REVEALED PREFERENCES FOR RISK AND AMBIGUITY

By

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Revealed Preferences for Risk and Ambiguity

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Abstract

We replicate the essentials of the Huettel et al. (2006) experiment on choice under uncertainty with 30 Yale undergraduates, where subjects make 200 pairwise choices between risky and ambiguous lotteries. Inferences about the independence of economic preferences for risk and ambiguity are derived from estimation of a mixed logit model, where the choice probabilities are functions of two random effects: the proxies for risk-aversion and ambiguity-aversion.

Our principal empirical finding is that we cannot reject the null hypothesis that risk and ambiguity are independent in economic choice under uncertainty. This finding is consistent with the hypothesized independence of the neural mechanisms governing economic choices under risk and ambiguity, suggested by the double dissociation-fMRI study reported in Huettel et al.

JEL Classification Numbers: C14, C25, C91, D03, D81

Keywords: Mixed logit, Risk-aversion, Ambiguity-aversion

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1 Introduction

Knight and Keynes in their classic monographs offer two independent but overlapping discussions of estimating probabilities for decision-making under risk and uncertainty. In this regard Keynes is probably best known for his chapter on "The state of long-run expectation" in *The General Theory of Employment, Interest and Money* (1936) and Knight for his chapter on "The meaning of risk and uncertainty" in *Risk, Uncertainty and Profit* (1921). A central contribution in the cited works of Knight and Keynes is the distinction between risk and uncertainty. Here is a quotation from Keynes (1937):

> By uncertain knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty; nor is the prospect of a Victory bond being drawn. Or, again, the expectation of life is only slightly uncertain. Even the weather is only moderately uncertain. The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence, or the obsolescence of a new invention, or the position of private wealth owners in the social system in 1970. About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know.

This distinction is absent in the expected utility (EU) model of decision-making under risk, due to von Neumann and Morgenstern (1944), and Savage's (1954) model of decision-making under uncertainty, but it is the genesis of Ellsberg's (1961) seminal critique of Savage's theory of subjective expected utility (SEU). In his analysis, Ellsberg proposes ambiguity or "irreducible uncertainty" as it is called by Keynes, as another aspect of decision-making under uncertainty. In Ellsberg's two-color thought experiment, subjects make pair-wise choices between a risky urn, where the relative frequencies of the two outcomes are 1/2, and an ambiguous urn, where the relative frequencies are unknown. In the first trial, if the subject chooses an urn and draws a black ball then she receives $100, the "good" outcome, but if she draws a white ball then she receives zero dollars, the "bad outcome." In the second trial the payoffs are reversed. Subjects that choose the ambiguous urn on both trials are said to be ambiguity-seeking and subjects that choose the risky urn on both trials are said to be ambiguity-averse. Ambiguity-seeking subjects in the Ellsberg experiment "act as if," the perceived probability of the "good" outcome is greater than the relative frequency of the "good" outcome. Ambiguity-averse subjects in the Ellsberg experiment "act as if" the perceived probability of the "bad" outcome is greater than the relative frequency of the "bad" outcome.

The dependence of perceived probabilities on payoffs is inconsistent with Savage's axiomatic model of decision-making under uncertainty, i.e., subjective expected utility (SEU) theory. See Savage, page 68: "... the view sponsored here does not leave room for optimism or pessimism to play any role in the person's judgement," or Ellsberg's (1961) explanation of the two color Ellsberg paradox: "... we would have to regard the subject's subjective probabilities as being dependent upon his payoffs, his evaluation of the outcomes ... it is impossible to infer from the resulting behavior a set of probabilities for events independent of his payoffs."

