Firm Level Hiring Policy with Culturally Biased Testing.

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Abstract

This paper explores the implications for labor market outcomes of systematic testing of applicants in the hiring process. A matching model in which productivity is a worker's private information is used. Both wages and hiring rates are endogenous. A minority is defined as a group for whom the test is less precise in identifying individual productivity. Welfare and employment outcomes across various hiring policies are compared. Simulations suggest that tests are typically too accurate so that in a *laissez faire* economy minority group members fair better than the majority group members. Rules requiring equity in hiring reverse this result.

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1 Introduction

This paper looks at the implications of firm level hiring policy for the labor market when aptitude tests form part of the hiring decision. The analysis is used to assess the role of cultural bias in the test for the labor market outcomes of ethnic minorities.

The baseline model studied here is a variant of the search and matching approach of Pissarides [2000] with a single ethnic group. Individuals are either qualified (*i.e.* productive) or unqualified (unproductive). Qualification is private information to the worker so that firms prefer to use an imprecise test of worker ability over no test at all. Firms set a threshold value of the test score and hire those workers that achieve above the threshold. For a given proportion of qualified workers in the unemployment pool, greater test precision improves the rate at which firms identify qualified workers. This causes the average quality of the unemployment pool to fall. The eventual consequence of universal adoption of all but the least accurate tests is that, over time, everyone is made worse off.

Autor and Scarborough [2004] argue that formal testing is sufficiently widespread to justify a study of its effect. At least 30% of firms are reported to use some form of aptitude test as part of the hiring process. They study a data set obtained from a large US retail company and find a significant increase in the effectiveness of hiring since the adoption of a computerized test. They also find that fears of increased ethnic inequity stemming from formal testing are unfounded, largely because firms are already using some form of statistical discrimination. Given their results and the current ubiquity of computers, testing ought to be even more common. One source of concern that they abstract from, however, is that the tests may be less able to determine the ability of workers from certain ethnic or cultural groups. This gives rise to the fear of systematic discrimination in the hiring process (or at least fear of litigation that asserts the same).

To assess the basis for such fears, I extend the baseline model to incorporate different ethnic groups. An ethnic minority is defined as visually identifiable group for whom the employment test is less precise. As long as firms are permitted to recognize the bias in the test and set different thresholds for different ethnic groups, the minority group members are typically better off than their majority group counterparts. If fear of litigation causes the firms to impose the same threshold and wage profile on ethnic minorities as they use to hire majority group workers the welfare results are reversed. This remains true if color-blind testing is used (*i.e.* when the employment decision is made ignorant of the applicant's ethnicity).

The implication is that concerns about cultural bias in the test (as long as it is recognized) may be misplaced. A greater cause for concern is the use of testing *per se.* Consistent with the results of Autor and Scarborough, individual firms always benefit from the use of a more accurate tests than their competitors use. Also, qualified workers *initially* benefit from the universal adoption of a more accurate test. But, in the long-run the success rate of the hiring process in identifying qualified workers is necessarily equal to their inflow proportions. The greater test accuracy simply serves to slowdown the matching rate for everyone leading to higher levels of unemployment.

Of course, being able to draw such conclusions requires analysis of a specific modeling environment and in doing so the generality of the results comes into question. One assumption made here for the purpose of tractability is that a worker's duration of unemployment is private information. This assumption restricts the direct applicability of the model to people with a low attachment to the labor market, young people, homemakers etc.; the kind of market studied by Autor and Scarborough [2004]. The more usual assumption is that workers can always provide evidence of how long they have been unemployed. But this is not true of everyone in any market. Furthermore, even when unemployment spell length is known, how hard an individual has been seeking work is not. Both assumptions represent polar cases. In any event, I will argue that making the worker's duration of unemployment common knowledge at worst dilutes my results rather than reverses them.

Analysis of a model of hiring with signal extraction is provided by Phelps [1972]. Aigner and Cain [1977] extend his model to allow for the error in the signal of productivity to be more dispersed for minority workers. In their basic model, the hiring decision is completely exogenous. Workers are simply paid their expected marginal productivity. As all workers get hired, the average wage does not differ across ethnic groups (wage dispersion does). They conclude that there is no economic discrimination. In an extension, they point out that if firms required workers' expected productivity to exceed some lower bound as a requirement of employment, discriminatory outcomes could emerge. Their analysis says nothing about how the lower bound is determined nor do they discuss its effect on unemployment.

Cornell and Welch [1996] provide a market based model similar in structure to that studied by Burdett *et al* [2001] (except that wages are exogenous). Society consists of two ethnic groups. Individuals making hiring decisions have less dispersed signals as to the productivity of an individual from their own group. This approach essentially endogenizes the lower bound on expected productivity suggested by Aigner and Cain [1977] and leads to the discriminatory hiring practices that they had predicted. Cornell and Welch [1996] go on to look at persistence of discrimination in this environment. They allow the last generation's hires be the next generations recruiters and start from a position where one group has complete control over hiring. They find that selective hiring leads to less favorable distribution of productivities among the unemployed for the ascendant group which leads in the long-run to the elimination of discrimination. While the paper provides a compelling narrative of the evolution of discrimination the current paper suggests that making wages exogenous is not an innocuous assumption. The initial preparedness of the minority group to accept lower wages would lead to differential hiring thresholds and faster integration than they find.

Another paper that uses signal extraction in a market context is Sattinger [1998]. In his paper the uncertainty is over an individual's quit rate. He supposes that for unmodelled reasons minority group members might have a higher expected quit rate than the majority group members. He examines how hiring practices will differ in this situation. Firms practice statistical discrimination and hire people from the faster quitting group more slowly. He argues that this kind of discrimination is economic because firms are influenced by the group's quitting rate in making their assessment about an individual's propensity to quit. Again his paper uses exogenous wages.

The reason both Sattinger [1998] and Cornell and Welch [1996] impose exogenous wages in their models is that with private information, wage formation is hard to address. In both of their models an inter-firm wage-posting game (as in Masters [1999]) would lead to a wage of zero; a consequence of the Diamond Paradox (Diamond [1971]). Bargaining is possible and plausible in a positive sense but opens a can of worms from a modeling perspective. One model of discrimination that does have bargained wages is Mailath *et al* [2000]. In their model, however, there is no private information. Discriminatory outcomes emerge when firms coordinate their recruitment efforts toward one ethnic group over another. This is justified because this favoritism leads the favored group to acquire skills more readily than the disfavored group.

The paper presented here is the first to embed bargaining into a market context with signal extraction. I use the single round bargaining game of Mailath *et al* [2000]: Nature picks either party with equal probability to make a take-it-or-leave-it offer. Given private information on worker productivity (and duration of unemployment), there are a large number of outcomes that can be supported as equilibria. The outcome used here is the natural counterpart to the solution that emerges in the full information world: either party always asks for the whole (expected) match surplus.

In another departure from the signal extraction literature, I do not use normally distributed errors. Instead I use the approach of Coate and Loury [1993] in their model of endogenous statistical discrimination. The use of normally distributed errors has the benefit of being able to associate test accuracy with error variance. But, it is hard to reconcile this with the often reported lower average performance of minorities on standardized tests. In my model people are either qualified or not with productivities 1 and 0 respectively. The test gives a score between 0 and 1, uniformly distributed for unqualified workers, distributed with an upward sloping density for qualified workers. Test accuracy is synonymous with first-order stochastic dominance of the distribution of test scores for the qualified workers. When a test is more accurate for one group it therefore means that qualified workers in that group are more likely to be identified as such. This means that changes in the test that fit with the concept of accuracy can also change the mean of the test scores.

Evidence on the relationship between standardized tests scores and productivity is provided in Hartigan and Wigdor [1989]. They provide a *meta*-analysis of previous studies and find that the slope of the regression of productivity measure on test scores is typically steeper for nonminorities than it is for blacks. They also confirm an earlier result of Wigdor and Garner [1982] that equations estimated on pooled data tend to overestimate the productivity of black workers. The data they use is largely based on testing current workers and productivity is taken from supervisors' reports. In my model, at least in steady state, the composition of the employed workforce is identical to that of the labor market entrants. Testing current, employed workers is the same as taking a random sample from new entrants. By construction, therefore, my model reproduces Hartigan and Wigdor's first result. As productive majority group members in the model are more likely to get high test scores than productive minority group members, at high test scores the majority worker's probability model will over predict minority worker productivity. At low test scores the opposite is true. By virtue of the way probability densities necessarily integrate to one, these effects cancel out. Indeed, that my model does not reproduce Hartigan and Wigdor's second result demonstrates the sense in which my model focuses on test accuracy.[1]

The rest of the paper is structured as follows. The next section lays out the baseline model of worker testing with a single ethnic group and derives analytical results in some detail. This is because many of the results are directly relevant for the extended model of Section 3 in which ethnic diversity is introduced. Section 3 also contains an analysis of various wage policies. Section 4 provides a general discussion of the issues raised in Sections 2 and 3 and illustrates with some numerical examples. Section 5 Concludes.

[1] Of course, their use of linear regression analysis on data generated from my model would be miss-specified. The actual results could generate over or under prediction of minority productivity.

2 The Baseline (single ethnic group) Model

2.1 Environment

Time passes continuously with an infinite horizon. The economy comprises of a large number (formally a continuum) of unemployed workers and a large number of unfilled jobs. The mass of unemployed workers is normalized to 1. The mass of vacancies is v (controlled by a free-entry condition). Participants on both sides of the market have (potentially) infinite lives, are risk neutral and discount the future at rate r.

Workers are either born qualified or unqualified for employment in the homogeneous jobs. A qualified worker produces 1 unit of output per period. The output from an unqualified worker is zero. Qualification is private information to the worker. Although the distribution of unemployment durations is common knowledge, an individual worker's duration of unemployment is also his private information. This last assumption is important for simplifying the analysis. Its role in driving the results is reviewed at the end of the paper.

Workers seek employment but do not know where the firms are. I assume that unemployed workers encounter vacancies at a Poisson arrival rate α. The arrival rate of workers to vacancies is then α/v. The implied meeting function is non-standard in the search and matching literature (Petrongolo and Pissarides [2001]). This was a deliberate choice. The focus of this paper is on the role of testing for individual productivity. Any inefficiencies that emerge in equilibrium should be attributable to the role of private information and the institutions set up to overcome it - not to externalities arising from the matching function.[2] This formulation also assists with the extension of the model incorporating different ethnic groups. As both groups will be in the same labor market, random matching will imply that both groups should have the same meeting rate. Comparing outcomes for

[2] In this arrangement, firms do face a congestion externality; they do not take account of the reduction in matching rate experienced by other vacancies when creating jobs of their own. However, as the workers' matching rate is exogenous, there is no spillover effect onto workers who are the primary concern of this study.

the different groups will amount to comparative statics in this baseline single ethnic group model.

