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# **Is the Discount on the Secondary Market a Case for LDC Debt Relief?**

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**A discount in the secondary market is a case for debt service relief but not necessarily for a write-off. The author derives a "maximum repayment" rescheduling program, which trades off higher current investment for lower current debt service.**

**Proposition 1:** The “maximum repayment” program the lenders would like to monitor involve a fixed investment rate that is smaller than the socially optimal rate and larger than the post-default rate. It involves a transfer of resources from the debtor that is a fixed fraction of GDP — a fraction that is smaller than the cost of default.

**Proposition 2:** When the debt-to-GDP ratio is above a floor value ( $h^*$ ), the lenders can capture the “maximum repayment” value ( $V^*$ ) by fictitiously splitting the debt into performing and nonperforming components. Each period, they should ask the borrower to service the performing component of the debt only, and let the performing component grow at a rate equal to the economy’s expected growth rate. Meanwhile, the nonperforming asset is automatically capitalized at the riskless rate. When the actual growth rate of the economy is above (below) its expected level, the performing part of the debt is scaled up (down). When this “maximum repayment” rescheduling strategy is undertaken, the equilibrium market value of the debt is equal to  $V^*$ .

**Proposition 3:** When the debt-to-GDP ratio is above the threshold  $h^*$ , the debt can be written down to  $h^*$  GDP without impairing the lender’s return. If the write-off is repeated each time the economy declines, and if the rescheduling is undertaken according to Proposition 2, the lenders capture the “maximum repayment”

while the market price of the debt is stabilized at a constant equilibrium price below par.

*(Implication:* Observing a discount on the debt does not automatically warrant a write-off. The discount implies the possibility of default, but lenders should not write the debt off until the possibility materializes. But the service of the debt should always be scaled down by its market value rather than kept in line with its face value.)

**Proposition 4:** When the lenders reschedule the debt on a period-by-period basis, they induce the country to follow a growth pattern that exactly mimics the post-default path. The lenders capture each period the penalty they could impose on the defaulting country. As a result, they get more on a period-by-period basis, but less on average than under the “maximum repayment” schedule. Under such a (“time consistent”) rescheduling strategy, a write-off and multiyear rescheduling may prove beneficial, but the gains fall short of the strategy defined in Proposition 2.

How relevant is the idea of “debt overhang” (according to which the market value of the debt may depend negatively upon its face value)? Empirical evidence presented here indicates that, at a 75 percent confidence level, 9 of 33 countries studied may suffer from a debt overhang problem. At a 90 percent confidence level, only 4 of them may be affected by it.

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by  
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Table of Contents

Introduction.....	1
I. The Setup.....	5
(a) Production.....	5
(b) Preferences.....	7
(c) External Debt.....	7
II. The Optimal Growth Rate: The (Totally) Open Case and the Post-Default Case.....	9
(a) The open economy case.....	9
(b) The post-default case.....	10
III. The "Maximum Repayment" Which can be Extracted from an Indebted Country.....	11
IV. How to Implement the "Maximum Repayment" Scheme.....	14
V. The "Debt Overhang" Problem Revisited.....	17
VI. Empirical Evidence of the "Debt Overhang" Problem....	20
Appendices.....	24
References.....	31

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## Introduction

In 1988, the price on the secondary market of LDC debt averaged 50 cents per dollar of face value. This figure is certainly an indication that the lenders do not expect (on average) to be repaid the full value of their outstanding claims on LDCs and indeed that they expect perhaps no more than half the value of these claims to be serviced. 1/ From the observation of such a discount in the secondary markets, can one go one step further and argue that the debt should be written down in order to account for the discrepancy between the face and the market value of the debt? It is this question that this paper tries to shed some light on, both theoretically and empirically.

Theoretically, the rough answer is as follows. A discount in the secondary market can be the effect of two distinct causes: one is that past shocks may have impaired the capability of a debtor to service its debt. Another cause is that future shocks may be expected to impair, when they occur, the servicing capacity of the country. For instance, the fall in the price of oil that occurred in 1986, if viewed as permanent, is a shock which (at that time) certainly reduced the expected ability of Mexico to service its external debt, and, to some extent, was translated into a fall of the market price of Mexico's debt. On the other hand, the prospect of say, a Middle-Eastern peace settlement, which brings the expectation of increased oil

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1/ The secondary market is a thin market in which, until recently, swap transactions have predominated. Hence secondary market prices may not accurately reflect market expectations. For the purpose of the analysis here, we assume that the secondary market price does reflect the expected value of discounted future debt service.

supplies, is also part of Mexico's debt value, but its implications for debt relief are dramatically different. As I will indicate in the theoretical part of this paper, only the shocks of the first kind--the "backward shocks"--are open to debt relief. As for the others--"forward shocks"--writing off and forgiveness is only optimal after the shocks occur but not beforehand.

Even though it may not always be a good thing to write off the debt in order to account for the discrepancy between its face and its market value, I will show that the service of the debt should always be scaled down by its market value rather than kept in line with its face value.

