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What do “residuals” from first-order conditions reveal about DGE models?

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Abstract

The first-order condition (FOC) associated with labour in many dynamic general equilibrium models involves only current period variables. Residuals constructed from this FOC are inconsistent with aggregate US data in that they are very large and highly persistent. The persistence suggests that models which introduce dynamic terms in the labour FOC may be more consistent with the data. Three such models (one with learning by doing, one with habit formation, and one with labour adjustment costs) confirm that they can reduce the persistence in the residuals making the models more consistent with the joint dynamics of consumption, output and hours.

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1 Introduction

DGE models are usually evaluated by comparing the predicted behaviour of individual aggregate series such as output, hours and consumption to their empirical counterparts. Most studies of business cycle behaviour focus on one or both of two prevalent evaluation techniques: matching a small set of second moments and comparing predicted and empirical impulse responses. Gregory and Smith (1991) as well as Christiano and Eichenbaum (1992) propose methods to test the difference between historical and predicted moments. Other authors (*e.g.* Cogley and Nason, 1995) suggest looking at impulse responses and autocorrelation functions to analyze the properties of RBC models. These diagnostics have demonstrated many shortcomings in the standard RBC model and sparked a significant amount of research. The goal of many extensions of the baseline model has been to address these shortcomings and success has been mixed so far (see Hansen and Wright, 1992, and King and Rebelo, 1999, for a discussion of some of these issues).

While moment matching has proved to be extremely useful as a diagnostic tool to test the ability of DGE models to capture basic features of the business cycle, there always remains the possibility that focusing on a limited set of moments obscures more than it reveals. Even worse, it is possible that while we appear to be making progress on bringing the model closer to the data on one or two dimensions we may actually be moving further away in many other unexplored dimensions. One possibility is to expand the set of moments used in the matching exercise but which moments should one add and where does one stop this process? Instead, we argue that it may be useful to ask if the first-order conditions (FOCs) generated by the model, that are supposed to explain the joint behaviour of macroeconomic aggregates, are in fact consistent with the data. While this is obviously not a novel suggestion it is not currently popular in evaluating DGE models. The hope is that the form and nature of the inconsistency could potentially provide clues as to the direction in which models need to be modified to make them more compatible with the data. Indeed the FOCs often suggest which additional moments may offer new insights as we illustrate below.

In this paper, we confine our attention to real business cycle models and suggest a simple way of evaluating them based on the consistency of their FOCs with the data. We show that the method is indeed helpful in revealing new shortcomings of these models and in suggesting directions to fruitfully modify the models. The diagnostic is easy to apply and involves evaluating the residuals obtained from estimated or calibrated FOCs. For example, consider the FOC associated with the labour input in the centralized version of the standard RBC model. In theory this requires that the marginal rate of substitution between consumption and leisure be exactly equal to the marginal product of labour in every period. In practice this is not true so that one can construct a "residual" which captures the extent to which the two measures deviate from each other.

Is this residual a meaningful measure that we should pay attention to? Certainly models are simplified representations of reality, so one might expect to find movement in the residuals due to the combined influence of all excluded economic forces.[1] Conceptually one can divide these into three categories: 1. major elements in the data that are not relevant for business cycle theory; 2. forces that operate at business cycle frequencies but have only a minor influence on the aggregate data; and finally 3. major influences at business cycle frequencies that theory should explain but can not do so. If the only excluded forces are relatively minor, one would expect their combined influence to be reflected in small residuals consisting mainly of random deviations around the value predicted by our model (zero in this case). If the residuals display large and systematic deviations, one would need to examine whether these are coming from forces that might be relevant to business cycle theory. If they are then this suggests the model is mis-specified and the properties of the residuals may suggest how to modify it.

Returning to our example, the residuals from the FOC associated with the labour input are large in the sense that they are even more volatile than total hours. Visually, the deviations appear to have systematic tendencies. To determine if these tendencies are relevant for business cycles we focus on three features of business cycles emphasized by macroeconomists: co-movement of the residuals with aggregate output and hours; persistence in the residuals especially as measured by the auto-correlation co-efficient; and the size of the deviations of the residuals from their predicted value as measured by their standard deviation relative to the standard deviation of

[1] We will discuss measurement error later.

hours. We find that the residuals strongly co-move with the business cycle whether we use filtered or unfiltered data. For example, the correlation of the residuals with HP filtered consumption, output and hours is above 0.74 in all cases.[2] The residuals are also highly auto-correlated and highly correlated with lagged aggregate variables. In addition to these "relevant" systematic features there appear to be others: the graph of the residual also displays evidence of structural breaks and low frequency trends. Since these features do not appear to be relevant to business cycle analysis we do not view their presence as a failure of the model. Instead we execute some robustness exercises to ensure these low frequency features are not entirely responsible for the presence of the three business cycle frequency features that we are interested in exploring.

What can we learn from these business cycle properties of the residuals that is new? Macroeconomists have been attempting to improve the labour market predictions of business cycle models for decades. For example, early moment matching exercises revealed that the basic RBC model does not generate sufficiently volatile hours. Is the behaviour of our residuals merely a symptom of the same disease? We argue that it is not. Our answer hinges on the high auto-correlation of the labour residuals. To show this we calculate residuals from the labour FOCs associated with the indivisible labour version of the basic RBC model as well as a model with labour hoarding. Both have been shown to increase the volatility of hours but fail to significantly reduce the persistence in the residuals.[3] This is not surprising since all three models have labour FOCs that are static in that all terms involve only contemporaneous aggregate variables. We conclude that models that do involve terms with leads and/or lags will have a better chance of explaining the dynamics in the aggregate data and reducing the persistence of the residuals.[4]

Is the persistence in the residuals merely a reflection of the lack of an endogenous propagation mechanism, a problem highlighted in Cogley and Nason (1995) by comparing predicted and estimated impulse responses for aggregate output? We show that it is not. Cooper and Johri (1997) show that a RBC model modified to allow external learning-by-doing generates a huge amount of endogenous persistence in aggregate output as well as hump-shaped impulse

[2]The correlation of residuals and hours is above 0.8 using unfiltered data.

[3]As expected both models do reduce the relative volatility of the residuals.

[4]Note that all the models we consider in this paper formally imply that the residuals from the corresponding labour FOC should have an auto-covariance of zero.

responses. Since the model implies essentially the same hours FOC as the basic RBC model, the residuals are virtually identical and inherit all the persistence.

Why might FOC residuals offer different implications from earlier model evaluation exercises? We believe the answer lies in that the FOCs direct us towards evaluating particular dynamic co-movement in the data. Typically model evaluation using impulse responses focuses on comparing the estimated and theoretical responses of *individual* series while our hours FOC makes predictions about the joint behaviour of aggregate output, consumption and hours. Similarly most moment matching exercises focus on correlations of individual variables with output and sometimes with hours. Looking at FOC residuals forces us to require consistency with the data not in individual series but in the joint behaviour of all the variables that appear in the FOC. Consider the labour input FOC associated with the indivisible labour model to illustrate this point. The FOC implies that a one percent increase in hours should be accompanied by a one percent fall in the consumption output ratio in the same period. Clearly then it does not suffice to look at the relative volatility of either consumption or hours with output. A model may do well on these dimensions and yet one might find that the movement of hours is too big relative to the movement of the consumption output ratio.

The residuals do in fact suggest such a discrepancy. Moreover the extremely high autocorrelation in the residuals suggests that the models are missing a dynamic element in this joint behaviour. To capture this it appears to be necessary to use models in which the labour input FOC involves dynamic terms rather than just current period variables. We look at three such models. The first modifies the standard model by including internal learning by doing, the second uses non-separable preferences based on habit formation in consumption and the last model introduces convex costs of adjusting labour. We show that these models are all able to reduce the persistence in the labour input FOC residuals. Formal J-tests of the overall fit of these models are also extremely successful.

An alternative approach to modifying the model so that the FOC is consistent with the data would be to allow for preference shocks. In this case the entire residual series from the FOC can be defined as a sequence of preference shocks with the appropriate amount of persistence. In a study of sources of fluctuations, Hall (1997) works with a log-linearized version of this FOC and concludes that the sheer size of these residual points to mis-specification of the labour side of the RBC model. In contrast to us he recommends modifying the intratemporal aspects of the model. While this may be useful it will not account for the persistence in the residuals. Our view of the large and persistent residuals as evidence of specification error rather than as evidence for large shifts in preferences is also motivated by the argument that it is unsatisfactory to leave such a large fraction of the variation in the data to be accounted for by unexplained exogenous forces on which no independent evidence exists.

The overall consistency of the models discussed above is evaluated with reference to aggregate US data. The parameters of these models are estimated using generalized method of moments (GMM) applied to moment conditions obtained from the FOCs associated with each model. These point estimates are then used to construct residuals from these FOCs and our focus is on their dynamics. It may be worth emphasizing that properties of residuals can be studied without estimating the parameters of the model. Once parameter values have been picked (by estimation or calibration) the residuals can be constructed and evaluated. The choice of instruments used or estimation strategies have no further impact on the procedure. Indeed it is easy and useful to explore the impact of changing parameter values on the dynamics of the residuals, as we show below in our sensitivity analysis.

Our work is related to early studies that estimated FOCs related to RBC type models using GMM procedures and formally tested the overall fit of the model using overidentifying restrictions tests. Two notable examples are Eichenbaum, Hansen and Singleton (1988) and Mankiw, Rotemberg and Summers (1985). Moreover Euler equation estimation is common in the asset-pricing literature and goes back to the work of Hansen and Singleton (1982). A number of studies use generalized method of moments procedures to estimate parameters of RBC models but evaluate the models using formal or informal moment-matching exercises. (A few examples of the former are Christiano and Eichenbaum, 1992, Burnside, Eichenbaum and Rebelo, 1993, and Burnside and Eichenbaum, 1996). There also exist other studies which estimate dynamic general equilibrium models using procedures other than GMM. For example, Altug (1989), McGrattan, Rogerson and Wright (1997) and Chow and Kwan (1998) estimate their model using a maximum likelihood procedure based on linearized decision rules and DeJong, Ingram and Whiteman (2000) use a Bayesian approach to estimate a model with multiple shocks. In addition there is work on evaluating RBC type models using spectral analysis as in Watson (1993), Wen (1998) and Diebold, Ohanian and Berkowitz (1998) and Bayesian econometric procedures as in Schorfheide (2000).

The remainder of this paper is organized as follows. Section 2 contains a discussion of the models that we explore. Section 3 offers brief concluding remarks.

2 Model Evaluation

In this section we present results from evaluating several different versions of RBC type models. Initially the key parameters of the model are estimated using a GMM procedure using the FOCs for hours worked as well as capital and the law of motion for the capital stock. This is backed up with sensitivity analysis in which we report the results of varying parameters on the properties of the residuals.[5]

2.1 Standard RBC model

In our standard RBC model, the central planner maximizes

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + B \ln(1 - L_t)\right] \tag{1}$$

where C denotes consumption and L denotes hours worked, subject to the accumulation equation for capital (K)

$$K_{t+1} = (1 - \delta) K_t + I_t \tag{2}$$

and the resource constraint

$$C_t + I_t = Y_t \tag{3}$$

[5]Our discussion of the performance of various models tends to focus on the hours FOC but we briefly point out discrepancies in other FOCs when they appear.

where I denotes investment and Y denotes output. The production function is Cobb-Douglas with constant-returns-to-scale

$$Y_t = K_t^{1-\alpha}(L_t X_t)^{\alpha} \tag{4}$$

where the level of technology (X) evolves according to the law of motion

$$X_t = X_{t-1}\exp(\tau + \upsilon_t)$$

where υ_t is an iid random variable with mean zero and standard deviation σ_υ and τ is the growth rate of the economy. The exact structure of the technology shocks is unimportant for our work.

