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### Poverty-Reducing Tax Reforms with Heterogeneous Agents

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**Abstract:** The poverty impact of indirect tax reforms is analyzed using sequential stochastic dominance methods. This allows agents to differ in dimensions that cannot always be precisely captured within the usual money-metric indicators of living standards. Examples of such dimensions include household size and composition, temporal or spatial variation in price indices, and individual needs and “merits”.

**Keywords:** Poverty, Efficiency, Tax Reform, Stochastic Dominance, Equivalence Scales

**JEL Classification:** D12, D63, H21, I32

# 1 Introduction

Analyzing the social impact of tax reforms can be done in two ways. The first specifies a particular form for the social evaluation function (SEF) and then searches for an optimal tax structure. The nature of that optimal tax structure is then conditional on the choice of the specific SEF. The second route assumes that we do not know precisely what the SEF is (or should be). The search is then for tax reforms that can improve on the existing tax system for a whole class of SEF – *viz*, it is a search for “socially-efficient” reforms.

Searches for socially-efficient *marginal* tax reforms have been recently carried out using marginal stochastic dominance tests. Two tests have been proposed. The first test was introduced by Yitzhaki and Slemrod (1991) and compares the concentration curves of two commodities. The second test was proposed by Makdissi and Wodon (2002) and Duclos, Makdissi and Wodon (2002) and compares “consumption dominance” curves. The first of these two tests covers the class of all SEF that are averse to inequality – namely, it tests for marginal second-order dominance. The second test can be designed to cover the social impact of tax reforms at any order of dominance.

This note extends the poverty results of Makdissi and Wodon (2002) to allow agents to differ in dimensions that cannot always be precisely captured within the usual money-metric indicators of living standards. Examples of such dimensions include temporal or spatial variation in price indices, varying individual needs, or heterogenous social or political “merits”. All such dimensions have the common property of often being difficult to measure precisely. For expositional simplicity, we follow in this paper much of the previous literature on multidimensional dominance and focus on the difficulty involved in *equivalizing* living standards in the presence of heterogenous household size and composition (on this, see *inter alia* Buhman *et al.* (1987), Atkinson and Bourguignon (1982, 1987), Atkinson (1991), and Jenkins and Lambert (1993)).

In the spirit of Mayshar and Yitzhaki (1996), we thus derive sequential stochastic dominance conditions that can be used to check whether an indirect tax reform is efficient over classes of SEF and over general assumptions on the shape of equivalence scales. Unlike Mayshar and Yitzhaki (1996), who consider second-order dual social-welfare tests, we consider poverty dominance of arbitrary ethical order using primal tests. Robustness is then also obtained over ranges of poverty lines for each household type.

## 2 Measuring poverty

Individuals are members of households that are heterogenous in size  $k$ . There are  $n$  different household sizes. For expositional simplicity, we assume that individuals are otherwise homogenous. We denote by  $f_k(y)$  the density function of *per capita* income among individuals who are members of group  $k$ , and assume it to be nil outside of the interval  $[0, a]$  – where  $a$  is greater than the maximum *per capita* income of all household types and is also greater than the maximum conceivable individual poverty line. Hence,  $\int_0^a f_k(y) dy = 1$  for all  $k$ . Denote by  $\theta_k$  the population share of individuals who are members of households of size  $k$ . We suppose that the government wishes to reduce an additive aggregate poverty index denoted generally by

$$P = \sum_{k=0}^n \theta_k \int_0^a p_k(y^E(q, y, k; q^R)) f_k(y) dy, \quad (\text{A1})$$

where  $p_k(y^E)$  is the finite contribution to aggregate poverty of an individual who is member of group  $k$  and whose real *per capita* income is given by  $y^E$ . For anyone in poverty,  $p_k(y^E) \geq 0$ , but  $p_k(y^E) = 0$  whenever  $y^E$  exceeds the poverty line of group  $k$ , given by  $z_k$ . We further denote by  $z_k^+$  the maximum conceivable poverty line that we may wish to consider for each group  $k$ . It must therefore be that  $z_k \leq z_k^+$  for all  $k$ .

