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Persistent disequilibrium dynamics and economic policy

Luca Colombo* Gerd Weinrich†

February 12, 2009

Abstract

We develop a theoretical model involving temporary equilibria with quantity rationing in each period and price adjustment between periods. The resulting dynamic system may present a variety of dynamic behaviors, ranging from the convergence to stationary or quasi-stationary states, to complex or even chaotic dynamics. In particular, our framework has the property of being able to endogenously allow for the characterization of persistent disequilibrium phenomena – such us unemployment or deflation. It provides therefore for an ideal setup to investigate the effects and persistency of recessionary phases, and to study the effectiveness of different economic policies aimed at resolving them.

JEL classification: D45, D50, E32, E37

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1 Introduction

Disequilibrium phenomena seem to be common occurrences of many advanced economies. Starting with the Great Depression, the high unemployment rates in Europe spanning over a decade since the mid-Seventies, or the high inflation over the Eighties are just prominent examples. The Japanese recession started in the mid-Nineties and not yet fully resolved has put the combination of unemployment and deflation on the spotlight,

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making clear that a liquidity trap cannot be easily discarded as a purely theoretical possibility. Most recently, the current global recession prompted by the financial sector crisis seems to share similar features, and many observers and policy makers are invoking Keynesian type remedies.

Nonetheless, many economists still consider the representative agent flexible price model as the workhorse of macroeconomics, despite its conclusions are irremediably at odds with all the evidence on the persistence of disequilibrium phenomena (see, e.g., Blanchard, 2000).

For instance, a key result of the flexible price approach is that of money neutrality, which goes against the observed long lasting effects on output and employment of monetary shocks. The New Keynesian literature that developed over the Nineties (see, e.g., Ball and Romer, 1990, and Blanchard, 1990) has emphasized the role of nominal and real rigidities in the wage and price adjustment processes in determining large aggregate effects of monetary shocks. However, most economists maintain that the price level eventually adjusts so that money neutrality is restored. We claim instead that the result of money neutrality in the long run should not be taken for granted, and that the economy may remain stuck in a quasi-stationary state of permanent unemployment, in the absence of appropriate policy interventions.

Building on previous work (e.g. Colombo and Weinrich, 2003a, and Bignami, Colombo and Weinrich, 2004), we develop a conceptual framework that endogenously allows for the emergence of disequilibrium situations (such us unemployment or deflation), and provides therefore for an ideal setup to investigate the effects and persistency of shocks, and the effectiveness of different economic policies. In particular, we consider an economy consisting of overlapping generations consumers, firms producing by means of an atemporal production function, and a government financing public expenditure through a tax on firms' profits. Within each period prices are fixed, and consistent allocations are obtained by means of temporary equilibrium with stochastic rationing. Prices are then adjusted between successive periods according to the strength of rationing on each market in the previous period.

The gradual adjustment of prices and wages (and hence their inability to function as instantaneous and perfect allocation devices and the need for quantity adjustments to complement them in making trades feasible) is the primary mechanism to explain the propagation and the persistent real effects of shocks. It is worth stressing that we do not account endogenously for the reasons behind different degrees of wage and price stickiness, but we rather rely on exogenously given rules. Although in this perspective our approach is obviously *ad hoc*, it is on the other hand consistent with several underlying conceptual models of price rigidities (be they in the New Keynesian or in the Neoclassical tradition), and it allows us to effectively parametrize the different degrees of wage and price stickiness that are observed in reality.[1]

The role of price and wage rigidities is complemented by that of other factors amplifying the importance of the spillovers among markets and affecting the reaction of consumers and firms to shocks and policy interventions. Following Colombo and Weinrich (2006, 2008), we focus in particular on the role of consumers' expectations and firms' inventories. Expectations are especially important since they influence consumers' choices and hence the response of the economy to a shock. For instance, a restrictive shock may determine an aggregate demand deficit and lead the economy into a state of (Keynesian) unemployment. Finding ways to convince consumers to hold inflationary expectations – in this way increasing their current consumption and hence aggregate demand – may prove a powerful tool for recovering from the recession. At the same time, the explicit consideration of firms' inventories is important for fully assessing the impact of a shock on the economy. Focusing again on a restrictive shock, inventories have in fact an obvious reinforcement effect: by increasing the reduction in labor demand following the shock, inventories contribute to further depress the aggregate demand and to favor the convergence of the economy to a quasi-stationary state with permanent unemployment.

In the second part of the paper, we will discuss in detail the direct effects and the interplay of expectations and inventories in the propagation of shocks and in the explanation of their persistence, as well as their influence on the outcomes of different economic policies aimed at resolving or mitigating the effects of deflationary recessions.

The paper is organized as follows. In Section 2 we outline our base model, define a temporary equilibrium, show its existence and uniqueness, and study the dynamics of the economy. In Section 3 we investigate by means of numerical simulations the

[1] The New Keynesian literature, in particular, has investigated many possible causes for the real rigidity of prices and wages ranging from efficiency wages (see, for example, Shapiro and Stiglitz, 1984) and countercyclical mark-ups (e.g. Rotemberg and Woodford, 1991), to coordination failure (e.g. Ball and Romer, 1991) and credit markets imperfections (e.g. Bernanke and Gertler, 1989, and Kiyotaki and Moore, 1997). Attention has been devoted as well to the sources of nominal stickiness focusing, for instance, on menu cost (e.g. Mankiw, 1985), near rationality (e.g. Akerlof and Yellen, 1985) and staggered contracts (Calvo, 1983).

dynamic behavior of the economy, focusing on the effects of the adjustment of prices and wages, and showing the possibility of chaotic dynamic behavior as well as the emergence of a Phillips curve as an attractor of our dynamic system. In Section 4 we extend our base model to encompass the role of consumers' expectations and firms' inventories, and in Section 5 we study the dynamic behavior of the extended economy by focusing especially on the possible policy remedies to deflationary recessions. Section 6 summarizes and suggests avenues for future research.

2 The Base Model

Following Colombo and Weinrich (2003a) and Bignami, Colombo and Weinrich (2004), we focus on an economy composed of n OLG-consumers offering labor inelastically when young and consuming a composite consumption good in both periods of their life. The consumption good is produced by n' firms, using an atemporal production function whose only input is labor. The public sector of the economy is represented by a government that levies a proportional tax on firms' profits to finance its expenditure for goods. Budget deficits and surpluses may arise through money creation or destruction.

The timing of the model is such that the aggregate profit Π_{t-1} realized by firms in period $t-1$ is distributed at the beginning of period t in part as tax to the government ($tax\Pi_{t-1}$) and in part to young consumers ($(1-tax)\,\Pi_{t-1}$), where $0 \leq tax \leq 1$. Also at the beginning of period t old consumers hold a total quantity of money M_t – consisting of savings generated in period $t-1$ – that allows households to transfer purchasing power between periods.

We denote with X_t the aggregate quantity of the good purchased by young consumers in period t, p_t its price, w_t the nominal wage and L_t the aggregate quantity of labor. Then we get $M_{t+1} = (1-tax)\,\Pi_{t-1} + w_t L_t - p_t X_t$.

Letting G be the quantity of goods purchased by the government and taking into account that old households want to consume all their money holdings in period t, the aggregate consumption is $Y_t = X_t + \frac{M_t}{p_t} + G$. Since $\Pi_t = p_t Y_t - w_t L_t$, denoting with $\Pi_t - \Pi_{t-1} = \Delta M_t^P$ and $\Delta M_t^C = M_{t+1} - M_t$ the variation in the money stock held by producers before they distribute profits and by consumers, respectively, we obtain $\Delta M_t^C + \Delta M_t^P = p_t G - tax\Pi_{t-1} =$ budget deficit.

Young households first visit the labor market where they either can sell their inelastic labor supply ℓ^s, or they are rationed to zero. Then on the goods market they may be rationed according to the stochastic rule

$$x_t = \begin{cases} x_t^d & \text{with prob.} \quad \rho\gamma_t^d \\ c_t x_t^d & \text{with prob. } 1-\rho\gamma_t^d \end{cases},$$

where x_t^d is the quantity demanded, $\rho \in [0,1]$ a fixed structural parameter of the rationing mechanism, $\gamma_t^d \in [0,1]$ a rationing coefficient which the household perceives as given but which will be determined in equilibrium and $c_t = \frac{\gamma_t^d - \rho\gamma_t^d}{1-\rho\gamma_t^d}$. These settings are chosen such that the expected value of x_t is $\gamma_t^d x_t^d$, that is, $Ex_t = \gamma_t^d x_t^d$.