The parameters are estimated using a GMM estimator. We use the GMM code written in GAUSS by Hansen, Heaton and Ogaki. The discount rate is set equal to the average real three-month US treasury-bill rate over the sample used in our empirical work (1955:1 to 1992:4). The resulting discount factor is $\beta = 1/1.00268$. A detailed description of the data used in this paper is included in a data appendix.

The standard RBC model implies the following FOCs for hours and capital respectively:

$$\alpha\frac{Y_t}{C_t L_t} - \frac{B}{1-L_t} = 0 \tag{5}$$

$$\mathrm{E}_t\left\{\beta\frac{C_t}{C_{t+1}}\left[(1-\alpha)\frac{Y_{t+1}}{K_{t+1}} + 1 - \delta\right] - 1\right\} = 0. \tag{6}$$

Let $\xi^s_{L,t}$ denote the left-hand side of equation (5), $\xi^s_{K,t+1}$ denote the expression inside braces in equation (6) and $\xi^s_{\delta,t} \equiv 1 + (I_t - K_{t+1})/K_t - \delta$. FOCs (5) and (6) together with the accumulation equation (2) yield the following moment restrictions:

$$\mathrm{E}\{\xi^s_{L,t}\} = 0 \qquad \mathrm{E}\{\xi^s_{K,t+1}\} = 0 \qquad \mathrm{E}\{\xi^s_{\delta,t}\} = 0. \tag{7}$$

We use these three moment conditions to just-identify the parameters B, α and δ. [6]

[6]The first and third moment conditions may appear unusual in that the model requires equations (2) and (5) to hold exactly in each period but we have imposed a weaker requirement that they hold true only on average. This is based on the view that models are simplifications of reality and must necessarily abstract from some influences present in the data but not central to the issues addressed by the model. This restriction basically allows us to obtain estimates of the parameters that appear in the FOCs. An alternative approach would be to calibrate the parameters from other studies but the GMM approach used above has the advantage that the average size of the residuals are minimized.

The estimates of B, δ and α are presented in the second column of Table 1 and are quite close to those estimated in the literature. We evaluate the overall ability of the model to "explain" the data by looking at Figure 1 which plots the residuals from the FOCs. The theoretical model predicts that $\hat{\xi}^s_{L,t}$ should always be zero whereas $\hat{\xi}^s_{K,t+1}$ should be zero in expected terms. Figure 1 shows that the residuals $\hat{\xi}^s_{L,t}$ deviate away from zero for long periods of time. Moreover these deviations are quite large in magnitude. Clearly, a lot of variation in the data remains unexplained by equation (5). To get a metric for the magnitude of the residuals, we compare their standard deviation to the standard deviation of hours. According to this measure, the residuals from the labour FOC are clearly large since their standard deviation is 1.64729 times the standard deviation of hours.[7] Figure 1 also suggests that the residuals $\hat{\xi}^s_{L,t}$ are highly autocorrelated. This is confirmed in Table 2 which presents the first-order autocorrelation coefficients for $\hat{\xi}^s_{L,t}$ and $\hat{\xi}^s_{K,t+1}$. The first-order autocorrelation is 0.24 in the residuals from the capital FOC and 0.99 in the residuals from the labour input FOC. [8]

While past work by Hall (1997) has highlighted the size of these residuals, the high degree of persistence in the residuals is surprising. We tried to account for this finding in a number of ways but it seems quite robust:

- Figure 1 suggests that there may be a downward trend in the residuals. However, the high persistence in $\hat{\xi}^s_{L,t}$ is not an artifact of the apparent downward trend since adding a linear trend to the autoregression reduces the autocorrelation coefficient to 0.96 only (reported in Table 2) while allowing for a quadratic trend reduces it to 0.93.[9] Also, if we use only the subsample 1955:1 to 1969:4 in our estimation, the apparent downward trend in $\hat{\xi}^s_{L,t}$ disappears but the residuals $\hat{\xi}^s_{L,t}$ are still large (standard deviation of residuals relative to hours is 0.92) and highly serially correlated (autocorrelation coefficient of 0.89). Similarly,

[7]This comparison of standard deviations is rendered meaningful by re-writing the hours FOC in such a way that the residuals are in the same units as L: $L_t - \alpha Y_t(1 - L_t)/(BC_t) = 0$.

[8]In a decentralized version of the model, the labour FOC (5) would be replaced by two conditions, one equating the marginal product of labour (labour demand) to the wage rate and the other equating the marginal rate of substitution (labour supply) to the wage rate. The residuals (not reported) from these conditions are both highly persistent with AR(1) coefficients greater than 0.9. Since completing a previous version of this paper, we have become aware of a recent study by Gali, Gertler and Ló pez-Salido that focuses on linearized versions of precisely these two residuals and attempts to account for them with time-varying markups.

[9]All of the first ten autocorrelation coefficients are between 0.994 and 0.97.

if we work with the subsample 1964:1-1992:4 or 1984:1-1992:4 the residuals $\hat{\xi}^s_{L,t}$ are still large (standard deviation of residuals relative to hours of at least 1.2) and highly serially correlated (autocorrelation coefficient of at least 0.9)

- The large size and high degree of persistence of the residuals are not an artifact of the data used to construct Figure 1. We also calculated the residuals from an alternative and longer dataset covering the period 1947:1-1999:4 (see data appendix) and found very similar properties. These results are available upon request.

- We also consider the impact of parameter choice on the properties of the residual by varying the values of α and B. Figure 2 shows that the persistence and size of $\hat{\xi}^s_{L,t}$ as measured by the autocorrelation coefficient and the ratio $SD(\hat{\xi}^s_{L,t})/SD(L_t)$ depend very little on the values chosen for α and B. Thus it is clear that neither the large size of the residuals nor their high autocorrelation is an artifact of the particular values chosen for the parameters by the GMM estimator.

- One may think that the high persistence in the residuals is due mainly to the large and systematic deviations of the residuals away from zero which in turn are caused by the presence of systematic low frequency movements in the data. These trends could be associated with demographic transitions or changes in labour laws affecting say the average number of hours worked in a week. One way to control for these trends is to focus on business cycle frequency movements in the data by detrending the data using the Hodrick-Prescott filter (with the smoothing parameter set to 1600). Once the low frequency noise is removed, the serial correlation in $\hat{\xi}^s_{L,t}$ is 0.87 (0.14 for $\hat{\xi}^s_{K,t+1}$) while $SD(\hat{\xi}^s_{L,t})/SD(L_t) = 1.25$. Another way to detrend the data is to use frequency domain techniques to filter the data, keeping only fluctuations with periodicities between 6 and 32 quarters. When we do so, the residuals $\hat{\xi}^s_{L,t}$ have no trend, are still highly persistent (first-order autocorrelation of 0.86) and large in size $SD(\hat{\xi}^s_{L,t})/SD(L_t) = 0.75$.

Having made the case for evaluating models using the fit of the key FOCs that are supposed to describe the joint behaviour of the aggregate series, it may appear surprising that we have offered no formal tests of the overall fit of the model. Indeed, it is well known that the basic RBC model is strongly rejected using the usual J-test statistic based on testing over-identifying restrictions. An example of this rejection can be found in Mankiw, Rotemberg and Summers (1985) for both the hours FOC as well as the Euler equation. However both of these equations are estimated using wage and interest rate data instead of the marginal product of capital and labour used in our study. As discussed in the next paragraph, we have chosen not to report the results of over-identifying restriction tests (which also reject the model) because the results are not reliable when there is a high degree of persistence in the residuals. Since $\hat{\xi}^s_{L,t}$ has an autocorrelation coefficient close to 1 we chose to use a just-identified estimator. However we later offer formal tests of models whenever they are appropriate.

As discussed in Andrews (1991), Altonji and Segal (1996) and Christiano and den Haan (1996), estimating the covariance matrix of the empirical moments is difficult when working with a short sample of persistent data. This often leads to noticeable bias in the estimate of the covariance matrix of the empirical moments. Since this estimate plays a central role in constructing the GMM weighting matrix, we prefer to work with a just-identified estimator so that the potential bias in the covariance matrix (and the weighting matrix) do not bias our parameter estimates. The J statistic too would be severely biased in this situation so we eschew using it whenever the residuals are highly persistent. The standard errors attached to the parameter estimates do depend on the weighting matrix and must be interpreted with caution. For this reason, we do not discuss the significance of individual parameter estimates but report them in the tables for completeness.[10]

2.2 RBC model with indivisible labour

Early moment matching exercises indicated that the behaviour of the labour market in the standard RBC model was at odds with empirical observations. In their survey article, Hansen and Wright (1992) document that the ratio of the standard deviation of hours to the standard deviation of average labour productivity (σ_L/σ_{APL}) is 1.37 in the US data (based on the household survey) and that hours and ALP are not correlated. However, their simulation of the standard model

[10]We use the quadratic spectral heteroskedasticity and autocorrelation consistent (HAC) estimator with prewhitening and automatic bandwidth selection suggested by Andrews and Monahan (1992) to estimate the covariance matrix of the empirical moments.

yielded $\sigma_L/\sigma_{APL} = 0.94$ and a correlation of 0.93. To correct for the former problem, Hansen (1985) suggested a model where labour is indivisible. These Hansen-Rogerson (Rogerson, 1988) preferences imply that equation (1) is simply replaced by

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + B(1 - L_t)\right] \tag{8}$$

and the FOC for hours (5) is replaced by

$$\alpha \frac{Y_t}{C_t L_t} - B = 0. \tag{9}$$

Hansen and Wright's (1992) simulation of the indivisible labour model yields a ratio $\sigma_L/\sigma_{APL} = 2.63$ and a correlation of 0.76 between hours and labour productivity.

While it is immediately obvious from equation (9) that the residuals in this model will behave very much like the residuals of the previous model, it is nonetheless interesting to study this model because it highlights the fact that looking at the properties of residuals identifies problems other than those discussed in the moment matching literature. Denoting the left-hand side of equation (9) $\xi^i_{L,t}$ and noting that $\xi^i_{K,t+1} = \xi^s_{K,t+1}$ and $\xi^i_{\delta,t} = \xi^s_{\delta,t}$ we use the GMM moment conditions

$$\mathrm{E}\{\xi^i_{L,t}\} = 0 \qquad \mathrm{E}\{\xi^i_{K,t+1}\} = 0 \qquad \mathrm{E}\{\xi^i_{\delta,t}\} = 0. \tag{10}$$

to estimate B, α and δ. Parameter estimates are presented in the third column of Table 1. The residuals from the labour FOC are plotted in Figure 3 together with their counterparts in the standard RBC model. Our measures of persistence are presented in the third column of Table 2. The residuals from the labour input FOC are still large relative to hours (ratio of standard deviation is 1.45 vs 1.65 in the standard model) and very highly persistent (autocorrelation coefficient of 0.99).

Despite making little progress in reducing persistence, the model does improve the two moments emphasized in the literature. This improvement is also seen in a reduction in the relative volatility of the residuals compared to the baseline model.

While the indivisible labour model is able to generate more volatility in hours worked, there clearly remains mis-specification in the modeling of the labour market as demonstrated by the dynamics existing in the residuals from the labour input FOC. We view this as an illustration of the usefulness of FOC residuals as a complementary diagnostic procedure in our toolkit.

