Nickell bias corrected		Time aggregation bias corrected		Nickell and time aggregation bias corrected	
	$\hat{\rho}$	Half-life	$\hat{\rho}$	Half-life	$\hat{\rho}$	Half-life	$\hat{\rho}$	Half-life
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Relative prices in 19 cities (Sample period:1918-1995)	0.916 (0.894, 0.883)	7.90	0.947 (0.922, 0.931)	12.73 (8.535, 9.695)	0.877	5.28	0.906	7.02
Relative prices in 17 cities (Sample period: 1918-2006)	0.929	9.41	0.950	13.51	0.887	5.78	0.912	7.52

Note: The values in brackets are estimated ρ and half-life reported by Cecchetti et al. (2002).