Let  $q$  be the vector of consumption prices; it equals production prices  $e$  (normalized to one and assumed constant) plus indirect taxes  $t$ , that is,  $q = e + t$ . Denote as  $q^R$  the consumption price vector for a reference period. Real income,  $y^E$ , is the level of income that would have yielded under  $q^R$  the same level of utility as income  $y$  under  $q$ . Letting  $v(\cdot)$  be the indirect utility function,  $y^E$  can then be implicitly defined by

$$v(q^R, y^E, k) = v(q, y, k). \quad (1)$$

In order to develop sequential stochastic dominance conditions of order  $s$ , we further require that the poverty measure  $p_k(\cdot)$  be a continuous function that is  $s$ -time differentiable over  $[0, a]$ . This, we call assumption A2. We also require that

$$(-1)^l p_1^{(l)}(y^E) \geq (-1)^l p_2^{(l)}(y^E) \geq \dots (-1)^l p_n^{(l)}(y^E) \geq 0 \quad \forall l \in \{1, 2, \dots, s\} \quad (\text{A3})$$

where  $p_k^{(l)}(\cdot)$  is the  $l$ -th derivative of  $p_k(\cdot)$ . The class of all poverty measures that obey assumptions A1, A2 and A3 is denoted by  $\Pi^s$ .<sup>1</sup> Note that this formulation is very general with respect to the choice of poverty lines  $z_k$ . All that is required is that  $z_k \leq z_k^+$  and that the choices of the implicit  $z_k$  in the  $p_k(\cdot)$  do not contradict assumptions A1, A2 and A3.

Note that the un-normalized version<sup>2</sup> of the popular Foster, Greer and Thorbecke (1984) indices satisfy assumption A3 for  $\alpha \geq s$ . For these FGT indices we have

$$p_k^{(s)}(x) = (-1)^s \left[ \prod_{i=0}^{s-1} (\alpha - i) \right] (z_k - x)^{\alpha-s}. \quad (2)$$

We can check that  $p_k^{(s)}$  is negative when  $s$  is odd and positive when  $s$  is even. Moreover,  $|p_k^{(s)}|$  increases when  $z_k$  increases. Other well-known additive indices also satisfy assumption A3 weakly for  $s = 1$ , such as the Watts (1968) index – for which  $p_k(x) = \ln(z_k) - \ln(x)$  – and the indices of Clark, Hemming and Ulph (1981)<sup>3</sup> where  $p_k(x) = \frac{1}{c}[(z_k)^c - x^c]$ ,  $c \leq 1$ . For these latter indices, the first derivatives at a given level of income are equal for all types of households. Furthermore, the signs of the derivatives are as assumed in A3. These indices thus satisfy weakly assumption A3.

The normative interpretation of the various orders of *unidimensional* poverty dominance is discussed in some detail in Duclos, Makdissi and Wodon (2002). It is also illustrated in Figures 1 and 2, both showing 3 panels of poverty-decreasing transfers each of 1 unit of income, either simultaneously in two groups  $k$  and  $k + 1$  (Figure 1) or within a single group  $k$  (Figure 2). Note that assumption A3 for  $s = 1$  implies that an increase in *per capita* real income weakly reduces poverty, whatever the value of  $k$ . Living in small households presumably reduces, however, the opportunities for sharing resources and thus for benefitting from economies of scales in the production of welfare. At a given value of *per capita* income  $y^E$ , individuals living in

<sup>1</sup>The continuity assumption A2 together with the assumption that  $p_k(y^E) = 0$  for the non-poor excludes  $s$ -related levels of discontinuities at the poverty line, a point which is clearly discussed in Zheng (1999).

<sup>2</sup>The original formulation proposed by Foster *et al.* (1984) involves a normalization by  $z_k$  – assumption A3 would not be satisfied for these normalized indices.

