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Are Stable Agreements for Sharing International River Waters Now Possible?

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Proposed here is a new scheme for allocating international river water that accounts for the stochastic nature of water supply and the *dynamic nature* of its demand. The suggested scheme is expected to improve the efficiency of river basins' water allocation and the riparians' welfare.

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Summary findings

International river and lake basins constitute about 47 percent of the world's continental land area, a proportion that increases to about 60 percent in Africa, Asia, and South America. Because water is a scarce and increasingly valuable resource, disputes about water allocation within these basins often contribute to regional tensions and conflicts.

Many principles of international law have been developed to allocate water within a water basin and to prevent or resolve international water disputes. Unfortunately, they rarely are easy to apply and often are contradictory. Sharing river water is particularly difficult because the effects are one-way, with upstream-downstream supply disputes have been among the most common. Agreements about the allocation of river water often last only until the first drought, when reduced flow denies some their full shares.

Kilgour and Dinar develop a simple formal model of water allocation among states within a river basin. They analyze the model in the context of variable flow rates, to project the behavior of riparian states during periods of above-normal and below-normal flow.

Their objective: to understand when, where, and how much the economic interests of the states conflict, to develop principles guaranteeing efficient allocations of scarce water supplies, and to identify when stable (self-enforcing) allocation agreements are possible. They also consider the possibility of using alternative sources of supply and of accommodating growth in demand.

Satellite technology will soon dramatically improve the ability of riparian states to predict annual flow volumes. In addition, water basin authorities will have real-time data on riparians' water use. These developments will have important implications for the enforceability and the flexibility of river water allocation systems.

This model shows how flexibility can be used to construct more durable systems for sharing water among riparian states. The new allocation methods proposed here should contribute to the better management of scarce water supplies, a crucial issue in an increasingly thirsty world.

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International River Waters Now Possible?**

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D. Marc Kilgour & Ariel Dinar

1. Introduction

Many of the world's river and lake basins and ground water aquifers are shared by two or more countries. In a United Nations study, international river and lake basins were estimated to comprise about 47 percent of the world's continental land area (United Nations, 1978). In Africa, Asia, and South America, this proportion rises to at least 60 percent (Barrett, 1994). Of the 200 international rivers in a survey cited by Barrett (1994), 148 are shared by two countries, 30 by three, 9 by four, and 13 by five or more countries--as many as ten.

Most countries that share ownership of water resources also depend on those resources. Thus, the size of a country's share of a commonly-held water body has important direct and indirect effects on the country's well-being. If the common waters are rivers, these direct and indirect effects are unidirectional, and can create problems are particularly difficult to resolve.

The direct effects of water allocation in international river basins are crucial because water is an essential input for many productive processes, and alternative sources may be impractical. But neither is it possible to ignore the indirect outcomes (externality effects) of water allocation. These considerations may include not only reduction of water level and flow volume (because of excessive withdrawals upstream), but also deterioration of water quality (as a result of pollution in return flows upstream). It is hardly surprising that international rivers have often been the focus of regional tensions and conflicts.

Examples of upstream-downstream disputes (Vlachos, 1990; Barrett, 1994; Kirmany and Rangeley, 1994) include the proposal for out-of-basin diversion of the Mekong River (Thailand and Laos); the operation of the Farraka Barrage diversion of the Ganges (India and Bangladesh); the proposed desalination plant near Morales Dam on the Colorado River (Mexico and U.S.A.); and the recent dispute over the 1959 Nile water agreement (Egypt, Sudan, and now Ethiopia).

In the literature on international water disputes, several principles have been proposed to prevent or resolve disputes within an international water basin (Sofer 1992):

- The *Harmon Doctrine*, that a state has absolute sovereignty over the area of the river basin within it. This principle clearly favors upstream states.
- The principle of *Territorial Integration of all Basin States*. Symmetrically, this principle favors downstream states, to which it accords "equal" use, without regard to their contribution to the flow.

- *Equitable Utilization of River Water*. According to this principle, each riparian state can use the river water unless this use negatively affects other riparians.
- The *Mutual Use Principle*, that a riparian state may object to another riparian state's use of river water, unless it receives reasonable direct compensation.
- The *Linkage Principle*, that, as a condition for agreeing to a particular water allocation, a state may request compensation in a non-related area (such as a special trade agreement).

These principles all refer to appealing notions of fairness and rights; regrettably, however, contradictions and inconsistencies seem to be inevitable when they are put into practice.

There is no international law governing the allocation of waters in international rivers. Nonetheless, the 1966 Helsinki Document (Henkin *et al.* 1987) provides a list of considerations to be included in the determination of a riparian state's water allocation. These include:

- Geography - the state's land area;
- Hydrology - the state's relative water contribution;
- Climate;
- History - the past and present use of water;
- Economic and Social needs of the state;
- Cost of obtaining water from alternative sources;
- Availability of other resources;
- Efficiency of water use;
- Capability of compensating other states;
- Pareto Optimality.

(The last point is that no allocation should be considered unless it is Pareto-efficient, which means that any alternative allocation that increases the welfare of any individual state must reduce the welfare of at least one other state.)

Yet even the apparently straightforward ideas of the Helsinki Document can be difficult to apply. One reason is that most of them are not constant over time--they change with flow in the river basin, and with national and international events and trends.

Although conditions in a river basin, and in the riparian countries, are usually characterized by constant change, traditional water allocation schemes make no effort to accommodate changes. The literature (Food and Agricultural Organization 1978, 1984) lists 3707 international water utilization agreements, virtually all of which exhibit this inflexibility.

Sometimes, renegotiation of in-force river-water allocation agreements takes place. For instance, the Nile River is shared by nine countries (Ethiopia, Zaire, Tanzania, Burundi, Uganda, Rwanda, Kenya, Sudan, and Egypt), but only Egypt and Sudan were Nile water recipients according to the 1929 water-sharing agreement. Egypt was to receive 48 Million Cubic Meters

(MCM) per year, and Sudan just 4 MCM. Thirty years later, renegotiation produced the 1959 Nile Treaty, in which Egypt was allocated 55.5 MCM per year, and Sudan 18.3 MCM. In turn, this agreement has recently been challenged by other riparian countries, who are interested in their own shares of Nile water (Whittington *et al.*, 1994).

But political instability over water supplies, including possibly war, is always possible. Renegotiation of water-sharing agreements typically takes place in a tense atmosphere, sometimes exacerbated by threats and hostile actions. Two of the river basins in which riparians have recently objected to current water allocations--the Nile (Whittington *et al.*, 1994) and the Jordan (Wolf, 1992)--lie in a politically volatile region--the Middle East.

Another aspect of water-sharing agreements that can lead to acrimony is enforcement. A state may have agreed to limit itself to a certain fixed volume of river water, but who or what is to stop it from increasing its consumption? Such unilateral violations of an agreement could take place openly or clandestinely. In fact, actual violations may be less damaging than the fear that other states are secretly engaging in routine violations.

In general, enforcement--comprising activities and structures designed to prevent or deter individual decisionmakers from acting in their immediate interests rather than complying with a specific regime--can be difficult and expensive. This is particularly true for environmental enforcement in an international context, where the prerequisites for domestic success, reliable information about behavior and credible threats of sanctions or penalties, are especially difficult to attain (Kilgour *et al.*, 1992; Kilgour, 1994).

