Law of Demand, forthcoming in the *New Palgrave Dictionary of Economics*

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Abstract: We formulate several laws of individual and market demand and describe their relationship to neoclassical demand theory. The laws have implications for comparative statics and stability of competitive equilibrium. We survey results that offer interpretable sufficient conditions for the laws to hold and we refer to related empirical evidence. The laws for market demand are more likely to be satisfied if commodities are more substitutable. Certain kinds of heterogeneity across individuals make the laws more likely to hold in the aggregate even if they are violated by individuals.

law of demand

The most familiar version of the law of demand says that as the price of a good increases the quantity demanded of the good falls. The principal use of the law of demand in economic theory is to provide sufficient, and in some contexts, necessary, conditions for the uniqueness and stability of equilibrium, and for intuitive comparative statics. To guarantee such properties in equilibrium models with more than one good, the familiar one-good law of demand just stated is not sufficient—some multi-good version of the law is needed. In its multi-good form, the law of demand is said to hold for a particular change in prices if the prices and the quantities demanded move in opposite directions; in formal terms, the vector of price changes and the vector of resulting demand changes have a negative inner product.

In this essay, we will examine different formulations of the law of demand. They differ principally in the domain of price changes over which the law applies. It is not always the case that the law of demand is required to hold for *all* price changes: the version of the law which is required for stability analysis and comparative statics varies from one context to another. For each formulation of the law of demand, we discuss the conditions which are sufficient to guarantee that it is satisfied.

To point out the obvious, the law of demand, in whatever form, is not a universal law at all, but a condition which may hold in some situations and not others. It is well known that in transactions where asymmetric information is an important consideration, violations of the law can occur. For example, lowering the price of a set of used cars does not necessarily lead to higher demand if potential buyers think that the lower price reflects the quality of the cars being offered. (For a discussion of violations of the law of demand and other issues which arise when price has an impact on the quality of the good being exchanged, see Stiglitz (1987).) In this essay we make the classical assumption that the features of the good being transacted are commonly known and independent of the price. As we shall see, even in this classical setting, various forms of the law of demand will hold only under conditions which are often neither obviously onerous nor obviously innocuous; in these cases, one must necessarily turn to empirical work to ascertain whether or not the law holds.

We will use the notation and terminology of Mas-Colell et al. (1995, chapters 2, 3, 5) and assume that the reader is familiar with the basic consumer and producer theory described there. We assume that there are L commodities and that the consumer is a price-taker. The demand of a consumer of type α with income w at price vector $p = (p_\ell)_{\ell=1}^L \gg 0$ is the vector $x(p,w,\alpha) = (x_\ell(p,w,\alpha))_{\ell=1}^L$ in $\mathbb{R}^L_+$, satisfying the budget identity $p \cdot x(p,w,\alpha) = w$ for all p and w. Unless stated otherwise, we assume the demand function $x(\cdot,\cdot,\alpha)$ to be C^1. Then it has a Slutsky matrix of substitution effects $S(p,w,\alpha)$ with ℓj element $S_{\ell j}(p,w,\alpha) = \partial x_\ell(p,w,\alpha)/\partial p_j + [\partial x_\ell(p,w,\alpha)/\partial w]x_j(p,w,\alpha)$. The Slutsky matrix $S(p,w,\alpha)$ is the Jacobian matrix of the Slutsky-compensated demand function x^*, defined by $x^*(q) = x(q, q \cdot x(p,w,\alpha),\alpha)$, evaluated at $q = p$. The term $[\partial x_\ell(p,w,\alpha)/\partial w]x_j(p,w,\alpha)$ is called an income effect since it approximates the effect on the demand for good ℓ when income rises enough to compensate for a unit increase in the price of good j. If the consumer chooses demand bundles by maximizing a well-behaved utility function then the Slutsky matrix is symmetric and negative semidefinite. The latter means that $v \cdot S(p,w,\alpha)v \leq 0$ for all $v \in \mathbb{R}^L$; in particular, the diagonal terms of the Slutsky matrix are nonpositive.