2.3 Measurement error

One potential explanation for the large and persistent residuals is the presence of measurement error in some of the aggregate series, most likely in aggregate hours. In this section we consider three possible ways to model measurement error in observed hours. We begin with classical measurement error which requires that the error be uncorrelated with the true value of the series. Suppose observed hours L_t are related to the true hours series by $L_t = l_t + u_t$, where u_t is measurement error. The indivisible labour model implies that

$$l_t - \frac{\alpha Y_t}{BC_t} = 0 \tag{11}$$

Then our measure of the residual in the indivisible labour model can be written as,

$$L_t - \frac{\alpha Y_t}{BC_t} = l_t + u_t - \frac{\alpha Y_t}{BC_t} = u_t. \tag{12}$$

where the second equality follows from equation (11). Clearly under this interpretation, the residual is entirely measurement error and the indivisible labour model is correct. Using the relationship between true hours, measured hours and the error, we can uncover the implied series for true hours, l_t. If the residual were entirely measurement error in hours it would be uncorrelated with l_t, in fact the correlation coefficient is -0.7. In order to extract the classical measurement error series from u_t, we regress u_t on l_t. The residual from this regression is uncorrelated with the true hours series and can be viewed as the true measurement error, u_t^*. Subtracting u_t^* from u_t gives us a residual series purged of the influence of classical measurement error.[11] This series is also very volatile and persistent, having a volatility roughly equal to aggregate hours (0.007) and a AR(1) coefficient equal to 0.96 which suggests that classical measurement error can not on it's own account for the behaviour of the residual.

[11] Plots for this section can be obtained from the authors upon request.

An alternative way to define measurement error is that it should be uncorrelated with the observed value of the series. Hyslop and Imbens (2001) refer to this as optimal prediction error. We proceed as before to obtain an alternative residual series purged of the influence of the optimal prediction error. In this case, we regress u_t on L_t, subtract the residual from this regression from u_t. Our findings are broadly consistent with the previous results: the residual has a standard deviation of 0.007 and a persistence of 0.92.

2.3.1 RBC model with variable labour effort

So far we have looked at measurement error that is supposed to be uncorrelated with the hours series. Here we suppose that the measurement error has systematic tendencies and is therefore correlated with the observed variables. In order to purge this systematic measurement error from the residual we need to model the behaviour of the measurement error. We use a model with unobserved variable labour effort (Burnside, Eichenbaum and Rebelo, 1993) as our model of procyclical measurement error. The question then is to what extent do the residuals purged of "measurement error" compare to those from the two previous models.

In our RBC model with variable labour effort, a central planner seeks to maximize

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \left[\ln C_t + BN_t \ln(T - \eta - fe_t) + B(1 - N_t)\ln(T)\right] \tag{13}$$

where N denotes the number of workers, T is an agent's time endowment, η is the fixed cost of going to work, f is the fixed shift length and e is effort. Output is produced according to the production function

$$Y_t = K_t^{1-\alpha}(N_t f e_t X_t)^{\alpha}. \tag{14}$$

The planner's optimization is subject to the accumulation equation (2) and the resource constraint (3).

From the planner's problem we get a FOC for employment

$$\mathrm{E}_{t-1}\left\{B\ln(T - \eta - fe_t) - B\ln(T) + \alpha\frac{Y_t}{N_t C_t}\right\} = 0, \tag{15}$$

a FOC for effort

$$\alpha \frac{Y_t}{C_t e_t} - B \frac{f N_t}{T - \eta - f e_t} = 0 \tag{16}$$

and a FOC for investment in capital given by equation (6).

Since labour effort is unobservable, we use FOC (16) to substitute e_t out of FOC (15). The result is

$$\mathrm{E}_{t-1} \left\{ \alpha \frac{Y_t}{N_t C_t} - B \ln \left(\frac{T}{T - \eta - \frac{\alpha(T-\eta)}{BC_t N_t / Y_t + \alpha}} \right) \right\} = 0 \tag{17}$$

In our estimation, we follow Burnside, Eichenbaum and Rebelo (1993) and set $T = 1369$, $f = 324.8$ and $\eta = 60$. This leaves three parameters (B, α and δ) to be estimated using three equations. The moment conditions used in the estimation are

$$\mathrm{E}\{\xi^{lh}_{L,t}\} = 0 \qquad \mathrm{E}\{\xi^{lh}_{K,t+1}\} = 0 \qquad \mathrm{E}\{\xi^{lh}_{\delta,t}\} = 0 \tag{18}$$

where $\xi^{lh}_{L,t}$ denotes the expression in braces in FOC (17),[12] $\xi^{lh}_{K,t+1} = \xi^{s}_{K,t+1}$ and $\xi^{lh}_{\delta,t} = \xi^{s}_{\delta,t}$. Parameter estimates are presented in Table 1. These estimates are very close to those reported in other studies.

The residuals purged of measurement error, $\xi^{lh}_{L,t}$ are plotted in Figure 4. The resemblance between Figures 3 and 4 is striking. Taking account of unobserved labour effort does not remove the dynamic element existing in $\xi^{s}_{L,t}$ and $\xi^{i}_{L,t}$. The serial correlation in $\xi^{lh}_{L,t}$ is still 0.99 as documented in Table 2. For all practical purposes, the only difference between Figures 3 and 4 is the scale of the vertical axis. It indicates that the residuals from the labour FOC are smaller in the variable labour effort model than in the previous two models but still quite large: the standard deviation of $\xi^{lh}_{L,t}$ relative to hours is 0.96. [13]

This result is not surprising because variable effort does not induce any new dynamic elements into the labour FOC equation. Equation (17) is quite similar to the FOC in the fixed effort model

[12] In estimating the variable effort model, we make use of the fact that hours worked are equal to fN_t so that equation (17) is actually estimated using hours data, as it was the case for the previous models.

[13] Unlike the previous two models, the FOC for hours (17) requires that the residuals be zero only on average. Thus the model no longer makes a formal prediction about the size of the residuals. Note however another implication of equation (17) is that the residuals (which now have the interpretation of expectational errors) should be uncorrelated which is clearly violated by the presence of serial correlation in the residuals.

with the only difference appearing in the term multiplying B. This term involves an expression for the unobservable variable effort with only current period variables appearing. The effort series recovered from FOC (16), is depicted in Figure 5. The series closely follows the dynamic pattern of the residuals from the labour FOC. Looking across Figures 3, 4 and 5, it is apparent that some of the residual in the indivisible labour model in Figure 3 is being relabeled as effort in Figure 5 with a corresponding reduction in the residual in Figure 4. This relabeling does not imply that the variation in effort is spurious. After all if (17) is the true FOC but we estimate (9), then the "true" effort series will be dumped into the residual. However if (17) is mis-specified then the possibility exists that any unobservable series introduced into the FOC will at least partly be a spurious proxy for the missing element. Cooper-Johri (2002) construct a constant effort model with a dynamic labor FOC and use this as the data generating process. Under the assumption that effort is variable, it is possible to obtain an expression for effort using a condition like (16). Using the artificial data generated from the model, a spurious effort series can be constructed. Some important moments of this effort series are an autocorrelation coefficient of 0.98 and a contemporaneous correlation with output of 0.97.

Having argued that the observed persistence in the hours FOC residuals cannot be explained away as measurement error, we conjecture that models that generate additional dynamics in the hours FOC will be more consistent with the data. The next sub-sections explore three such types of models.

2.4 Three models of dynamic labour supply

The standard labour supply equation implied by (5) has proved to be a huge stumbling block for business cycle theory. This has been recognized at least since the work of Dunlop and Tarshis. One way to interpret the problem is to replace the marginal product of labour with the real wage rate in these equations so that we can more generally write the labour FOC as a static labour supply equation:

$$w_t = L^s(C_t, L_t). \tag{19}$$

It is immediately apparent that this labour supply function will run afoul of the data because of the tight link between movement in wages and hours worked. Models with such a labour supply function predict a high absolute correlation between wages and hours as well as roughly equal volatility in the two variables. Neither of these predictions are supported by the data. One solution, has been to break this tight link by introducing a random variable on the right hand side which shifts the labour supply curve at business cycle frequencies. Models that incorporate preference shocks (see Parkin, 1988), or government spending shocks (see discussion in Christiano and Eichenbaum, 1992) are examples of this idea.

The idea of labour supply shifts helps to interpret the residuals this paper is concerned with. In the data, there appear to be movements in labour supply in response to movements in wages (or marginal product of labour) that are not captured by the right hand side of (19). These unexplained shifts in labour supply lead to the presence of a residual. While it is possible to write models that modify (19) by introducing additional contemporaneous terms, the observed persistence in the residuals persuades us to explore models which will introduce leads and/or lags into (19). As a result we present three models of dynamic labour supply below. All three models give rise to an additional term in (19) which involves non-contemporaneous variables. This additional term acts as a shift factor for the static labour supply equation and has the potential to account for the missing dynamics in the data which appear as a residual. In all cases the additional term appears because the representative agent not only equates the contemporaneous costs of working with the contemporaneous benefits but takes into account the inter-temporal links built into the problem.

2.4.1 Learning by doing

We begin our study of dynamic labour supply models with a model that incorporates learning by doing (LBD) into the indivisible labour model studied above. Our example is based on Cooper and Johri (2002) in which the representative agent learns to be more productive from past production. [14] Since the learning is internalized by the representative agent, he is aware that working harder today and producing more will result in higher productivity tomorrow. As a result the agent will choose to equate the current disutility of work not only with the current marginal utility of the additional goods produced today but also the future marginal utility of the additional goods produced tomorrow which arise from the higher productivity induced by learning by doing. The model was developed to build an additional propagation mechanism into RBC models. Cooper and Johri (2002) shows that the model is able to generate considerable persistence in output as reflected by hump-shaped impulse responses in output and two positive autocorrelation coefficients in output growth. Other moments look very similar to the standard indivisible labour model[15].

We briefly sketch the model without any discussion of the modeling assumptions (which can be found in Cooper-Johri). In the model, a central planner maximizes utility (8) subject to the accumulation equation for physical capital (2) and resource constraint (3). The crucial change occurs in the production technology which is now subject to LBD. Learning influences productivity through the stock of organizational capital, H, with the technology being given by the following production function:

$$Y_t = K_t^{1-\alpha-\varepsilon} H_t^{\varepsilon} (L_t X_t)^{\alpha}. \tag{20}$$

The stock of organizational capital itself evolves according to a log-linear accumulation equation and depends on past production as well as past organizational capital as follows:

$$H_{t+1} = H_t^{\gamma} Y_t^{1-\gamma}. \tag{21}$$

where $\gamma \in [0, 1)$ and $\varepsilon > 0$.

The FOCs corresponding to the planners problem are

$$\mathrm{E}_t \left\{ \frac{\alpha Y_t}{C_t L_t} - B + \beta \left(B \left(\gamma + \varepsilon(1-\gamma) \right) \frac{L_{t+1}}{L_t} - \alpha\gamma \frac{Y_{t+1}}{C_{t+1} L_t} \right) \right\} = 0 \tag{22}$$

[14] Also see Chang, Gomes and Schorfeide (2002) for an alternative specification of a learning by doing model.

[15] Details of the model including a discussion of the labour supply response to shocks as well as simulation results and moment matching exercises can be found in the paper and are not discussed here.

and

$$\mathrm{E}_t\left\{\beta\frac{C_t}{C_{t+1}}\left[(1-\alpha-\varepsilon)\frac{B}{\alpha}\frac{L_{t+1}C_{t+1}}{K_{t+1}}+1-\delta\right]-1\right\}=0. \tag{23}$$

where these have been re-written for ease of comparison with the earlier models.

We immediately see that the LBD model generates two dynamic FOCs that are different from the equations we have seen so far. For our discussion of dynamic labour supply we focus on equation (22). Note that the first two terms in (22) are actually the two terms appearing in (9). This is the same current period comparison of the disutility of work with the utility of consumption. The discounted third term incorporates the new dynamics introduced into the labour supply decision and can be seen as endogenously shifting labour supply. Once again replacing the marginal product of labour with the real wage rate, (22) may be written as $w_t = L^s(C_t) + Z^{lbd}(C_t, L_t, L_{t+1}, C_{t+1}, W_{t+1})$. There are two factors that modify the labour supply decision of the agent relative to the indivisible labour model creating the appearance of shifts. First, the additional output created by working in period t generates more organizational capital for period $t+1$. This additional organizational capital in $t+1$ is beneficial in two ways. First, it directly adds to production in period $t+1$, raising the marginal utility of consumption in that period. Second, it implies that less needs to be produced in period t+1 to maintain productivity in $t+2$. This reduction in output yields a net utility gain since the disutility of leisure falls more than the utility of consumption. Given a wage rate, (22) implies the agent would work more in the presence of LBD than in the indivisible labour model.