Fortunately, new technology will soon alleviate information problems for international agreements about the sharing of river water. Data on water levels gathered by satellite will be made available publically, permitting flow volumes and withdrawal rates to be calculated. (SADC, WB, and WMO, 1993) In addition, much more accurate forecasts of annual flow volumes will become available to all riparians.

Considerable political stability may be achieved simply through this sharp increase in the volume and accuracy of flow information. One likely consequence is much simpler and more effective enforcement. Furthermore, the availability of accurate one-year forecasts will play a fundamental role, permitting questions related to the stochastic nature of the flow to be sidestepped. There is no need to ensure that allocations are "on average" fair or welfare-maximizing if they can be modified annually so that they possess these properties every year.

Annual adjustment of allocations is certainly a new and promising idea. Studies by the UN Food and Agriculture Organization (1978, 1984) indicate that most water allocation agreements include static descriptions of both the supply of and the demand for water. For example, the 1959 Nile Agreement, predicated on the construction of the Aswan Dam, explicitly incorporates the assumption that "there will be an annual mean flow at Aswan of 84 MCM." Losses were estimated at 10 MCM, and the entire balance was confidently allocated to Egypt and Sudan with little or no explicit provision for years of below-normal flow.

The annual volume of river water may conceivably be constant in a few cases, but usually it is stochastic. River flow is well known to be affected by weather, for example, and fluctuations of 25% above or below the mean annual flow volume are quite common. As well, some authorities have predicted that climatic changes and long-term drought events will alter mean annual flow by up to 70% in some rivers (ILRI, 1993). Equally, the demand for river water may change over time. Dynamic demand patterns may reflect population growth, changes in national priorities, long-term planning, adoption of new technologies, and changes in tastes.

A water-sharing scheme that accounts for the stochastic nature of water supply and the dynamic nature of water demand will almost certainly produce more political stability. Destabilizing shortages are already a risk in river basins where average water consumption approaches average flow volume. In addition to reducing political tensions, the ability to match allocations to current conditions may reduce regional water-allocation transactions costs, allow states to plan water-related investments more effectively, and increase regional social welfare.

The purpose of this paper is to characterize stable water-sharing agreements in the presence of accurate foreknowledge of annual supply, identify allocations that are Pareto-efficient, and show how flexible rather than fixed allocation schemes reduce costs. Section 2 discusses the definitions and assumptions on which the water-allocation models are based. Section 3 provides a rather general analysis, and some examples, of how two-state water-allocation agreements can be structured optimally. Section 4 illustrates how the method could be applied to certain three-state problems, and Section 5 provides a summary evaluation of the effects of stochastic flow on the allocation structures proposed here. Some conclusions are gathered in Section 6.

2. Modeling Definitions and Assumptions

The models used below to analyze water sharing among states in a river basin treat states' internal economies and their external economic relations as independent. Internal models represent states' individual needs for water as well as alternative (out-of-basin) sources of water supply, if any; each state must have its own internal model. In contrast, only one external model covers all states--it is geographic and not economic, and represents the relevant aspects of the river basin, i.e. the relative positions of the states with respect to river flow.

2.1 Individual State Models

A model of an individual state describes the state's internal need for river water. It also describes the state's opportunities (if any) to obtain water from sources external to the river basin.

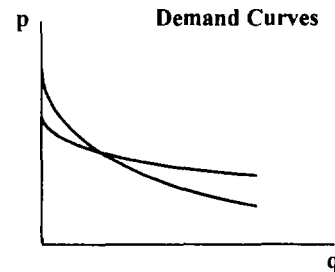
State i 's water demand function characterizes its need for water by showing the amount of water that would be purchased within the state, denoted q_i , as a function of the (internal) unit price of water, p_i . As usual, the demand function for state i is given in the form

$$p_i = f_i(q_i) \quad .$$

For technical reasons, this report will concentrate on demand functions of the exponential form,

$$p = a e^{-bq} \quad .$$

Two demand curves of this form are shown in the diagram at the right. The parameter $a > 0$ equals the value of p at $q = 0$, and the parameter $b > 0$ measures the rate of decrease of p as q increases. Of the two curves at the right, the one with the larger value of a also has the larger value of b . Water-sharing problems involving two states with demand functions similar to these are analysed in section 3.



A state may also have the option of purchasing water from an outside source, i.e. water that does not originate in the river basin. In some models discussed below, a given state may be able to purchase water at a fixed price per unit, in any amount up to a fixed ceiling. Generalizations to variable price purchases are possible, but have not been attempted at this stage.

The general objective of the water-sharing schemes designed below is to determine ways to share water that maximize welfare insofar as possible. Only two commodities are included--water and money. A state's total welfare is considered to equal its consumers' surplus, calculated according to its water demand function, plus its net money receipts with respect to water. Thus, if state i benefits from total water consumption q_i , its total welfare is

$$\begin{aligned} WELFARE_i &= CONSUMPTION SURPLUS_i + NET TRANSFERS_i \\ &= \int_0^{q_i} f_i(q) dq + X_i \quad , \end{aligned}$$

where X_i , the net transfer to state i , equals the net receipts from other within-basin states for the sale or purchase of river water, reduced by the total of any payments for out-of-basin water.

The exponential form of the demand functions given above has certain implications for a state's internal water economy.

- (1) States have no absolute minimum water requirements, i.e. there is no demand level $q_m > 0$ below which price becomes infinite; and
- (2) No state's demand is ever saturated, i.e. in the absence of other arrangements, all available

water will be purchased and consumed internally, although the purchase price may be very low.

The second feature may not be a practical restriction in view of recent experience in many river basins where water demand consistently exceeds flow. The first feature would not normally hinder the model's ability to represent the situation of source states (see section 2.2), which may simply withdraw as much water as they need. For non-source states, this assumption is innocuous if internally-sourced water, which is not included in the flow model of section 2.2, is adequate to meet any such absolute requirements. (Internally-sourced water is within-basin water available to the state in question but not received from any upstream state. If not consumed internally, it would be added to total river flow.) Another way to interpret the model is to measure only those flow volumes in excess of the minimum necessary to support the river ecology. Taking this approach means that these models can never imply the loss of ecological communities; at the same time, however, it renders the models incapable of dealing with flows that are so low as to threaten such losses.

2.2 Flow Model

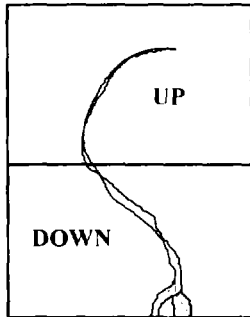
The flow model represents the geography of water flow within the river basin. Essentially, the flow model identifies the states involved, and specifies the water-flow relationship of each to the others. In the simplified models used here, all states are either source states or non-source states.

A source state is a state from which water flow originates; a non-source state is a state which receives its water from one or more other states. In the models used here, each source state passes water to exactly one non-source state; each non-source state receives water from one or more states (which may be source or non-source states). Thus, in the Graph Theory sense, a source state has no predecessors and one successor, and a non-source state has at least one predecessor. It will also be assumed that there are at least two states (otherwise there would be no sharing problem) and that there is exactly one non-source state with no successors; this is called the outlet state.

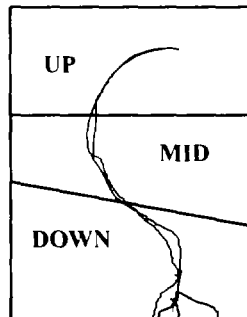
Source states have a special advantage, in that they have the first opportunity to withdraw water from the river system. Indeed, for technical convenience, and to simplify the diagrams, all of the technical calculations below are carried out according to the Harmon Doctrine--each state owns all the water it receives--but, as noted in Section 3.2, below, this assumption is not essential by any agreed-upon *status quo* water rights.