One-good and multi-good laws of demand

The term 'law of demand' most often refers to the effect of price changes on consumers with fixed incomes. The law for a single good ℓ and a single consumer of type α is

$$(p_\ell - \bar{p}_\ell)(x_\ell(p,w,\alpha) - x_\ell(\bar{p},w,\alpha)) \leq 0, \tag{1}$$

for p and $\bar{p}$, with $p_i = \bar{p}_i$ for $i \neq \ell$ and income w fixed. (In the *strict* version of the law, the weak inequality in (1) is replaced by strict inequality when $p \neq \bar{p}$; all the laws of demand discussed in this article can be stated in their corresponding strict forms, though we generally do not do so.) The inequality (1) is equivalent to

$$0 \geq \frac{\partial x_\ell}{\partial p_\ell}(p, w, \alpha) = S_{\ell\ell}(p, w, \alpha) - x_\ell(p, w, \alpha)\,\frac{\partial x_\ell}{\partial w}(p, w, \alpha),\ \forall (p, w).$$

It holds if the substitution effect $S_{\ell\ell}$ is negative and larger in magnitude than the income effect $x_\ell(p, w, \alpha)\ \frac{\partial x_\ell}{\partial w}(p, w, \alpha)$. If the consumer is utility-maximizing, then $S_{\ell\ell} \leq 0$, so a sufficient condition for good ℓ to obey the law of demand is that the demand for this good is normal ($\partial x_\ell(p, w, \alpha)/\partial w \geq 0$). If the demand for good ℓ is not normal, the price effect $\partial x_\ell/\partial p_\ell$ may be positive. This is called a Giffen effect and good ℓ is called a Giffen good. All goods are normal and Giffen effects are ruled out if the demand function is generated by homothetic preferences or by a concave additive utility function ($u(x) = \sum_{\ell=1}^{L} u_\ell(x_\ell)$), or, more generally, by a supermodular concave function u, i.e., one in which all commodity pairs are Auspitz-Lieben-Edgeworth-Pareto complements: $\partial^2 u(x)/\partial x_j \partial x_\ell \geq 0$ for all $j \neq \ell$ (Chipman, 1985).

Giffen goods are rarely observed. Sometimes demand for a durable good like oil may increase with its current price if traders expect an even higher price in the future. However, if commodities are distinguished by date, this is not a Giffen effect since a future price changes along with the current price. A possible example of a Giffen good is proposed by Baruch and Kannai (2002). They give evidence suggesting that, in Japan of the 1970s, shochu, a cheap (and, by some accounts, nasty) alcoholic drink fits the definition. One may explain the demand for shochu in the following way. A consumer chooses between sake (good 1) and shochu (good 2). He always prefers sake to shochu, but he also *must have* a minimum alcohol intake (which we fix at 1). Formally, his utility is $u(x_1, x_2) = x_1$, subject to the "survival" constraint $x_1 + x_2 \geq 1$. If the consumer is sufficiently poor, both the budget and survival constraints bind, with the consumer consuming as much sake - and as little shochu - as possible. A fall in the price of shochu allows him to buy less shochu and more sake and still meet his alcohol requirement; this he chooses to do since he always prefers sake to shochu.

Turning now to multi-good laws of demand, let $P \subseteq \mathbb{R}^L_{++}$ be a set of prices and let $X : P \to \mathbb{R}^L$ be a function representing individual or aggregate demand of firms or of consumers. The natural multi-good generalization of the one-good law in (1) is

$$(p - p') \cdot (X(p) - X(p')) \leq 0 \tag{2}$$

for all (p, p') in some subset of $P \times P$. If P is convex and open and X is C^1, (2) holds on $P \times P$ if and only if the Jacobian matrix $\partial X(p)$ is negative semidefinite at each p (Hildenbrand and Kirman, 1988).

