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Voting over Taxes and Expenditure: The Role of Home Production

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Abstract

This paper investigates, first, how allowance for subsistence activities, or home production, affects the standard results in models involving the majority choice of the tax rate in a flat tax–basic income scheme. The paper extends the analysis of home production to choices regarding the composition of government expenditure, in situations where there is a tax-financed pure public good in addition to a transfer payment, conditional on a given tax rate. The effect of home production is to reduce the transfer payment in each model, but the effect is small.

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1 Introduction

Early approaches to examining the democratic choice of the size of government involved a simple model of a transfer payment in the form of a basic income combined with a proportional tax, with endogenous labour supply responses; these include Roberts (1977) and Meltzer and Richard (1978).[1] The government budget constraint implies that voting is over only one dimension, the tax rate. Despite the existence of double-peaked preferences, a majority-voting equilibrium is known to exist as long as there is 'hierarchical adherence', such that the ordering of individuals by income is independent of the tax rate.[2] The median voter's preferred tax rate can be established as a function of the ratio of the median wage rate to the arithmetic mean wage, such that an increase in the skewness of the wage rate distribution (a reduction in the ratio) is associated with a higher equilibrium tax rate, that, is a more redistributive tax-transfer system.[3] However, empirical evidence regarding such a relationship, involving cross-country data, has been mixed; for a review, see Borck (2007) and Harms and Zink (2003). In the context of time series evidence for particular countries, the variation in inequality is typically too small to establish an effect.

The aim of this paper is to investigate how allowance for subsistence activities, or home production, in the model affects the standard results. For example, there may be a larger 'tax base effect' of an increase in the tax rate, where individuals can substitute home production for goods purchased in markets as well as substituting between leisure and work. This may result in lower tax rates, *ceteris paribus*. Tridimas and Winer (2005) consider the quasi-linear utility function with home production. They only concentrate

[1] Extensions within the Robert-Meltzer-Richard framework include, for example, Galasso (2003) who considers fairness and redistribution.

[2] Double-peaked preferences exist for some individuals because, after the point where they move to the non-participation corner solution, they prefer to see the tax rate increase until total revenue reaches a maximum. Results regarding the existence of voting equilibria were established by Roberts (1977).

[3] It can be shown that the majority choice satisfies the condition, $1 - \frac{y_m}{\bar{y}} = \left|\eta_{\bar{y},\tau}\right|$, where y_m and $\bar{y}$ are respectively the median and arithmetic mean gross *income*, and $\left|\eta_{\bar{y},\tau}\right|$ is the absolute elasticity of average income with respect to the tax rate, τ. The left hand side of this expression can be interpreted as a measure of income inequality.

on public goods and linear income tax and use the probabilistic voting model to find collective choice.

This paper also extends the analysis of home production to demographic choices regarding the composition of government expenditure, in situations where there is a tax-financed pure public good in addition to a transfer payment, conditional on a given tax rate. Hence, as so often in practice, there is a separation between taxing and expenditure decisions.[4] Again the role of the government budget constraint means that voting is unidimensional.[5] In this context, Creedy and Moslehi (2009) established a positive relationship between inequality and the proportion of expenditure devoted to the (inequality reducing) transfer payment, corresponding to the result mentioned above concerning the choice of tax rate. The influence of home production on this relationship is thus examined here.

Section 2 considers voting over the tax rate in a simple model in which there are two goods, one of which is produced at home, in addition to leisure. Section 3 considers the case where voting concerns the division of government expenditure between transfer payments and a pure public good, conditional on a given tax rate. Brief conclusions are in Section 4.

2 Marketed and Home Produced Goods

This section extends the widely used basic model of a pure tax and transfer system to allow for home production. To obtain some idea of likely orders of magnitude, it is useful to obtain explicit solutions for the majority choice of

[4] Bearse *et al.* (2001), who examine majority voting over a uniform transfer and public education, also assume that the tax rate is given exogenously. After pointing out that this is a common assumption, Tridimas (2001, p. 308) suggests that, 'This is less restrictive than it first appears, since in practice governments are often constrained in the policy instruments that they may vary at anyone time'. Tridimas and Winer (2005) consider voting over only tax-financed public goods. On difficulties raised by multidimentional voting, see Mueller (2003, pp. 87-92).

[5] In some models, a two-stage process is envisaged in which voting over the tax rate takes place, where voters have information about the conditional choice of government expenditure.

the tax rate, using a specific form for the utility function.[6] Of course, some results can easily be obtained without the need to assume specific forms. Thus in a diagram with the basic income, or transfer payment, on the vertical axis and the proportional tax rate on the horizontal axis, individuals with lower wage rates have flatter indifference curves. Each voter's preferred position involves a tangency between the highest (upward sloping) indifference curve and the concave government budget constraint. Hence the lower the median relative to the mean, the higher is the majority choice of tax rate.[7]

To obtain the majority voting equilibrium tax rate in a pure transfer system consisting of a basic untaxed income and a proportional tax rate, it is necessary to derive each individual's indirect utility function expressed in terms of the tax rate. The government budget constraint means that there is just one degree of freedom in the choice of the two tax parameters, so that voting is unidimensional. The indirect utility function, along with the budget constraint, is derived in subsection 2.1, and the voting equilibrium is obtained in subsection 2.2. The effects on the choice of tax rate of variations in preferences for, or the efficiency of, home production are examined in subsection 2.3.