It is clear that the new capabilities for enforcement under discussion here are not helpful in geographic situations in which a portion of the river itself constitutes an international boundary. Assuming that this does not occur, there can be only one two-state flow model, represented in the diagram on the left, below. Several models incorporating this geography are presented and analyzed in section 3. Likewise, there are two three-state models, the I-geography (shown in the middle diagram below), and Y-geography (shown at the right). The I-geography models the

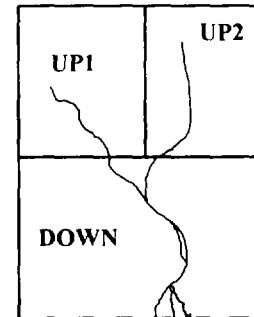
Jordan River basin, and the Y-geography is a very rough model of the Nile. A simple three-state I-geography model is presented and analyzed in section 4.



Two-State Geography



Three-State I-Geography



Three-State Y-Geography

3. Two-State Models

Throughout this section there are two states, called UP and DOWN, which share a river basin, as shown schematically by the diagram on the left, above. Their demands for water are given by

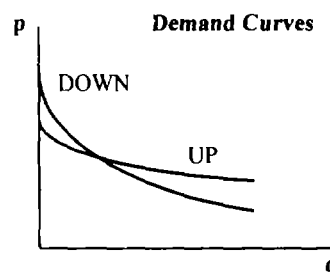
$$UP: \quad p_U = a_U e^{-b_U q_U} \quad ;$$

$$DOWN: \quad p_D = a_D e^{-b_D q_D} \quad .$$

The example to be analyzed in detail in section 3.2 has the following parameters:

$$\text{Example 1: } a_U = 6.0, b_U = 0.5; \\ a_D = 8.0, b_D = 1.0$$

Example 1 demand curves are sketched in the diagram on the right.



3.1 Specification of the Two-State Problem

States UP and DOWN are to share the river water, which originates entirely within UP, in some optimal way. Essentially, sharing takes place when UP does not consume the entire volume, instead passing some of it on to DOWN. DOWN, in turn, compensates UP with a money payment. In accordance with the general objectives of this research, what will be determined below is a schedule for sharing water, and making payments, that depends explicitly on total flow volume. Through sections 3.1 and 3.2, the problem is simplified by assuming no out-of-basin sources. This aspect will be included in section 3.3.

Let Q denote the total flow from UP to DOWN if there were no withdrawals by UP. Then UP's withdrawal amount, q_U , must satisfy

$$0 \leq q_U \leq Q \quad .$$

and DOWN's withdrawal, q_D , must satisfy

$$0 \leq q_D \leq Q - q_U \quad .$$

The non-saturation assumption is relevant at this point. First, it implies that DOWN will withdraw all available water, in other words that

$$q_D = Q - q_U \quad .$$

But it also implies that, in the absence of other considerations, UP will do likewise.

Thus, $q_U < Q$ only if UP receives some compensation, which is modeled here as an amount, x , transferred from DOWN to UP. Such a transfer is feasible provided the values of Q and q_U are known to be public knowledge. But notice that the amount transferred, x , is in fact the total price to DOWN of the water volume $Q - q_U$. So what is required here are two functions,

$$q(Q) \text{ and } x(Q),$$

such that, if total volume is Q , then the withdrawal by UP of $q_U = q(Q)$ units of water (leaving $Q - q(Q)$ units for DOWN), and the payment of $x(Q)$ by DOWN to UP, maximizes welfare in some sense.

In fact, the problem is largely solved as soon as all schedules of withdrawals and payments,

$$(q(Q), x(Q)) \quad ,$$

that are efficient in the Pareto sense, have been identified. For any such schedule, it is guaranteed

that any other schedule that offers a higher level of welfare to one state must offer a lower level of welfare to the other.

In accordance with the definitions above, the welfare of UP is given by

$$W_U(q_U, x) = \int_0^{q_U} f_U(q) dq + x = \frac{a_U}{b_U} [1 - e^{-b_U q_U}] + x ,$$

and the welfare of DOWN by

$$W_D(q_D, x) = \int_0^{q_D} f_D(q) dq - x = \frac{a_D}{b_D} [1 - e^{-b_D q_D}] - x .$$

For Example 1, these functions are

$$W_U(q, x) = 12 [1 - e^{-q/2}] + x ,$$

$$W_D(q, x) = 8 [1 - e^{-(2-q)}] - x .$$

In section 3.2, the two-state problem will be solved in general, and for the specific case of Example 1. Section 3.2 will also introduce the closely related Example 2. Example 3, introduced in section 3.3, is identical to Example 1 except that UP is allowed the opportunity to purchase up to a fixed amount of water, at a fixed price, from an outside source. In this case, the function W_U must be modified to reflect that any payments made by UP for out-of-basin water reduce UP's net proceeds, and therefore UP's welfare.

3.2 The Two-State Problem with No Out-of-Basin Sources: Solution and Examples

We now proceed to the identification of optimal schedules, (q, x) , for the fundamental two-state problem introduced above. As noted in section 2.2, we assume for convenience that the source state, UP, "owns" the water, and therefore has the option of withdrawing the entire flow volume, Q . We also assume that, currently, there are no direct payments for water (from DOWN to UP). Thus we take $(q, x) = (Q, 0)$ to be the Status Quo schedule. Note that the technical calculations are unchanged under different models of ownership of river water; as well, current water-related payments could be included in the model by using a Status Quo schedule, (q_0, x_0) , with $0 < q_0 < Q$ and $x_0 \neq 0$.

Our first objective is to identify all schedules that are at least as preferable for both states as the Status Quo. To do this, we will identify in turn all schedules (q, x) such that $W_U(q, x) \geq W_U(Q, 0)$, and then all schedules (q, x) such that $W_D(q, x) \geq W_D(Q, 0)$. Any schedules satisfying both these conditions must be at least equally preferred to the Status Quo by both states. Note that this intersection must be non-empty, because it must contain the Status Quo schedule, $(Q, 0)$.

This process is represented in Figure 1, which shows the Status Quo as an open circle within the (two-dimensional) space of all schedules. The two thick lines emanating from the Status Quo are the *Basic Indifference Curves*. They represent all schedules indifferent to UP (lower line) and indifferent to DOWN (upper line). Figure 2 shows the Basic Indifference Curves for Example 1, for the particular case $Q = 1$.

Figure 1 shows not only the Basic Indifference Curves, but also other indifference curves of both UP and DOWN. These are obtained by shifting either of the Basic Curves directly up or down. A few of UP's and DOWN's indifference curves, obtained in this way, are shown in Figure 1 as thin curves parallel to the Basic Indifference Curves.

Note that UP always prefers higher indifference curves, and DOWN always prefers lower curves. This observation implies that any schedules that both sides prefer to the Status Quo lie above UP's Basic Indifference Curves, and below DOWN's. This is the situation illustrated in Figure 1. Figure 2 shows the Basic Indifference for Example 1 when $Q = 1$; clearly, such jointly preferred schedules do exist in this case. If UP's Basic Indifference Curve were to lie entirely above DOWN's, then there would be no jointly preferable schedules--the states could not do better than to accept the Status Quo--UP gets all the water; DOWN gets no water, but pays nothing. Example 2, below, will be used to show that it is possible that there are no schedules jointly preferred to the Status Quo.