Suppose that the supply vector of the L goods changes from ω to ω'. Let p and p' be corresponding equilibrium prices so $X(p) = \omega$ and $X(p') = \omega'$. Then if X obeys (2) for all prices, we obtain $(p - p') \cdot (\omega - \omega') \leq 0$. It is clear that this comparative statics property and the law of demand on X are essentially two sides of the same coin. Note also that, according to this property, an increase in the supply of good k, with the supply of all other goods held fixed, will lead to a fall in the price of k.

Suppose that P is open and X obeys the *strict* law of demand; that is, X satisfies (2) with strict inequality for all p and p' in P. This implies in particular that X is 1-1 and that, for each $\bar{\omega}$ in $X(P)$, there is a unique equilibrium price vector $\bar{p} = X^{-1}(\bar{\omega})$. A tâtonnement path for the function $X - \bar{\omega}$ is the solution to $dp/dt = X(p(t)) - \bar{\omega}$ for some initial condition $p(0) = p'$ in P. We say that $X - \bar{\omega}$ is monotonically stable if each of its tâtonnement paths $p(t)$ satisfies $d|p(t) - \bar{p}|^2/dt < 0$ whenever $p(t) \neq \bar{p}$. It is easy to check that $X - \bar{\omega}$ is monotonically stable for every $\bar{\omega}$ in $X(P)$ if and only if X obeys the strict law of demand. Furthermore, because P is open, a tâtonnement path for $X - \bar{\omega}$ which begins at a price sufficiently close to $\bar{p} = X^{-1}(\bar{\omega})$ stays in P for all $t > 0$. Lyapunov's second theorem then guarantees that the tâtonnement path converges to $\bar{p}$.

Laws of demand are thus useful as intuitive sufficient conditions for the uniqueness and stability of equilibrium and for comparative statics. We will examine, in different contexts, circumstances under which they hold.

Law of demand for competitive firms and consumers with quasilinear utility

For a firm with production set Y, profit maximizing net output vector y at price vector p and $\bar{y}$ at $\bar{p}$ satisfy $p \cdot y \geq p \cdot \bar{y}$ and $\bar{p} \cdot \bar{y} \geq \bar{p} \cdot y$. The net demand vectors $x = -y$ and $\bar{x} = -\bar{y}$ satisfy $p(x - \bar{x}) \leq 0$ and $\bar{p}(x - \bar{x}) \geq 0$, hence satisfy the law of demand: $(p - \bar{p}) \cdot (x - \bar{x}) \leq 0$. Similarly, a consumer with utility function $u(x_0, x) = x_0 + \phi(x_1, \ldots, x_L)$ (*quasilinear* with respect to good 0) and with sufficiently high income w satisfies the law of demand on a restricted domain, where the price of good 0 is fixed (say at 1). This is a special case of the law for firms. The consumer's optimal demand for goods 1 through L at p (the price vector for goods 1 to L) and income w maximizes $w - p \cdot x + \phi(x)$. This is equivalent to profit maximization with x an input vector and $\phi(x)$ the value of output.

Bewley (1977) shows that a long-lived consumer with a random income stream and a random but stationary time-separable utility, who is constrained from borrowing, will accumulate savings so that the marginal utility of income is nearly constant. In the short run, this consumer acts as if its utility is quasilinear with respect to money, and its short run demands for other goods satisfy the law of demand. Vives (1987) formalizes Marshall's idea (in his *Principles*) that consumer demands for goods with small expenditure shares are close to demands generated by quasilinear utility.

