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Abstract
In a two-player stag hunt with asymmetric information, players may lock each other into requiring a large number of confirmations and confirmations of confirmations from one another before eventually acting. This intuition has been formalized in the electronic mail game (EMG). The literature provides extensions on the EMG that eliminate inefficient equilibria, suggesting that no formal rules are needed to prevent players from playing inefficiently. The present paper investigates whether these results extend to the multi-player EMG. We show that standard equilibrium refinements cannot eliminate inefficient equilibria. While two players are predicted to play efficiently, many players need formal rules telling them when who talks to whom.

Keywords: Multi-Player Electronic Mail Game, Collective Action, Communication Networks.

JEL classification: D82, D85, D71.

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1. Introduction

E-mail has made it cheaper and faster to communicate within firms. Taking the stance from the organization literature that a firm can be seen as being involved in coordination problems (for a recent overview, see Calvó-Armengol and de Marti, 2009), it seems at first sight that increased communication through e-mail would make it easier for firms to solve their coordination problems. Hand in hand with the arrival of these technologies, sociologists have argued in favor of a new view on firms, which rather than as hierarchies should be seen as non-hierarchical networks of employees (e.g. Powell, 1990). Yet, in contradiction to this intuition, some firms have introduced measures that limit internal e-mail exchanges, such as e-mail-free Fridays (Washington Post, 2007). While other explanations may contribute to this phenomenon, we show in this paper that one rationale for this phenomenon is that allowing employees to freely communicate may decrease rather than increase the probability of coordination.

Marschak (1955) and Radner (1962) have induced a large literature that treats the problem of organization as a problem of aggregating information that is dispersed over many individuals (see Calvó-Armengol and de Marti, 2009). The question arising in this literature is: what communication network optimally aggregates this dispersed information? In this paper, we take a somewhat different approach, and see firms as being involved in repeated collective-action problems, where in each collective-action problem, one employee or player is informed about the opportunity to benefit from collective action. Collective action is only successful if the informed player is able to rally a sufficient number of other players; for simplicity, we assume this to be all the players in the game. Such a game has an "I'll go if a sufficient number of other players go" feature: as long as a sufficient number of other players act, the individual player acts.[1] Thus collective action is a possible equilibrium outcome. Yet, at the same time, acting with an insufficient number of players is risky. Before players actually act, they will not only want to know whether there is an opportunity for collective action, but also – if there is even a small probability that information does not get through – they will want to know whether everyone else knows about this opportunity. Furthermore, they will want to know whether everyone knows that everyone knows it; and so on. It thus seems that players may engage in such an extensive amount of checking and double-checking each other's knowledge that they never actually come to act.[2] Thus, if when we talk about a communication network in this paper, contrary to what is the case in the organization literature, we refer to the exchange of messages, confirmations of messages, etc. between players.

The fear of excessive communication is confirmed by Rubinstein (1989) in his electronic mail game (henceforth EMG), which in its standard form is a two-player game where an

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[1] In sociology, such models of collective action are known as threshold models (see Granovetter, 1978), where an agent's threshold refers to the number of other agents who need to participate in the collective action for benefits to arise for the agent. See Chwe (2000) for a formal model of such threshold models. In these models, players' thresholds may differ. In the current paper, we take the simplifying assumption that all players share the same threshold, and that this threshold is equal to the number of players in the game.

[2] In general, there are two mechanisms by which players attempt to reduce the risk of acting alone, as listed by Chwe (1995), namely *reconfirmation* and *redundancy*. In Chwe's work, however, redundancy means that an identical message is sent several times, whereas in the current paper, it means that players are on multiple chains on which players are ordered in a different way, where these chains form multiple alternative channels along which information can travel.

informed player knows whether or not a stag hunt[3] is played; if a stag hunt is played, there are benefits in cooperating. If the underlying game is a stag hunt, Rubinstein assumes that an automatic communication protocol sends a message to the uninformed player. If this message gets through, which happens with large probability, the automatic communication protocol sends a proof of receipt from the uninformed to the informed player. It this proof of receipt arrives, the protocol sends a proof of receipt of the proof of receipt from the informed to the uninformed player – and so on until a message gets lost. Rubinstein shows that the only equilibrium for this game is one where no player ever acts.

More recently, some authors have extended the two-player EMG to come to equilibria where cooperation still takes place. Binmore and Samuelson (2001) assume that communication is voluntary. Players both decide on how many proofs of receipt to send, and on how many proofs of receipt to pay attention to. Furthermore, players incur more costs the more proofs of receipt they send and pay attention to. These costs put a cap on how many messages to sent back and forth. Moreover, the fact that communication is voluntary ensures the existence of efficient equilibria where at most the uninformed player sends a proof of receipt to the informed player. Nevertheless, players' mutual expectations may still lock them into equilibria where a large number of proofs of receipt, and proofs of proofs of receipt are required. Thus, Binmore and Samuelson's analysis continues to give credence to Geanakoplos' (1992) lesson, taken from the two-player EMG: it justifies strict rules of communication, such as a military rule between fighter pilots to only confirm each message once, and not confirm the confirmation.

De Jaegher (2008a) extends Binmore and Samuelson's paper by assuming that the individual player is able to falsely pretend having received a message. While Nash equilibria where players send a large number of messages back and forth continue to exist under such modified assumptions, efficient equilibria either are not sequential equilibria or do not meet the intuitive criterion. Long exchanges of confirmations are not sequentially rational because a player who is planning not to act should not send any costly messages. Suppose that a player does not receive a message, but still pretends to have received it (= false acknowledgement). At worst, when detected, the other player will punish such a false acknowledgement by not replying to it and not acting. But the cheating player who receives a confirmation of his false acknowledgement then knows that his cheating was not detected. Moreover, applying the intuitive criterion, consider a Nash equilibrium where the informed player sends a message to the uninformed player, who then sends a confirmation, after which the informed player sends a confirmation of this confirmation. Three messages then need to arrive for collective action to take place. The uninformed player does not run any risk from acting, whereas the informed player is not completely sure whether his or her last message reached the uninformed player. Both players are better off if the informed player simply sends a single message to the uninformed player. By the intuitive criterion, if the informed player deviates from the inefficient described equilibrium by staying quiet at the first stage and sending a message at the third stage, the uninformed player, who then appears to receive a confirmation of a message that he or she never sent, should realize that the informed player is trying to move to a more efficient equilibrium. Intuitively, two players involved in everyday situations such as meeting each other for lunch, will not engage in an endless exchange of confirmations, and do not need strict rules or institutions telling them how to communicate.[4]

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[3] Skyrms (2004) recently stresses the importance of stag hunt games for the analysis of collective action. On the importance of interactive knowledge in collective action problems, see Chant and Ernst (2008).

[4] A small literature investigates the robustness of Rubinstein's results to modifications other than introducing multiple players. Dulleck (2007) shows that boundedly rational players with imperfect recall can still coordinate on requiring only a few messages. Dimitri (2004) shows that when messages from different players get lost with different probabilities, coordinate action can still occur, as the player whose messages arrive with high

The purpose of the current paper is to investigate whether this intuition about voluntary communication without proofs of receipt extends to the *multi*-player electronic mail game.[5] Let it be the case that most of the time, there is no benefit from collective action. When the opportunity for beneficial collective action arises, let an informed player find out about this and be able to send messages to other uninformed players. Let messages not get through with small probability, and let acting with less than the required number of players be risky. De Jaegher (2008b) investigates such a multi-player game with *in*voluntary communication (i.e., an automatic communication protocol), and shows that equilibria exist where players get locked into requiring a large number of proofs of receipt and proofs of proofs of receipt from one another. The question we here seek to answer is whether inefficiency is maintained in case of voluntary communication, where messages need not take the form of proofs of receipt.

We show that, as such, a powerful mechanism is at work in the multi-player game that eliminates the most inefficient equilibria of the game *with involuntary communication* as Nash equilibria of the game *with voluntary communication*. As communication is voluntary, each message sent by the individual player confirms that this player received every single required message so far, and thus sent all of his or her messages so far. Thus, player j does not necessarily need to receive multiple messages from player i; the last message received from player i suffices, as this shows that player i also sent all previous messages. It follows that cases where players reoccur on separate strings of confirmations are often eliminated. Also, such cases are further eliminated by application of sequential rationality and of the intuitive criterion, in arguments similar to those for the two-player EMG.

Yet, we also show that inefficient Nash equilibria exist that survive equilibrium refinements. To see why, suppose that the informed player informs each uninformed player separately, and then requires a confirmation from each uninformed payer; for N players, this means that $2(N-1)$ messages must arrive for collective action to take place. Note that, in such an equilibrium, the informed player in this case does not run any risk from acting, and he or she knows with certainty that all other players are going to act. Each uninformed player, however, is never completely certain that any other player acts. All players are now better off in an efficient equilibrium where they send messages to one another ordered in a chain, where the last uninformed player in the chain sends a message to the informed player. This means that only N messages must arrive for collective action to take place. The informed player continues to know with certainty that all other players act; the uninformed player who is at position X in the line knows that $(X-1)$ other players act.

