Overlapping Labour Markets

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Abstract

Overlapping labour markets arise when some types of workers do not meet employers with some types of jobs. For example, skilled workers could seek high-skill or low-skill jobs, but low skill workers could be limited to low-skill jobs. The paper derives conditions for equilibrium and efficiency, distinguishes reducible from irreducible overlapping labour markets, and describes distributional impacts of proportional demand shifts and technological change. Many labour models incorporate the structure of overlapping labour markets, so that the results have widespread applicability.

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1 Introduction

A labour market can be regarded as a collection of workers and employers that interact to allocate workers to jobs on the basis of wages and qualifications. Overlapping Labour Markets depart from this simple description. An Overlapping Labour Market (hereafter OLM) arises in a labour market with heterogeneous workers and jobs when direct interactions between particular types of workers and jobs do not occur. The paper describes conditions when OLM's are inefficient and the resulting misallocation of workers and jobs. It also shows how OLM's differ from labour markets where all agents interact. In particular, demand shifts, consisting of proportional changes in numbers of jobs, have distributional impacts. Skill biased technological change has consequences in an OLM that differ from responses with ordinary markets.

In the OLM considered here, unemployed workers and employers with vacancies meet to determine whether the worker should be employed. There are two types of workers, s_1 and s_2, and two types of jobs, k_1 and k_2. Type s_1 workers seek either k_1 or k_2 jobs, and employers with k_2 jobs seek either s_1 or s_2 workers. The distinguishing feature of OLM's is that s_2 workers do not seek

k_1 jobs, presumably because production would be too low or the workers lack qualifications.

For example, s_1 workers could be college graduates while s_2 workers are high school graduates, and k_1 jobs could require high skills while k_2 jobs only require low skills. Then s_1 workers could get either k_1 or k_2 jobs, but s_2 workers would be limited to k_2 jobs. More specifically, high school graduates and college graduates take jobs as salespeople, but only college graduates take jobs as managers. High school and college graduates work as clerks, and college graduates work as teachers. Native-born and immigrants could take manual labour jobs in a country, while management jobs could go only to native-born workers. In the context of Spanish labour markets, Dolado, Jansen and Jimeno (2002) analyze a market where workers with high education and low education take unskilled jobs, while skilled jobs go to the workers with high education. Workers with land in India use their own labour and hire landless workers, while landless workers only work for landed workers (Mandal, 2001). Labour market structures related to OLM's have been studied previously. Sattinger (1985) describes a labour market where workers set reservation wages and firms set reservation skill levels, generating OLM's. With internal and external labour markets, both currently employed and external workers can take port of entry jobs, while only current workers can take other jobs (Doeringer and Piore, 1971). In dual labour markets, primary workers can seek jobs in both the primary and secondary labour markets, while rationing limits secondary workers from entering the primary labour market (Saint-Paul, 1996). In the analysis of overeducation, it is implicitly assumed that some workers with more education take jobs requiring less education (Sicherman, 1991; Hartog, 1992, Chapter 7). Gautier (2002) analyzes the impact on low-skilled workers when high-skilled workers temporarily accept simple jobs. Albrecht and Vroman (2002) analyze wage inequality in a model where low-skill workers cannot take high-skill jobs. Acemoglu (1999) considers a model in which the skill composition of workers generates differentiation in jobs, with high quality jobs unavailable to unskilled workers. Moen (2002) considers the relative welfare of good and bad workers in a labour market where they search in different markets. Shi (2002) considers a directed search model in which high tech firms receive both skilled and unskilled workers, but low tech firms only receive low skill workers. Shi concludes that the low skill workers will receive different wages in the different jobs. These examples show that many labour markets share a common internal structure, so that general results for OLM's have widespread applicability.