Multigood Laws of demand for a consumer

Suppose the demand of a consumer of type α is determined by maximizing a utility function u^α. The Hicksian compensated demand $h(p,\bar{u},\alpha)$ is a bundle that minimizes $p \cdot x$ subject to $u^\alpha(x) \geq \bar{u}$. Keeping the utility level fixed at $\bar{u}$, this Hicksian demand function satisfies the multi-good law of demand: (2) holds for $X(p) = h(p,\bar{u},\alpha)$. Utility maximization also guarantees that $x(\cdot,\cdot,\alpha)$ satisfies the *weak weak axiom of revealed preference*: $p \cdot x(p',w',\alpha) \leq w \Rightarrow p' \cdot x(p,w,\alpha) \geq w'$. Equivalently, for any fixed w, $X(p) = x(p,w,\alpha)$ satisfies (2) on the restricted domain with $p \cdot X(p') = w$. This is also called the compensated law of demand since the demand vector $X(p')$ remains barely affordable when the price vector changes from p' to p. The weak weak axiom is satisfied so long as the consumer maximizes a *complete* preference relation; the preferences need not be transitive. When $x(\cdot,\cdot,\alpha)$ is C^1, the following are equivalent: (i) $x(\cdot,\cdot,\alpha)$ obeys the weak weak axiom; (ii) its Slutsky matrix $S(p,w,\alpha)$ is negative semidefinite (but not necessarily symmetric); (iii) its Jacobian matrix $\partial_p x(p,w,\alpha)$ is negative semidefinite on the hyperplane orthogonal to $x(p,w,\alpha)$ (Kihlstrom et al., 1976; Brighi, 2004).

When we say that $x(\cdot,\cdot,\alpha)$ obeys the unrestricted law of demand (or law of demand, for short) we mean that for each w, $X(p) = x(p,w,\alpha)$ satisfies (2) for *all* price changes. Since this is equivalent to negative semidefiniteness of the Jacobian $\partial_p x(p,w,\alpha)$ for all p, it is stronger than simply saying that the diagonal terms of the matrix are nonpositive. Thus it is not equivalent to the one-good law of demand for every good and does not follow from the assumption that the demand for every good is normal.

Let $M(p,w,\alpha)$ be the income effects matrix, with ℓj component $[\partial_w x_\ell(p,w,\alpha)]x_j(p,w,\alpha)$. From the Slutsky decomposition, $\partial_p x(p,w,\alpha) = S(p,w,\alpha) - M(p,w,\alpha)$, we see that type α satisfies the law of demand if it satisfies the weak weak axiom and $M(p,w,\alpha)$ is positive semidefinite at each p. However, the latter condition is strong; it occurs if and only if demand is linear in income for all goods, which excludes the possibility of luxuries or necessities.

A more promising approach is to find conditions under which the Slutsky matrix always "dominates" the income effects matrix even when the latter "misbehaves." Assuming that type α has a concave utility function u^α, a sufficient and (in a sense) necessary condition for the law of demand is $-[x^T \partial^2 u^\alpha(x)x]/(\partial u^\alpha(x)x) \leq 4, \forall x$. This result was obtained independently by Milleron (1977) and Mitjuschin and Polterovich (1979) (see also Mas-Colell et al. (1995, page 145) and an alternative formulation in Kannai (1989)).

An important application of this result is in the theory of portfolio decisions. In that case, the demand bundle is the consumer's contingent consumption over L states of the world; it is standard to assume that the consumer has a von Neumann-Morgenstern utility function $u^\alpha(x) = \sum_{i=1}^{l} \pi_i v^\alpha(x_i)$, where π_i is the subjective probability of state i and $v^\alpha : R_{++} \to R$ is the Bernoulli utility function. Suppose the coefficient of relative risk aversion, $-y{v^\alpha}''(y)/{v^\alpha}'(y)$, does not *vary* by more than four on the domain of v^α. Then the consumer's demand for contingent consumption at different state prices will obey the law of demand; this in turn implies that the law of demand holds for the consumer's demand for securities, whether or not the market is complete (Quah, 2003).

Laws of market demand when the income distribution is independent of price

Consider a large economy with consumers drawn at random from a probability space $A \times \mathbb{R}_+$ of consumer types and their incomes, with distribution μ. The expected aggregate (market) demand vector at prices p is $X(p) = \int_{A\times\mathbb{R}_+} x(p, w, \alpha) d\mu$. We are interested in conditions under which X obeys the unrestricted law of demand, i.e., (2) holds for all price changes; equivalently, $\partial X(p)$ is negative semidefinite for all p. If $x(\cdot, \cdot, \alpha)$ obeys the law of demand for all α, then, clearly, so will X. One justification for studying the law of demand at the individual level is that it is preserved by aggregation.