Inspection of equation (22) immediately suggests that this model may have the potential to capture the auto-correlation in $\frac{Y_t}{C_t L_t}$ that creates problems for static labour FOCs. However numerical confirmation of this point requires us to estimate the parameters of the model and construct the appropriate residual series. A useful by-product of this exercise is a formal test of the Cooper and Johri (2002) model.

Denoting the expressions in braces in equations (22) and (23) by $\xi^{lbd}_{L,t+1}$ and $\xi^{lbd}_{K,t+1}$ respectively, and noting that $\xi^{lbd}_{\delta,t} = \xi^{s}_{\delta,t}$ we can write the moment restrictions

$$\mathrm{E}\left\{\xi^{lbd}_{L,t+1}\cdot Q_t\right\}=0 \qquad \mathrm{E}\left\{\xi^{lbd}_{K,t+1}\cdot Q_t\right\}=0 \qquad \mathrm{E}\{\xi^{s}_{\delta,t}\cdot Q_t\}=0 \tag{24}$$

where Q_t is an instrument set.

Since the parameters B and α can almost always be written in a ratio (to see this, divide all terms in (22) by α) it is difficult to identify them separately. For this reason, we set one of the parameters and estimate the other. Given the strong evidence on total labour input share of around two-thirds, we chose to set $\alpha = 0.55$ and estimate B. We picked a lower value of α than usual because it represents the returns to raw labour excluding the effect of organizational capital.[16]

Since the LBD model has more than three parameters to be estimated, the instrument set must include more than a constant. To select the instruments, and therefore the moments employed in the GMM optimization, we use Andrews (1999) moment selection procedure. Since we are using a relatively small sample (152 data points), we restrict our attention to small instrument sets. Monte Carlo work by Hansen, Heaton and Yaron (1996), Kocherlakota (1990) and Smith (1999) suggests using small instruments sets in small samples. In applying Andrews procedure, we look at instrument sets including two, three or four variables only. The variables we included in our testing are: a constant, consumption growth, output growth and output to consumption ratio.[17] We use the GMM-AIC, GMM-BIC, GMM-HQIC criteria as well as the upward and downward procedures suggested by Andrews. [18] All these methods selected the instrument set

$$Q_t = \left\{1, \frac{Y_t}{C_t}\right\}. \tag{25}$$

The GMM estimates obtained using this instrument set are presented in Table 1. Most parameter estimates are close to those estimated earlier. $B = 6.11$, similar to the indivisible labour case and the depreciation rate remains close to two percent per quarter. The point estimate of the share of organizational capital in the production function, $\varepsilon = 0.24$. Interestingly this implies a learning rate of 18 percent which is very close to the benchmark rate of 20% reported in a large number of industry studies. [19] The point estimate of γ, the parameter from the accumulation equation for

[16]The implied share of labour augmented by organizational capital at the point estimates obtained by us is slightly above two-thirds. In addition, picking B equal to the value estimated in the indivisible labour model and estimating α delivers a point estimate of 0.55.

[17]Using lagged consumption growth, lagged output growth and lagged output to consumption ratio has a trivial effect on parameter estimates but does not change the results in any way.

[18]We follow the recommendations made in Andrews (1999) and perform the tests using an optimal weighting matrix and centering the contributions to the empirical moments when constructing the weighting matrix.

[19]See Cooper-Johri (2002) for an extensive discussion of the empirical learning by doing literature.

organizational capital is 0.95 which is somewhat high relative to earlier estimates of the model. Note that Table 1 also presents the value of the over-identifying restrictions test-statistics and its associated p-value. The test does not reject the model and the instruments at conventional significance levels. The use of an over-identified estimator is not subject to the qualification made earlier because the residual series from the LBD model, have little persistence. As a result the computation of the GMM weighting matrix is less of a problem.

The two series depicted in Figure 6 are strikingly different. The figure suggests that the residuals from the labour input FOC are much less autocorrelated in the LBD model than in the RBC model. Actually, we cannot reject (at any conventional level of significance) the null hypothesis that the residuals $\hat{\xi}^{lbd}_{L,t+1}$ have zero serial correlation. This is easily seen by looking at the serial correlation coefficients reported in Table 2: $\rho_1(\hat{\xi}^{lbd}_{L,t+1}) = 0.04979$ with a standard error of 0.12687. The residuals from the modified FOC for capital have a marginally higher persistence than in the previous three models but the difference is not significant. The results of a sensitivity analysis for the effect of the parameter values on the persistence of the residuals for the the labour input FOC are presented in Figure 7. We see that B, α and ε have a very small effect on the degree of persistence in $\hat{\xi}^{lbd}_{L,t+1}$ and $\hat{\xi}^{lbd}_{K,t+1}$ while γ appears to be an important determinant of the serial correlation in the residuals for only the labour input FOC.

As demonstrated by Cooper and Johri (2002), learning-by-doing acts as an effective internal propagation mechanism, generating hump shaped impulse responses and realistic autocorrelation functions for output growth. This raises the question whether our initial finding of persistence in the residuals of standard RBC models is merely another symptom of the weak internal propagation built into those models. Put another way, if this were true, then our procedure would add no new information about the model not already revealed by looking at impulse responses or moments that capture the persistence of the simulated series.[20] In order to demonstrate that this is not the case we consider an example of a DGE model that generates persistent aggregate variables while leaving the hours first order condition unchanged. This model is a straight-forward modification of the LBD model discussed above with one crucial change. Instead of internal learning, the model has a learning externality. Since the learning is external (see Cooper and

[20]We thank an anonymous referee for encouraging us to explore this point.

Johri, 1997, for details) it is easy to see that the representative agent does not take into account that his productivity will increase if he produces more. As a result he bases his decision regarding how many hours to work only on the current disutility of work and the current marginal utility of the additional output produced by the marginal hour of work. So the FOC in this model is in fact the same as in the indivisible labour model, equation (9), and therefore must have identical residuals as well.[21]

The results from the LBD model suggest that we need to incorporate elements that generate dynamic labour supply functions into RBC type models in order to explain the strong dynamic cross-correlations displayed by the data. While learning by doing is one such mechanism, another source of introducing dynamics comes from abandoning time separability of preferences. We explore this issue in the next sub-section.

2.4.2 Habit Formation in an RBC model

This section begins with a sketch of the basic RBC model modified to include non separabilities in preferences over consumption in adjoining time-periods. The FOC associated with this model are estimated and the residuals evaluated as before.[22] While we restrict ourselves to discussing the case where past consumption raises current marginal utility (habits), we do not restrict the estimation procedure in this way.

Our habit formation specification is a special case of Constantinides (1990). [23] We assume that consumption in period $t-1$ affects the marginal utility of consumption in period t. More specifically, in our habit formation model, a central planner maximizes

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - \lambda C_{t-1}) + B \ln(1 - L_t)\right] \tag{26}$$

subject to the usual constraints (2), (3) and (4). There are two dynamic FOCs associated with this

[21]The model generates highly persistent series. A discussion can be found in Cooper and Johri (1997). Detailed simulation results for this model are available upon request.

[22]We present similar results for habit formation in leisure in an appendix available from the authors.

[23]Lettau and Uhlig (2000) study the moments and impulse response function of the indivisible model augmented with habit formation à la Campbell and Cochrane (1999).

habit formation model. The condition for hours is

$$\mathrm{E}_t\left\{\alpha\frac{Y_t}{L_t}\left[\frac{1}{C_t-\lambda C_{t-1}}-\frac{\beta\lambda}{C_{t+1}-\lambda C_t}\right]-\frac{B}{1-L_t}\right\}=0 \tag{27}$$

and the condition for capital is

$$\begin{aligned}&\mathrm{E}_t\Bigg\{\frac{1}{C_t-\lambda C_{t-1}}-\frac{\beta\lambda}{C_{t+1}-\lambda C_t}\\ &-\beta\left((1-\alpha)\frac{Y_{t+1}}{K_{t+1}}+1-\delta\right)\left(\frac{1}{C_{t+1}-\lambda C_t}-\frac{\beta\lambda}{C_{t+2}-\lambda C_{t+1}}\right)\Bigg\}=0.\end{aligned} \tag{28}$$

Note once again the presence of an additional dynamic term in (27) which acts like a shift factor (this time multiplicative). In order to compare (27) to (19) we can re-write it as before as

$$w_t = Z^{habits}(C_t, C_{t-1}, C_{t+1})L^s(C_t, L_t). \tag{29}$$

The mechanism involved in shifting the labour supply function is quite different here compared to the LBD model. The contemporaneous benefit of working is only being compared to the contemporaneous cost of working, however the marginal benefit of consumption itself involves a dynamic interlinkage between consumption in adjoining periods. To get a sense for this, note that the marginal utility of consumption in period t is increasing in C_{t-1}. This itself introduces a wedge between the wage rate and the static labour supply function given by L^s in (19). Note too that the extra consumption made possible by working an additional hour has two benefits. First, the agent gets extra utility in the current period. Second, the extra consumption in period t increases the marginal utility of consumption in period $t+1$. As a result (27) differs from (5) because of two terms. The first is the marginal utility of consumption in period t with respect to C_t which is influenced by C_{t-1} as discussed above. The second is the discounted marginal utility of consumption in period $t+1$ which is influenced by C_t.While both terms are potentially important sources of shifts in L^s, it turns out that the latter effect dominates in terms of accounting for the persistence of the residual as we discuss below.

There are two ways to see why the dynamic interlinkage appears to shift the labour supply equation in (19). Most directly, one can view the increase in marginal utility in period $t+1$ as a preference shock which shifts labour supply. More intuitively, the dynamic influence of consumption on future marginal utility of consumption makes the agent choose a different level of consumption than in the absence of habit formation. Given the marginal product of output, this leads the agent to choose a different number of hours to work relative to the basic divisible labour model.

As before we begin by estimating the model. Denoting the expressions in braces in equations (27) and (28) by $\xi^{hf}_{L,t+1}$ and $\xi^{hf}_{K,t+1}$ respectively, and noting that $\xi^{hf}_{\delta,t} = \xi^{s}_{\delta,t}$ we can write the moment restrictions

$$\mathrm{E}\ \{\xi^{hf}_{L,t+1} \cdot Q_t\} = 0 \qquad \mathrm{E}\ \{\xi^{hf}_{K,t+1} \cdot Q_t\} = 0 \qquad \mathrm{E}\{\xi^{hf}_{\delta,t} \cdot Q_t\} = 0 \tag{30}$$

where Q_t is an instrument set.

For consistency with the estimation of the LBD model, we use the instrument set in equation (25) when estimating the habit formation model. The estimates of the parameters are presented in Table 1. Our estimate of λ is 0.97 and is close to the value estimated by Boldrin, Christiano and Fisher (2001) for their one-sector model that is closest to ours $(\lambda = 0.9)$. Table 1 also reports the J-test statistics and its associated p-value. Clearly, the model and instruments are not rejected at any conventional level of significance.

The residuals from the labour FOC are plotted in Figure 8 while the usual persistence measures are reported in Table 2. The persistence in the residuals from the FOCs is now significantly smaller than in the standard RBC model. In fact the autocorrelation coefficient is now too low for both $\xi^{hf}_{L,t+1}$ (-0.40) and $\xi^{hf}_{K,t+1}$ (-0.58).[24] A glance at Figure 8 makes clear that the reduction in the autocorrelation of the residuals from the labour FOC is accompanied by a huge increase in volatility. The standard deviation is 1.492. [25]

[24]We also look at a model where utility depends on current and lagged consumption in ratio form in the spirit of Abel (1990). Persistence in residuals remained a problem.

