POLICY RESEARCH WORKING PAPER 1511

Default Risk and the Effective Duration of Bonds

David F. Babbel
Craig Merrill
William Panning

Estimates of how much credit risk shortens the effective duration of corporate bonds based on observable data and easily estimated bond pricing parameters.

The World Bank
Financial Sector Development Department
September 1995

Summary findings

Basis risk is the risk attributable to uncertain movements in the spread between yields associated with a particular financial instrument or class of instruments, and a reference interest rate over time. There are seven types of basis risk: Yields on

- Long-term versus short-term financial instruments.
- Domestic currency versus foreign currencies.
- Liquid versus illiquid investments.
- Bonds with higher or lower sensitivity to changes in interest rate volatility.
- Taxable versus tax-free instruments.
- Spot versus futures contracts.
- Default-free versus non–default-free securities.

Basis risk makes it difficult for the fixed-income portfolio manager to measure the portfolio's exposure to interest rate risk, heightens the anxiety of traders and arbitrageurs who are hedging their investments, and compounds the financial institution's problem of matching assets and liabilities.

Much attention has been paid to the first type of basis risk. In recent years, attention has turned toward understanding the relation between credit risk and duration. Babbel, Merrill, and Panning focus on that, emphasizing the importance of taking credit risk into account when computing measures of duration.

The consensus of all work in this area is that credit risk shortens the effective duration of corporate bonds. The authors estimate how much durations shorten because of credit risk, basing their estimates on observable data and easily estimated bond pricing parameters.

An earlier version of this paper — a product of the Financial Sector Development Department — was presented at the Second Biennial Conference on the Financial Dynamics of the Insurance Industry, New York University. Copies of this paper are available free from the World Bank, 1818 H Street NW, Washington, DC 20433. Please contact Susana Coca, room G8-017, telephone 202-473-7474, fax 202-522-3198, Internet address scoca@worldbank.org. September 1995. (15 pages)

Financial Sector Development Department
The World Bank

Default Risk and the Effective Duration of Bonds

by

David F. Babbel
Financial Sector Development Department, The World Bank
Wharton School, University of Pennsylvania

Craig Merrill
Marriott School of Management, Brigham Young University

William Panning
Met Life Investment Management Company

The authors thank Hayne Leland, Jerome Fons, and Edward Altman for helpful comments. An earlier version of this paper was presented at the Second Biennial Conference on *The Financial Dynamics of the Insurance Industry,* Salomon Center, New York University.

Default Risk and the Effective Duration of Bonds

I. Introduction

Basis risk refers to the risk attributable to uncertain movements in the spread between yields associated with a particular financial instrument or class of instruments, and a reference interest rate over time.[1] There are seven basic types of basis risk.

1) Yields on long-term vs. short-term financial instruments
2) Yields in domestic currency vs. yields in foreign currencies
3) Yields on liquid vs. illiquid investments
4) Yields on bonds with high vs. low sensitivity to changes in interest rate volatility
5) Yields on taxable vs. tax-free instruments
6) Yields on spot vs. futures contracts
7) Yields on default-free vs. nondefault-free securities

One problem with basis risk is that it creates difficulties for the fixed income portfolio manager to measure the portfolio's exposure to interest rate risk. It heightens the anxiety of traders and arbitrageurs who are hedging their investments. And it compounds the problem of matching assets and liabilities for the financial institution.

Much attention has been given to the first type of basis risk. Several authors have provided theoretical models and empirical estimates of bond duration when interest rates of different maturities do not fluctuate in lockstep. Some of these authors have given a unitary measure of duration,[2] while others have opted for a multi-factor model and produced separate duration measures with respect to fluctuations in the short-term and the long-term interest rates.[3] Similar efforts have been underway for accommodating other kinds of basis risk. In recent years, attention has turned toward understanding the relation between credit risk and duration. That is the focus of the present study.

To underscore the importance of taking credit risk into account when computing duration measures, we have provided a numerical example in Exhibit 1. The example relates to the insurance field, but is equally applicable to other financial institutions. To simplify the analysis, we will assume that each of the duration measures takes into account any and all options and interest-sensitive cash flows stemming from the assets and liabilities.[4] The first column gives the grade of the assets, *A*. As shown in the second column, assets are split evenly between AA and BB-grade bonds. The duration of assets, *DA*, computed with respect to the reference yield on each segment of the portfolio, is shown in the third column. Note that both grades of bonds have durations of 10, so the asset portfolio duration is also 10. The next column in the upper half of the table gives the liabilities, *L*, and the duration of liabilities, *DL*, where the value of liabilities and duration are computed with respect to the blended discount rate of the asset portfolio.

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[1]The concept of basis risk has been broadened in recent years to include more than what was originally entailed in the term. One narrower definition of basis risk refers only to the uncertain movements in the spread between spot and futures prices.

[2]See, for example, Khang [1979], Cox, Ingersoll and Ross [1979], and Babbel [1983, 1989].

[3]See, for example, Brennan and Schwartz [1983], Nelson and Schaefer [1983], and Bierwag, Kaufman, and Latta [1987, 1988].

[4]This, of course, is a heroic assumption insofar as it relates to life insurance companies, which sometimes use *ad hoc* rules of thumb to adjust durations for call provisions on corporate bonds, while ignoring altogether the lapse options in life insurance policies. See Babbel [1990] for a discussion of these problems.

(While this procedure for valuation and duration estimation is not strictly correct, a number of firms use it.) The liability duration is assumed to be 11. The final two columns in the upper half of the table give the value of surplus (net worth is the analog for non insurance companies), *S*, and the duration of surplus, *DS*. In this example, the duration of surplus is zero.[5] The problem with the duration estimates shown in the upper half of the table is that each is derived with respect to a different reference rate of interest.

