APPLICATION OF A PLANT LOCATION MODEL TO AN AREA'S COTTON GINNING INDUSTRY*

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The conventional cotton marketing system includes characteristics that impair its efficiency. This paper reports on a study which examined the potential operational efficiency gains in that portion of the system which involved the flow of seed cotton from the field through the ginning process. Up to 70 percent of the annual production is harvested in three to four weeks; the rest is harvested and processed during the remaining 3 1/2 to 4 months of the ginning season [3].

The maximum output of harvested cotton in any single time period (Figure 1) is associated with the week, on the accumulative harvested cotton relationship $\left(H(t)\right)$ where $\left(\frac{H(t)}{t}\right)$ is a maximum (t_1). It is during this week that the maximum output occurs, and it is this output that the industry is required to process during that time period; i.e., the industry must adjust its capacity to this peak demand. The industry's required processing capacity per week is represented by the slope of the accumulative harvested cotton relationship $\left(\frac{dH(t)}{dt}\right)$ where $\left(\frac{H(t)}{t}\right)$ is a maximum; i.e., in Figure 1, by the slope of the ray OA. On either side of the tangency between the accumulative harvest $\left(H(t)\right)$ and processing $\left(P(t)\right)$ relationships, actual processing capacity exceeds required capacity; subsequently, excess plant capacity exists during the off-peak, or the major portion of the ginning season.

Principal shortcomings of the existing marketing system are: (1) substantial capital investment is required to create processing capacity capable of accommodating peak demands, and (2) inefficient use of variable inputs occur because of the industry's excess plant capacity during much of the season's duration.

Because seed cotton can be successfully stored, a feasible alternative involves storage of seed cotton and then its processing over an extended time. This would require less capital per processed unit, and the orderly flow of seed cotton from storage would permit more efficient use of the variable inputs. With the introduction of seed cotton storage, the industry's processing capacity can be determined independent of harvest. For a fixed quantity of area seed cotton production, there is an inverse relationship between required processing capacity (slope of ray OB) and length of processing season; i.e., as length of processing season increases, the required capacity of the area industry decreases.[1] In this study, costs were examined for processing

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[1] With the extended season the quantity of cotton requiring processing (Pe) at the end of the regular harvest season in Figure 1 is:

$$Pe = \int_{0}^{t2} [H(t)-P(t)]\,d(t).$$

The maximum quantity of cotton placed into and remaining in storage during any week is where $d\frac{\left(H(t)\right)}{dt} - d\frac{\left(P(t)\right)}{dt} = 0$.

