Bayesian Inferences on Fourier Flexible Functional Form in Agricultural Production

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Abstract

Flexible functional forms are used to examine the characteristics of production technologies. The Fourier functional form is capable of approximating any function globally, with the specified expansion. Unfortunately the exact form of the expansion is not known. The regularity conditions are likely to be violated without the exact form of expansion. In this paper we use a Bayesian approach to impose regularity conditions locally on a Fourier flexible functional from using agricultural production data. Monotonicity, concavity, convexity and elasticities are compared.

Key words: Cost function, Flexible functional form, Fourier series, Markov Chain Monte Carlo

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1. Introduction

In recent years flexible functional forms have been used to examine production technology. Duality theory facilitates the use of more “flexible” functional forms to accommodate the production of multiple outputs using many inputs. Either an indirect profit or cost function can be used to examine the underlying production technology of firms. Flexible functional forms are derived approximating the primary cost function to its second order Taylor series expansion. Among the alternative forms flexible functions used are the normalized quadratic (Diewert and Wales, 1987), translog (Christensen at el., 1971; O’Donnel and Woodland, 1995) and generalized Leontief (Diewert, 1972; Lopez 1980).

The Fourier function is capable of representing a multivariate function when the true functional form is unknown (Dym and McKean, 1972). Mutually orthogonal sine and cosine functions help the functional form to behave as an n-vector linear combination of n-mutually orthogonal, function space-spanning basis vectors (Mitchell and Onvural, 1996). Though the Fourier functions can be expanded to have infinite sine and cosine terms, unbounded expansion is less useful because the parameters in the Fourier function have less economic interpretation, as the expansion increases and more importantly finite observation constrains such expansions. Therefore, the approximate level of expansion given the number of observations and other econometric considerations needs to be considered.

Gallant (1981) suggests that a second order polynomial in the explanatory variables can facilitate such approximation and infer the properties of the underlying function. Validity and reliability of the estimates can be improved by adding trigonometric terms to the flexible function. Limiting the expansion of the trigonometric terms may limit the theoretical properties of the functional form. Imposition of regularity conditions may help to solve the problem partially. It is well known that curvature restrictions can be imposed using Choleskey factorization and eigenvalue decomposition in certain flexible functional forms. Diewert and Wales (1987) reported that imposing global concavity in translog cost functions may result an upward bias among input substitutes and a similar imposition in Leontief cost functions eliminates complementarity between inputs, while Caves at el. (1980) have shown restriction of parameters in quadratic cost function may cause it to lose the flexibility of the functional form itself.

The complexity of imposing global regularity conditions in flexible functional forms without the loss of econometric properties can be overcome by imposing those conditions locally or in the region in which inferences will be drawn. Such methods are widely used in the econometric literature. For example Lau (1978), and Gallant and Golub (1984) used numerical methods, while Chalfant and Wallace (1992) and Terrell (1996) have used a Bayesian approach.

This paper aims at examining the regularity conditions of the Fourier flexible functional form. A system of seemingly unrelated cost and factor share equations will be analyzed using the Markov Chain Monte Carlo (MCMC) method. In the first step, the input and output elasticities will be estimated without the MCMC method for the Fourier extensions. Following, the Metropolis-Hastings algorithm is used on the estimated coefficients to impose the regularity conditions and then elasticities are estimated.

Fourier Functional Form

The first step of our analysis consists in modeling the cost function with a Fourier series. There have been a few applications of the Fourier series in the agricultural economics literature. Gallant (1981, 1982, 1984); Elbadawi, Gallant and Souza (1983) and Chalfant and Gallant (1985) discuss the functional form in greater detail, so we present a brief description here. It is known that the production possibilities faced by a firm can be represented by a cost function. A correctly specified cost function meets the known set of assumptions; nonnegative for all positive prices and output, monotonic, linearly homogenous in prices and, concave in input prices and convex in output prices.

The existing literature on Fourier series is embedded with a translog cost function in a classical statistical framework. Unfortunately, the flexibility of these functional forms is achieved with the cost of forgoing the global regularity conditions (Barnett at el., 1991). Gallant's contribution of the Fourier series as a semi-parametric approach has reinvigorated the discussion of flexible functional forms which can attain the global flexibility property. The Fourier extension gives a better approximation of the unknown "true" functional form than the translog form (McAllister and McManus, 1993; Mitchell and Onvural, 1996; Berger and Mester, 1997). Further, Gallant also suggests that the approximation error can by minimized having fewer trigonometric terms along with a second order polynomial in the explanatory variable. The functional representation is translog when the second order polynomial is expressed as the log-log funtion (1981).

In many cases with a large number of observations, it's difficult to specify the correct expansion, which may result in the estimated models being inflexible and irregular. The most appropriate approach would be to have global regularity conditions in the Fourier functional form with a fewer numbers of parameters. Often it is constrained by finite observation and forgoing some of the other econometric properties. However, it is possible to overcome the lack of global regularity by employing techniques that enable the function to attain the regularity condition locally at each observation.