<sup>3</sup>Again, the original formulation proposed by Clark *et al.* (1981) involves a normalization by  $z_k$ , which means that assumption A3 would not be satisfied for these indices.

small households can then be considered to have a lower overall level of welfare than those living in larger ones. As depicted by the first panel of Figure 1, for  $s = 1$  and at a given real income  $y^E$ , the potential for significant poverty reduction is then assumed by A3 to be greater for individuals living in smaller households. This is also a relatively straightforward application of Sen’s (1997) Weak Equity Axiom – as pointed out in Duclos and Makdissi (2001).

For  $s = 2$ , assumption A3 says that the potential for poverty reduction decreases as we move toward the poverty line. This implies that an equalizing transfer from a richer to a poorer individual of the same group will weakly decrease poverty, as depicted by the first panel of Figure 2. As displayed in the second panel of Figure 1, Assumption A3 also implies that (*ceteris paribus*) this decreasing effect is stronger for equalizing transfers across individuals who are members of smaller households. This constitutes a “weak version” of a “favorable composite transfer” (see Kolm (1976) and Kakwani (1980)). Poverty indices that are members of  $\Pi^3$  are sensitive to “favorable composite transfers” across individuals of the same group (see the second panel of Figure 2), and the more so, the smaller the size of the households to which the individuals concerned by these transfers belong (see the third panel of Figure 1). The normative content of higher orders of  $\Pi^s$  (such as  $\Pi^4$  in the third panel of Figure 2) can be interpreted using the generalized transfers of Fishburn and Willig (1984) together with a “weak version” (analogous to Sen’s Weak Equity Axiom) of a higher-order ( $s + 1$ ) normative principle.

It can be shown that moving to higher-order classes of poverty indices increases the analyst’s ability to identify poverty-reducing tax reforms, although as the above discussion will have shown, it also restricts the size of these classes. As noted above, we may also wish to use this framework to identify reforms that decrease poverty for a population of individuals that differ, for instance, in difficult-to-measure prices, needs or merits. In such cases, one would rank groups of individuals in decreasing values of price indices, merits or needs, denoting by  $k = 1$  the group facing the largest (but not precisely measurable) price level, or the group suffering from the highest needs, or the group exhibiting the greatest political or social merits<sup>4</sup>. The force of the normative principles illustrated by the panels of Figure 2 would

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<sup>4</sup>An example of the latter practice might stem from some discriminatory political behavior – one that favors more a decrease in poverty for some ethnic or regional group than for others.

then be assumed to be greatest among those with the lowest value of  $k$ .

### 3 Poverty-reducing fiscal reforms

We wish to find a commodity  $j$  whose tax rate can be increased for the benefit of a revenue-neutral fall in the tax rate of a commodity  $i$ . This fiscal reform should reduce poverty. Let  $x_{ik}(q, y)$  be the consumption of the  $i$ th commodity by an individual with *per capita* income  $y$  in a household of size  $k$  and facing prices  $q$ . Average consumption of the  $i$ th commodity is denoted by  $X_i$ :

$$X_i = \sum_{k=1}^n \theta_k \int_0^a x_{ik}(q, y) dF_k(y). \quad (3)$$

*Per capita* commodity tax revenue,  $R$ , can be expressed as

$$R = \sum_{l=1}^I t_l X_l \quad (4)$$

where  $I$  is the number of commodities in the economy. Revenue neutrality of the marginal tax reform requires that

$$dR = \left[ X_i + \sum_{l=1}^I t_l \frac{\partial X_l}{\partial q_i} \right] dq_i + \left[ X_j + \sum_{l=1}^I t_l \frac{\partial X_l}{\partial q_j} \right] dq_j = 0. \quad (5)$$

It is simple to show that equation (5) may be rewritten as

$$dq_j = -\alpha \left( \frac{X_i}{X_j} \right) dq_i, \quad (6)$$

where

$$\alpha = \frac{1 + \frac{1}{X_i} \sum_{l=1}^I t_l \frac{\partial X_l}{\partial q_i}}{1 + \frac{1}{X_j} \sum_{l=1}^I t_l \frac{\partial X_l}{\partial q_j}}. \quad (7)$$

Following Wildasin (1984), we may interpret the parameter  $\alpha$  as the differential economic efficiency cost of raising one dollar of public funds by taxing the  $j$ th *versus* the  $i$ th commodity.