Exhibit 1
Balance Sheet Conversion of Duration to Effective Duration

Grade	*A*	*DA*	*L*	*DL*	*S*	*DS*
AA	55	10				
BB	55	10	100	11	10	0
Total:	110	10	100	11	10	0

Grade	*A*	*DA*	Basis Risk Adjustment Factor	*EDA*	*L*	*DL*	Basis Risk Adjustment Factor	*EDL*	*S*	*DS*	*EDS*
AA	55	10	0.95	9.5							
BB	55	10	0.80	8.0	100	11	1.15	12.65	10	0	-30.25
Total:	110	10		8.75	100	11		12.65	10	0	–30.25
Liabilities adjusted for correct discount rate:					103	11	1.15	12.65	7		-48.64

In the lower half of the table, the previous elements are repeated, but adjustments are introduced to account for basis risk in the duration measures. These adjustment factors reflect the fact that yields on AA bonds may fluctuate only 95% as much as Treasury yields, whereas yields on BB bonds may fluctuate only 80% as much as Treasury yields. (The adjustment numbers used were chosen to illustrate the issue, and are not precise.) When the adjustments are applied to the duration measures, we produce effective duration of assets measures, *EDA*, for the two grades of assets held by the firm. The overall asset portfolio effective duration, defined with respect to movements in the reference Treasury rate, is now 8.75. Furthermore, the effective duration of liabilities, *EDL*, when taken with respect to the reference Treasury rate, is now increased by 15% and is 12.65. (For guaranteed insurance liabilities, the appropriate discount rate is very close to Treasury rates and independent of the asset portfolio yield.) When taken together, and levered by the assets-to-surplus ratio, the effective duration of surplus, *EDS*, is now –30.25. Thus, what initially looked like an immunized balance sheet is actually highly exposed to interest rate risk.

But we must not stop there. Not only does the liability duration measure need to be augmented by 15% to reflect the fact that the driving interest rate for liabilities is the Treasury rate, but the value of liabilities themselves must also reflect the lower discount rates.[6] Since the hypothetical insurer was using the blended yield on its asset portfolio to discount the liabilities, the valuation using lower Treasury rates will result in a higher lia-

[5] The duration of surplus is equal to the difference in asset and liability durations, multiplied by the assets-to-surplus ratio, and augmented by the duration of liabilities.

[6] This applies equally whether a single discount rate is used to value insurance liabilities, or an entire lattice of stochastic interest rates is used to value them.

bilities value, shown here to be 103. This will result in a lower economic surplus value of 7, an increased leverage, and consequently a higher (in absolute terms) duration measure of –48.64. Thus, our ostensibly immunized insurance company actually exhibits a rather extreme exposure to interest rate risk when credit basis risk is taken into account.

II. Review of the Literature

As in the case of long-term vs. short-term interest rate basis risk, two approaches have been taken with credit basis risk. In one approach, an attempt is made to collapse the impact of credit risk on interest rate risk into a single duration measure,[7] while the other approach involves using multi-factor models to develop separate risk measures.[8]

Bierwag and Kaufman [1988] develop simple single-factor measures of duration for option-free bonds subject to default risk for hypothetical stylized stochastic processes governing the time pattern of default losses. To obtain an accurate measure of duration, they must stipulate the stochastic process governing the timing of the losses from default for a given expected present value of future payments, as well as the stochastic process governing interest rates. Because relatively little is currently known about the timing of reductions in payments for bond issues after default, they suggest that much additional research on the bankruptcy process will be needed in order to use their model effectively. Their study is purely analytical, with no empirical verification.

Jerome Fons [1990] conducted an empirical examination of the effective duration[9] of risky bonds. He found that the effective duration of corporate bonds is always less than their Macaulay duration, and that the gap between the two widens as one moves from Aaa to B rated bonds. However, he finds that the behavior of this gap is not monotonic in the bond rating. Ambarish and Subrahmanyam [1990] suggest that this may be due to the underlying non-monotonicity in the coefficient of correlation between the riskless interest rate and the portfolios of market values of the firms issuing the bonds. Moreover, for a pair of bonds within a given rating category, even when the coupons, maturities, and call provisions are similar, there can be a huge difference in their perceived riskiness, as manifest by their different yields.[10] Because it is market perception of risk that influences price movements over time, to the extent that it is not captured in the bonds' ratings, a more contemporaneous market-based risk measure will be more useful when estimating effective durations.

Ambarish and Subrahmanyam [1990] provide a methodological critique of empirical studies done on the riskiness of corporate bonds. They suggest more attention be paid to the contingent claims element in default-prone bonds. In their analytical model they show that because of the attributes of the embedded put option on the market value of the firm, a lower grade corporate bond will have a lower effective duration than a higher grade bond. While the authors provide a simple numerical analysis to demonstrate their points, they do not include an empirical study to validate their assertions.

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[7] See, for example, Chance [1990], Fons [1990], and Leland [1994].

[8] See, for example, Bierwag and Kaufman [1988], Longstaff and Schwartz [1993], and Jarrow and Turnbull [1995].

[9] The term "effective duration" is used to refer to the percentage change in the price of a bond occasioned by a small movement in a reference rate of interest. Effective duration measures are designed to take into account call and prepayment provisions as well as default risk. Traditional duration measures typically ignore these features.

[10] See Jerome Fons [1994], p. 30.

Chance [1990] develops a closed-form solution to the duration of default-prone zero-coupon bonds. He incorporates default risk into his pricing model by using the Merton [1974] framework and derives his duration measure using a comparative statics approach to interest rate changes. This allows him some insight into the effect of default risk on interest rate sensitivity. He finds that default risk decreases the interest rate sensitivity of a bond. However, his model also suffers from the same limitations as Merton's model, i.e., only simple capital structures and pure discount securities can be analyzed,[11] and the market value of firm assets must be known. Because the assets underlying the financial security are often not tradable, their values are not observable. This makes application of the theory and estimation of the relevant parameters problematic. Additionally, all of the other liabilities of the firm senior to the corporate debt must first (and simultaneously) be valued. This generates significant computational difficulties (see Jarrow and Turnbull, [1995, pp. 53-4]). While he does not state it explicitly, Chance's duration measures are based on changes in the short rate of interest.