Eastwood and Gallant (1991) suggest that the number of parameters to be included in the Fourier series expansion should equal to number of observations raised to the power two thirds for producing consistent and unbiased estimates. Further, the increased expansion has the potential to represent the unknown "true" cost function and more importantly to be consistent with the Sobolov norm. The Fourier function can be written as Gallant (1982, p.309);

$$\ln TC = \alpha_o + \sum_{p=1}^{8}\beta_p \ln w_p + \sum_{k=1}^{2}\gamma_k \ln Y_k + \frac{1}{2}(\sum_{p=1}^{8}\sum_{q=1}^{8}\beta_{pq}\ln w_p \ln w_q + \sum_{k=1}^{2}\sum_{l=1}^{2}\gamma_{kl}\ln Y_k \ln Y_l) + \sum_{p=1}^{8}\sum_{k=1}^{2}\lambda_{ik}\ln w_p \ln Y_k + \sum_{z=1}^{\alpha} 2\left[u_j \cos\left(jw_p\right) - v_j \sin\left(jw_p\right)\right] + \sum_{j=1}^{\alpha} 2\left[s_j \cos\left(jY_k\right) - t_j \sin\left(jY_k\right)\right] \quad (1)$$

Gallant and Golub (1984) used a restrictive form of the Fourier model to fit the model with observed data while restricting the Fourier function to satisfy the regularity conditions at those data points. Terrell (1996) used the translog, generalized Leontief and symmetric generalized McFadden flexible functional forms to estimate the posterior moments of elasticities imposing restrictions to the prior distribution. Since then there has been a growing literature on use of Bayesian inference with flexible functional forms.

Bayesian Statistics

Let β be the parameter vector to be estimated. The β can be represented as a probability distribution or density function, P(β). The likelihood of observing the data (y) conditional on the probability density function of β, P(y| β). Based on Bayes theorem, one can express the probability of β as;

$$P(\beta \mid y) = \frac{P(y \mid \beta)P(\beta)}{P(y)} \qquad (3)$$

where P(y| β) is the posterior distribution and reflected jointly by the observed data and prior distribution P(β). Since our β is random and assuming P(y) is independent of β, the above equation 3) can be rewritten as

$$P(\beta \mid y)\, \alpha\, P(y \mid \beta)P(\beta) \qquad (4)$$

where the posterior distribution [Py| β)] is proportional (α) to the product of conditional and prior distribution of β. [(Py| β) Py] is the probability of y averaged over the parameters of interest or marginal distribution of β. Py can be estimated as

$$P(y) = \int P(y, \beta) d\beta \qquad (5)$$

$$= \int P(y \mid \beta) p(\beta) d\beta \qquad (6)$$

The likelihood principle asserts that the function, P(β), contains all relevant information. The advantage of this assertion is that the sample also contains the information about the data and the parameters. Though the function is of unknown parameters, one can specify the probability of the sample observed on the basis of known parameters. Using sampling literature, it is possible to specify the sampling distribution of the estimated parameter β as the function of observed data, $\hat{\beta} = f(y)$. The sampling procedure provides prior information about the parameter β before observing the data. Spall (2003) states that drawing samples from the density P(β) is not always feasible because the density may be complicated and often times analytically intractable. The Markov Chain Monte Carlo (MCMC) method offers an alternative to produce a dependent (y) sequence containing P(β) without sampling from P(β).

The Bayesian approach is based on drawing samples from a MCMC simulation. MCMC methods provide a criterion for generating samples from joint distributions based on conditional distributions. In the past, the application of the Bayesian approach in econometric literature has increased considerably and the advancement computer technology has facilitated its use. The Gibbs sampler and Metropolis-Hastings algorithms are widely used in MCMC. In our analysis, we use the Metropolis-Hastings algorithm, which is capable of producing a sequence of parameters $(\beta_1, \beta_2, \beta_3 ...)$ based on the some initial condition, β_0. The next state of the parameter β_n is chosen from a point in an appropriate proposal distribution, which may be arbitrarily chosen by the researcher.

Data

The translog cost function consists of 2 aggregated outputs and 8 inputs. The inputs and outputs were defined based on physical input-output analysis. Data for the estimation was obtained from the Kansas Farm Management Association (KFMA) data base. A total of 2756 observations were used in the analysis. The outputs were aggregated into crops (y1) and livestock (y2) and the inputs were the prices of seed (w1), fertilizer (w2), pesticide (w3), feed (w4), energy (w5), labor (w6), land (w7), and machine (w8). Summary statistics for the raw data is found in table 1.

Table 1: Summary Statistics of Raw Data

Variable	Average	Standard deviation	Minimum	Maximum
Crop - output	782.3624	662.1689	0.6434	5758.1200
Livestock- output	598.7168	827.9419	0.0000	11537.6900
Seed – input	140.2692	33.7729	64.0000	194.0000
Fertilizer – input	129.9615	25.3203	56.0000	173.0000
Pesticide – input	126.2692	28.0813	67.0000	173.0000
Feed – input	120.5769	16.8512	86.0000	159.0000
Energy – input	170.5385	50.7599	57.0000	234.0000
Labor – input	160.0385	51.9691	69.0000	253.0000
Land – input	24.3587	6.9783	9.5639	35.5000
Machine – input	170.9615	59.8269	58.0000	271.0000

Empirical Specification

The Fourier flexible functional form for 8 inputs and 2 outputs is specified in equation (1). In this paper the trigonometric expansion term (J) is arbitrarily set to 1. The multi-indices for the Fourier expansion is presented in table 1.