2.1 Indirect Utility

Suppose that individual i buys an amount, x_i, of a marketed good at price, p, and produces y_i of a home produced good using h_i units of time, according to:

$$y_i = \theta h_i^{\delta} \tag{1}$$

It is assumed that other inputs into home production, arising from endowments of the individual, are fixed and therefore subsumed into the term, θ. These endowments may include, for example, a fixed holding of land and capital goods in the form of tools. If the production function were to involve

[6] Hindricks and Myles (2006, pp. 503-5) discuss majority voting in the simple case where utility is consumption less (half) the square of labour supply, and show that the median voter's preferred tax rate is $(1 - y_m/\bar{y}) / \{2 - y_m/\bar{y}\}$. Persson and Tabellini (2000, chapter 6) also give an example using quasi-linear preferences.

[7] For example, see Mueller (1989, pp. 512-514)

inputs of amounts of the market-purchased good, x, the model would become significantly more complex.[8] The individual consumes ℓ_i units of leisure and the total endowment of time is 1, so that the time devoted to paid work is $1 - \ell_i - h_i$. The utility function can be written:[9]

$$U = x_i^\alpha y_i^\phi \ell_i^\gamma \tag{2}$$

so that, after substituting for y_i:

$$U = x_i^\alpha \left(\theta h_i^\delta\right)^\phi \ell_i^\gamma \tag{3}$$

Writing $\beta = \delta\phi$ and ignoring the constant θ^ϕ, this can be rewritten as:

$$U = x_i^\alpha h_i^\beta \ell_i^\gamma \tag{4}$$

It is convenient below to write $\alpha + \beta + \gamma = \rho$. The standard model, which excludes home production, is thus obtained by setting $\beta = 0$. Utility therefore takes the basic Cobb-Douglas form in terms of the consumption of a market-purchased good and the time devoted, separately, to leisure and home production.[10] The latter does not generate utility directly but does so via the production function in (1).

With a tax and transfer system involving a proportional tax applied to all earnings at the rate, τ, and a basic income of b, the individual's budget constraint, where w_i is the wage rate, is given by:

$$px_i + w_i (1 - \tau)(h_i + \ell_i) = w_i (1 - \tau) + b = M_i \tag{5}$$

[8] Greenwood *et al.* (1995) allow for the purchase of inputs, in a real business cycle model.

[9] An alternative way of looking at home production is to suppose that, instead of having two goods, one of which can be produced at home, there is just one good which may either be produced at home or purchased at price p. From the point of view of consumption, they are otherwise the same. Home and market amounts consumed are x_s and x_p respectively. Utility is thus $U_i = (x_p + x_s)^\alpha \ell^{1-\alpha}$, where $x_s = \theta h_s^\delta$. A problem is that this formulation becomes intractable.

[10] If a CES utility function were used instead of (2), this would not, when combined with (1), give rise to an equivalent CES in terms of hours of home production, as does (4). Furthermore, the CES does not give rise to a linear relationship between earnings and the wage rate, so the government budget constraint is considerably more complex than the case below.

where M_i is 'full income'.[11] Using the standard properties of the Cobb-Douglas utility function, involving fixed expenditure proportions, the individual's optimum values are given by:

$$x_i = \frac{\alpha}{\rho}\frac{M_i}{p} \tag{6}$$

$$h_i = \frac{\beta}{\rho}\frac{M_i}{w_i\,(1-\tau)} \tag{7}$$

$$\ell_i = \frac{\gamma}{\rho}\frac{M_i}{w_i\,(1-\tau)} \tag{8}$$

Thus, as expected, high wage individuals devote relatively more time to working in the labour market, rather than taking leisure or engaging in home production. Where the opportunity cost of time is lower, it is better to spend more time in home production. The indirect utility function is obtained by substituting individual's optimum values into (4), so that:

$$V_i = k\,\{w_i\,(1-\tau)\}^{\alpha-\rho}\,(w_i\,(1-\tau)+b)^{\rho} \tag{9}$$

The gross earnings, y_i, of each individual is obtained from optimum values. Therefore:

$$y_i = w\,(1-h_i-\ell_i) \tag{10}$$

and substitution gives:

$$y_i = \left(\frac{\alpha}{\rho}\right) w_i - \left(\frac{\rho-\alpha}{\rho}\right)\left(\frac{b}{1-\tau}\right) \tag{11}$$

This expression takes precisely the same form as the case where there is no home production: the only difference concerns the value of the coefficients on the wage rate and the basic income. This clearly applies only if w_i exceeds a minimum wage, $w_{\min}$, required to induce positive labour supply, where:

$$w_{\min} = \left(\frac{\rho-\alpha}{\alpha}\right)\left(\frac{b}{1-\tau}\right) \tag{12}$$

[11] This of course envisages the individual selling all the endowment, that is one unit, of labour time at the going wage and 'buying back' the time required for leisure and home production at a price given by the net wage.