It is assumed that when a worker and firm choose to match, it is forever. Thus, matched pairs effectively leave the market. To make the environment stationary, every worker hired is replaced by a new entrant. A proportion η of new entrants are qualified. The implied proportion of the unemployment pool that is qualified, μ, is therefore endogenous.[3] Unemployed workers receive no income. On the other hand, a vacancy costs a per unit time to keep open.

Following Coate and Loury [1993], when a firm and worker meet, the firm will test (or interview) the worker which generates a noisy signal $\theta \in [0, 1]$ as to her ability. If the worker is qualified, the probability distribution function over θ is $F(\theta)$. The associated density function $f(.)$ is assumed to be continuously differentiable and increasing on $[0, 1]$ with $f(1) < \infty$. If the worker is unqualified the distribution of signals is normailized to being uniform on $[0, 1]$.[4] Given a worker's type, the signals are independent draws from the appropriate distribution. The outcome of the test is assumed to be observable to the worker and, if necessary, verifiable in a court of law. The test results remains the property of the firm and it is assumed that test results are not shared with other firms.[5]

As $f' > 0$, the distribution of signals for qualified workers first-order stochastically dominates the distribution of signals for unqualified workers (*i.e.* $F(\theta) < \theta$). The restriction on the upper endpoint of f means that even if someone gets the maximum grade on the test, there is still a positive chance that he is unqualified.

From Bayes' rule, the expected productivity of a worker given signal θ

[3] An alternative here would be the "cloning" assumption, that individuals are replaced by someone the same as themselves. In my notation, this would make μ exogenous and η endogenous.

[4] To see why this is a normalization, suppose instead that the signal generated from testing unqualified candidates was distributed G. Then we can use $G(\theta)$ as the signal and simply call the distribution of $G(\theta)$ among the qualified candidates F.

[5] It is typically illegal for firms to share information about applicants.

and model consistent prior μ, is

$$\pi(\theta, \mu) = \frac{\mu f(\theta)}{\mu f(\theta) + (1-\mu)} \tag{1}$$

This forms the basis for the wage negotiation. Straightforward differentiation shows that π is increasing in both arguments.

2.2 Bargaining

Following Mailath *et al* [2000], I assume that wages are determined by a single round of strategic bargaining. Once a worker has met a firm with a vacancy, the worker takes the test and nature chooses either party to make a take-it-or-leave-it wage offer. If the offer is accepted the contract is struck and, as described earlier, both parties leave the market. The presence of private information complicates the formal analysis (see Appendix) of this bargaining game beyond that of Mailath *et al* [2000]. The solution that emerges is still quite simple. Whoever makes the offer will get all the expected match surplus based on the continuation value of a qualified worker, V_q :

$$\frac{\pi(\theta, \mu)}{r} - V_q - V_f$$

where V_f is the continuation value of a vacancy.[6] Essentially, it will never be in the interest of a firm to offer the worker a wage less than rV_q. Wage formation therefore has the flavor of efficiency wages (as in Weiss [1980]). Firms try to impute the productivity of workers from the wage they will accept.

2.3 Search

As free-entry drives the value of V_f to zero, a qualified worker and a firm will match if the realized value of θ means that $\pi(\theta, \mu) \geq rV_q$. As π is increasing in θ and any worker and firm who meet each other take V_q and μ as given, there exists a unique (threshold) signal, θ^*, such that $\pi(\theta^*, \mu) = rV_q$. If the

[6] The continuation value for unqualified workers is V_u.

realized value of θ exceeds θ^* the worker is hired. As the wage, w, divides the effective surplus equally between the worker and the firm,

$$w(\theta) = \frac{1}{2}\left(\pi(\theta,\mu) - rV_q\right) + rV_q = \frac{1}{2}\left(\pi(\theta,\mu) + rV_q\right) = \frac{1}{2}\left(\pi(\theta,\mu) + \pi(\theta^*,\mu)\right) \tag{2}$$

Now, the standard continuous time asset value equation for V_q is

$$rV_q = \alpha\left[1 - F(\theta^*)\right]\left(\mathbb{E}^q_{\{\theta\geq\theta^*\}}\left[\frac{w(\theta)}{r}\right] - V_q\right)$$

where $\mathbb{E}^q_{\{\theta\geq\theta^*\}}$ is the expectation with respect to $F(.)$ given $\theta \geq \theta^*$. Then,

$$rV_q = \alpha\left[1 - F(\theta^*)\right]\left(\mathbb{E}^q_{\{\theta\geq\theta^*\}}\frac{\pi(\theta,\mu) - rV_q}{2r}\right)$$

Replacing rV_q by $\pi(\theta^*,\mu)$ implies

$$\pi(\theta^*,\mu) = \frac{\alpha}{2r}\int_{\theta^*}^{1}\left(\pi(\theta,\mu) - \pi(\theta^*,\mu)\right)dF(\theta)$$

The preceding analysis has assumed that unqualified people will accept any job acceptable to qualified people (i.e. $V_q \geq V_u$). Showing that this is always true is left as an exercise for the reader.

2.4 Steady-state

As $1-F(\theta^*) > 1-\theta^*$ qualified people are hired more frequently than unqualified people. The proportion of qualified people, μ, in the unemployment pool is obtained from the steady-state population flow equations. For qualified workers,

$$\delta\eta = \alpha\left[1 - F(\theta^*)\right]\mu \tag{3}$$

where δ is the (endogenous) steady-state birth rate. Equation (3) therefore equates the inflow of qualified people to the rate at which they acquire jobs. Similarly for unqualified people,

$$\delta(1-\eta) = \alpha\left[1 - \theta^*\right](1-\mu)$$

Eliminating δ also eliminates α and yields

$$\frac{\mu}{1-\mu} = \left(\frac{\eta}{1-\eta}\right)\left(\frac{1-\theta^*}{1-F(\theta^*)}\right)$$

Clearly, $\mu < \eta$; the qualification rate is lower in the steady-state market population than it is among the new entrants.

2.5 Vacancies

Free-entry (with zero set-up cost) means that the asset value of vacancies, V_f is driven to zero. The present discounted flow benefit to holding open a vacancy has to equal to the cost, a of keeping it open.

Half of the qualified workers a firm could meet will extract the whole of the expected match surplus. The expected profit from a match with a qualified worker is therefore $[1-\pi(\theta^*,\mu)]/2$ which occurs with probability $\mu[1-F(\theta^*)]$. Similarly, the expected profit from matching with an unqualified worker is $-\pi(\theta^*,\mu)/2$ which happens with probability $(1-\mu)[1-\theta^*]$. As firms meet workers at the rate α/v. We have

$$\frac{\alpha}{2rv}\left\{\mu\left[1-\pi(\theta^*,\mu)\right]\left[1-F(\theta^*)\right]-(1-\mu)\,\pi(\theta^*,\mu)\left[1-\theta^*\right]\right\}=a \tag{4}$$

Note that the contents of the curly brackets are the *ex ante* (*i.e.* pre-test) match surplus.

2.6 Equilibrium

Definition 1 *A Market Equilibrium is a list $\{\theta^*,\mu,v\}$ that satisfies:*

free-entry, $V_f=0$:

$$2rva=\alpha\left\{\mu\left[1-\pi(\theta^*,\mu)\right]\left[1-F(\theta^*)\right]-(1-\mu)\,\pi(\theta^*,\mu)\left[1-\theta^*\right]\right\} \tag{5}$$

efficient match formation:

$$2r\pi(\theta^*,\mu)=\alpha\int_{\theta^*}^{1}\left(\pi(\theta,\mu)-\pi(\theta^*,\mu)\right)dF(\theta) \tag{6}$$

steady-state condition:

$$\frac{\mu}{1-\mu}=\left(\frac{\eta}{1-\eta}\right)\left(\frac{1-\theta^*}{1-F(\theta^*)}\right) \tag{7}$$

where

$$\pi(\theta,\mu)=\frac{\mu f(\theta)}{\mu f(\theta)+(1-\mu)}.$$

The system is block recursive. It should be clear that given θ^* and μ, as long as RHS of (5) is positive (verified below), the implied value of v is unique. Equations (6) and (7) jointly determine θ^* and μ.[7] In the sequel equilibria will be referred to simply as a pair, (θ^*, μ).

Figure 1 depicts the determination of equilibrium in (θ, μ) space. The diagram is restricted to values of μ less than η. The curve labeled SS represents the schedule of values for which θ and μ are consistent with steady state. It should be clear that the curve passes through the points $(0, \eta)$ and $(1, \hat{\mu})$ where

$$\hat{\mu} = \frac{\eta}{(1-\eta)\, f(1) + \eta}$$

Moreover, the *SS curve* is downward sloping over the whole region.[8] This is because a higher test score threshold means that a higher proportion of the workers who pass are qualified. This reduces the average quality of the unemployment pool.[9]

The curve labeled *EM* represents efficient matching (equation (6)). The Appendix establishes that the implied reaction function, $\theta^*(\mu)$ is downward sloping, that $\theta^*(\eta) \geq 0$ and $\theta^*(0) < 1$. As the quality of the unemployment pool worsens, matching becomes more stringent. Existence of equilibrium is then a consequence of the intermediate value theorem.

Multiplicity of steady-states has not in general been ruled out. However, as η does not enter equation (6), monotonicity of $\theta^*(\mu)$ implies that for η small enough there must be a unique crossing. In any case, the concern here is with the dynamically stable steady-states which will have qualitatively

[7] If the level of vacancy creation were to affect the matching rate of workers (as occurs with more usual matching arrangements), this recursiveness disappears.

[8] To see this, notice that from (7)

$$\left.\frac{d\mu}{d\theta^*}\right|_{SS} = \frac{(1-\eta)\eta\left[(1-\theta^*)f(\theta^*) - (1-F(\theta^*))\right]}{\left[\eta(1-\theta^*) + (1-\eta)\,(1-F(\theta^*))\right]^2}$$

and

$$(1-\theta^*)f(\theta^*) - (1-F(\theta^*)) = \int_{\theta^*}^{1} \left[f(\theta^*) - f(\theta)\right] d\theta < 0$$

[9] A similar effect was identified by Arrow [1973] who called it the "filtering effect" of education.

