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# Econometric Analysis of Fiscal Policy Budget Constraints in Endogenous Growth Models 

Yoshihiko Tsukuda<br>Tohoku University

Tatsuyoshi Miyakoshi<br>Osaka University


#### Abstract

Kneller et al. (1999) examined the predictions of the public-policy endogenous growth models of Barro (1990) and others that suggest that unlike distortionary taxation and productive expenditures, nondistortionary taxation and nonproductive expenditures have no direct effect on the rate of growth. This paper provides an econometric theory with their empirical methodology and applies to work by Kneller et al. (1999) as a numerical example to show how the econometric theory works in practice. This paper also confirms from the viewpoint of econometric analysis that their study supports the Barro (1990)'s predictions.


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## 1. Introduction

The public-policy endogenous growth models of Barro (1990) provide mechanisms by which fiscal policy can determine the economic growth. His models classify fiscal variables of the government budget into one of four categories: distortionary or nondistortionary taxation and productive or nonproductive expenditures. In Barro's prediction, nondistortionary taxation and nonproductive expenditures have no effect on the rate of growth, while distortionary taxation and productive expenditures have direct effects. Subsequently, many empirical studies have considered his predictions and employs linear regression: Devarajan et al. (1996), Zhang and Zou (1998), Davoodi and Zou (1998), Xei et al.(1999), Kneller et al. (1999), Gupta et al. (2005), Shelton (2007), Baldacci, et al. (2008).

Let a panel data linear regression be:

$$
\begin{equation*}
y_{i t}=\beta_{0}+\sum_{j=1}^{k} \beta_{j} Y_{j i t}+\sum_{j=1}^{m} \gamma_{j} X_{j i t}+u_{i t}, \quad i=1, \cdots, I ; t=1, \cdots N, \tag{1}
\end{equation*}
$$

where $y_{i t}, X_{j i t}$ and $Y_{j i t}$ respectively denote the per capita growth rate of GDP, fiscal variables and conditioning (nonfiscal) variables, and $u_{i t}$ are i.i.d. $N\left(0, \sigma^{2}\right)$. Assuming that all elements of the budget (including the deficit/surplus) are included, the fiscal variables are subject to a linear constraint:

$$
\begin{equation*}
\sum_{j=1}^{m} X_{j i t}=0, \quad i=1, \ldots, I, \quad t=1, \ldots, N \tag{2}
\end{equation*}
$$

This assumption is crucial to the following arguments of the paper. If we exclude some elements of the budget from the regression analysis on a priori ground, the constraint of equation 2 is no more satisfied and the problem of multicollinearity does not arise.

The standard estimation procedure is not applicable for equation (1) because of perfect collinearity. In order to avoid collinearity, one variable (say, the last variable $X_{\text {mit }}$ ) must be omitted from equation (1). The equation actually estimated is:

$$
\begin{equation*}
y_{i t}=\beta_{0}+\sum_{j=1}^{k} \beta_{j} Y_{j i t}+\sum_{j=1}^{m-1} \gamma_{j, m} X_{j i t}+u_{i t}, \tag{3}
\end{equation*}
$$

where $\gamma_{j, m}=\gamma_{j}-\gamma_{m}(j=1, \ldots, m-1)$. We can only estimate $\gamma_{j, m}$ for $j=1, \ldots, m-1$ but cannot obviously estimate individual parameter of $\gamma_{j}$ for $j=1, \ldots, m$ even though running all possible regressions: e.g., when $\mathrm{j}=1$ and 2 , we can get only two coefficients by running two regressions, $\gamma_{1,2}=\gamma_{1}-\gamma_{2}, \gamma_{2,1}=\gamma_{2}-\gamma_{1}$. One unit increase of the particular variable $\gamma_{j}$ accompanies one unit decrease of an omitted variable $\gamma_{m}$ in the budget constraint (2). The estimated coefficients $\hat{\gamma}_{j, m}$ are different depending on the omitted variables.