In an interesting and provocative experiment, Huettel et al. (2006) test a new model of decision-making under uncertainty with proxies for risk-aversion and ambiguity-aversion, consistent with Ellsberg's explanation of the two color Ellsberg paradox, where agents choose actions and beliefs. The proxies are β for risk-aversion, where β is the coefficient of relative risk-aversion for the utility function $u(w) = w^{\beta}$, and α for ambiguity-aversion in the α-max min expected utility model. The α-max min expected utility of an ambiguous lottery, $x = (x_1, x_2)$ is $(1-\alpha)u(x_1 \vee x_2) + \alpha u(x_1 \wedge x_2)$. If $\beta \in (0, 1)$ then u is concave and the subject is risk-averse. If $\beta = 1$ then u is linear and the subject is risk-neutral. Finally, if $\beta > 1$ then u is convex and the subject is risk-loving. Huettel et al. interpret α as a measure of ambiguity-aversion, where $\alpha \in [0, 0.5)$ denotes ambiguity-seeking, $\alpha = 0.5$ is ambiguity-neutral, and $\alpha \in (0.5, 1]$ denotes ambiguity-averse. The utility of an ambiguous lottery in the Huettel et al. model is the α-max min expected utility and the utility of a risky lottery is the expected utility. Huettel et al. assume that subjects maximize utility in choosing between a pair of lotteries. Before reviewing their experiment, we show that the Huettel et al. model is consistent with Ellsberg's explanation of the two-color paradox. The utility of the risky urn is $[u(0) + u(100)]/2$ and the utility of the ambiguous urn is $(1-\alpha)u(100) + \alpha u(0)$, where $u(0) = 0$. If the agent is ambiguity-averse then $(1-\alpha) \in [0, 0.5)$. Hence $u(100)/2 > (1-\alpha)u(100)$ and the agent chooses the risky urn on both trials. If the agent is ambiguity-seeking then $(1-\alpha) \in (0.5, 1]$. Hence $u(100)/2 < (1-\alpha)u(100)$ and the agent chooses the ambiguous urn on both trials.

Returning to the experiment of Huettel et al. Using $fMRI$ data from pairwise choices between risky lotteries, where the probabilities of the payoffs are known to the subjects, and ambiguous lotteries, where the probabilities of the payoffs are unknown to the subjects, Huettel et al. conclude that the neural mechanisms governing choice under risk and choice under ambiguity are independent. Briefly, they asked 13 subjects to make pair-wise choices between lotteries with different degrees of uncertainty, i.e., certain, risky and ambiguous, and used the $fMRI$ data to identify regions in the brain that are activated during the choice process. For each subject, β is estimated to maximize the number of correct predictions in the risky-risky and risky-certain trials, using the expected utility model. The $fMRI$ data identified a region of the brain that is activated during the choice process, call it region R Given the estimated $\widehat{\beta}$, α for each subject is estimated to maximize the number of correct predictions in the ambiguous-risky and ambiguous-certain trials , using the α-max min expected utility model. The $fMRI$ data identified a different region of the brain that is activated during this choice process, call it region A. Moreover, A is inactive when R is active and R is inactive when A is inactive. As is common in the neural science literature, this double dissociation$fMRI$ study is interpreted as independence of the two choice behaviors.

Unfortunately, the estimation procedure in the Huettel et al. study is not identified, i.e., there are several values of α and β that maximize the number of correct predictions — see the sub-section on Behavioral Data Acquisition and Analysis in the section on Procedures in Huettel et al. Recently, Levy et al. (2010) offered an alternative explanation of the hypothesized finding of differential activation in parts of the brain as a consequence of choice under risk and ambiguity. In the Huettel et al. study subjects were told, ex post, the probabilities defining ambiguous lotteries, possibly allowing learning of the ambiguous probabilities. In the Levy et al. study, where subjects were not told the ambiguous probabilities, the levels of neural activation resulting from choice under risk and ambiguity were comparable. Given the limitations of the Huettel et al. study, in determining the independence of economic preferences for risk and ambiguity, it is important to replicate their experiment and estimate α and β with an econometric model that is identified, using an experimental design, as in Levy et al., where subjects are not told the ambiguous probabilities.

To that end, we recast the Huettel et al. model as a random utility model, more specifically a mixed logit model. The mixed logit model allows us to estimate a bivariate distribution over α and β from pair-wise choices between risky and ambiguous lotteries of subjects randomly selected from the population. The random utility model was first proposed in psychology by Thurstone (1927) in a form now called the binomial probit model, and subsequently introduced in economics by Marschak (1960) who investigated the properties of choice probabilities for utility functions subject to random perturbations. McFadden (1974) introduced the conditional logit model. In the binomial case, this is the well-studied logistic model in biostatistics. See McFadden's Nobel Lecture for a brief history of the origins of the random utility model.