Table 1: Multi-indices for Fourier expansion

Crop	1	0	0	0	0	0	0	0	0	0
Livestock	0	1	0	0	0	0	0	0	0	0
Seed	0	0	1	0	0	0	0	0	0	0
Fertilizer	0	0	0	1	0	0	0	0	0	0
Pesticide	0	0	0	0	1	0	0	0	0	0
Feed	0	0	0	0	0	1	0	0	0	0
Energy	0	0	0	0	0	0	1	0	0	0
Labor	0	0	0	0	0	0	0	1	0	0
Land	0	0	0	0	0	0	0	0	1	0
Machine	0	0	0	0	0	0	0	0	0	1
α	1	2	3	4	5	6	7	8	9	10

Since the translog cost function is nested in a Fourier series, the theoretical properties such as symmetry, homogeneity and concavity in input prices and convexity in output can be easily imposed in equation (1) by satisfying the conditions and examining the second order derivative of Hessian matrices for inputs and outputs.

a. $\beta_{ij} = \beta_{ji} \ \forall i, j$, $\gamma_{kl} = \gamma_{lk} \ \forall \ k, l \ and \ \lambda_{ik} = \lambda_{ki} \ \forall i, k$ (7)

b. $\sum_{i=1}^{8} \beta_i = 1; \ \sum_{i=1}^{8} \beta_{ij} = 0 \ \forall \ j; \sum_{i=1}^{8} \lambda_{ik} = 0 \ \forall \ k$ (8)

In the Fourier function the diagonal elements for the Hessian matrix for inputs and outputs are the function of the respective diagonal parameters and trigonometric expansion of the variable as well. For example the diagonal element for the Hessian matrix for input price i is obtained by twice differentiating the log cost function with respect log input price. The resulted diagonal element is presented in equation (11). Using Shephard's lemma, the input demand for factor i is obtained by differentiating the cost function with respect to the input price of factor i. In a translog cost function factor share equations are obtained differentiating the log cost function with respect to log input price.

$$s_i = \partial LnC(w, y) / \partial Lnw_i = w_i x_i / C(w, y) \qquad (9)$$

Bur for the Fourier series (nested translog) results the factor share equation is;

$$s_i = \lambda_i \left[\sum_{i=1}^{8} \beta_{ip} \ln w_p + \sum_{k=1}^{2} \lambda_{ik} \ln Y_k + \sum_{j=1}^{\infty} -u_{ji} \cos\left(jw_i \right) - v_{ji} \sin\left(jw_i \right) \right] \qquad (10)$$

where λ_i is defined in scaling procedure.

$$\beta_{iid} = \lambda_i^2 \left[\beta_{ii} + u_{ji} \cos\left(jw_i \right) - v_{ji} \sin\left(jw_i \right) \right] \qquad (11)$$

A system of eight equations is to be estimated assuming the errors in the cost function and factor share equations are independently identically distributed. The eighth factor input share equation is dropped in recognizing the homogeneity condition and to avoid the singularity of the error covariance matrix. Gallant (1980) suggests that the independent variables should be scaled as the Fourier series is a periodic function while the flexible cost function is continuous. The proposed scaling procedure by Gallant (1980) is similar to the scaling procedure used discussed in this paper except that the scaling factor λ is estimated as;

$$\lambda = \frac{2\pi - \varepsilon}{Max(l_i : i = 1, 2, ..., N)} \qquad (12)$$

where l_i is the scaled input price and N is the number of inputs used in he cost function.

Further Gallant also has shown that the Fourier approximation can be made accurate in a desired region only when the variables are between 0 and 2π (1980). Further the scaling of the observations also reduces the approximation problems near the endpoints as discussed by Gallant

(1981). There are two scaling methods Gallant, (1982) and, Mitchell and Onvural, (1996) were used in the literature and we used the method proposed by Mitchell and Onvural (1996, p.188). This method is;

$p_i^{\min}$ = sample minimum value of the ith input price, i = 1,2,...,8
$p_i^{\max}$ = sample maximum value of the ith input price, i = 1,2,...,8
$y_k^{\min}$ = sample minimum value of the kth output quantity, k = 1,2
$y_k^{\max}$ = sample maximum value of the kth output quantity, k = 1,2
$\mathrm{w}_{\mathrm{pi}} = 0.0001 - \ln p_i^{\min}$
$\mathrm{w}_{\mathrm{yk}} = 0.0001 - \ln y_k^{\min}$
M_i = sample maximum value of $\mathrm{lnp}_{\mathrm{i}}^{\max} + w_{pi}$
M_k = sample maximum value of $\mathrm{lnp}_{\mathrm{i}}^{\max} + w_{pk}$
$\lambda_i = 6 / M_i$
$\lambda_k = 6 / M_k$
$\eta_k = 6 / (Lny_k^{\max} + w_{yk})\, \lambda_k$
$\mathrm{w}_{\mathrm{y1}} = 0.1916$
$\mathrm{w}_{\mathrm{y2}} = 3.0001$
$\mathrm{w}_{\mathrm{p1}} = \text{-}1.8061$
$\mathrm{w}_{\mathrm{p2}} = \text{-}1.7481$
$\mathrm{W}_{\mathrm{p3}} = \text{-}1.8260$
$\mathrm{w}_{\mathrm{p4}} = \text{-}1.9344$
$\mathrm{w}_{\mathrm{p5}} = \text{-}1.7558$
$\mathrm{w}_{\mathrm{p6}} = \text{-}1.8387$
$\mathrm{w}_{\mathrm{p7}} = \text{-}0.9805$
$\mathrm{w}_{\mathrm{p8}} = \text{-}1.7633$
$l_i = (\ln w_1 + w_{pi})\, \lambda_i$, for i = 1,2...8
$o_k = (\ln y_k + w_{yk})\, \mu_k \lambda_k$, for k = 1, 2

Table 2: Summary Statistics of Scale Data

Variable	Average	Standard deviation	Minimum	Maximum
o_1 – Crop	4.4449	0.6179	0.0002	6.0000
o_1 – Livestock	4.0496	1.7372	0.0001	6.0000
l_1 – Seed	3.1210	1.1408	0.0010	4.6106
l_2 – Fertilizer	3.1167	0.8701	0.0009	4.2923
l_3 – Pesticide	2.6073	1.0193	0.0010	4.0732
l_4 – Feed	1.7162	0.7401	0.0012	3.2132
l_5 – Energy	3.9875	1.4470	0.0009	5.4351
l_6 – Labor	3.4301	1.5696	0.0010	5.6986
l_7 – Land	1.5885	0.6003	0.0004	2.3534
l_8 – Machine	3.9140	1.6115	0.0009	6.0000