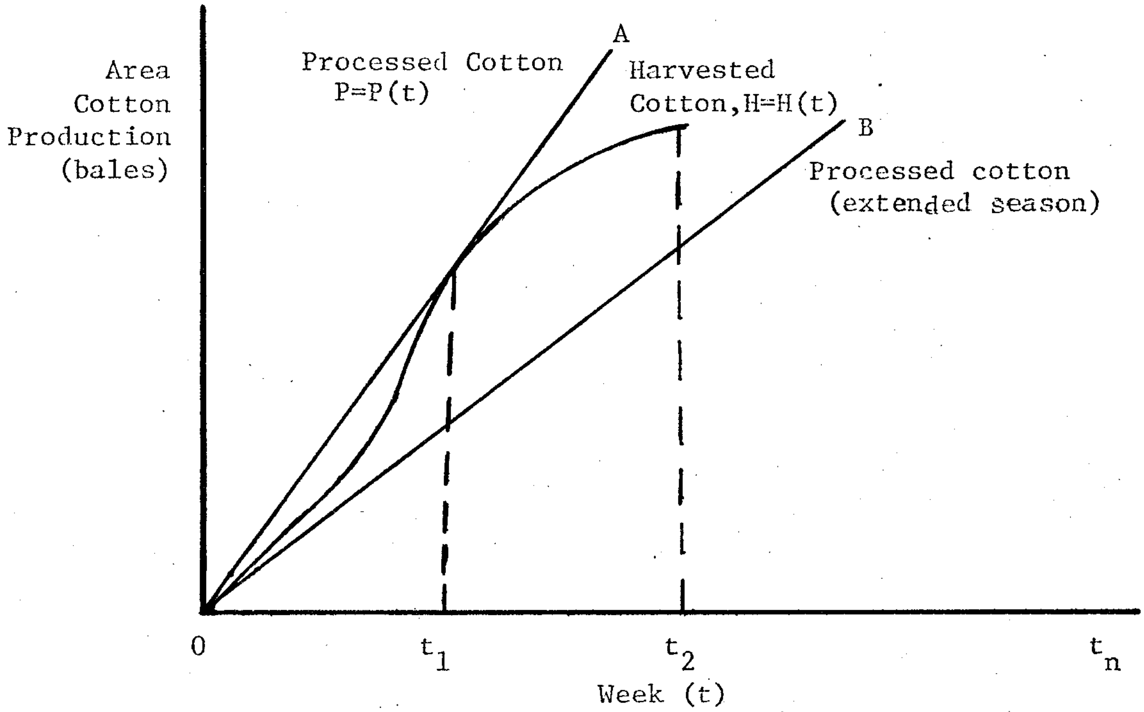

Figure 1. ACCUMULATION OF AREA HARVESTED AND PROCESSED COTTON BY WEEK

seasons extended to four, six, and nine months.

This paper reports the empirical results obtained when a long-run spatial model was applied to the cotton ginning industry in a Southwestern irrigated valley. A solution was obtained for each of three alternative processing season lengths and then each was compared to resolve the least-cost solution. In addition, the sensitivity of the solution to variations in model parameters was determined.

AN ALTERNATIVE STORAGE-ASSEMBLY -PROCESSING SYSTEM

To reduce the interdependence between the harvesting and processing operations of the conventional system, field storage was introduced into the alternative system. Seed cotton is unloaded from the harvesters into a slip-form (ricker) that is moved along the turnrow. The cotton is moderately compressed in the ricker and is then covered with plastic. The stacks remain in the field until needed for processing, when they are loaded for transporting to the gin plant.[2]

The assembly system is made up of a fleet of trucks, trailers, and seed cotton containers. Each container is capable of a 30,000-pound capacity. The containers are distributed to the fields where the stored cotton is transferred from the stack to the containers with front-end loaders. Each container unit is transported to a processing location on a specially designed trailer pulled by a truck. At the processing location, the container is removed from the trailer for later ginning while the truck-trailer combination is free to deliver empty containers to the field and retrieve loaded container units to the processing location.[3] The capacity of the assembly system is dictated by the bale-per-hour processing capacity of the gin plant, i.e., the assembly fleet's capacity must coincide with the gin plant's processing capacity.

The processing portion of the system approximates the conventional system except that the processing season is extended to four, six, and nine months. This is made possible with field storage.

PLANT LOCATION AND SENSITIVITY MODEL

The problem is to determine simultaneously the number, size, and location of gin plant(s) that minimize the transportation and processing costs

[2] This storage system has been extensively researched by engineers at Texas Tech. University and is now widely used in portions of the Texas High Plains.

[3] This storage-assembly-processing system has been successfully implemented by a firm in the High Plains of Texas.

involved in assembling and processing the seed cotton in the study area. The Stollsteimer plant location model was used, since it is designed to solve such problems [8]. A recent modification of this model by Chern and Polopolus [1] involves the substitution of a discontinuous plant cost function. However, the original formulation that specifies economies of scale in plant operation but plant costs independent of plant location was found to be appropriate for this problem.

Field-storage, which was assumed to exist in the alternative system, does not influence the least-cost solution, because the storage costs are independent of processing or transportation costs or those factors which affect optimum number, size, and location of plants. In contrast, if the storage system physical requirements were affected by gin plant size and if these facilities displayed economies of scale, then this cost should be entered into the plant location model.