The impact of the tax reform on the poverty indicator  $p_k (y^E (q, y, k; q^R))$  is

$$dp_k (y^E (q, y, k; q^R)) = p_k^{(1)} (y^E (q, y, k; q^R)) \left[ \frac{\partial y^E (q, y, k; q^R)}{\partial t_i} dt_i + \frac{\partial y^E (q, y, k; q^R)}{\partial t_j} dt_j \right] \quad (8)$$

Setting  $q^R$  to  $q$ , the change in real income induced by a marginal change in the tax rate of good  $i$  is given by (see among others Besley and Kanbur (1988))

$$\left. \frac{\partial y^E (q, y, k; q^R)}{\partial t_i} \right|_{q^R=q} = -x_{ik} (q, y). \quad (9)$$

Using (6) and (7), we can then rewrite (8) as

$$dp_k (y^E) = -p_k^{(1)} (y^E (q, y, k; q^R)) \left[ \frac{x_{ik} (q, y)}{X_i} - \alpha \frac{x_{jk} (q, y)}{X_j} \right] X_i dt_i. \quad (10)$$

We can now introduce Consumption Dominance ( $CD$ ) curves,  $CD_{ik}^s (y)$ . We start with  $s = 1$  and define

$$CD_{ik}^1 (y) = \frac{x_{ik} (q, y)}{X_i} f_k (y), \quad (11)$$

which gives the density of the consumption of good  $i$  by individuals of type  $k$  (this is because  $\sum_k \theta_k \int_0^a CD_{ik}^1 (y) dy = 1$ ). Next, we define

$$CD_{ik}^s (y) = \int_0^y CD_{ik}^{s-1} (u) du, \quad s = 2, 3, \dots \quad (12)$$

We can then show that

$$CD_{ik}^s (y) = \int_0^y (y - u)^{s-2} \frac{x_{ik} (q, u)}{X_i} f_k (u) du, \quad s = 2, 3, \dots \quad (13)$$

Note that  $\theta_k CD_{ik}^2 (y)$  gives the share of the total consumption of commodity  $i$  found among individuals whose income is less than  $y$  and who are members of group  $k$ . Since the impact of the tax reform on aggregate poverty is

$$dP = \sum_{k=1}^n \theta_k \int_0^a dp_k (y^E (q, y, k; q^R)) f_k (y) dy, \quad (14)$$



we can, using (10) and (11), rewrite (14) as

$$dP = -X_i dq_i \sum_{k=1}^n \theta_k \int_0^a p_k^{(1)}(y^E(q, y, k; q^R)) [CD_{ik}^1(y) - \alpha CD_{jk}^1(y)] dy. \quad (15)$$

(15) suggests that the difference between two  $CD$  curves can be important in determining whether a tax reform involving the two goods can be deemed good for poverty alleviation for a particular  $P$ . This result is in fact considerably more general, as the following theorem formalizes (its proof is shown in the appendix).

**Theorem 1** *A revenue-neutral marginal tax reform,  $dq_j = -\alpha \left(\frac{X_i}{X_j}\right) dq_i > 0$ , will reduce poverty for all  $P \in \Pi^s$ , for any given  $s = 1, 2, 3, \dots$ , and for all  $z_k \leq z_k^+, k = 1, \dots, n$ , if*

$$\sum_{k=1}^l \theta_k [CD_{ik}^s(y) - \alpha CD_{jk}^s(y)] \geq 0, \quad \forall y \leq z_l^+, \quad \forall l \in \{1, 2, \dots, n\}. \quad (16)$$

Note that, for a given  $s$ , Theorem 1 allows for considerable freedom in the choice of particular poverty indices to use for each subgroup  $k$ , in the selection of poverty lines to apply to each of these subgroups, in the precise ways in which household size enters into the  $p_k(\cdot)$  functions, and in the manner in which *per capita* income and household size combine to generate individual welfare. Yet, if condition (16) holds, we are assured that the tax reform will be beneficial whatever these precise choices may be, so long as they conform to the general assumptions that define  $\Pi^s$ .