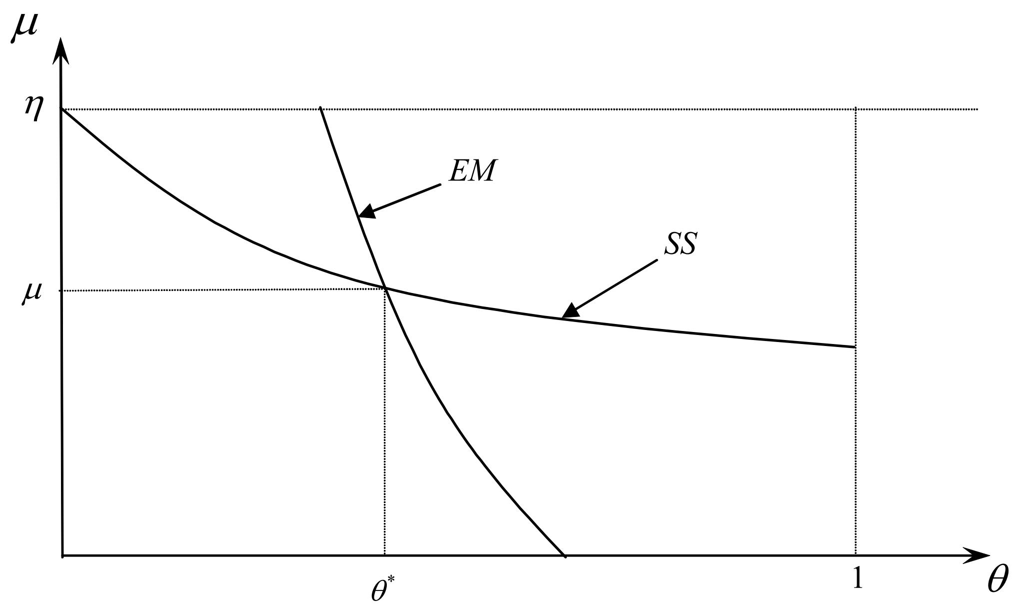

Figure 1: Steady-state equilibrium in baseline model

similar comparative statics. No example of multiple steady-state has been found.

Whenever $f(0)$ is sufficiently close to 1, an equilibrium with $\theta^* = 0$ and $\mu = \eta$ is possible. This happens whenever $\pi(0, \eta)$, is high enough to warrant match formation. To explore this possibility further we need to analyze what happens when there is no testing.

2.7 No-test world

When firms have no means of distinguishing workers, every meeting leads to a match. Let V represent the value to being a worker and w the expected wage (formed by the usual bargaining protocol). Then

$$rV = \alpha\left(\frac{w}{r} - V\right)$$

and

$$w = \frac{1}{2}(\eta + rV)$$

so

$$w = \frac{(r+\alpha)\eta}{2r+\alpha}, \quad rV = \frac{\alpha\eta}{2r+\alpha} \tag{8}$$

Any test such that $\pi(0, \eta) < rV$ will have no effect. From the definition of $\pi(.,.)$ and the solution for rV, this means that a *viable* test requires

$$f(0) < \frac{\alpha(1-\eta)}{2r + \alpha(1-\eta)} \tag{9}$$

By restricting attention to viable tests, we rule out the $(0, \eta)$ equilibrium.

2.8 Changes in F

Here we consider a change in the accuracy of the test. Attention is restricted to variations in F that satisfy the monotone likelihood property (MLP). That is, test 2 is deemed more *accurate* than test 1 if the likelihood ratio, $f_2(\theta)/f_1(\theta)$, is increasing in θ. This implies that F_2 first-order stochastically dominates F_1 which means that for any given pass threshold, the probability that a qualified worker passes test 1 is lower than the probability that a qualified worker passes test 2.

It should be clear from equation (7) that moving to a more accurate test decreases steady-state μ for every given value of θ^* – the SS curve shifts down. Essentially as the test works better, firms are more able to distinguish the qualified workers leaving less of them among the unemployed. Equation (6) implies that for any given value of μ, an increase in test accuracy makes matching more selective, θ^* increases and the EM curve shifts to the right (see Appendix). These shifts are demonstrated in Figure 2 (for a unique steady-state). The solid lines represent the EM and SS curves prior to the change in F. The dashed lines represent the EM and SS curves after the change in F. The combined effect of the increased accuracy of the test (Point 1 to Point 2) leads to a fall in μ and increase in θ^*.

In terms of the outcomes for individuals in the model, $\pi(\theta^*, \mu)$, the probability that the marginal individual is qualified, is more important than θ^* or μ taken individually. This is because $V_q = \pi(\theta^*, \mu)/r$ and the average wage, $\bar{w} = (\pi(\theta^*, \mu) + \eta)/2$. The latter equality comes from the fact that in steady-state, the proportion of productive individuals among those who get hired must equal η.

In Figure 2, point X represents the immediate impact of the universal adoption of the more accurate test (*i.e.* what happens when μ is held constant). It is shown in the Appendix that such a move necessarily makes qualified workers better off; $\pi(\theta^*, \mu)$ increases. This is because qualified workers' expected posterior probability of being shown to be qualified increases.

At Point X, however, as qualified workers would be getting jobs faster than they enter the market, μ has to fall. It is also shown in the Appendix that the implied South Easterly movements along the EM curve (from point X to point 2) make all workers worse off; $\pi(\theta^*, \mu)$ falls. This is because the implied reduction in the matching rate from the worsening prior out-weighs the benefits from increased selectivity even for the qualified workers. So, in general, the overall impact of increased accuracy has an ambiguous effect on $\pi(\theta^*, \mu)$. More simply put, more accurate testing is initially good for qualified workers but the ensuing dilution of the unemployment pool can lead in steady-state to a reduction in the rate at which they get jobs. The

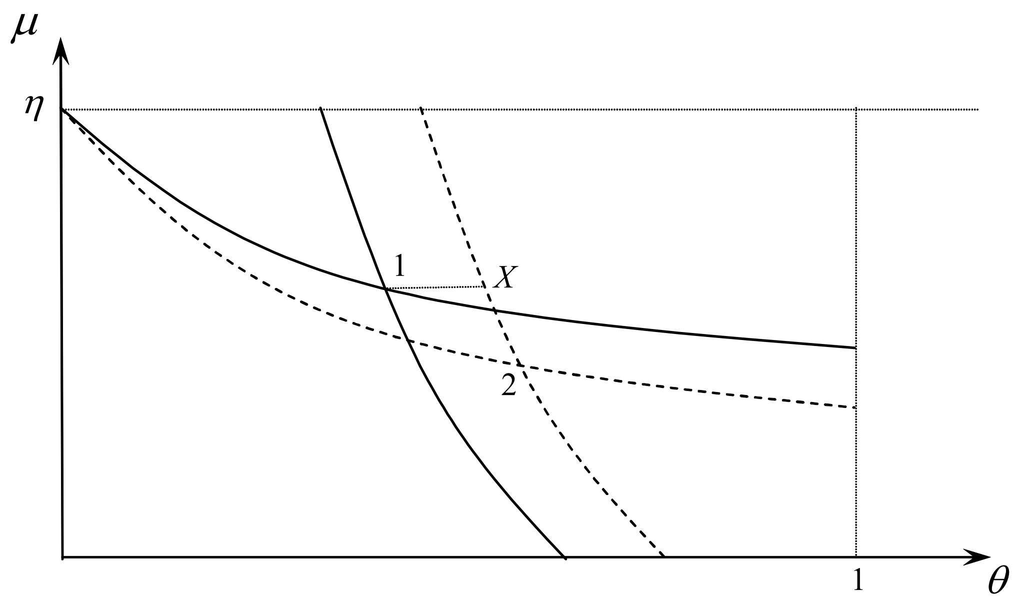

Figure 2: Changes in F

ambiguity of the effect of changes in test accuracy on $\pi(\theta^*, \mu)$ will be revisited in the simulations.

2.9 Other comparative statics

An increase in α (or a decrease in r) shifts the EM curve to the right at every value of μ. Matching becomes more selective (θ^* rises) which lowers the steady-state proportion of qualified workers (μ falls). An increase in η shifts the SS curve upwards for every value of θ^*. With more qualified workers around, matching becomes less selective which leads to a further increase in μ.

2.10 Efficiency

While the well-being of qualified workers can improve with the accuracy of tests (see simulation section for further discussion), the lot of the unqualified workers will only get worse as tests get more accurate. This is essentially because θ^* increases with accuracy.[10] Here we consider the issue as to whether there is a best test.

Risk-neutrality means that Utilitarian Welfare is simply output minus costs. When there is no testing, output is $\alpha\eta$.[11] In a world with testing, output is $\alpha(1 - F(\theta^*)\mu$. As $\eta \geq \mu$, output is always higher in the no-test world. A remaining issue is what happens to costs. The only source of cost in the model is the advertising cost, va, paid by the firms. Under free-entry, the profits of firms just cover those costs so that welfare equals the flow output that goes to workers. Whenever

$$\frac{(r+\alpha)\eta}{2r+\alpha} > \mu(1 - F(\theta^*)),$$

[10] The wage profile changes too but increased accuracy tends to work against the unqualified here as well. This is because the MLP means the expected test score, contingent on passing is higher for productive workers. The average wage $\bar{w} = (\pi(\theta^*, \mu) + \eta)/2$ only increases if the wage profile shifts sufficiently in favour of the qualified workers.

[11] More precisely, the description of the environment means that $\alpha\eta$ is the increase in output per unit time. As the individuals who are already matched have effectively left the market, this increase in output is all that matters.

total output under a test is less than that which accrues to workers in the absence of a test - testing reduces utilitarian welfare. As θ^* increases with α, for any $F(.)$ the condition will be true for α/r large enough. It is possible to construct tests that generate a level of welfare higher than from no-testing but they necessarily need to be such that μ is close to η and θ^* is close to 0.

It is also possible to show that in any steady-state equilibrium

$$\pi(\theta^*, \mu) < \frac{\alpha\eta(1 - F(\theta^*))}{2r\eta + \alpha(1 - F(\theta^*))}$$

Comparison with (8) implies that whenever $(1 - F(\theta^*)) < \eta$, everyone is worse off under testing than under the absence of testing. In the simulations that follow, it will be shown that steady-states in which everyone is worse off under testing are easy to construct even when this condition does not apply. Also, for the class of tests used, whenever testing is a bad idea, further increases in test accuracy make everyone even worse off.

Of course, this analysis ignores transitions and we know that qualified workers are better off immediately after the introduction of the new test. However, as long as α is large relative to r, the transition is relatively short and steady-state welfare strongly influences the value of introducing the new test. In any case, when we come to compare outcomes for different ethnic groups in the same market we re-interpret the difference in accuracy across tests as the difference in accuracy for the same test across different groups. There, transitions will not be an issue.