The effective demand x_t^{di}, $i = 0, 1$, is obtained from solving

$$\underset{x_t}{max}\ \rho\gamma_t^d u\left(x_t, \frac{\omega_t^i - x_t}{\theta_t^e}\right) + \left(1-\rho\gamma_t^d\right) u\left(c_t x_t, \frac{\omega_t^i - c_t x_t}{\theta_t^e}\right)$$

subject to the constraints $0 \le x_t \le \omega_t^i, 0 \le x_{t+1} \le \left(\omega_t^i - x_t\right)\frac{p_t}{p_{t+1}}, i = 0, 1,$ where $\omega_t^0 = \frac{1-tax}{p_t}\frac{\Pi_{t-1}}{n}$ and $\omega_t^1 = \omega_t^0 + \frac{w_t}{p_t}\ell^s$ are real income in case of rationing and no rationing on the labor market, respectively, and $\theta_t^e = p_{t+1}^e/p_t$ is the expected relative price for period t.

The aggregate supply of labor is $L^s = n\ell^s$. Denoting with L_t^d the aggregate demand of labor and with $\lambda_t^s = \min\left\{\frac{L_t^d}{L^s}, 1\right\}$ the fraction of young consumers that will be employed, the aggregate demand of goods of young consumers is

$$X_t^d = \lambda_t^s n x_t^{d1} + (1-\lambda_t^s)\, n x_t^{d0} \equiv X^d\left(\lambda_t^s; \frac{w_t}{p_t}, \frac{(1-tax)\,\Pi_{t-1}}{p_t}\right),$$

where x_t^{d0} and x_t^{d1} are the effective quantities demanded in case of rationing and no rationing, respectively, on the labor market. From the maximization of the expected utility $\gamma_t^d x_t^h \left(\left(\omega_t^i - x_t\right)/\theta_t^e\right)^{1-h}$ we obtain that the effective demand x_t^{di} is equal to $h\omega_t^i$, which is independent of γ_t^d and θ_t^e, although it depends on the real income ω_t^i.The total aggregate demand of the consumption sector is then obtained by adding old consumers' aggregate demand M_t/p_t and government demand G:

$$Y_t^d = X^d\left(\lambda_t^s; \alpha_t, (1-tax)\,\pi_t\right) + m_t + G_t,$$

where $\alpha_t = w_t/p_t, \pi_t = \Pi_{t-1}/p_t$ and $m_t = M_t/p_t$.

Each of the n' identical firms uses an atemporal production function $y_t = f(\ell_t)$. As with consumers, firms too may be rationed, by means of a rationing mechanism analogue to that assumed for the consumption sector. Denoting the single firm's effective demand of labor by ℓ_t^d, the quantity of labor effectively transacted is

$$\ell_t = \begin{cases} \ell_t^d & \text{with prob.} \quad \lambda_t^d \\ \\ 0 & \text{with prob. } 1 - \lambda_t^d \end{cases},$$

where $\lambda_t^d \in [0,1]$. On the goods market the rationing rule is

$$y_t = \begin{cases} y_t^s, & \text{with prob.} \quad \sigma\gamma_t^s \\ \\ d_t y_t^s, & \text{with prob. } 1 - \sigma\gamma_t^s \end{cases}, \tag{1}$$

where $\sigma \in (0,1)$, $\gamma_t^s \in [0,1]$ and $d_t = \frac{(\gamma_t^s - \sigma\gamma_t^s)}{(1-\sigma\gamma_t^s)}$. σ is a fixed parameter of the mechanism whereas λ_t^d and γ_t^s are perceived rationing coefficients taken as given by the firm the effective value of which will be determined in equilibrium. The definition of d_t ensures that $Ey_t = \gamma_t^s y_t^s$. It is obvious that $E\ell_t = \lambda_t^d \ell_t^d$.

The firm's effective demand $\ell_t^d = \ell^d(\gamma_t^s; \alpha_t)$ is obtained from the expected profit maximization problem

$$\max_{\ell_t^d} \gamma_t^s f\left(\ell_t^d\right) - \alpha_t \ell_t^d$$

subject to

$$0 \leq \ell_t^d \leq \frac{d_t}{\alpha_t} f\left(\ell_t^d\right).$$

The aggregate labor demand is $L_t^d = n'\ell_t^d(\gamma_t^s; \alpha_t) \equiv L^d(\gamma_t^s; \alpha_t)$ and, because only a fraction λ_t^d of firms can hire workers, the aggregate supply of goods is

$$Y_t^s = \lambda_t^d n' f\left(\ell^d(\gamma_t^s; \alpha_t)\right) \equiv Y^s\left(\lambda_t^d, \gamma_t^s; \alpha_t\right).$$

2.1 Temporary Equilibrium Allocations

For any given period t a feasible allocation can be described as a temporary equilibrium with rationing as follows.

Definition 1 : Given a real wage α_t, a real profit level π_t, real money balances m_t, a level of public expenditure G and a tax rate *tax,* a list of rationing coefficients $\left(\gamma_t^d, \gamma_t^s, \lambda_t^d, \lambda_t^s, \delta_t, \varepsilon_t\right) \in [0,1]^6$ and an aggregate allocation $\left(\overline{L}_t, \overline{Y}_t\right)$ constitute a *temporary equilibrium with rationing* if the following conditions are fulfilled:

(C1) $\overline{L}_t = \lambda_t^s L^s = \lambda_t^d L^d(\gamma_t^s; \alpha_t)$;
(C2) $\overline{Y}_t = \gamma_t^s Y^s(\lambda_t^d, \gamma_t^s; \alpha_t) = \gamma_t^d X^d(\lambda_t^s; \alpha_t, (1-tax)\pi_t) + \delta_t m_t + \varepsilon_t G$;
(C3) $(1-\lambda_t^s)(1-\lambda_t^d) = 0; (1-\gamma_t^s)(1-\gamma_t^d) = 0$;
(C4) $\gamma_t^d(1-\delta_t) = 0;\ \delta_t(1-\varepsilon_t) = 0$.

Conditions (C1) and (C2) require that expected aggregate transactions balance. This means that all agents have correct perceptions of the rationing coefficients. Equations (C3) formalize the short-side rule according to which at most one side on each market is rationed. The meaning of the coefficients δ_t and ε_t is that also old households and/or the government can be rationed. However, according to condition (C4) this may occur only after young households have been rationed (to zero).

Depending on which market sides are rationed, we can characterize different types of equilibrium. More precisely, we indicate with *Keynesian Unemployment* $[K]$ an equilibrium in which there is excess supply on both markets ($\lambda_t^s < 1,\ \gamma_t^s < 1$); with *Repressed Inflation* $[I]$ one in which there is excess demand on both markets ($\lambda_t^d < 1$, $\gamma_t^d < 1$); with *Classical Unemployment* $[C]$ one where there is excess supply on the labor market and excess demand on the goods market ($\lambda_t^s < 1,\ \gamma_t^d < 1$); and finally with *Underconsumption* $[U]$ an equilibrium with excess demand on the labor market and excess supply on the goods market $(\lambda_t^d < 1,\ \gamma_t^s < 1)$.

The existence and uniqueness of temporary equilibrium is shown in Colombo and Weinrich (2003a, pp. 9-12). In particular it is shown that the notion of temporary equilibrium with rationing defines a unique temporary equilibrium allocation given by $(\overline{L}_t, \overline{Y}_t) = (\mathcal{L}(\alpha_t, \pi_t, m_t, G, tax), \mathcal{Y}(\alpha_t, \pi_t, m_t, G, tax))$.