Specifically, I will show that the optimal rescheduling of the debt should proceed as follows. The lenders should split the debt into two components: a performing and a non-performing part. They should act "as if" the debt amounted to the performing component and scale how much money the borrower should pay in debt service on that part only (while the non-performing part is automatically capitalized at the riskless rate). The performing component of the debt should reflect the market value of the debt but it is important that the lenders calculate it themselves. If they were to rely on the market estimate, the borrower would have an incentive to bring down the market price through poor policies or through confrontations with creditors. Nevertheless, at equilibrium, the lenders' and the market's evaluation should coincide. 2/

On the other hand, the non-performing asset should not necessarily be written down. Good outcomes can occur (or expected bad outcomes may fail to materialize) and, conditionally on the good news, the size of the performing

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2/ See the caveat in footnote 1.

asset may be scaled up. In brief, the answer to the question in the title of this paper is as follows: (1) Yes, a discount on the secondary market implies that the service of the debt should be scaled down in line with its market value; (2) No, such a strategy does not necessarily imply that the face value should be written down. A discount in the secondary market is a case for debt service relief, not necessarily for a write-off.

Obviously, there may come a point in time when the non-performing asset becomes so big as to warrant a write-off. However, if creditors operate optimally (along the lines sketched above), the write-off does not modify the market value of the debt; it simply raises the market price. This should come as no surprise. If they behave optimally, the lenders should not lose by having more nominal claims than less.

How does this result relate to the "debt overhang" idea, according to which the market value may depend negatively upon the face value of the debt? The link may come as follows. In order to achieve their first best outcome, I will show that the lenders must reschedule the performing component of the debt generously enough to allow for the country's investment needs and, on average, they should let the performing asset grow along with GDP. While a strategy along these lines is shown to be optimal for the lenders, it is not however the case that such a strategy is "time-consistent" (as initially defined by Calvo and Kydland and Prescott). It is not a strategy which can be implemented on a period-by-period basis.

Indeed, if dealt with on a period-by-period basis (without setting out the rules of future reschedulings) I will show that a self-fulfilling downward spiral is bound to appear: one in which the fear that the lender will not acknowledge the investment needs of the debtor immediately raises the

cost of capital in the debtor country, reducing investment immediately and making it ex post-optimal for the lenders to tighten their rescheduling strategy.

In the previous literature on the "debt overhang" problem (originated by and Krugman (1987) Sachs (1988)), only these second best equilibria have been examined (for the technical reason that most of the models examined in this literature are two-period models in which the commitment issue could not be addressed). In such a case, when the lenders fail to commit themselves to their first best strategy, a write-off may indeed prove beneficial, by helping the lenders to commit themselves to let the country invest. In no case, however, can it help them get the first best repayment stream.

Whether the "debt overhang" problem is empirically relevant has remained an open question (see Claessens (1988) for a review). Following an idea in Krugman (1988), one may check whether the elasticity of the market price of the debt with respect to its face value is larger than one in absolute value. In the last section of the paper, I will indicate that, at the 75 per cent degree of confidence, 9 countries (in a sample of 33) may suffer (according to Krugman's test) from a debt overhang problem, while at the 90 percent degree of confidence, only 4 of them may be affected by it.

Before closing this introduction, I should emphasize that the case for a write-off which is explored in this paper only rests upon the question of knowing whether the private lenders may find it in their best interest to do so. This is obviously only a narrow way of dealing with the overall question. A write-off may prove beneficial to the lending countries as a whole, when all the relevant spillovers are taken into account, and not to the private lenders themselves. (For such a broader viewpoint see Dornbusch

(1988).) In any case, if the industrialized countries wanted to help the debtors (for whatever reason: altruism or educated selfishness), then a partial forgiveness of the debt may be a crucial preliminary step that the industrialized countries would want to encourage--perhaps through regulatory or tax measures--on the part of private lenders. Without partial forgiveness, the lenders may tend to be the main beneficiaries of the public funds poured into the debtor country.) All these crucial issues are outside the scope of this paper but should be kept in mind before any policy conclusion is drawn.

Section 1 spells out the model. It is a stochastic version of the model examined in Cohen and Sachs (1986). Section 2 calculates the socially efficient and the post-default growth rates of the economy. Section 3 shows that the lenders, if they were to monitor the investment and the consumption strategy of the borrower (in order to maximize their return) would choose a lower investment strategy than the socially efficient one. Section 4 shows how an optimal rescheduling (based upon the distinction between performing and non-performing assets) can achieve the equilibrium described in Section 3. Section 5 shows the dynamic inconsistency of the optimal strategy spelled out in Section 4, and shows the link with the "debt overhang" literature. Section 6 investigates the empirical relevance of the "debt overhang."