Thus, the Status Quo, $(Q, 0)$, is optimal when UP's Basic Indifference Curve lies entirely above DOWN's. But what if there are many schedules jointly preferred to the Status Quo, as suggested by Figure 1? In fact, Figure 2 makes clear that there could be a jointly preferred schedule (q, x) for every $q \in [0, Q]$. With such a profusion of schedules available, can some be better than others?

The answer lies again in indifference curves. Begin in Figure 1 at any schedule slightly above and to the left of the Status Quo. By moving upward and to the left, and staying between the two indifference curves passing through the initial schedule, new schedules preferred by both sides are obtained. Shifting in this way to a jointly preferred schedule is possible as long as DOWN's indifference curve is more steeply sloped than UP's. Likewise, any schedule lying between the two Basic Indifference Curves in Figure 1, with a very small value of q , can be improved by moving downward and to the right, as long as UP's indifference curve at the starting point is steeper than DOWN's. Clearly any schedule between the two Basic Indifference Curves can be jointly improved in this way, except those where the two states' indifference curves have equal slopes.

The equal slope condition defines the *Contract Curve*, which constitutes the set of all schedules that are

- (1) preferred or indifferent, for both states, to the Status Quo; and
- (2) Pareto-efficient within the set of schedules jointly preferred to the Status Quo.

The latter property means that whenever an initial schedule lies above UP's Basic Indifference Curve and below DOWN's, then another schedule preferred by both states exists if and only if the initial schedule is not on the Contract Curve. Thus the schedules on the Contract Curve solve the problem--they are improvements on the Status Quo that cannot themselves be improved upon. The idea of a Contract Curve goes back to Edgeworth, and is related as well to the von Neumann-Morgenstern Stable Set.

Remarkably, in water-sharing problems of this type, the Contract Curve is always a vertical line segment, as shown in Figure 1. Define q_0 , x_m , and x_M so that the points on the Contract Curve constitute the set

$$\{(q, x) : q = q_0, x_m \leq x \leq x_M\} .$$

Figure 1 shows the locations of q_0 , x_m , and x_M in general. Note that all three are functions of Q .

It is in general possible to determine specific functional forms for q_0 , x_m , and x_M , provided the demand functions are given explicitly. For instance, for two-state exponential demand problems,

$$q_0 = \begin{cases} 0 & \text{if } Q \leq \frac{1}{b_D} \ln \left(\frac{a_D}{a_U} \right) \\ Q & \text{if } Q \geq \frac{1}{b_U} \ln \left(\frac{a_U}{a_D} \right) \\ \frac{b_D Q - \ln \left(\frac{a_D}{a_U} \right)}{b_U + b_D} & \text{otherwise} \end{cases}$$

For Example 1,

$$q_0 = \frac{2}{3} \left[Q - \ln \left(\frac{4}{3} \right) \right]$$

provided $Q \geq \ln(4/3) \approx 0.288$, and $q_0 = 0$ otherwise.

The Contract Curve division of water volume for Example 1 is shown in Figure 3. Note that, for each value of Q , the amount of water to be shared is represented by the 45° line. Thus, Figure 3 shows how the volume of water consumed by UP, and the volume received by DOWN, depend on the total flow, Q . In this case, the division is represented by a straight line which intersects the 45° line at $Q = \ln(4/3)$, the threshold level at which UP's optimal consumption level, q_0 , drops to zero. Thus, if the total volume is below this threshold, all of the water is sold by UP to DOWN. If the volume is above this threshold, the amount sold equals a fixed amount, $\ln(4/3)$, plus $2/3$ of the balance. This is implied by the formula above, and is illustrated by the slope ($1/3$) of the division line in Figure 3.

The Contract Curve implies not just a water division, but a "bargaining range" for the amount transferred from DOWN to UP, $[x_m, x_M]$. The quantities x_m and x_M , the endpoints of the Contract Curve shown in Figure 1, are of course functions of Q . Figure 4 shows this bargaining range for Example 1. The sharp changes in slope of both the minimum and the maximum transfers occur at exactly $Q = \ln(4/3)$, the threshold value of Q below which UP consumes no river water. In general, the bargaining range can be calculated explicitly provided the demand functions are known.

At this stage, Example 2 can be mentioned briefly. Example 2 is identical to Example 1 except that UP's and DOWN's demand functions are reversed. Thus

Example 2: $a_U = 8.0$, $b_U = 1.0$; $a_D = 6.0$, $b_D = 0.5$.

The solution of Example 2 is straightforward, and will not be described here. However, the optimal water allocations to UP and DOWN, shown in Figure 5, do have an important property. Note that, at low volumes (small values of Q), all of the water is allocated to UP. In this interval, the Status Quo schedule, $(Q, 0)$, is in fact optimal--there are in fact no schedules jointly preferred or indifferent to it. Thus, as noted earlier, it is possible that there is no transfer of water from UP to DOWN, and no payments from DOWN to UP. In the case of Example 2, it occurs when total volumes are extremely low, because very small quantities of water are more valuable to UP than DOWN.

3.3 The Two-State Problem with Out-of-Basin Sources: Solution of an Example

The extended two-state problem studied here, called Example 3, is identical to Example 1, analyzed in section 3.2, except for one provision--the upstream state, UP, has the option of purchasing water from an out-of-basin source. The amount of water that can be purchased is limited and the (unit) purchase price is fixed. **Example 3** is thus identical to Example 1, except that UP has the option to purchase up to 1 unit of water, at a cost of 2 per unit.

Before a schedule for sharing water and making compensating transfers can be determined, UP's optimal policy for purchasing water must be determined. If UP purchases m units and extracts q units from the river flow, UP's total welfare is

$$W_U(q, m, x) = 12 \left[1 - e^{-(q+m)/2} \right] - 2m + x .$$

When this expression is maximized with respect to m , UP's optimal purchase policy is determined, as follows:

$$m^*(q) = \begin{cases} 1 & \text{if } q \leq \ln(3) - 1 \\ \ln(3) - q & \text{if } \ln(3) - 1 \leq q \leq \ln(3) \\ 0 & \text{if } q \geq \ln(3) \end{cases}$$

Thus, UP should purchase as much external water as possible, subject to a maximum of $\ln(3) - q$ units. It is easy to check that, when $q_U = \ln(3)$, $p_U = 2$. UP's purchasing policy is illustrated at right.

Substitution of $m = m^*(q)$ into $W_U(q, m, x)$ yields a welfare function for UP that depends only on q and x . This welfare function,

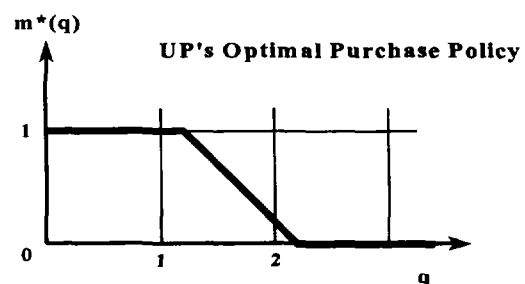
$$W_U^*(q, x) = W_U(q, m^*(q), x) ,$$

incorporates the assumption that UP chooses its optimal purchase policy. When combined with DOWN's welfare function in the usual way, it produces a schedule that takes into account UP's out-of-basin purchase option.

