A Search Version of the Roy Model*

Michael Sattinger
Department of Economics
University at Albany
Albany, NY 12222
Email: m.sattinger@albany.edu

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Abstract

This paper considers the decisions of workers to search in different labor markets. In the basic model, a worker can search in one labor market or another but not both. With non-pecuniary benefits, a worker chooses the labor market offering the highest reservation utility level. Conditions for simultaneous search in two markets are also derived under the assumption that workers suffer a reduction in wage offers. Decisions of where to search are relevant to self-selection into sectors and self-selection biases, the formation of interview networks, and generation of overlapping markets.

1. Introduction

This paper examines a worker's decision of where to search. A worker can search in one labor market or another labor market, or in both. The model extends search theory by introducing a prior sectoral choice of where to search, instead of having all workers in an aggregate labor market searching in a common market. The decision of where to search could result in the worker ending up in either of two types of jobs (or occupations), or moving back and forth between two types of jobs.

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A worker's decision concerning where to search can play a significant role in the assignment of workers to jobs (see Sattinger, 1993, for a survey). In the model developed by A.D. Roy (1951), workers choose sectors on the basis of income maximization, leading to a self-selection assignment of workers to productive activities. In the envelope model developed by Jan Tinbergen (1951), there are continuous distributions of job and worker attributes. In the equilibrium, the offer curves of cost-minimizing employers form an envelope, and workers choose the wage-attribute combination that maximizes their utilities. The Roy and Tinbergen models do not incorporate search. In search models, workers find jobs by considering job offers sequentially (in most models), stopping when the wage offer exceeds a reservation wage determined optimally by the worker (Dale Mortensen and Christopher Pissarides, 1999, provide a survey). With heterogeneous workers and jobs, this can result in an assignment that differs from frictionless exact assignments (Sattinger, 1995). The paper presented here fits into the assignment literature by combining the sectoral choice approach with the search approach. Prior to engaging in search, the worker decides in which labor market he or she will search.

Sectoral search choice can play a significant role in determining the structure and operation of labor markets. In product markets with rationing (Denis Carlton, 1978, John Gould, 1978), buyers are assumed to choose where to seek a good on the basis of a combination of price and likelihood of getting the good. The ability of buyers to choose where to seek the good gives sellers price-setting ability, since an increase in price reduces the number of buyers until the value of seeking the good at that firm returns to its former level. In labor markets where firms announce wages (known as wage posting) and workers can determine the likelihood of getting a job, equilibrium wages eliminate search congestion (Sattinger, 1990). With a continuum of labor markets differing by ratio of unemployed workers to vacancies, worker choice of labor markets yields complete markets and a return to the envelope models of Tinbergen (Espen Moen, 1995; Shouyoung Shi, 2001; Mortensen and Randall Wright, 2002). Robert Shimer (2001) has shown that decisions of where to apply for a job (combined with firm decisions to offer jobs) determine the assignment of workers to jobs in an economy with frictions. Worker or buyer decisions analogous to sectoral search choice have therefore appeared previously in various models.

To investigate a worker's decision to search in one market versus another, a search version of the Roy model is developed (Roy, 1951). As is well known, in the simple Roy model, people choose between fishing for trout or hunting for rabbits

on the basis of income maximization. The Roy model provides a simple way of looking at self-selection of occupations. There are no employers in the Roy model and no wage offers, so search would not seem to be a natural extension of the model. However, it can be assumed that there is a distribution of nonpecuniary benefits associated with fishing or hunting locations. James Heckman and Guillaume Sedlacek (1985) extend the Roy model to include nonpecuniary benefits in the form of occupational preferences but not in a search context. Sattinger (1996) develops a search model in which workers consider both wage and nonpecuniary benefit. In the model developed here, a worker examines a particular site in a search process (say, while looking for a fishing spot) and determines the satisfaction level associated with the site. Sites for trout fishing can vary for a number of reasons, such as view, sunshine, vegetation, slope of the bank, or overhanging branches. It is assumed that these features do not affect productivity, and that no congestion for other fishers occurs. Likewise, sites for hunting rabbits can vary. In less bucolic workplaces, a number of features affect worker satisfaction, such as noise, heat, room, fresh air, and comfort. With variation in satisfaction, workers no longer choose sectors simply on the basis of income maximization.