Blume, Keim and Patel [1991] focus primarily on the sample mean returns and standard deviations of return of junk bonds relative to Treasury and high-grade bonds. They note that junk bonds can be expected to have lower durations than default-free bonds of similar maturities. The lower durations are said to stem from higher coupon rates (and hence, accelerated payments) along with higher discount rates. In comparing the sample standard deviations of junk bonds with default-free bonds, they create matched synthetic Treasury bonds, including some control for call provisions, but they do not control for differences in effective duration.

In a recent paper, Leland [1994] has examined optimal capital structure with default risk. A byproduct of his theoretical analysis is a measure of effective duration for default-prone bonds. He finds that default-prone bonds have effective durations that are *much* shorter than their Macaulay durations. Indeed, under some conditions, he even finds that their effective durations turn negative.

Leland's study is attractive in providing closed-form solutions to the pricing and duration of coupon-bearing default-prone corporate debt. He achieves this by sacrificing some generality. First, his corporate debt pricing model has a single stochastic state variable: the value of the firm. Interest rates are nonstochastic. Second, Leland's model prices only corporate debt with a constant cash flow stream; i.e., his debt must have a constant coupon coupled with a time homogeneous repayment of principal, as well as reissue of new principal. Third, Leland's study is a purely theoretical analysis. There is no empirical verification offered. Fourth, his model assumes a single class and structure of outstanding bonds. Fifth, in comparing effective durations with Macaulay durations, his effective duration measures are taken with respect to a movement in the riskless rate, whereas his Macaulay duration measures are taken with respect to a movement in the risky bond yield. Thus, the differences between effective duration and Macaulay duration are not solely related to default risk, but also stem from using different reference interest rates, and from the assumed behavior of those reference rates.

III. Risky Debt Pricing Model

In this study we attempt to remedy a number of the limitations of previous studies regarding the impact of default risk on effective duration. Our starting point is the creation of a bond pricing model that takes into account default risk. We then calibrate our

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[11] In an unpublished appendix to his paper, Chance derives a more complex duration measure for risky bonds featuring two payments. The complexity of his duration formula increases substantially by including a second payment.

bond pricing model to a large set of price observations that are based on the universe of noncallable corporate bonds. This allows us to avoid the entanglement of call option and sinking fund risks with credit risk in our duration measures. We then estimate effective durations for a wide variety of yield spreads over matched synthetic Treasury bonds. These synthetic bonds are identical to the corporate bonds in terms of coupon rate and time to maturity. Two effective duration measures are offered. The first is taken with respect to a movement in the yield on a matched Treasury, while the second is taken with respect to a movement in the instantaneous riskless rate of interest. These duration measures are compared to those of their matched Treasury counterparts. We show that the effective duration measures for default-prone bonds are shorter than their matched synthetic Treasury counterparts. Finally, we demonstrate the portion of the reduction in duration that was occasioned by credit risk, versus the reduction that stemmed purely from higher coupon rates and higher discount rates.

The risky debt pricing model we employed in this study is described fully in Merrill [1994]. Here we will briefly review its features. The model is based on two underlying state variables which display partial correlation with each other: the value of the firm and the instantaneous interest rate. The movement of the value of the firm through time evolves according to a general diffusion process, and the instantaneous interest rate follows a Cox-Ingersoll-Ross process. Features of corporate bonds, such as their time to maturity, par value, coupon rate, and priority in the overall debt structure are incorporated as parameters. The overall capital structure of the firm, market value of equity, and dividend rate are also key elements to the valuation. In addition, to complete the model, we must estimate the instantaneous return and volatility of the returns on common stock, the volatility of interest rates, the long-run average of the instantaneous interest rate, and the speed of mean reversion.

The full functional dependence of the value of a bond may be expressed as follows:

$$B(r,S,\tau,c_B,c_S,F_B,V_R,\kappa,\mu,\sigma,\sigma_S,\rho)$$

where:

$r =$ the instantaneous riskless rate of interest
$S =$ the value of the common stock
$\tau =$ the time to maturity of the bond
$c_B =$ the coupon payment on the bond
$c_S =$ the instantaneous rate of cash flow paid as a dividend
$F_B =$ the face value of the bond
$V_R =$ expected residual value of the firm after all claims have been paid that are senior to the bond being valued
$\kappa =$ the speed with which the interest rate r reverts to the long - run mean rate
$\mu =$ the long - run average of the instantaneous interest rate
$\sigma =$ a parameter used in the instantaneous variance of dr, $\left(\sigma\sqrt{r}\right)^2$
$\sigma_S =$ the square root of the instantaneous variance of the return on S
$\rho =$ the instantaneous correlation between the two diffusion processes

As can be seen from the above expression, this model uses only variables which can be observed directly or can be easily estimated from observable data. (The value of V_R is based on Altman [1989, 1993]). Therefore, it is amenable to an empirical test of the contingent claims pricing methodology, and can be applied to estimating the duration of risky bonds.

IV. Empirical Validation

Model-based prices were compared to actual prices given in a data set of all non-callable corporate bonds issued between January 1977 and July 1991. These bonds are also free of other features such as sinking funds or convertibility that might complicate the analysis.[12] Only those bonds that showed a sufficient trading volume were included in our test, to avoid basing our pricing model on stale prices. The use of the simple bonds in this data set allows the focus to be on default risk and interest rate risk only. In order to estimate the other parameters of the model, default-free bonds have been synthesized. This was accomplished by estimating the term structures using "on-the-run" Treasury bonds. The default-free value of the corporate bond's cash flows was determined by discounting these promised cash flows using the Treasury term structures. They give the default-free price of the corporate cash flows and provide the necessary yield-to-maturity information that serves as a reference point from which to compute effective durations of default-prone bonds. Finally, equity data (which are used in the bond pricing model) for each of the firms in the data set were taken from the CRSP tapes.