The price responsiveness of inputs can be measures by estimating the price elasticity of demand (η_{ij}). Huang and Wang (2001) discuss the elasticity estimation using the Fourier function. In our analysis the price elasticity of conditional demand (η_{ij}) was estimated using the proposed method by Huang and Wang (2001, p.220).

$$\sigma_{ii} = \frac{\beta_{ii} + S_i^2 - S_i}{S_i^2},\ \sigma_{ij} = \frac{\beta_{ij} + S_i S_j}{S_i S_j} \tag{13}$$

and

$$\eta_{ii} = S_i \sigma_{ii},\ \eta_{ij} = S_j \sigma_{ij} \tag{14}$$

where σ_{ij} is Allen-Uzawa partial elasticities of substitutions. The Morishima elasticity of substitution can be estimated using

$$M_{ij} = \eta_{ij} - \eta_{ii}$$

For detail derivation and discussion of elasticities in translog function refer Binswanger (1974) and Morishima elasticities Thomson and Taylor (1995).

The total number of simulations run was set to 350,000 and the acceptance rate was about 58.06%. About 30 percent of the simulations were set for initial burning period. After the burning period, if the candidate parameters hold the input and output curvature and monotonicity conditions then the parameters were retained for elasticity and substitution estimations.

Results

The results from the empirical analysis are presented in two sections. The first section discusses the results from estimates without the Bayesian analysis. Results from the Bayesian MCMC analysis are presented section two. Estimated parameter values and their significance, price elasticities and Morishima elasticities of substitutions are discussed in the both sections. The model without the MCMC was estimated using the GAUSS OPTMUM procedure. The OPTMUM procedure minimizes the objective function choosing the parameter values. For Bayesian MCMC, the Seemingly Unrelated Regression (SUR) model was used in the estimation. In the Bayesian estimation significance of the parameters and the elasticities were estimated based on a 90% confidence interval.   The upper bounds of parameters/elasticities were estimated by trimming the top 5% of the sorted parameters/elasticities and bottom 5% of the parameters/elasticities in the Bootstrap framework. If the upper and lower bound values contain zero, then the parameter/elasticity is considered as not significant.

The parameters for the cost function and factor share equations are reported in table 3. The parameter estimates of crops and livestock quantities were positive and significant, which is consistent with the economic theory. But the squared quantity of livestock was positive only for livestock. The estimated parameters of the input prices for land and feed and the squared input prices of seed, fertilizer, feed, and land were positive and significant, which is a violation of economic theory. The maximum eigen value of the Hessian matrix for the input prices was positive. This indicates that the curvature condition (concavity) for the input prices was violated. Similarly the minimum eigen values of the Hessian matrix for the output quantities was negative, which is also violation of output curvature (convexity) condition.

Monotonicity of the function can be tested by examining the predicted factor shares. This regularity condition holds only if the predicted factor shares for all the factor share equations are positive. The predicted factor shares for pesticide and labor were negative, indicating that the monotonicity condition was not satisfied in the model.

Table 4 presents the own and cross price elasticity for the inputs. All inputs were price inelastic at sample mean price, except pesticide. The machine's own price elasticity was positive, while for the other inputs it was negative. A percent increase in mean price of seed increases the use of fertilizer, feed and land, but for the other inputs it decreases the input use. A percent increase in mean price in fertilizer results in an increased use of seed by 0.74 percent, while a similar increase in pesticide price, results an increase in the seed use by 1.212 percent. Interestingly a percent increase in mean price in feed results an increase use of all the inputs. The change in energy price has negative impact on usage of other inputs except for fertilizer and feed. A percent increase in mean price of labor increase the use of seed by 1.95 percent. Increase in land price is associated with decrease in use of fertilizer, pesticide and energy. A percent increase in mean price of machine likely to reduce the use of seed by about 3.92 percent and increasing the use of pesticide by 2.42 percent.