[25]While equation (27) does not directly make any predictions about the volatility of the residual, the model as a whole can be used to generate one. Using estimated parameters and actual US data, a linearized version of the habit formation model was used to simulate consumption, output and hours series. Residuals from equation (27) had a volatility roughly 26 times smaller than that of the observed residuals, $\xi^{hf}_{L,t+1}$. This evidence suggests that the habit formation model may not be able to account for the dynamics of aggregate data. This probably occurs due to the tremendous desire to smooth consumption. This counter-factual smoothness leads to huge swings in the residual series. Note however that this evidence is not conclusive since the simulated data is generated from a linearized model.

In the habit formation model, the estimates and the degree of persistence left in the residuals turn out to depend on the instrument set used. [26] When using

$$Q_t = \left\{1, \frac{Y_t}{Y_{t-1}}\right\} \quad \text{or} \quad Q_t = \left\{1, \frac{C_t}{C_{t-1}}\right\},$$

the estimates of B, α and δ are very close to the estimates in the standard RBC model and the estimates of λ are 0.30 and 0.46 respectively. These estimates of λ are not sufficiently large to remove the persistence in the residuals from the labour FOC but they do reduce the volatility of the residuals. Evidentally there is a trade-off between controlling the volatility and persistence of residuals in the habit formation model. To fully remove the persistence in the residuals from the labour FOC, the parameter λ must be set to 0.7741 (see Figure 9).[27] Interestingly, the residuals from the capital FOC still have a sizeable negative serial correlation (-0.60) when $\lambda = 0.7741$. These properties of the residuals are highly sensitive to varying λ between 0.5 and 0.9 as is evident in Figure 9.

Returning to the issue of which term in (27) is responsible for removing the persistence in the residuals, consider a variant of the model with external rather than internal habits. In this case (27) reduces to

$$\mathrm{E}_t \left\{ \alpha \frac{Y_t}{L_t} \left[\frac{1}{C_t - \lambda C_{t-1}} \right] - \frac{B}{1 - L_t} \right\} = 0. \tag{31}$$

The absence of the second term significantly hampers the ability of the model to account for the persistence in the residual from equation (31). For example, when $\lambda = 0.77$, the autocorrelation of the residual is 0.74 and when $\lambda = 0.98$ it is 0.25.

2.4.3 Labour adjustment costs

In this section we consider a model in which the representative agent faces quadratic costs of adjusting his labour supply. The specific form of adjustment costs is taken from Cooper and Willis (2004) and adapted to the simple DGE setting for ease of comparison with the other

[26] This sensitivity to the choice of instruments does not arise in the estimation of the LBD model. Whenever the GMM algorithm converges to estimates that are economically meaningful, those estimates are similar to the ones presented in Table 1 and the residuals from the labour FOC always have very low persistence.

[27] This value of λ was estimated by using a just identified GMM estimator using the moment conditions $\mathrm{E}\{\xi^{hf}_{L,t+1}\} = 0$, $\mathrm{E}\{\xi^{hf}_{K,t+1}\} = 0$, $\mathrm{E}\{\xi^{hf}_{\delta,t}\} = 0$ and $\mathrm{E}\{\xi^{hf}_{L,t+1} \times \xi^{hf}_{L,t}\} = 0$. We thank Martin Browning for suggesting this estimator.

models discussed in this paper. Relative to the indivisible labour model, there is only one change: the representative agent must subtract the cost of changing the labour supply from current output. As a result equation (4) is replaced with

$$Y_t = K_t^{1-\alpha}(L_t)^{\alpha} - \frac{q}{2}(\frac{L_t - L_{t-1}}{L_{t-1}})^2 L_{t-1}. \tag{32}$$

This results in a dynamic FOC for hours which may be written as

$$0 = \mathrm{E}_t \left\{ \frac{\alpha Y_t}{C_t L_t} - B + q \left[\frac{\beta}{2C_{t+1}} \left(\frac{L_{t+1}^2 - L_t^2}{L_t^2} \right) - \left(\frac{L_t - L_{t-1}}{C_t} \right) \right] \right\}. \tag{33}$$

Once again the dynamic terms involving leads and lags of hours and consumption appear as an additive shift factor so that (33) may be written as $w_t = L^s(C_t) + Z^{ac}(C_t, L_t, L_{t-1}, C_{t+1}, L_{t+1})$. As usual there are two factors of importance here. First there is an incentive to keep hours close to their past value due to the presence of quadratic adjustment costs which reduce the marginal benefit of working. Second, working more this period reduces the costs of increasing hours in the future and the agent takes this into account when choosing how to respond to the wage rate.

As can be seen by varying the adjustment cost parameter q in Figure 10, the model has the ability to reduce the persistence in the residual, but not to remove it for plausible values of adjustment costs. For example, the median value of estimates reported by Cooper and Willis is $q = 2$, whereas we show that the persistence in the residual remains high even for values of $q = 200$.

The results from the last three sections confirm the intuition that adding dynamics in the labour FOC can potentially help to reduce the persistence in the residuals and improve the "fit" of this FOC but the ability of these models to capture the joint dynamics of the data vary widely.[28]

[28] This result is related to the findings of Chow and Kwan (1998) who showed that adding lagged dependent variables (investment and hours) to the state vector of an unrestricted reduced form regression constructed from the standard RBC model helps reduce the serial correlation in the residuals of the linearized investment and hours decision rules.

3 Concluding remarks

In this paper we argue that the traditional first-order condition (FOC) for the labour input in RBC type models which equates the marginal rate of substitution between consumption and leisure to the marginal product of labour is fundamentally inconsistent with aggregate US data. Residuals constructed from this FOC are very large (over one and a half times more volatile than aggregate hours) and highly persistent (autocorrelation of 0.99). Based on robustness exercises, we argue that the likely explanation for these properties is that the labour market side of the RBC model is not properly specified.

The high degree of persistence in the residuals leads us to hypothesize that models that introduce not just current period variables but also additional dynamic terms in the labour FOC would be more consistent with the data. We test this hypothesis by looking at three alternative models of dynamic labour supply embedded in the RBC structure. The first is a learning by doing model in which the representative agent realizes that current production is an input into future productivity. The second involves using variants of non-separable preferences in either consumption or leisure or both. The third involves quadratic adjustment costs in changing hours worked. In each case we argue that the new dynamic terms play the role of endogenous shifts of the static labour supply curve associated with the basic RBC model. We show that these models have the potential to remove or at least substantially reduce the persistence in the residuals. As a result, the models are much more consistent with the joint dynamics of aggregate consumption, output and hours as compared to models that involve static FOCs. This improved ”fit” of the models with aggregate US data is reflected in the fact that formal tests of the model and instruments using over-identifying restrictions do not reject either the learning by doing model or the model with habit formation in consumption. This is quite surprising given how well known and widespread is the rejection of earlier RBC type models using overidentifying restrictions. Unfortunately, for the model with habit formation in consumption, the removal of persistence in the residuals from the labour FOC comes at the expense of a fairly high degree of negative autocorrelation in the residuals from the capital FOC.

We argue that looking at the graphs and dynamic patterns of the residuals from the key first-order conditions of the model can be a useful tool for its evaluation, complementary to moment matching and impulse response graphs which are currently popular techniques. A clear advantage of the proposed simple evaluation techniques are that they do not rely on specific assumptions about the properties of shocks. Boileau and Normandin (2002) shows that the business cycle implications of many of the models investigated in this paper depend importantly on the specification of the law of motion for the shocks so that conventional tests based on moment matching exercises are joint tests of individual behaviour and the assumed laws of motion of exogenous processes. Moreover, the proposed techniques do not rely on specific identifying restrictions required to carry out impulse response comparisons. They focus attention on the joint behaviour of macro-economic aggregates as opposed to the behaviour of individual series and they do not rely on simulation of models linearized around balanced growth paths.

Even though we have emphasized estimation procedures, and graphs of residuals from estimated relationships, the procedure can be used profitably for calibrated models as well which is illustrated by sensitivity analysis throughout the paper. While we have focused on representative agent RBC type models in this paper, it is clear that our results have wider applicability because similar static labour market FOC are embedded in many classes of models popular in the literature. As a by-product of our exploration of a number of dynamic general equilibrium models, the paper offers a set of estimates of the structural parameters of these models based on the same estimation procedure and data sets.

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Data appendix

Except for the wage series, we use the same data set as Burnside and Eichenbaum (1996). See their paper for more details. We thank Craig Burnside who provided the data.

Sample period: 1955Q1-1992:Q4

Real Wages

Wages and Salaries from the Bureau of Economic Analysis (mnemonic wascur) divided by lhours and the GDP deflator.

Capital

Sum of the net stocks of consumer durables, producer structures and equipment, and government and private residential capital plus government nonresidential capital.

Private consumption

Sum of private-sector expenditures on nondurable goods plus services plus the imputed service flow from the stock of consumer durables.

Output

Measured as $C_t + G_t + I_t$ plus net exports and time-t inventory investment.

Hours worked

Seasonally adjusted household hours series obtained from Citibase (mnemonic LHOURS).

Gross investment

Purchases of consumer durables, gross private nonresidential investment (structures and equipment) and residential investment, as well as the change in the gross stock of government capital.

Population

Data are converted to *per capital* terms using the civilian noninstitutional population aged 16 and over.

Alternative Dataset

Sample period: 1947Q1-1999:Q4

Capital

Net stocks of nonresidential (producer structures and equipment) and residential capital. From NIPA table 15 (Bureau of Economic Analysis).

Private consumption

Sum of private-sector expenditures on nondurable goods plus services. Data are from NIPA tables (Bureau of Economic Analysis).

Output

Gross domestic product. Data are from NIPA tables (Bureau of Economic Analysis).

Hours worked

Before 1964: U.S manhours of nonfarm employees, seasonally adjusted (BLS, NBER Macrohistory data). From 1964, the total hours series is constructed using average weekly hours of production workers (seasonally adjusted) and employees on nonfarm payrolls (seasonally adjusted)

Gross investment

Purchases of consumer durables, gross private nonresidential investment (structures and equipment) and residential investment. Data are from NIPA tables (Bureau of Economic Analysis).

Population

Data are converted to *per capital* terms using the civilian noninstitutional population aged 16 and over (Citibase).

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Table 1: Parameter Estimates

	RBC Standard	RBC Indivisible	Labour Effort	Learning By Doing	Habit Form. in C
B	4.90801 (0.10639)	6.18325 (0.16057)	4.18368 (0.06460)	6.11239 (0.08887)	3.57770 (2.36293)
δ	0.01952 (0.00063)	0.01952 (0.00069)	0.01952 (0.00023)	0.02006 (0.00013)	0.02019 (0.00015)
α	0.72994 (0.00703)	0.72994 (0.00785)	0.72994 (0.00500)	0.55 -	0.51485 (0.29647)
ε				0.23925 (0.00371)	
γ				0.94952 (0.01785)	
λ					0.97397 (0.00869)
J-test (p-value)				0.86836 (0.64779)	0.69342 (0.70701)

Note to Table 1: Standard errors are in parentheses.