2.11 Firm adoption of tests

To reduce the complexity of the preceding analysis we have so far imposed the test on the environment. Yet, a positive analysis should ask whether the adoption of any viable test is in a firm's private interest. There are 2 questions here. First, will firms adopt any viable test over no-test? Second, given some test is in common usage would a firm necessarily prefer to use a more accurate test than the other firms?[12]

[12] It may appear that the first question is a special case of the second as no-test might be viewed as a very low accuracy test. But this need not be true, for instance the class of

To answer the first question recall that, by definition, if a firm applies a viable test when no other firm is using any test, there exists some strictly positive threshold, θ_t, of the test score below which matches will not form. Also, firms receive half of the expected pre-test match surplus which, when no other firm is testing, can be written as

$$S_t \equiv \eta\,[1-\pi(\theta_t,\eta)]\,[1-F(\theta_t)] - (1-\eta)\,\pi(\theta_t,\eta)\,[1-\theta_t]$$

where θ_t solves $\pi(\theta_t,\eta) = rV$ (as defined in (8)). As $f(.)$ is upward sloping, $\theta_t f(\theta_t) > F(\theta_t)$. Substitution shows that $S_t > \eta - rV$, the match surplus without the test. This sheds light on the efficiency of testing considered above. Compared with the no-test world, a firm will use any viable test because it increases the expected match surplus. However, in the long run, as other firms adopt the test, they can only be better off than before the test was first introduced if the adjustment to steady-state has a small effect on the rate at which they hire qualified workers. This will happen only if the test is sufficiently imprecise.

To see why firms always prefer more accurate tests consider what happens if a firm unilaterally adopts test F_n such that $F_n < F$ for all θ. The implied expected surplus under the same threshold, θ^*, would be

$$\begin{aligned} S_n &\equiv \mu\,[1-rV_q]\,[1-F_n(\theta^*)] - (1-\mu)\,rV_q\,[1-\theta^*] \\ &> \mu\,[1-rV_q]\,[1-F(\theta^*)] - (1-\mu)\,rV_q\,[1-\theta^*] \end{aligned}$$

Of course, imposing a threshold equal to θ^* would either cause this firm to form some matches with negative expected (*post-test*) surplus or prevent some matches with positive (*post-test*) surplus from forming. The pair-wise efficient value of the threshold, θ_n, is such that under the new test, $\pi_n(\theta_n,\mu) = rV_q$.[13] Using θ_n can only further increase the expected (*pre-test*) surplus.

densities used to characterize the test in the simulations is θ^n, $n > 0$ on $[0,1]$. These do not uniformly converge to the uniform density.

[13] Where $\pi_n(\theta,\mu) \equiv \frac{\mu f_n(\theta)}{\mu f_n(\theta)+1-\mu}$.

As firms do not take account of the impact of their adoption choice on other market participants, firms will always adopt the most accurate test even when it is not in their collective long-term interest to do so.

3 Ethnic Minorities

Here we extend the model to incorporate individuals in the economy for whom the hiring process works less well. These individuals will be designated (ethnic) minorities. Specifically, it will be assumed that minorities are identifiable by appearance and the employment test is less accurate for them. This is meant to reflect the extent of cultural bias in the test. If the test is designed to best identify productive individuals in the majority group, questions will be written in a way that individuals raised in a different culture may find ambiguous

Notationally, the subscript A will be used for workers who are members of the majority group and subscript I will be used for the minority workers. The proportion of minority individuals among the inflow to the labor market is ϕ. The endogenous steady-state proportion of the pool of unemployed workers that are minority group members is ψ. As above, the proportion of qualified workers in each group is η. Following from the preceding analysis, for both groups, the distribution of the test scores for unqualified workers is assumed to be uniform.[14] The difference in F_j, $j = A, I$, the distribution of scores for qualified workers, is such that the test is more accurate (in the sense of MLP) for the majority group members.

We look at how differential test-score performance interacts with different policy positions with respect to the treatment of minorities in the hiring process.

Different treatment is *laissez faire* in the extended model: firms and workers are free to arrive at their pair-wise efficient hiring threshold and wage profile.

[14] This restriction is not made without loss of generality. It essentially states that the test is equally uninformative about the productivity of unqualified workers of either ethnic group.

Equal treatment dictates that minorities should be hired as if they were majority group members. That is, the pair-wise efficient test threshold and wage profiles that emerge from the hiring of majority group members are imposed in the hiring decision with respect to minorities.

Color blind hiring means that the matching and wage offer decisions have to be made before the firm knows the ethnicity of the worker.

Random matching means that in this extended environment the rate at which workers of both ethnic groups meet firms will be the same. Even if I had assumed a more standard meeting technology, α would be the same for both groups. This section will also assume that α is exogenous.This helps to focus the analysis on equity across ethnic groups under each policy regime. This approach, however, does abstract from how the effect of the different policy regimes on vacancy creation might impact the meeting rate of workers and firms.

3.1 Different treatment (DT)

Here, comparison of outcomes across ethnic groups within the same market therefore amounts to comparative statics with respect to $F(.)$ in the baseline model. An equilibrium of the extended model will be a list $\{\theta_A^*, \mu_A, \theta_I^*, \mu_I, \psi, v\}$ such that the market for labor is in steady-state, and subject to efficient matching and free-entry of vacancies.

The steady-state equations are therefore:

$$
\begin{aligned}
\delta\eta(1-\phi) &= \alpha(1-\psi)[1-F_A(\theta_A^*)]\mu_A \\
\delta(1-\eta)(1-\phi) &= \alpha(1-\psi)[1-\theta_A^*](1-\mu_A) \\
\delta\eta\phi &= \alpha\psi[1-F_I(\theta_I^*)]\mu_I \\
\delta(1-\eta)\phi &= \alpha\psi[1-\theta_I^*](1-\mu_I)
\end{aligned}
$$

which means that

$$
\frac{\mu_J}{1-\mu_J} = \left(\frac{\eta}{1-\eta}\right)\left(\frac{1-\theta_J^*}{1-F_J(\theta_J^*)}\right) \text{ for } J = A, I.
$$

and ψ is obtained from

$$\left(\frac{\psi}{1-\psi}\right) = \left(\frac{\phi}{1-\phi}\right)\left(\frac{1-\theta_A^*}{1-\theta_I^*}\right)\left(\frac{1-\mu_A}{1-\mu_I}\right) \tag{10}$$

As general uniqueness has not been shown there is an implicit assumption that both ethnic groups are in the same equilibrium. While it may not be obvious from (10), $\psi < \phi$. This is because steady-state requires that people get jobs in the same proportions at which they enter the market. As $\theta_A^* > \theta_I^*$ unqualified majority workers get jobs more slowly than do unqualified minority workers. As discussed above this means the matching rates for qualified workers also shows a similar pattern and consequently in steady-state there will be relatively more majority workers in the unemployment pool.

As α is exogenous, efficient matching means

$$2r\pi_J(\theta_J^*, \mu_J) = \alpha \int_{\theta_J^*}^{1} \left(\pi_J(\theta, \mu_J) - \pi_j(\theta_J^*, \mu_J)\right) dF(\theta) \text{ for } J = A, I \tag{11}$$

where

$$\pi_J(\theta, \mu) = \frac{\mu f_J(\theta)}{\mu f_J(\theta) + (1-\mu)}.$$

Further analysis and discussion is left to the simulations section.

3.2 Equal treatment (*ET*)

The majority threshold for matching and the wage as a function of the test score is imposed on the minority.[15] A tilde ($\tilde{}$) over variable symbols is used to represent those variables that will change under this policy prescription.

The steady-state equations are therefore:

[15] Equilibrium here is entirely specified by what happens in the market for majority workers. Indeed, it might appear that knowing they have to hire minority workers at the same rate and with the same wages as majority workers, firms would have a different optimal hiring strategy than that implied by the single ethnic group model. This not true. Firms will produce less vacancies but, because the matching rate by workers in unaffected by the presence of different ethnic groups, efficient matching for majority group members is unaffected by this hiring policy.

$$\begin{aligned}
\delta\eta(1-\phi) &= \alpha(1-\tilde{\psi})[1-F_A(\theta_A^*)]\mu_A \\
\delta(1-\eta)(1-\phi) &= \alpha(1-\tilde{\psi})[1-\theta_A^*](1-\mu_A) \\
\delta\eta\phi &= \alpha\psi[1-F_I(\theta_A^*)]\tilde{\mu}_I \\
\delta(1-\eta)\phi &= \alpha\psi[1-\theta_A^*](1-\tilde{\mu}_I)
\end{aligned}$$

which means that

$$\frac{\mu_J}{1-\mu_J} = \left(\frac{\eta}{1-\eta}\right)\left(\frac{1-\theta_A^*}{1-F_j(\theta_A^*)}\right) \text{ for } J = A, I.$$

and $\tilde{\psi}$ is obtained from

$$\left(\frac{\tilde{\psi}}{1-\tilde{\psi}}\right) = \left(\frac{\phi}{1-\phi}\right)\left(\frac{1-\mu_A}{1-\tilde{\mu}_I}\right)$$

As $F_A(\theta) < F_I(\theta)$ for all θ, $\mu_A < \tilde{\mu}_I$ and so $\tilde{\psi} > \phi$. Equal treatment necessarily leads to more unemployment among the minority group.

3.3 Color-blind hiring (CB)

The Boston Philharmonic Orchestra, now famously, conducts auditions behind a screen to avoid the candidate's appearance influencing the result. In the context of this paper, this will amount to the ethnicity of the worker being private information. Hiring will necessarily be consistent with equal treatment. The difference from the previous arrangement is that absent information as to the ethnic group of the worker, the prior reflects the proportion of productive individuals in the whole market rather than simply that of the majority group. Also, the posterior probabilities of being productive will be calculated based on the appropriately combined distribution of test scores.

For what follows we will use the carat (^) to signify variables that have the same meaning as above but are potentially different under color-blind hiring. The analysis follows that of the original model so that given a test score of θ the posterior probability that the worker is productive is

$$\hat{\pi}(\theta, \hat{\psi}, \hat{\mu}_A, \hat{\mu}_A) = \frac{(1-\hat{\psi})\hat{\mu}_A f_A(\theta) + \hat{\psi}\hat{\mu}_I f_I(\theta)}{(1-\hat{\psi})\hat{\mu}_A f_A(\theta) + (1-\hat{\psi})(1-\hat{\mu}_A) + \hat{\psi}\hat{\mu}_I f_I(\theta) + \hat{\psi}(1-\hat{\mu}_I)}$$

The analysis of the extended bargaining problem is in the Appendix. The implied wage is

$$w(\theta) = \tfrac{1}{2}[\hat{\pi}(\theta, \hat{\psi}, \hat{\mu}_A, \hat{\mu}_A) - r\hat{V}_f + r\hat{V}_q^A]$$

for $\theta \geq \hat{\theta}_A^*$ where $\hat{\theta}_A^*$ solves

$$\hat{\pi}(\hat{\theta}_A^*, \hat{\psi}, \hat{\mu}_A, \hat{\mu}_A) - r\hat{V}_f = r\hat{V}_q^A$$

For values of θ below $\hat{\theta}_A^*$ there is no match. As is pointed out in the Appendix, it is possible for qualified minorities to accept jobs when $\theta < \hat{\theta}_A^*$. In the simulations that follow we always check that they would prefer not to match in that situation.