With a distribution of nonpecuniary benefits, workers set a reservation level of total benefits that they could get from a given occupation. If workers could only choose between the two occupations, they would choose the occupation that yields the highest reservation level of total benefits, since the reservation level also measures the expected long run flow of benefits. In this way, choice of occupation under search would parallel choice of occupation in the original Roy model, and questions concerning the distribution of workers in each occupation could be considered as in the original Roy model and extensions. However, it is also possible for workers to search in both markets, depending on the search technology. For example, it is possible to assume that the number of sites workers could visit in each of the occupations would be a proportion a of the number that could be visited if only one occupation were considered. A worker would only consider searching in both markets if doing so yields a higher reservation level of total benefits than concentrating in either of the two occupations. If some workers search in both markets, then a simple form of overlapping markets arises. In the "trout" market, some workers search for sites in the "rabbit" market, but some do not. In the "rabbit" market, some workers search for trout sites, but some do not. The two markets overlap for those workers searching in both markets. With the Roy production technology, no externalities arise if the number of workers choosing an occupation does not affect the number of catches or the distribution

of nonpecuniary benefits available to other workers.

The next section develops the formal model, and section 3 discusses implications of the model for self-selection biases and overlapping markets.

2. Model

2.1. Conditions in Sectors

Suppose there are two sectors, indexed by $j = 1, 2$. Let x_{ij} be the output of person i in sector j. For the purposes of this section, it will not be necessary to specify the distribution of the x_{ij} values. Let p_j be the price paid to a worker per unit of output in sector j. The income of person i in sector j would then be $p_j x_{ij}$. Let s be a particular satisfaction level, measured in monetary units, and let $G_j(s)$ be the cumulative distribution function of satisfaction levels in sector j. Let $g_j(s)$ be the corresponding probability density function of satisfaction levels.

Let z_{ijt} be the utility level for person i from site t in sector j:

$$z_{ijt} = p_j x_{ijt} + s_{ijt} \tag{2.1}$$

where s_{ijt} is the satisfaction level from site t in sector j. Since $p_j x_{ij}$ does not vary from site to site in sector j, selection of a reservation utility level is equivalent to selection of a reservation satisfaction level in sector j. Let s_{0j} be the reservation satisfaction level if a worker is searching in sector j.

Suppose workers move into and out of employment. Let γ be the transition rate from employment to unemployment. (For simplicity, γ will be assumed to be the same in both sectors.) When unemployed, a worker could search in sector j for sites at which to work. Let $\lambda_j(s_{0j})$ be the transition rate from unemployment to employment for a worker searching in sector j with reservation satisfaction level s_{0j}. Let θ_j be the rate at which a worker comes across sites in sector j, if the worker is only looking in sector j. Then

$$\lambda_j(s_{0j}) = \theta_j(1 - G_j(s_{0j})) \tag{2.2}$$

Now consider the determination of the reservation satisfaction level if the worker only searches in sector j. In the Markov process describing the worker's movements between employment and unemployment, it is possible to specify the asset values in the two states of employed and unemployed.[1] If the worker chooses

[1] The following derivations are described in more detail in Sattinger (1985) for the standard search problem in a Markov process.

a reservation satisfaction level s_{0j}, the flow of asset value while unemployed is

$$rV_{ij}(s_{0j}) = \frac{\lambda_j(s_{0j})}{\lambda_j(s_{0j}) + \gamma + r}\left(p_j x_{ij} + s_e(s_{0j})\right) + \frac{\gamma + r}{\lambda_j(s_{0j}) + \gamma + r} b_j \tag{2.3}$$

where r is the discount rate, b_j is the level of benefits (if any) while unemployed, and $s_e(s_{0j})$ is the expected satisfaction level if the reservation satisfaction level is s_{0j} :

$$s_e(s_{0j}) = \frac{\int_{s_{0j}}^{\infty} s g_j(s) ds}{1 - G(s_{0j})} \tag{2.4}$$

The reservation satisfaction level s_{0j} is chosen to maximize the flow of asset value $rV_{ij}(s_{0j})$. By taking the derivative of $rV_{ij}(s_{0j})$ with respect to s_{0j}, it can be shown that the optimal reservation satisfaction level s_{0j} occurs when

$$p_j x_{ij} + s_{0j} = rV_{ij}(s_{0j}) \tag{2.5}$$

Setting $p_j x_{ij} + s_{0j}$ equal to the right side of 2.3 and solving yields the reservation value s_{0j}.

2.2. Choice Between Sectors

Now suppose that the worker is constrained to choose one sector or the other and has calculated the reservation satisfaction levels in each sector. The expected utility levels in the two sectors are then $p_1 x_{i1} + s_{01}$ and $p_2 x_{i2} + s_{02}$. The worker chooses the sector that yields the higher utility level. In particular, the worker chooses sector 1 whenever $p_1 x_{i1} + s_{01} > p_2 x_{i2} + s_{02}$ or

$$s_{01} > (p_2 x_{i2} - p_1 x_{i1}) + s_{02} \tag{2.6}$$

That is, the reservation satisfaction level in the sector chosen must exceed the difference in incomes plus the reservation satisfaction level in the other sector.