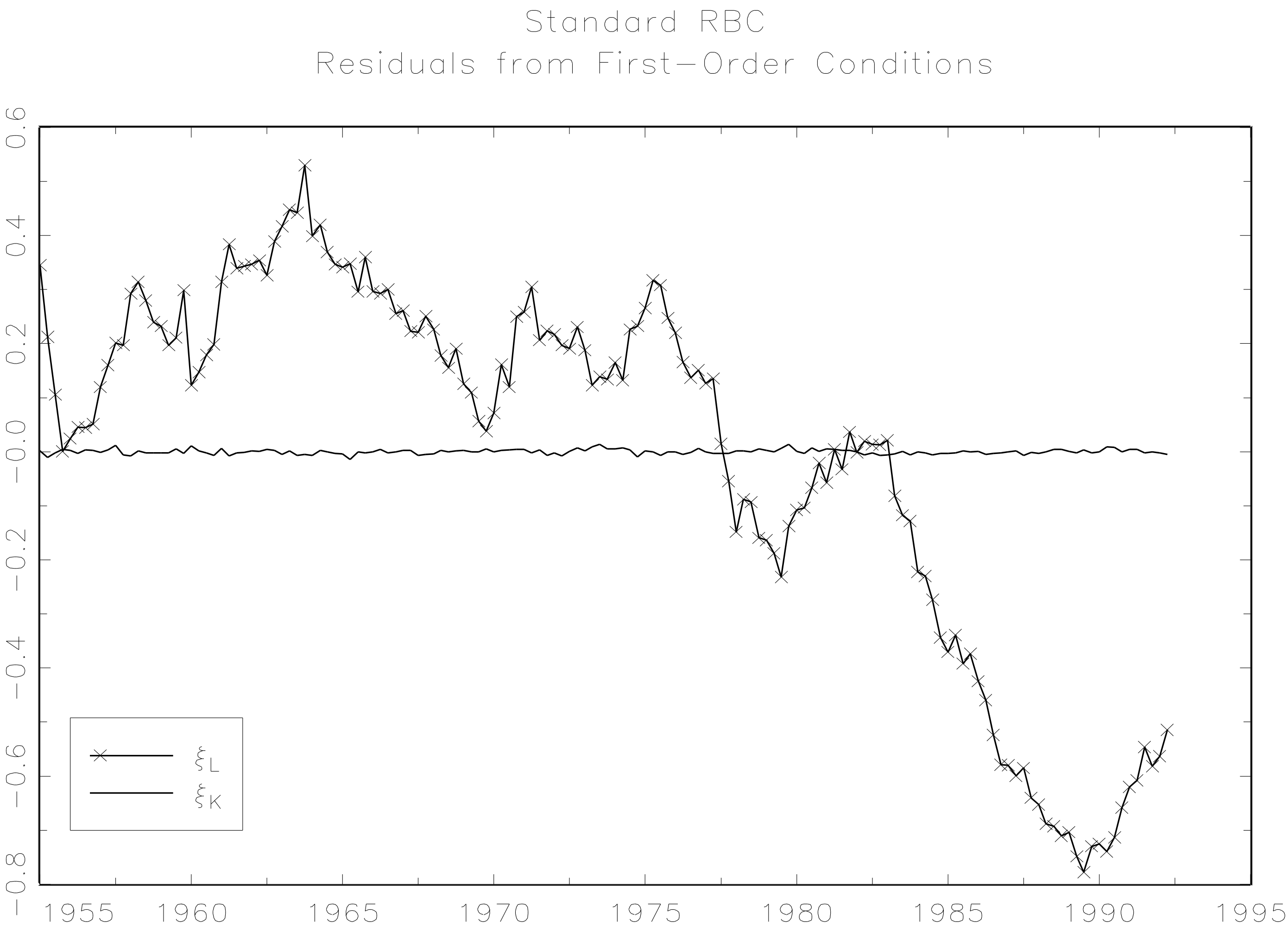
Table 2: Persistence in Residuals from First-Order Conditions

	RBC Standard	RBC Indivisible	Labour Effort	Learning By Doing	Habit Form. in C	
$\rho_1(\hat{\xi}_K)$	0.23592 (0.08312)	0.23592 (0.08312)	0.23592 (0.08312)	0.31981 (0.12434)	-0.58365 (0.06575)	
$\rho_1(\hat{\xi}_L)$	0.99302 (0.01357)	0.99112 (0.01195)	0.99030 (0.01166)	0.04979 (0.12687)	-0.39821 (0.04828)	
$\rho_{1,tr}(\hat{\xi}_L)$	0.96357 (0.02588)	0.95409 (0.02367)	0.95147 (0.02395)	0.01909 (0.12142)	-0.39821 (0.04827)	

Notes to Table 2:

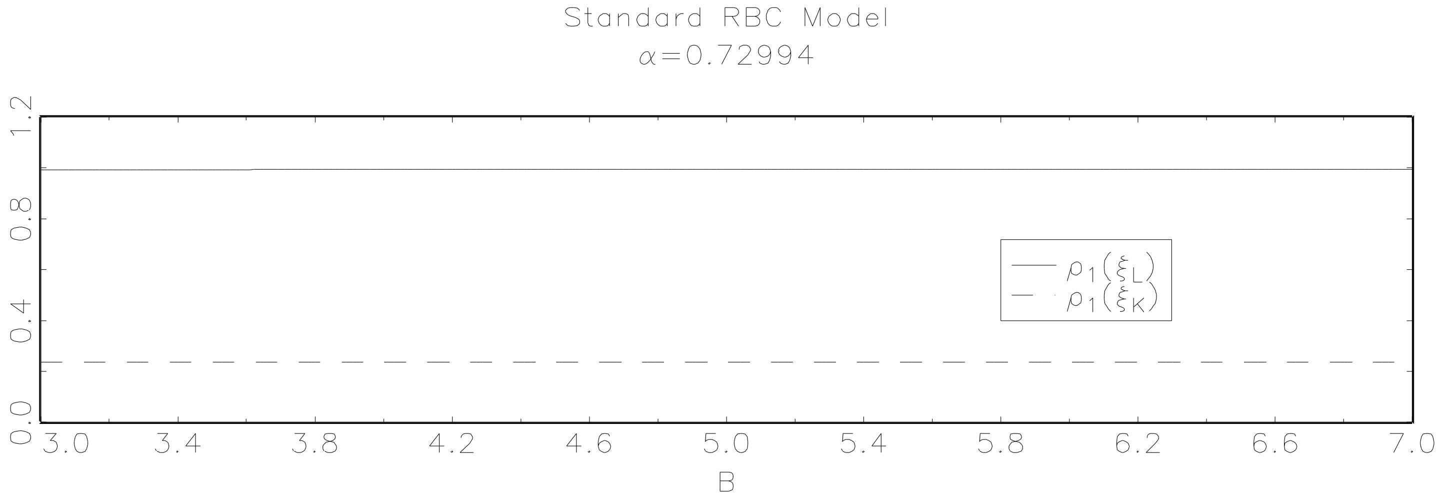
Standard errors are in parentheses.
$\hat{\xi}_K$: residuals from the capital first-order condition.
$\hat{\xi}_L$: residuals from the labour first-order condition.
ρ_1: first-order autocorrelation coefficient.
$\rho_{1,tr}$: first-order autocorrelation coefficient, including a linear trend in the autoregression.

Figure 1



Standard RBC
Residuals from First–Order Conditions
0.6
0.4
0.2
–0.0
–0.2
–0.4
–0.6
–0.8
1955
1960
1965
1970
1975
1980
1985
1990
1995
ξ_L
ξ_K

Figure 2


Standard RBC Model
α=0.72994
0.0
0.4
0.8
1.2
3.0
3.4
3.8
4.2
4.6
5.0
5.4
5.8
6.2
6.6
7.0
B
$\rho_1(\xi_L)$
$\rho_1(\xi_K)$

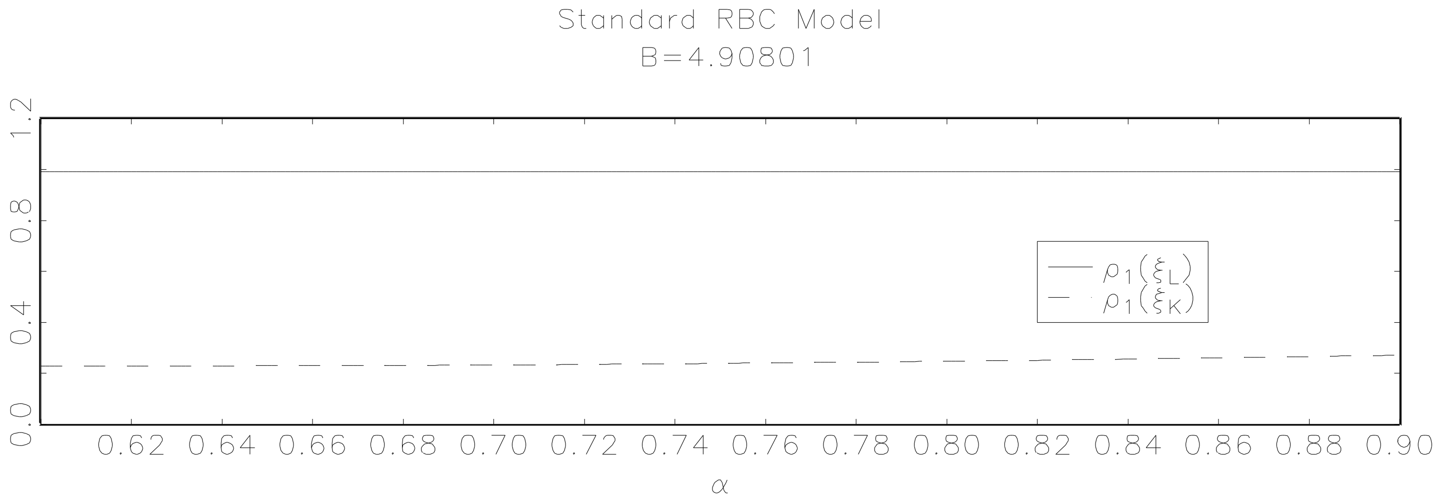

Standard RBC Model
B=4.90801
0.0
0.4
0.8
1.2
0.62
0.64
0.66
0.68
0.70
0.72
0.74
0.76
0.78
0.80
0.82
0.84
0.86
0.88
0.90
α
$\rho_1(\xi_L)$
$\rho_1(\xi_K)$

Figure 3

Residuals from Hours First-Order Conditions
0.6
0.4
0.2
-0.0
-0.2
-0.4
-0.6
-0.8
1955
1960
1965
1970
1975
1980
1985
1990
1995
RBC Standard
RBC Indivisible