The hypothesis of a zero coefficient of $X_{j i t}$ for equation (3) is:

$$
\begin{equation*}
\mathrm{H}_{0}: \gamma_{j, m}=0 \text { vs } \mathrm{H}_{1}: \gamma_{j, m} \neq 0 \text { for } j=1, \ldots, m-1 \tag{4}
\end{equation*}
$$

We should note that the null is $\gamma_{j}-\gamma_{m}=0$ rather than $\gamma_{j}=0$. A standard test statistic is given by:

$$
\begin{equation*}
t_{j, m}=\frac{\hat{\gamma}_{j, m}}{\left\{\hat{\operatorname{Var}}\left(\hat{\gamma}_{j, m}\right)\right\}^{1 / 2}}, \tag{5}
\end{equation*}
$$

where $\hat{\gamma}_{j, m}$ denotes the OLS estimator of $\gamma_{j, m}$, and $\hat{\operatorname{Var}}\left(\hat{\gamma}_{j, m}\right)$ is its estimated variance. The statistic has a $t$-distribution with degrees of freedom $\nu_{j, m}=\mathrm{N} \cdot \mathrm{I}-(\mathrm{k}+1)-(\mathrm{m}-1)$ under the null.

The previous researches implicitly recognized the omitted variables, perceived the estimated coefficient $\gamma_{j, m}$ of particular variable as a direct effect $\gamma_{j}$, the estimated coefficients in each regression were different (depending on the implicitly omitted variable). Therefore, there has never been a uniform estimation result for direct effect $\gamma_{j}$ in Barro (1990). However, they discussed which regressions are better for estimation.
Kneller et al. (1999) showed that it is complicate to identify which regressions with omitted variables are better in statistical sense. Considering the final goal being to find a direct effect $\gamma_{j}$, they recommend that we should omit a neutral category where economic theory suggests that $\gamma_{m}=0$ if we wish to test the null $\gamma_{j}=0$ of against $\gamma_{j} \neq 0$. They also recommend finding more neutral categories. However, Kneller et al. (1999) did not explicitly explore the implications of econometric methodology.

The purpose of this paper is to provide an econometric theory with Kneller et al. (1999). We prove: (i) An econometric theory alone does not provide any criteria to determine which variables to omit, i.e., which regressions are better for estimation. Barro (1990)'s model predicts that nondistortionary taxation and nonproductive expenditures have no effect on the growth rate. It is Barro (1990)'s prediction that provides a criterion. Because the coefficients for these variables are zero, the omission of these variables does not change the coefficients for the remaining variables. We can get a direct effect $\gamma_{j}$. (ii) The regression with two omitted variables provides estimates that are more efficient and more powerful test statistics than a regression with just one omitted variable when an economic theory indicates that two different coefficients are simultaneously zero. We also apply the analysis to work by Kneller et al. (1999) as a numerical example to show how the econometric theory works in practice. Finally, we confirm that their study supports Barro(1990)'s predictions.

Section 2 provides some propositions for the estimation and testing results. Section 3 provides a numerical example to illustrate how econometric analysis works in practice.

## 2. Effects of omitted variables on a linear regression with fiscal policy budget constraints

### 2.1 Estimates of all other regressions reproduced

We prove that the estimates of all other regressions can be reproduced using only the estimates of the regression equation originally chosen.

If we omit alternative omitted fiscal variable (say $X_{n i t}(n \neq m)$ ) instead of the last variable, the equation to be estimated is:

$$
\begin{equation*}
y_{i t}=\beta_{0}+\sum_{j=1}^{k} \beta_{j} Y_{j i t}+\sum_{j=1(j \neq n)}^{m} \gamma_{j, n} X_{j i t}+u_{i t}, \quad n=1, \cdots, m-1, \tag{6}
\end{equation*}
$$

where $\gamma_{j, n}=\gamma_{j}-\gamma_{n} ; j=1, \ldots, m(j \neq n)$. The coefficients of (6) with an alternative omitted variable $X_{n i t}$ are completely determined by those of (3) with an originally omitted variable $X_{m i t}$; conversely, the coefficients of (3) are also completely determined by those of (6) as indicated in the following proposition. The proofs of all propositions are given in Tsukuda and Miyakoshi (2008, Appendix).