Given m islands of production, each of which produces Xi of seed cotton to be processed at N or less locations, the problem is to solve for the number of plants $n \leqslant N$, that should be used, the locational configuration (Ln) for the n plant locations, and the size of plant at each chosen location. Algebraically, the objective is to minimize:

$$\underset{(n,Ln)}{TC} = \underset{(n,Ln)}{TPC} + \underset{(n,Ln)}{TTC}$$ [4]

where:

TC = total transportation and ginning costs for the study area industry,

TPC = total ginning or processing cost for the study area industry, and

TTC = total seed cotton transportation or assembly cost incurred in the study region.

With N potential plant locations, the objective is to find the optimum or least-cost locational pattern (Ln*) for each n subset. ($n = 1, 2 \ldots, n \leqslant N$.)
For each n location there are $(N!/(N-n)!n!)$ locational patterns (Ln).[5] To determine the optimum locational pattern (Ln*) and allocation of raw product, the following transportation cost function is minimized.

$$\text{Minimize: } TTC = \sum_{i=1}^{m} \sum_{j=1}^{n} XijCij|Ln$$

for each Ln, subject to

$$\sum_{j=1}^{n} Xij = Xi$$

= quantity of seed cotton available at production island i annually,

$$\sum_{i=1}^{m} Xij = Xj$$

= quantity of seed cotton shipped to plant location j annually,

Cij = cost of transporting one lint bale equivalent from production island i to plant j, and

$$Xj, Xij \leqslant 0 \quad Cij > 0.$$

The industry's total ginning cost is

$$TPC = \sum_{j=1}^{n} PjXj|Ln.$$

Processing cost for any jth plant is represented as

$$PjXj = \alpha + \beta Xj,$$

or, alternately, the industry's total ginning cost is

$$TPC = \sum_{j=1}^{n} (\alpha + \beta Xj) = n\alpha + \beta X$$

where βX becomes a constant and TPC is a function of n plant numbers. The value of the α parameter is the increase in cost of adding an additional plant into the system.

Ladd and Halvorson [4] developed the necessary logic to test the sensitivity of the Stollsteimer model's solution to changes in parameters. They show n* plants to be the optimum number when:

$$-\Delta TTC|n^* \geqslant \alpha \geqslant -\Delta TTC|\, n^*+1.$$

When the conditions of the above formulations are met, the reduction in plant cost (α) associated with the use of n*-1 plants is less than the additional

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[4] If the adopted storage system were to affect the optimal solution, it should also be included among those activities comprising total costs.

[5] Given N potential plant sites, the total number of locational patterns is

$$\sum_{n=1}^{N} \frac{N!}{(N-n)!n!} = 2^N - 1.$$

transportation cost incurred with n*-1 gin plants. $(\frac{dTTC}{dn} < 0)$. Likewise, the addition of a plant, n*+1, will result in an increase in total systems cost since the transportation cost savings associated with using n*+1 plants is less than the cost of an additional plant (α). Extending the above logic gives rise to a formulation capable of testing the sensitivity of the least-cost solution:

$$\frac{-\Delta TTC|(n+\delta n)}{R} \geqslant \frac{\alpha + \delta\alpha}{\epsilon R} \geqslant \frac{-\Delta TTC|(n+\delta n+1)}{R}.$$

For any value of $(\alpha+\delta\alpha) / \epsilon R$ satisfying the above, $n+\delta n$ is the optimum plant numbers. The R parameter represents the assembly system's transportation cost per mile while ϵ represents any change in these costs.[6]

SYSTEM COST FUNCTIONS

The economic-engineering technique was used to synthesize system costs. This involved the breakdown of the system into subsystems and then stages where the estimation of empirical input-output relationships were made. The resulting building blocks were then synthesized into hypothetical "models" of each subsystem. Cost data were used in combination with input-output relationship to estimate system costs.