The resulting schedule is shown as Figure 6. Note the following properties:

- (1) when total volume is very low, all river water is sold by UP to DOWN--in fact, the threshold volume below which UP consumes no river water is much higher as a result of UP's out-of-basin purchase option;
- (2) when total flow is somewhat greater, but still low, the base volume plus 1/3 of the excess is sold to DOWN;
- (3) at a somewhat higher threshold, DOWN's volume is fixed--it does not increase when total volume increases; and
- (4) when total volume is very large, UP again extracts 2/3 of the incremental flow.

A comparison of Figures 3 and 6 is instructive. Note that in both cases DOWN receives all of the river water when the total volume is extremely low. But the threshold level at which UP starts to consume river water is considerably higher in Figure 6. In general, UP receives a



reduced allocation of river water in Example 3, except that when water volume is very high ($Q \geq \ln(36) \approx 3.584$) the allocation to UP is unchanged--i.e the divisions in Examples 1 and 3 coincide. Of course, when flow volumes are very high, UP's outside purchase option is not used because the "internal" price of water is much lower than the fixed "external" (purchase) price.

Comparison of the bargaining ranges with (Figure 7) and without (Figure 4) out-of-basin purchase is also instructive. For very large volumes ($Q \geq \ln(36)$ again), the ranges are identical. For extremely small volumes ($Q \leq \ln(4/3) \approx 0.288$), the upper boundary of the range is identical, but the lower boundary moves down when the option is available. This reflects that, because of its option, the river water is not as valuable to UP. Over most of the intervening values of total flow, UP sells more of the river water to DOWN, and receives more for it--although the "unit price" actually declines. (Since the Contract Curve method determines a specific volume of water to be sold by UP to DOWN, but determines only a range of payments, only an average or expected unit price can be calculated.) Nonetheless it is clear that the out-of-basin purchase option benefits both sides at all volumes up to the level at which there is so much river water that the optimal schedule is the same in both examples.

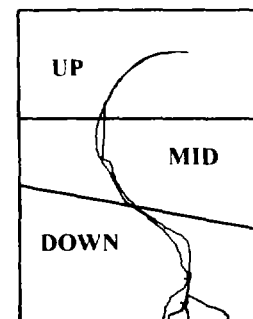
This completes the analysis of the two-state problem. It is important at this point to review the properties of the solution that have been identified. Given the states' water demands, it is possible to determine a schedule which gives, *for any level of total flow*, the optimal consumption level for each state. Furthermore, a bargaining range, specifying bounds on the amount of money to be transferred to balance this transaction, can also be determined. Thus, under this system states can plan for a particular volume of water, and a particular level of transfers, as soon as the total annual flow can be accurately predicted. The next section shows similar results for a three-state problem.

4. Three-State Models

As noted in section 2.2, there are two distinct three-state geographies, the I-geography and the Y-geography. In the following, only I-geography models will be considered; models based on Y-geography will be addressed in future work.

4.1 The Three-State I-Geography Problem with No Out-of-Basin Sources

In this section, Example 4, a three-state generalization of Example 1. The three states, called UP, MID, and DOWN, are assumed to exhibit I-geography, as defined in section 2.2 and



Locations of UP, MID, and DOWN

illustrated by the sketch at the right. River flow begins in the source state UP, and then passes through MID to the outlet state DOWN.

The specific water demands for two of the three states are identical to those of the two states of Example 1. Specifically, the water demands in **Example 4** are

$$UP: \quad p_U = 6 e^{-q_U/2} \quad ;$$

$$MID: \quad p_M = 8 e^{-q_M} \quad ;$$

$$DOWN: \quad p_D = 14 e^{-2 q_D} \quad .$$

Thus, UP's demand in Example 4 is the same as in Example 1, and MID's demand in Example 4 is identical to DOWN's in Example 1. A further property shared with earlier examples is that the further downstream a state, the greater its need for small amounts of water, and the less its need for large amounts. Thus DOWN in Example 4 is even more desirous of low volumes of water than DOWN in Example 1. As noted above, this feature tends to assure that, when total volume is very low, all of the water is sold by upstream states to downstream states.

A water-sharing and compensation schedule must specify how much water is to be consumed, and how much compensation is to be received (or paid), by each state. Since there are three states in this problem, a schedule is a 6-tuple

$$(q_U, q_M, q_D; x_U, x_M, x_D)$$

satisfying

$$q_U \geq 0, q_M \geq 0, q_D \geq 0 \quad ,$$

$$q_U + q_M + q_D = Q \quad ,$$

and

$$x_U = x_M + x_D \quad .$$

In this notation, positive values of x_D are treated as *payments made* by DOWN, positive values of x_M as *payments made* by MID, and positive values of x_U as *payments received* by UP.

4.2 An Example Three-State I-Geography Problem with No Out-of-Basin Sources

The determination of the Contract Curve and the Bargaining Range for the problem introduced in section 4.1 is similar to the calculations in section 3.2, and will not be given here in detail. It can be shown, for example, that when all three states are consuming positive amounts of river water, then the amount received by UP is

$$q_U = \frac{b_D b_M Q - b_D \ln(a_M/a_U) - b_M \ln(a_D/a_U)}{b_D b_U + b_U b_M + b_M b_D}$$

Furthermore, the amounts to MID and DOWN are symmetric: they can be obtained from the expression for q_U by applying (once or twice) the cyclic permutation $(U, M, D) \rightarrow (M, D, U)$.

A complete schedule for the three states is shown in Figure 8. For very low volumes, all the water is consumed by DOWN. As volume increases, the water is shared between DOWN and MID. At large volumes, all three states consume positive quantities of water. Note that the line dividing the shares of DOWN and MID has a sharp corner at $Q = \ln(4/3) + \frac{1}{2}\ln(7/3) \approx 0.711$, which is the threshold above which all three states consume water.

The complete compensation schedule for the three states is represented in Figure 9. Of the four lines,

- the lowest (on the left) is the maximum amount that MID can be required to pay--note that it equals zero at very low volumes, where MID receives no water;
- the lowest (on the right) is the minimum amount that UP can receive, which equals the minimum total payment of DOWN and MID;
- the second highest is the maximum amount that DOWN can be required to pay; and
- the highest is the maximum amount that UP can receive, which of course equals the total of what DOWN and MID can be required to pay. Note that, at very low volumes, the maximum paid by MID is zero, so the maximum amount that UP can receive equals the maximum amount that DOWN can be required to pay.

In fact, the lowest three lines contain all of the information needed to construct the Bargaining Zone for any value of Q .

Figure 10 illustrates the Bargaining Zones for three specific values of Q . The shaded triangles represent all feasible combinations of amounts paid by MID (vertical axis) and DOWN (horizontal axis). Note that the minimum to be received by UP determines the 45° line that forms the lower left-hand boundary of each Bargaining Zone. Note that x_{MID} may be negative or zero at $Q = 0.25$, which is within the interval where all of the water is consumed by DOWN. In this

interval, DOWN pays a positive amount, UP receives a positive amount, and MID receives either a positive amount or zero, depending on the specific bargaining outcome achieved. For the other values of Q illustrated, x_{MID} may be positive, negative, or zero.

In summary, for Example 4 we have determined a complete schedule for consuming water, and paying appropriate compensation, for each possible value of Q , the total volume of water available. Because this schedule is essentially the Contract Curve for the problem, a Pareto-efficient schedule of water allocation and payments is guaranteed. As previously, the Contract Curve determines water volumes precisely; it does not, in general, determine compensation levels exactly, but rather establishes bounds that create a Bargaining Zone. A similar calculation could be carried out for more complex models, in which one or more of the states has an option to purchase out-of-basin water, similar to Example 3.