The proxies for ambiguity-aversion and risk-aversion, α and β, are treated as random effects, i.e., random variables uncorrelated with the explanatory variables, in the mixed logit model presented in this paper. For a detailed discussion of the mixed logit model see chapter 6 of Train (2009). The following example from McCulloch et al. (2008) illustrates the differences between fixed and random effects:

> Consider a clinical trial to treat epileptics, in which a drug is administered at four different dose levels. y_{ij} is the number of seizures experienced by patient j receiving dose i, where $E[y_{ij}] = \mu + \alpha_i$, μ is a general mean and α_i is the effect on the number of seizures due to treatment i. In this model of the expected value of y_{ij}, μ and each α_i are considered fixed and unknown constants, that we wish to estimate. These are the only treatments being used and we are considering no others, thus the α_i are fixed effects.
>
> Suppose now the clinical trials were conducted at 20 different clinics in New York City, where y_{ij} is the number of seizures experienced by patient j receiving treatment at the i the clinic. Now $E[y_{ij}] = \mu + a_i$. The clinics have been chosen randomly with the object of treating them as a representation of the population of all clinics in New York City and inferences can and will be made about that population. This is characteristic of random effects, thus the a_i are random effects.

There are two criteria for using a random effects model in lieu of a fixed effects model. First, the data is generated by taking a random sample from some fixed population. Our sample is randomly selected from the population of Yale students, matriculating in the summer session and fall term of 2009. Second, the explanatory variables — the payoffs and probabilities defining the lotteries – must be uncorrelated with the random effects, α and β. This is certainly true in our experiment in which the payoffs and probabilities defining the lotteries in the pair-wise comparisons are generated randomly and independently for each subject.

We replicate the essentials of the Huettel et al. experiment with 30 randomly chosen Yale undergraduates. One modification is that we asked the subjects to make some pairwise comparisons between ambiguous lotteries — this was not the case in the Huettel et al. experiment. In our experiment, each subject makes 200 pair-wise choices between risky and ambiguous lotteries. In the Huettel et al. analysis, α and β are interpreted as parameters and the choice probability, p_A, for x_A in the pair-wise comparison between lotteries x_A and x_B is defined as the percent correctly predicted. This is not the case for the mixed logit model that we present. In our model, the choice probability, p_A, for x_A in the pairwise choices between lotteries x_A and x_B is interpreted as the proportion of individuals in the population, with the same preferences for risk and ambiguity, that choose x_A or is interpreted as the proportion of times that a single individual chooses x_A in repeated pairwise choices between options x_A and x_B. This is our other modification of the Huettel et al. experiment.

We interpret α and β as random effects with a bivariate log-normal distribution, parameterized by unknown hyper-parameters Ψ. Using the Bayesian perspective, we can first estimate Ψ by simulated maximum likelihood and then estimate the individual random effects α_j and β_j for each subject $j = 1, 2, ..., 30$, by simulating the posterior distribution of α_j and β_j conditional on the subject's pair-wise comparisons of risky and ambiguous lotteries, using Bayes theorem. The posterior means are consistent estimates of the individual-level random effects, α_j and β_j — see chapter 11 of Train (2009) for the details. The Bernstein–von Mises theorem in chapter 12 of Train provides an alternative classical method of estimating the individual-level random effects. That is, maximum likelihood estimation of α_j and β_j. The Bernstein–von Mises theorem shows that the Bayesian and classical estimates of the individual-level random effects α_j and β_j are asymptotically equivalent.

We estimate α_j and β_j by maximizing the log-likelihood of each subject's pair-wise choices in risky and ambiguous lotteries. Following Huettel et al., we use a two-step procedure to estimate α_j and β_j for each subject $j = 1, 2, ..., 30$. That is, our estimator is two-step maximum likelihood estimation. To estimate β_j, we ask each subject to choose between 40 risky-certain pairs and 40 risky-risky pairs of lotteries. Here we assume that each subject is maximizing expected utility, which only depends on β_j. To estimate α_j, we ask each subject to choose between 40 ambiguous-certain pairs and 40 pairs of ambiguous-ambiguous lotteries. Here we assume that each subject is maximizing α-max min expected utility, which depends on both α_j and β_j, where we use the previously estimated value of β_j and need only estimate α_j. It is well known that these estimates are consistent under the standard conditions for maximum likelihood estimation, but any standard estimator of the asymptotic covariance matrix for asymptotic normality of the maximum likelihood estimate of α requires a correction. For specifics, see Theorem 17.8 in Greene (2003) due to Murphy and Topel (1985). This correction is not necessary under the null hypothesis that ambiguity and risk are statistically independent.