Six "hypothetical" model ginning plants were synthesized for the estimation of plant costs. To determine the effect of a longer ginning season, plants were assumed to extend operation to four (Model I), six (Model II), and nine (Model III) months, which gave rise to three different long-run total annual processing costs. Plants were assumed to operate 15 hours per day for five days per week. Total processing cost for any jth plant (TPCj) was of the following general form:

	TPCj =	$\alpha + \beta Xj$
Model I:	TPCj =	\$124,948 + \$8.82Xj
Model II:	TPCj =	\$175,059 + \$7.91Xj
Model III:	TPCj =	\$229,545 + \$7.42Xj

A recent study by Moore [5] estimated similar cost relationships for gin plants. The constant term (α) may be interpreted as the minimum average long-run cost of establishing and maintaining a plant, while βXj represents those costs which are due to volume processed per plant (Xj).

Input-output data on the proposed transportation system were obtained from a firm which recently adopted an analogous system. Estimated per-unit costs of the proposed system are approximately 25 percent of those of the existing trailer transportation system. Transportation cost from the ith island of production to the jth plant site was found to be represented as follows:

$$TTCi = Xi(b_1 + b_2 Mij).$$

The b_1 parameter represents constant per-bale assembly costs or those costs not affected by length of haul, while the b_2 parameter is a proxy for those costs which are affected by miles of assembly (Mij) and represent costs per bale-mile.

Model I:	TTCi =	Xi(\$1.39 + \$.0228Mij)
Model II:	TTCi =	Xi(\$1.20 + \$.0228Mij)
Model III:	TTCi =	Xi(\$1.17 + \$.0228Mij)

The fixed and variable per-bale cost of field storing the valley's production was estimated to be \$1.16 and \$3.26, respectively. A study by Smith [7] verifies similar costs.

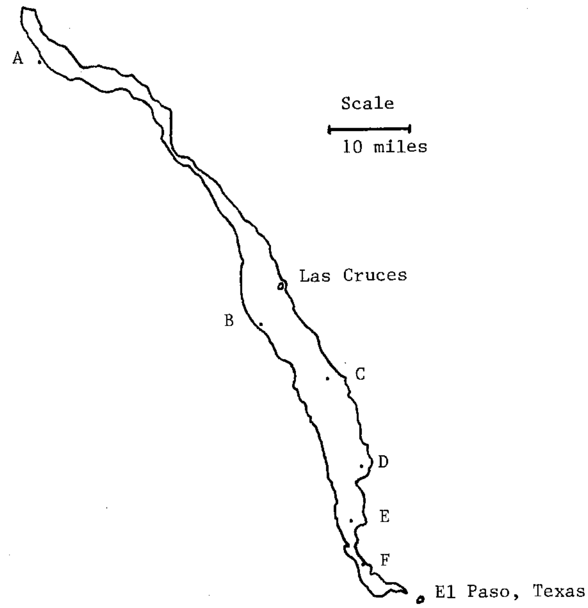
STUDY AREA

The study area was the irrigated, cotton-producing portion of the Rio Grande Valley in New Mexico (Figure 2). The valley is approximately 85 miles long and averages 2.5 miles in width. With the use of Agricultural Stabilization and Conservation Service aerial photos and production data, the valley was divided into 139 islands of production. Twelve potential plant locations were selected on the basis of accessibility, zoning laws, and concentration of cotton production. Actual plant locations were considered as potential plant sites except when the actual location was near a concentrated population center. A measurement of distance (Mij) between each production island i and each potential plant location j resulted in a 139 x 12 mileage matrix, which was converted to a transportation cost matrix by multiplying each matrix cell (Mij) with the b_2 parameter ($Cij = b_2 Mij$).

STUDY RESULTS

The optimal or least-cost plant locational pattern (Ln*) for one to four plants is shown in Table 1. If one plant were to be located in the study area, a plant located at site C would minimize total assembly costs. And, of all two-plant combinations $(\frac{12!}{10!2!} = 66)$, those located at sites A and D minimized total assembly cost. As plant numbers increase the average distance of assembly decreases and, as shown in Table

[6] If the value of Cij changes to $Cij + \delta Cij$, then $Cij + \delta Cij = \epsilon Cij + \lambda i > 0, \epsilon > 0$.