Aggregating the Slutsky decomposition across all agents, the law of demand requires

$$v \cdot \partial X(p)v = v \cdot \left[\int_{\alpha \in A} x(p, w, \alpha) d\mu \right] v = v \cdot \bar{S}(p)v - v \cdot \bar{M}(p)v \leq 0, \quad \forall\, v \qquad (3)$$

where $\bar{S}(p) = \int S(p, w, \alpha) d\mu$ is the mean Slutsky matrix, and $\bar{M}(p)$ is the mean income effects matrix, with ℓj element $\int [\partial x_\ell(p, w, \alpha)/\partial w] x_j(p, w, \alpha)\, d\mu$. (We assume here and below that these integrals exist.) If all consumers obey the weak weak axiom, which they do if they are utility maximizers, then $S(p, w, \alpha)$ and hence $\bar{S}(p)$ are negative semidefinite; so $\partial X(p)$ is negative semidefinite if $\bar{M}(p)$ is positive semidefinite.

The matrix $\bar{M}(p)$ is determined by the consumers' Engel curves $x(p, \cdot, \alpha)$ at p. Positive semidefiniteness of this matrix is known as *increasing spread* (Hildenbrand, 1994). To see why, note that

$$2v \cdot \bar{M}(p)v = \partial_t \int [v \cdot x(p, w + t, \alpha)]^2 d\mu(\alpha, w)|_{t=0}. \tag{4}$$

We can interpret $v \cdot x(p, w, \alpha)$ as α's demand for a commodity (call it T_v), which is consumed when the other goods are consumed; specifically, the consumption of one unit of good j requires v_j units of T_v. Then $\int [v \cdot x(p, w, \alpha)]^2 d\mu$ measures the spread of the consumers' demands for T_v around the origin. By (4), $\bar{M}(p)$ is positive semidefinite if and only if for every v the consumers' demands for T_v spread out from 0 as their incomes rise. This is the multi-good generalization of normality, where the consumers' demands for a single good increase (spread from 0) as their incomes rise.

We now consider various interpretable conditions on the distribution of consumer characteristics which guarantee increasing spread (and thus the law of demand). This property holds if consumers have the same demand function and income is distributed with a nonincreasing density function ρ on $[0, \bar{w}]$ (Hildenbrand, 1983). In that case, integrating by parts, (4) becomes $2v \cdot \bar{M}(p)v = [v \cdot x(p, \bar{w}, \alpha)]^2 \rho(\bar{w}) - \int [v \cdot x(p, w, \alpha)]^2 \rho'(w) dw \geq 0$. While the nonincreasing density condition is strong, imposing some weak restrictions on the Engel curves will guarantee increasing spread for a significantly larger class of income density functions (Chiappori, 1985). However, to guarantee increasing spread for *every* non-trivial income distribution requires stringent conditions on the consumers' Engel curves: $x(p, \cdot, \alpha)$ must lie in a single plane (depending on p) and the demand for each good is either a concave or convex function of income (Freixas and Mas-Colell, 1987; Jerison, 1999).

Increasing spread is also implied by certain kinds of behavioral heterogeneity across consumers. We consider consumers with the same income w and demands of the form $x_\ell(p, w, \alpha) = e^{\alpha_\ell} \hat{x}(e^{\alpha_1} p_1, \ldots, e^{\alpha_L} p_L, w)$, where $\hat{x}$ is an arbitrary demand function and $\alpha = (\alpha_1, \ldots, \alpha_L) \in R^L$. If $\hat{x}$ is generated by some utility function $\hat{u}$, then $x(\cdot, \cdot, \alpha)$ is generated by the utility function $u^\alpha(x) = \hat{u}(e^{-\alpha_1} x_1, \ldots, e^{-\alpha_L} x_L)$. Increasing spread is guaranteed if α has a sufficiently flat density over $\mathbb{R}^L$. This condition also ensures that the mean Slutsky matrix $\bar{S}(p)$ is negative semidefinite even if $\hat{x}$, hence each $x(\cdot, \cdot, \alpha)$, violates the weak weak axiom (and so is *not* generated by a utility function). Thus when α has a sufficiently flat density, X satisfies the law of demand; in fact it can be shown that X is nearly generated by Cobb-Douglas preferences (Grandmont, 1992). Whether flatness of the α density implies heterogeneity (in some meaningful sense) of the consumers' demands depends on the behavior of $\hat{x}$ (Giraud and Quah, 2003).