Proposition 1: The parameters of equations (3) and (6) hold the following relationships for $n=1, \ldots, m-1$ :

$$
\begin{equation*}
\gamma_{j, n}=\gamma_{j, m}-\gamma_{n, m} \quad j=1, \ldots, m-1(j \neq n) ; \text { and } \gamma_{m, n}=-\gamma_{n, m}, \tag{7}
\end{equation*}
$$

The following proposition indicates that the OLS estimates for the coefficients of equations (3) and (6) carry the same relations as in (7).

Proposition 2 : Let the OLS estimates of (3) and (6) be $\hat{\gamma}_{j, m}$ and $\hat{\gamma}_{j, n}$ respectively. Then, the estimates $\hat{\gamma}_{j, m}$ and $\hat{\gamma}_{j, n}$ hold the following relationships for $n=1, \ldots, m-1$ :

$$
\begin{equation*}
\hat{\gamma}_{j, n}=\hat{\gamma}_{j, m}-\hat{\gamma}_{n, m} ; j=1, \ldots, m-1(j \neq n) \text { and } \hat{\gamma}_{m, n}=-\hat{\gamma}_{n, m}, \tag{8}
\end{equation*}
$$

Proposition 2 shows that all coefficient estimates for the regression with any other single omitted variable are completely determined by the estimates of the coefficients for the equation (3) with an originally chosen omitted variable. The empirical estimates of parameters might be very sensitive to the variables included and the time span specified. However, a key message of Proposition 2 carries that the relationship between the estimates expressed in equation (8) always holds regardless of the variables included and the sample periods specified.

### 2.2 T-statistics of all other regressions reproduced

We consider the effects of alternative omitted variable on the test for a zero coefficient of $X_{j i t}$ for equation (6):

$$
\begin{equation*}
\mathrm{H}_{0}: \gamma_{j, n}=0 \text { vs } \mathrm{H}_{1}: \gamma_{j, n} \neq 0 \text { for } j=1, \ldots, m(j \neq n) . \tag{9}
\end{equation*}
$$

The test statistic is:

$$
\begin{equation*}
t_{j, n}=\frac{\hat{\gamma}_{j, n}}{\left\{\hat{\operatorname{Var}}\left(\hat{\gamma}_{j, n}\right)\right\}^{1 / 2}}, \tag{10}
\end{equation*}
$$

where $\hat{\operatorname{Var}}\left(\hat{\gamma}_{j, n}\right)$ is the estimated variance of $\hat{\gamma}_{j, n}$. The following relations between the test statistics of (5) and (10) hold.

Proposition 3: The formula of (10) is written in terms of the quantities used only for estimating equation (3) as:

$$
\begin{equation*}
t_{j, n}=\left\{\frac{\hat{\operatorname{Var}}\left(\hat{\gamma}_{j, m}\right)}{\hat{\operatorname{Var}}\left(\hat{\gamma}_{j, n}\right)}\right\}^{1 / 2} t_{j, m}-\left\{\frac{\hat{\operatorname{Var}}\left(\hat{\gamma}_{n, m}\right)}{\hat{\operatorname{Var}}\left(\hat{\gamma}_{j, n}\right)}\right\}^{1 / 2} t_{n, m} \tag{11}
\end{equation*}
$$

where:

$$
\begin{equation*}
\hat{\operatorname{Var}}\left(\hat{\gamma}_{j, n}\right)=\hat{\operatorname{Var}}\left(\hat{\gamma}_{j, m}\right)-2 \hat{\operatorname{Cov}}\left(\hat{\gamma}_{j, m}, \hat{\gamma}_{n, m}\right)+\hat{\operatorname{Var}}\left(\hat{\gamma}_{n, m}\right), \tag{12}
\end{equation*}
$$