5. The Consequences of Variations in Flow Volume

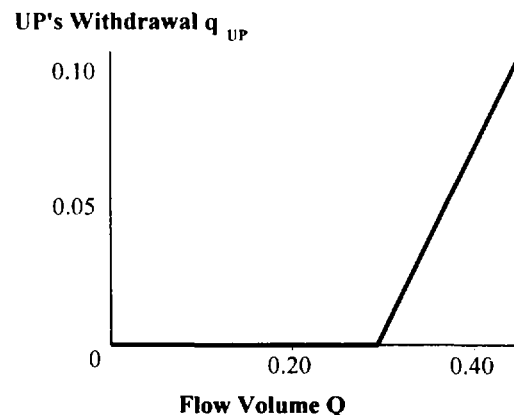
A simplified model of stochastic flow will now be used to assess the affects on welfare of the allocation scheme proposed in the preceding sections. The object is to gain some insight into how much benefit can be achieved, first, by imposing a fixed allocation in conditions of annually varying flow, and, second, by adjusting each year's allocation so that it is optimal with respect to that year's flow volume.

All calculations will be carried out for Example 1 of Section 3.2. The measure of benefit used will be total regional welfare. In Example 1, if total flow volume is Q and the withdrawal-payment schedule is (q, x) , then the total welfare of states UP and DOWN is

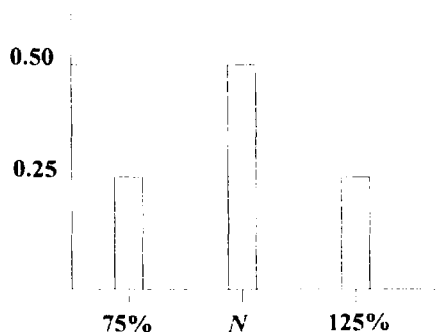
$$W(q) = W_U(q, x) + W_D(q, x) = 20 - 12e^{-q/2} - 8e^{-Q \cdot q} .$$

Note that total welfare depends only the allocation amounts q (to UP) and $Q - q$ (to DOWN), and not on x , the payment level. This is convenient, because the allocations are precisely determined under this system, whereas all that is known about x is that it must lie somewhere in the bargaining range.

The 25-50-25 stochastic flow model is a very stylized normal distribution, shown in the probability histogram below left. This model assumes that, in any year, flow volume equals its long-term average (denoted N , for Normal) with probability 0.50; volume is 25% below its long-term average with



probability 0.25, and 25% above its long-term average with the complementary probability, 0.25. All years are assumed independent.



To model high water demand conditions, assume that $N = 0.36$. The relevant region of UP's optimal consumption function is shown in the figure above right. To summarize, this function combined with the 25-50-25 flow model produces flow volume $Q = 0.36$ and optimal allocation $q_0 = 0.048$ with probability $\frac{1}{2}$, $Q = 0.27$ and $q_0 = 0$ with probability $\frac{1}{4}$, and $Q = 0.45$ and $q_0 = 0.108$ with probability $\frac{1}{4}$. In these three cases, total welfare $W(Q)$ equals 2.429,

1.893, and 2.948, respectively; expected total welfare is 2.425.

In contrast, suppose that UP's withdrawal level, q_{UP} , is fixed at 0.048 regardless of total flow volume, Q . Note that 0.048 is in fact the optimal value of q_{UP} under normal flow conditions, i.e. when $Q = 0.36$; this withdrawal level will be suboptimal whenever flow is abnormally low or high, which occurs half the time in the 25-50-25 model. The above formula for $W(Q)$ still applies however, and shows that total welfare is 2.429, 1.877, and 2.933, respectively, in the three cases. Expected total welfare is 2.417. Thus this calculation suggests that the capacity to adjust current withdrawal rates to flow has a marginal effect, increasing welfare by only 0.3%.

Some idea of the extent of this benefit is shown by Figure 11, which shows the effects of variable withdrawal, and various fixed withdrawal rates, on total welfare. Note that, among fixed rates, the one which maximizes total welfare is $q_{UP} = 0.048$, the optimal rate for normal flow. It is clear that, if a fixed withdrawal rate is used, then choosing the optimal rate can increase expected welfare by 10% - 20%.

There are several reasons to believe that the above calculation understates the benefits of a variable withdrawal schedule on welfare. First, the 25-50-25 flow model implies that half the time, flow is precisely normal, in which case the best fixed rate coincides with the variable rate. A more realistic model would show that the variable rate produces some benefits, even when flow is close to normal. More important, the 25-50-25 model does not allow for low-probability extreme events, when the costs of a far-from-optimal fixed rate would be expected to be greatest. Finally, the high-water-demand scenario represented by the assumption of $N = 0.36$ also biases the calculation, since in the 75%-of-Normal event, the optimal variable rate ($q_0 = 0$) is not as far from the best fixed rate (0.048) as it would be if flow conditions were such that both states were optimally allocated positive water volumes.

To test this latter assertion, the calculation was repeated for $N = 6.0$ in the 25-50-25 flow model. Expected total welfare then equals 16.714 using the best fixed schedule, and 17.148 using

the variable schedule. Thus the variable schedule makes a somewhat more significant improvement of about 2.6%. Again, the best fixed schedule represents an improvement of about 10% - 20% over suboptimal schedules.

It is conjectured that in more realistic flow models the variable allocation system will result in improvements on the order of 10% relative to the best fixed allocation. This conjecture will be tested as part of a future project.

6. Conclusions

The objective of this paper has been to indicate how to assess whether stable water-sharing agreements in international river basins are possible, and to identify and describe, insofar as possible, any stable agreements that do exist. A method for determining stable allocations was developed and applied to some simple two- and three-country models. For stability, and to maximize welfare over the long term, this method depends on the operation of new systems that will make current flow data available to each riparian, permitting each to learn the others' withdrawal rates. These systems will also provide accurate annual flow volume predictions, making feasible annual adjustments in allocations (and payments). Using some simple examples, it was shown that an adjustable allocation scheme that reflects total flow volume results in improved total welfare, relative to the best fixed scheme.

But there are other reasons, beyond this increase in total welfare, for recommending a system that adjusts the allocation according to the total annual flow volume. The best fixed allocation may be difficult to determine, not just because of the stochastic nature of the flow, but because long term trends may be changing the flow distribution. As well, demands for water may change with time, and the particular allocation that maximizes welfare may depend on the location and shape of the demand curves. And, as noted in the Introduction, a more fundamental problem is that an allocation system that is far out of line relative to the supply of and demands for water may be challenged by one or more riparians, leading to regional tensions and, possibly, war.

Future research will include the application of these ideas to an existing international river basin, using more realistic models of the flow and the riparians' water demands. It is hoped that these methods will permit full utilization of current flow level data, which is about to become available. This important information, if efficiently utilized, may be crucial to the social and economic development of many countries around the world.