2.2 Dynamics

In order to investigate the dynamic behavior of the economy, we need to link successive periods, which is done by the adjustment of prices and by the changes in the stock of money and in profits. As for the latter, by definition of these variables one immediately obtains

$$\Pi_t = p_t \mathcal{Y}(\alpha_t, \pi_t, m_t, G, tax) - w_t \mathcal{L}(\alpha_t, \pi_t, m_t, G, tax)$$

and

$$M_{t+1} = (1-tax)\,\Pi_{t-1} + w_t\overline{L}_t - p_t\overline{X}_t = (1-tax)\,\Pi_{t-1} + w_t\overline{L}_t - p_t\overline{Y}_t + \delta_t M_t + \varepsilon_t p_t G =$$
$$(1-tax)\,\Pi_{t-1} - \Pi_t + \delta_t M_t + \varepsilon_t p_t G.$$

Regarding the adjustment of prices we make the standard assumption that whenever an excess of demand (supply) is observed, the price rises (falls). In terms of the rationing coefficients observed in period t, this amounts to

$$p_{t+1} < p_t \Leftrightarrow \gamma_t^s < 1;\; p_{t+1} > p_t \Leftrightarrow \gamma_t^d < 1;$$

$$w_{t+1} < w_t \Leftrightarrow \lambda_t^s < 1;\; w_{t+1} > w_t \Leftrightarrow \lambda_t^d < 1.$$

More precisely, in our numerical analysis, these adjustments have been specified by means of the non-linear rules:[2]

$$p_{t+1} = (\gamma_t^s)^{\mu_1} p_t,\text{ if } \gamma_t^s < 1;\; p_{t+1} = \left(\frac{\gamma_t^d + \delta_t + \varepsilon_t}{3}\right)^{-\mu_2} p_t,\text{ if } \gamma_t^d < 1;$$
$$w_{t+1} = (\lambda_t^s)^{\nu_1} w_t,\text{ if } \lambda_t^s < 1;\; w_{t+1} = \left(\lambda_t^d\right)^{-\nu_2} w_t,\text{ if } \lambda_t^d < 1,$$

where μ_1, μ_2, ν_1 and ν_2 are nonnegative parameters for the speeds of adjustment.

Then the adjustment equations for the real wage are

$$\alpha_{t+1} = \frac{(\lambda_t^s)^{\nu_1}}{(\gamma_t^s)^{\mu_1}}\alpha_t \text{ if } \left(\overline{L}_t, \overline{Y}_t\right) \in K \cup U,$$

$$\alpha_{t+1} = \frac{\left(\lambda_t^d\right)^{-\nu_2}}{\left(\frac{\gamma_t^d+\delta_t+\varepsilon_t}{3}\right)^{-\mu_2}}\alpha_t \text{ if } \left(\overline{L}_t, \overline{Y}_t\right) \in I,$$

$$\alpha_{t+1} = \frac{(\lambda_t^s)^{\nu_1}}{\left(\frac{\gamma_t^d+\delta_t+\varepsilon_t}{3}\right)^{-\mu_2}}\alpha_t \text{ if } \left(\overline{L}_t, \overline{Y}_t\right) \in C,$$

whereas $\theta_t = p_{t+1}/p_t$ is given by

$$\theta_t = (\gamma_t^s)^{\mu_1} \text{ if } \left(\overline{L}_t, \overline{Y}_t\right) \in K \cup U,$$

[2]The rules we consider here are the same used in Bignami, Colombo and Weinrich (2004). In Colombo and Weinrich (2003a) we consider instead linear adjustment rules. The different formulations of the adjustment mechanisms are without implications in terms of the qualitative results emerging from the numerical analysis of the economy dynamics.

$$\theta_t = \left(\frac{\gamma_t^d + \delta_t + \varepsilon_t}{3}\right)^{-\mu_2} \text{ if } \left(\overline{L}_t, \overline{Y}_t\right) \in I \cup C.$$

The dynamics of the model in real terms is given by the sequence $\{(\alpha_t, m_t, \pi_t)\}_{t=1}^{\infty}$, where α_{t+1} is as above and

$$\pi_{t+1} = \frac{[\mathcal{Y}(.) - \alpha_t \mathcal{L}(.)]}{\theta_t},$$

$$m_{t+1} = \frac{1}{\theta_t}\left[\delta_t m_t + \varepsilon_t G + (1 - tax)\,\pi_t\right] - \pi_{t+1}.$$

3 Complex Dynamics and the Phillips Curve as an Attractor

The dynamic behavior of the economy outlined above is described by a non-linear three-dimensional dynamical system with state variables α_t, m_t and π_t that entails three subsystems (corresponding to the three nondegenerate equilibrium regimes) each of which may become effective through endogenous regime switching.[3] In order to investigate the model dynamics one needs therefore to use numerical simulations.[4] We use the utility function $u(x_t, x_{t+1}) = x_t^h x_{t+1}^{1-h}$ and the production function $f(\ell) = a\ell^b$, and we specify the following parameter set, corresponding to a stationary Walrasian equilibrium as a benchmark case:

$$\begin{gathered} a = 1,\ b = 0.85,\ h = 0.5,\ L^s = 100,\ n' = 100,\ \alpha_0 = 0.85, \\ m_0 = 46.25,\ \pi_0 = 15,\ G = 7.5,\ tax = 0.5. \end{gathered} \tag{2}$$

As shown by Bignami, Colombo and Weinrich (2004), the dynamic behavior of the system is very sensitive to the choice of relevant parameters, such as the economic policy instruments. The complexity of the economy dynamics is clearly illustrated by the bifurcation diagram in Figure 1, showing the periodic (cycles of different orders) and non-periodic (chaotic) long-run characteristics of the system dynamics for different

[3] The underconsumption regime is degenerate in the sense that it can be seen as a limiting case of both the Keynesian and the inflationary regime.

[4] Our numerical simulations are based on the software MACRODYN that has been developed by Volker Böhm at the University of Bielefeld.

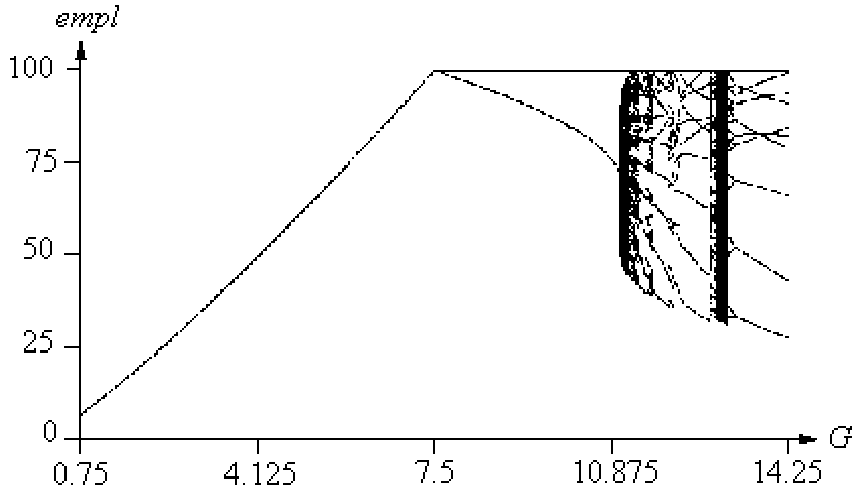

Figure 1: Bifurcation diagram for employment over government demand.

values of the government public expenditure G.[5] It is immediate to note that for G smaller than 7.5 (the Walrasian value) the economy converges to quasi-stationary states with (Keynesian) unemployment, which illustrates the possibility of permanent unemployment although prices and wages are flexible. It is also worth stressing that, as expected from textbook theory, an expansionary fiscal policy can help driving the economy towards full employment. However, a too expansionary fiscal policy ends up being destabilizing, as it induces an highly cyclical and irregular dynamic behavior, a feature that has to be added to the risk of inflation that is traditionally associated with expansionary policy measures.

The dynamic response of our economy to a shock also crucially depends on the values of prices and wage adjustment speeds. For instance, Bignami, Colombo and Weinrich (2004) have shown that following a restrictive monetary shock (e.g. a reduction to $m_0 = 40$) to allow for some wage flexibility downwards helps the system to return to the Walrasian equilibrium, as expected from textbook theory. However, further increasing the downward flexibility of wage over a certain threshold gives rise to irregular (chaotic) behavior with frequently high unemployment rates.[6] As analyzed in detail by Colombo

[5] The figure has been obtained by using the benchmark parameter set, except for G that is allowed to change, and by letting the nominal wages to be rigid downwards, i.e $\nu_1 = 0$. All other adjustment speeds are set equal to $\mu_1 = \mu_2 = \nu_2 = 0.4$.