## I. The Setup

### (a) Production

I will consider a one-good economy, in which the same good can be used for export, consumption or investment. In each period, the available stock of capital is a pre-determined variable. The production,  $Q_t$ , is a linear function of existing capital:



$$(1) Q_t = K_t$$

Capital can be increased through investment, and investment itself is a costly process. Let us assume that an increase  $I_t$  of capital costs  $J_t$ :

$$(2) J_t = I_t \left( 1 + \frac{1}{2} \theta \frac{I_t}{Q_t} \right)$$

The investment decision  $I_t$ , while taken at time  $t$ , increases the capital stock at time  $t + 1$ , according a stochastic law of motion:

$$(3) K_{t+1} = [ K_t(1-d) + I_t ] (1+\theta_{t+1})$$

in which  $d$  is the rate of depreciation of installed capital and  $\theta_t$  is an iid stochastic variable which is worth:

$$(4) \begin{aligned} \theta_t &= u \text{ with probability } p \\ \theta_t &= v \text{ with probability } 1-p. \end{aligned}$$

Here, the investment decision  $I_t$  must be taken before its productivity ( $\theta_{t+1}$ ) is known. One can think of  $\theta_t$  as a stochastic shock which exogeneously increases (or decreases) the productivity of installed capital stock. I shall refer to the event of probability  $p$  (where the rate of growth of the productivity of capital is  $u$ ) as of the "good state" and to the event of probability  $1 - p$  (when productivity experiences a slower, possibly negative,

growth rate) as the "bad state." I will call  $\theta$  the expected rate of growth of the productivity of capital:

$$(1+\theta) = p (1+u) + (1-p) (1+v).$$

(See Genotte, Kharas and Sadeq (1987) for a model with a similar structure.)

(b) Preferences

I will assume that the country is managed by a social planner who can impose on the country an investment and consumption decision. The planner's preferences are represented by an intertemporal expected utility function:

$$(5) \quad U_0 = E_0 \sum_0^{\infty} \beta^t u(C_t)$$

in which  $C_t$  is the aggregate consumption of the country at time  $t$ ; and  $u(C) = \frac{1}{\gamma} C^\gamma$ ,  $1 < \gamma < 1$  and  $\gamma \neq 0$ , or  $u(C) = \text{Log } C$  when  $\gamma = 0$ .

(c) External Debt

In order to focus on the question raised in the title, I will simply assume that the country inherits an initial debt  $D_0$  (assumed to be short-term) which is large enough to be quoted below par on secondary markets; and I will investigate the optimal rescheduling strategy for the lenders. It is not difficult to show how the framework which is used here could imply that the optimal borrowing strategy does involve such a risk. But some technical issues (such as that of calculating the optimal maturity of the debt) would take this paper too far afield. (See e.g. Cohen (1988) for an analysis, in a

three period model, of the difficulties at hand.)

Following the Eaton and Gersovitz (1981) approach and my earlier work with Sachs, I shall assume that the country always has the ability to repudiate its stock of outstanding debt while the lenders can retaliate and impose on the borrower the following two sanctions:

(a) A defaulting country is forced to financial autarky forever after it has defaulted.

(b) The productivity of capital of the defaulting country is reduced by a factor  $\lambda$  so that the post-default technology of production is:

$$(6) Q_t = (1 - \lambda) K_t.$$

In all that follows, I will assume that the lenders are risk-neutral, act competitively, and have access to a riskless rate of interest which stays constant all along. I will assume that  $\beta$  is low enough to insure that the country will be constrained on the borrowing side. Furthermore, I will leave aside all bargaining issues and assume that the lenders can credibly make (at each point in time, but not necessarily for the entire future) a take-it-or-leave-it offer to the debtor.

II. The Optimal Growth Rate: The (Totally) Open Case and the Post-Default Case

In this section, I would like to calculate the optimal investment strategy in the two extreme cases when the country has a free access to the world financial markets on the one hand and when it is forced to a post-default path on the other hand.

(a) The open economy case

Assume in this sub-section that  $\lambda = 1$  in equation (6) above, i.e. assume that the country cannot repudiate its external debt (because it is too costly). With that assumption, the model boils down to the standard Fisherian case where the investment decision can be separated from the consumption decision. I will simply solve, here, the optimal investment decision. The country wants to maximize its productive wealth when the return on its investment is taken to be the world riskless rate of interest. Mathematically, this amounts to solving the following program:

$$(7) \quad W_0 = \text{Max}_{(I)_t \geq 0} E_0 \left\{ \sum_{t=0}^{\infty} \frac{-1}{(1+r)^t} [K_t - I_t (1 + \frac{1}{2} \phi \frac{I_t}{Q_t})] \right\}.$$

The solution to this program is given in Appendix 1. Given the linearities in the model,  $W_0$  is shown to be a linear function of initial output:

$$(8) \quad W_0 = \bar{w} Q_0$$

and is obtained by picking up a fixed investment rate:

$$(9) \quad \bar{x} = \frac{I_t}{Q_t}$$

associated to a fixed rate of gross investment

$$(10) \quad \bar{y} = \frac{J_t}{Q_t} = \bar{x} \left( 1 + \frac{\dot{i}}{2} \phi \bar{x} \right)$$

(All the technical conditions for the equilibrium to exist are spelled out in appendix). The equilibrium growth rate of the economy oscillates. It is high in the good state of nature  $[(1+u)(1+\bar{x}-d)]$  and low in the bad state of nature  $[(1+v)(1+\bar{x}-d)]$ .

From here on, I will refer to this equilibrium as the socially efficient equilibrium.