Let $\hat{\pi}^* \equiv \hat{\pi}(\hat{\theta}_A^*, \hat{\psi}, \hat{\mu}_A, \hat{\mu}_A)$. Recognizing that vacancy creation leads to $\hat{V}_f = 0$ implies that in equilibrium

$$w(\theta) = \tfrac{1}{2}[\hat{\pi} + \hat{\pi}^*]$$

So that a steady-state color-blind equilibrium is a tuple $\{\hat{\theta}_A^*, \hat{\psi}, \hat{\mu}_A, \hat{\mu}_A\}$ such that

$$\begin{aligned} \hat{\pi}^* &= \frac{\alpha}{2r}\int_{\hat{\theta}_A^*}^{1} (\hat{\pi} - \hat{\pi}^*)\, dF_A && (12) \\ \frac{\hat{\mu}_J}{1-\hat{\mu}_J} &= \left(\frac{\eta}{1-\eta}\right)\left(\frac{1-\hat{\theta}_A^*}{1-F_J(\hat{\theta}_A^*)}\right) \text{ for } J = A, I. \\ \left(\frac{\hat{\psi}}{1-\hat{\psi}}\right) &= \left(\frac{\phi}{1-\phi}\right)\left(\frac{1-\hat{\mu}_A}{1-\hat{\mu}_I}\right) \end{aligned}$$

notice that although inference is made with respect to the whole population, ultimately it is the matches with the majority workers that determines the threshold test score for a match to form.[16]

Given the complexity of these conditions no general proof of existence has been found. In the numerical examples below, existence of equilibrium has been established for a wide range of parameter values. No case of multiple equilibrium has been found. Analysis of this model is deferred until the simulations.

[16] There is also an equation which controls the formation of vacancies. As we saw above, exogeneity of α means that the magnitude v does not affect the equilibrium variables of interest.

r	Discount rate	0.04
α	Meeting rate	0.2
η	Proportion of productive workers	0.8
ϕ	Proportion of minority workers	0.2
n_A	Distribution function index (majority)	2
n_I	Distribution function index (minority)	1.5

Table 1: Parameters for first example

4 Simulations and Discussion

4.1 Example 1

For the numerical analysis, the distribution of test scores among qualified workers is restricted to be of the form

$$F(\theta;n) = \begin{cases} 0 \text{ for } \theta < 0 \\ \theta^{n+1} \text{ for } \theta \in [0,1] \\ 1 \text{ for } \theta > 0 \end{cases}$$

This generates a class of functions parameterized by $n > 0$. As for any $n > 0$, the density evaluated at a test score of zero, $f(0;n) = 0$, every test in this class is viable (see (9)). Also for any two values, $n_1 > n_2$ means that test n_1 is a more accurate than test n_2 (*i.e.* the MLP condition is satisfied).

The parameters for the first example are provided in Table 1.[17] The appropriateness of the parameter values will be discussed in more detail with the presentation of the results from the Leading Example. For now, to add concreteness, it may be helpful to think about the time unit as one year (α has been chosen purposefully low). Also the chosen distribution function parameters mean that the expected test score of a qualified majority worker is 75% while the expected test score of a qualified minority worker is 71.4%. (As the test score for unqualified workers from both groups is uniformly distributed, their expected result is 50%.)

[17] In the interest of space, the sensitivity analysis will not be reported here. The Matlab™ code is available from the author (it requires Matlab 6.1 or higher and the optimization toolbox).

-	μ	θ^*	V_u	V_q	ψ	$\bar{w}$
Majority ET/DT	0.7162	0.4139	12.432	14.115	-	0.6823
Minority DT	0.7428	0.3200	12.929	14.161	0.1921	0.6832
Minority ET	0.7249	0.4139	12.432	13.920	0.2050	0.6790
Majority CB	0.7200	0.3973	12.549	14.176	-	0.6841
Minority CB	0.7280	0.3973	12.549	13.990	0.2047	0.6808
No test	0.8000	-	14.286	14.286	0.2000	0.6857
Best test, $n = 0.2$	0.7998	0.0017	14.243	14.304	0.2000	0.6861

Table 2: Table Caption

Table 2 summarizes the results.[18] The definitions of the column headings are consistent with their use in the preceding analysis. Each row represents the outcomes for the specified group under the specified scenario. Given the particular test accuracies, there is no interaction between the outcomes for different ethnic groups in the *laissez faire* economies. Consequently, the rows marked 'Majority ET/DT', 'Minority DT', 'No test' and 'Best test' simply represent the outcomes from the implied testing regime in the basic model.

The best test is the value of n that achieves the highest value of $\mu V_q + (1-\mu)V_u$. This occurs for $n = 0.2$. Under this test, qualified workers are expected to score 54.5%. For n larger than 0.3, utilitarian welfare is lower using the test than without it. For n larger than 0.6, even qualified workers would be better off without the test. The results demonstrate that when testing is more selective than this, further increases in accuracy make both the qualified and unqualified workers worse off.[19] Viewed as the same test applied with varying efficacy to different ethnic groups, the minority group is typically be better off than the majority group. The result does not depend

[18] While multiple steady-states have not been ruled out, plotting the SS and EM curves over the whole feasible region reveals a distinct unique crossing point. In this way, only unique equilibria have been observed even for extreme parameter values.

[19] While this has not been proven in general, $\pi(\theta^*, \mu)$ decreases monotonically with accuracy for $n > 0.3$. This pattern is robust to a wide range of parameter values. That is for each arrangement studied, $\pi(\theta^*, \mu)$ falls with n once n exceeds some critical value.

Test score %	Majority ET/DT (Minority ET)	Minority DT	Color-Blind Testing
20	-	-	-
40	-	0.6063	0.5686
60	0.6481	0.6684	0.6535
80	0.6968	0.7021	0.6989
100	0.7240	0.7224	0.7248

Table 3: Equilibrium Wage Profiles

on the size of the minority group. Instead the intuition follows straight from that of the basic model. When presented with workers from each group, the firms realize that the person for whom the test is more accurate is less likely to be qualified. This also leads to more unemployment among the majority group as evidenced by $\psi < 0.2$.

Imposing equal treatment leaves the outcome for the majority workers unaffected. The outcome for the unqualified minority workers is exactly the same as for their majority group counterparts. The minority qualified workers however are left much worse off for two reasons. Firstly, they are less likely to get hired, $1 - F_I(\theta_A^*) < 1 - F_A(\theta_A^*)$. This means that unemployment is also higher among the minority group. Second, while they get paid the same wage at each test score, their expected test score contingent on being hired is lower.

Minorities fair better under color-blind testing than under equal treatment. This is because the efficient matching threshold and the wage profile take account of their impact on the set of workers any firm meets. Still they are bound to do worse than their majority group counterparts (and experience higher levels of unemployment) for exactly the same reasons as before. What is notable about the results is that the majority group members do better under color-blind testing than they do in the *laissez faire* economy. This is because being lumped in with the minority group raises the firms' prior on the probability that a majority group worker is qualified.

Table 3 summarizes the implied wage profiles. As $V_q^I > V_q^A$ we know that on average the minorities get paid higher wages under different treatment. However, the wage profiles reveal that some majority group workers can receive more than any minority worker. This does not happen with color-blind testing as every one faces the same wage profile. As with equal-treatment, under color-blind testing, minorities get lower average wages because the cultural bias means that their test score distribution is skewed more to the left.

4.2 Opportunity cost of vacancies

So far the theory has followed the approach typical to the literature in assuming that vacancies have no establishment cost. Free-entry drives the *ex ante* expected value of a vacancy to zero. The *ex post* profit of firms is therefore just sufficient to cover the flow advertising cost. The major benefit of this approach is that vacancies have zero opportunity cost which typically simplifies the bargaining solution.

While the addition of opportunity costs has no qualitative impact, it is quantitatively important.[20] To see this, consider what happens if firms have to pay a one-time set-up cost, k, for each vacancy they create in addition to the flow advertising cost a. Unrestricted vacancy creation will now drive the value of vacancies down to k. That is $V_f = k$. Analysis of the bargaining game is unchanged as it was carried out for a generic value of V_f (see Appendix). Equation (2) becomes

$$w(\theta) = \frac{1}{2}\left[\pi(\theta,\mu) + \pi(\theta^*,\mu) - 2rk\right]$$

Efficient matching means that for positive k, fewer matches will form but the decision to match is still mutual. The matched pair simply have to cover the set-up cost and then divide up any remaining surplus. The quantitative importance comes from the implied value of capital share of income. With

[20] It seems reasonable to think that testing is more likely to be socially beneficial when there is an opportunity cost to hiring. Indeed, matching does become more selective. In steady state, however, a proportion $1-\eta$ of those who do get jobs are inevitably unproductive.

r	Discount rate	0.04
α	Meeting rate	60
η	Proportion of productive workers	0.8
ϕ	Proportion of minority workers	0.2
n_A	Distribution function index (majority)	2
n_I	Distribution function index (minority)	1.5
k	Vacancy creation cost	7

Table 4: Parameters for Leading Example

$V_f = 0$ firms get a very small share of output in markets with realistic meeting rates for workers.