Even when $\bar{M}(p)$ is not positive semidefinite, i.e., $v \cdot \bar{M}(p)v < 0$ for some v, it is clear from (3) that $v \cdot \partial X(p)v < 0$ can hold provided the substitution effects are large enough, i.e., $v \cdot \bar{S}(p)v$ is sufficiently negative. This feature can be exploited; for example, one can substantially weaken the nonincreasing density condition in Hildenbrand (1983; described above) and still obtain the law of demand if substitution effects are accounted for through restrictions on the utility function (Quah, 2000). Similarly, a large enough positive income effect can compensate for consumers' violations of the weak weak axiom, i.e., situations where, for some v, $v \cdot \bar{S}(p)\tilde{v} > 0$.

Whether the substitution effect $v\bar{S}(p)v$ dominates the income effect $v\bar{M}(p)v$ is an empirical question. The sizes of the effects must be estimated. Haerdle et. al. (1991) show how this can be done with cross section data under standard econometric assumptions, without restrictions on the functional forms of the consumer demands. In most empirical demand analyses, consumers are grouped according to observable attributes other than income, and within a group, a, the consumers' budget share vectors are assumed to have the form $b^a(p, w) + \epsilon$, where ϵ is a mean 0 random variable with distribution independent of income w. Under this assumption, a consumer's type is its attribute group and a realized value of ϵ. Within group a, the distribution of types with income w, denoted $\mu^a(\alpha|w)$, does not vary with w. Thus, if the income distribution in the group has a density ρ^a, then

$$\int \{\partial_w [v \cdot x(p, w, \alpha)]^2\} d\mu^a = \int \{\partial_w \int [v \cdot x(p, w, \alpha)]^2 d\mu^a(\alpha|w)\} \rho^a(w) dw, \forall v \in \mathbb{R}^L. \quad (5)$$

The left side of (5) equals $2v \cdot M^a(p)v$, where $M^a(p)$ is the mean income effect matrix of the consumers in group a. The right side of (5) is the mean of the derivative of $\int [v \cdot x(p, w, \alpha)]^2 d\mu^a(\alpha|w)$ with respect to w. It can be efficiently estimated by the nonparametric method of average derivatives (Stoker, 1991). The mean income effect matrix $\bar{M}(p)$ is a weighted average of the matrices $M^a(p)$, weighted by the shares of the population in the groups a. Condition (5), called metonymy, is weaker than the assumption that the budget shares have the form $b^a(p, w) + \epsilon$, so weak, in fact, that it is not potentially refutable with infinite cross section data (Evstigneev et al., 1996; Jerison, 2001). Income effect matrices estimated in this way using cross section expenditure data from several countries are all positive semidefinite (Haerdle et al., 1991; Hildenbrand and Kneip, 1993).

Laws of demand in private ownership economies

In the last section, we assumed consumer incomes to be exogenously given independent of prices. This is plainly not true in general equilibrium. For example, consider a private ownership economy with consumers drawn randomly from a distribution μ over types, where type α has the demand function $x(\cdot,\cdot,\alpha)$ and an endowment vector ω^α. If the consumers receive no profits, the income of type α at price vector p is $p\cdot\omega^\alpha$. We are interested in laws of demand that can be satisfied by the consumer sector's aggregate demand $\tilde{X}(p) = \int x(p, p\cdot\omega^\alpha, \alpha)\, d\mu$ or aggregate excess demand $\zeta(p) = \tilde{X}(p) - \bar{\omega}$, where $\bar{\omega} = \int \omega^\alpha d\mu$ is the aggregate endowment.