A market where all workers are the same and all jobs are the same is termed here an ordinary market with homogeneous agents. Labour markets with heterogeneous agents can often be analyzed as collections of ordinary markets with homogeneous agents. In frictionless assignment models, workers self-select jobs so each worker is in an ordinary market with homogeneous agents (Tinbergen, 1951, 1956; Sattinger, 1975; Teulings, 1995). For example, in Tinbergen's analysis (1956), labour market compartments form that consist of a single type of worker and a single type of job. More recently, labour markets have been considered in which a given type of worker only searches for a given type of job

(Moen, 1997; Shi, 2001; Mortensen and Wright, 2002). Each worker is again in an ordinary market with homogeneous agents, but the labour market (or compartment, to use Tinbergen's term) is now characterized as having search or matching frictions.

Ordinary markets with heterogeneous agents have also been considered in which each type of worker seeks all types of jobs and each type of employer seeks (or interviews) all types of workers. Then all workers are pooled to form the labour supply, and all jobs are pooled to form the labour demand (Sattinger, 1995; Morgan, 1995; Lu and McAfee, 1996; Shimer and Smith, 2000).

The next section establishes the equilibrium in an OLM. Section 3 relates efficiency in an OLM to efficiency in ordinary markets with homogeneous agents. If the OLM is irreducible, so that separate markets for s_1 and s_2 workers at k_2 jobs do not arise, then efficiency results when a particular condition holds. Section 4 analyzes distribution. A proportional increase in all jobs can reduce the wage differential between more skilled and less skilled workers. Production changes corresponding to skill biased technological change are also considered. Section 9 summarizes differences between results for OLM's and previous analysis and indicates future extensions.

2 Equilibrium Assignment

2.1 Solution for Ordinary Market

As a preliminary to derivation of the equilibrium assignment in an OLM, this section derives the solution for an ordinary market with homogeneous agents using the same underlying assumptions and notation as for the OLM. Suppose there are N_s workers and N_k jobs. Production occurs when a worker is matched with a job, yielding production of a. Let u be the unemployment rate for workers and let v be the vacancy rate for jobs. Workers find jobs through a matching technology described by a matching function. Let $M(vN_k, uN_s)$ be the rate at which matches occur per period if there are uN_s unemployed workers and vN_k vacant jobs (see Mortensen and Pissarides, 1999; Pissarides, 2000; Petrongolo and Pissarides, 2001; and Rogerson and Wright, 2002, for discussions of the matching function). Assume M has constant returns to scale, is an increasing function of its arguments, has decreasing marginal returns, and is zero if either uN_s or vN_k is zero. Assume matches between workers and jobs break up at the exogenous rate γ.

The steady state values of u and v are determined from

$$\begin{aligned} (1-v)N_k &= (1-u)N_s \\ \gamma(1-v)N_k &= M(vN_k, uN_s) \end{aligned} \tag{1}$$

The first condition arises because the number of matched (or non-vacant) jobs must equal the number of matched (or employed) workers in a steady state. The second condition arises because the flow of jobs becoming vacant must equal the rate of formation of new matches. The steady state conditions in 1 can be reexpressed in terms of the ratio of vacancies to unemployed, often referred to as market tightness. Let $\theta = vN_k/(uN_s)$. Since it has constant returns to scale, $M(vN_k, uN_s)/(uN_s) = M(vN_k/uN_s, 1)$. Let $m(\theta) = M(\theta, 1)$. Then $m(\theta)$ is the rate at which unemployed workers get matches, and $m(\theta)/\theta$ is the rate at which vacancies get matches. Assume $m(\theta)$ is invertible, i.e. that $m(\theta) = x$ can be solved for θ as a function of x. Using θ and $m(\theta)$, 1 becomes

$$\begin{aligned}(1-v)u\theta/v &= 1-u \\ \gamma(1-v)\theta/v &= m(\theta)\end{aligned} \tag{2}$$

Solving 2 yields the steady state unemployment and vacancy rates given θ:

$$u = \frac{\gamma}{\gamma + m(\theta)}, \quad v = \frac{\gamma}{\gamma + m(\theta)/\theta} \tag{3}$$

From differentiation and assumptions about the matching function, it can be shown that $\partial u/\partial\theta < 0$ and $\partial v/\partial\theta > 0$. The measure of market tightness, θ, depends on the ratio of jobs to workers in the market. Define $\eta = N_k/N_s$ as the market ratio. Then

$$\theta = v\eta/u \tag{4}$$

where u and v are functions of θ determined by 3. Implicit differentiation yields[1]

$$\frac{d\theta}{d\eta} = \frac{(\gamma + m(\theta))^2}{\gamma(\gamma + m'(\theta) + m(\theta) - \theta m'(\theta))} > 0 \tag{5}$$

Comparisons between markets are summarized as follows, using subscripts to denote different markets.