RIO GRANDE VALLEY, NEW MEXICO

Figure 2. LEAST-COST PLANT LOCATION CONFIGURATION, 1 TO 4 PLANT LOCATIONS

1, total transportation cost decreases, i.e., $\frac{dTTC}{dn} < 0$. The empirically determined optimal plant locational pattern reveals a tendency for plants to be concentrated in the most intensive production area – an anticipated locational pattern.

Total system cost (assembly plus processing cost) is minimized for all three models when the area's cotton production is assembled to a single plant location at site C. Given the area's production level, increasing plant numbers to two (optimally located at A and D) would necessitate a decrease in average plant size and a loss of total economies of scale for the industry. As shown in Table 1, the industry's savings in transportation cost associated with the operation of two plants does not offset the total loss in economies of scale for the processing industry.

At the valley's current production level, total processing costs are minimized with the four-month processing season (Model I). However, total systems costs (assembly and processing costs) are lowest with the six-month processing season (Model II). This is due to the intensified use of the assembly system and the resulting lower fixed cost per bale of assembly (b_1). The small savings of the six-month processing season over the four-month processing season may disappear if farmers' opportunity costs and possible additional risk of the extended season were included in the analysis.

The sensitivity of the least-cost solution was determined with respect to changes in the per bale-mile assembly cost parameter (b_2) and the α value – the minimum annual long-run cost of establishing and maintaining a plant. With the estimated cost parameters, the one-plant operation was least-cost. If plant costs in Models I, II, and III remain unchanged ($\delta\alpha$=0), but the per bale–mile assembly cost is varied, the least-cost number of plants changes as shown in Table 2. The single-plant solution is optimal for the three models as long as the value of b_2 is less than \$.3386, \$.4742, and \$.6220, respectively. The calculated b_2 parameter was \$.0228; ceteris paribus, the single plant solution is not sensitive to changes in the per bale-mile assembly

Table 1. OPTIMAL PLANT SIZE AND PLANT LOCATIONAL CONFIGURATION WITH ASSOCIATED PROCESSING AND TRANSPORTATION COSTS FOR 1 THROUGH 4 PLANT LOCATIONS, MODEL I, II, AND III.

			Model I				Model II				Model III			
Number of Plants	Optimum Location Set	Bales Processed per Plant	Bale/Hour Processing Capacity	Processing Cost	Trans-portation Cost	Total System Cost	Bale/Hour Processing Capacity	Processing Cost	Trans-portation Cost	Total System Cost	Bale/Hour Processing Capacity	Processing Cost	Trans-portation Cost	Total System Cost
	(Ln)			..... dollars				..... dollars				..... dollars		
1	C	50,000	46	565,948	90,898	656,846	30	570,559	81,398	651,957	20	600,545	79,898	680,443
2	D	38,921	35				24				16			
	A	11,079	10	690,896	82,480	773,376	7	745,618	72,980	818,598	5	830,091	71,480	901,571
3	B	18,770	17				12				8			
	A	8,830	8				5				4			
	E	22,400	20	815,845	77,955	893,800	14	920,677	68,455	989,132	9	1,059,637	66,955	1,126,592
4	D	13,975	13				9				6			
	B	16,590	15				10				7			
	A	8,830	8				5				4			
	F	10,605	10	940,793	76,181	1,016,974	6	1,095,736	66,681	1,162,417	4	1,289,183	65,181	1,354,364

Table 2. **RELATION BETWEEN NUMBER OF PLANTS IN LEAST-COST SOLUTION AND PER BALE-MILE COST OF ASSEMBLY (b_2) MODEL I, II, III**