and $\hat{\operatorname{Cov}}\left(\hat{\gamma}_{j, m}, \hat{\gamma}_{n, m}\right)$ is the estimate of covariance between $\hat{\gamma}_{j, m}$ and $\hat{\gamma}_{n, m}$.
We note that knowledge of the covariances $\operatorname{Cov}\left(\hat{\gamma}_{j, m}, \hat{\gamma}_{n, m}\right)$ for $j=1, \cdots, m-1(j \neq n)$ is necessary for producing $\hat{\operatorname{Var}}\left(\hat{\gamma}_{j, n}\right)$ from the regression results of equation (3).

The analysis in the two subsections reveals the estimates and test-statistics are reproduced by alternative regressions. In this sense, any additional use of the regression with an alternative omitted variable cannot extract further information from the given data set. This implies that we are indifferent to the choice of omitted variable and cannot provide any criteria to determine which variables to omit from a purely statistical point of view.

On the other hand, Barro (1990) predicts that nondistortionary taxation and nonproductive expenditures have no effect on the growth rate. It is this prediction that provides a criterion to determine which variables to omit. Because the coefficients for
these variables are zero, the omission of these variables does not change the coefficients for the remaining variables. We can get a final goal being to find a direct effect $\gamma_{j}$.

### 2.3 Effects of two simultaneously omitted variables

We prove that the regression with two omitted variables provides estimates that are more efficient and more powerful test statistics than a regression with just one omitted variable when economic theory indicates that two different coefficients are simultaneously zero.

Suppose it is true that two different coefficients are simultaneously zero (say, $\gamma_{m}=0$ and $\gamma_{n}=0$ ), in equation (1). This assumption is justified by economic theory on the basis of the public-policy endogenous growth models in Barro (1990). This is not a statistical hypothesis to be tested by empirical data. The true regression equation is:

$$
\begin{equation*}
y_{i t}=\beta_{0}+\sum_{j=1}^{k} \beta_{j} Y_{j i t}+\sum_{j=1(j \neq n)}^{m-1} \gamma_{j} X_{j i t}+u_{i t}, \quad n=1, \cdots, m-1 \tag{13}
\end{equation*}
$$

The regression equations (3) and (6) are misspecified because one of the zero coefficient restrictions is ignored. The parameters to be estimated are equal among the three equations, i.e. $\gamma_{j}=\gamma_{j, m}=\gamma_{j, n}$ for $j=1, \ldots, m-1(j \neq n)$.

Proposition 4: Let $\hat{\gamma}_{j}, \hat{\gamma}_{j, m}$ and $\hat{\gamma}_{j, n}$ be the OLS estimates of equations (13), (3) and (6) respectively. Then, we have for $j=1, \ldots, m-1(j \neq n)$ :
(i) $E\left\{\hat{\gamma}_{j, m}\right\}=E\left\{\hat{\gamma}_{j}\right\}=E\left\{\hat{\gamma}_{j, n}\right\}=\gamma_{j}$,
(ii) $\operatorname{Var}\left\{\hat{\gamma}_{j}\right\}<\operatorname{Min}\left\{\operatorname{Var}\left\{\hat{\gamma}_{j, m}\right\}, \operatorname{Var}\left\{\hat{\gamma}_{j, n}\right\}\right\}$.

All three estimators of the coefficients are unbiased. The estimator of the true model is the most efficient among the three in the sense that estimator ( $\hat{\gamma}_{j}$ ) has the smallest variance. Proposition 4 analytically implies the claim that when economic theory suggests that there is more than one neutral category, more precise parameter estimates can be obtained by omitting both categories.