By substituting the optimal choices of s_{01} and s_{02} into the expressions for the expected utility levels, it is possible to divide the $(x_{i1,}x_{i2})$ plane into two groups of workers, one choosing sector 1 and the other choosing sector 2. By setting the expected utility levels equal, it is possible to determine the dividing line between the two sectors. This is done in Figure 2.1 using particular assumptions.[2] Figure

[2]Figure 2.1 assumes that the distributions of satisfaction levels are given by the Pareto functions $g_1(s) = g_2(s) = 2s^{-3}$. The parameter values are $b_1 = .1$, $b_2 = .2$, $r = .1$, $\gamma = .05$, $\theta_1 = \theta_2 = 1$, $p_1 = 1$ and $p_2 = 2$.

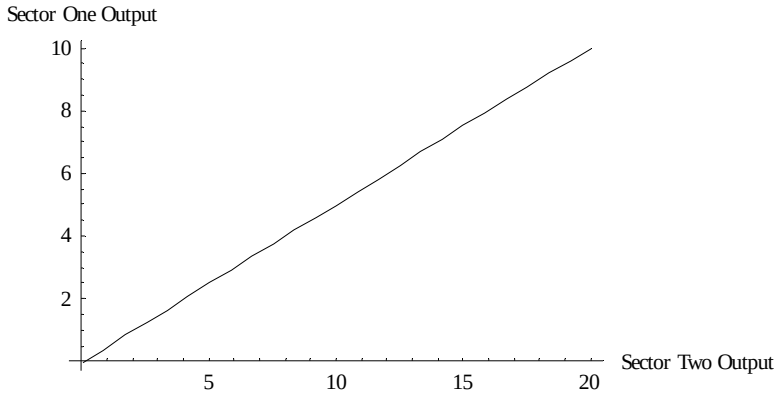

Figure 2.1: Choice Between Sectors

2.2 assumes $\theta_2 = 4$, making sector 2 more attractive and yielding a different dividing line. Note that the dividing line is no longer a straight line as it is in the standard Roy model.

2.3. Choice of Both Sectors

In this section, the worker is allowed a third choice: the worker can search in both sectors. Let s_{01} and s_{02} again be the reservation satisfaction levels when looking in sectors 1 and 2, respectively. Suppose there is some loss in the number of sites searched in the two sectors as a result of going back and forth between the two sectors. Let $a\lambda_j(s_{0j})$ be the rate at which acceptable jobs are found in sector j, where $a < 1$. When the worker searches in both markets, the transition rate out of unemployment is then $a\lambda_1(s_{01}) + a\lambda_2(s_{02})$. The worker now moves between three states, employed in sector 1, employed in sector 2, and unemployed. Specifying asset value functions for each state and solving yields an expression for the asset value while unemployed, as a function of s_{01} and s_{02}. The reservation satisfaction levels s_{01} and s_{02} can be expressed as functions of a common reservation utility level. Setting the common reservation utility level equal to the flow of asset value when unemployed and solving yields the common reservation utility level.

The worker chooses to search in both sectors if doing so yields a higher level of expected utility then if the worker only searched in one of the other sectors.

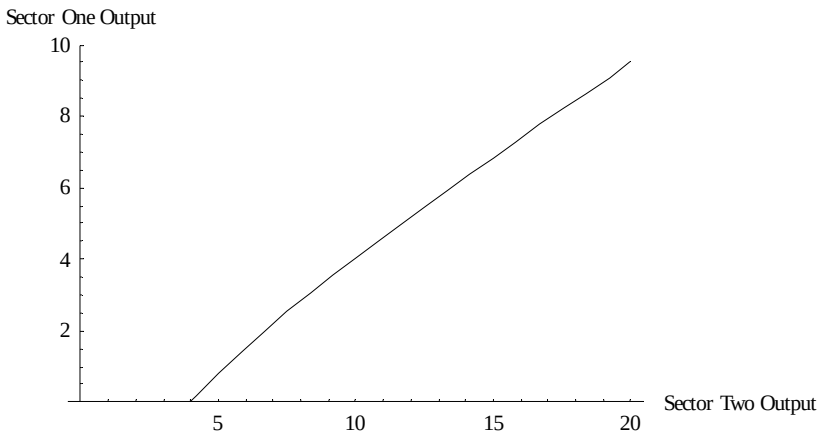

Figure 2.2: Choice Between Sectors: Alternative Parameters

This occurs if the common reservation utility level exceeds the reservation utility levels if the worker only searched in a single sector.[3]

It is not necessary to calculate the common reservation utility level to determine if a worker should search in both markets. Taking s_{01} and s_{02} from the individual sector optimization problems, it is possible to calculate the resulting flow of asset values from searching in both markets. If this value exceeds the utility values implied by s_{01} and s_{02} from searching in only one market, the worker should search in both markets. If the value falls below the utility value for either sector 1 or 2, the worker should search in only one sector; raising the common reservation utility level will only lower the flow of asset value from searching in both markets.