The government's budget constraint in this 'pure' transfer scheme is given by:

$$b = \tau \bar{y} \tag{13}$$

where $\bar{y}$ is arithmetic mean earnings. From (11):

$$\bar{y} = \frac{1}{n} \sum_{w > w_{\min}} \left\{ \left(\frac{\alpha}{\rho} \right) w_i - \left(\frac{\rho - \alpha}{\rho} \right) \left(\frac{b}{1 - \tau} \right) \right\} \tag{14}$$

and letting $F_1 (w_{\min})$ and $F (w_{\min})$ denote respectively the proportion of total wage (rates) and the proportion of people with $w < w_{\min}$:[12]

$$\bar{y} = \bar{w} \frac{\alpha}{\rho} \left\{ 1 - F_1 (w_{\min}) \right\} - \left(\frac{\rho - \alpha}{\rho} \right) \left(\frac{b}{1 - \tau} \right) \left\{ 1 - F (w_{\min}) \right\} \tag{15}$$

This expression is highly nonlinear in view of the fact that $w_{\min}$ depends on b and τ, so it is not possible to express b as a convenient function of τ. However, the analysis is tractable if it is assumed that all individuals work, that is if all $w_i > w_{\min}$ and therefore F_1 and F are equal to zero. This produces a linear relationship between arithmetic means of y and w, such that:

$$b = \frac{\frac{\alpha}{\rho} \tau \bar{w}}{1 + \left(\frac{\rho - \alpha}{\rho} \right) \left(\frac{\tau}{1 - \tau} \right)} = \alpha \bar{w} \frac{\tau (1 - \tau)}{\rho - \alpha \tau} \tag{16}$$

The assumption that all wages are sufficient to avoid the non-participation corner solution for relevant tax rates therefore simplifies the form of the government's budget constraint but otherwise has little effect on the model: as mentioned earlier, the possibility of double-peaked preferences does not prevent a voting equilibrium from arising.

Substituting (16) into full income gives:

$$M_i = w_i (1 - \tau) \left(1 + \left(\frac{\bar{w}}{w_i} \right) \frac{\frac{\alpha}{\rho} \tau}{1 - \frac{\alpha}{\rho} \tau} \right) \tag{17}$$

Substituting optimal values and (17) into the individual's utility function gives indirect utility, V_i, in terms of the tax rate, τ, as:

$$V_i = k \left\{ w_i (1 - \tau) \right\} \left(1 + \left(\frac{\bar{w}}{w_i} \right) \frac{\frac{\alpha}{\rho} \tau}{1 - \frac{\alpha}{\rho} \tau} \right)^{\rho} \tag{18}$$

[12] These correspond to the ordinate and abscissa of the Lorenz curve of wage rates at the point where $w = w_{\min}$.

Where $k = \alpha^\alpha \beta^\beta \gamma^\gamma / (\rho^\rho p^\alpha)$ depends on the price of goods in the market and the parameters of the utility function.

2.2 The Majority Choice of Tax Rate

This model is known to satisfy hierarchical adherence, so the median voter theorem can be invoked and, denoting the median wage by w_m, the majority choice is the solution to $dV_m/d\tau = 0$. This gives the condition:[13]

$$\begin{aligned}\frac{\frac{\alpha}{\rho}\left(1+\left(\frac{\bar{w}}{w_i}\right)\frac{\frac{\alpha}{\rho}\tau}{1-\frac{\alpha}{\rho}\tau}\right)}{1-\tau} &= \frac{d\left(1+\left(\frac{\bar{w}}{w_i}\right)\frac{\frac{\alpha}{\rho}\tau}{1-\frac{\alpha}{\rho}\tau}\right)}{d\tau} \\ &= \left(\frac{\bar{w}}{w_m}\right)\frac{\frac{\alpha}{\rho}}{\left(1-\frac{\alpha}{\rho}\tau\right)^2}\end{aligned} \tag{19}$$

Rearrangement of (19) gives the majority choice as the appropriate root of the following quadratic:[14]

$$\tau^2\left(\frac{\alpha}{\rho}\right)^2\left(1-\frac{w_m}{\bar{w}}\right)-\tau\left(1+\frac{\alpha}{\rho}-2\frac{\alpha}{\rho}\frac{w_m}{\bar{w}}\right)+\left(1-\frac{w_m}{\bar{w}}\right)=0 \tag{20}$$