The reason that the inefficient equilibrium, where the informed player talks bilaterally to each uninformed player, cannot be eliminated is that a switch to the efficient equilibrium would require the informed player to stop sending messages to all but one uninformed player. But just as in the inefficient equilibrium, this uninformed player then receives a message from the informed player, and cannot observe that the informed player is trying to deviate to the efficient equilibrium. Concluding, the mechanisms that are at work in the two-player EMG to eliminate inefficiencies in a single string of confirmations, do not always work to eliminate inefficiencies across several strings of confirmations. We conclude that, while two players involved in a stag-hunt like collective action problem do not need hierarchies that tell them

probability can then be quite sure that his or her message arrived, and that it is the confirmation of the other player that got lost. Coles (2007) provides a similar result for the two-player EMG. Binmore and Samuelson (2001) investigate the effect of communication being voluntary instead of automatic. They show that, while efficient equilibria now exist, players may still coordinate on inefficient equilibria where a large number of messages are sent back and forth.

[5] A related multi-player stag hunt game is treated by Van Damme and Carlsson (1993). These authors treat the multi-player stag hunt in the context of equilibrium selection of the efficient, collective-action equilibrium rather than the inefficient equilibrium without action.

when to confirm which message, multiple players do need such hierarchies. Intuitively, two is company, but three is a crowd.

The paper is structured as follows. Section 2 treats the multi-player EMG with voluntary communication by means of proofs of receipt (cf. Binmore and Samuelson, 2001). Section 3 treats the multi-player EMG with voluntary communication by means of messages that can take on the meaning of confirmations of receipt in equilibrium (but can otherwise be sent even if no preceding message was received, cf. De Jaegher 2008a). We end with an interpretation in Section 4.

2. Multi-player electronic mail game with voluntary communication consisting exclusively of proofs of receipt

2.1 Model

Our N-player electronic mail game takes the following form. There are two states of nature, state a and state b. State a occurs with probability $(1-p)>1/2$. The N players can choose from two actions, namely actions A and B. Taking action A yields payoff zero whatever the state of nature, and whatever the action of the other player. If players choose different actions, then those who choose action A obtain 0, and those who choose action B incur a loss of L. If all N players choose action B in state b, then each player obtains payoff M. If they all choose action B in state a, then each player incurs a loss of L. It is assumed that $L>M>0$.[6] In two-player form, the game looks as follows:

	A	B
A	(0, 0)	(0, –L)
B	(–L, 0)	(–L, –L)

G_a (probability (1–p) > ½)

	A	B
A	(0, 0)	(0, –L)
B	(–L, 0)	(M, M)

G_b (probability p)

Tabel 1. Two-player electronic mail game

Only player 1 knows the state of nature. At stage 0, Nature decides whether the state is a or b, and player 1 observes Nature's choice. At stage 1, for each uninformed player, when having observed state b, player 1 decides whether or not to send an e-mail to him/her. For each e-mail sent, player 1 incurs a cost of d.

[6] This is Morris and Shin's (1997) version of the electronic mail game in N-player form. It differs from Rubinstein's (1989) original game in two aspects, both concerned with the game played in state a. First, in Rubinstein, each player obtains a payoff of M when all players play A in state a. Second, in Rubinstein, when all players play B in state a, each player obtains a payoff of zero. This version of the electronic mail game is also used in Morris (2002a, 2002b).

Simultaneously at stage 1, for each e-mail sent by player 1, Nature decides whether or not to let the e-mail arrive. Each e-mail arrives with probability $(1-\varepsilon)$, and gets lost with probability ε. When having received an e-mail at stage 2, an uninformed player i can forward this e-mail to each of the $(N-1)$ other players (which includes the informed player), where each sent e-mail again comes at a cost d. Again, Nature at stage 2 simultaneously decides whether or not to let each e-mail arrive, with the same probabilities as given above. At stage 3, again at cost d, each e-mail that player j received at stage 2 (note that an uninformed player can receive up to $(N-2)$ e-mails at stage 2; an informed player up to $(N-1)$ e-mails) can be forwarded to the $(N-1)$ other players. Nature again decides for each such e-mail whether or not to let it arrive. Etc. Players can forward e-mails up to stage z. The action decisions (A or B) are taken at stage $(z+1)$. The payoffs are obtained at stage $(z+2)$.

By scrolling down a received e-mail, a player observes a sequence of players through which a message was forwarded. Thus, when player n receives a particular e-mail from player j at stage t, player n observes that player 1 sent an e-mail to player i, who forwarded this e-mail to player… j, who forwarded this e-mail to player k, who forwarded this e-mail to player l, who finally forwarded this e-mail to player n. Each e-mail received can thus be seen as an observed *message string* of the form $1\overset{1,x}{\rightarrow}i\overset{2,x}{\rightarrow}...\overset{t-3,x}{\rightarrow}j\overset{t-2,x}{\rightarrow}k\overset{t-1,x}{\rightarrow}l\overset{t,x}{\rightarrow}n$, where the numbers above the arrows refer to the stage at which an e-mail was sent, and x labels the message string. Note that when n observes $1\overset{1,x}{\rightarrow}i\overset{2,x}{\rightarrow}...\overset{t-3,x}{\rightarrow}j\overset{t-2,x}{\rightarrow}k\overset{t-1,x}{\rightarrow}l\overset{t,x}{\rightarrow}n$, he/she also observes $1\overset{1,x}{\rightarrow}i\overset{2,x}{\rightarrow}...\overset{t-3,x}{\rightarrow}j\overset{t-2,x}{\rightarrow}k\overset{t-1,x}{\rightarrow}l$, $1\overset{1,x}{\rightarrow}i\overset{2,x}{\rightarrow}...\overset{t-3,x}{\rightarrow}j\overset{t-2,x}{\rightarrow}k$, $1\overset{1,x}{\rightarrow}i\overset{2,x}{\rightarrow}...\overset{t-3,x}{\rightarrow}j$, etc. The latter message strings are called *sub message strings* of $1\overset{1,x}{\rightarrow}i\overset{2,x}{\rightarrow}...\overset{t-3,x}{\rightarrow}j\overset{t-2,x}{\rightarrow}k\overset{t-1,x}{\rightarrow}l\overset{t,x}{\rightarrow}n$. The short-hand notation for stating that message string $m_{j,\tau,x}$, received by player j at stage is a sub message string of message string $m_{i,t,x}$ received by player i at stage t is $m_{j,\tau,x}\subset m_{i,t,x}$; x labels these message strings because a player may receive several message strings at one and the same stage.

An action strategy $e_i^{z+1}(.)$ for player i is a mapping from an event, namely an observed set of strings of messages received and not received, to the set (A, B). As an event, consisting of a number of messages received and a number of messages not received, is fully characterized by the messages received, we only write down the messages received to characterize an event (meaning that all message strings not denoted are then not received). E.g., $e_i^{z+1}(\{m_{i,t,x}\wedge m_{i,s,y}\wedge ....\})=B$ means that observance of a set of message strings $\{m_{i,t,x}\wedge m_{i,s,y}\wedge ....\}$ leads to the playing of B by player i.

Given a message string $1\overset{1,x}{\rightarrow}i\overset{2,x}{\rightarrow}...\overset{t-2,x}{\rightarrow}j\overset{t-1,x}{\rightarrow}k$ received by player k, a signaling strategy for player k is a mapping from an event, namely an observed set of strings of messages received and not received, to the set (0, 1) denoting whether or not player k forwards message string $1\overset{1,x}{\rightarrow}i\overset{2,x}{\rightarrow}...\overset{t-2,x}{\rightarrow}j\overset{t-1,x}{\rightarrow}k$ to player l. E.g., $s_{1\overset{1,x}{\rightarrow}i\overset{2,x}{\rightarrow}...\overset{t-2,x}{\rightarrow}j\overset{t-1,x}{\rightarrow}k\overset{t,x}{\rightarrow}l}(\{m_{k,y,t}\wedge m_{k,z,s}\wedge ....\})=1$ means that player k forwards message string $1\overset{1,x}{\rightarrow}i\overset{2,x}{\rightarrow}...\overset{t-2,x}{\rightarrow}j\overset{t-1,x}{\rightarrow}k$ to player l whenever having previously observed message strings $m_{k,y,t}$, $m_{k,z,s}$, … The inverse mapping $s^{-1}_{1\overset{1,x}{\rightarrow}i\overset{2,x}{\rightarrow}...\overset{t-2,x}{\rightarrow}j\overset{t-1,x}{\rightarrow}k\overset{t,x}{\rightarrow}l}(1)\rightarrow\{.\},\{.\},...$ gives a list of all events (received message strings) for which player k forwards message string $1\overset{1,x}{\rightarrow}i\overset{2,x}{\rightarrow}...\overset{t-2,x}{\rightarrow}j\overset{t-1,x}{\rightarrow}k$ to player l.