Treating these estimates as realizations of the random variables α and β, we examine the correlation, ρ, between α and β by regressing α on β. Despite the limitations of the study reported in Huettel et al. cited above, our results are consistent with their hypothesized finding on economic preferences for risk and ambiguity, that the neural processes governing choice under risk are independent of the neural processes governing choice under ambiguity. That is, we cannot reject the reject the null hypothesis that $\rho = 0$ at the 5% significance level.

2 The Mixed Logit Model

To replicate the essentials of the Huettel et al. experiment, we consider pair-wise choices in 200 monetary gambles made by 30 randomly chosen Yale undergraduates in 2009. As in the Huettel et al. experiment, each lottery involves choices between a known payoff, payoffs with known probabilities, and payoffs with unknown probabilities. We refer to these lotteries as certain, risky and ambiguous lotteries, respectively. In our experiment, each subject chooses between 40 risky-certain pairs, 40 risky-risky pairs, 40 ambiguous-certain pairs, 40 ambiguous-ambiguous pairs and 40 risky-ambiguous pairs. All ambiguous lotteries have two positive payoffs and all certain lotteries have one positive payoff. In the risky-certain pairs and the risky-risky pairs, all risky lotteries have one zero payoff and one positive payoff, but in the risky-ambiguous pairs both ambiguous and risky lotteries have two positive payoffs. Expected values of lotteries are chosen as random, whole-dollar amounts between \$5 and \$25, and expected values of pairs of lotteries are matched within 20%. The probability of winning the amount presented in a certain lottery is always 1, and the probabilities of winning amounts presented in risky and ambiguous lotteries are chosen randomly between 0.25 and 0.75, and varied across gambles.

At the start of each trial, subjects are given a pair-wise choice between lotteries, represented by two pie charts. Subjects are instructed to choose the lottery on the left or right by typing "*L*" or "*R*." Once a choice is made, a box appears around the chosen lottery and the other lottery disappears. Finally, the payoff of the lottery is displayed at the bottom of the screen. Figure 1 displays pairs of risky, certain and ambiguous lotteries. After completion of 200 trials, subjects are paid winnings from 4 randomly selected trials. Winnings ranged from \$0 to \$93 in a single trial, and \$35 to \$99 overall. The results of the experiment are summarized in the appendix. Ex post, subjects are not told the probability of outcomes in an ambiguous lottery.

Here is a brief description of the parametric mixed logit models we use to analyze our data. For each pair of risky lotteries: $X \equiv (x_1, x_2; \pi_1, \pi_2)$ and $Y \equiv (y_1, y_2; \eta_1, \eta_2)$, p_X is the probability of choosing X, and p_Y is the probability of choosing Y in a pair-