[6] The threshold level of ν_1 at which irregular behavior appears is about 0.14 (see Bignami, Colombo

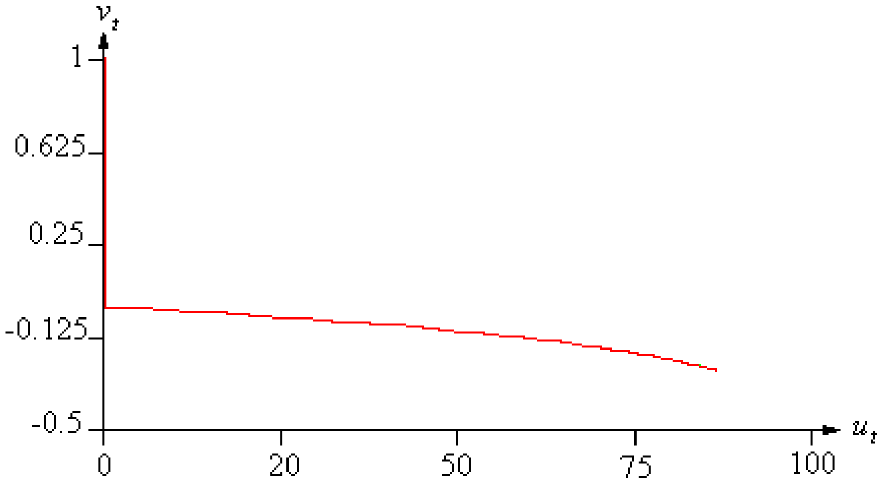

Figure 2: A long run Phillips-curve.

and Weinrich (2003a), a too high downwards adjustment speed of the wage has striking implications in terms of the dynamic behavior of employment and prices. This is evident from Figure 2, showing an attractor in the unemployment rate ($u_t = \left(L^s - \overline{L}_t\right)/L^s$) - inflation rate ($v_t = \left(w_{t+1} - w_t\right)/w_t$) plane. In an economic perspective, it is apparent that this attractor describes a Phillips curve. However, it is impossible to interpret this Phillips curve as a policy instrument in terms of a trade-off between unemployment and inflation as is commonly done. Any point on the curve is in fact but one element of a trajectory of pairs of rates of unemployment and wage inflation, and successive points of this trajectory may lie far away one from the other. Thus, even if the government tried to select a specific point on the curve in one period, in the next period already the system may go to a very different point on the curve.

4 A Model with Consumers' Expectations and Firms' Inventories

The main contribution of the setup presented in Section 2 consists in developing (and characterizing the dynamic behavior of) a framework in the Keynesian tradition able to account for the emergence of endogenous business cycles, and for the possible con-

and Weinrich, 2004, for a more detailed analysis).

vergence of the economy to quasi-stationary states characterized by disequilibrium situations (e.g. long-run unemployment or capacity under-utilization), even when prices and wages are allowed to adjust over time. Although providing useful results both in a methodological and in an economic perspective, this basic framework neglects several features that would be useful in using the above setup to investigate real world situations, and evaluate the effects of different economic policies. Improving over the modeling shortcomings of the basic setup becomes especially important if one has the ambition to use the above approach as a lens to interpret and investigate the effects of crises like the one we are experiencing these days, for which Keynesian type remedies are invoked and being adopted.

In a series of recent papers, that in a policy perspective are motivated mainly by the study of deflationary recessions (as the one that hit Japan over the Nineties), we extend our framework to embed features allowing us to investigate the effects of variables that provide important channels for the propagation and the persistency of shocks, and play a prominent role in the current policy debate. In particular, in Colombo and Weinrich (2006) we add to the basic model presented in Section 2 the possibility of inventories holding by firms, and in Colombo and Weinrich (2008) we focus on the role played by consumers expectations. In the following, we outline a model combining both these features.

We consider the same economy as in the basic setup introduced in Section 2. However, as we now explicitly consider the role of consumer expectations, whether old consumers hold a total quantity of money M_t (consisting of the savings generated in period $t-1$) at the beginning of period t depends on their price expectations for their second period of life. Since consumers may store the consumption good bought in the first period, they will voluntarily hold money only if they expect the good's price to decrease. They may be forced, however, to do this in case they are rationed in their consumption goods purchases in the first period. Firms too may now transfer unsold units of the consumption good into the future, as we allow for inventories holding by firms. Denoting with S_t the aggregate amount of inventories carried over by firms to period t, with Y_t^p the aggregate amount of goods produced and with Y_t the quantity sold in period t, there results $S_{t+1} = Y_t^p + S_t - Y_t$.

Taking expectations into account, young consumers have to decide whether to buy the quantities x_t and x_{t+1} in periods t and $t+1$, respectively, or buy the total quantity $x_t + x_{t+1}$ in period t and transfer x_{t+1} to period $t+1$. This in turn depends on the price expectation $\theta_t^e \equiv p_{t+1}^e/p_t$. If $\theta_t^e < 1$, then the consumer expects a decrease in the goods price and hence prefers to buy x_{t+1} in his second period of life, while if $\theta_t^e > 1$ he buys everything in his first period.

The case $\theta_t^e < 1$ is identical to the consumer's problem discussed in the base model of Section 2. As for the case $\theta_t^e > 1$, the consumer wants to buy the total quantity $x_t + x_{t+1} \equiv \widehat{x}_t$ in his first period of life, and thus has to meet the budget constraint

$$x_t + x_{t+1} \leq \omega_t^i \ , \ i = 0, 1 \ .$$

Monotonicity of the utility function implies that his effective demand is $\widehat{x}_t^{di} = \omega_t^i$. Hence, the aggregate demand of goods by young consumers in case of deflationary expectations $\theta_t^e < 1$ is

$$\begin{aligned} X_t^d &= \lambda_t^s n x_t^{d1} + (1 - \lambda_t^s)\, n x_t^{d0} \\ &= h\left[(1 - tax)\,\frac{\Pi_{t-1}}{p_t} + \frac{w_t}{p_t}\lambda_t^s L^s\right] \equiv X^d\left(\lambda_t^s; \frac{w_t}{p_t}, \frac{(1 - tax)\,\Pi_{t-1}}{p_t}, h\right) \ , \end{aligned} \tag{3}$$

whereas in case of inflationary expectations $\theta_t^e > 1$ it is

$$\begin{aligned} \widehat{X}_t^d &= \lambda_t^s n \widehat{x}_t^{d1} + (1 - \lambda_t^s)\, n \widehat{x}_t^{d0} \\ &= (1 - tax)\,\frac{\Pi_{t-1}}{p_t} + \frac{w_t}{p_t}\lambda_t^s L^s = X^d\left(\lambda_t^s; \frac{w_t}{p_t}, \frac{(1 - tax)\,\Pi_{t-1}}{p_t}, 1\right) . \end{aligned} \tag{4}$$

From (3) and (4) it is evident that the only difference in the aggregate effective demand by young consumers implied by different expectations $\theta_t^e <$ or > 1 lies in the multiplicative factor $\tau \in \{h, 1\}$. Therefore, we identify the value of τ with the corresponding expectation type.

The total effective aggregate demand of the consumption sector is now obtained, as in our base setup, by adding old consumers' aggregate demand $m_t = M_t/p_t$ and government demand G:

$$Y_t^d = X^d\left(\lambda_t^s; \alpha_t, (1 - tax)\,\pi_t, \tau\right) + m_t + G$$

where $\alpha_t \equiv w_t/p_t$ and $\pi_t \equiv \Pi_{t-1}/p_t$.

Turning now to the production sector, we continue to assume that all firms are identical and produce according to the production function $y_t^p = f(\ell_t) = a\ell_t^b, a, b > 0$. Denoting with s_t the inventories held at the beginning of period t, the total amount supplied by a firm is $y_t^s = y_t^p + s_t$. As for firms' rationing, we assume again 0-1 rationing in the labor market and thus, recalling that ℓ_t^d is the single firm's effective demand of labor and $\lambda_t^d \in [0,1]$ is the probability that the firm is not rationed on the labor market, it follows that $E\ell_t = \lambda_t^d \ell_t^d$. On the goods market the rationing rule is the same proposed in the base model (see (1)), where the firm's effective supply becomes now $y_t^s = f\left(\ell_t^d\right) + s_t$.