(b) The post-default case

Assume now, as another extreme case, that the country has defaulted upon its external debt. In that case, the social planner must choose its investment decision so as to allocate consumption optimally over time. Mathematically, the planner must solve the following program:

$$(11) \quad U_d(Q_0) = \text{Max}_x \left\{ u [Q_0 [1 - \lambda - x (1 + \frac{\dot{i}}{2} \phi x) ] ] \right. \\ \left. + \beta p U_d [Q_0 (1+u) (1+\bar{x}-d)] + \beta(1-p) U_d [Q_0 (1+v)(1+\bar{x}-d)] \right\}$$

in which  $U_d$  is the utility level that the planner can reach when the available output is  $Q_0$  at the initial time.

The solution is spelled out in Appendix 1 where it is shown that the solution  $U_d(Q_0)$  can be written:

$$(12) \quad U_d(Q_0) = C_\gamma Q_0^\gamma \quad \text{if } \gamma \neq 0 \quad \text{and} \quad \frac{1}{1-\beta} \log Q_0 + C_0 \quad \text{if } \gamma = 0$$

in which  $C_\gamma$  is a constant. The solution is also shown to involve a fixed investment rate:

$$(13) \quad x_d = \frac{I_t}{Q_t}$$

which is smaller than the socially efficient investment rate (obtained in the open economy case).

### III. The "Maximum Repayment" Which Can Be Extracted from an Indebted Country

In this section, I will consider the following simple problem. I will assume that the lenders can monitor both the investment and the repayment strategy of the debtor in such a way as to maximize the value of the transfers made abroad by the country. While the borrower will be assumed to give up its sovereignty over its consumption and investment decision, it will nevertheless keep its sovereignty over the matter of defaulting: at any point in time, the borrower will stay free to break the lenders' rule and to follow afterwards the post-default path defined by equation (12). In other words, the rules of the game in this section are as follows: the lenders monitor the debtor's economy so as to maximize the value of the transfers channelled abroad by the debtor, subject to the constraint that the program is never expected (neither today nor later on) to be dominated by a post-default path. Clearly, under

this set of hypotheses, the value of the transfers channelled abroad by the debtor will provide an upper bound to the market value of any debt accumulated by the country.

Formally, the problem can be written as follows. Call  $P_t$  the amount of transfers abroad made by the debtor,  $y_t$  the gross investment rate (inclusive of the cost of installation) achieved by the country, and  $C_t$  the consumption left to the country. One has:

$$C_t = Q_t (1 - y_t) - P_t.$$

Call:

$$(14) \quad U_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} U_s (C_s) \right\}$$

the level of utility which the lenders' program is expected to deliver to the country. With this notation, the program that the lenders must solve is as follows.

$$(15) \quad \text{Maximize } E_0 \sum_{t=0}^{\infty} \frac{P_t}{(1+r)^t}$$

subject to  $U_t \geq U_d(Q_t)$ , for all  $t$ .

in which  $U_d(Q_t)$  is the post-default level of utility (as defined in equation (12)).

This problem is solved in Appendix 2. Given the many linearities built in this model, the problem boils down to finding a fixed (gross)

investment rate  $y$  and a fixed debt service ration  $P_t/Q_t$  which solves the problem (15). The solution is shown to involve an investment rate which lies between the socially efficient rate and the post-default rate. One can state:

Proposition 1: The "maximum repayment" program which the lenders would like to monitor involves a fixed investment rate which is smaller than the socially optimum one and larger than the post-default one. It involves a transfer of resources from the debtor which is a fixed fraction of GDP, a fraction which is smaller than the cost of default.

From Proposition 1, we therefore see that the idea according to which the debt may have a "pro-incentive" effect is not granted in the context of the exercise which is carried through here. (For another approach see also Corden (1988) or Helpman (1988).) Even when it is the banks themselves that design the investment and consumption policy of the borrower, they will choose a lower investment rate than is socially desirable. The reason is that the banks must take care to avoid a situation in which the country may one day choose to default. A too rapid path of capital accumulation, even while socially desirable, will raise the post-default utility of the country and, if not carefully balanced, can be counterproductive to the banks.

From here on, I will call  $V_t^*$  the "maximum repayment" that the lenders can expect to receive from the debtor. Due to the linearities involved,  $V_t$  can be written as a linear function of current output:

$$(16) \quad V_t^* = z^* Q_t$$



In appendix 2, I also show that the fraction of GDP which is channelled abroad can be written:

$$(17) \quad b^* = z^* [(1 + r) - (1 + \theta) (1 + x^* - d)]$$

in which  $x^*$  is the net investment rate that is described in Proposition 1.

#### IV. How to Implement the "Maximum Repayment" Scheme

I will now indicate how the lenders can indeed capture the "maximum repayment" even when they do not monitor the investment and consumption choice of the borrower. Consider the following decomposition of the debt:

$$(18) \quad D_t = V_t^* + R_t$$

in which  $D_t$  is the face value of the debt,  $V_t^*$  is the maximum value calculated above, and  $R_t$  is the residual. Assume that the lenders fictitiously regard  $R_t$  as a non-performing asset and only insist on  $V_t^*$  being serviced (while  $R_t$  is automatically capitalized). Furthermore, assume that, each period, they ask the borrower to transfer an amount  $P_t$  which is the amount necessary to keep  $V_t^*$  growing at the expected rate of growth of the economy.