The two roots of this quadratic equation are examined in Appendix A where it is shown that the largest root can be ruled out as it is greater than one. The majority choice of the tax rate, τ_m, is thus:

$$\tau_m = \frac{1+\frac{\alpha}{\rho}-2\frac{\alpha}{\rho}\frac{w_m}{\bar{w}}-\sqrt{\left(1-\frac{\alpha}{\rho}\right)\left(1+3\frac{\alpha}{\rho}-4\frac{w_m}{\bar{w}}\frac{\alpha}{\rho}\right)}}{2\left(\frac{\alpha}{\rho}\right)^2\left(1-\frac{w_m}{\bar{w}}\right)} \tag{21}$$

A standard result in the literature on majority voting is that an increase in wage rate inequality is associated with an increase in the median voter's

[13] Alternatively, indirect utility can be written as $V_i = k\{w_i(1-\tau)\}^\alpha\left\{1+\frac{b}{w_i(1-\tau)}\right\}^\rho$ and the first-order condition for $dV_i/d\tau = 0$ can be expressed as $\frac{db}{d\tau} = \frac{\alpha}{\rho}w_i - \frac{(\rho-\alpha)b}{\rho(1-\tau)}$. If there is no home production, so that $U = x_i^\alpha \ell_i^{1-\alpha}$, it can be seen that the right hand side is simply y_i. Hence the choice of τ implies that $\frac{\tau}{b}\frac{db}{d\tau} = \frac{\tau y_i}{b}$, or the elasticity of b with respect to τ is equal to the ratio of tax paid to benefit received.

[14] The terms $1+\frac{\alpha}{\rho}-2\frac{\alpha}{\rho}\frac{w_m}{\bar{w}} = \left(1-\frac{\alpha}{\rho}\right)+2\frac{\alpha}{\rho}\left(1-\frac{w_m}{\bar{w}}\right)$, $\left(\frac{\alpha}{\rho}\right)^2\left(1-\frac{w_m}{\bar{w}}\right)$ and $\left(1-\frac{w_m}{\bar{w}}\right)$ are all positive.

desired tax rate and thus transfer payment, making the system more redistributive. Redistribution in this model is of course across the arithmetic mean, since the effective average tax rate is negative for $y_i < \bar{y}$ and positive for $y_i > \bar{y}$. Hence, as the median wage tends to the arithmetic mean wage, the majority choice of tax rate tends to $\tau_m = 0$.

It is therefore of interest to see if this result is modified where home production exists. In this context the ratio $w_m/\bar{w}$, which is more clearly a measure of skewness of the wage rate distribution, is directly related to inequality. For example, if w is lognormally distributed as $\Lambda\left(w|\,\mu, \sigma^2\right)$ where μ and σ^2 are respectively the mean and variance of logarithms, it can be shown that $w_m/\bar{w}$ depends only on σ^2. The derivative of the tax rate, τ_m, with respect to $w_m/\bar{w}$ is:

$$\frac{\partial \tau_m}{\partial\left(\frac{w_m}{\bar{w}}\right)} = \frac{\left(1-\frac{\alpha}{\rho}\right)\left\{1-\frac{\left(1+\frac{\alpha}{\rho}-2\frac{w_m}{\bar{w}}\frac{\alpha}{\rho}\right)}{\sqrt{\left(1-\frac{\alpha}{\rho}\right)\left(1+3\frac{\alpha}{\rho}-4\frac{w_m}{\bar{w}}\frac{\alpha}{\rho}\right)}}\right\}}{2\left(\frac{\alpha}{\rho}\right)^2\left(1-\frac{w_m}{\bar{w}}\right)^2} < 0 \tag{22}$$

It can be shown that the term in curly brackets in (22) is negative, since this reduces to the condition that $0 < \left(\frac{\alpha}{\rho}\right)^2\left(1-\frac{w_m}{\bar{w}}\right)^2$. Hence, $\partial\tau_m/\partial\left(\frac{w_m}{\bar{w}}\right)$ is negative so that increasing the degree of equality, that is, reducing inequality, reduces the majority choice of tax rate.

2.3 Variations in Beta

The question of interest here is how the existence of home production affects the choice of tax rate. Differentiation of both (21) and (22) with respect to β, bearing in mind that $\rho = \alpha+\beta+\gamma$, does not yield unequivocal results. However, further insight can be obtained by considering individuals' preference in (b, τ) space. The majority voting equilibrium is characterised by tangency between the median voter's indifference curve and the government budget constraint. With b and τ on vertical and horizontal axes respectively, any change leading indifference curves of the median voter to become steeper, and the government budget constraint (over the relevant – that is upward sloping – range) to become flatter, has the effect of unambiguously reducing the choice of tax rate.