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Figure 2: Schedules Indifferent to Status Quo when $Q = 1.0$, Example 1

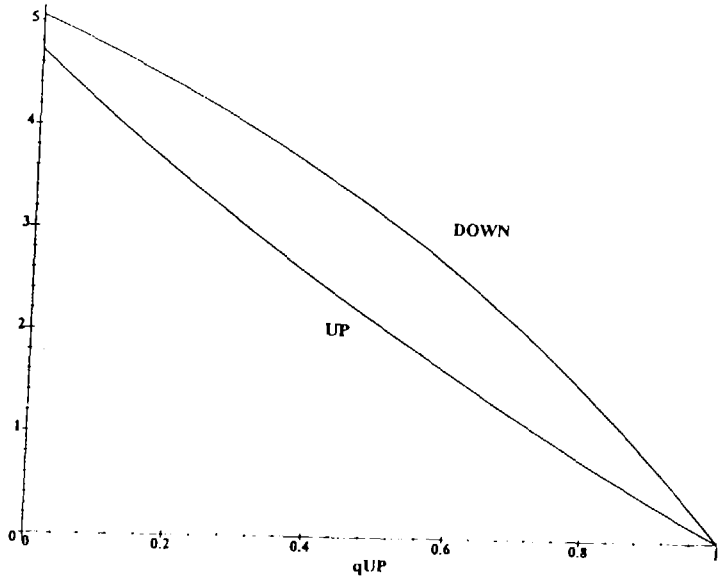


Figure 3: Allocations to UP and DOWN, Example 1

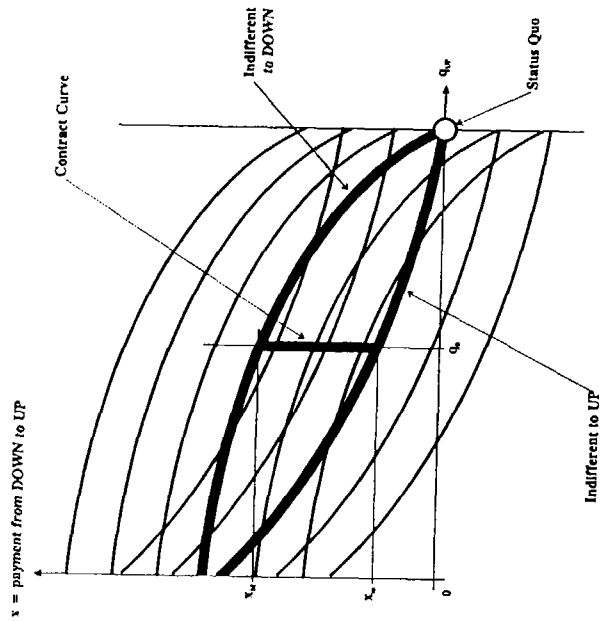
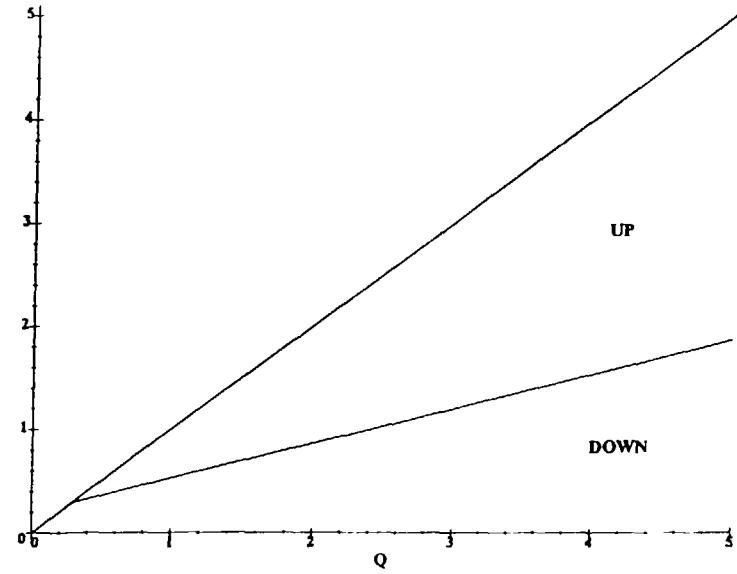