Under these assumptions  $P_t$  must solve:

$$(19) \quad V_{t+1}^* = (1+r) V_t^* - P_t = (1+\theta) (1+x-d) V_t^*$$

in which  $(1+\theta) (1+x-d) = p (1+u) (1+x-d) + (1-p) (1+u) (1+x-d)$  is the expected growth rate of the economy when the investment rate  $x$  has been selected by the debtor.  $P_t$  is then given by:

$$P_t = [(1+r) - (1+\theta) (1+x-d)] V_t^*$$

$$P_t = z^* [ (1+r) - (1+\theta) (1+x-d) ] Q_t$$

and the optimum investment decision chosen by the country will coincide with the "maximum repayment" strategy designed in equation (17). (See Appendix 2, and Portes (1987) for a suggestion in the same spirit.)

Provided that the non-performing asset is initially large enough, which amounts to assuming that  $D/Q \geq h^*$  with  $h^*$  a given threshold, this scheme can be shown to be repeated for ever and indeed deliver the "maximum repayment" scheme (see Appendix 2 for further details). If  $D/Q$  is below  $h^*$ , then the non-performing asset should be charged a larger interest rate until the face value of the debt reaches the  $h^* Q$  ceiling.

It is crucial to note that this fictitious decomposition of the debt into a performing and a non-performing part is updated each period. Indeed, along equation (19)  $V^*$  is only left to grow at a rate  $(1+\theta) (1+x-d)$  which is the average growth rate of the economy. If things go well the actual growth rate will be larger and  $V_{t+1}$  must be scaled up; conversely,  $V_{t+1}$  will be scaled down if the bad state occurs.

The second crucial remark to make is the following: the performing

asset is not calculated from the observation of the market value of the debt but from the theoretical computation of the maximum repayment scheme. Even though they do coincide at the equilibrium, it is crucial that the lenders do not let  $P_t$  depend upon the observed market value of  $D_t$ . Indeed, if they were to do so, they would ask to be repaid:

$$P_t = z(x) [ (1+r) - (1+\theta)(1+x-d) ] Q_t$$

and the country would be induced to bring down the market value of the debt. These results can be summarized as follows:

**Proposition 2:** When the debt to GDP ratio is above a floor value  $h^*$ , the lenders can capture the "maximum repayment" value  $V^*$  by proceeding as follows. They should fictitiously split the debt into a performing and a non-performing component, the performing component being equal to  $V^*$ . Each period, they should ask the borrower to service the performing component of the debt only, and let the performing component grow at a rate equal to the expected growth rate of the economy. Meanwhile the non-performing asset is automatically capitalized at the riskless rate. When the actual growth rate of the economy is above (below) its expected level, the performing part of the debt is scaled up (scaled down). When this rescheduling strategy is undertaken, the equilibrium market value of the debt is equal to  $V^*$ .

Now obviously, as time passes, the size of the non-performing asset grows relative to the performing one, and some write-off of the debt may become possible without impairing the lenders' ability to capture  $V^*_t$ . One can actually show:

**Proposition 3:** When the debt-to-GDP ratio is above the threshold  $h^*$ , the debt can be written-down to  $h^*$  GDP without impairing the lenders' return. If the write-off is repeated each time the economy goes into the bad state and if the rescheduling is undertaken according to Proposition 2, the lenders capture the "maximum repayment" while the market price of the debt is stabilized at a constant equilibrium price below par.

One important implication of Proposition 3 is that it is not enough to observe a discount on the debt to warrant a write-off. The intuition is that hinted at in the introduction: the discount on the debt takes into account the possibility that the economy may go into a bad state. But lenders have no reason to write-off the debt before that prediction materializes. It is only in the deterministic case when  $u = v$  that the optimal strategy is indeed to write-off the debt "once and for all" (in order to erase whatever backward shocks may have lifted the debt-to-GDP ratio above  $h^*$ ) and let the debt be quoted at par.

#### V. The "Debt Overhang" Problem Revisited

In view of Proposition 2, it appears that the face value of the debt is of little importance in assessing the optimal rescheduling strategy of the

debt. This should come as no surprise: when they behave optimally, lenders get as much as the country can transfer and more nominal claim cannot imply less actual payments. (See also Bulow and Rogoff (1988).) This result, however, contradicts the "debt overhang" argument according to which too large a nominal claim may excessively discourage investment and reduce the market value of the debt. I would now like to indicate how these two conflicting views can be reconciled.

A key feature of the optimal rescheduling strategy described in Proposition 2 is that lenders should let the performing asset grow at the expected growth rate of this economy. As apparent from equation (17) this implies that the service of the debt is negatively correlated with the investment decision of the borrower. Even though such behavior is in the lenders' self-interest, I now want to show that this is not a "time-consistent" decision, that is: it is a decision which is an optimal one to take only if the lenders can commit themselves (in whatever way: sophisticated contracting or a built-in reputation) to implement it later on. In order to see why such a commitment is necessary, assume instead that the lenders operate on a period-by-period basis and simply reschedule the debt each period to the best of their ability, taking for granted that they will do the same (and will be expected to do so) later on. Such a policy can be characterized as a "time-consistent" policy: it is one which is found to be optimal to implement today, when it is expected to be implemented in the future. Since the work of Kydland and Prescott (1977) and Calvo (1978), it is well known that such a policy maybe intertemporally sub-optimal (even though it is pointwise optimal). Let us see what the outcome of such a "time-consistent" rescheduling strategy would be.