The indirect utility function for workers in terms of b and τ can be written (dropping subscripts for the median voter) as:

$$V_i = k\left\{w_i\left(1-\tau\right)\right\}^{\alpha}\left(1+\frac{b}{w_i\left(1-\tau\right)}\right)^{\rho} \tag{23}$$

The slope of an indifference curve is:

$$\begin{aligned}\left.\frac{db}{d\tau}\right|_{V_i} &= -\frac{\partial V_i/\partial\tau}{\partial V_i/\partial b} \\ &= \frac{w_i}{\rho}\left(\alpha+\frac{\left(\alpha-\rho\right)b}{w_i\left(1-\tau\right)}\right) \end{aligned} \tag{24}$$

The sign of the first derivative is undetermined; however it can be shown that over the relevant range of taxes it is increasing. On the other hand the negative sign of the second derivative, $-\left(\rho-\alpha\right)b/\rho\left(1-\tau\right)^2$, shows that indifference curves are concave in $(b,\,\tau)$ space.[15] This property does not seem to have been recognised in the literature; for instance, Mueller (2003, p. 514) draws upward sloping convex indifference curve for workers.

From (24), the effect of a change in β on the slope of indifference curves is:

$$\frac{d}{d\beta}\left(\left.\frac{db}{d\tau}\right|_{V_i}\right) = \frac{-\alpha w_i}{\rho^2}\left(1+\frac{b}{w_i\left(1-\tau\right)}\right) < 0 \tag{25}$$

and for a given τ the indifference curves get flatter. A change in β also causes the government budget constraint, $b=\tau\bar{y}$, to change. The slope of this is:

$$\left.\frac{db}{d\tau}\right|_{R} = \bar{y}+\tau\frac{d\bar{y}}{d\tau} \tag{26}$$

and the effect of a change in β on this slope is:

$$\frac{d}{d\beta}\left(\left.\frac{db}{d\tau}\right|_{R}\right) = \frac{d\bar{y}}{d\beta}+\tau\frac{d}{d\beta}\left(\frac{d\bar{y}}{d\tau}\right) \tag{27}$$

[15] Alternatively, writing the equation of the indifferenc curve as $b = \left(\frac{\bar{V}_i}{k}\right)^{\frac{1}{\rho}}\left\{w_i\left(1-\tau\right)\right\}^{\frac{\rho-\alpha}{\rho}} - w_i\left(1-\tau\right)$, the first derivative is $\left.\frac{db}{d\tau}\right|_{\bar{V}_i} = -\left(\frac{\bar{V}_i}{k}\right)^{\frac{1}{\rho}}\left(\frac{\rho-\alpha}{\rho}\right)w_i^{\frac{\rho-\alpha}{\rho}}\left(1-\tau\right)^{\frac{-\alpha}{\rho}}+w_i$. The sign of this is undetermined. However, it applies only for the range of τ for which labour supply is positive, and is there positive. For τ beyond the point where the individual does not work, the indifference curves become horizontal. The second derivative is $\left.\frac{d^2b}{d\tau^2}\right|_{\bar{V}_i} = -\frac{\alpha}{\rho}\left(\frac{\bar{V}_i}{k}\right)^{\frac{1}{\rho}}\left(\frac{\rho-\alpha}{\rho}\right)w_i^{\frac{\rho-\alpha}{\rho}}\left(1-\tau\right)^{\frac{-\alpha}{\rho}-1} < 0$. This is negative, implying that indifference curve are actually slightly concave in b, τ space.

From the expression for $\bar{y}$ above:

$$\frac{d\bar{y}}{d\tau} = -\left(\frac{\rho - \alpha}{\rho}\right)\frac{b}{(1-\tau)^2} \tag{28}$$

and:

$$\frac{d}{d\beta}\left(\frac{d\bar{y}}{d\tau}\right) = -\frac{\alpha}{\rho^2}\frac{b}{(1-\tau)^2} \tag{29}$$

Furthermore:

$$\frac{d\bar{y}}{d\beta} = -\frac{\alpha}{\rho^2}\left(\bar{w} + \frac{b}{1-\tau}\right) \tag{30}$$

Hence:

$$\frac{d}{d\beta}\left(\left.\frac{db}{d\tau}\right|_R\right) = -\frac{\alpha}{\rho^2}\left(\bar{w} + \frac{b}{(1-\tau)^2}\right) < 0 \tag{31}$$

Hence the government budget constraint also becomes flatter. This means that there are opposing tendencies on the preferred value of τ. The flattening of the indifference curves leads towards an increase in τ while the flattening of the budget constraint leads towards a reduction in τ. Thus the question, in determining whether the change in β leads to a reduction in the median voter's choice of τ, is whether the change (in absolute terms) in the slope of the budget constraint is greater than that of the indifference curve, at the initial τ. Since $w_m < \bar{w}$ and $0 < 1 - \tau < 1$, it can be seen that:

$$\left|\frac{d}{d\beta}\left(\left.\frac{db}{d\tau}\right|_R\right)\right| > \left|\frac{d}{d\beta}\left(\left.\frac{db}{d\tau}\right|_{V_i}\right)\right| \tag{32}$$

An increase in β therefore reduces τ_m. This is illustrated in Figure 1, which shows the variation in τ_m with $w_m/\bar{w}$ for a range of values of β. In producing the figure, the value of α is set to 0.7 and it is convenient to set $\gamma = 1 - \alpha$ (so that $\rho = 1 + \beta$). The introduction of home production, or an increase in β, not only reduces the value of τ_m but also involves a very slight reduction in the extent to which it varies with $w_m/\bar{w}$. An increase in β can arise from either an increase in preferences for the home produced good, ϕ, or an increase in the productivity of time spent in home production, δ. In each case there is a stronger incentive to devote more time to home production, involving a greater opportunity cost of working. The median voter thus wishes to compensate by having a slightly lower income tax rate.