Number of Plants in Minimum Cost Solution	Model I Cost of Assembly Operation per bale-mile (dollars)	Model II Cost of Assembly Operation per bale-mile (dollars)	Model III Cost of Assembly Operation per bale-mile (dollars)
1	$.3386 \geq b_2$	$.4742 \geq b_2$	$.6220 \geq b_2$
2	$.6295 \geq b_2 \geq .3386$	$.8819 \geq b_2 \geq .4742$	$1.1501 \geq b_2 \geq .6220$
3	$1.6100 \geq b_2 \geq .6295$	$2.2410 \geq b_2 \geq .8819$	$2.9501 \geq b_2 \geq 1.1501$
4	$2.8100 \geq b_2 \geq 1.6100$	$3.9405 \geq b_2 \geq 2.2410$	$5.1703 \geq b_2 \geq 2.9501$

Table 3. **RELATION BETWEEN NUMBER OF PLANTS IN LEAST-COST SOLUTION AND $\alpha, \delta\alpha$ FOR MODELS I, II, and III**

Number of Plants in Minimum Cost Solution	Interval Value for α (dollars)	Value of $\delta\alpha$ Model I (dollars)	Value of $\delta\alpha$ Model II (dollars)	Value of $\delta\alpha$ Model III (dollars)
1	$\alpha \geq 8418$	$\delta\alpha \geq -116530$	$\delta\alpha \geq -166641$	$\delta\alpha \geq -221128$
2	$8418 \geq \alpha \geq 4525$	$-116530 \geq \delta\alpha \geq -120423$	$-166641 \geq \delta\alpha \geq -170534$	$-221128 \geq \delta\alpha \geq -225021$
3	$4525 \geq \alpha \geq 1774$	$-120423 \geq \delta\alpha \geq -123174$	$-170534 \geq \delta\alpha \geq -173285$	$-225021 \geq \delta\alpha \geq -227772$
4	$1774 \geq \alpha \geq 1011$	$-123174 \geq \delta\alpha \geq -123937$	$-173285 \geq \delta\alpha \geq -174048$	$-227772 \geq \delta\alpha \geq -228535$

cost parameter.

To resolve the sensitivity of the least-cost solution to changes in the minimum annual long-run cost of establishing and maintaining a gin plant (α), it was necessary to fix b_2 or R and find those values of $\delta\alpha$ that would give rise to alternative optimal plant numbers. The results of this analysis are shown in Table 3. For any value of $\alpha \geqslant \$8{,}418$, the one-plant solution would remain optimal, i.e., the one-plant solution exists for Models I, II, and III as long as $\delta\alpha \geqslant \$116{,}530$, $\$-166{,}641$, and $\$-221{,}128$, respectively. The two-plant solution becomes optimal only if α were greater than $4,525 or less than $8,418. Because this would require at least the minimum, a 93 percent decrease in the α value, then, ceteris paribus, the optimal solution, for the study area, does not appear sensitive to changes in $\alpha(\delta\alpha)$.

If the above-described system were introduced into the study area, the storage, assembly, and processing costs were estimated to be $4.42, $1.62, and $11.41 per bale, respectively, or to total $17.45 per bale. Accomplishment of the same functions by the conventional system is estimated to cost $29.23 [2, 6]. At the present production level, the savings from the alternative system would represent a 28 percent return on capital investment.

Because the alternative system offers sizable savings and profitability, its eventual adoption would seem a reality. This study employed a static, partial equilibrium analysis which limits the ability to predict the ramifications of adoption. One would expect the primary beneficiary to be area cotton farmers; however, not all affected groups would gain, i.e., conditions of Pareto criterion would not be met. Potential direct losers are displaced employees and owners of ginning firms which will be forced out of business due to their cost disadvantage.

In an effort to evaluate the potential effects of this marketing technology, a new study has been initiated by the authors in cooperation with a sociologist. Its purpose is to determine the benefits and costs to affected parties and to examine institutional arrangements which may permit a more equitable distribution of costs and benefits.

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