From the maximization of expected profits, $\gamma_t^s\left[f\left(\ell_t^d\right) + s_t\right] - \alpha_t \ell_t^d$ we get each firm's effective labor demand as

$$\ell_t^d = \ell^d\left(\gamma_t^s; \alpha_t\right) = \left(\frac{\gamma_t^s ab}{\alpha_t}\right)^{\frac{1}{1-b}}, \tag{5}$$

which is independent of s_t. The aggregate labor demand then is $L_t^d = n'\ell^d\left(\gamma_t^s; \alpha_t\right) \equiv L^d\left(\gamma_t^s; \alpha_t\right)$ and, because only a fraction λ_t^d of firms can hire workers, the aggregate supply of goods is

$$Y_t^s = \lambda_t^d n' f\left(\ell^d\left(\gamma_t^s; \alpha_t\right)\right) + S_t \equiv Y^s\left(\lambda_t^d, \gamma_t^s; \alpha_t, S_t\right). \tag{6}$$

4.1 Temporary Equilibrium Allocations

For any t, the definition of a temporary equilibrium with rationing is now described by the following

Definition 2 Given a real wage α_t, a real profit level π_t, real money balances m_t, inventories S_t, a level of public expenditure G, a tax rate tax and an expectation type $\tau \in \{h, 1\}$, a list of rationing coefficients $\left(\gamma_t^d, \gamma_t^s, \lambda_t^d, \lambda_t^s, \delta_t, \varepsilon_t\right) \in [0,1]^6$ and an aggregate allocation $\left(\overline{L}_t, \overline{Y}_t\right)$ constitute a *temporary equilibrium with rationing* if the following conditions are fulfilled:

(C1) $\overline{L}_t = \lambda_t^s L^s = \lambda_t^d L^d\left(\gamma_t^s; \alpha_t\right);$

(C2) $\overline{Y}_t = \gamma_t^s Y^s\left(\lambda_t^d, \gamma_t^s; \alpha_t, S_t\right) = \gamma_t^d X^d\left(\lambda_t^s; \alpha_t, (1 - tax)\,\pi_t, \tau\right) + \delta_t m_t + \varepsilon_t G;$

(C3) $\left(1 - \lambda_t^s\right)\left(1 - \lambda_t^d\right) = 0; \left(1 - \gamma_t^s\right)\left(1 - \gamma_t^d\right) = 0;$

(C4) $\gamma_t^d\left(1 - \delta_t\right) = 0;\ \delta_t\left(1 - \varepsilon_t\right) = 0.$

The four conditions in the above definition have exactly the same interpretation as in the base model of Section 2, from which they differ just for the explicit consideration of the role of expectations and inventories. Colombo and Weinrich (2008, Proposition 1) show the existence of a unique temporary equilibrium allocation given by $\left(\overline{L}_t, \overline{Y}_t\right)$ $= \left(\mathcal{L}\left(\alpha_t, \pi_t, m_t, S_t, G, tax, \tau\right), \mathcal{Y}\left(\alpha_t, \pi_t, m_t, S_t, G, tax, \tau\right)\right)$.

4.2 Dynamics

In this extended framework with expectations and inventories, the link between successive periods is given by the adjustment of prices, by the changes in the stock of money and in profits and by possible changes in the expectation type. For given τ, the adjustment of prices and wages is again such that the price rises (falls) whenever an excess of demand (supply) is observed. More precisely, our simulations are based on the following adjustment mechanisms:[7]

$$p_{t+1} = \begin{cases} \left[1 - \mu_1 \left(1 - \gamma_t^s\right)\right] p_t & \text{if } \gamma_t^s < 1 \\ \left[1 + \mu_2 \left(1 - \frac{\gamma_t^d + \delta_t + \varepsilon_t}{3}\right)\right] p_t & \text{if } \gamma_t^d < 1 \end{cases} \tag{7}$$

$$w_{t+1} = \begin{cases} \left[1 - \nu_1 \left(1 - \lambda_t^s\right)\right] w_t & \text{if } \lambda_t^s < 1 \\ \left[1 + \nu_2 \left(1 - \lambda_t^d\right)\right] w_t & \text{if } \lambda_t^d < 1 \end{cases} \tag{8}$$

where μ_1, μ_2, ν_1, $\nu_2 \in [0, 1]$ can be interpreted as the degree of flexibility of adjustment. The dynamics of the real wage is then described by the following equations:

$$\alpha_{t+1} = \begin{cases} \frac{1-\nu_1(1-\lambda_t^s)}{1-\mu_1(1-\gamma_t^s)} \alpha_t & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in K \\ \frac{1-\nu_1(1-\lambda_t^s)}{1+\mu_2\left(1-\frac{\gamma_t^d+\delta_t+\varepsilon_t}{3}\right)} \alpha_t & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in C \\ \frac{1+\nu_2\left(1-\lambda_t^d\right)}{1+\mu_2\left(1-\frac{\gamma_t^d+\delta_t+\varepsilon_t}{3}\right)} \alpha_t & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in I \\ \frac{1+\nu_2\left(1-\lambda_t^d\right)}{1-\mu_1(1-\gamma_t^s)} \alpha_t & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in U \end{cases} \tag{9}$$

whereas the inflation factor $\theta_t = p_{t+1}/p_t$ is given by

$$\theta_t = \begin{cases} 1 - \mu_1 \left(1 - \gamma_t^s\right) & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in K \cup U \\ 1 + \mu_2 \left(1 - \frac{\gamma_t^d + \delta_t + \varepsilon_t}{3}\right) & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in C \cup I \end{cases} . \tag{10}$$

[7]Differently than in the base model of Section 2, we consider now linear adjustment rules, to explicitly show that our results do not depend on the non-linearity of the adjustment mechanisms.

The dynamics of profits, money and inventories follow from the definition of these variables and equations (9) - (10), i.e.

$$\pi_{t+1} = \frac{\overline{Y}_t - \alpha_t \overline{L}_t}{\theta_t},$$

$$\begin{aligned} m_{t+1} &= \frac{M_{t+1}}{p_{t+1}} = \frac{1}{p_{t+1}} \left[(1 - tax) \Pi_{t-1} + w_t \overline{L}_t - p_t \overline{Y}_t + \delta_t M_t + \varepsilon_t p_t G \right] \\ &= \frac{1}{\theta_t} \left[(1 - tax) \pi_t + \alpha_t \overline{L}_t - \overline{Y}_t + \delta_t m_t + \varepsilon_t G \right] \\ &= \frac{1}{\theta_t} \left[\delta_t m_t + \varepsilon_t G + (1 - tax) \pi_t \right] - \pi_{t+1} \end{aligned}$$

and

$$S_{t+1} = Y^s \left(\lambda_t^d, \gamma_t^s; \alpha_t, S_t \right) - \overline{Y}_t = \lambda_t^d n' a \left(\frac{\gamma_t^s ab}{\alpha_t} \right)^{\frac{b}{1-b}} + S_t - \overline{Y}_t \ ,$$

where

$$\overline{Y}_t = \mathcal{Y} \left(\alpha_t, \pi_t, m_t, S_t, G, tax, \tau \right) \text{ and } \overline{L}_t = \mathcal{L} \left(\alpha_t, \pi_t, m_t, S_t, G, tax, \tau \right) \ .$$

It follows that the dynamics of the model is then given by the sequence $\{(\alpha_t, m_t, \pi_t, S_t)\}_{t=1}^{\infty}$.

We now need to consider the possibility of expectation switching, which should occur whenever it is required in order to keep expectations correct along a trajectory of the system. To illustrate the point, consider the case in which consumers have deflationary expectations in period t ($\theta_t^e \leq 1$ or, equivalently, $\tau = h$) but the equilibrium in period t is such that there is excess demand on the goods market and thus $p_{t+1} > p_t$. The assumption $\tau = h$ in period t has then been incorrect and we substitute it by $\tau = 1$, i.e. $\theta_t^e > 1$. Obviously then a different equilibrium arises in period t but we claim that the type of equilibrium is still such that there is excess demand on the goods market. Thus, expectations have been adjusted so as to become correct. Analogously we correct the expectations in case $\theta_t^e > 1$ but the equilibrium in period t involves excess supply on the goods market. The rationale for doing this is given by the following lemma, that is proven in Colombo and Weinrich (2008).