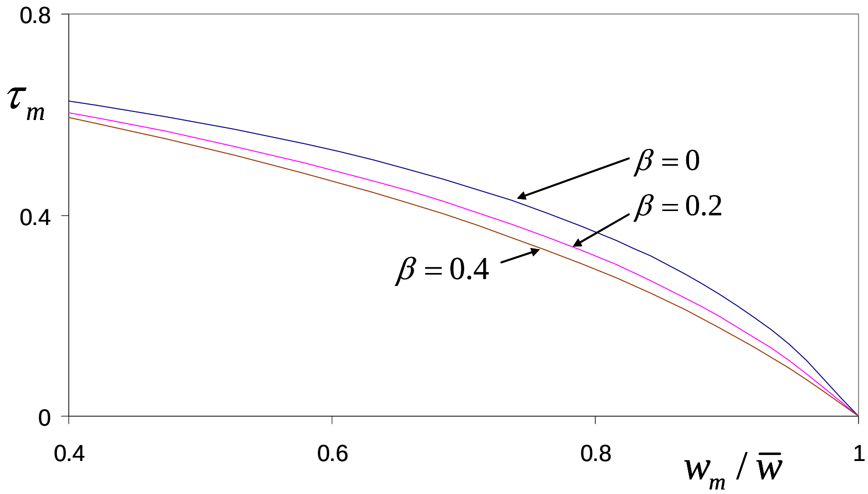

Figure 1: Median Voter's Preferred Tax Rate and Ratio $w_m/\bar{w}$

It is also of interest to consider the way in which time allocation varies as β increases. The introduction of home production, or an increase in labour productivity in home production, is expected to involve a shift away from leisure. It is shown here that it also leads to a small reduction in labour supply. First, the partial effects on leisure, ℓ, and time in home production, h, of an increase in β can be seen by differentiating the above expressions for optimal choices, giving:

$$\frac{\partial \ell}{\partial \beta} = -\frac{\ell}{\rho} \tag{33}$$

and:

$$\eta_{\ell,\beta} = \frac{\beta}{\ell}\frac{\partial \ell}{\partial \beta} = -\frac{\beta}{\rho} \tag{34}$$

Similarly:

$$\frac{\partial h}{\partial \beta} = h\left(\frac{1}{\beta} - \frac{1}{\rho}\right) \tag{35}$$

with:

$$\eta_{h,\beta} = \frac{\beta}{h}\frac{\partial h}{\partial \beta} = 1 - \frac{\beta}{\rho} \tag{36}$$

Hence $\eta_{\ell,\beta} + \eta_{h,\beta} = 1$. An increase in β therefore leads to a shift from leisure towards home production, but the two changes are not equal: there is a small effect on labour supply. Thus:

$$\begin{aligned}\frac{\partial(1-\ell-h)}{\partial\beta} &= -\left(\frac{\partial\ell}{\partial\beta}+\frac{\partial h}{\partial\beta}\right) \\ &= -\frac{\alpha M_i}{\rho^2 w_i(1-\tau)}<0 \qquad (37)\end{aligned}$$

Hence, the partial effect of an increase in β is to reduce labour supply for all wage groups. However, the increase in β has been seen above to lead to a reduction in the majority choice of τ and a reduction in the value of b, since the government budget constraint becomes flatter. The latter reduction has the effect of increasing labour supply. Hence the change in labour supply resulting from both changes depends on the individual's wage rate.

3 Composition of Expenditure with a Public Good

This section extends the model of Section 2 by introducing a pure public good which is tax financed, and examine the majority choice of the composition of government expenditure; that is, it derives the median voter's preferred allocation of tax revenue between transfer payments and the public good.[16] In concentrating on the composition of expenditure, the tax rate is considered to be exogenously determined, as mentioned in the introduction. This means that there is again only one degree of freedom in choosing the transfer and public good expenditure and voting concerns just one dimension.

Consider the model in Section 2 which has two goods, one of which is produced at home. Suppose that, in addition, there is a tax-financed amount of a pure public good, Q_G, where the cost of production per unit is constant and equal to p_G, (the price of the private marketed good is p, as above). The augmented utility function is thus:

$$U_i = x_i^{\alpha} h_i^{\beta} \ell_i^{\gamma} Q_G^{\eta} \qquad (38)$$

The budget constraint facing each individual is the same as in (5). The utility maximising amounts, x_i, h_i and ℓ_i are exactly the same as in equations (6)

[16] This section therefore extends the results of Creedy and Moslehi (2009), whose model does not include home production.

to (8) in Section 2. Similarly, individual i's earnings are the same as given in (11).