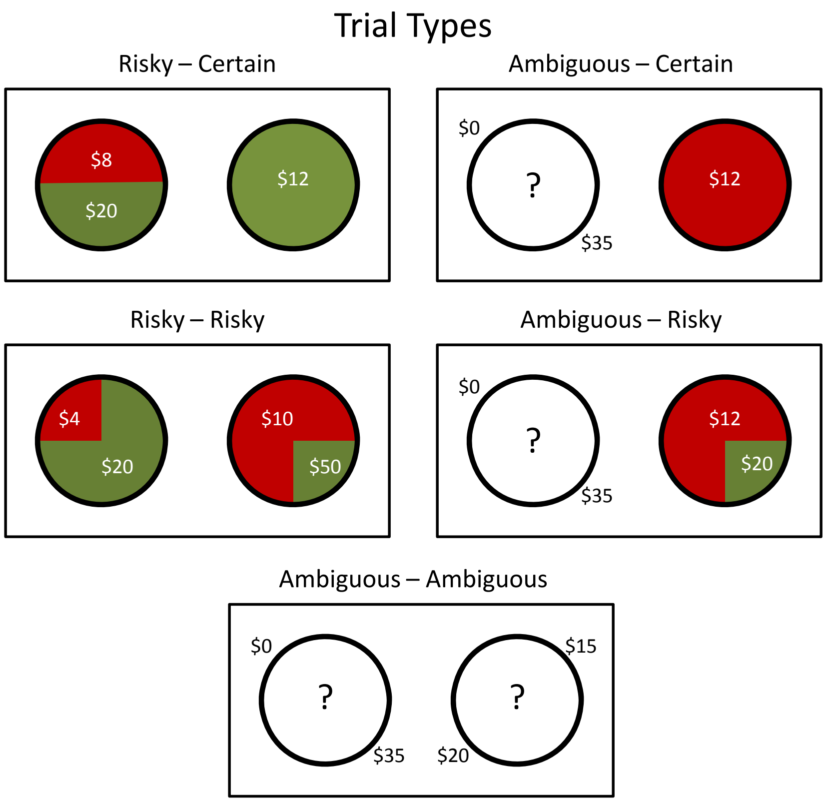

Figure 1. Experimental Design

(A) Subjects made decisions between pairs of gambles, drawn from the following types: *certain*, with a known outcome; *risky*, with two outcomes with known probabilities; and *ambiguous*, with two outcomes with unknown probabilities. Probabilities and reward values varied across trials, and expected value was roughly matched between the gambles.

(B) At the beginning of each trial, two gambles were presented and the subjects indicated their preference by pressing a joystick button. A square then appeared around the selected gamble.

wise comparison between X and Y, where $p_X + p_Y = 1$.

To estimate β, we use the multiplicative random utility model, where the expected utility of the risky option X is given by

$$EU(X) = \pi_1 (x_1)^\beta + \pi_2 (x_2)^\beta.$$

The logit choice probability for choosing X, $p_X(\beta)$, is defined by the logistic cdf

$$\Lambda[\eta] \equiv \frac{\exp \eta}{1 + \exp \eta}$$

That is,

$$p_X(\beta) \equiv \frac{\exp[\ln EU(X) - \ln EU(Y)]}{(1 + \exp[\ln EU(X) - \ln EU(Y)])}.$$

— see Fosgerau and Bielaire (2009) for a discussion of the multiplicative random utility model. In our data set, the pairs of risky-certain and risky-risky lotteries, $x_2 = y_2 = 0$. Hence the logit choice probability for choosing X as a function of β is

$$p_X(\beta) = \frac{\exp[\beta \ln(x_1) + \ln \pi_1 - \beta \ln(y_1) - \ln \eta_1]}{(1 + \exp[\beta \ln(x_1) + \ln \pi_1 - \beta \ln(y_1) - \ln \eta_1])}.$$

We denote the chosen lotteries as X^j in each pair of 40 risky-certain and 40 risky-risky lotteries. The likelihood of the observed risky choices in the 80 pair-wise comparisons$\{(X^j, Y^j)\}_{j=1}^{j=80}$ as a function of β is

$$\prod_{j=1}^{j-80} p_{X^j}(\beta).$$

The log-likelihood

$$\begin{aligned} & \frac{1}{80} \sum_{j=1}^{j=80} \ln p_{X^j}(\beta) \\ = \; & \frac{1}{80} \sum_{j=1}^{j=80} \frac{\ln(\exp[\beta \ln(x_1^j) + \ln \pi_1^j - \beta \ln(y_1^j) - \ln \eta_1^j]}{(1 + \exp[\beta \ln(x_1^j) + \ln \pi_1^j - \beta \ln(y_1^j) - \ln \eta_1^j]))}. \end{aligned}$$

McFadden has shown that the log-likelihood function with these choice probabilities is globally concave in β. Hence the MLE for $\hat{\beta}$ is identified. We estimate $\hat{\beta}$ by numerically maximizing the log-likelihood of the logit choice probabilities.

Our null hypothesis is that economic preferences for risk and ambiguity are independent, where β is a measure of the subject's tolerance for risk and α is a measure of the subject's attitude towards ambiguity. The alternative hypothesis is that economic preferences for risk and ambiguity are correlated. Under the null hypothesis, every function of α and every function of β are independent. In particular, the LL for β and risky-certain or risky-risky data is independent of the LL for α and ambiguous-certain or ambiguous-ambiguous data, for every fixed value of β, e.g., $\hat{\beta}$, the estimate of β.