In an empirical test by Merrill [1994], the model was shown to fit the corporate bond price data better than previous single-factor models, although it was biased slightly high with the bias increasing at higher prices. Therefore, the model prices were fine tuned to the data so that the model could be used with greater confidence when describing the relationship between risky bond value and interest rates. This results in unbiased model prices, which are depicted in Exhibit 2. (A perfect model fit would have all data points line up along the 45° line.) This corporate bond pricing model, fine-tuned to reflect actual corporate bond prices, was then used to estimate the effective corporate bond durations disclosed in the remainder of this study.

V. Computation and Comparison of Duration Measures

In Exhibit 3, we show the effective durations of default-prone bonds relative to those of matched synthetic Treasury bonds. Note that higher yield spreads over the matched Treasuries are associated with lower effective durations relative to those on the matched Treasuries. These effective duration measures are analogous to the modified duration measure in one way, but different in two key respects. They are similar in that they relate the percent change in bond price to change in yield. They are different in that the percent change in this effective duration measure is estimated numerically based on a price/yield curve that incorporates default risk, rather than based on a simple derivative of price relative to yield. Moreover, the effective duration measures of both the corporate and the synthetic Treasury bonds are derived with respect to a change in the yield of the matched synthetic Treasury bonds, whereas Modified duration is typically related to a bond's own yield. By having the effective durations based on changes in the same reference yields, it is easier to compare the interest rate sensitivities of the two kinds of bonds, to isolate the impact of credit risk, and to compute portfolio duration when both kinds of bonds are present.

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[12] The problem of call options, sinking funds, and convertibility has rendered previous empirical studies of risky bond duration, which have not been based on this kind of data set, unable to isolate the impact of pure default risk on duration. See, for example, Fons [1990]. In his sample of 702 "low-rated" corporate bonds, Fons [1987] found that over 95% had call provisions. Another difference between this study and that of Fons is that he uses bond ratings to categorize the riskiness of corporate bonds, whereas we use the yield spread between the corporate bond and the synthetic Treasury bond. This allows our duration measures to reflect the most recent market information available about the credit riskiness of the corporate bonds, and also accounts for changing spreads between the Treasury yields and corporate bond yields of a given credit category.

Exhibit 4 is based on exactly the same data as Exhibit 3, but displays the data differently. On the vertical axis we show the ratio of effective corporate bond duration to effective synthetic Treasury bond duration, as a function of effective synthetic Treasury bond duration and yield spread. This chart shows that to compute the effective durations of default-prone bonds, one merely needs to compute the effective durations of matched synthetic Treasury bonds and then reduce them by the factors displayed on the vertical axis. (Recall again that the ratios do not reflect any call options that might be present. These would have to be taken into account separately.) In the case of bonds manifesting yield spreads as high as 600 b.p., the reduction in effective duration below that of the matched Treasury appears to approach 20 percent. Most of this reduction is due to credit risk, as will be shown later in Figure 7.

Exhibit 5 is constructed similarly to Exhibit 3, except that the duration measures are taken with respect to a different reference rate of interest. Here we show effective durations based on movements in the instantaneous riskless rate of interest. The term structure of interest volatility is based on the CIR model. The pattern shown is very similar to that in Exhibit 3, with the lowest effective corporate bond durations associated with the highest yield spreads. However, the effective duration measures are much lower, owing to the low volatility of long-term yields relative to that of the short rate. Exhibit 6 is similar to Exhibit 4, but is drawn with respect to movements in the short rate.

Finally, in Exhibit 7 we demonstrate for a 10-year bond how effective duration differs from a simpler derivative-based measure of modified duration. The top curve shows the reduction in duration purely from using increasing discount rates. The middle curve reflects the decline in modified duration that would stem from both increasing discount rates and rising coupon rates. Default risk is not included in the analysis of these top two curves apart from its impact on the yields and coupons. The lowest curve shows the effective duration measure on par corporate bonds, with the reference interest rates being the yields on matched synthetic Treasury bonds. The substantial difference between the lowest curve and the middle curve shows how biased upward the traditional measure of modified duration is for bonds subject to default. We suspect that had this difference been taken into account in the Blume, Keim and Patel [1991] study, their assessment of the relative riskiness of junk bonds would have been increased.

VI. Epilogue

The consensus of all work in this area is that credit risk shortens effective duration of corporate bonds. In this paper we have provided estimates of how much durations shorten due to credit risk. Our estimates are based on observable data and easily estimated bond pricing parameters. Because our duration measures are taken with respect to movements in a common reference rate of interest, they can be used with greater confidence when attempting to compute the duration of a portfolio of bonds subject to varying degrees of credit risk.

Over the past three years, several hundred additional noncallable corporate bonds have been issued and traded. This should provide an even richer data set with which to refine and tune our corporate bond pricing model, and will strengthen our estimates for the impact of credit risk on effective duration. We are currently engaged in assembling these data and plan to report our results in a subsequent paper.