However, the form the government budget constraint in (13) must be modified to allow for the need to raise extra revenue to finance expenditure of $G = p_G Q_G$ on the public good. The government budget constraint becomes:

$$b = \tau \bar{y} - G/N \tag{39}$$

Where N is the number of individuals. Hence (16) is easily modified by the inclusion of the term in G/N, so that:

$$b = \frac{\frac{\alpha}{\rho}\tau\bar{w} - G/N}{1 + \left(\frac{\rho-\alpha}{\rho}\right)\left(\frac{\tau}{1-\tau}\right)} \tag{40}$$

where again $\rho = \alpha + \beta + \gamma$. Instead of looking for the individual's preferred tax rate, the problem here is to obtain the preferred expenditure levels of G and b for a given tax rate. The indirect utility function modified by the addition of the public good is:

$$\begin{aligned} V_i &= k\frac{M_i^{\rho}}{\{w_i(1-\tau)\}^{\rho-\alpha}}Q_G^{\eta} \\ &= k\frac{(w_i(1-\tau)+b)^{\rho}}{\{w_i(1-\tau)\}^{\rho-\alpha}}Q_G^{\eta} \end{aligned} \tag{41}$$

By substituting the transfer payment from the government budget constraint (40) the indirect utility function is written in term of one policy variable, Q_G. Therefore the indirect utility function becomes:

$$V_i = \frac{k\left(\frac{1-\tau}{\rho-\alpha\tau}\right)^{\rho}}{\{w_i(1-\tau)\}^{\rho-\alpha}}\left\{\rho w_i + \alpha\tau(\bar{w} - w_i) - \frac{p_G Q_G}{N}\right\}^{\rho} Q_G^{\eta} \tag{42}$$

It can be shown that $d^2V_i/dQ_G^2 < 0$ if $\alpha+\beta+\gamma$ is less than one. Hence preferences are singled-peaked and the majority choice of expenditure on public goods is obtained from $dV_m/dQ_G = 0$. This gives, after some manipulation:

$$\frac{G_m}{N} = \frac{p_G Q_{G,m}}{N} = \bar{w}\left(\frac{\eta}{\rho+\eta}\right)\left\{\frac{w_m}{\bar{w}} + \frac{\alpha}{\rho}\tau\left(1 - \frac{w_m}{\bar{w}}\right)\right\} \tag{43}$$

Hence the expenditure per capita on the public good, as a proportion of $\bar{w}$, depends on the preference parameters, the tax rate, and the ratio $w_m/\bar{w}$. It increases linearly with τ and $w_m/\bar{w}$. The resulting value of b_m is given by appropriate substitution of G_m/N into (40):

$$b_m = \bar{w}\frac{\left(\frac{1}{\rho+\eta}\right)\left\{\alpha\tau - \eta\frac{w_m}{\bar{w}}\left(1-\frac{\alpha}{\rho}\tau\right)\right\}}{1+\left(\frac{\rho-\alpha}{\rho}\right)\left(\frac{\tau}{1-\tau}\right)} \tag{44}$$

and $b_m/\bar{w}$ is also a linear function of $w_m/\bar{w}$, but a nonlinear function of the exogenous tax rate, τ. The ratio of expenditure on transfers to expenditure on the public good is therefore a function of $w_m/\bar{w}$. An important implication of the Cobb-Douglas preferences is that this does not depend on the cost of the public good per unit relative to the price of the marketed private good. Combining (44) and (43) shows that the majority choice of the ratio of the transfer payment to public good expenditure per capita, R_m, depends on the given tax rate, the preference parameters and, importantly, the ratio, $w_m/\bar{w}$. Further analysis shows that $dR_m/d\left(w_m/\bar{w}\right) < 0$, so that increasing equality is associated with a lower R_m and hence a reduced emphasis on a redistributive expenditure share.[17]

Figure 2 shows the relationship between R_m and $w_m/\bar{w}$, again for $\alpha = 0.7$ and $\gamma = 1-\alpha$, for three different values of β. It can be seen that home production, as modelled here, has little effect on this relationship. Just as it involved a slightly lower tax rate, and hence transfer payment, when considering voting over the tax rate, it implies a slightly lower ratio of expenditure on transfers relative to the public good.

4 Conclusions

This paper has examined the implications of allowing for home production in modelling two types of democratic choice. First, majority voting over tax and benefit levels was examined in a pure transfer system with endogenous labour supply. Second, the choice of the share of transfer payments in total

[17] Empirical support for this, for the case where $\beta = 0$, is reported in Creedy and Solmaz (2009). Also see Lind (2005) for problems raised in testing this type of model empirically.