Lemma 1 *Assume that for $\tau = h$ in period t an equilibrium with $\gamma_t^d < 1$ occurs. Then this inequality is preserved when switching in period t to $\tau = 1$. Conversely, assume that for $\tau = 1$ in period t an equilibrium with $\gamma_t^s < 1$ occurs. Then this inequality is preserved when switching in period t to $\tau = h$.*

Taking expectations switching into account, a trajectory of the dynamic system is given by a sequence $\{(\alpha_t, m_t, \pi_t, S_t, \tau_t)\}_{t=1}^{\infty}$. It is important to stress that these state variables are perfectly foreseen by economic agents in any period t, so that our dynamic system is a truly forward looking one.[8]

5 Numerical Analysis: the Study of Deflationary Recessions

The non-linear dynamic system representing the economy outlined in the previous section is substantially more complicated than the three dimensional one investigated in Section 3. In particular, we now have to deal with a five-dimensional system – with state variables $\alpha_t, m_t, \pi_t, S_t$ and τ_t – composed of four non-degenerate subsystems each of which may become effective through endogenous regime switching. In analyzing the dynamics of the model, we therefore need to resort to numerical simulations. For this purpose, we consider the same benchmark parameter set used for the base model, that is (2), corresponding to the stationary Walrasian equilibrium, with the addition of $S_0 = 0$ and $\tau_0 = h$, with trading levels $L^* = Y^* = 100$.

Several factors are shown to complement the role of price and wage stickiness (flexibility) – that we illustrated in Section 3 – in determining the dynamic effects of a shock. The model outlined in Section 4, by considering explicitly firms' inventories and consumers' expectations, adds further propagation mechanisms for shocks and helps explaining their persistence.

Consumers' expectations, in particular, have played an important role in the policy debate. For instance, the 2008 Nobel Laureate Paul Krugman, in a series of influential articles on the Japanese deflationary recession (see, e.g. Krugman, 1998), stressed the importance of creating inflationary expectations to overcome the liquidity trap in which the country got stuck. Although our setup does not offer insights on how to create inflationary expectations in the first place, it theoretically supports this claim. To see why, let us start from our benchmark parameter set (2) and consider a restrictive monetary shock determined by a reduction in the initial money stock to $m_0 = 40$, keeping

[8] In most cases of dynamic systems in economics, they are given by a system of implicit difference equations, in which case an explicit solution in the sense of a (local) flow of mappings cannot be computed analytically. On the other hand, models that avoid this problem - giving rise to truly forward looking dynamic systems - typically are not compatible with perfect foresight outside the stationary state. For a systematic discussion of this issue see e.g. Böhm and Wenzelburger (1999).

all other parameters and initial values at their Walrasian levels. Letting $\nu_1 = 0.025$ and $\nu_2 = 0.1$ be the downward and upward adjustment speeds of wages, respectively, and $\mu_1 = \mu_2 = 0.1$ the adjustment speeds of prices out of the Walrasian equilibrium, Figure 3 shows the convergence of the dynamic system to a deflationary recessionary quasi-stationary state $(\overline{\alpha}, \overline{m}, \overline{\pi}, \overline{S}, \overline{\tau})$ entailing a permanent decrease in employment and output.[9]

Consider now a change in consumers' expectations from $\tau = h$ to $\tau = 1$, that is $(\alpha_0, m_0, \pi_0, S_0, \tau_0) = (\overline{\alpha}, \overline{m}, \overline{\pi}, \overline{S}, 1)$. Expecting inflation for the next period, young consumers demand all their planned life-time consumption in the first period, which boosts aggregate demand and can potentially lead the economy out of the recession. This is shown in Figure 4 for $\nu_1 = 0.025$ and $\nu_2 = 0.1$, where the economy returns immediately to full employment and the inflationary expectations are confirmed. More precisely, the percentage price inflation, $\phi_t = 100\,(p_{t+1} - p_t)\,/p_t$, remains positive over a prolonged period of time, meaning that the economy finds itself each period in a state of repressed inflation, which confirms the validity of Krugman's policy suggestion.

It is also interesting to note that the degree of stickiness of the nominal wage (and price) seems to affect the ability of inflationary expectations to restore full employment. Figure 5 shows that, when a higher downward flexibility of the wage rate is assumed ($\nu_1 = 0.06$ in the figure), the increase in aggregate demand due to anticipated purchases by consumers is not sufficient to overcome the recession, and expectations return from inflationary to deflationary after one period. This confirms that inflationary expectations need to be sustained over time to help escaping a deflationary recession; something in the spirit of the 'irresponsible' monetary policy – i.e., a monetary policy remaining expansionary even when prices start rising – advocated by Krugman (1998).

It is finally worth noting that, for intermediate values of downward nominal wage flexibility, multiple equilibria with self-confirming expectations will emerge (see Colombo and Weinrich, 2008), which suggests that imposing downward wage rigidity may be a useful measure to overcome a recession when inflationary expectations alone are not enough to do so.

Further measures that can be efficiently analyzed in our framework and complement

[9] The downward speed of the wage adjustment plays a crucial role. Until approximately $\nu_1 = 0.018$ the economy is able to return to full employment after the monetary shock, whereas for speeds of wage adjustment larger than this it gets trapped in the underemployment situation shown in Figure 3 (see Colombo and Weinrich (2008) for a detailed bifurcation analysis).