As shown in Cohen and Michel (1988) calculating a time-consistent policy simply amounts to finding a feed-back decision rule which, here, can be written:

$$(19) P_t = bQ_t$$

in which  $b$  is the largest amount that the lenders can ask at time  $t$  when it is expected that future payments will be set according to another rule:

$$(20) P_{t+s} = b^* Q_{t+s}$$

which they take as given. A time consistent strategy is one for which, at the equilibrium,  $b^* = b$ .

The equilibrium is calculated in Appendix 3. It is shown that the equilibrium growth rate is nothing else but the post-default path and that  $b^* = \lambda$ . In other words the "time-consistent" policy is simply one in which the lenders take every period the costs that the borrower would incur by defaulting and, as a result, their rescheduling strategy simply mimics the post-default path that the country could follow on its own.

As apparent from equation (19) a time consistent rescheduling strategy act as a tax on output: the borrower expects that the lenders will ask for as much as it can pay and this is an amount which, it can foresee, will be proportional to how much output it can generate. These expectations increase the shadow cost of capital in the debtor country and reduce investment immediately, making it optimal for the lenders to do what they are expected to: disregard the incentive to invest and ask for as much as they can.

It is this downward spiral that most people (I think) have in mind when discussing the debt overhang problem: debt acts as a tax which inefficiently discourages investment and less annual payment from the debtor would imply more overall income to the lenders. Under these circumstances, a write-off may help the lenders. In fact, a write-off cum a multi-year rescheduling can perform even better inasmuch as it helps the lenders commit themselves to put an explicit ceiling on how much money they will ask for each period to come. It should be clear, however, that neither a write-off nor a multiyear rescheduling can help the lenders get the first best, unless, the rescheduling is made contingent upon the investment decision of the borrower. (See Appendix 4 for a formal proof of these statements).

To summarize, one can state:

Proposition 4: When the lenders reschedule the debt on a period-by-period basis, they induce the country to follow a growth pattern which exactly mimics the post-default path. The lenders capture each period the penalty that they could impose on the defaulting country. As a result they get more on a period-by-period basis, but less on average than under the "maximum repayment" scheme. Under such a ("time-consistent") rescheduling strategy, a write-off and a multi-year rescheduling may prove beneficial, but the gains necessarily fall short of the optimal strategy defined in Proposition 2.

#### VI. Empirical Relevance of the "Debt Overhang" Problem

Let us now investigate whether the "debt-overhang" problem is or not empirically relevant.

Krugman (1987) has suggested that we regard the "debt-overhang" as a "Debt Laffer Curve" problem, the question at hand being: does more nominal debt imply a lower market value for this debt? A test of the "debt overhang," according to this formulation, therefore amounts to deciding whether the elasticity of the market price of the debt with respect to its face value is strictly larger than one (in absolute value). Certainly if this elasticity is larger than one, then one can make the case that the lenders operate inefficiently. However, an elasticity equal to or smaller than one is not in itself sufficient to accept the hypothesis that the lenders reschedule the debt efficiently. In this section, we shall stick to Krugman's test, but certainly more work is needed in order to investigate the efficiency of the rescheduling process which has been undertaken since 1982.

Previous attempts to measure the elasticity of the price of the debt with respect to its nominal value systematically found a low estimate. A study by Purcell and Orlanki, following a previous estimate by Sachs and Huizinga, reported an elasticity of 0.34. We have estimated an equation, representative of these earlier studies, as follows:

$$(21) \text{ Log } \rho = 5.06 - 0.653 \log D/X - 2.231 A/D - 1.016 R/D$$

(0.152)                      (0.603)                      (0.373)

$$- 0.274 \text{ Dummy } 1987.12$$

(0.132)

$R^2 = 0.560$  pooled equations for 1986.12 and 1987.12 data; 60 degrees of freedom. (Standard errors in parenthesis).

$\rho$ : price of the debt (cents on the dollar).

D: debt; X: exports; A: arrears; R: amount of rescheduling since 1982.



From this equation, one would tend to reject at the 95 percent level of confidence that the elasticity of the debt was larger than one. Before commenting on the insufficiency of such an equation, it is interesting to report that the price of the debt seems to be very poorly correlated to macroeconomic data related to the country. For instance, the most important of these macroeconomic data (one would guess), such as the non-interest current account or the domestic inflation rate, never appeared to be significantly correlated with the price. On the other hand, arrears or rescheduling data (as we can see from equation (21)) always perform extremely well.

These results are summarized in diagrams 1 to 3. They tend to indicate that the market is extremely sensitive to the "punctuality" of payments and pay little attention to overall macroeconomic performance. Finally, one also sees from equation (21) that a dummy separating the 1986 and 1987 data appears to be significant. This may be a reflection of Citibank's decision to build up \$3 billion of reserves against developing country exposure, a move which significantly influenced the market.