We consider only candidate equilibria for which it is the case that only a *single* set of message strings $\{m_{i,x,t} \wedge m_{i,y,s} \wedge ...\}$ leads player i to play B, where each message string in this set is then crucial for player i playing B.

Definition 1

Define as an *equilibrium with crucial message* strings a Nash equilibrium with the following features:

(i) the equilibrium can be described as a set M of sets of message strings M_i, where M_i = $\{m_{i,x,t}, m_{i,y,s},...\}$, with one such set for each player i. A typical message $m_{i,x,t}$ in the set M_i takes the form $1 \overset{1}{\rightarrow} j \overset{2}{\rightarrow} ... \rightarrow k \overset{t-1}{\rightarrow} l \overset{t}{\rightarrow} i$.

(ii) For each individual player i, as soon as he/she does not receive one of the message strings $m_{i,x,t}, m_{i,y,s},...$, he/she does not act. Put otherwise, each player i only acts when receiving every message string in the set $\{m_{i,x,t}, m_{i,y,s},...\}$.

In De Jaegher (2008b), equilibria are also described where players consider several alternative sets of message strings as sufficient for doing B. The existence of such equilibria is logical when communication is assumed to be involuntary and costless. If communication is voluntary and costly, however, players may not be willing to incur the costs of sending multiple messages if a few messages suffices most of the time to induce the other players to act. For this reason, we here focus on equilibria with crucial messages.

An important concept that we will treat throughout is the one of a *final node message string*.

Definition 2

In an equilibrium where all message strings are crucial, define as a *final node message string* any message string $m_{i,x,t} \in M$ such that $\nexists\, m_{j,x,s} \in M : m_{i,x,t} \subset m_{j,x,s}$. Define M_F as the set of all final node message strings in M.

2.2 Nash equilibria

We now come to a first proposition that identifies all candidate equilibrium networks as "trees" with three properties. A *first* property is that each uninformed player must occur at least once in the tree; this follows naturally from the assumption that all players are required to act, and therefore to be informed, for benefits to arise. A *second* property is that individual final node message strings in a tree need not run all the way to stage z. With involuntary communication, if all e-mails are automatically sent, then players always require information that a crucial e-mail has arrived, causing all strings of crucial messages to run to stage z. This is the case both in the two-player electronic mail game with involuntary communication (Rubinstein, 1989), and in the multi-player electronic mail game with involuntary communication (De Jaegher, 2008b). Simply, if confirmations are available, players require them. Yet, with voluntary communication, if nobody sends a confirmation, then the potential recipients do not let their decision whether or not to act depend on the receipt of such a confirmation; in turn, the senders do not send the confirmations. Thus, players' mutual expectations can lead them to keep communication shorter. A *third* property is that a player i who requires a message string $m_{i,t,x}$ in equilibrium, for any message string $m_{j,\tau,x}$ with $m_{j,\tau,x} \subset m_{i,t,x}$, will not require a message $m_{i,(\tau+1),y}$ such that $m_{j,\tau,x} \subset m_{i,(\tau+1),y}$. Graphically, this means that for any final node message string in an equilibrium tree, if a single message confirms receipt of a sub message string of this final node, this message can only be sent at the same stage as the final node.

Proposition 1.

Assume that an equilibrium exists that is described by a set of crucial messages M. Let M_F be the subset of M that consists of all final nodes in M. Denote by $(M - M_F)$ the set of all messages in M that are *not* final nodes. Then

(i) Each uninformed player must occur at least once in M;

(ii) $\forall m_{i,t,x} \in M_F : \forall m_{j,\tau,x} \subset m_{i,t,x} : m_{j,\tau,x} \in M$;

(iii) $\forall m_{i,t,x} \in (M - M_F) : \exists! m_{j,\tau,x} \in M_F$ such that $m_{i,t,x} \subset m_{j,\tau,x}$;

(iv) $\forall m_{i,t,x} = ...i \overset{t}{\rightarrow} j \in M_F : t \le z$;

(v) $\forall m_{i,t,x} \in M_F : \forall m_{j,\tau,x} \subset m_{i,t,x} :$ $\forall m_{i,(\tau+1),y}$ with $m_{j,\tau,x} \subset m_{i,(\tau+1),y} :$ $m_{i,(\tau+1),y} \notin M_F$.

Proof:

(i) This is the consequence of the assumption that all players must act, and thus must be informed, for the collaborative payoff to be achieved.

(ii) Suppose that a message string $m_{i,t,x} \in M_F$ takes the form $1 \overset{1,x}{\rightarrow} ...j \overset{t-2,x}{\rightarrow} k \overset{t-1,x}{\rightarrow} l \overset{t,x}{\rightarrow} i$. Then it is easy to see that $1 \overset{1,x}{\rightarrow} ...j \overset{t-2,x}{\rightarrow} k \overset{t-1,x}{\rightarrow} l \in M_l$, $1 \overset{1,x}{\rightarrow} ...j \overset{t-2,x}{\rightarrow} k \in M_k$, $1 \overset{1,x}{\rightarrow} ...j \in M_j$. Simply, if a final node message string is crucial to player i, then a player, who does not receive a message string that is a sub message string of this final node message string, knows that player i does not act, and will not act him- or herself.

(iii) Each message string $m_{i,t,x}$ in $(M - M_F)$ must be a sub message string of at least one message in M_F. If this would not be the case, then either the message $m_{i,t,x}$ itself would be a final node (a contradiction); or would be a sub message string of a crucial final node message string not in M_F (but this contradicts the definition of M_F).

(iv) Let no player require a proof of message string $... \overset{t-1,x}{\rightarrow} i \overset{t,x}{\rightarrow} j$. Given that sending an e-mail is costly, j does not send a proof of this message string then. Given that such a proof is never received, it is a best response for other players not to let their decision on whether or not to play B depend on whether they receive such a proof from player j. This in turn justifies player j's strategy of not sending a proof.

(v) If player i receives message string $m_{i,t,x}$, then player i knows that player j received message string $m_{j,\tau,x}$, and does not need a confirmation $m_{i,(\tau+1),y}$ of this to play B. It follows that message string $m_{i,(\tau+1),y}$ cannot be crucial if message string $m_{i,t,x}$ is crucial.

QED

An example of a tree that is eliminated by Proposition 1 is given in the left part of Figure 1. If Bob requires a message from Carl that Carl heard from Alice, then Bob does not need to hear from Alice directly. Similarly, Alice need not hear directly from Carl that he found out her information, if Alice already finds this out through Bob. The messages crossed through are thus eliminated (where it should be noted that one can make an opposite exercise where the crossed through messages are maintained, at the cost of all other messages except the message at stage 1 from Alice to Carl). The right part of Figure 1 presents a tree that is not eliminated by Proposition 1 as an equilibrium. This is because further confirmations are now asked of the messages that are crossed through in the left part of Figure 1.

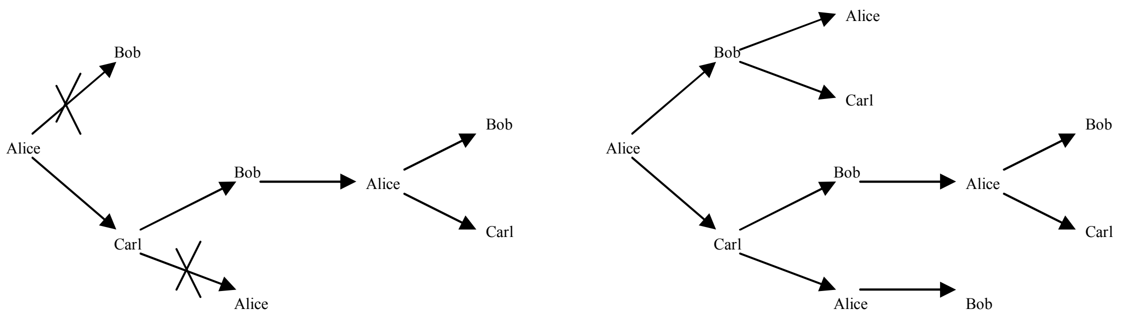

Figure 1. Tree eliminated by Proposition 1 (left), and tree not eliminated by Proposition 1 (right).

Nevertheless, as we will now go on to show, the right part of Figure 1 is still eliminated as an equilibrium, and Proposition 1 does not describe all aspects of Nash equilibria. A first step to realizing this is to see that in the multi-player EMG with voluntary communication, in any equilibrium with crucial messages, when sending an e-mail a player does not only confirm receipt of a string of messages, but confirms receipt of every message he or she has received so far. In the right part of Figure 1, *if* a Nash equilibrium would correspond to this tree (which we will show is not the case), then Bob only sends a message at stage 3 when having received a message at stage 1, and Alice only sends messages at stage 4 when having received all messages at stage 2. If earlier messages are crucial to a player, then the player plays *A* as soon as not receiving any of those messages; given that *A* always yields payoff zero, the player then does not have any reason to send any further costly messages. Proposition 2 generalizes this principle.