For the purpose of the leading example, I have incorporated a set-up cost. As became clear earlier, only the Steady-State and Efficient Matching conditions are important for deriving the variables of interest. Only the Efficient Matching conditions are affected by the set-up cost. In particular equation (11) becomes

$$2r\left[\pi_J(\theta_J^*, \mu_J) - rk\right] = \alpha \int_{\theta_J^*}^{1} \left(\pi_J(\theta, \mu_J) - \pi_j(\theta_J^*, \mu_J)\right) dF(\theta) \text{ for } J = A, I$$

and equation (12) becomes

$$2r\left[\hat{\pi}^* - rk\right] = \alpha \int_{\hat{\theta}_A^*}^{1} \left(\hat{\pi} - \hat{\pi}^*\right) dF_A$$

4.3 Leading Example

The parameters, based on a time unit of one year, are reported in Table 4. The discount rate is standard. The choice of η and ϕ are arbitrary but seem reasonable. The results do not depend much on them. Given r, η and ϕ, the choices of n_A, n_I, k and α were made to fit with the following outcomes: 13 weeks average duration of unemployment, workers share of income of 2/3, acceptance rate of workers by firms of 1/11 (as reported by Autor and Scarborough [2004]). The most significant change from Example 1 is that the meeting rate, α, is 300 times higher. The new rate corresponds to

-	μ	θ^*	V_u	V_q	ψ	$\bar{w}$
Majority ET/DT	0.5807	0.9624	12.696	12.843	-	0.5169
Minority DT	0.6239	0.9526	12.745	12.851	0.1811	0.5170
Minority ET	0.6221	0.9624	12.696	12.828	0.2172	0.5168
Majority CB	0.5809	0.9614	12.703	12.846	-	0.5169
Minority CB	0.6223	0.9614	12.703	12.832	0.2171	0.5169
No test[21]	0.8000	-	12.983	12.983	0.2000	0.5197

Table 5: Results for Leading Example

workers making 5 applications per month. This makes matching much more selective. (The introduction of the opportunity cost of vacancies mainly affects the wage.)

The results for the leading example are reported in Table 5. Qualitatively, the results are similar to those of Example 1. However, some of the features highlighted there are less prominent and some are more so. Under color-blind testing and equal treatment the unemployment rate for minorities is 13% higher than for the majority group workers. Under equal-treatment it is 11% lower.[22] A feature of Example 1 that almost disappears is wage dispersion. As matching is very selective, there is very little variation in the expected productivities of those hired. As such, there is very little variation in the wages they receive.[23]

4.4 Alternative model formulations

Making the meeting rate, α, exogenous was a deliberate simplifying assumption. In terms of the basic model, with a more usual constant-returns to scale matching function (*a la* Pissarides [2000]) changes in the test accuracy will affect the meeting rate through the effect on vacancy creation. We know

[22] These differences become more pronounced as the size of the minority approaches 50%.

[23] This does not mean that I might as well have made the wage exogenous (or even imposed the same endogenous wages on both groups). While the equilibrium appears to exhibit the law-of-one-price, the test thresholds and therefore the aggregate outcomes are very sensitive to the realized wage profiles.

that for any α, firms will adopt more accurate tests over less accurate tests. When the original test is such that increased accuracy leads to a lower match surplus with constant α, firms would create less vacancies and α would fall. Endogenizing α would not reverse the welfare implications of testing.[24]

When the model is used to compare predicted outcomes under cultural bias in the test, random matching implies that α should be the same for each group. Still, when we look at a different hiring policies, one should expect α to adjust. However, the focus of the paper is on the different outcomes faced by the ethnic groups within any regime. Also, with realistic elasticities of meeting with respect to vacancies, changing the hiring policy would not alter α much.

Perhaps the strongest simplifying assumption is that workers cannot credibly reveal how long they have been unemployed. If durations of unemployment were observable, workers would have types indexed by their current spell-length. The prior probability that any worker is qualified would then be a function of spell-length. As qualified workers would have typically shorter durations of unemployment, the results described above would certainly be diluted. However, for a given meeting rate, α, at least under the class of densities used for the simulations, we know that increased accuracy slows down matching. For sufficiently accurate tests (which firms would readily adopt) further increases in test accuracy would make everyone worse off.

Other informational assumptions are potentially interesting. For instance, the test score could be private information to the firm. In this case, however, wage formation would be difficult to model. We know from the work on the Coase conjecture that (see Ausubel and Deneckere [1989]) stationary bargaining equilibria lead to the firm getting all the surplus. That would be equivalent to posting wages. In the absence of any match-specific shock to the workers preferences (as in Masters [1999]) the Diamond [1971] paradox would apply and the wage would be driven to zero. Another possibility is that workers do not know their own productivities. Again the

[24] This is simple to show numerically. However, in the interest of space, the results are not reported here.

objections to this variant are technical rather than economic. The difficulty here would be that each test the worker took would reveal something about his true productivity. This would lead to an unmanageable degree of heterogeneity.

A major component of the contribution of this work has been to allow the hiring decisions to influence the population of unemployed workers and *vice-versa.* Had I chosen to use the 'Cloning' assumption whereby workers are replaced by their own type, μ would have been exogenous.

A further approach would have been to have the true productivity of the worker revealed after some period of employment (most tractably according to a Poisson process). Combining this with an exogenous birth/death process would generate an endogenous inflow rate to unemployment. In that world, the steady-state proportion of unqualified workers in the unemployment pool would be higher than occurs in my model. Still, in steady-state, workers have to be hired in the same proportions that they enter the unemployment pool. More accurate testing would still lead to a worse prior on an individual's productivity as occurs here.

A large literature on discrimination (*e.g.* Coate and Loury [1993]) is concerned with understanding discrimination as an equilibrium phenomenon. In that approach, qualification occurs as the consequence of an unobserved investment. In the current model I have taken qualification as exogenous. An extension of my framework could allow for investment. To this end it is of interest to note that even when more accurate tests lead to lower welfare, they increase the incentive to become qualified. In the examples above, $V_q - V_u$ always increases with test accuracy.

5 Conclusion

This paper provides a model of the labor market in which only the workers know their true productivity. Firms adopt any testing technology that helps to distinguish qualified from unqualified workers. At low levels of accuracy, testing can help to overcome the informational problems. Even more accurate tests are initially beneficial. However, when all firms adopt the test, everyone can end up worse off. Greater test accuracy leads to a dilution of the quality of the workers in the unemployment pool which means that in steady-state, even the qualified workers can spend longer unemployed and have lower lifetime utility than when a less accurate test is used.

It was shown that the environment is easily extended to address the possible impact of systematic hiring tests on inequity across racial groups. An ethnic minority is defined to be a group for whom a given test is less accurate. For more selective (and I have argued more realistic) tests, the outcomes depend on the wage policies imposed on the firms. When firms are allowed to recognize a worker's background, test thresholds used to hire individuals will differ across ethnicities. In this case, the minority group members experience lower unemployment and higher average wages than the majority group members. When "equity" is imposed on the hiring decision, either through equal treatment or color-blind hiring, the cultural bias in the test causes inequity in outcomes precisely in the direction that the wage policies were intended to prevent.

The recommendation that emerges is that firms should be allowed to recognize the difference between individuals and adapt their hiring accordingly. Of course, in practice this is exactly what people call affirmative action. It is an apparent lowering of recruitment standards directed at minority groups. In fact it is simply a recognition that the tests may not work the same way for people with differing backgrounds. An alternative route is to adapt the tests to ensure a similar degree of accuracy across ethnic groups, my results suggest that simply recognizing differences across groups negates this requirement.

The results are complementary with those of Autor and Scarborough [2004] with respect to equity. Both papers suggest that fears of systematic discrimination against minorities arising from testing my not be well founded.[25] An individual employer, as they identify, may be better off from adopting a more accurate test. However, the implied reduction in matching

[25] Recall that they conclude that the adoption of systematic hiring tests tend not to exacerbate ethnic inequity. This is because, in the absence of a test, firms would already use statistical discrimination.

rates can eventually lead all workers to be worse off in terms of unemployment and welfare.

6 Appendix

6.1 Analysis of Bargaining game.

We analyze the game in two parts corresponding to the choice by nature as to who makes the wage offer (after the realization of the test score, θ and hence the posterior probability $\pi = \pi(\theta, \mu)$ that the worker is of type q). A worker's duration of unemployment should also be relevant here. For instance, knowing a person to be a new entrant would imply a prior of η rather than μ. We seek an equilibrium which is stationary with respect to the workers' unemployment spell length. To do this we impose stationarity of the allocation and argue at the end that the implied equilibrium is an equilibrium of the true game. We assume that if either party is indifferent between accepting an offer and rejecting it, they accept. This assumption is common in bargaining theory. Here, it rules out some kinds of mixed strategy equilibria.

6.1.1 Firm makes offer (screening model)

Firm action: wage offer $w_f \in [0, 1]$

Worker action: picks probability of acceptance[26], $a_i \in [0, 1]$, $i = q, u$

Let V_i $i = q, u, f$ be the disagreement value value to qualified workers, unqualified workers and firms respectively. Then,

Firm pay-offs: $\begin{cases} a_q(1 - w_f) + (1 - a_q)rV_f \text{ if worker type } q \\ -a_u w_f + (1 - a_u)rV_f \text{ if worker type } u \end{cases}$

Type i worker pay-offs: $a_i w_f + (1 - a_i)rV_i$, $i = q, u$

Firm strategies: $w_f(\pi) : [0, 1] \to [0, 1]$

Type i worker strategies: $a_i(\pi, w) : [0, 1] \times [0, 1] \to [0, 1]$

[26] Although we assume acceptance in the case of indifference, using the continuous selection eases the notation.

A sub-game perfect equilibrium is a triple $\{w_f^*, a_q^*, a_u^*\}$ such that

$$\begin{aligned} a_i^* &= \arg\max_{a\in[0,1]} \left\{ a w_f^* + (1-a) r V_i \right\} \\ w_f^* &= \arg\max_{w\in[0,1]} \left\{ \pi a_q^*(1-w) - (1-\pi) a_u^* w + \left[\pi(1-a_q^*) + (1-\pi)(1-a_u^*) \right] r V_f \right\} \end{aligned}$$

Equilibria have the form

$$w_f^*(\pi) = \begin{cases} rV_q \text{ for } \pi \geq r(V_f + V_q) \\ \omega < rV_u \text{ otherwise} \end{cases}$$

$$a_i^* = \begin{cases} 1 \text{ if } w \geq rV_i \\ 0 \text{ otherwise} \end{cases} \text{ for } i = q, u$$

There is a continuum of equilibria; one for every $\omega \in [0, rV_u)$ but they are all pay-off equivalent.

6.1.2 Worker makes offer (signalling model)

Type i worker action: makes wage offer $w_i \in [0,1]$

Firm action: picks probability of acceptance, $a_f \in [0,1]$,

Then,

Firm pay-offs: $\begin{cases} a_f(1-w_q) + (1-a_f)rV_f \text{ if worker type } q \\ -a_f w_u + (1-a_f) r V_f \text{ if worker type } u \end{cases}$

Type i worker pay-offs: $a_f w_i + (1-a_f) r V_i$, $i = q, u$

Type i worker strategies: $w_i(\pi) : [0,1] \to [0,1]$

Firm strategies: $a_f(\pi, w) : [0,1] \times [0,1] \to [0,1]$ where w is wage offered by worker

In formulating their strategies firms will up-date their beliefs as to the productivity of the worker based on the wage the worker offers.