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Exhibit 2
Market Prices vs. Model Prices

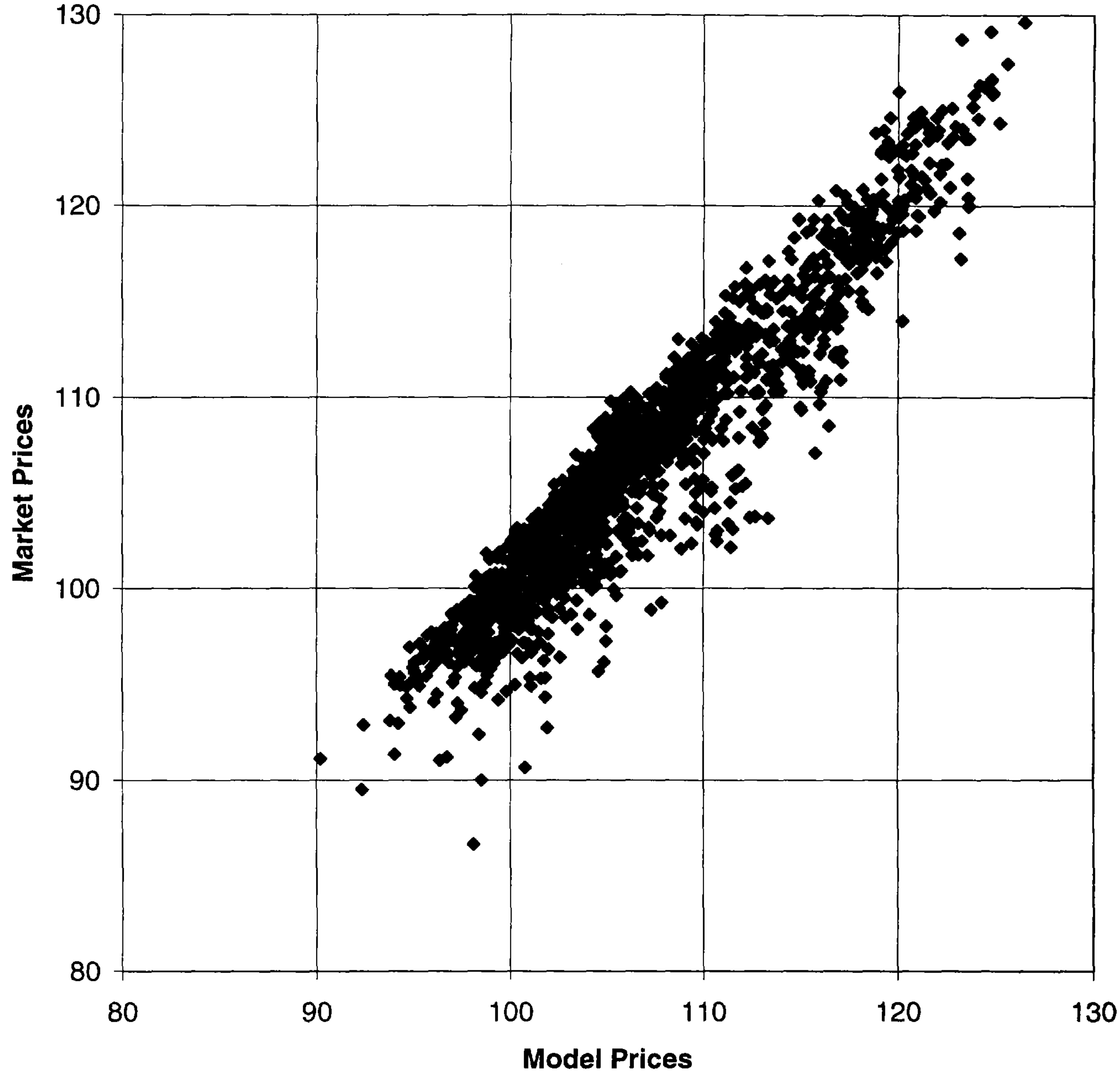
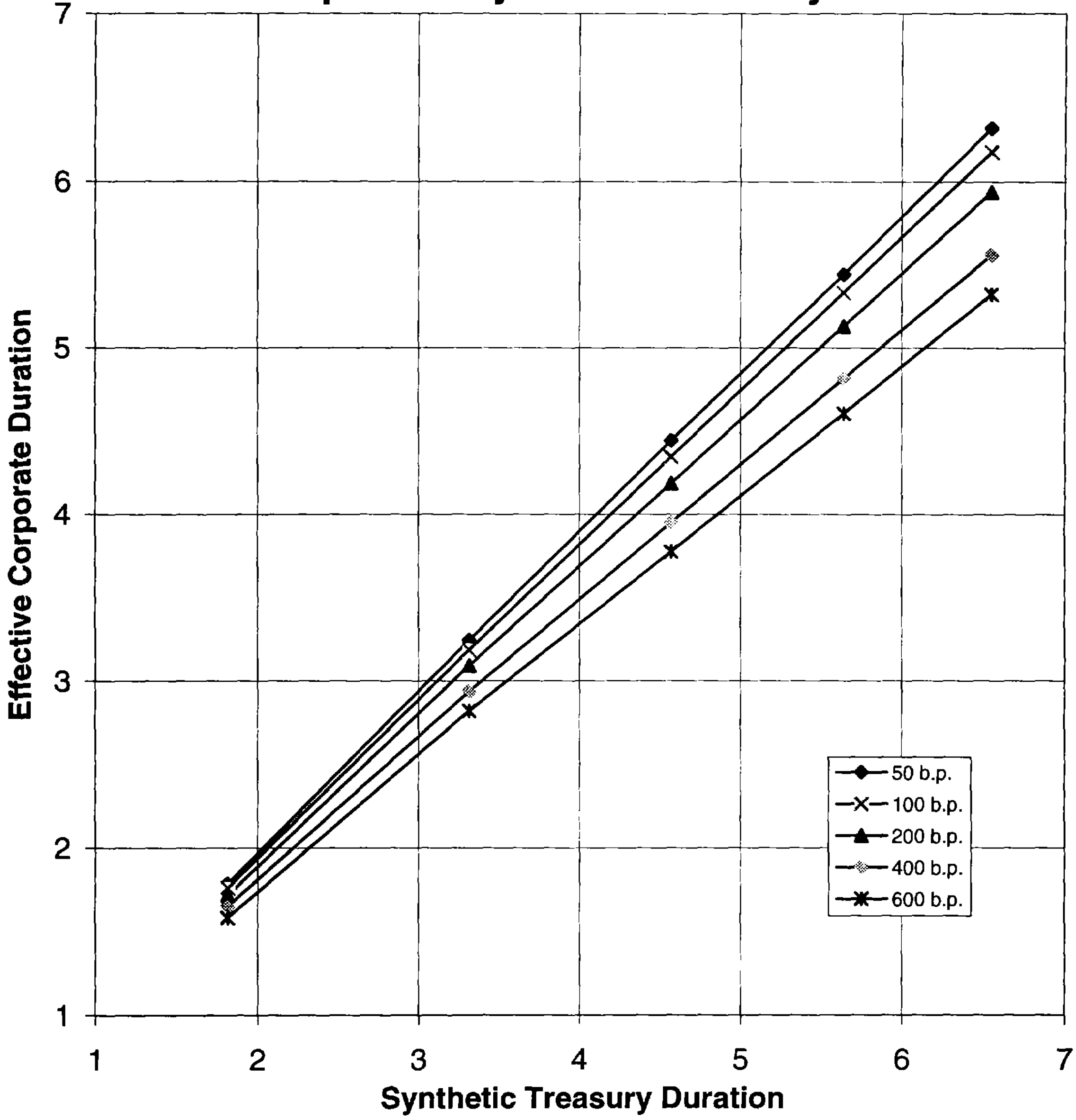
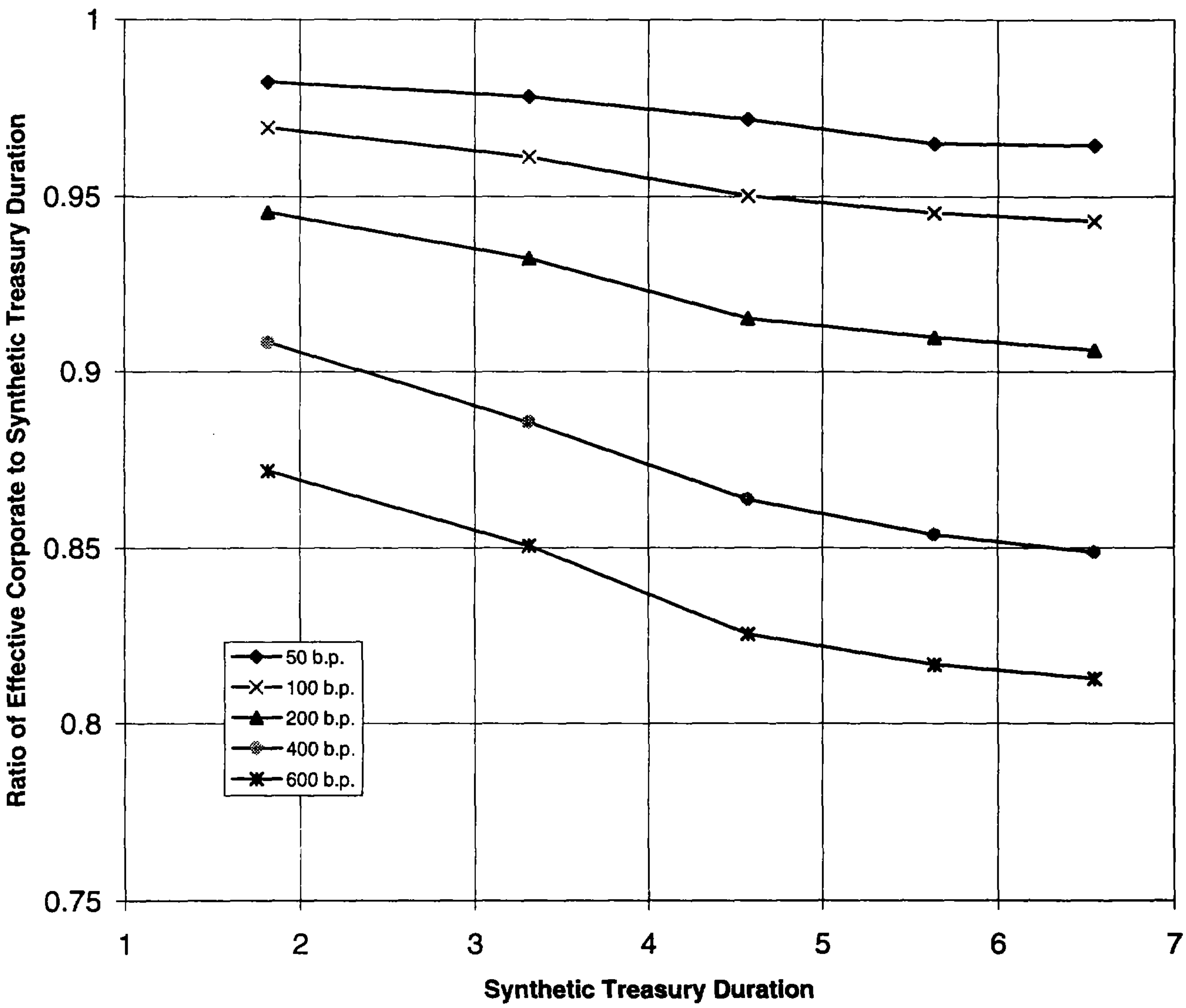