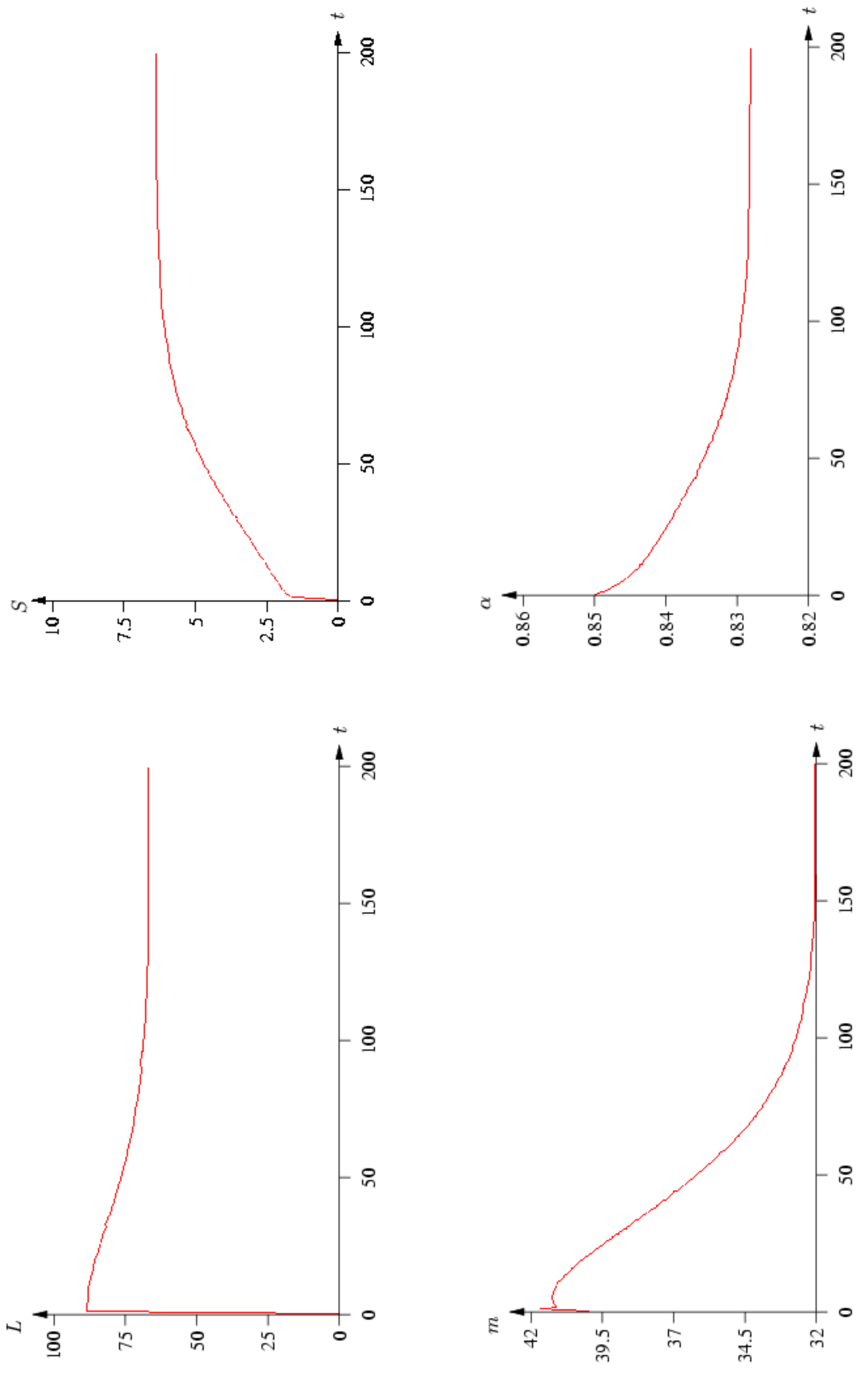

Figure 3: Time series when $\nu_1 = 0.025$ and $m_0 = 40$.

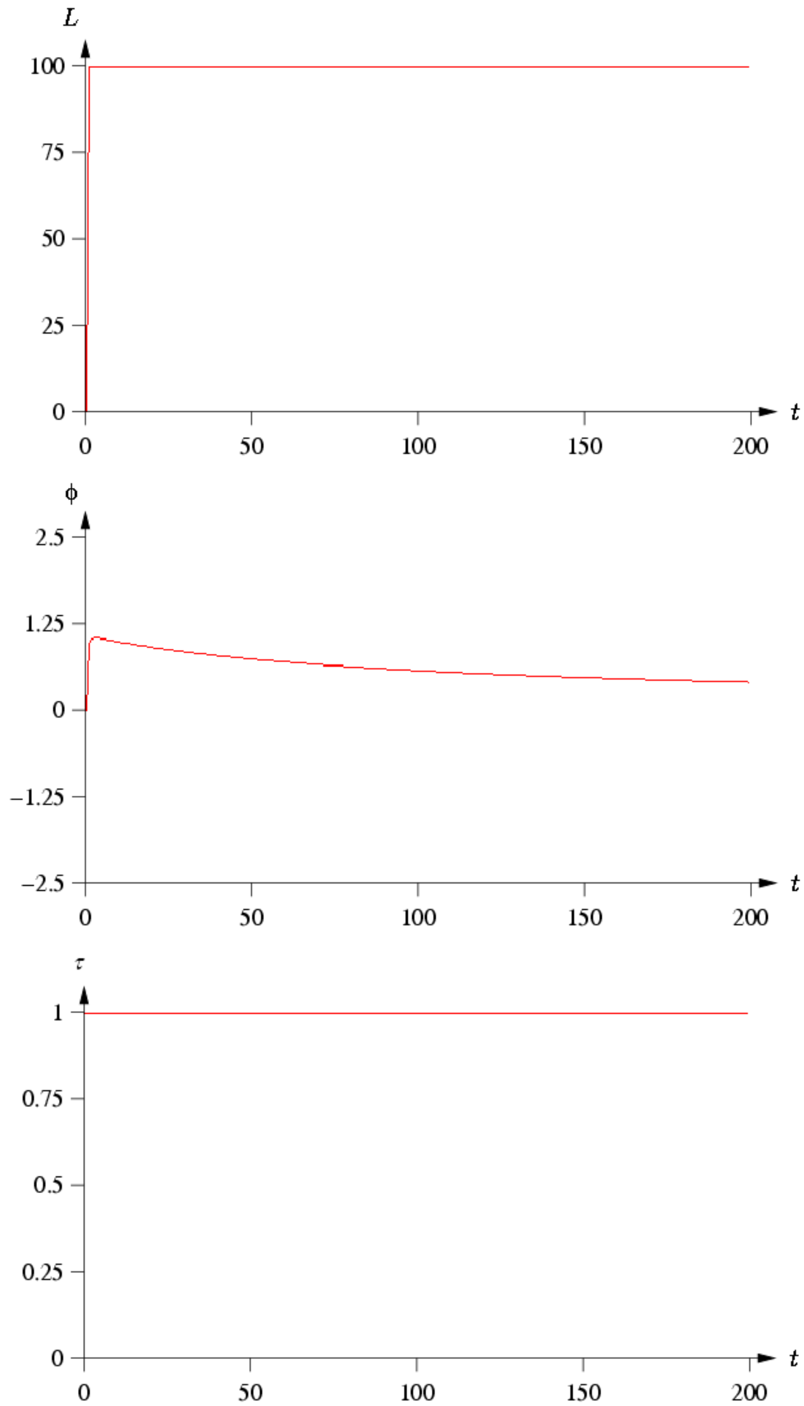

Figure 4: Effect of inflationary expectations if $\nu_1 = 0.025$.

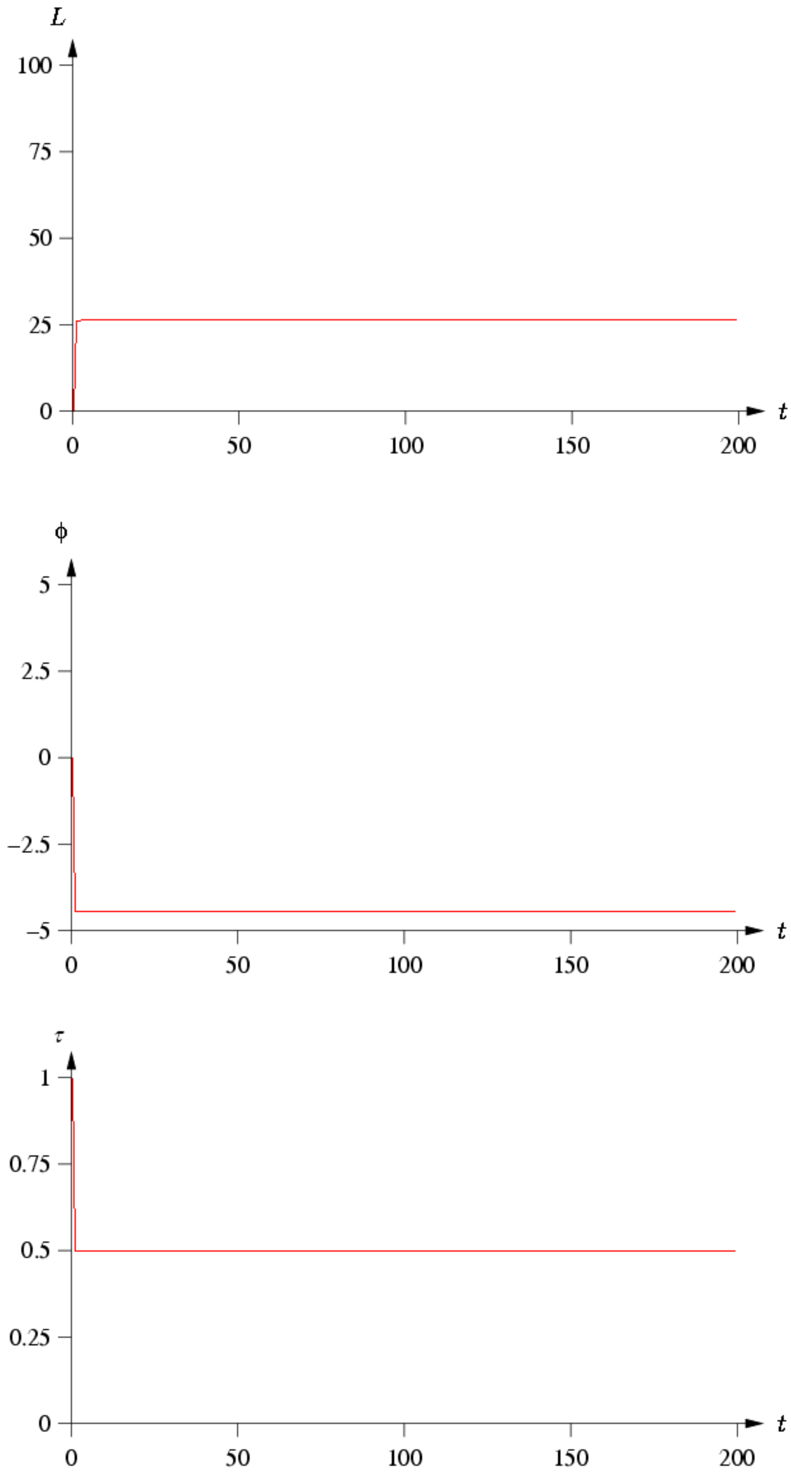

Figure 5: Effect of inflationary expectations if $\nu_1 = 0.06$.

the role of inflationary expectations are based on expansionary fiscal and monetary policies. Specifically, one such measure that received considerable attention in the policy debate has been proposed in 2003 by Ben Bernanke for the Japanese recession. It consists in a tax reduction accompanied by a transfer of funds from the Central Bank to the government to compensate for the loss in tax revenue. In Colombo and Weinrich (2003b) we formally analyze this policy and show under which conditions it can effectively help to overcome the crisis.