Despite its appeal and its simplicity, an equation such as (21) is extremely misleading. First, it leads us to reject the hypothesis that the elasticity of the price with respect to debt is larger than one for the entire sample. But it may very well be the case that only a sub-group of countries was hit by the debt-overhang problem. Running, for instance, the same regression for the sub-sample of countries for which the debt-to-export ratio is larger than 3 (a sub-sample of 16 countries) would yield a larger elasticity, which we estimated to be at 1.183 (with a standard error of 0.339). Second, and perhaps more importantly, an equation such as (21) takes

the arrears and the rescheduling variables as exogeneous, while these variables obviously depend upon debt and perhaps upon the price itself. In order to overcome these two difficulties (to which one should also add a more technical one which is that the price being smaller than one hundred,  $\log \rho$  cannot be normally distributed), we have estimated a reduced form equation in which the dependent variable has the logistic form  $\log (\rho / 100 - \rho)$ , so as to let the elasticity depend upon the level of the price. The result comes as follows:

$$(22) \quad \text{Log } \frac{\rho}{100-\rho} = \quad 2.152 \quad - \quad 1.509 \quad \text{log D/X} \\ \quad \quad \quad \quad (0.378) \quad \quad \quad (0.305)$$

$$-0.048 \text{ X growth} - 0.583 \quad \text{Dummy 87.12}$$

$$(0.024) \quad \quad \quad (0.288)$$

$R^2 = 0.389$ ; pooled equation for 1986.12 and 1987.12 data;

60 degrees of freedom; X growth: rate of growth of exports.

According to this equation the elasticity of the price with respect to debt ( $100 - \rho$ ) is 1.509 (with a standard error of 0.305). This indicates that the debt overhang problem could not be rejected at the 95 percent level degree of confidence for these countries in the sample for which the price was almost zero (such as Sudan). More generally, Table 1 indicates the countries for which the debt-overhang problem could not be rejected at various degrees of confidence. At the 90 percent level of confidence, only 4 countries pass the test.

Table 1  
Countries with a Potential Debt  
Overhang Problem  
(as of 1987.12)

At the 50% level of confidence: $\rho \leq 34$ 1	Argentina (34) Jamaica (33) Nigeria (29)
At the 75% level of confidence: $\rho \leq 23$	Dominican Republic (23) Congo (23) Zaire (19) Zambia (17) Costa Rica (15)
At the 90% level of confidence: $\rho \leq 11$	Bolivia (11) Peru (7) Nicaragua (4)
At the 95% level of confidence: $\rho = 0$	Sudan (2)

(The numbers in parenthesis are secondary market prices in cents per dollar).

**Appendix 1. Optimal Growth in the (Totally) Open and In the Post-Default Economy Cases.**

**A) The open economy case**

From equation (7), doubling  $Q_0$  would also double  $W_0$  so that one can look for  $\bar{w}$  such as in equation (8).  $\bar{w}$  is the solution to the following Bellman equation underlying the definition of  $W_0$  in equation (7):

$$(A1.1) \quad \bar{w} = \text{Max}_x \left\{ 1 - x \left( 1 + \frac{1}{2} \theta x \right) + \frac{\bar{w}}{1+r} [ p(1+u) + (1-p)(1+v) ] (1+x-d) \right\}$$

The equilibrium value of  $\bar{x}$  is

$$(A1.2) \quad \bar{x} = \frac{1}{\phi} \frac{(1+\theta)\bar{w}}{1+r} \quad , \text{ with } 1 + \theta = p(1+u) + (1-p)(1+v).$$

We shall assume  $\bar{x}$  to be positive.

Equation (A1.1) yields that  $\bar{x}$  is a solution to:

$$(A1.3) \quad \frac{1}{2} x^2 - x \left[ \frac{r-\theta}{1+\theta} + d \right] + \frac{1}{\phi} \left( 1 - d - \frac{r-\theta}{1+\theta} \right) = 0$$

The solution that is socially efficient is:

$$(A1.4) \quad \bar{x} = \left( \frac{r-\theta}{1+\theta} + d \right) \left[ 1 - \sqrt{1 - \frac{2}{\phi} \left( 1 - d - \frac{r-\theta}{1+\theta} \right)} \right]$$

which exists and is positive if:

$$(A1.5) \quad \phi > 2 \left( 1 - \frac{r-\theta}{1+\theta} - d \right) / \left( \frac{r-\theta}{1+\theta} + d \right); \quad \frac{r-\theta}{1+\theta} + d > 0$$

a condition which we shall assume to hold.

**B) The Post-Default Case**

Let us "guess" that the solution to equation (11) can indeed be written:

$$(A1.6) \quad U_d(Q_0) = C_Y Q_0^Y$$

Then the "guess" will prove to be the right one if

$$(A1.7)$$

$$C_Y = \text{Max}_X \{ [1-\lambda-y]^Y + \beta p [(1+u)(1+x-d)]^Y C_Y + \beta [(1-p)(1+v)(1+x-d)]^Y C_Y \}$$

By the envelope theorem, the derivative of the right - hand side is smaller than one when  $\beta$  is small enough to induce the country to be in the borrowing side.