To estimate α, we use the additive random utility model, where subjects evaluate ambiguous lotteries, using α-max min expected utility. Given the pair of ambiguous lotteries $W \equiv (w_1, w_2)$ and $Z \equiv (z_1, z_2)$, the logit choice probability for choosing W as a function of α, for fixed $\hat{\beta}$, is

$$p_W(\alpha, \hat{\beta}) = \frac{\exp\{[\alpha(w_1 \wedge w_2)^{\hat{\beta}} + (1-\alpha)(w_1 \vee w_2)^{\hat{\beta}}] - [\alpha(z_1 \wedge z_2)^{\hat{\beta}} + (1-\alpha)(z_1 \vee z_2)^{\hat{\beta}}]\}}{[1 + \exp\{[\alpha(w_1 \wedge w_2)^{\hat{\beta}} + (1-\alpha)(w_1 \vee w_2)^{\hat{\beta}}] - [\alpha(z_1 \wedge z_2)^{\hat{\beta}} + (1-\alpha)(z_1 \vee z_2)^{\hat{\beta}}]\}}$$

— see Train (2009) for a discussion of the additive random utility model. We denote the chosen lotteries as W^j in each pair of 40 ambiguous-certain and 40 ambiguous-ambiguous lotteries. The likelihood of the observed ambiguous choices in the 80 pair-wise comparisons $\{(W^j, Z^j)\}_{j=1}^{j=80}$ as a function of α, for fixed $\hat{\beta}$, is

$$\prod_{j=1}^{j=80} p_{W^j}(\alpha, \hat{\beta}).$$

The log-likelihood

$$\frac{1}{80} \sum_{j=1}^{j=80} \ln p_{W^j}(\alpha, \hat{\beta})$$

$$= \frac{1}{80} \sum_{j=1}^{j=80} \frac{\ln \exp\{[\alpha(w_1 \wedge w_2)^{\hat{\beta}} + (1-\alpha)(w_1 \vee w_2)^{\hat{\beta}}] - [\alpha(z_1 \wedge z_2)^{\hat{\beta}} + (1-\alpha)(z_1 \vee z_2)^{\hat{\beta}}]\}}{[1 + \exp\{[\alpha(w_1 \wedge w_2)^{\hat{\beta}} + (1-\alpha)(w_1 \vee w_2)^{\hat{\beta}}] - [\alpha(z_1 \wedge z_2)^{\hat{\beta}} + (1-\alpha)(z_1 \vee z_2)^{\hat{\beta}}]\}}.$$

If $\hat{\beta} \neq 0$, then the log-likelihood function is globally concave in α, $\hat{\alpha}$ is not identified for subjects where $\hat{\beta} = 0$. That is, if $\hat{\beta} = 0$, then for all $\hat{\alpha} \in [0, 1]$: $p_{W^J}(\hat{\alpha}, \hat{\beta}) = 1/2$. Hence $\hat{\alpha}$ is indeterminate and the six subjects with indeterminate $\hat{\alpha}$ "act as if" they flip a fair coin to choose between any pair of ambiguous lotteries. These 6 subjects were excluded from our analysis. The MLE for α is identified for the remaining 24 subjects. We estimate $\hat{\alpha}$ for each of these 24 subjects with the 40 ambiguous-certain and the 40 ambiguous-ambiguous lotteries, by numerically maximizing the log-likelihood of the logit choice probabilities, for fixed $\hat{\beta}$.

To estimate the correlation between risk and ambiguity, we consider several specifications. In the five specifications, where we exclude the six subjects with unidentified $\hat{\alpha}$, the slope coefficient of the regression is not significantly different from 0 at the 0.05 level, indicating linear independence between risk and ambiguity. Hence we cannot reject the null hypothesis of independence of economic preferences for risk and ambiguity. The regression coefficients, regression statistics and confidence intervals are in the appendix on parametric data analysis. All the statistical and numerical analysis was done with Matlab. This empirical finding only shows that risk aversion and ambiguity aversion are not linearly dependent. Hence we test directly for independence by constructing a 2×2 contingency table, where the columns are labeled AA, for ambiguity aversion and AS, for ambiguity seeking,and the rows are labeled RA, for risk averse, and RS , for risk seeking. Here is the table, where we have omitted the 6 subjects with unidentified $\hat{\alpha}$:

	AA	AS
RA	5	0
RS	19	0

We see that the cells in the second column are both zero. Consequently, Prob($AA|RA$) = Prob($AA|RS$) = 1. Hence in our choice experiment it follows from Fisher's exact test that risk and ambiguity are independent — see section 4.6 in Lehmann and Romano (2005).