A Bayesian perfect equilibrium is a triple $\{w_q^*, w_u^*, a_f^*\}$ such that

$$\begin{aligned} a_f^* &= \arg\max_{a\in[0,1]} \{ a [\tilde{\pi}(1-w) - (1-\tilde{\pi})w] + (1-a) r V_f \} \\ w_i^* &= \arg\max_{w\in[0,1]} \left\{ a_f^* w + (1-a_f^*) r V_i \right\} \end{aligned}$$

where $\tilde{\pi}$ are the updated beliefs of the firm based on the wage offered by the worker. As unqualified workers can freely emulate their qualified counterparts, there can be no separating equilibria, $w^* \equiv w_q^* = w_u^*$.[27]

As perfect Bayesian equilibrium does not restrict out-of-equilibrium beliefs, whenever $\pi > r(V_f + V_q)$ there are a very large number of equilibria. For instance, there is one for every $w^* \in [r(V_f + V_q), \pi]$ supported by the belief of the firm that wage offers other than the equilibrium wage will only be made by unqualified workers.

The equilibrium chosen here is

$$
\begin{aligned}
w^* &= \begin{cases} \pi - rV_f \text{ if } \pi \geq r(V_f + V_q) \\ rV_q \text{ if } \pi < r(V_f + V_q) \end{cases} \\
a_f^* &= \begin{cases} 1 \text{ if } \tilde{\pi} - rV_f - w \geq 0 \\ 0 \text{ otherwise} \end{cases} \\
\tilde{\pi} &= \begin{cases} \pi \text{ if } w \geq V_q \\ 0 \text{ if } w < V_q \end{cases}
\end{aligned}
$$

This equilibrium is Pareto dominant for the workers, and it is the only equilibrium that is "prior consistent". That is, the beliefs do not discriminate with respect to who takes out-of-eqilibrium actions except when those actions could never be optimal for one type. (It is also consistent with the outcome of the complete information version of Mailath *et al* [2000] in which the worker simply demands the whole surplus.)

Returning to the issue of unemployment spell length. In the screening game, firms will have stationary strategies as long as μ, V_q and V_u are constant. As long spell-length workers can freely emulate short spell-length workers, equilibria of the true signalling game must also be pooling with respect to spell length. This, with stationary offers by firms imply that μ, V_q and V_u are constant. (Non-stationary equilibria may well be possible.)

[27] If we allowed firms to follow a mixed strategy semi-separating equilibria are possible.

6.2 Analysis of bargaining game under color blind testing.

Here there is an additional dimension of private information; workers know their own ethnicity.[28] This generates 4 types of worker. Additional to above notation we will use superscript $J = A, I$ to indicate the ethnic group respectively majority and minority.

As above, we analyze the game in two parts corresponding to the choice by nature as to who makes the wage offer (after the realization of the test score, θ and hence the posterior probability $\pi(\theta, \hat{\psi}, \hat{\mu}_A, \hat{\mu}_I)$ that the worker is of type q).

6.2.1 Firm makes offer (screening model)

Firm action: makes wage offer $w_f \in [0, 1]$

Worker action: picks probability of acceptance, $a_i^J \in [0, 1]$, $i = q, u$ $J = A, I$

Firm pay-offs: $\begin{cases} a_q^J(1 - w_f) + (1 - a_q^J)r\hat{V}_f \text{ for worker type } q,\ J = A, I \\ -a_u^J w_f + (1 - a_u^J)r\hat{V}_f \text{ for worker type } u,\ J = A, I \end{cases}$

Type iJ worker pay-offs: $a_i^J w_f + (1 - a_i^J)r\hat{V}_i^J$

Firm strategies: $w_f(\pi) : [0, 1] \to [0, 1]$

Type iJ worker strategies: $a_i^J(\pi, w) : [0, 1] \times [0, 1] \to [0, 1]$

A sub-game perfect equilibrium is a tuple $\{\hat{w}_f, \hat{a}_i^J\}$ $i = q, u$; $J = A, I$ such that

$$\begin{aligned} \hat{a}_i^J &= \arg\max_{a \in [0,1]} \left\{ a\hat{w}_f + (1 - a)r\hat{V}_i^J \right\} \\ \hat{w}_f &= \arg\max_{w \in [0,1]} \left\{ \sum_{J=A,I} \left[\hat{\pi}_q^J \hat{a}_q^J (1 - w) - \hat{\pi}_u^J \hat{a}_u^J w \right] + r\hat{V}_f \sum_{J=A,I,\ i=q,u} \hat{\pi}_i^J (1 - \hat{a}_i^J) \right\} \end{aligned}$$

where $\hat{\pi}_i^J$ is the posterior probability that the worker is of type iJ. For

[28] This is intrinsically different from duration of unemployment. In any equilibrium a person's ethnic group affects the wage distribution he faces whether it is revealed or not. A person's duration of unemployment can only affect the wage distribution if it is revealed. We take the stance as before with respect to duration of unemployment.

example

$$\hat{\pi}_q^A(\theta,\psi,\mu^A,\mu^I) = \frac{(1-\psi)\mu_A f_A(\theta)}{(1-\psi)\mu_A f_A(\theta) + (1-\psi)(1-\mu_A) + \psi\mu_I f_I(\theta) + \psi(1-\mu_I)}$$

We focus on equilibria of the form

$$\hat{w}_f(\pi) = \begin{cases} r\hat{V}_q^A \text{ for } \sum_{J=A,I} \hat{\pi}_q^J \geq r(\hat{V}_f + \hat{V}_q^A) \\ \omega < r\min\{V_u^A, V_u^I\} \text{ otherwise} \end{cases}$$

$$\hat{a}_i^J = \begin{cases} 1 \text{ if } w \geq r\hat{V}_i^J \\ 0 \text{ otherwise} \end{cases}$$

When they exist, there is a continuum of these equilibria; one for every $\omega \in [0, r\min\{\hat{V}_u^A, \hat{V}_u^I\})$ but they are all pay-off equivalent. Any equilibrium of this type may fail to exist if

$$\hat{\pi}_q^I > r(\hat{V}_f + \hat{V}_q^I)$$

because it then becomes worthwhile to offer a wage below $r\hat{V}_q^A$ at certain test score values on the basis that even though anyone who accepts is not type qA, there is sufficient reason to believe that the applicant is of type qI.

6.2.2 Worker makes offer (signalling model)

Type iJ worker action: makes wage offer $w_i^J \in [0,1]$

Firm action: picks probability of acceptance, $a_f \in [0,1]$,

Then,

Firm pay-offs: $\begin{cases} a_f(1-w_q^J) + (1-a_f)rV_f \text{ if worker type } q,\ J = A, I \\ -a_f w_u^J + (1-a_f)rV_f \text{ if worker type } u,\ J = A, I \end{cases}$

Type iJ worker pay-offs: $a_f w_i^J + (1-a_f)rV_i^J$, $i = q, u$, $J = A, I$

Type iJ worker strategies: $w_i^J : [0,1] \to [0,1]$.

Firm strategies: $a_f : [0,1] \times [0,1] \to [0,1]$.

In formulating their strategies firms will up-date their beliefs as to the productivity of the worker based on the wage the worker offers and the implication of that offer for the worker's ethnicity.

A Bayesian perfect equilibrium is a tuple $\{\hat{w}_i^J, \hat{a}_f\}$, $i = q, u$, $J = A, I$ such that

$$
\begin{aligned}
\hat{a}_f &= \arg\max_{a\in[0,1]} \left\{ a \left[\sum_{J=A,I} \tilde{\pi}_q^J (1-w) - \sum_{J=A,I} \tilde{\pi}_u^J w \right] + (1-a) r \hat{V}_f \right\} \\
w_i^* &= \arg\max_{w\in[0,1]} \left\{ a_f^* w + (1-a_f^*) r \hat{V}_i \right\}
\end{aligned}
$$

where $\tilde{\pi}_i^J$ is the updated belief of the firm that the worker is of type iJ based on the wage offered by the worker. As unqualified workers can freely emulate their qualified counterparts, there can be no symmetric separating equilibria in pure strategies.

The equilibrium that I focus on is

$$
\begin{aligned}
\hat{w}_i^J &= \begin{cases} \sum_{J=A,I} \hat{\pi}_q^J - r\hat{V}_f \text{ if } \sum_{J=A,I} \hat{\pi}_q^J \geq r(\hat{V}_f + \hat{V}_q^A) \\ rV_q^A \text{ otherwise} \end{cases} \\
\hat{a}_f &= \begin{cases} 1 \text{ if } \sum_{J=A,I} \tilde{\pi}_q^J - r\hat{V}_f - w \geq 0 \\ 0 \text{ otherwise} \end{cases} \\
\tilde{\pi}_q^A &= \begin{cases} \hat{\pi}_q^A \text{ if } w \geq V_q^A \\ 0 \text{ if } w < V_q^A \end{cases} \\
\tilde{\pi}_q^I &= \begin{cases} \hat{\pi}_q^I \text{ if } w \geq V_q^I \\ 0 \text{ if } w < V_q^I \end{cases}
\end{aligned}
$$

This equilibrium fails to exist if

$$
\sum_{J=A,I} \hat{\pi}_q^J < r(\hat{V}_f + \hat{V}_q^A) \text{ and } \hat{\pi}_q^I > r(\hat{V}_f + \hat{V}_q^I)
$$

in this case it may be worth while for a type qI workers to ask for less than $r\hat{V}_q^A$. When this happens depends on parameter values. The simulations of this equilibrium check that this condition is not violated.

Again within the relevant parameter range there are a lot of other Bayesian perfect equilibria. The chosen one is Pareto dominant for the workers, and is "prior consistent".

6.3 Slope of EM curve.

Substituting for $\pi(\theta,\mu)$ into (6) and dividing through by $f(\theta^*)$ yields

$$\Psi(\theta^*,\mu) \equiv 2r - \alpha(1-\mu)\int_{\theta^*}^{1} \frac{f(\theta)-f(\theta^*)}{[\mu f(\theta)+(1-\mu)]\, f(\theta^*)} dF(\theta) = 0 \qquad (13)$$

then

$$\frac{d\theta^*}{d\mu} = -\frac{\frac{\partial\Psi}{d\mu}}{\frac{\partial\Psi}{d\theta^*}}$$

where

$$\frac{\partial\Psi}{d\mu} = \alpha\int_{\theta^*}^{1} \frac{f(\theta)}{[\mu f(\theta)+(1-\mu)]^2\, f(\theta^*)} dF(\theta) > 0$$

$$\frac{\partial\Psi}{d\theta^*} = \alpha(1-\mu)\int_{\theta^*}^{1} \frac{f'(\theta^*)f(\theta)}{[\mu f(\theta)+(1-\mu)]\, f^2(\theta^*)} dF(\theta) > 0 \qquad (14)$$

Simple inspection of (13) indicates that $\theta^*(0) < 1$. $\square$

6.4 How Changes in F affect the EM curve.

This analysis makes use of the $\Psi(.,.)$ function as defined in (13). As $\Psi(\theta^*,\mu)$ is increasing in θ^* the effect of increased accuracy on θ^* will be the negative of its effect on $\Psi(\theta^*,\mu)$.