We consider the testing of a zero coefficient for $\mathrm{X}_{\mathrm{jit}}$ in equation (13):

$$
\begin{equation*}
\mathrm{H}_{0}: \gamma_{j}=0 \text { vs } \mathrm{H}_{1}: \gamma_{j} \neq 0 \text { for } j=1, \ldots, m-1(j \neq n), \tag{16}
\end{equation*}
$$

where the test statistic is given by:

$$
\begin{equation*}
t_{j}=\frac{\hat{\gamma}_{j}}{\left\{\hat{\operatorname{Var}}\left(\hat{\gamma}_{j}\right)\right\}^{1 / 2}} . \tag{17}
\end{equation*}
$$

Both (5) and (10) can be used for testing the hypothesis (16) in addition to (17).

## Proposition 5:

(i) All three statistics for (5), (10) and (17) have a t-distribution under the null and a noncentral t-distribution with noncentrality parameters $\delta_{i}$ under the alternative where $\delta_{i}=\frac{\gamma_{i}}{\left\{\operatorname{Var}\left(\hat{\gamma}_{i}\right)\right\}^{1 / 2}}$ for $i=j,(j, m),(j, n) ; \nu_{j}=N \cdot I-k-m+1$, and $\nu_{j, m}=\nu_{j, n}=N \cdot I-k-m$, respectively;
(ii) The expected values of $t_{i}$ are respectively given by:

$$
\begin{equation*}
E\left\{t_{i}\right\}=\left(\frac{1}{2} \nu_{i}\right)^{1 / 2} \frac{\Gamma\left(\frac{1}{2}\left(\nu_{i}-1\right)\right)}{\left\{\Gamma\left(\frac{1}{2} \nu_{i}\right)\right\}^{1 / 2}} \delta_{i} \tag{18}
\end{equation*}
$$

where $E(\bullet)$ denotes an expectation operator, and $\Gamma(p)$ is a Gamma function with p degree of freedom ${ }^{1}$.

The expected value is zero under the null and an increasing function of the noncentrality parameter $\delta_{i}$ under the alternative.

Proposition 6:The following inequality holds for any $\gamma_{j} \neq 0$ for $j=1, \ldots, m-1(j \neq n)$ :

$$
\begin{equation*}
\left|E\left\{t_{j}\right\}\right| \geq \max \left\{\left|E\left\{t_{j, m}\right\}\right|,\left|E\left\{t_{j, n}\right\}\right|\right\} \tag{19}
\end{equation*}
$$

up to the order of $O\left((N \cdot I)^{-2}\right)$ when the number of observations $(\mathrm{N} \cdot \mathrm{I})$ increases.
The absolute value of the expectation of the $t$-statistic of equation (17) for testing a zero coefficient is always the highest among the three tests under the alternative hypothesis ( $\gamma_{j} \neq 0$ ) when the sample size is large. This suggests that among the three tests, the test of (17) has the highest power for testing the hypothesis (16). The above statement may be justified as follows. When the sample size NI is large, the test statistic for (5), (10) or (17) is approximated by $t_{i} \doteq z_{i}+\mu_{i}$ where $z_{i}=\frac{\hat{\gamma}_{i}-\gamma_{i}}{\left\{\hat{\operatorname{Var}}\left(\hat{\gamma}_{i}\right)\right\}^{1 / 2}}$ has an approximate standard normal distribution and $\mu_{i}=E\left(t_{i}\right)$.

[^1]The test statistic of (17) shifts the standard normal distribution as much as $\mu_{i}$ under the alternative hypothesis. Therefore, the probability that $t_{i}$ falls in the critical region is larger when the magnitude of shift is higher. Thus, we should omit two variables when economic theory indicates that two different coefficients are simultaneously zero.

## 3. A numerical example

Kneller et al. (1999, p.180) summarize the basic results about the growth effects of fiscal policy in their Table 3. In order to examine how the analysis presented in the previous section works in practice, we reproduce their results in Table 1 after adjusting for the sign of the fiscal taxation variables ${ }^{2}$.

The $i-t h$ column in Table 2 shows the estimates of the coefficients for the regression with the $i$-th variable omitted. These are calculated by utilizing the relations in Propositions 2 and 3 based upon the original regression with the last fiscal variable (nonproductive expenditures in column 8 of Table 2) omitted. The parameter estimates in column 6 of Table 2 are the same as those in column 1 of Table 1 . The $t$-values in column 6 are almost the same as those in column 1 of Table 1. Any differences between the corresponding $t$-values are from rounding errors ${ }^{3}$.