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4 Appendix: Data Analysis

Estimation Results

- Beta measures risk-attitude.
  - Beta < 1: risk-averse.
  - Beta > 1: risk-seeking.
- Alpha measures ambiguity-attitude.
  - Alpha < 0.5: ambiguity-seeking.
  - Alpha > 0.5: ambiguity-averse.

Subject No.	**Beta**		**Alpha**	
1	0.8846	risk-averse	0.8031	ambiguity-averse
2	0	risk-averse	0	ambiguity-seeking indeterminate
3	1.0600	risk-seeking	0.6750	ambiguity-averse
4	1.6342	risk-seeking	0.8210	ambiguity-averse
5	0	risk-averse	0	ambiguity-seeking indeterminate
6	1.1975	risk-seeking	0.8007	ambiguity-averse
7	1.8955	risk-seeking	0.8504	ambiguity-averse
8	0.4385	risk-averse	1.0000	ambiguity-averse
9	2.6677	risk-seeking	0.7104	ambiguity-averse
10	1.2111	risk-seeking	0.7788	ambiguity-averse
11	2.7591	risk-seeking	0.8493	ambiguity-averse
12	0	risk-averse	0	ambiguity-seeking indeterminate
13	2.8378	risk-seeking	0.7460	ambiguity-averse

Subject No.	Beta		Alpha	
14	1.9467	risk-seeking	0.8322	ambiguity-averse
15	2.0150	risk-seeking	0.8966	ambiguity-averse
16	1.5265	risk-seeking	0.7113	ambiguity-averse
17	2.8376	risk-seeking	0.8056	ambiguity-averse
18	0.7023	risk-averse	0.8883	ambiguity-averse
19	2.9604	risk-seeking	0.8521	ambiguity-averse
20	1.3524	risk-seeking	0.8067	ambiguity-averse
21	0.4946	risk-averse	1.0000	ambiguity-averse
22	0	risk-averse	0	ambiguity-seeking indeterminate
23	0.7039	risk-averse	0.9863	ambiguity-averse
24	1.8134	risk-seeking	0.8421	ambiguity-averse
25	2.0643	risk-seeking	0.9536	ambiguity-averse
26	2.3418	risk-seeking	0.8417	ambiguity-averse
27	1.6675	risk-seeking	0.8341	ambiguity-averse
28	1.5246	risk-seeking	0.8102	ambiguity-averse
29	0	risk-averse	0	ambiguity-seeking indeterminate
30	0	risk-averse	0	ambiguity-seeking indeterminate

Summary Excluding Subjects with Zero Beta

- 6 subjects excluded because their beta is zero.
- 24 out of remaining 24 individuals are ambiguity-averse.
- Out of 24 ambiguity-averse individuals:
    - 5 individuals are risk-averse and ambiguity-averse
    - 19 individuals are risk-seeking and ambiguity-averse

Regression Analysis

Scenario I a: Regression Analysis Including Subjects with Zero Beta

LHS: alpha, RHS: constant, beta

Regression Coefficients (constant, beta)

0.3629
0.2272

95 % Confidence Intervals

- Each row contains the left and right endpoint of the 95% confidence interval for the corresponding coefficient.
- If zero is inside the confidence interval, the coefficient is not significantly different from zero.

0.1849	0.5409
0.1196	0.3347

Regression Statistics (R-squared, F-test, p-value for F-test)

- F-test: Null hypothesis states that both regression coefficients equal zero.
- Null hypothesis rejected at 95% level if p less than 0.05.

0.4008	18.7285	0.0002

Scenario I b: Regression Analysis Including Subjects with Zero Beta

LHS: beta, RHS: constant, alpha

Regression Coefficients (constant, alpha)

0.1693
1.7644

95 % Confidence Intervals

- Each row contains the left and right endpoint of the 95% confidence interval for the corresponding coefficient.
- If zero is inside the confidence interval, the coefficient is not significantly different from zero.