Table 3: Parameter Estimates without Bayesian Statistics

	Parameter	Estimates	Std. Error		Parameter	Estimates	Std. Error
a0	**Constant**	-6.5678*	0.2362	**b67**	**Labor/land**	0.0026	0.002
a1	**Crops**	6.1733*	0.1281	**b77**	**Land/land**	0.0490*	0.0029
a2	**Livestock**	0.0430*	0.0152	**ab11**	**Crop/seed**	0.0001	0.0008
b1	**Seed**	0.0081	0.0044	**ab12**	**Crop/fertliz.**	-0.0011*	0.0005
b2	**Fertilizer**	0.0021	0.0037	**ab13**	**Crop/pesti.**	0.0038*	0.0008
b3	**Pesticide**	-0.0236*	0.0045	**ab14**	**Crop/feed**	0.0002	0.0007
b4	**Feed**	0.0112*	0.0037	**ab15**	**Crop/energy**	-0.0016*	0.0008
b5	**Energy**	-0.0262*	0.0041	**ab16**	**Crop/labor**	0.0019*	0.0008
b6	**Labor**	-0.0462*	0.0048	**ab17**	**Crop/land**	-0.0103*	0.0008
b7	**Land**	0.1160*	0.0046	**ab22**	**Livest./fertliz.**	-0.0015*	0.0006
a11	**Crops/crops**	-0.8830*	0.0305	**ab23**	**Livest./pesti.**	-0.0041*	0.0006
a12	**Crops/livest.**	-0.3988*	0.0018	**ab24**	**Livest./feed**	0.0011*	0.0005
b11	**Seed/seed**	0.0128*	0.0025	**ab25**	**Livest./energy**	0.0083*	0.0006
b12	**Seed/fertliz.**	0.0110*	0.0017	**ab26**	**Livest./labor**	0.0013*	0.0006
b13	**Seed/pesti.**	0.0188*	0.0013	**ab27**	**Livest./land**	-0.0071*	0.0006
b14	**Seed/feed**	0.0009	0.0008	**c11**	**Sin. Seed**	0.0016*	0.0007
b15	**Seed/energy**	-0.0023	0.0013	**c12**	**Cos. Seed**	0.0108*	0.0007
b16	**Seed/labor**	0.0304*	0.0019	**c21**	**Sin. Fertiliz.**	-0.0001	0.0005
b17	**Seed/land**	0.0013	0.0019	**c22**	**Cos. Fertiliz.**	0.0144*	0.0007
b22	**Fertliz./fertliz.**	0.0176*	0.0019	**c31**	**Sin. Pesti.**	-0.0136*	0.0004
b23	**Fertliz./pesti.**	-0.0009	0.0012	**c32**	**Cos. Pesti.**	0.0042*	0.0005
b24	**Fertliz./feed**	0.0004	0.0008	**c41**	**Sin. Feed**	0.0042*	0.0005
b25	**Fertliz./energy**	0.0047*	0.0011	**c42**	**Cos. Feed**	-0.0035*	0.0003
b26	**Fertliz./labor**	0.0053*	0.0016	**c51**	**Sin. Energy**	0.0273*	0.0005
b27	**Fertliz./land**	-0.0035	0.0019	**c52**	**Cos. Energy**	0.0248*	0.0006
b33	**Pesti./pesti.**	0.0053*	0.0013	**c61**	**Sin. Labor**	-0.0239*	0.0008
b34	**Pesti./feed**	-0.0066*	0.0006	**c62**	**Cos. Labor**	-0.0067*	0.0005
b35	**Pesti./energy**	0.0188*	0.0011	**c71**	**Sin. Land**	0.0054*	0.0005
b36	**Pesti./labor**	-0.0186*	0.0013	**c72**	**Cos. Land**	0.0163*	0.0007
b37	**Pesti./land**	0.0109*	0.0014	**c81**	**Sin. Machi.**	-0.0388*	0.0014
b44	**Feed/feed**	0.0099*	0.0008	**c82**	**Cos. Machi.**	0.2372*	0.0013
b45	**Feed/energy**	0.0170*	0.0006	**d11**	**Sin. Crops**	-0.6775*	0.0155
b46	**Feed/labor**	-0.0104*	0.0008	**d12**	**Cos. Crops**	0.3173*	0.0095
b47	**Feed/land**	0.0020*	0.0008	**d21**	**Sin. Livest.**	-0.3152*	0.0021
b55	**Energy/energy**	0.0382*	0.0011	**d22**	**Cos. Livest.**	-0.4005*	0.0051
b56	**Energy/labor**	0.0198*	0.0013	**a22**	**Livest./livest.**	0.7391*	0.0051
b57	**Energy/land**	-0.0153*	0.0014	**ab21**	**Livest./seed**	-0.0043*	0.0007
b66	**Labor/labor**	-0.0345*	0.0036				

* significant at 5% level

Table 4: Own and Cross Price Elasticities

	Price							
Quantity	Seed	Fertilizer	Pesticide	Feed	Energy	Labor	Land	Machine
Seed	-0.1429	0.7453	1.2165	0.0885	-0.0862	1.952	0.1502	-3.9236
Fertilizer	0.4655	-0.2553	-0.0544	0.047	0.2541	0.1757	-0.0786	-0.554
Pesticide	-1.0393	0.0744	-1.3121	0.4008	-0.9919	0.9982	-0.548	2.418
Feed	0.0446	0.0379	-0.2362	-0.6432	0.6232	-0.3865	0.1284	0.4319
Energy	-0.0212	0.1003	0.2862	0.3051	-0.3201	0.2776	-0.1827	-0.4452
Labor	-0.6908	-0.0995	0.4135	0.2716	-0.3985	-0.2423	0.0035	0.7425
Land	0.0358	-0.03	0.153	0.0608	-0.1768	-0.0024	-0.1696	0.1291
Machine	-0.0692	-0.0156	-0.0499	0.0151	-0.0318	-0.037	0.0095	0.1789

Table 5 presents the Morishima elasticities of substation. About 81% of the estimated elasticities were positive, indicating that most of the inputs are substitutable. All the inputs wre net substitutes for seed, fertilizer, pesticide, feed and labor except pesticide and labor for seed, and feed for labor. Although pesticide and labor were not net substitutes for energy, but other inputs were substitutable. Seed, fertilizer, feed, labor and machine were net substitutes for land, but for machine only pesticide, feed and labor were net substitutes.