Suppose, $\hat{\theta} > 0$ is any threshold value of the test score and θ is any test score such that $\theta > \hat{\theta}$. Define $\Delta(.)$ to be any small change in f such that $\Delta + f$ is a density function, continuously differentiable and more accurate than f in the sense of MLP. The implied restrictions on Δ are

$$\int_0^1 \Delta d\theta = 0 \quad \text{and for any } \theta > \hat{\theta}, \quad \frac{\Delta(\theta)}{f(\theta)} > \frac{\hat{\Delta}}{\hat{f}} \qquad (15)$$

where $\hat{\Delta} \equiv \Delta(\hat{\theta})$ and $\hat{f} = f(\hat{\theta})$. Also MLP implies that there is a unique $\tilde{\theta}$ such that $\Delta(\tilde{\theta}) = 0$. To see this, recall that MLP requires

$$\frac{d}{d\theta}\left(\frac{\Delta(\theta)+f(\theta)}{f(\theta)}\right) > 0$$

so (using the prime to represent differentiation with respect to θ), $\Delta' f - \Delta f' > 0$. $\Delta = 0$ therefore implies $\Delta' > 0$ which precludes multiple crossings. Notationally, we use $\tilde{f} \equiv f(\tilde{\theta})$.

We wish to obtain the effect of the change $\varepsilon\Delta$ to f on $\Psi(\hat{\theta},\mu)$ where ε is any scalar such that $\varepsilon > 0$. First, is simple to see that $\varepsilon\Delta + f$ is also a density function and more accurate than f.

Defining the functional $I(f)$ by

$$I(f) \equiv \int_{\hat{\theta}}^{1} \frac{(f-\hat{f})f}{(\mu f + 1 - \mu)\hat{f}} d\theta$$

We have

$$I(\varepsilon\Delta+f)-I(f) \equiv \int_{\hat{\theta}}^{1} \frac{(\varepsilon\Delta + f - \varepsilon\hat{\Delta} - \hat{f})\,(\varepsilon\Delta + f)}{[\mu\,(\varepsilon\Delta + f) + 1 - \mu]\left(\varepsilon\hat{\Delta} + \hat{f}\right)} d\theta - \int_{\hat{\theta}}^{1} \frac{(f-\hat{f})f}{(\mu f + 1 - \mu)\hat{f}} d\theta \tag{16}$$

Define

$$I'(f|\Delta) \equiv \lim_{\varepsilon\to 0} \frac{I(\varepsilon\Delta + f) - I(f)}{\varepsilon}$$

as the derivative of $I(f)$ with respect to Δ. $I'(f|\Delta)$ is how the functional $I(f)$ changes as f moves infinitesimally toward $\Delta + f$ under the restrictions imposed by (15).

A first order Taylor series expansion of (16) around $\varepsilon = 0$ implies[29]

$$I'(f|\Delta) = \int_{\hat{\theta}}^{1} \left[\frac{\mu f^2 + (1-\mu)(2f - \hat{f})}{(\mu f + 1 - \mu)^2 \hat{f}}\right] \Delta d\theta - \hat{\Delta} \int_{\hat{\theta}}^{1} \frac{f^2}{(\mu f + 1 - \mu)\hat{f}^2} d\theta. \tag{17}$$

The sign of $I'(f|\Delta)$ is not yet obvious as Δ is negative for $\theta < \tilde{\theta}$.

Now suppose Δ is chosen so that so that $\tilde{\theta} > \hat{\theta}$. In this case $\hat{\Delta}$ is negative and the second term in (17) is positive. Furthermore, as

$$\frac{d}{d\theta}\left[\frac{\mu f^2 + (1-\mu)(2f - \hat{f})}{(\mu f + 1 - \mu)^2 \hat{f}}\right] = \frac{2(1-\mu)(\mu\hat{f} + 1 - \mu)}{(\mu f + 1 - \mu)^3 \hat{f}} > 0,$$

the integrand in the first term of (17) is positive and increasing in θ for all $\theta > \hat{\theta}$. So

$$\frac{\mu f^2 + (1-\mu)(2f - \hat{f})}{(\mu f + 1 - \mu)^2 \hat{f}} \left\{ \begin{array}{c} \geq \\ \leq \end{array} \right\} \frac{\mu \tilde{f}^2 + (1-\mu)(2\tilde{f} - \hat{f})}{(\mu \tilde{f} + 1 - \mu)^2 \hat{f}} \left\{ \begin{array}{c} \theta \geq \tilde{\theta} \\ \hat{\theta} \leq \theta \leq \tilde{\theta} \end{array} \right.$$

[29] An alternative aproach to the same result is the Voltara Derivative used by Ryder and Heal (1976).

As $\Delta(\theta) < 0$ for $\theta < \tilde{\theta}$

$$\int_{\hat{\theta}}^{1} \left[\frac{\mu f^2 + (1-\mu)(2f - \hat{f})}{(\mu f + 1 - \mu)^2 \hat{f}} \right] \Delta d\theta > \frac{\mu \tilde{f}^2 + (1-\mu)(2\tilde{f} - \hat{f})}{(\mu \tilde{f} + 1 - \mu)^2 \hat{f}} \int_{\hat{\theta}}^{1} \Delta d\theta > 0.$$

For $\tilde{\theta} > \hat{\theta}$ both terms in (17) are positive.

If Δ is chosen so that so that $\tilde{\theta} \leq \hat{\theta}$, $\hat{\Delta} \geq 0$ and the second term of (17) is negative. However, as indicated above in (15), in this case $\Delta \geq \hat{\Delta} f / \hat{f}$. So

$$\begin{aligned} I'(f|\Delta) &> \int_{\hat{\theta}}^{1} \left\{ \left[\frac{\mu f^2 + (1-\mu)(2f - \hat{f})}{(\mu f + 1 - \mu)^2 \hat{f}} \right] \hat{\Delta} \frac{f}{\hat{f}} - \frac{\hat{\Delta} f^2}{(\mu f + 1 - \mu) \hat{f}^2} \right\} d\theta \\ &= \hat{\Delta} \int_{\hat{\theta}}^{1} \frac{(1-\mu) f (f - \hat{f})}{(\mu f + 1 - \mu)^2 \hat{f}^2} d\theta > 0 \end{aligned}$$

Consequently, $I(f)$ increases with the accuracy of f and so $\Psi(\hat{\theta}, \mu)$ is decreasing with accuracy of f for every $\hat{\theta}$. This means that $\Psi(\theta^*, \mu)$ must also fall with accuracy and θ^* therefore rises at every value of μ.

6.5 How Changes in F affect $\pi^* \equiv \pi(\theta^*, \mu)$

Maintaining the notation from the preceding analysis, we examine the impact of a change in the density function of the form $\varepsilon \Delta(\theta)$ where ε is an infinitesimal scalar and Δ is subject to restrictions (15). We define θ^*_ε to be the value of θ^* that solves (6) and therefore (13). Also define π^*_ε to be the value of π^* associated with the change in F while holding μ fixed. Then

$$\pi^*_\varepsilon - \pi^* = \frac{\mu \left(\varepsilon \Delta(\theta^*_\varepsilon) - f(\theta^*_\varepsilon) \right)}{\mu \left(\varepsilon \Delta(\theta^*_\varepsilon) - f(\theta^*_\varepsilon) \right) + 1 - \mu} - \frac{\mu f(\theta^*)}{\mu f(\theta^*) + 1 - \mu}$$

A first-order Taylor series expansion around $\varepsilon = 0$, using (17) implies

$$\lim_{\varepsilon \to 0} \frac{\pi^*_\varepsilon - \pi^*}{\varepsilon} = \left[\frac{\mu (1-\mu)}{(\mu f(\theta^*) + 1 - \mu)^2} \right] \left\{ f'(\theta^*) \frac{\alpha (1-\mu) I'(f|\Delta)}{\frac{\partial \Psi}{\partial \theta^*}} + \Delta(\theta^*) \right\} \tag{18}$$

The first term in the curly brackets represents the indirect effect of the change in F on θ^* and then on $\pi(\theta^*, \mu)$. The second term represents the direct effect of f on $\pi(.,.)$. Simple substitution from (17) and (14) into

(18) implies that under efficient matching with fixed μ, increased accuracy increases $\pi(\theta^*, \mu)$.

The other assertion is that movements along the EM curve towards the South-West which occur as the market converges to steady state, reduce $\pi(\theta^*, \mu)$. This is because

$$\frac{d\pi^*}{d\mu} = \frac{\partial \pi^*}{\partial \theta^*} \left.\frac{d\theta^*}{d\mu}\right|_{EM} + \frac{\partial \pi^*}{\partial \mu}$$

So that, using $f'^* \equiv f'(\theta^*)$ and $f^* \equiv f(\theta^*)$

$$\begin{aligned} \operatorname{sgn}\left\{\frac{d\pi^*}{d\mu}\right\} &= \operatorname{sgn}\left\{ f^* - \mu\left(1-\mu\right) f'^* \left[\frac{\int_{\theta^*}^1 \frac{f}{[\mu f + (1-\mu)]^2 f^*} dF}{\int_{\theta^*}^1 \frac{(1-\mu) f'^* f}{[\mu f + (1-\mu)] f^{*2}} dF} \right] \right\} \\ &= \operatorname{sgn}\left\{ \int_{\theta^*}^1 \frac{f\left[\mu f + 1 - 2\mu\right]}{\left[\mu f + (1-\mu)\right]^2} dF \right\} \end{aligned}$$

(Here sgn{.} is the sign operator.)

Now

$$\int_{\theta^*}^1 \frac{f\left[\mu f + 1 - 2\mu\right]}{\left[\mu f + (1-\mu)\right]^2} dF > \int_{\theta^*}^1 \frac{\pi^2(\theta, \mu)}{\mu} (f-1) d\theta > 0$$

the last inequality comes from the fact that $\pi(\theta, \mu)$ is increasing in θ and that $1 - F(\theta^*) > 1 - \theta^*$.

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