Theorem 1 *Let η_1 and η_2 be the market ratios for two ordinary markets with homogeneous agents. If $\eta_1 = \eta_2$ then $\theta_1 = \theta_2$, $u_1 = u_2$ and $v_1 = v_2$. If $\eta_1 > (<)\eta_2$ then $\theta_1 > (<)\theta_2$, $u_1 < (>)u_2$ and $v_1 > (<)v_2$.*

Proof. Since θ is a monotonic function of η, two markets will have the same market tightness whenever the market ratios in the two markets are the same. If $\eta_1 > (<)\eta_2$, comparisons between θ_1 and θ_2 arise from 5, and then comparisons between unemployment and vacancy rates arise from 3, completing the proof.

Wages and profits are determined through generalized Nash bargaining over the surplus from a match as follows (Pissarides, 2000, pp. 15-17).[2] Let w be the wage rate and let W_u and W_e be a worker's expected present discounted values of income from being unemployed and employed, respectively. Then W_u and W_e satisfy

$$\begin{aligned}rW_U &= m(\theta)(W_E - W_U) \\ rW_E &= w + \gamma(W_U - W_E)\end{aligned} \tag{6}$$

[1] The derivatives of M with respect to its arguments are positive by assumption so that $m'(\theta) = M_1(\theta, 1) > 0$ when $\theta > 0$. By Euler's theorem, since M is homogeneous of degree one, $M(\theta, 1) = \theta M_1(\theta, 1) + M_2(\theta, 1)$. Then $M_2(\theta, 1) = M(\theta, 1) - \theta M_1(\theta, 1) = m(\theta) - \theta m'(\theta) > 0$. The denominator in $d\theta/d\eta$ is therefore positive, so that $d\theta/d\eta > 0$.

[2] It would also be possible to analyze OLM's assuming wage posting.

where r is the discount rate for workers. In 6, the flow of value for unemployed workers, given by rW_U, equals the transition rate into employment times the gain in value. The flow of value for employed workers, rW_E, equals the wage rate plus the transition rate into unemployment times the loss in value from the transition. Similarly, let W_F and W_V be the employers' present discounted values for a job that is filled and vacant, respectively. Then W_F and W_V satisfy

$$\begin{aligned} rW_V &= \frac{m(\theta)}{\theta}(W_F - W_V) \\ rW_F &= a - w + \gamma(W_V - W_F) \end{aligned} \tag{7}$$

In generalized Nash bargaining, the flow value rW_U serves as the reservation wage for workers and the flow value rW_V serves as the reservation profit for firms. Let β be the relative measure of labour's bargaining strength. Then the wage determined by bargaining satisfies

$$w = rW_U + \beta(a - rW_U - rW_V) \tag{8}$$

Substituting the solutions for W_U from 6 and W_V from 7 into 8 and solving yields

$$w = \frac{a\beta(r + \gamma + m(\theta))}{\beta(r + \gamma + m(\theta)) + (1 - \beta)(r + \gamma + m(\theta)/\theta)} \tag{9}$$