Proposition 2
In equilibrium, let player *i* send an e-mail at stage *t*. Then player *i* only sends this e-mail when having received *each* message $m_{i,\tau,x} \in M_i : \tau < t$.

Proof:
By definition, each message in M_i is a crucial message, in the sense that when player *i* does not receive it, he/she plays *A*. But playing *A* yields a zero payoff with certainty. It follows that, as soon as a player does not receive a crucial message, she does not send any further messages, as sending messages is costly.
QED

We now define the concept of a *path* that will be important for eliminating communication networks as equilibria.[7]

Definition 3.
In a set of crucial message strings *M*, consider all parts of these messages strings from stage *s* to stage *t*. Then we say that a *path* exists from player *a* to player *i* between stage *s* and stage *t*

[7] For readers familiar with the network literature, it should be stressed that our concept of a path differs from the one treated in that literature, as our paths can jump from one string of links to the other.

if an order of (sub) message strings $\overset{...}{\rightarrow} a \overset{\rho,u}{\rightarrow} b, \overset{...}{\rightarrow} b \overset{\sigma,v}{\rightarrow} c, \overset{...}{\rightarrow} c \overset{\tau,w}{\rightarrow} d, ..., \overset{...}{\rightarrow} g \overset{\varsigma,x}{\rightarrow} h, \overset{...}{\rightarrow} h \overset{\omega,y}{\rightarrow} i$ exists in *M*, where $a,...,i$ refer to players, where $u,...,y$ refers to message strings, and where $\rho,\sigma,\tau,\varsigma,\omega$ denote stages. The message strings $u,...,y$ may or may not be different. It is the case that $\rho < \sigma < \tau < \varsigma < \omega$.

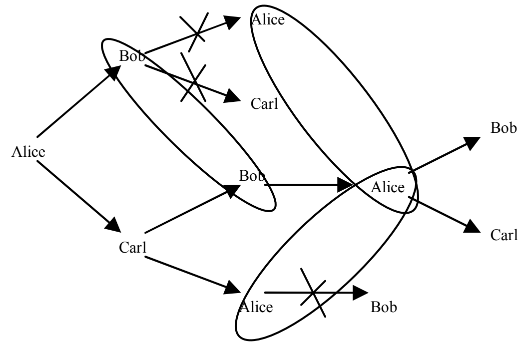

Figure 2. Paths, and tree eliminated by Proposition 3.

In Figure 2, which is the right-part of Figure 1, the following paths exist. Note first that all message strings are also paths. The most interesting paths are those that jump across message strings, where such jumps are indicated by ovals. In Figure 2, these are Alice → Bob, Bob → Alice, Alice → Bob; Alice → Bob, Bob → Alice, Alice → Carl; Alice → Carl → Alice, Alice → Bob; and Alice → Carl → Alice, Alice → Carl. We now provide a result about the information that a player receives when getting a message in a path.

Lemma 1. Across message strings in an equilibrium with crucial messages, let a path $a \overset{\rho,u}{\rightarrow} b, \overset{...}{\rightarrow} b \overset{\sigma,v}{\rightarrow} c, \overset{...}{\rightarrow} c \overset{\tau,w}{\rightarrow} d, ..., \overset{...}{\rightarrow} g \overset{\varsigma,x}{\rightarrow} h, \overset{...}{\rightarrow} h \overset{\omega,y}{\rightarrow} i$ exist from player *a* to player *i*. Then, when receiving an e-mail from player *h* at stage *ω* in message string *y*, player *i* knows that player *b* observed message string $a \overset{\rho,u}{\rightarrow} b$, player *c* observed message string $\overset{...}{\rightarrow} b \overset{\sigma,v}{\rightarrow} c$, player *d* observed message string $\overset{...}{\rightarrow} c \overset{\tau,w}{\rightarrow} d$, etc.

In the example of Figure 2, e.g. when Bob receives an e-mail from Alice at stage 4, he knows that Alice received an e-mail from Carl at stage 2. We now define as a *message set* all the message strings that a player knows to have been realized when receiving an e-mail in an equilibrium with crucial messages.

Definition 4. Consider message string $... \overset{(t-1),y}{\rightarrow} h \overset{t,y}{\rightarrow} i$, and consider all paths from stage 1 to stage $(t-1)$ between players 1 and *h*. Then by Lemma 1, in any equilibrium with crucial messages, when player *i* receives message string $... \overset{(t-1),y}{\rightarrow} h \overset{t,y}{\rightarrow} i$, player *i* knows the information contained in all the message strings which are part of a path from stage 1 to stage $(t-1)$ between players 1 and *h*. We refer to this information, which takes the form of a set of messages, as player *i*'s *message set* when receiving message $... \overset{(t-1),y}{\rightarrow} h \overset{t,y}{\rightarrow} i$ in an equilibrium with crucial messages. This message set is denoted $\mu_{i,t,y}$.

In Figure 2, when Bob or Carl receive an e-mail from Alice at stage 4, their individual message sets contain all messages in the tree except the message sent by Bob to Carl at stage 2, and the message sent by Alice to Bob at stage 3. While *message sets* can span different *message strings*, they in fact have similar properties to message strings. Just like message strings, message sets can be seen as a chain of messages starting at stage 1 (this is because paths can always be made to start at stage 1 – see above). Also, just as message strings, message sets aggregate information. Let message string $\overset{...}{\rightarrow} k \overset{\sigma,u}{\rightarrow} l$ be an element in player *i*'s message set $\mu_{i,t,y}$ when receiving message $... \overset{(t-1),y}{\rightarrow} h \overset{t,y}{\rightarrow} i$. Denote by $\mu_{l,\sigma,u}$ player *l*'s message set when receiving message $\overset{...}{\rightarrow} k \overset{\sigma,u}{\rightarrow} l$. Then all messages in $\mu_{l,\sigma,u}$ are elements of $\mu_{i,t,y}$. $\mu_{l,\sigma,u}$ can thus be seen as a *sub message set* of message set $\mu_{i,t,y}$.

Finally, just as a message string can only materialize if all of its sub message strings are realized, so is the realization of a message set only possible when all of its sub message sets have been realized. To see why, consider one path $a \overset{\rho,u}{\rightarrow} b, \overset{...}{\rightarrow} b \overset{\sigma,v}{\rightarrow} c, \overset{...}{\rightarrow} c \overset{\tau,w}{\rightarrow} d, ..., \overset{...}{\rightarrow} g \overset{\varsigma,x}{\rightarrow} h, \overset{...}{\rightarrow} h \overset{\omega,y}{\rightarrow} i$. If *b* does not receive a message from *a* at stage *ρ*, player *b* does not send a message at stage *σ*, player *c* does not send a message at stage *τ*,…, player *g* does not send a message to player *h* (and *h* will not send a message to *i*). The same applies wherever you start in a path. Moreover, the same applies for any path arriving at *i* at stage *t*. Summarising:

Lemma 2. Consider a message $\overset{...}{\rightarrow} h \overset{\omega,y}{\rightarrow} i$. If *any* message contained in *i*'s message set $\mu_{i,\omega,y}$ accumulated in message $\overset{...}{\rightarrow} h \overset{\omega,y}{\rightarrow} i$ is not received, then *i* will not send any confirmations of message string $\overset{...}{\rightarrow} h \overset{\omega,y}{\rightarrow} i$ in an equilibrium with crucial message strings.

In the example of Figure 2, consider the message set that contains all the messages in the tree except the message sent by Bob to Carl at stage 2, and the message sent by Alice to Bob at stage 3. Then, if any message in this message set gets lost, Bob and Carl do not receive a message from Alice at stage 4. We now come to a simple generalization of part (v) of Proposition 1:

Proposition 3.

Let an equilibrium tree contain a crucial message $\overset{...}{\rightarrow} h \overset{\omega,y}{\rightarrow} i$. Consider the message set $\mu_{i,\omega,y}$ about which player *i* is informed when receiving this message in an equilibrium. Then for any message $\overset{...}{\rightarrow} a \overset{\sigma,u}{\rightarrow} b$ in $\mu_{i,\omega,y}$, we have $\overset{...}{\rightarrow} a \overset{\sigma,u}{\rightarrow} b \overset{\sigma+1,u}{\rightarrow} i \notin M_F$.