Table 5: Morishima Elasticities of Substitution

	Price							
Quantity	Seed	Fertilizer	Pesticide	Feed	Energy	Labor	Land	Machine
Seed	---	1.0006	2.5286	0.7317	0.2339	2.1943	0.3198	-4.1025
Fertilizer	0.6084	---	1.2577	0.6902	0.5742	0.418	0.0909	-0.7328
Pesticide	-0.8964	0.3297	---	1.044	-0.6718	1.2405	-0.3784	2.2391
Feed	0.1875	0.2932	1.0758	---	0.9433	-0.1442	0.298	0.253
Energy	0.1217	0.3556	1.5983	0.9483	---	0.5199	-0.0132	-0.6241
Labor	-0.5479	0.1558	1.7256	0.9148	-0.0784	---	0.173	0.5636
Land	0.1787	0.2253	1.4651	0.704	0.1433	0.24	---	-0.0498
Machine	0.0737	0.2397	1.2622	0.6583	0.2883	0.2053	0.1791	---

In this section, we present the results from Bayesian MCMC estimates. Table 6 reports the parameter estimates based on imposing curvature on input prices, output quantities and monotonicity. The parameter estimates for squared output quantities for crop and livestock were negative and positive respectively, but for crop it was not significant. The coefficients of squared input prices of seed, fertilizer and land were positive, but for the other inputs it was negative. The mean, upper and lower bound values for the parameters are presented in appendix 1. Only about 1.3 percent of the coefficients upper and lower bound parameter values contain zero, which indicates that about 98.7 percent of the parameters are significant at 5 percent level.

Table 6: Parameter Estimates with Bayesian Statistics

	Parameter	Estimates	Std. error		Parameter	Estimates	Std. error
a0	**Constant**	-6.568*	0.2362	**b67**	**Labor/land**	0.003	0.0020
a1	**Crops**	6.173*	0.1281	**b77**	**Land/land**	0.049	0.0029
a2	**Livestock**	0.043	0.0152	**ab11**	**Crop/seed**	0	0.0008
b1	**Seed**	0.008	0.0044	**ab12**	**Crop/fertliz.**	-0.001	0.0005
b2	**Fertilizer**	0.002	0.0037	**ab13**	**Crop/pesti.**	0.004	0.0008
b3	**Pesticide**	-0.024	0.0045	**ab14**	**Crop/feed**	0	0.0007
b4	**Feed**	0.011	0.0037	**ab15**	**Crop/energy**	-0.002	0.0008
b5	**Energy**	-0.026	0.0041	**ab16**	**Crop/labor**	0.002	0.0008
b6	**Labor**	-0.046	0.0048	**ab17**	**Crop/land**	-0.01	0.0008
b7	**Land**	0.116	0.0046	**ab22**	**Livest./fertliz.**	-0.001	0.0006
a11	**Crops/crops**	-0.883*	0.0305	**ab23**	**Livest./pesti.**	-0.004	0.0006
a12	**Crops/livest.**	-0.399*	0.0018	**ab24**	**Livest./feed**	0.001	0.0005
b11	**Seed/seed**	0.013	0.0025	**ab25**	**Livest./energy**	0.008	0.0006
b12	**Seed/fertliz.**	0.011	0.0017	**ab26**	**Livest./labor**	0.001	0.0006
b13	**Seed/pesti.**	0.019	0.0013	**ab27**	**Livest./land**	-0.007	0.0006
b14	**Seed/feed**	0.001	0.0008	**c11**	**Sin. Seed**	0.002	0.0007
b15	**Seed/energy**	-0.002	0.0013	**c12**	**Cos. Seed**	0.011	0.0007
b16	**Seed/labor**	0.03	0.0019	**c21**	**Sin. Fertiliz.**	0	0.0005
b17	**Seed/land**	0.001	0.0019	**c22**	**Cos. Fertiliz.**	0.014	0.0007
b22	**Fertliz./fertliz.**	0.018	0.0019	**c31**	**Sin. Pesti.**	-0.014	0.0004
b23	**Fertliz./pesti.**	-0.001	0.0012	**c32**	**Cos. Pesti.**	0.004	0.0005
b24	**Fertliz./feed**	0	0.0008	**c41**	**Sin. Feed**	0.004	0.0005
b25	**Fertliz./energy**	0.005	0.0011	**c42**	**Cos. Feed**	-0.003	0.0003
b26	**Fertliz./labor**	0.005	0.0016	**c51**	**Sin. Energy**	0.027	0.0005
b27	**Fertliz./land**	-0.003	0.0019	**c52**	**Cos. Energy**	0.025	0.0006
b33	**Pesti./pesti.**	0.005	0.0013	**c61**	**Sin. Labor**	-0.024	0.0008
b34	**Pesti./feed**	-0.007	0.0006	**c62**	**Cos. Labor**	-0.007	0.0005
b35	**Pesti./energy**	0.019	0.0011	**c71**	**Sin. Land**	0.005	0.0005
b36	**Pesti./labor**	-0.019	0.0013	**c72**	**Cos. Land**	0.016	0.0007
b37	**Pesti./land**	0.011	0.0014	**c81**	**Sin. Machi.**	-0.039	0.0014
b44	**Feed/feed**	0.01	0.0008	**c82**	**Cos. Machi.**	0.237*	0.0013
b45	**Feed/energy**	0.017	0.0006	**d11**	**Sin. Crops**	-0.678*	0.0155
b46	**Feed/labor**	-0.01	0.0008	**d12**	**Cos. Crops**	0.317*	0.0095
b47	**Feed/land**	0.002	0.0008	**d21**	**Sin. Livest.**	-0.315*	0.0021
b55	**Energy/energy**	0.038	0.0011	**d22**	**Cos. Livest.**	-0.4*	0.0051
b56	**Energy/labor**	0.02	0.0013	**a22**	**Livest./livest.**	0.739*	0.0051
b57	**Energy/land**	-0.015	0.0014	**ab21**	**Livest./seed**	-0.004	0.0007
b66	**Labor/labor**	-0.035	0.0036				

* significant at 5% level

Table 7 presents the mean elasticity estimates based on Bayesian simulations. Own price elasticities pesticide, feed, energy and labor were elastic, for the other inputs it was inelastic. The values of own price elasticities were higher compared to the previous estimates. A percent increase in mean price of seed results in an increase use of other inputs, except fertilizer and land. A percent in increase in mean price machine also increase the usage of all other inputs. Lower and upper bound elasticities are presented appendix 2. Own price elasticities at upper and lower bound was negative which indicates that those were significant and about 7.1 percent of the cross price elasticities contains zero, which implies that about 92.9 percent of the cross price elasticities were significant at 90 percent confidence interval..