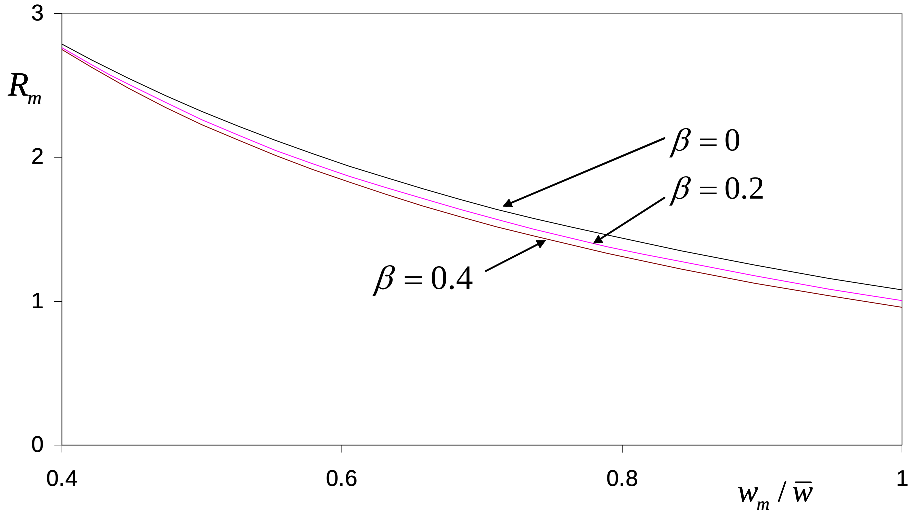

Figure 2: Expenditure Share and Wage Ratio

expenditure was considered in a model in which the tax rate is exogenously fixed but there is also a tax-financed pure public good. The specification of home production implies that a Cobb-Douglas utility function in terms of amounts consumed of a marketed good and a home produced good (along with leisure) can be re-expressed as a function of the time devoted to home production. The analysis was simplified by the assumption that the minimum wage in the population is sufficient to ensure that all individuals work, producing a convenient form of government budget constraint which allows explicit solutions to be obtained. Both the tax rate in the first model and the expenditure share in the second model were found to depend on the ratio of the median voter's wage to the arithmetic wage. This general property has of course been established earlier for models which make no allowance for home production.

The effect of introducing home production in these models was found to have little effect on the democratic choice of tax and transfer levels and on the choice of expenditure composition. Attempts to examine these relationships empirically using cross-sectional data for a range of democratic countries, even where the extent of home production may be expected to vary significantly, are therefore not likely to be significantly biased by ignoring home production. This negative result is in fact convenient for empirical work, given the difficulty of obtaining information regarding the time spent in home production.

Appendix A: Majority Voting and Two Roots of the Quadratic

In order to find the majority choice of tax rate the two roots of the quadratic equation (20) need to be examined. Writing this quadratic as $A\tau^2+B\tau+C=0$, the roots are given by the standard expression $\left\{-B \pm \sqrt{B^2-4AC}\right\}/2A$. The term B^2-4AC is given by:

$$B^2-4AC=\left(1+\frac{\alpha}{\rho}-2\frac{\alpha}{\rho}\frac{w_m}{\bar{w}}\right)^2-4\left(\frac{\alpha}{\rho}\right)^2\left(1-\frac{w_m}{\bar{w}}\right)^2 \tag{A.1}$$

which, after rearranging, becomes:

$$B^2-4AC=\left(1-\frac{\alpha}{\rho}\right)\left(1+3\frac{\alpha}{\rho}-4\frac{w_m}{\bar{w}}\frac{\alpha}{\rho}\right) \tag{A.2}$$

So that the two roots are:

$$\tau_m=\frac{1+\frac{\alpha}{\rho}-2\frac{\alpha}{\rho}\frac{w_m}{\bar{w}} \pm \sqrt{\left(1-\frac{\alpha}{\rho}\right)\left(1+3\frac{\alpha}{\rho}-4\frac{w_m}{\bar{w}}\frac{\alpha}{\rho}\right)}}{2\left(\frac{\alpha}{\rho}\right)^2\left(1-\frac{w_m}{\bar{w}}\right)} \tag{A.3}$$

It can be shown that the largest root is greater than unity since:

$$\begin{aligned} & 1+\frac{\alpha}{\rho}-2\frac{\alpha}{\rho}\frac{w_m}{\bar{w}}+\sqrt{\left(1-\frac{\alpha}{\rho}\right)\left(1+3\frac{\alpha}{\rho}-4\frac{w_m}{\bar{w}}\frac{\alpha}{\rho}\right)} \\ > \; & 2\left(\frac{\alpha}{\rho}\right)^2\left(1-\frac{w_m}{\bar{w}}\right) \end{aligned} \tag{A.4}$$

After much manipulation it can be shown that this condition reduces to:

$$0>\left(1-\frac{w_m}{\bar{w}}\right)\left(\frac{\alpha}{\rho}-1\right)\left\{\frac{\alpha}{\rho}+\frac{w_m}{\bar{w}}\left(1-\frac{\alpha}{\rho}\right)\right\} \tag{A.5}$$

Of the three terms in parentheses, only the middle term is negative. Hence this condition always holds. Therefore only the lowest root needs to be considered.

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