Proof:

By Lemma 1, message string $\overset{...}{\rightarrow} h \overset{\omega,y}{\rightarrow} i$ already contains the information that message $\overset{...}{\rightarrow} a \overset{\sigma,u}{\rightarrow} b$ was received. It follows that if message string $\overset{...}{\rightarrow} h \overset{\omega,y}{\rightarrow} i$ is crucial, message string $\overset{...}{\rightarrow} a \overset{\sigma,u}{\rightarrow} b \overset{\sigma+1,u}{\rightarrow} i$ cannot be crucial. QED

In the example of Figure 2, if the equilibrium tree involves the e-mails sent by Alice to Bob and Carl at stage 4, then it cannot contain the e-mails that are crossed out. Note that after these messages are deleted, further messages need to be deleted in order to keep the e-mails sent by Alice to Bob and Carl at stage 4 as crucial messages. This generalizes the principle in the left part of Figure 1. There, if players consider a message string as crucial, they cannot consider final node confirmations of sub message strings of this message string as crucial. In Figure 2, if players consider a message set as crucial, they cannot consider final node confirmations of sub message sets of this message set as crucial.

In order to have an idea of both the strength and the limits of the principle expanded in Proposition 3 for eliminating inefficient equilibria, we provide an additional result.

Corollary 1. In the multi-player EMG with voluntary communication by means of proofs of receipt, equilibria with crucial messages cannot contain more than $N(N-1)$ final node message strings.

Proof: It can be checked that any tree with more than $N(N-1)$ final node messages violates Proposition 3. QED

In comparison to the result in Corollary 1, in the largest possible communication network, $(N-1)^z$ messages are sent at stage z. An example of an equilibrium tree with a maximal number of final node message strings for the three-player EMG with proofs of receipt is given in Figure 3. The final node message strings end in all possible e-mails that the three players can send to one another. Note that, when Carl receives an e-mail from Bob at stage 3, Carl also knows that Bob sent an e-mail to him at stage 2. Yet, the e-mail at stage 2 continues to be crucial because Alice and Bob require a confirmation of it. It follows that Carl still considers the e-mail from Bob at stage 2 crucial, since otherwise Carl cannot send a proof of receipt to Alice and Bob. For this reason, equilibrium trees where a lot of messages are required continue to exist in this version of the game.

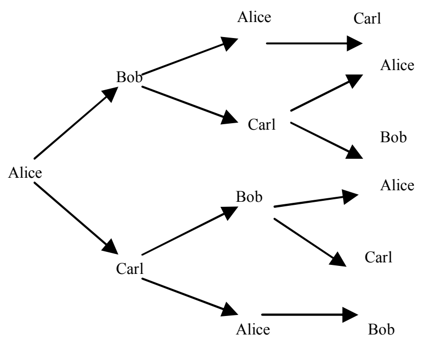

Figure 3. Equilibrium tree with maximal amount of final node message strings in multi-player EMG with proofs of receipt.

We have so far shown what form Nash equilibrium trees with crucial message strings (in the multi-player EMG with proofs of receipt) take if they do exist. It remains to be shown that the trees with the properties derived in Propositions 1 and 3 are indeed Nash equilibria. We separately check whether the players' action strategies are best responses, given that they have followed the equilibrium signaling strategies (Proposition 4). Next, we check whether the players' signaling strategies are best responses.

Proposition 4.
Consider any set of crucial message strings M with the properties derived in Proposition 1 and Proposition 3. Consider an individual player i. For a tree as characterized in Propositions 1 and 3, let all other players than i only play B when receiving all their e-mails in this tree, and only send a proof of receipt when having received all e-mails in the tree so far. Then:

(i) if player i does not receive a message string of which player i's proof of receipt is crucial to at least one other player, then player i plays A;
(ii) if player i does not receive a crucial final node message string, then player i plays A;
(iii) small levels of ε with $\varepsilon \in [0, \varepsilon_C[$ exist such that player i prefers to play B when player i receives all crucial messages. ε_C, the critical value of ε such that some player receiving a message string in M is indifferent between playing A and B, is a function of z.

Proof:
(i) By definition, if player i does not receive a message string, she cannot send a proof of receipt of this message string. If such a proof of receipt is crucial to another player, then this other player will play A. Given that player i incurs a loss as soon as one other player plays A, he/she plays A.
(ii) By Corollary 1, if player i does not receive a final node crucial message string from player h, he/she does not receive any other message containing the same information. At best, player i knows that a message was sent to player h. Player i's expected utility from playing B is then $\frac{(1-\varepsilon)\varepsilon}{\varepsilon+(1-\varepsilon)\varepsilon}M - \frac{\varepsilon}{\varepsilon+(1-\varepsilon)\varepsilon}L$. Since $L > M$, this is smaller than the payoff zero when playing A. As player i plays A even under these best circumstances, she always plays A when she does not receive a final crucial message.
(iii) In any candidate equilibrium, given that all other players follow the candidate equilibrium strategy, a player who has received all crucial messages faces uncertainty whether X other crucial messages arrive. Each such player therefore faces a decision of weighing $(1-\varepsilon)^X M - \left[1-(1-\varepsilon)^X\right]L$ against zero. For any X, a small ε can be found such that the player prefers to play B.

QED

Intuitively, the potential receiver of a non-final node message string plays A when not receiving this message string, as this automatically blocks at least one other player from receiving a crucial proof of receipt. The potential receiver of a final node message string, when not receiving this message string, plays A because of the large loss of playing B when not all other players are playing B.

Proposition 5.
By Proposition 4 a level of ε denoted as ε_C exists such that for each ε with $\varepsilon \in [0, \varepsilon_C[$, each player in any tree characterized by a set of crucial message strings M that has the properties derived in Propositions 1 and 3, strictly prefers to play B when having received all crucial message strings in M. Given such an ε_C, levels of d with $d \in [0, d(\varepsilon_C)[$ exist such that each receiver of a non-final node message string in M strictly prefers to send the proof of receipt implied by M when having received all messages so far in M.
Proof:
The general form of the expected payoff to a player of sending any candidate equilibrium message, given that all other players follow the candidate equilibrium, can be derived in the following way. Assume that, when a player sends a current message, this implies that the

player will also send all further messages, if the opportunity presents itself. By Proposition 4, the player will only consider doing B when having been able to send all further messages in M. Let X_1, X_2, X_3,…, X_F messages need to arrive for the player to send his or her first, second, third,… final message after having sent the current message. The player's expected payoff when sending the current message then takes a nested form:

$$(1-\varepsilon)^{X_1}\left\{\left[(1-\varepsilon)^{X_2}\left((1-\varepsilon)^{X_3}(\ldots-d)-d\right)\right]-d\right\} \tag{1}$$

where the player's decision to send the final message takes the form

$$(1-\varepsilon)^{X_F}M-\left[1-(1-\varepsilon)^{X_F}\right]L-d\,, \tag{2}$$

and is contained into the latter expression. It is easy to see that if the player is willing to send the earliest message (along with all future ones (see (1)), he also will want to send all future messages. For $d=0$, given that $\varepsilon\in\left[0,\varepsilon_C\right[$, (2) is larger than zero. Moreover (1) is larger than zero as well. It follows that levels $d\in\left[0,d(\varepsilon_C)\right[$ exist such that all expressions such as (1) and (2) are strictly larger than zero. QED

2.3 Efficient equilibria

Having characterized the equilibria with crucial message strings of the multi-player EMG with proofs of receipt, we now investigate what are the efficient equilibria. In order to disentangle the effect of the fact that the e-mails are costly, and of the fact that they are noisy, we first treat efficiency separately for a model with costly, noiseless e-mails, and with costless, noisy e-mails. In the model with costly, noiseless messages, all trees that have $(N-1)$ messages are efficient. In the model with costless, noisy messages, lines with $(N-1)$ messages are efficient and best to the last uninformed player receiving a message; lines with N messages, where the informed player receives a final message, are also efficient and are best to the informed player.

Proposition 6. In the absence of noise, any tree starting at the informed player with exactly one message sent to each of the $(N-1)$ uninformed players is Pareto-efficient.
Proof:
Players are certain that all messages arrive, and therefore, compared to trees with $(N-1)$ messages, cannot become better off if confirmations are added to these trees. Given that messages are costly, some players would become worse off without anyone getting better off.

Thus, many Pareto-efficient trees exist for the game without noise, and players differ on their preferences over these trees depending on the message sending costs they incur. In the absence of noise, there only is a conflict over who bears the cost of sending messages. In a star, the informed player bears all the message sending costs. In a line, each player except the last player in line sends one message. There are many other Pareto-efficient trees between these two extreme cases. We next treat noisy communication without costs.

Proposition 7. Consider equilibria where each message is crucial. Let sending messages be costless, but let there be noise. Then the Pareto-efficient trees are any line of size $(N-1)$, and any line of size *N* where the informed player receives at most one confirmation of earlier messages (anywhere between stage 2 and the final stage).

Proof:

The proof consists of three parts. In (i), we first show that no more than one final message can be situated at a stage *before* the final stage at which any message is sent. In (ii) we show that only one message can be sent at the final stage. In (iii), we show that uninformed players can only occur once in each message tree, whereas the informed player can occur twice.