**Appendix 2: The "Maximum Repayment Scheme"**

Because of the linear structure of the model, one has to find  $b^*$  and  $x^*$  such that

$$V_0 = \text{Max}_{b, x} E_0 \sum_{t=0}^{\infty} \frac{b Q_t}{(1+r)^t} \equiv z^* Q_0$$

subject to  $E_0 \left( \sum_{t=0}^{\infty} \beta^t u [(1-b-y) Q_t] \right) \geq U_d(Q_0)$

(The stationarity of the problem implies that this inequality, if it holds at time 0, will also hold at later times).

Call  $\omega(x)$  the solution to:

$$(A2.1) \quad W_0 = \omega(x)Q_0 = E_0 \sum_0^{\infty} \frac{Q_t}{(1+r)^t} \equiv \frac{1}{(1+r)-(1+\theta)(1+x-d)} Q_0$$

when the investment rate is  $x$ . The problem at hand is therefore simply that of finding:

$$(A2.2) \quad z^* = \text{Max}_x \quad b \omega(x) \\ b ; x$$

subject to:

$$A2.3) \quad E_0 \sum_0^{\infty} \beta^t (1-b-y)^Y Q_t^Y \geq U_d(Q_0)$$

Let

$$A2.4) \quad \mu_t \equiv \beta^t E_0 \left[ \sum_{i=0}^t (1+\theta_i)^Y \right]$$

(A2.3) can be written:

$$(A2.5) \quad \sum_0^{\infty} \mu_t (1-b-y)^Y (1+x-d)^{tY} > U_d(Q_0)$$

By duality, maximizing  $z^*$  in (A2.2) subject to (A2.5) amounts to finding  $z^*$  which is a solution to:

$$U_d(Q_0) = \text{Max}_0 \sum_0^{\infty} \mu_t \left( 1 - \frac{z^*}{\omega(x)} - y \right)^Y (1+x-d)^{tY}$$

From the definition of  $w(x) = (1+r) - (1+\theta)(1+x-d)$  in equation (A2.1), this amounts to asking the country to transfer:

$$P_t = b^* Q_t = z^* [(1+r) - (1+\theta)(1+x-d)] Q_t$$

in which  $x$  is freely chosen by the country so as to maximize its utility.

$$\text{Since } U_d(Q_0) = \int_0^{\infty} u_t (1-\lambda-y_d)^t (1+x_d-d)^t$$

one can see from (A2.5) that

$$b^t < \lambda \text{ and } x^* > x_d.$$

The investment rate is larger under the optimal scheme than under default.

### Appendix 3: The Time Consistent Path

For equation (17), the lenders want to induce the country to repay in each period:

$$(A3.1) P_t = z^* [(1+r) - (1+\theta)(1+x^*-d)] Q_t$$

in which  $z^*$  and  $x^*$  are the optimal choice defined in Proposition 1. Let us show that Proposition 2 solves this problem when  $h^*$  is defined as

$$(A3.2) h^* = z^* \frac{(1+r) - (1+x^*-d)(1+\theta)}{(1+r) - (1+x^*-d)(1+u)}$$

and when the price of the debt is

$$(A3.3) \quad q^* = \frac{(1+r) - (1+x^* - d)(1+u)}{(1+r) - (1+x^* - d)(1+\theta)}$$

If (A3.1) and (A3.3) are satisfied, the market value of the debt is  $z^*Q_t$  (the maximum value) and Proposition 2 indicates that the borrower should repay  $P_t$  so that, when measured in market terms, the debt grows at the rate  $(1+\theta)(1+x-d)$ . This implies that  $P_t$  must be such that

$$q^* D_{t+1} = (1+r) q^* D_t - P_t = q^* D_t (1+\theta)(1+x-d)$$

so that

$$P_t = z^* [(1+r) - (1+\theta)(1+x-d)] Q_t$$

Given this rule of the game, the country must:

$$\text{Maximize} \quad \sum_0^{\infty} \mu_t (1+x-d)^t Y \{ (1 - z^* [(1+r) - (1+\theta)(1+x-d)] - y \}^Y$$

which is exactly the problem at hand in Appendix 2.

#### Appendix 4: MYRAs and Write-Offs

In order to see how a multi-year rescheduling agreement associated with a write-off can help time-consistent lenders, let us restrict the analysis to the deterministic case when  $u = v$ . Assume that the lenders are trapped into the time-consistent strategy by which they are expected to (and indeed do) levy  $P_t = b^* Q_t$  each period. Assume that they reschedule the debt on



a long run basis so as to let each period's payment falling due equal:

$$(A6.1) \quad P_t = P_0 (1 + g)^t$$

with  $g$  being some exogenous growth rate. The country now must:

$$(A4-1) \quad \text{Max} \quad \sum_0^{\infty} \beta^t u [Q_t - J_t - P_t]$$

and one sees that the disincentive to growth is eliminated (inasmuch as the borrower takes  $P_t$  as not contingent upon  $Q_t$ ). Clearly there exists a value of  $P_0$  and  $g$  for which the equilibrium growth rate of the economy coincides with  $g$  and for which the borrower is exactly indifferent between servicing the debt and defaulting. For this equilibrium one finds that lenders raise the value of their claim above the time-consistent pay-off (to the extent that the disincentive to grow has been eliminated) but fall short of the first best strategy (to the extent that the incentive to grow has not been optimally designed). In order to require (A4.1), the lenders must therefore write off part of the debt below the "maximum repayment" value.

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