Table 2 illustrates how the estimates of the other regression coefficients are produced using only the estimates of the originally chosen regression equation. This explains why running other regression equations does not provide additional information.

The estimated coefficients for each fiscal variable differ considerably column by column. For example, the coefficient for distortionary taxation in column 8 is 0.410 and significantly different from zero while the corresponding coefficient in column 4 is zero to the third decimal point and apparently insignificant. However, the econometric theory itself may not provide any criteria for determining the omitted variables. Instead, economic theory plays an essential role. According to Barro (1990)'s public-policy endogenous growth models, a neutral category (nondistortionary taxation and nonproductive expenditures) has no effect on the rate of growth. Economic theory suggests that $\gamma_{6}=0$ and $\gamma_{8}=0$ in this example.

[^2]Suppose it is true that nonproductive expenditures have no effect on the rate of growth ( $\gamma_{8}=0$ ). This is a prediction of Barro (1990)'s model. In statistical terminology, the constraint $\gamma_{8}=0$ is one of the maintained hypotheses, not a hypothesis to be tested. The hypothesis in (4) now turns out to be the null of $\gamma_{j}=0$ against $\gamma_{j} \neq 0$ for $j=1, \ldots, 7$. In particular, the estimate of nondistortionary taxation is not significantly different from zero. As expected, the estimated values in column 6 are very similar to those in column 8 because nondistortionary taxation is classified into a neutral category. Thus, as seen in Table 2, we cannot find evidences which deny the Barro (1990)'s prediction $\gamma_{6}=0$ and $\gamma_{8}=0$, because the estimated coefficients are mostly the same in column 6 and 8.

Suppose, as suggested by the fiscal policy endogenous growth model, that both nonproductive expenditures and nondistortionary taxation have no effect on the rate of growth ( $\gamma_{6}=0$ and $\gamma_{8}=0$ ). The estimated coefficients are presented in the first column of panels in Table 1. The entries in the last column of panel (a) and (b) in Table 1, respectively, show the $t$-values for the case with a single omitted fiscal variable, while the last column of panel (c) presents the $t$-values with two simultaneously omitted variables. The $t$-values in panel (c) are the highest of those in the other two panels. Proposition 6 confirms the facts empirically observed in Table 1 from the viewpoint of econometric theory.

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Table 1. Regression results
(Table 3 from Kneller et al. (1999, p.180))

| Omitted Fiscal Variable |  | Panel (a) |  |  | Panel (b) |  |  | Panel (c) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Non-dis. Taxation |  |  | Non-productive expenditures |  |  | Non-dis. Taxation and non-prod. Expenditures |  |  |
|  |  | Est. | s.d. | t-val. | Est. | s.d. | t-val. | Est. | s.d. | t-val. |
| 1. | Lending minus repayments | 0.417 | 0.229 | 1.820 | 0.380 | 0.178 | 2.130 | 0.384 | 0.176 | 2.180 |
| 2. | Other revenues | 0.154 | 0.190 | 0.810 | 0.117 | 0.104 | 1.120 | 0.118 | 0.104 | 1.130 |
| 3. | Other expenditures | 0.315 | 0.158 | 2.000 | 0.279 | 0.115 | 2.420 | 0.289 | 0.105 | 2.750 |
| 4. | Budget surplus | 0.446 | 0.160 | 2.790 | 0.410 | 0.089 | 4.600 | 0.416 | 0.084 | 4.930 |
| 5. | Distortionary taxation | 0.446 | 0.160 | 2.790 | 0.410 | 0.097 | 4.210 | 0.410 | 0.094 | 4.370 |
| 6. | Non-distortionary taxation | - | - | - | -0.037 | 0.161 | -0.230 | - | - | - |
| 7. | Productive expenditures | 0.290 | 0.146 | 1.980 | 0.253 | 0.130 | 1.950 | 0.268 | 0.110 | 2.430 |
| 8. | Non-productive expenditures | 0.037 | 0.161 | 0.230 | - | - | - | - | - | - |

Note: The entries in columns s.d. and $t$-val. are standard deviations and $t$-values, respectively.