(i) Consider a tree *g* in which stage *t* is the last stage with any final node message string, and let more than one message string be crucial at stage *s*, with $s < t$. Thus, we have a final node message string m_x ending at *t*, $1 \to ...m \xrightarrow{\tau} q \xrightarrow{\tau+1} r...f \xrightarrow{s} g...h \xrightarrow{t-1} i \xrightarrow{t} j$. Additionally, we have a message $k \xrightarrow{s} l$, with $l \neq g$, part of message string m_y. Denote by $\xrightarrow{...} m \xrightarrow{\tau} n$, with $\tau \leq s$ the earliest message string contained in m_y but *not* in m_x. Denote by $g_{m \to n}$ the tree consisting of all messages that can only be sent if $\xrightarrow{...} m \xrightarrow{\tau} n$ arrives. Consider a final node message $\xrightarrow{...} o \xrightarrow{\sigma} p$ of $g_{m \to n}$, with $\sigma \geq \tau$. Construct now a new tree $1 \to ...m \xrightarrow{\tau} q \xrightarrow{\tau+1} n...o \xrightarrow{\sigma+1} p \xrightarrow{\sigma+2} r...f \xrightarrow{s+\sigma-(\tau-1)} g...h \xrightarrow{t-1+\sigma-(\tau-1)} i \xrightarrow{t+\sigma-(\tau-1)} j$. This is done by taking the tree $g_{m \to n}$ out of the old tree, and reconnecting it, where player *q* at stage $(\tau + 1)$ sends a message to player *n* (and continues to send all other messages he/she sent in the old network); where all confirmations are further sent as in $g_{m \to n}$ (and thus now also confirm that *q* received a message from *m* at τ); and where *p*, when receiving a message from *o*, sends a confirmation at $(\sigma + 2)$ to *r*. Then all players up to stage τ have the same information as before. All players after τ have at least as much information. It follows that all players are better off in the newly constructed network. An example is in Figure 4. By repeating this procedure any number of times, one eventually obtains a tree in which not more than one message is sent at a non-final stage.

(ii) Consider two players *i* and *j* receiving a final node message string at the same stage *t*. By (i), this needs to be from one and the same player. Construct a new tree where *j* no longer receives such a message, but where *i* at stage $(t+1)$ sends a confirmation of all messages received before to *j*. Then *i* is equally well off, but *j* is better off.

(iii) Consider all candidate efficient trees, which by (i) and (ii) must be lines. Let a player *i* occur more than one time in such a line. The first time, let him receive a message from player *h*, and let him send a message to *j*. Construct a new tree where *h* confirms directly to *j*, without passing by *i*, and where everything otherwise stays the same. Then everyone is better off because less messages are sent over all, but players are still ordered in the same way according to their uncertainty.

QED

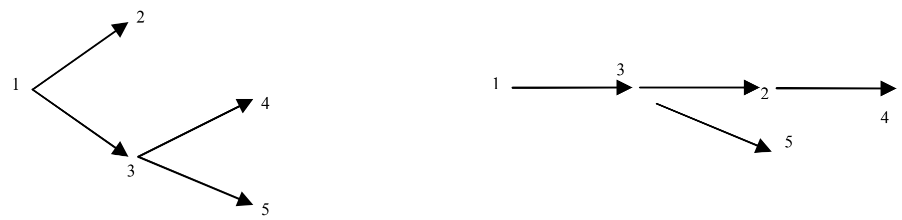

Figure 4. Example of part (i) of the proof of Prop. 7, where $1 = k = f = m$, $s = \tau$, $3 = g = q$, $2 = n$, $4 = r$.

Finally, we provide a result for the combination of noise and costly signals, based on the assumption that signaling costs are relatively small.

Proposition 8. Consider equilibria with crucial messages in the multi-player EMG with proofs of receipt. Let there be noise, and let sending messages be costly. Then relatively small ranges of d exist such that the efficient equilibrium trees are any tree of size $(N-1)$, and any tree of size N where the informed player receives at most one confirmation of earlier messages (anywhere between stage 2 and the final stage), with a common characteristic. The common characteristic in these trees is that multiple messages can only be sent at the unique final node stage.

Proof:

We follow the same structure as in the proof of Proposition 7, and check the extent to which the arguments set out there continue to apply with costly signals.

(i) Consider the procedure in part (i) of the proof of Proposition 7 for constructing a new tree from an inefficient tree. With costly signals, it continues to be the case that all players up to stage τ have the same information as before; also, they have the same signaling costs as before. All players after τ have at least as much information, and with the exception of player p, carry the same signaling costs. As player p has more information, for small signaling costs, she will be better off by sending a message in the newly constructed tree. It follows that all players are better off in the newly constructed tree.

(ii) Consider two players i and j receiving a final node message string at the same stage (from one and the same player). Construct a new tree where j no longer receives such a message, but where i at stage $(t+1)$ sends a confirmation of all messages received before to j. Then i is worse off, but j is better off. Therefore, there is no Pareto-superior move from networks with several final node messages sent at the same stage.

(iii) The argument set out in (iii) of Proposition 7 is reinforced because j additionally saves signaling costs from the Pareto superior move.

QED

By Proposition 8, the structure of all efficient equilibrium trees takes the form of a line, a star, or a combination of a line and a star. The last player in the line sends a message to each remaining player. Examples of such networks for the four-player case are given in Figure 5, where it should be noted that the positions of the uninformed players are interchangeable.

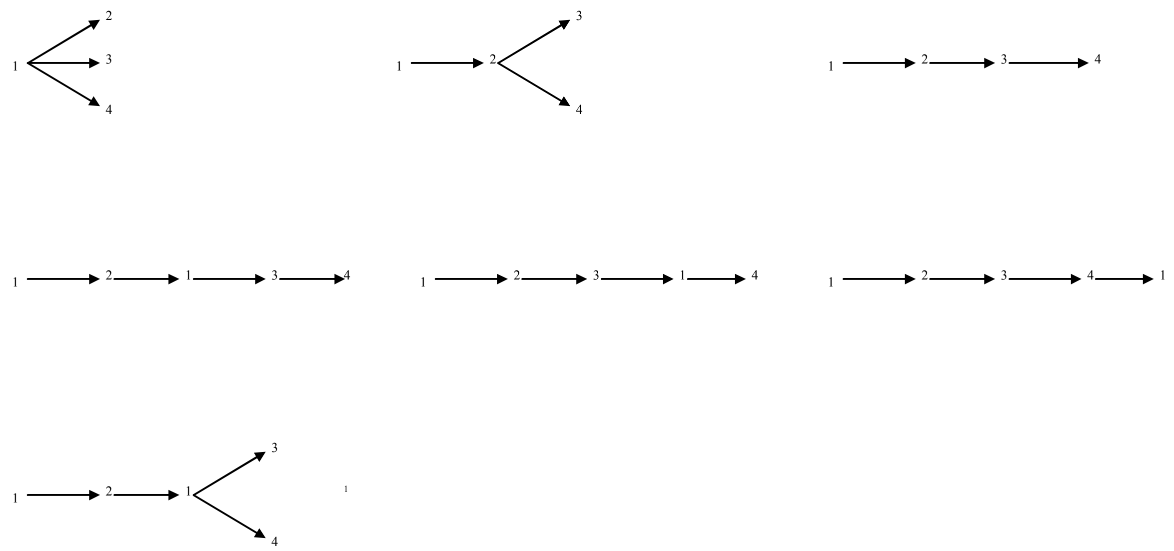

Figure 5. Efficient trees in the four-player EMG.

2.4 Equilibrium selection

We end this section by noting that equilibrium refinements have no cutting ground in the EMG with proofs of receipt. Eliminating some Nash equilibria is driven by players' responses in equilibrium to out-of-equilibrium events. Nash equilibria may not be sequential equilibria because players' best response to an out-of-equilibrium event cannot possibly be a best response as soon as this out-of-equilibrium event occurs with positive probability. Yet, because of noise, the event of not receiving a message already occurs in equilibrium with positive probability, limiting the available out-of-equilibrium events. Furthermore, because players can only send proofs of receipt, the only out-of-equilibrium messages that can be sent are extra proofs of receipt, thus extending rather than limiting the number of messages. For this reason, out-of-equilibrium messages cannot have the function of indicating a player's willingness to move to a Pareto-superior equilibrium. Equilibrium selection arguments can be at work, however, if players' messages are not literally proofs of receipt. This case is investigated in the next section.

3. Multi-player electronic mail game with voluntary communication: false acknowledgements can be sent

We now assume that messages are not literally proofs of receipt, and can only take on the meaning of confirmations of receipt in equilibrium. We assume each player to have a set of messages at his or her disposition. In equilibrium, a particular message sent at a particular time takes on the meaning of a particular message string. Note that the sender of such a message can then pretend to have observed a message string even when this is not true. While this extends the strategy space, Nash equilibria replicating the equilibria with proofs of receipt may continue to exist. In such replicating equilibria, while it is possible to tell other players that one has received a message string even if this is not true (referred to as a *false acknowledgement*), players find it a best response to be honest. This section investigates the extent to which such replicating equilibria continue to be Nash equilibria (Section 3.1), and whether any replicating equilibria survive standard refinements of the Nash equilibrium (sequential equilibrium, Section 3.2; intuitive criterion, Section 3.3).