Table 7: Bayesian simulated input price elasticity

	Price							
Quantity	**Seed**	**Fertilizer**	**Pesticide**	**Feed**	**Energy**	**Labor**	**Land**	**Machine**
Seed	-0.7852	-0.0121	0.1361	0.0327	0.0241	-0.2768	0.3898	0.4916
Fertilizer	-0.0072	-1.1008	0.1040	-0.1026	0.0005	0.0675	0.3119	0.7267
Pesticide	0.0784	0.1011	-1.0218	-0.0376	0.0615	0.0001	0.1199	0.6984
Feed	0.0261	-0.1381	-0.0521	-0.9254	-0.0230	-0.1504	0.9930	0.2699
Energy	0.0161	0.0006	0.0714	-0.0192	-0.9996	-0.0007	0.2140	0.7174
Labor	-0.2025	0.0832	0.0001	-0.1377	-0.0007	-1.2387	0.8059	0.6904
Land	0.0822	0.1110	0.0439	0.2623	0.0675	0.2324	-0.8309	0.0316
Machine	0.0112	0.0278	0.0275	0.0077	0.0243	0.0214	0.0034	-0.1232

Table 8 presents the Morishima elasticities of substitution based in Bayesian simulation. Values of all the estimates were positive, indicating all the inputs were substitutable.

Table 8: Bayesian simulated Morishima elasticities of substitution

	Price							
Quantity	**Seed**	**Fertilizer**	**Pesticide**	**Feed**	**Energy**	**Labor**	**Land**	**Machine**
Seed	0.0000	1.0887	1.1578	0.9580	1.0237	0.9619	1.2207	0.6148
Fertilizer	0.7780	0.0000	1.1258	0.8228	1.0001	1.3062	1.1427	0.8499
Pesticide	0.8636	1.2019	0.0000	0.8878	1.0611	1.2388	0.9508	0.8216
Feed	0.8113	0.9627	0.9696	0.0000	0.9766	1.0883	1.8239	0.3931
Energy	0.8013	1.1014	1.0932	0.9061	0.0000	1.2380	1.0449	0.8406
Labor	0.5827	1.1840	1.0219	0.7877	0.9989	0.0000	1.6367	0.8136
Land	0.8674	1.2118	1.0657	1.1877	1.0671	1.4711	0.0000	0.1548
Machine	0.7964	1.1286	1.0493	0.9330	1.0239	1.2601	0.8343	0.0000

Conclusion

In this paper, we have used the Fourier flexible functional form to examine the regularity conditions and to estimate the elasticities. Input and output curvature and the monotonicity conditions to hold with the Fourier expansion of one (J =1). We constrained the expansion of Fourier function in order to preserve the other econometric properties. The Bayesian theory provides an alternative methodology to impose the regularity conditions locally in all observations. The MCMC approach has the flexibility to impose the regularity condition, but also to produce other theoretically consistent estimates, such as negativity of own price elasticities.

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Appendix 1:

Bayesian simulated confidence intervals for parameters

	Parameter	Mean	Lower bound	Upper bound
a0	**Constant**	7.4157	6.7322	8.1415
a1	**Crops**	0.8418	0.5459	1.0900
a2	**Livestock**	-0.4372	-0.9374	0.0011
b1	**Seed**	0.0244	0.0229	0.0263
b2	**Fertilizer**	0.0240	0.0180	0.0296
b3	**Pesticide**	0.0277	0.0269	0.0288
b4	**Feed**	0.0460	0.0425	0.0494
b5	**Energy**	0.0180	0.0171	0.0191
b6	**Labor**	0.0235	0.0222	0.0247
b7	**Land**	0.1302	0.0775	0.1855
a11	**Crops/crops**	0.2099	0.1553	0.2883
a12	**Crops/livest.**	-0.2105	-0.2294	-0.1883
b11	**Seed/seed**	0.0020	0.0008	0.0033
b12	**Seed/fertliz.**	-0.0003	-0.0009	0.0002
b13	**Seed/pesti.**	0.0017	0.0014	0.0023
b14	**Seed/feed**	0.0002	0.0001	0.0004
b15	**Seed/energy**	-0.0002	-0.0005	0.0001
b16	**Seed/labor**	-0.0054	-0.0059	-0.0049
b17	**Seed/land**	0.0064	0.0050	0.0075
b22	**Fertliz./fertliz.**	-0.0057	-0.0067	-0.0039
b23	**Fertliz./pesti.**	0.0018	0.0013	0.0023
b24	**Fertliz./feed**	-0.0019	-0.0038	0.0000
b25	**Fertliz./energy**	-0.0008	-0.0013	-0.0004
b26	**Fertliz./labor**	-0.0013	-0.0032	0.0010
b27	**Fertliz./land**	0.0074	0.0056	0.0087
b33	**Pesti./pesti.**	-0.0014	-0.0018	-0.0010
b34	**Pesti./feed**	-0.0013	-0.0017	-0.0010
b35	**Pesti./energy**	0.0007	0.0005	0.0010
b36	**Pesti./labor**	0.0004	-0.0005	0.0011
b37	**Pesti./land**	-0.0001	-0.0010	0.0010
b44	**Feed/feed**	-0.0006	-0.0019	0.0007
b45	**Feed/energy**	-0.0007	-0.0011	-0.0002
b46	**Feed/labor**	-0.0026	-0.0038	-0.0016
b47	**Feed/land**	0.0152	0.0112	0.0200
b55	**Energy/energy**	-0.0001	-0.0007	0.0008
b56	**Energy/labor**	-0.0007	-0.0012	-0.0003
b57	**Energy/land**	0.0040	0.0024	0.0052
b66	**Labor/labor**	-0.0057	-0.0071	-0.0034
b67	**Labor/land**	0.0144	0.0093	0.0170
b77	**Land/land**	0.0014	-0.0086	0.0107
ab11	**Crop/seed**	-0.0001	-0.0004	0.0000