Figure 4

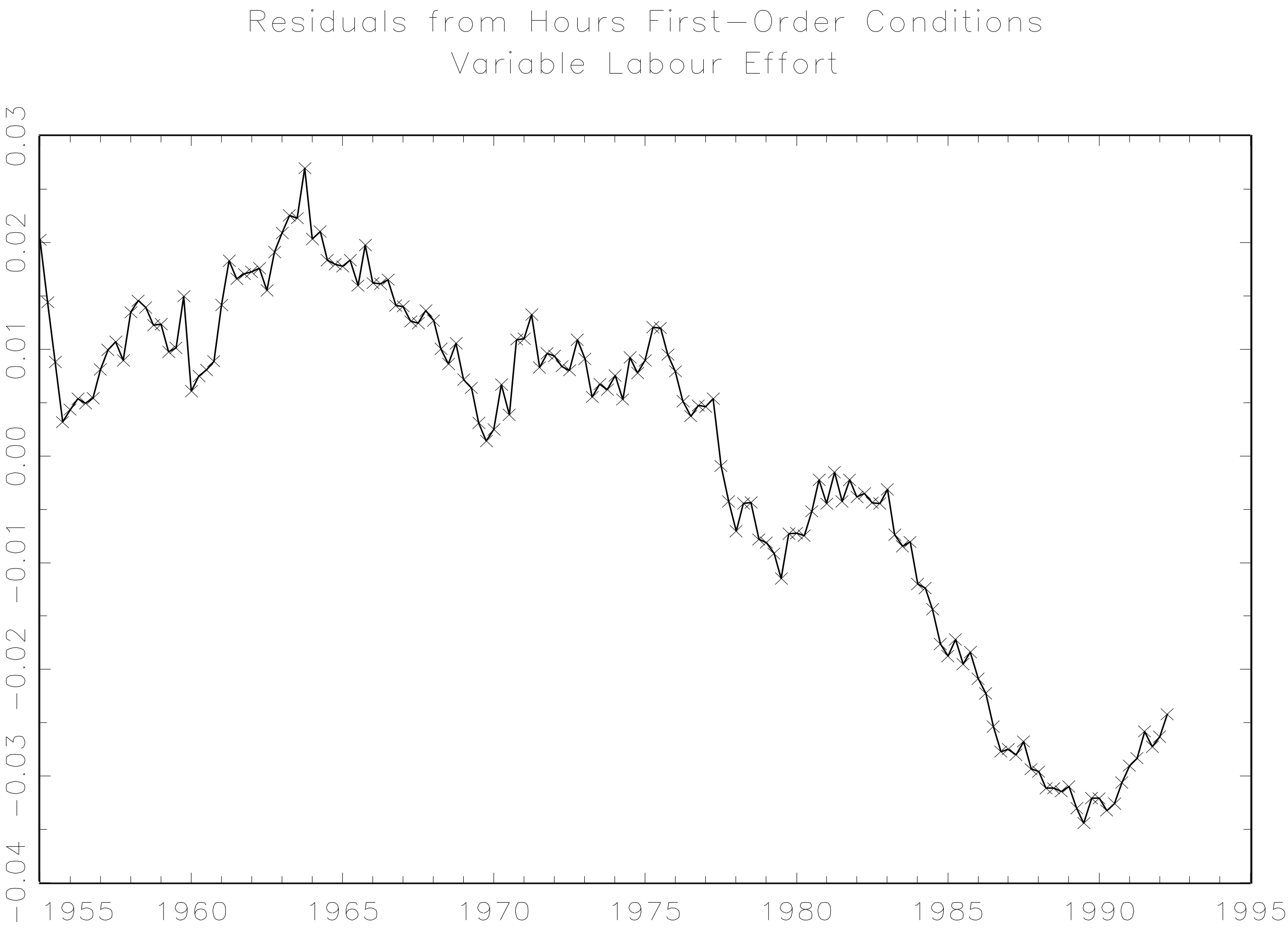
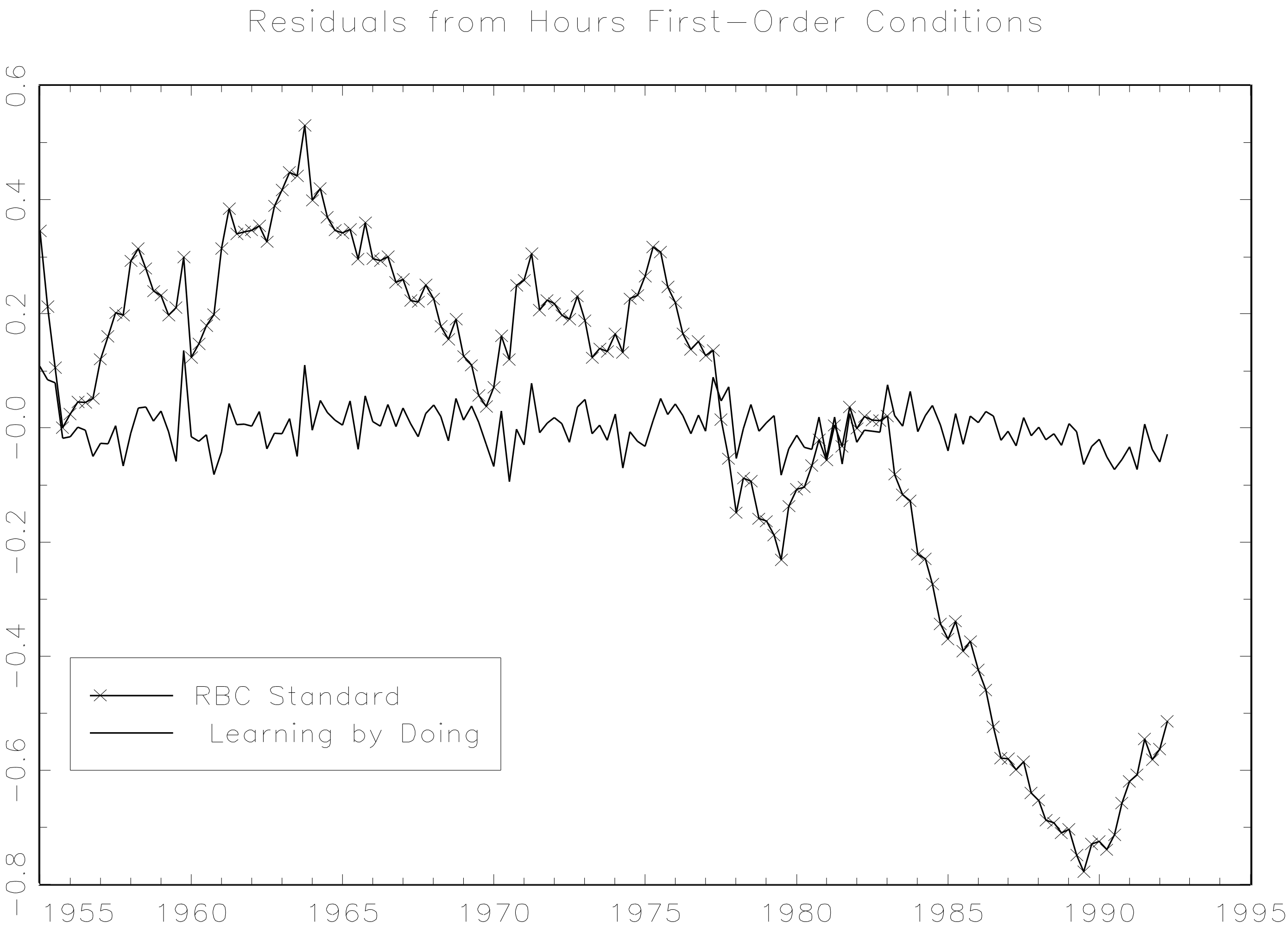

Residuals from Hours First–Order Conditions
Variable Labour Effort
0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
−0.04
1955
1960
1965
1970
1975
1980
1985
1990
1995

Figure 5

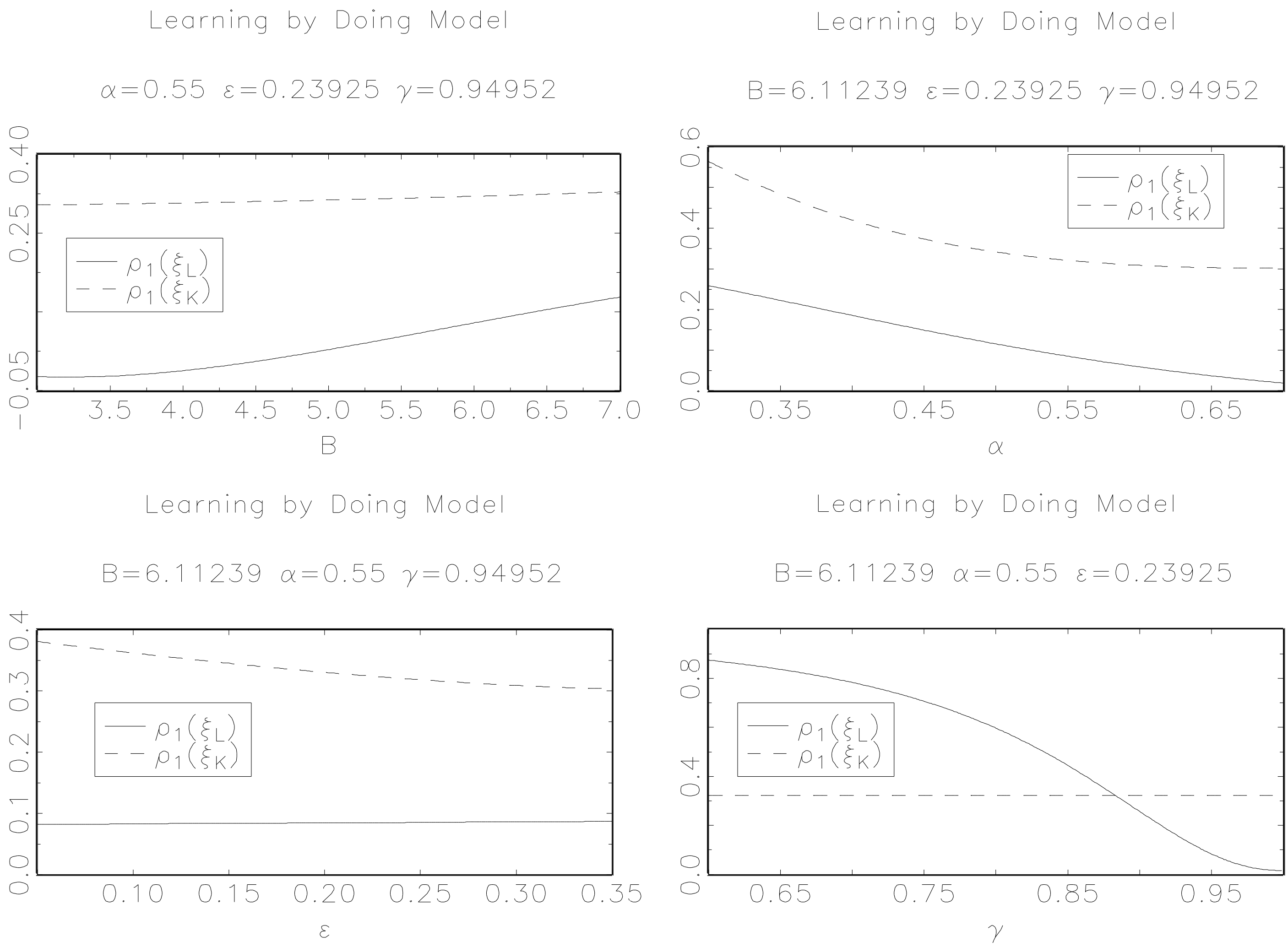
Variable Labour Effort — Effort Series
1.06
1.04
1.02
1.00
0.98
0.96
0.94
0.92
0.90
1955
1960
1965
1970
1975
1980
1985
1990
1995

Figure 6



Residuals from Hours First−Order Conditions
0.6
0.4
0.2
−0.0
−0.2
−0.4
−0.6
−0.8
1955
1960
1965
1970
1975
1980
1985
1990
1995
RBC Standard
Learning by Doing

Figure 7



Learning by Doing Model
α=0.55 ε=0.23925 γ=0.94952
0.40
0.25
−0.05
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0
B
ρ1(ξL)
ρ1(ξK)
Learning by Doing Model
B=6.11239 ε=0.23925 γ=0.94952
0.6
0.4
0.2
0.0
0.35
0.45
0.55
0.65
α
ρ1(ξL)
ρ1(ξK)
Learning by Doing Model
B=6.11239 α=0.55 γ=0.94952
0.4
0.3
0.2
0.1
0.0
0.10
0.15
0.20
0.25
0.30
0.35
ε
ρ1(ξL)
ρ1(ξK)
Learning by Doing Model
B=6.11239 α=0.55 ε=0.23925
0.8
0.4
0.0
0.65
0.75
0.85
0.95
γ
ρ1(ξL)
ρ1(ξK)