3.1 Nash equilibria

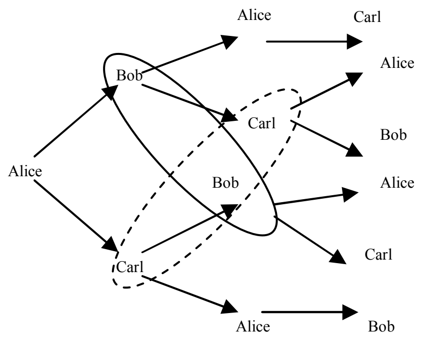

Figure 6. Non-Nash tree in the three-player EMG without proofs of receipt.

Figure 6 repeats Figure 3, and indicates two paths. In the EMG with proofs of receipt, the presence of these paths is irrelevant. Even though e.g. Carl, when receiving an e-mail from Bob at stage 3, knows that Bob sent an e-mail to him at stage 2, Carl still requires the e-mail at stage 2 as well, as Alice and Bob require a proof of receipt of it. Yet, in the EMG without proofs of receipt (i.e. with the possibility of false acknowledgements), when Carl does not receive an e-mail from Bob at stage 2, Carl can pretend to Alice and Bob that he received an e-mail from Bob. Carl does not know whether there is an opportunity for collective action at this point, but reasons that he may yet receive an e-mail from Bob at stage 3, indicating that Bob's message at stage 2 got lost. Carl thereby still gets an opportunity to benefit from collective action even though a message got lost. As long as the cost of sending a message is not too high (it is still probable that there is no opportunity for collective action), Carl may still turn out to play *B* even when not receiving an e-mail from Bob at stage 2. But if this is so, then the tree in Figure 6 is *not* a Nash equilibrium with crucial messages. In general, if players consider a message set as crucial in the multi-player EMG without proofs of receipt, they cannot consider any confirmations of sub message sets of this message set as crucial. Note that this is more general than the final node confirmations of Proposition 3.

Proposition 9. Consider a candidate equilibrium of the multi-player EMG *without* proofs of receipt replicating any of the equilibria derived for the case *with* proofs of receipt. Consider player *i*'s decision to send one ore more false acknowledgements at stage $(t+1)$, after not having received message string $\overset{\dots}{\rightarrow} h \overset{t,x}{\rightarrow} i$ from *h* at *t*, but having received all equilibrium messages at or before *t*. Consider all messages contained in $\mu_{i,t,x}$ in this case. Suppose (i) that there is a path from each of these messages to player *i* at stage *t* or later. Then, for sufficiently small *d*, the candidate equilibrium is *not* a Nash equilibrium.
Proof:

In a candidate equilibrium replicating an equilibrium of the multi-player EMG with proofs of receipt, let player i not receive message string $\overset{\dots}{\rightarrow} h \overset{t,x}{\rightarrow} i$ (but receive all other messages). Let player i send one or more false acknowledgements, pretending to have received message string $\mu_{i,t,x}$. By (i), it is still possible that player i receives information later on that all message strings contained in $\mu_{i,t,x}$ except $\overset{\dots}{\rightarrow} h \overset{t,x}{\rightarrow} i$ were received. If player i does not receive this information, by Proposition 4, player i plays A. If player i does receive this information, then player i plays B for the same reason that he/she plays B when $\overset{\dots}{\rightarrow} h \overset{t,x}{\rightarrow} i$ is received and the candidate equilibrium is followed. It follows that player i's expected payoff net of signaling costs of sending one or more false acknowledgements is positive. Therefore, for sufficiently small d, player i sends these, and the candidate equilibrium is not a Nash equilibrium.
QED

An application of Proposition 9 is that, as soon as a player i requires a message string from player j at stage t on string $m_{i,t,x}$, player i cannot require in equilibrium any messages of other message strings that have $m_{j,(t-1),x}$ as a sub message string. This is summarized in Corollary 2.

Corollary 2. In an equilibrium tree of the multi-player EMG without proofs of receipt, let player i require $m_{i,t,x}$. Then $\forall m_{j,(t-1),x} \subset m_{i,t,x} : \forall m_{i,\tau,x}$ with $m_{j,(t-1),x} \subset m_{i,\tau,x} : m_{i,\tau,x} \notin M$.

As an example of the relevance of Corollary 2 consider the three-player game. If Alice sends a message to Bob and Carl at stage 1, then Bob's message cannot (directly or indirectly) be forwarded to Carl, and vice versa. Thus, we have one message string on which only Alice and Bob can be positioned, and one message string on which only Alice and Carl can be positioned. An example of such a tree is given in Figure 7.

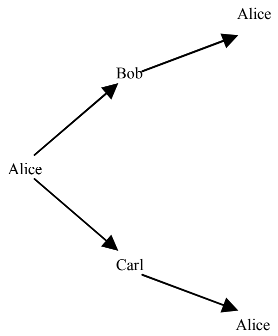

Figure 7. Equilibrium tree in the three-player EMG without proofs of receipt.

3.2 Sequential equilibria

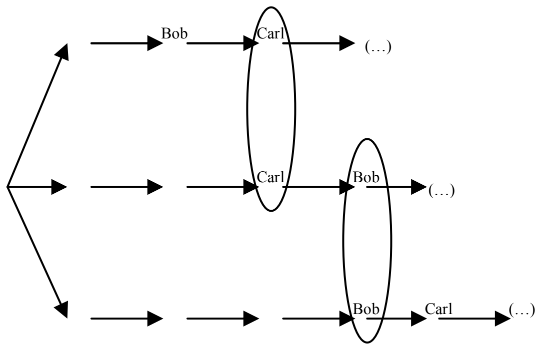

Figure 8 Nash tree that is not sequentially rational.

Contrary to what is the case in the game with proofs of receipt, in the game without proofs of receipt, out-of-equilibrium events can be generated by the fact that false acknowledgements can be sent. As we now show, this causes some paths that exist across Nash equilibrium trees to be eliminated for not being sequentially rational. Consider Figure 8. Let Carl not receive the e-mail from Bob at stage 3, but let Carl still send a false acknowledgement to Bob at stage 4. If Bob did not receive the e-mail at stage 2, then Bob finds out that Carl's acknowledgement is false. Bob could then punish Carl by playing *A* and not sending an e-mail. Yet, such a punishment does not keep Carl from sending a false acknowledgement: if Carl receives an e-mail from Bob at stage 5, Carl then knows that Bob did not detect the false acknowledgement, and can safely play *B*; if Carl does not receive an e-mail, Carl knows that it is likely that his false acknowledgement was detected, and does not run any risk by playing *A*. Therefore, this kind of punishment does not stop Carl from sending a false acknowledgement as long as the cost of sending a message is relatively low. A punishment that does stop Carl from sending a false acknowledgement is when Bob sends an e-mail at stage 5 even after having detected a false acknowledgement, and even though he is planning to do *A*. But such a punishment is not sequentially rational, as it is never a best response to send a costly message when one at the same time takes an action that always yields payoff 0. Note that the same principle is at work if Carl does not send a message directly to Bob, and next Bob does not send a message directly to Carl, but if there is instead a corresponding path between these players. The general result is stated in Proposition 10.

Proposition 10.
Consider a Nash equilibrium tree of the multi-player EMG without proofs of receipt. Then this tree is *not* a sequential equilibrium (Fudenberg and Tirole, 1991) if, for any message string $\stackrel{\dots}{\rightarrow} h \stackrel{t,x}{\rightarrow} i \in M$,

(i) there is a path in the tree from player i to player h between stage t and some stage $(t + x)$, with $x > 1$; *and*

(ii) there is a path in the tree from player h to player i starting at stage $(t + x)$.

Proof:
Let all other players follow the Nash equilibrium, and let player i not receive message string $\stackrel{\dots}{\rightarrow} h \stackrel{t,x}{\rightarrow} i$. If the last e-mail in this message got lost, and if player i sent a false acknowledgement, then for small noise player i is likely to get a message from player h.

Moreover, if player *h* does not send any messages when detecting a false acknowledgement, then a player *i*, who still receives a message from player *h,* knows that player *h* did not detect the false acknowledgement. At the same time, it is never a best response for player *h* to send a message and play *A*. Thus, when player *i* still receives a message from player *h*, player *i* knows that the false acknowledgement went undetected; when player *i* does not receive anymore messages from player *h*, player *i* avoids any risk by playing *A*. Net of signaling costs, the expected payoff of sending a false acknowledgement is therefore positive. It follows that for sufficiently small *d*, player *i* sends a false acknowledgement. The Nash equilibrium is supported by a response of player *h* to an out-of-equilibrium event that is never a best response when the out-of-equilibrium event actually takes place. It follows that such a Nash equilibrium is not a sequential equilibrium. QED

It should be noted that Proposition 10 generalizes a principle that is at work in the two-player EMG without proofs of receipt (De Jaegher, 2008a) to the multi-player setting. In the two-player game, this principle eliminates any sequence of the form 1 → 2 → 1 → 2. There, it is strictly a result about message strings. As pointed out above, in the multi-player game, message sets have the same function as message strings, and Proposition 10 eliminates orders of the type $h \rightarrow i$, $i \rightarrow h$, $h \rightarrow i$ in a message set (and thus in a path).