Table 2. Derived estimates

|  | Omitted Fiscal Variable | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | Lending minus repayments | - | 0.263 | 0.101 | -0.030 | -0.030 | $\begin{aligned} & 0.417 \\ & (1.82) \end{aligned}$ | $0.127$ | $\begin{aligned} & 0.380 \\ & (2.13) \end{aligned}$ |
| 2. | Other revenues | -0.263 | - | -0.162 | -0.293 | -0.293 | $\begin{aligned} & 0.154 \\ & (0.81) \end{aligned}$ | -0.136 | $\begin{aligned} & 0.117 \\ & (1.12) \end{aligned}$ |
| 3. | Other expenditures | -0.101 | 0.162 | - | -0.131 | -0.131 | 0.316 <br> (2.00) | 0.026 | $\begin{aligned} & 0.279 \\ & (2.42) \end{aligned}$ |
| 4. | Budget surplus | 0.030 | 0.293 | 0.131 | - | 0.000 | $\begin{aligned} & 0.447 \\ & (2.79) \end{aligned}$ | 0.157 | $\begin{aligned} & 0.410 \\ & (4.60) \end{aligned}$ |
| 5. | Distortionary taxation | 0.030 | 0.293 | 0.131 | 0.000 | - | $\begin{aligned} & 0.447 \\ & (2.79) \end{aligned}$ | 0.157 | $\begin{aligned} & 0.410 \\ & (4.21) \end{aligned}$ |
| 6. | Nondistortionary taxation | -0.417 | -0.154 | -0.316 | -0.447 | -0.447 | - | -0.290 | $\begin{gathered} -0.037 \\ (0.23) \end{gathered}$ |
| 7. | Productive expenditures | -0.127 | 0.136 | -0.026 | -0.157 | -0.157 | $\begin{aligned} & 0.290 \\ & (1.98) \end{aligned}$ | - | $\begin{aligned} & 0.253 \\ & (1.95) \end{aligned}$ |
| 8. | Non-productive expenditures | -0.380 | -0.117 | -0.279 | -0.410 | -0.410 | $\begin{aligned} & 0.037 \\ & (0.23) \end{aligned}$ | -0.253 | - |

Note: The $t$-statistics for columns 6 and 8 are in parentheses. The $t$-statistics for the other columns are unavailable.


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    Submitted: May 12 2010. Published: November 08, 2010.

[^1]:    ${ }^{1}$ See, for example, Johnson and Kotz (1970, p.203) for the properties of a noncentral $t$-distribution.

[^2]:    ${ }^{2}$ The coefficient for distortionary taxation in Table 3 of Kneller et al. (1999) is negative, indicating that an increase in the tax rate for the distortionary category induces a reduction in the growth rate. Though this way of treating the fiscal revenue variables is intuitively appealing, the fiscal variables do not sum to zero. Hence, the fiscal budget constraint of equation (2) is not satisfied. In order to avoid this inconsistency, we measure the fiscal revenue variables as negative values and the expenditure variables as positive values. This variable adjustment should change the signs of the coefficients for distortionary taxation and other revenues in Table 3.
    ${ }^{3}$ In general, we cannot apply Proposition 2 for calculating the $t$-values in columns 1 through 7 because Kneller et al. (1999) do not report $\hat{\operatorname{Cov}}\left(\hat{\gamma}_{j, 8}, \hat{\gamma}_{n, 8}\right)$. However, we can directly use $\hat{\operatorname{Var}}\left(\hat{\gamma}_{j, 6}\right)$ in column 2 of their Table 3 for calculating the $t$-values in column 6 of our Table 2.