The first thing to note is that under standard assumptions, both $\tilde{X}$ and ζ are zero-homogeneous, and essentially for this reason, satisfy the unrestricted law of demand only in exceptional cases (Hildenbrand and Kirman, 1988). However, if the consumers' endowments are collinear (i.e., if for each α, there is some $k \geq 0$ with $\omega^\alpha = k\bar{\omega}$) then the sufficient conditions for the law of market demand given in the previous section are also sufficient for $\tilde{X}$ (and hence ζ) to satisfy (2) for p and p' in $P = \{p \in \mathbb{R}^L_{++} : p\cdot\bar{\omega} = 1\}$; in other words, the law of demand holds for mean income preserving price changes. This is so because, when endowments are collinear, a price change which preserves mean income also preserves the income of *every* agent.

When we drop the strong assumption of collinear endowments, this restricted form of the law of demand is not guaranteed even if all consumers have homothetic preferences (Mas-Colell et al., 1995, page 598). However, it *does* hold when the consumer sector has two properties: (a) all agents have homothetic preferences and (b) the preferences and endowments are independently distributed. Quah (1997) shows that this scenario can be understood as the idealization of a more general situation. The crucial feature of homothetic preferences here is that they generate demand functions which are linear in income. Retaining the independence assumption (b), one can show that when substitution effects are non-trivial (in some specific sense), $\tilde{X}$ obeys the restricted law of demand provided the mean demand of agents with identical endowments is not 'too non-linear' in income. This last property can arise from an appropriate form of heterogeneity in demand behavior, which can be modelled using the parametric framework employed by Grandmont (1987, 1992).

It is interesting to ask when aggregate consumer excess demand ζ satisfies the weak weak axiom: $p\cdot\zeta(p') \leq 0 \Rightarrow p'\cdot\zeta(p) \geq 0$. This condition ensures that the set of equilibrium prices is convex in all competitive production economies with convex technology and constant returns to scale; furthermore, it is the weakest restriction on ζ guaranteeing this conclusion (Mas-Colell et al., 1995, page 609). The sufficiency of this condition hinges on the fact that the production side of the economy satisfies the law of demand. Since the equilibrium set is generically discrete, its convexity implies generic uniqueness of equilibrium (up to scalar multiple). When ζ satisfies the weak weak axiom it also satisfies the law of demand (2) on the restricted set with $p \cdot \zeta(p') = 0$. If (2) holds strictly on this set when p and p' are not collinear, then the unique equilibrium is globally stable under tâtonnement, and there are natural comparative statics.

Using a Slutsky decomposition, it can be shown that ζ satisfies the weak weak axiom if the mean Slutsky matrix $S(p)$ is negative semidefinite (as it is if the consumers are utility maximizing) and the consumers' excess demand vectors spread apart on average when their incomes rise. The latter condition is called nondecreasing dispersion of excess demand (NDED). To formalize it, define $z(p,t,\alpha) \equiv x(p, t + p \cdot \omega^\alpha) - \omega^\alpha$, the excess demand of type α with income transfer t. The corresponding aggregate excess demand is $Z(p,t) \equiv \int z(p,t,\alpha) d\mu$. NDED holds if $\partial_t \int \{v \cdot [z(p,t,\alpha) - Z(p,t)]\}^2 d\mu|_{t=0} \geq 0$ for every $p \in \mathbb{R}^L_{++}$ and every v with $v \cdot p = 0$ and $v \cdot \zeta(p) = 0$; in other words, the income transfers raise the variance of the composite excess demands $v \cdot z(p,t,\alpha)$ (Jerison, 1999). Quah's 1997 model (described above) is an example of an economy where NDED is satisfied approximately.

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Suggested Cross References: Tatonnement, Stability, Comparative Statics, General Equilibrium, Uniqueness of Equilibrium.