Exhibit 3
Corporate vs. Synthetic Treasury Duration with respect to Synthetic Treasury Yield



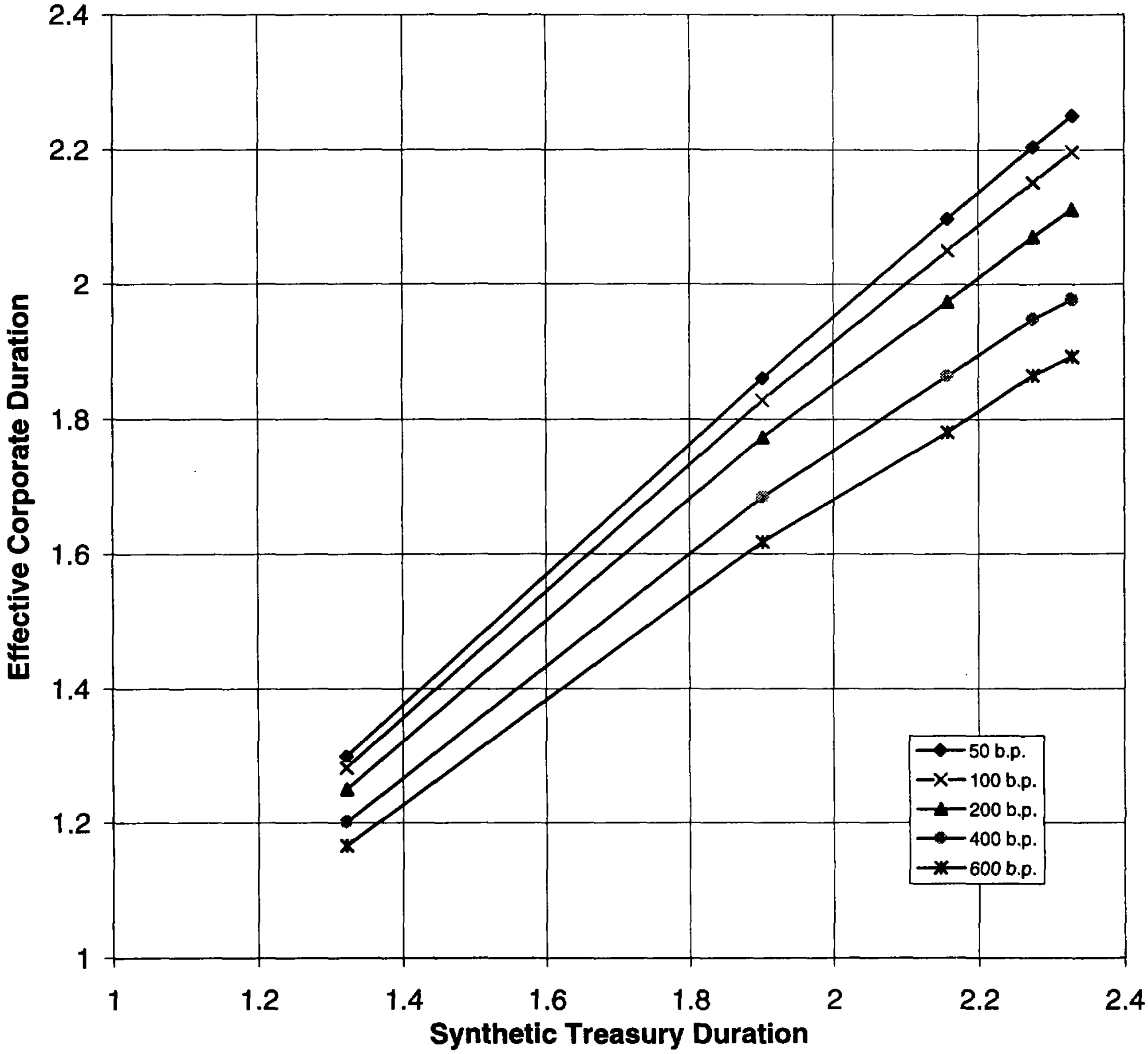
Note that the sample points represent bonds with maturities of 2, 4, 6, 8 and 10 years to maturity

Exhibit 4
Ratio of Corporate to Synthetic Treasury Duration with respect to Synthetic Treasury Yield



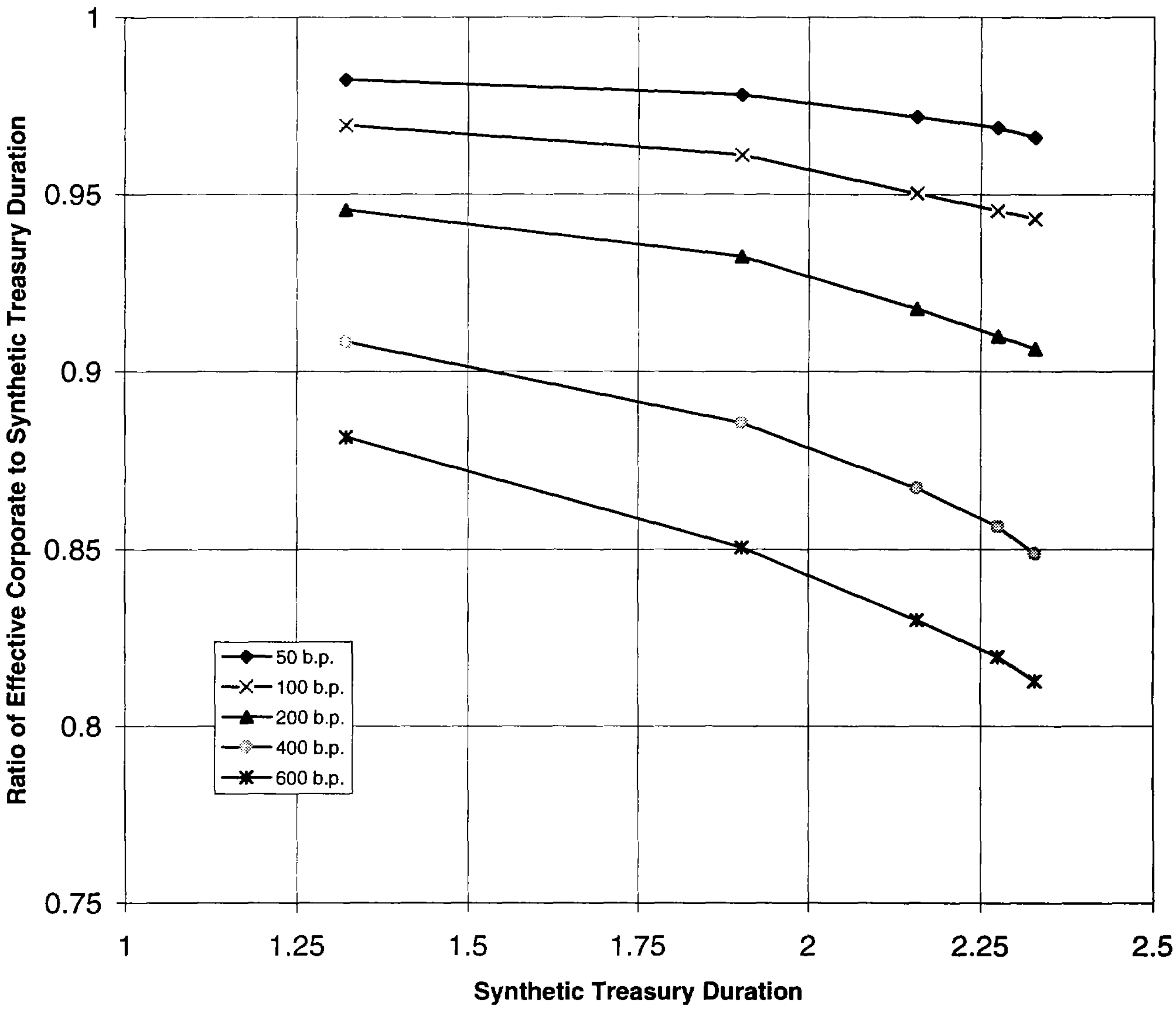
Note that the sample points represent bonds with maturities of 2, 4, 6, 8 and 10 years to maturity