-0.4593	0.7980
0.9293	2.5996

Regression Statistics (R-squared, F-test, p-value for F-test)

- F-test: Null hypothesis states that both regression coefficients equal zero.
- Null hypothesis rejected at 95% level if p less than 0.05.

0.4008	18.7285	0.0002

Scenario II a: Regression Analysis Excluding Subjects with Zero Beta

LHS: alpha, RHS: constant, beta

Regression Coefficients (constant, beta)

0.9047
-0.0399

95 % Confidence Intervals

- Each row contains the left and right endpoint of the 95% confidence interval for the corresponding coefficient.
- If zero is inside the confidence interval, the coefficient is not significantly different from zero.

0.8195	0.9898
-0.0859	0.0061

Regression Statistics (R-squared, F-test, p-value for F-test)

- F-test: Null hypothesis states that both regression coefficients equal zero.
- Null hypothesis rejected at 95% level if p less than 0.05.

0.1281	3.2320	0.0860

Scenario II b: Regression Analysis Excluding Subjects with Zero Beta

LHS: beta, RHS: constant, alpha

Regression Coefficients (constant, alpha)

4.3791
-3.2127

95 % Confidence Intervals

- Each row contains the left and right endpoint of the 95% confidence interval for the corresponding coefficient.
- If zero is inside the confidence interval, the coefficient is not significantly different from zero.

1.2602	7.4980
-6.9188	0.4934

Regression Statistics (R-squared, F-test, p-value for F-test)

- F-test: Null hypothesis states that both regression coefficients equal zero.
- Null hypothesis rejected at 95% level if p less than 0.05.

0.1281	3.2320	0.0860

Scenario III a: Regression Analysis of Log-Representation Excluding Subjects with Beta = 0 and Alpha = 1

LHS: log(alpha/(1-alpha)), RHS: constant, log(beta)

Regression Coefficients (constant, log(beta))

1.9250
-0.5002

95 % Confidence Intervals

- Each row contains the left and right endpoint of the 95% confidence interval for the corresponding coefficient.
- If zero is inside the confidence interval, the coefficient is not significantly different from zero.

1.4139	2.4360
-1.2746	0.2743

Regression Statistics (R-squared, F-test, p-value for F-test)

- F-test: Null hypothesis states that both regression coefficients equal zero.
- Null hypothesis rejected at 95% level if p less than 0.05.

0.0832	1.8149	0.1930

Scenario III b: Regression Analysis of Log-Representation Excluding Subjects with Beta = 0 and Alpha = 1

LHS: log(beta), RHS: constant, log(alpha/(1-alpha))

Regression Coefficients (constant, log(alpha/(1-alpha))

0.7826
-0.1663

95 % Confidence Intervals

- Each row contains the left and right endpoint of the 95% confidence interval for the corresponding coefficient.
- If zero is inside the confidence interval, the coefficient is not significantly different from zero.

0.3118	1.2535
-0.4239	0.0912

Regression Statistics (R-squared, F-test, p-value for F-test)

- F-test: Null hypothesis states that both regression coefficients equal zero.
- Null hypothesis rejected at 95% level if p less than 0.05.

0.0832	1.8149	0.1930

Scenario IV: Regression Analysis of scenario III without Outliers

- Only regression of log(beta) on constant and log(alpha/(1-alpha)) in scenario III a contains outliers.
- Outliers are determined according to criterion in MATLAB:
  - If zero is outside of residual-specific confidence-interval (95%), residual is considered an outlier.

LHS: log(alpha/(1-alpha)), RHS: constant, log(beta)

Regression Coefficients(constant, log(beta))

1.4643
0.0199

95 % Confidence Intervals

- Each row contains the left and right endpoint of the 95% confidence interval for the corresponding coefficient.
- If zero is inside the confidence interval, the coefficient is not significantly different from zero.

1.1658	1.7629
-0.4268	0.4667

Regression Statistics (R-squared, F-test, p-value for F-test)

- F-test: Null hypothesis states that both regression coefficients equal zero.
- Null hypothesis rejected at 95% level if p less than 0.05.

0.0005	0.0088	0.9263