3.3 Forward induction

When checking whether the Nash equilibria described in Section 3. 1 are sequential equilibria, the test is whether punishments of out-of-equilibrium messages such as false acknowledgements are indeed a best response when this false acknowledgement is actually sent. We now check whether an out-of-equilibrium move such as a false acknowledgement can be interpreted as an attempt of a player to move to a Pareto superior equilibrium. After all, if player *h* detects an out-of-equilibrium message sent by player *i*, player *h* should wonder why such a costly message was sent, given that player *i* could have safely obtained payoff zero by not sending any further message and playing *A*. It is clear that player *i* only has the intention to send a costly message if he/she believes that collective action is still possible. Thus, rather than interpreting an out-of-equilibrium message as an attempt to cheat, one could argue that it should be interpreted as an attempt to increase the probability of collective action. Yet, the question then is: how does the sender of an out-of-equilibrium message intend to coordinate on taking collective action (cf. Cho and Kreps, 1987)?

For example, consider the inefficient Nash equilibrium tree for the four player EMG in the top left part of Figure 9. The probability of collective action is larger in the efficient Nash equilibria on the top right and bottom left parts of Figure 9. Yet, if Alice deviates by sending an e-mail straight to Bob, it is unclear whether the other players will be able to coordinate on either the top right or bottom left tree. For instance, Bob may tell Carl what he heard from Alice, expecting Carl to forward this message to David. But Carl may think that Bob already informed David directly, in which case collective action fails.

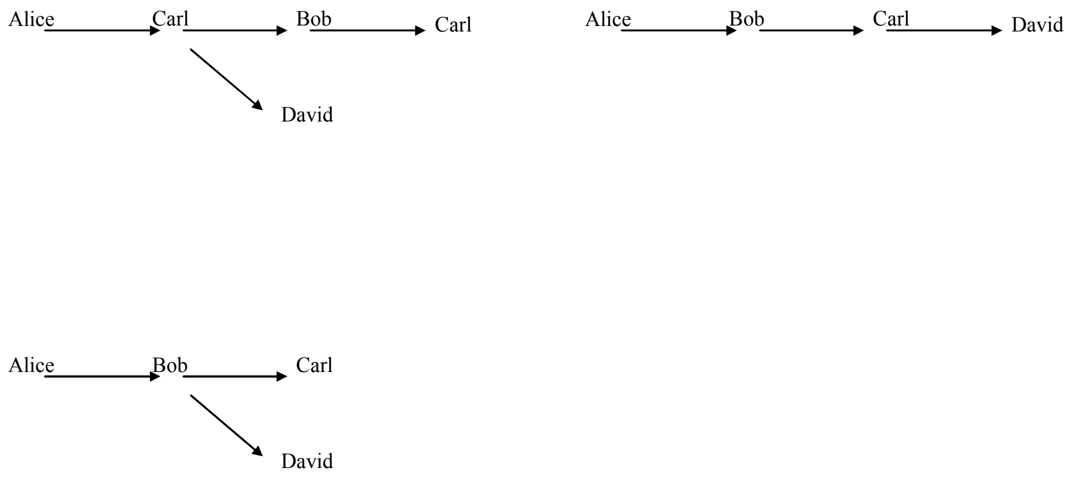

Figure 9 Inefficient and efficient Nash equilibria of the four-player EMG without proofs of receipt.

Because of the problem illustrated by Figure 9, we propose to eliminate trees by means of forward induction only if an out-of-equilibrium message allows a move to a Pareto-superior tree, where everyone continues to send messages to the same players at the same stages as before, but where part of the messages from the old tree are no longer sent. This leads us to the following conjecture.

Conjecture 1. Consider a Nash equilibrium of the multi-player EMG without proofs of receipt, with crucial message string set *M*. Consider the message string $\overset{...}{\rightarrow} h \overset{t,x}{\rightarrow} i \in M$, with corresponding message set $\mu_{i,x,t}$ for player *i*. Let the following conditions be valid:

(i) Consider player *h*'s message set $\mu_{h,x,(t-1)}$. Before a stage τ with $\tau \geq t$, there is a path from each message in $\mu_{h,x,(t-1)}$ to player *i*.

(ii) At a stage $\sigma \geq \tau$, there is a path from player *i* to player *h*.

Then, if player *i* receives all e-mails contained in *M* except the e-mail from *h* at stage *t*, player *i* sends all other e-mails in *M*, and player *h* sends all messages in *M* and plays *B* when receiving all messages in *M* except the message at stage (t – 1).

An application of this conjecture is represented in Figure 10. If Carl receives an e-mail from Eric but not from Bob, Carl sends a false acknowledgement to David, who confirms receipt of it to Bob. Even if Bob did not receive a message from Alice, Bob should still act. It is as if Carl were saying: "I don't need to hear from you that you know about the opportunity for collective action, because I already know this information through Eric. I know that you require information about this opportunity, but since I am communicating it to you, I am confident that you will receive it."

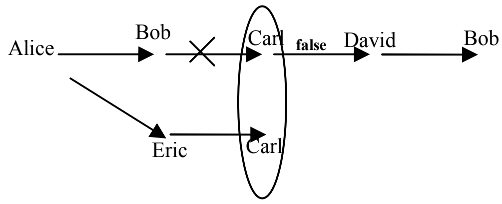

Figure 10 Forward induction

3.4 Remaining inefficient equilibria

Figure 9 already illustrates one way in which players may not be able to move away from inefficient equilibria. A player may be included more than once in a message string. As the player, the last time he receives a message, finds out the same information plus additional information contained in the message string, he/she does not need this information the first time. Nevertheless, if other players require a confirmation that he/she has also received this early information, then in equilibrium the player will still require this early information before he/she acts.

Another way in which players may get locked in inefficient equilibria is illustrated in Figure 7. As already shown in Section 2.3, all players are better off if they communicate in a line Alice → Bob → Carl → Alice. Yet, if Alice stops sending a message to Carl and only sends a message to Bob, then neither Bob nor Carl observe out-of-equilibrium events. Players may get locked into such an inefficient equilibrium whenever a player informs separate “cliques” of other players who do not talk to each other.

4. Interpretation

This paper shows that, contrary to what is the case in the two-player electronic mail game with voluntary communication, players of the *multi*-player game with voluntary communication may get locked into playing inefficient equilibria where they require too large an amount of messages from each other, thus reducing the probability of collective action. The most efficient communication protocol for multiple players would be one in which each player receives only one message. While sequential rationality helps to rule out sequences for messages send back and forth between any pair of two players also in the multiple-player situation, it does not help to rule out inefficient communication where redundant messages and confirmations are sent to separate subgroups, or “cliques”, of players. At the same time, the intuitive criterion has little cutting ground, because of the many players involved. When an individual player sends an out-of-equilibrium message in an attempt to move to a Pareto-superior equilibria, it is not clear which Pareto-superior equilibrium he wants to move to.

From this perspective, an argument can be made for what can broadly be described as institutionalized communication (for an overview, see Koessler, 2000). In a *first* type of institutionalized communication, players can take *leadership* in order to guide all players towards a Pareto-efficient outcome (for a broad perspective on leadership in resolving coordination problems, see Foss, 1999). For instance, in the four-player game, let players be stuck in the inefficient equilibrium on the top left of Figure 9. Alice could now take the lead and instruct Bob to instruct Carl to tell David that there is an opportunity for collective action, after which Bob indeed instructs Carl to tell David. Similarly, in Figure 7 Bob could take the lead and instruct Carl to confirm to Alice that both Bob and Carl know about the opportunity. It should be noted that such leadership requires a richer language than the one we have assumed in the body of the paper, where players could only tell the string of players through which a message was forwarded. In a *second* type of institutionalized communication, common knowledge gets generated by an event such as a public meeting (Chwe, 2001). In the example of Figure 9, when Alice finds out about the opportunity for collective action, she calls a meeting of the four players. The fact that all players observe each other in this meeting creates common knowledge of players' intentions to act. A third *type* of institutionalized communication is guided by strict protocols of who can talk to whom, and of who needs to know what (Chwe, 1995). Thus, collective action is successful because a *hierarchy* exists among the agents involved in the collection-action problem. Interpreting firms as collectives that (next to other things) need to solve coordination (collective-action) problems, this argument would seem to give credit to the so-called classical management's (e.g. Fayol, 1949) arguments in favor of hierarchically-organized firms.

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