An important implication of Theorem 1 is that evaluating the poverty impact of a marginal indirect tax reform involving two goods can be done quite straightforwardly even in a world of difficult-to-measure individual heterogeneity. All that is needed is disaggregated pre-reform consumption data on the two goods, the ratio of the marginal economic efficiency cost of the two goods, and a ranking of the individuals in terms of “needs”. In particular, there is no need for estimates of the parameters of individual demand functions or estimates of equivalence scales. One can also use the methodology proposed by Duclos and Makdissi (2001) to identify the social preferences underlying an observed tax reform. This method, which is analogous to the “inverse optimum problem” of Ahmad and Stern (1984, 1991), consists in estimating the preferences of a social planner that would support a given economic policy.

## 4 Conclusion

We have shown in this note how  $CD$  curves can be used to test for sequential stochastic dominance in an environment in which there are heterogeneous households as well as uncertainty on the exact equivalence scale to be used to compare the welfare of the individuals living in these households. More generally, the tools can be extended to deal with comparisons of social welfare (where we let the maximum poverty lines extend to infinity), and to cases in which there are uncertain cardinal differences in prices, needs or merits (by ordering individuals in decreasing order of price indices, needs or merits).

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## A Proof of Theorem 1

We first find a general expression for  $\int_0^a p_k^{(1)}(y) CD_{ik}^1(y) dy$ . Integrating by parts, we obtain

$$\begin{aligned} & \int_0^a p_k^{(1)}(y^E(q, y, k; q^R)) CD_{ik}^1(y) dy \\ &= p_k^{(1)}(y^E(q, y, k; q^R)) CD_{ik}^2(y) \Big|_0^a \\ & \quad - \int_0^a p_k^{(2)}(y^E(q, y, k; q^R)) CD_{ik}^2(y) dy. \end{aligned} \quad (17)$$

We know that  $CD_{ik}^2(0) = 0$  by definition and, from our assumptions,  $p_k^{(1)}(y^E(q, a, k; q^R)) = 0$ . The first term on the right hand side of the equation is then equal to 0. We can thus rewrite (17) as

$$\int_0^a p_k^{(1)}(y^E(q, y, k; q^R)) CD_{ik}^1(y) dy = - \int_0^a p_k^{(2)}(y^E(q, y, k; q^R)) CD_{ik}^2(y) dy. \quad (18)$$

Let us now assume that

$$\begin{aligned} & \int_0^a p_k^{(1)}(y^E(q, y, k; q^R)) CD_{ik}^1(y) dy \\ &= (-1)^{s-2} \int_0^a p_k^{(s-1)}(y^E(q, y, k; q^R)) CD_{ik}^{s-1}(y) dy. \end{aligned} \quad (19)$$

Integrating by parts (19), we obtain

$$\begin{aligned} & \int_0^a p_k^{(1)}(y^E(q, y, k; q^R)) CD_{ik}^1(y) dy \\ &= (-1)^{s-2} p_k^{(s-1)}(y^E(q, y, k; q^R)) CD_{ik}^s(y) \Big|_0^a \\ & \quad - (-1)^{s-2} \int_0^a p_k^{(s)}(y^E(q, y, k; q^R)) CD_{ik}^s(y) dy. \end{aligned} \quad (20)$$

We know that  $CD_{ki}^s(0) = 0$  by definition and, from our assumptions,  $p_k^{(s-1)}(y^E(q, a, k; q^R)) = 0$ . The first term on the right-hand side of (20) is then equal to 0. We can rewrite (20) as

$$\begin{aligned} & \int_0^a p_k^{(1)}(y^E(q, y, k; q^R)) CD_{ik}^1(y) dy \\ &= (-1)^{s-1} \int_0^a p_k^{(s)}(y^E(q, y, k; q^R)) CD_{ik}^s(y) dy. \end{aligned} \quad (21)$$