As a simplification, consider the wage in the limit as the discount rate approaches zero. Then

$$w = \frac{a\beta}{(1 - \beta)/\theta + \beta} \tag{10}$$

Then from 10 and 5,

$$dw/d\theta > 0, \quad dw/d\eta > 0 \tag{11}$$

Let

$$Y_s = (1 - u)w, \quad Y_k = (1 - v)(a - w) \tag{12}$$

Then Y_s is the expected income of a worker, calculated as the proportion of time employed times the wage rate. It can be shown that Y_s is the limit of rW_U as r approaches zero. Similarly, Y_k is the expected income (or profit) for a job. In terms of expected incomes, the equilibrium wage rate determined through bargaining satisfies

$$w = Y_s + \beta(a - Y_s - Y_k) \tag{13}$$

As θ increases, u decreases and v increases (from 3) and w increases (from 10). Then applying 5,

$$dY_s/d\theta > 0, \quad dY_k/d\theta < 0, \quad dY_s/d\eta > 0, \quad dY_k/d\eta < 0 \tag{14}$$

The matching function $m(\theta)$ drops out of the ratio of expected incomes, which simplifies to

$$Y_s/Y_k = \beta\theta/(1 - \beta) \tag{15}$$

2.2 Solution for OLM

Now suppose there are two types of workers, s_1 and s_2, and two types of jobs, k_1 and k_2. Suppose there are N_{s1} and N_{s2} workers of types s_1 and s_2, respectively, and N_{k1} and N_{k2} jobs of types k_1 and k_2, respectively. Let a_{ij} be the production rate when an s_i worker is matched with a k_j job. Matches between s_2 workers and k_1 jobs do not form, generating an OLM. Assume $a_{11} > a_{12} > a_{22}$, so that type s_1 workers are more productive than type s_2, and type k_1 jobs are more productive than type k_2. Then the more productive workers have access to both types of jobs, while the less productive workers can only get the less productive jobs.[3] Assume that the same matching function as in Section 2.1 operates in any submarket where workers of a single type are matched with jobs of a single type.

For the basic OLM model, suppose s_1 workers choose either to seek k_1 jobs or k_2 jobs, but not both. Similarly, suppose employers with k_2 jobs can choose either to seek s_1 workers or s_2 workers, but not both. (Section 3.3 will consider alternatives to these assumptions and their consequences.) Given these assumptions, the OLM reduces to three ordinary submarkets with homogeneous agents. Subscripts ij will be used to represent the value of a parameter or variable in the submarket where type s_i workers are matched with type k_j jobs. Let δ be the proportion of type s_1 workers seeking k_2 jobs, and let μ be the proportion of k_2 jobs for which employers seek s_1 workers. Then the market ratios in the three submarkets are

$$\eta_{11} = \frac{N_{k1}}{(1-\delta)N_{s1}}, \quad \eta_{12} = \frac{\mu N_{k2}}{\delta N_{s1}}, \quad \eta_{22} = \frac{(1-\mu)N_{k2}}{N_{s2}} \tag{16}$$

Given these market ratios, the market tightness ratios θ_{ij} in each submarket can be determined by solving 4, the unemployment and vacancy rates can be determined from 3, and the wage rates and expected incomes from 10 and 13. Equilibrium in the OLM requires that s_1 workers be indifferent between seeking k_1 jobs or k_2 jobs. This occurs when the expected incomes are equal. Equilibrium also requires that employers with k_2 jobs be indifferent between seeking s_1 and s_2 workers, so that the expected incomes must be equal. Equilibrium in the OLM therefore occurs when δ and μ are such that

$$Y_{s11} = Y_{s12}, \quad Y_{k12} = Y_{k22} \tag{17}$$

Equilibrium can be calculated and existence and uniqueness demonstrated using the following procedure. Consider a tentative level of market tightness in the second submarket, $\widehat{\theta}_{12}$. First, all of the other variables in the OLM, including δ and μ, can be derived from $\widehat{\theta}_{12}$. From 3 and 13, $\widehat{\theta}_{12}$ determines u_{12} and w_{12} and therefore $Y_{s12} = (1 - u_{12})w_{12}$. By the equilibrium condition 17, $Y_{s11} = Y_{s12}$. Then θ_{11} can be determined from Y_{s11} and η_{11} from 4. Since

[3] If instead $a_{11} < a_{12} < a_{22}$, an alternative configuration arises in which the less productive workers have access to both job types, while the most productive workers are limited to the most productive jobs.