Figure 8

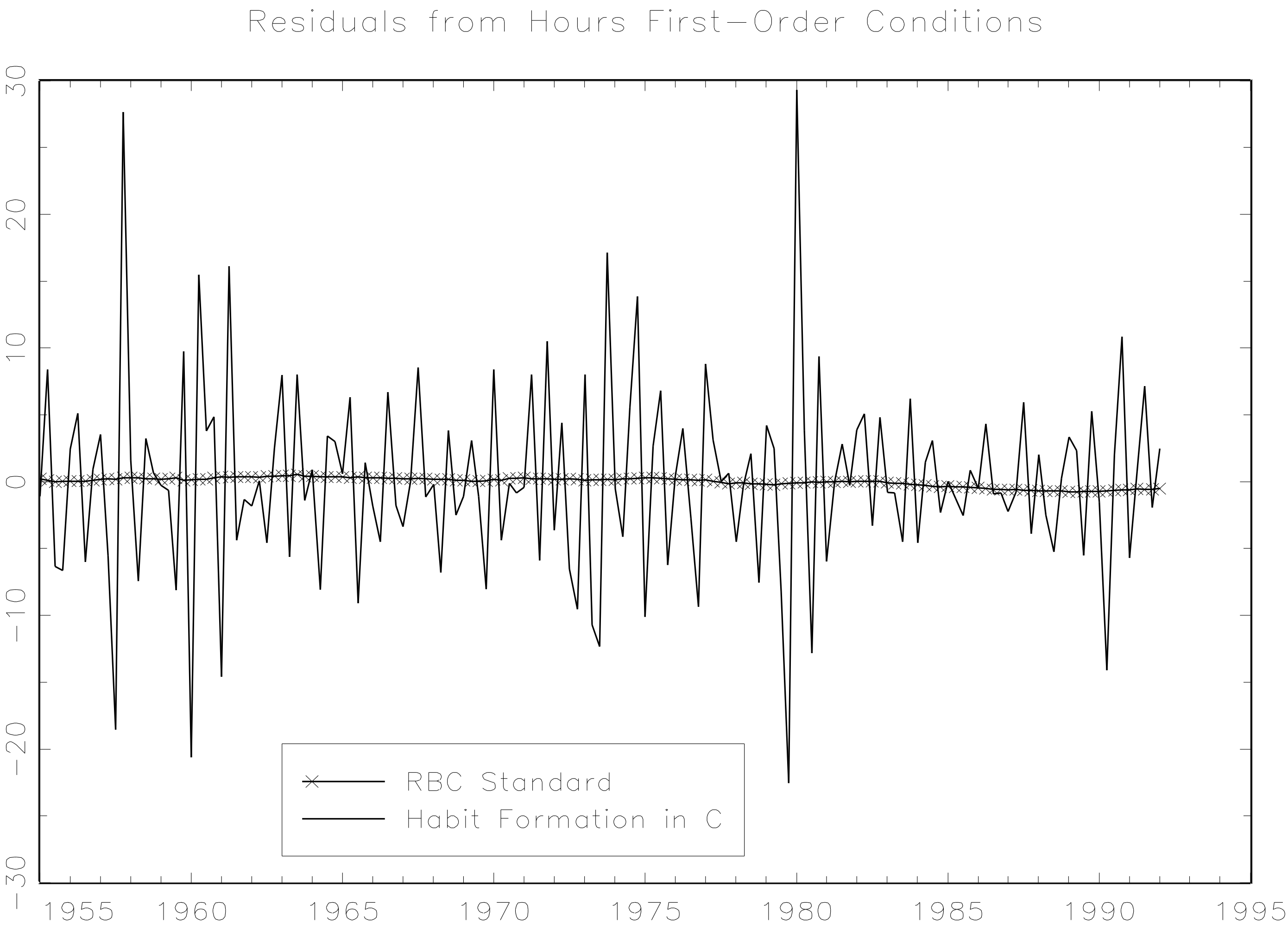
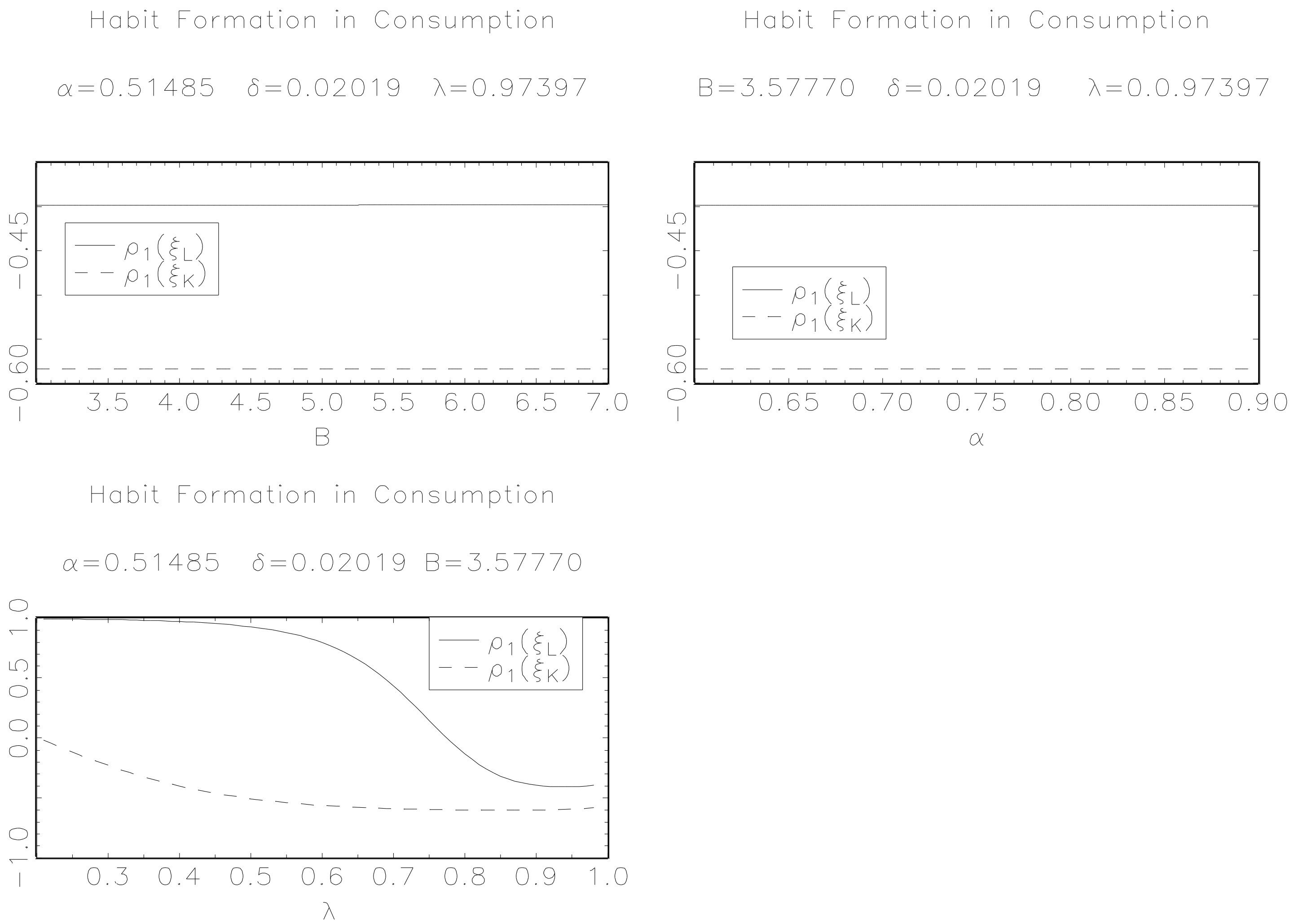
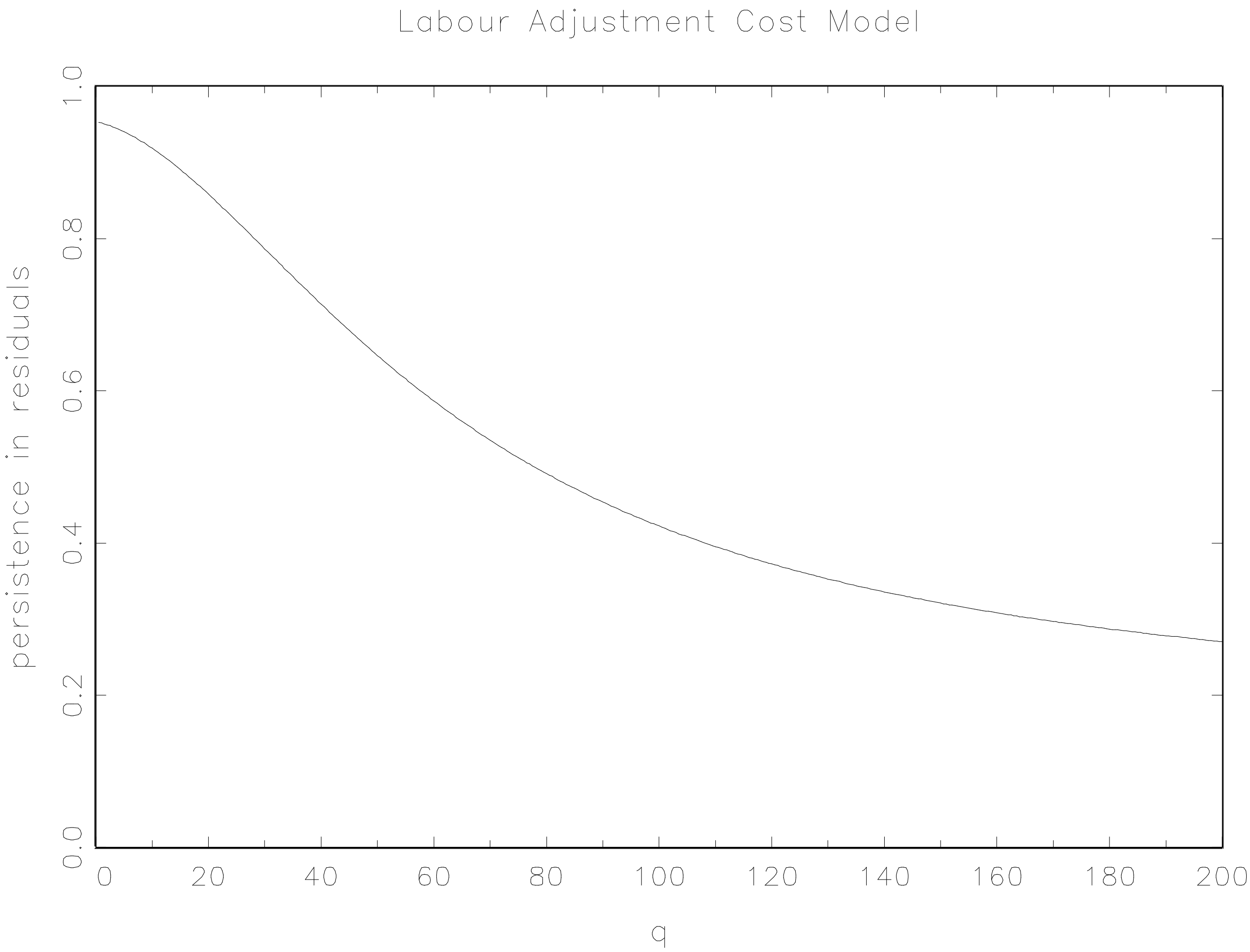

Residuals from Hours First-Order Conditions
30
20
10
0
−10
−20
−30
1955
1960
1965
1970
1975
1980
1985
1990
1995
RBC Standard
Habit Formation in C

Figure 9



Habit Formation in Consumption
α=0.51485 δ=0.02019 λ=0.97397
−0.60
−0.45
3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0
B
ρ1(ξL)
ρ1(ξK)
Habit Formation in Consumption
B=3.57770 δ=0.02019 λ=0.0.97397
−0.60
−0.45
0.65 0.70 0.75 0.80 0.85 0.90
α
ρ1(ξL)
ρ1(ξK)
Habit Formation in Consumption
α=0.51485 δ=0.02019 B=3.57770
−1.0
0.0
0.5
1.0
0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
λ
ρ1(ξL)
ρ1(ξK)

Figure 10


Labour Adjustment Cost Model
persistence in residuals
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100
120
140
160
180
200
q