ab12	**Crop/fertliz.**	0.0010	0.0008	0.0013
ab13	**Crop/pesti.**	-0.0011	-0.0013	-0.0010
ab14	**Crop/feed**	-0.0040	-0.0050	-0.0029
ab15	**Crop/energy**	0.0011	0.0007	0.0014
ab16	**Crop/labor**	0.0009	0.0007	0.0011
ab17	**Crop/land**	0.0018	-0.0007	0.0040
ab22	**Livest./fertliz.**	0.0013	0.0002	0.0026
ab23	**Livest./pesti.**	0.0015	0.0012	0.0017
ab24	**Livest./feed**	-0.0027	-0.0038	-0.0014
ab25	**Livest./energy**	0.0011	0.0006	0.0017
ab26	**Livest./labor**	-0.0021	-0.0027	-0.0015
ab27	**Livest./land**	0.0180	0.0076	0.0311
c11	**Sine Seed**	-0.0044	-0.0046	-0.0042
c12	**Cosine Seed**	-0.0037	-0.0039	-0.0035
c21	**Sine Fertiliz.**	-0.0045	-0.0054	-0.0031
c22	**Cosine Fertiliz.**	0.0012	0.0009	0.0014
c31	**Sine Pesti.**	-0.0003	-0.0006	0.0000
c32	**Cosine Pesti.**	0.0004	0.0000	0.0007
c41	**Sine Feed**	0.0046	0.0041	0.0057
c42	**Cosine Feed**	-0.0033	-0.0054	-0.0016
c51	**Sine Energy**	-0.0024	-0.0028	-0.0019
c52	**Cosine Energy**	0.0009	0.0006	0.0011
c61	**Sine Labor**	-0.0023	-0.0029	-0.0017
c62	**Cosine Labor**	-0.0013	-0.0022	-0.0007
c71	**Sine Land**	-0.0277	-0.0395	-0.0121
c72	**Cosine Land**	0.0164	0.0109	0.0203
c81	**Sine Machi.**	-0.1813	-0.1983	-0.1655
c82	**Cosine Machi.**	-0.0085	-0.0482	0.0257
d11	**Sine Crops**	-0.1146	-0.1672	-0.0624
d12	**Cosine Crops**	-0.0058	-0.0697	0.0366
d21	**Sine Livest.**	-0.3597	-0.4245	-0.2943
d22	**Cosine Livest.**	-0.4323	-0.6186	-0.2672
a22	**Livest./livest.**	0.7740	0.6055	0.9750
ab21	**Livest./seed**	-0.0956	-0.1199	-0.0753

Appendix 2

Bayesian simulated upper bound input price elasticity

	Price							
Quantity	**Seed**	**Fertilizer**	**Pesticide**	**Feed**	**Energy**	**Labor**	**Land**	**Machine**
Seed	-0.7852	-0.0121	0.1361	0.0327	0.0241	-0.2768	0.3898	0.4916
Fertilizer	-0.0072	-1.1008	0.1040	-0.1026	0.0005	0.0675	0.3119	0.7267
Pesticide	0.0784	0.1011	-1.0218	-0.0376	0.0615	0.0001	0.1199	0.6984
Feed	0.0261	-0.1381	-0.0521	-0.9254	-0.0230	-0.1504	0.9930	0.2699
Energy	0.0161	0.0006	0.0714	-0.0192	-0.9996	-0.0007	0.2140	0.7174
Labor	-0.2025	0.0832	0.0001	-0.1377	-0.0007	-1.2387	0.8059	0.6904
Land	0.0822	0.1110	0.0439	0.2623	0.0675	0.2324	-0.8309	0.0316
Machine	0.0112	0.0278	0.0275	0.0077	0.0243	0.0214	0.0034	-0.1232

Bayesian simulated lower bound input price elasticity

	Price							
Quantity	**Seed**	**Fertilizer**	**Pesticide**	**Feed**	**Energy**	**Labor**	**Land**	**Machine**
Seed	-0.9397	-0.0198	0.1060	0.0225	-0.0015	-0.3078	0.3766	0.4680
Fertilizer	-0.0114	-1.1960	0.0728	-0.1100	-0.0161	-0.0780	0.2867	0.7209
Pesticide	0.0630	0.0720	-1.0318	-0.0351	0.0422	0.0074	0.0719	0.6408
Feed	0.0208	-0.1434	-0.0473	-1.0730	-0.0244	-0.1460	0.6820	0.2744
Energy	-0.0010	-0.0192	0.0503	-0.0211	-1.0006	-0.0242	0.2251	0.6278
Labor	-0.2196	-0.0905	0.0092	-0.1378	-0.0245	-1.2698	0.4902	0.6961
Land	0.0662	0.0805	0.0210	0.1023	0.0433	0.0940	-0.9359	-0.0355
Machine	0.0105	0.0267	0.0266	0.0073	0.0211	0.0211	-0.0035	-0.2399