By focusing on consumers' expectations, in the above analysis we left in the background the effects of firms' inventories. Although a full exploration of the role of inventories is behind the scope of this paper (we refer the reader to Colombo and Weinrich, 2006), it is easily seen that inventories amplify the importance of the spillover effects among markets. More precisely, following a restrictive monetary shock as the one considered above and provided wages are flexible downward, the nominal wage diminishes, which (if the decrease in the nominal wage is large enough) implies a reduction of the real wage as well. The presence of inventories, by increasing the fall of labor demand that in turn depresses labor income and aggregate demand, crucially reinforces this reduction. Eventually the economy converges to a quasi-stationary state with permanent unemployment, a constant low real wage and permanent deflation of the nominal variables.[10]

6 Concluding Remarks

In this paper, we outlined a truly forward looking dynamic framework that generates a wide variety of dynamic behaviors, ranging from cycles of different orders and complex/chaotic behavior to convergence towards quasi-stationary states where the economy lies persistently far away from its Walrasian equilibrium.

Being capable to endogenously determine the emergence of disequilibrium situations, our setup provides an ideal framework to represent and explain the causes of

[10] The dynamic behavior of the economy following a restrictive shock becomes completely different whenever the nominal wage is rigid downward. In this case, the real wage and the real money stock increase up to the point in which there start to be excess demand on the goods market and excess supply on the labor market. At this point the goods price starts increasing again, which implies a reduction of the real wage and of the real money stock until the economy converges back to the Walrasian equilibrium. Therefore, unlike in the case where nominal wages are flexible downward, imposing downward nominal wage rigidity may be a measure limiting the effects of deflationary recessions.

prolonged economic crises, such as the recent Japanese deflationary recession, and to evaluate the impact of alternative economic policies aimed at resolving them. In this sense, our modeling may also prove useful in the analysis of the effects and consequences of the remedies that are being proposed to face the current global recession determined by the breakdown of the financial system.

Several extensions are possible that would extend the reach of our analysis. Three of them stand at the center of our current research agenda in this field. The first deals with the formation of expectations. In the current framework, we focus on the role of agents' expectations without investigating the process from which they arise. Accounting endogenously for the mechanism of expectations formation may enrich the scope of our analysis, by further highlighting the transmission channels of shocks and the effectiveness of alternative economic policies.

A second feature of the model that would benefit from an in-depth examination deals with inventories. In this paper, we focus essentially on their role as a propagation mechanism of shocks and as a source of spillovers between markets. At the beginning of each period, the stock of inventories carried by a firm is simply what remains unsold at the end of the previous period. In this capacity, inventories are a passive element in the decision problem of firms, while it would be more satisfactory to consider them as strategic choice variables as suggested in the literature (see, e.g., Blinder and Fischer, 1981, and Blinder, 1982, where firms have a target inventory level and want to keep a certain inventories-to-sale ratio).

Finally, a third aspect of our modeling that could be refined has to do with the adjustment mechanism of prices and wages. As we discussed in the Introduction, the mechanism by which prices and wages are adjusted between periods is exogenously given in the current formulation of the model. On the one hand, this adds to the flexibility of the approach, by making it consistent with a wide variety of possible explanations for rigidities. On the other hand, in many circumstances it may be interesting to focus on specific sources of stickiness, and endogenize them, to better explore their implications in terms of the feedback and spillover effects arising in the economy.

To explore the implications of these extensions and to generalize the scope of our approach lies at the core of our current research.

References

Akerlof, G. A. and J. L. Yellen, 1985. "A Near-Rational Model of the Business Cycle with Wage and Price Inertia", *Quarterly Journal of Economics* 100, 823-838.

Ball, L. and Romer, D., 1990. "Real Rigidities and the Non-Neutrality of Money", *The Review of Economic Studies* 57(2), 183-203.

Ball, L. and D. Romer, 1991. "Sticky Prices as Coordination Failure", *American Economic Review* 81, 539-553.

Bernanke, B. and M. Gertler, 1995. "Inside the Black Box: The Credit Channel of Monetary Policy Transmission", *Journal of Economic Perspectives* IX, 27-48.

Bignami, F., L. Colombo and G. Weinrich, 2004. "Complex Business Cycles and Recurrent Unemployment in a Non-Walrasian Macroeconomic Model", *Journal of Economic Behaviour and Organization*, 53, 173-191.

Blanchard, O.J., 1990. "Why Does Money Affect Output? A Survey", in B. Friedman and F. Hahn (eds.), *Handbook of Monetary Economics*, Amsterdam, North Holland.

Blanchard, O.J., 2000. "What Do We Know About Macroeconomics That Fisher And Wicksell Did Not?", *Quarterly Journal of Economics*, 1375-1409.

Blinder, A., 1982. "Inventories and Sticky Prices", *American Economic Review* 72, 334-348.

Blinder, A.S. and S. Fischer, 1981. "Inventories, Rational Expectations, and the Business Cycle", *Journal of Monetary Economics* 8, 277-304.

Böhm, V. and J. Wenzelburger, 1999. "Expectations, Forecasting and Perfect Foresight - A Dynamical Systems Approach", *Macroeconomic Dynamics* 3, 167-186.

Calvo, G., 1983. "Staggered Prices in a Utility-maximizing Framework", *Journal of Monetary Economics* 12, 383-398.

Colombo, L. and G. Weinrich, 2003a. "The Phillips Curve as a Long-Run Phenomenon in a Macroeconomic Model with Complex Dynamics", *Journal of Economic Dynamics and Control*, 28(1), 1-26.

Colombo, L. and G. Weinrich, 2003b. "Unemployment and Inventories in the Business Cycle", Contributi di ricerca in Econometria e Matematica n. 69, Università Cattolica - Milano, available at http://www.unicatt.it/docenti/weinrich/invent.pdf.

Colombo, L. and G. Weinrich, 2006. "Deflationary Recessions in a General Equilibrium Framework", in A. Namatame, T. Kaisouji and Y. Aruka (eds.), *The Complex*

Networks of Economic Interactions, Lecture Notes in Economics and Mathematical Systems, Springer-Verlag, Berlin, Heidelberg.

Colombo, L. and G. Weinrich, 2008. "The Role of Expectations and Wages in Deflationary Recessions", *mimeo,* downloadable at:
http://www.unicatt.it/docenti/weinrich/expdefrec.pdf.

Kiyotaki, N. and J. Moore, 1997. "Credit Cycles", *Journal of Political Economy* 105, 211-248.

Krugman, P., 1998. "It's Baaack: Japan's Slump and the Return of the Liquidity Trap", *Brookings Papers on Economic Activity*, 2, pp. 137-187.

Mankiw, N.G., 1985. "Small Menu Costs and Large Business Cycles", *Quarterly Journal of Economics* 100, 529-537.

Rotemberg, J.J. and M. Woodford, 1991. "Markups and the Business Cycle", *NBER Macroeconomics Annual*, O. Blanchard and S. Fischer (eds.), Cambridge, MIT Press, 63-128.

Shapiro, C. and J. Stiglitz, 1984. "Equilibrium Unemployment as a Worker Discipline Device", *American Economic Review* 74, 433-444.

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