The area of the $(x_{i1,}x_{i2})$ plane in which workers search in both markets is shown in Figure 2.3 using most of the assumptions used in Figure 2.1.[4] Workers are more likely to search in both markets if their expected utilities in the two markets are approximately the same. If the expected utility in one market is much greater than in the other market, they are likely to concentrate their search in that one market. (This would occur if a worker is much more productive in one sector.)

[3]That is, the common reservation utility level exceeds both $p_1x_{i1} + s_{01}$ and $p_2x_{i2} + s_{02}$, where s_{01} and s_{02} are calculated assuming the worker only searches in sector 1 or 2, respectively.

[4]Figure 2.3 assumes $a = .8$ and $b_3 = .1$

Figure 2.3: Search in Both Sectors

The shape of the relevant area depends on where the density of workers with respect to x_{i1} and x_{i2} occurs. For example, if there is a uniform distribution in the rectangle with x_{i1} from 0 to 5 and x_{i2} from 0 to 2, most workers will search in both sectors except for workers in the upper left and lower right of the rectangle. In these areas, performance in one sector is much greater than in the other sector. If instead there is a uniform density with x_{i1} from 5 to 20 and x_{i2} from 2 to 10, only a narrow strip of workers will search in both sectors.

In the analysis of the standard Roy model, sectoral choice is based on comparative advantage: workers in one sectoral activity have a comparative advantage at that activity compared to the workers in the other sector performing the other activity. With workers forced to choose between two sectors, the presence of satisfaction may lead to some workers (those with approximately equal incomes from the two sectors) choosing sectors that do not accord with their comparative advantage. Workers with substantial comparative advantage (with unequal potential incomes from the two sectors) would continue to choose sectors based on comparative advantage.

With the option of searching in both markets, workers with approximately equal incomes in the two markets would choose to search in both markets. Those choosing to search in only one sector are likely to have a comparative advantage in that sector's activity. Workers searching in both sectors would sometimes end

up in one sector and sometimes in the other. Their times in the two sectors would be related to their comparative advantage but some of the time they would be in the sector where they do not have a comparative advantage.

The parameter a, reflecting the relative efficiency of searching in two markets, affects the number of workers choosing to search in two markets. With a larger value of a, the time spent unemployed is smaller (for a given reservation common utility level), the gain from searching in the two markets is greater, and the reservation common utility level will be greater. The area of the (x_{i1},x_{i2}) plane searching in both markets will be greater.

3. Conclusions

This paper has outlined some of the determinants of the worker decision to search in one market or another or in both markets. The basis of choice is the reservation utility level from each alternative choice (which determines reservation satisfaction levels). Dividing lines between sectors occur when the reservation utility levels are equal. Workers are more likely to search in two markets simultaneously when their incomes from the two markets are approximately equal.

Self-selection biases may be substantially reduced with sectoral search choice. An important implication of the original Roy model was the presence of self-selection biases, examined by Heckman in a number of papers. The distribution of wages or earnings in a sector does not reflect the distribution of wages for the population since workers with low earnings in that sector are likely to have chosen the other sector. This occurs because the worker's outcomes in the two sectors (the productivity and income levels that the worker would receive in the two sectors) are determined and known to the worker at the same time as the choice of sectors. With sectoral search choice, this is no longer the case. The worker chooses the sector (or both sectors) prior to knowing the outcome of search in the two sectors. The worker choice is based on the distribution of all outcomes, as determined by the distributions of satisfaction levels as considered here, rather than the particular outcome that results from the search process. The distribution of satisfaction levels (or wage offers in a more general model) in a sector is then unaffected by the fact that the worker chose that sector. The distribution of satisfaction levels at accepted jobs for a worker will nevertheless depend on the worker's reservation utility level. If the worker's reservation utility level corresponds to a satisfaction level s^*, the distribution of satisfaction levels among accepted jobs will be $g_j(s)/(1 - G_j(s^*)$, $s \geq s^*$. There will still exist selection

biases since the reservation utility level will be determined simultaneously with the choice of sector. Estimation procedures for sectoral search choice will differ both from the standard self-selection procedures and from the procedures for estimating reservation wages in an aggregate search market.

The search version of the Roy model considered here does not incorporate employer decisions or congestion externalities. Without further assumptions about production in the two activities, it does not lend itself to questions concerning the efficiency of assignment. However, it is possible to incorporate sectoral search choice into an overlapping markets model (Sattinger, 2000).

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