Equations (18) and (21) obey the relation assumed in equation (19). We have shown that if (19) is true then (21) is true. This implies that equation (21) is true for all  $s \in \{2, 3, 4, \dots\}$ . From equation (15) and equation (21), we obtain

$$dP = (-1)^s X_i dq_i \int_0^a \sum_{k=1}^n p_k^{(s)}(y^E(q, y, k; q^R)) \theta_k [CD_{ik}^s(y) - \alpha CD_{jk}^s(y)] dy. \quad (22)$$

Using Abel's Lemma<sup>5</sup> and our assumptions on  $p_k(\cdot)$ , it is easy to see that if  $\sum_{k=1}^l \theta_k [CD_{ik}^s(y) - \alpha CD_{jk}^s(y)] \geq 0 \quad \forall y \leq z_l^+$  and  $\forall l \in \{1, 2, \dots, n\}$ , then  $dP \leq 0$ .

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<sup>5</sup> Abel's lemma is proved in Jenkins and Lambert (1993):

**Abel's Lemma:** If  $x_n \geq x_{n-1} \geq \dots \geq x_2 \geq x_1 \geq 0$ , a sufficient condition for  $\sum_{i=1}^n x_i y_i \geq 0$  is  $\sum_{i=j}^n y_i \geq 0$  for each  $j$ . If  $x_n \leq x_{n-1} \leq \dots \leq x_2 \leq x_1 \leq 0$ , the same condition is sufficient for  $\sum_{i=1}^n x_i y_i \leq 0$ .

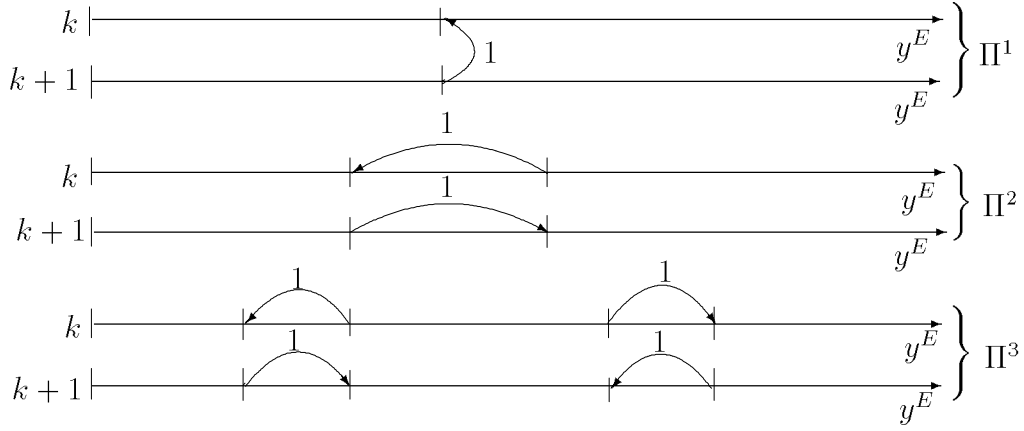


Figure 1: Two-group poverty-reducing income transfers

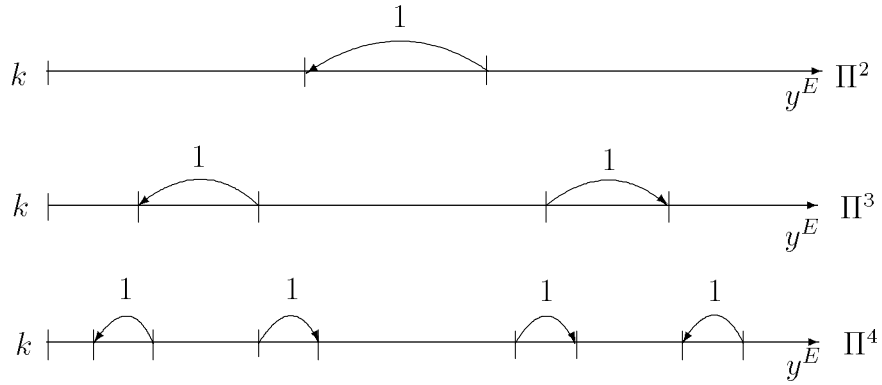


Figure 2: Single-group poverty-reducing income transfers