Exhibit 5
Corporate vs. Synthetic Treasury Duration With Respect to Short Rate



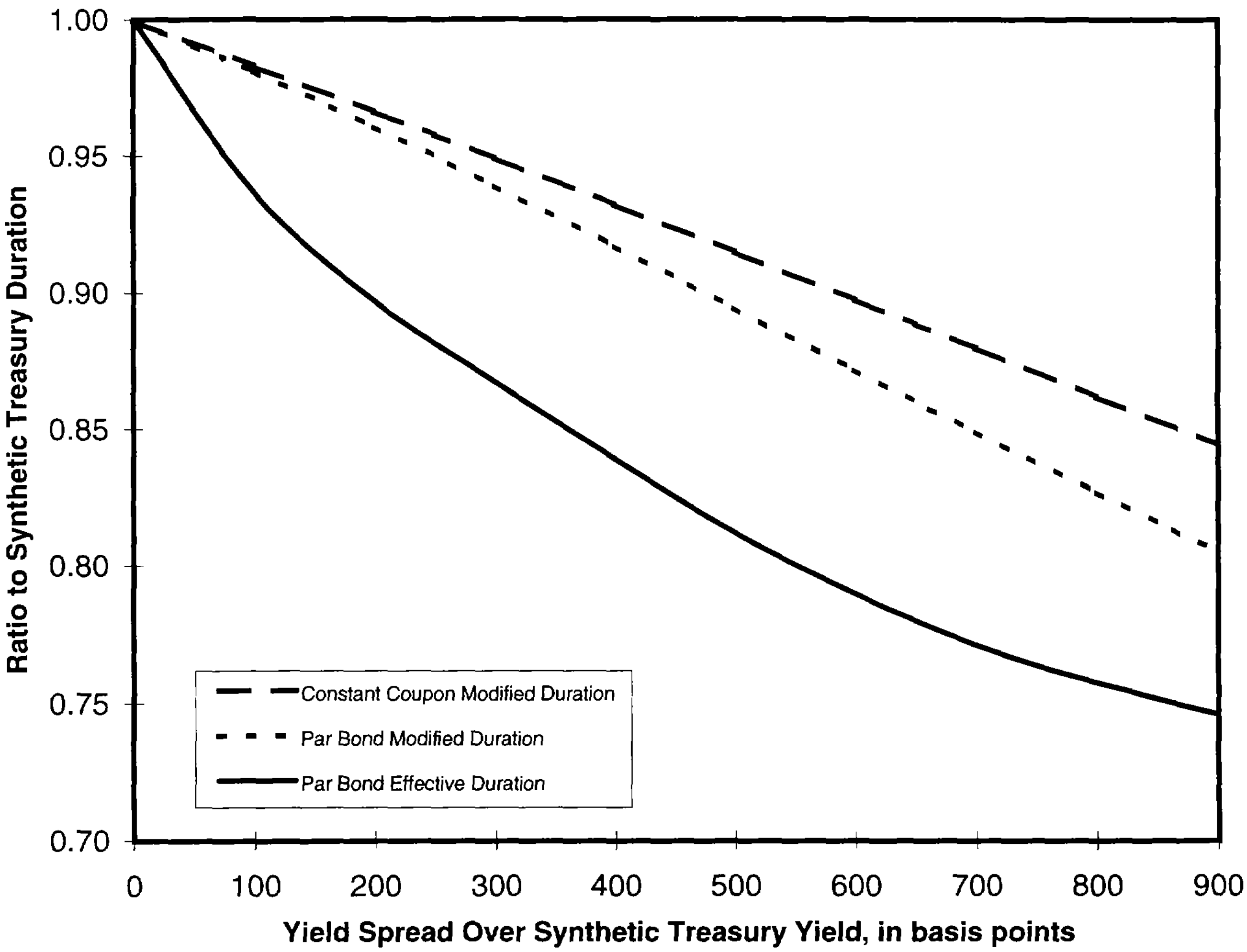
Note that the sample points represent bonds with maturities of 2, 4, 6, 8 and 10 years to maturity

Exhibit 6
Ratio of Corporate to Synthetic Treasury Duration with respect to Short Rate



Note that the sample points represent bonds with maturities of 2, 4, 6, 8 and 10 years to maturity

Exhibit 7
Various Methods of Calculating Corporate Bond Duration



10 year maturity bond, constant coupon is 5%

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