Figure 1: Determination of Contract Curve

Figure 4: Bargaining Range, Example 1

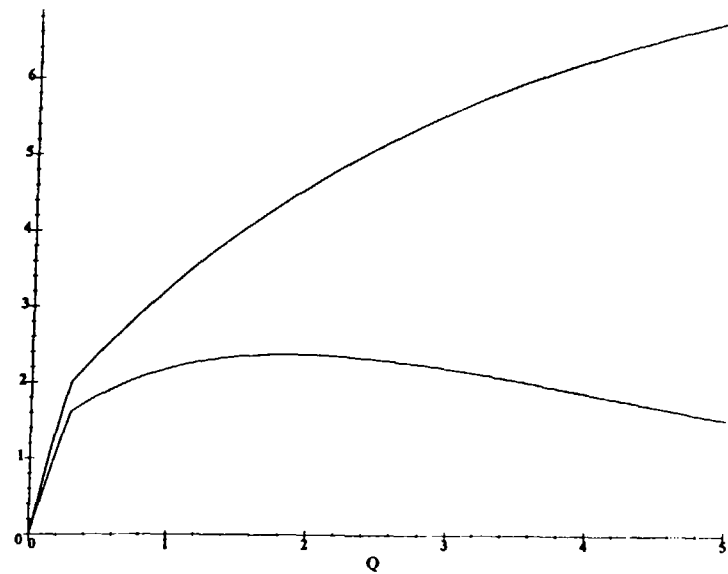


Figure 5: Allocations to UP and DOWN, Example 2

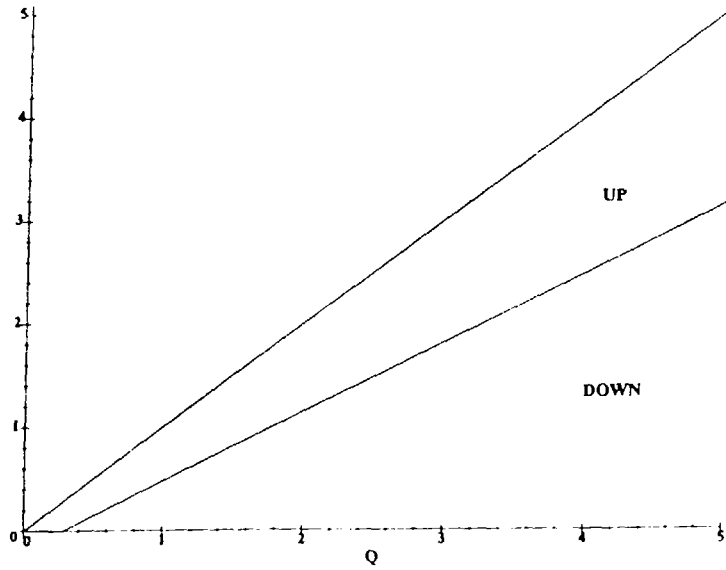


Figure 6: Allocations to UP and DOWN, Example 3

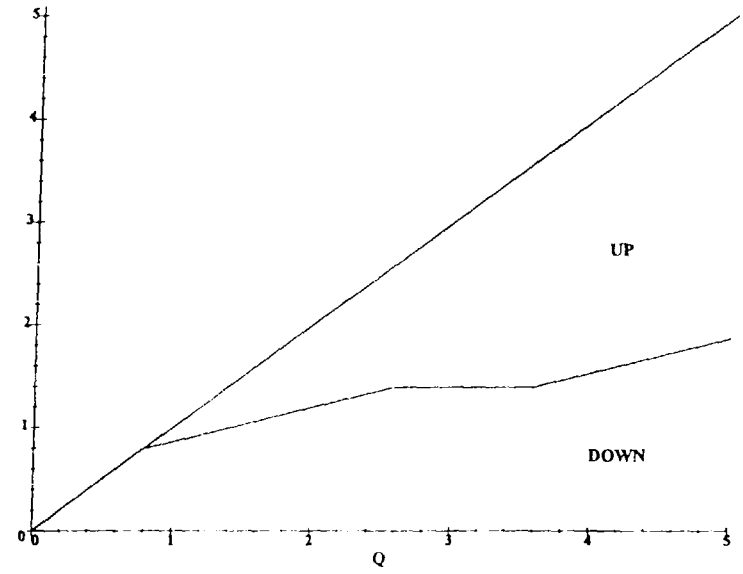


Figure 7: Bargaining Range, Example 3

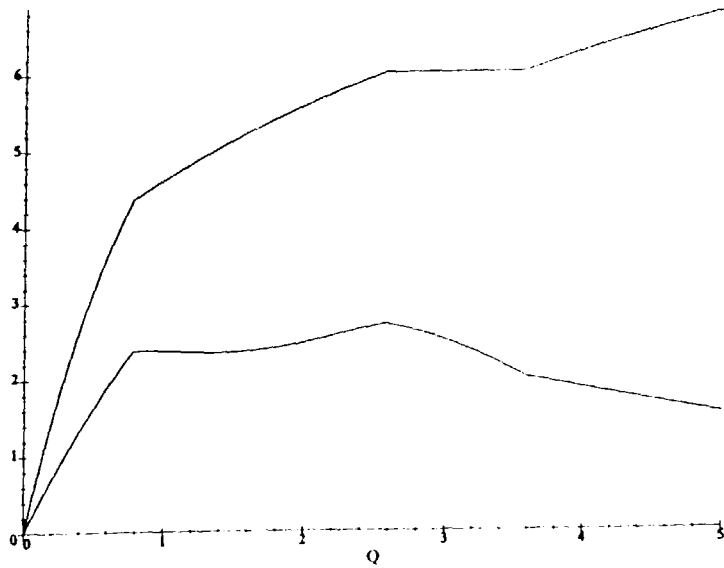


Figure 8: Allocations to UP, MID, and DOWN, Example 4

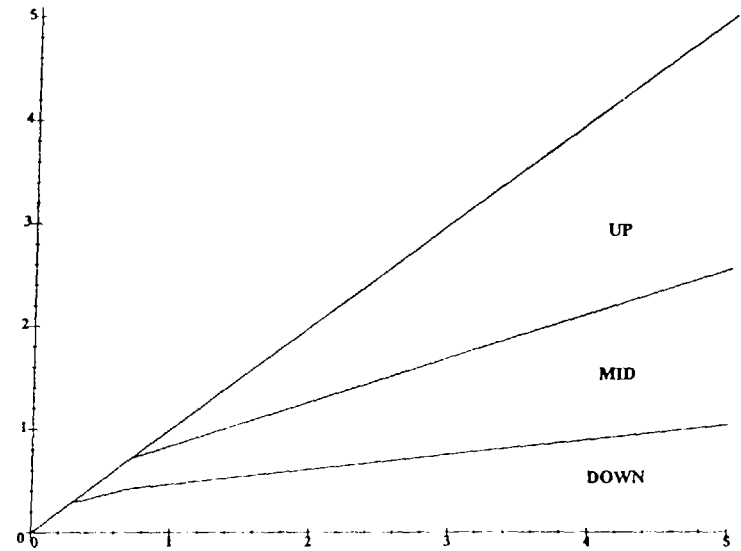


Figure 9: Bargaining Range, Example 4

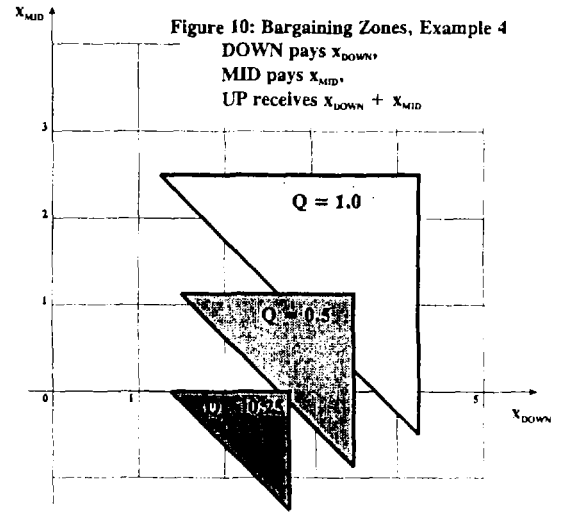
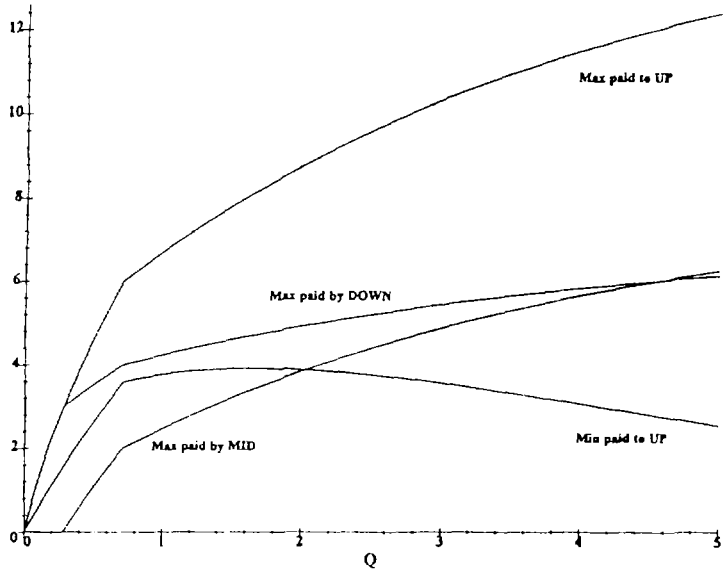
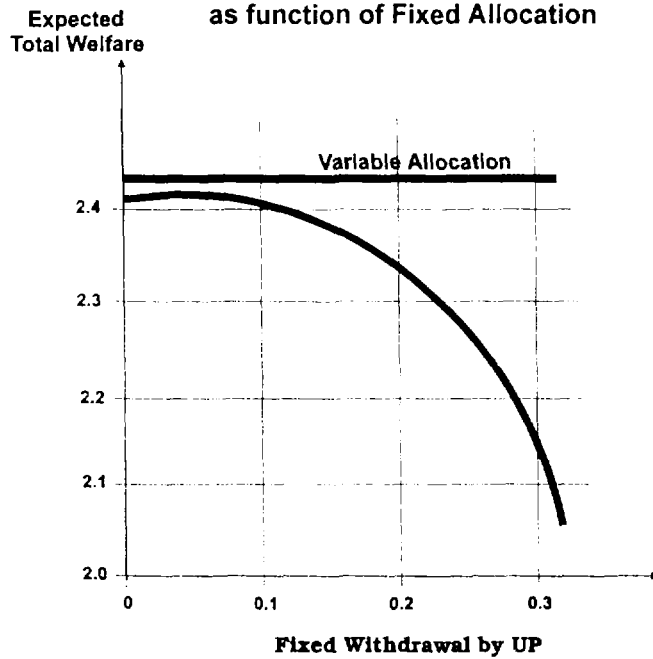


Figure 11: Expected Total Welfare under Variable Allocation and as function of Fixed Allocation



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