$\eta_{11} = N_{k1}/((1-\delta)N_{s1})$, δ can be found as $1 - N_{k1}/(\eta_{11}N_{s1})$. Similarly, $\widehat{\theta}_{12}$ determines v_{12} and $Y_{k12} = (1-v_{12})(a_{12}-w_{12}) = Y_{k22}$. Then Y_{22} determines θ_{22}, which determines η_{22}, and μ can be determined as $1 - N_{s2}\eta_{22}/N_{k2}$.

Second, with all the other variables in the OLM derived from $\widehat{\theta}_{12}$, it is also possible to derive a value of θ_{12}. Let $H(\widehat{\theta}_{12})$ be the derived value of θ_{12}. It is found by solving 4 for θ, with $\eta = \mu N_{k2}/(\delta N_{s1})$, using the derived values of δ and μ. Equilibrium occurs when $H(\widehat{\theta}_{12}) = \widehat{\theta}_{12}$.

Theorem 2 *If there is a level of $\widehat{\theta}_{12}$ at which the derived values of δ and μ are both positive, then an equilibrium for the OLM exists and is unique.*

Proof. From Section 2.1 and the derivations of δ and μ, δ is an increasing function of $\widehat{\theta}_{12}$ and μ is a decreasing function of $\widehat{\theta}_{12}$. When the derived values δ and μ are positive, $\mu N_{k2}/(\delta N_{s1})$ is a declining function of $\widehat{\theta}_{12}$, so that $H(\widehat{\theta}_{12})$ is also a declining function of $\widehat{\theta}_{12}$.[4] Using 17, it is possible to determine a value of $\widehat{\theta}_{12}$ at which $\delta = 0$ and a different value of $\widehat{\theta}_{12}$ at which $\mu = 0$. If there is a value of $\widehat{\theta}_{12}$ such that δ and μ are positive, there will be a lower value of $\widehat{\theta}_{12}$ at which $\delta = 0$ and a higher value of $\widehat{\theta}_{12}$ at which $\mu = 0$. The equilibrium value of $\widehat{\theta}_{12}$, if it exists, must lie between these values. As $\widehat{\theta}_{12}$ approaches the lower value (from above), $H(\widehat{\theta}_{12})$ approaches infinity; as $\widehat{\theta}_{12}$ approaches the higher value (from below), $H(\widehat{\theta}_{12})$ approaches zero. At an intermediate value, $H(\widehat{\theta}_{12}) = \widehat{\theta}_{12}$, establishing existence. Since $H(\widehat{\theta}_{12})$ is a declining function of $\widehat{\theta}_{12}$, the solution with positive δ and μ is unique, completing the proof.

Figure 1 shows the determination of the equilibrium value of θ_{12} at point A, from which all the other variables can be derived.[5] The downward sloping curve is $H(\widehat{\theta}_{12})$, and the upward sloping line is $\widehat{\theta}_{12}$ (i.e., the line with slope one).

3 Efficiency

3.1 Marginal Products

Production in the OLM can be calculated as

$$y = N_{s1}(1-\delta)(1-u_{11})a_{11} + N_{s1}\delta(1-u_{12})a_{12} + N_{s2}(1-u_{22})a_{22} \tag{18}$$

where unemployment and vacancy rates are functions of the numbers of different types of workers and jobs and the proportions δ and μ.

[4] The requirement that δ and μ be positive for some value of $\widehat{\theta}_{12}$ rules out the case where no s_1 workers seek k_2 jobs and no employers with k_2 jobs seek s_1 workers.

[5] Figure 1 assumes $N_{s1} = N_{k2} = 2$, $N_{k1} = N_{s2} = 1$, $a_{11} = 2$, $a_{12} = 1.5$, $a_{22} = 1$, $\beta_{11} = \beta_{12} = \beta_{22} = .5$, $\gamma = .05$, and $m(\theta) = \theta^{1/2}$. The equilibrium occurs at $\delta = .496$, $\mu = .508$, and $\theta_{12} = 1.594$. In Figure 1, the minimum value of $\widehat{\theta}_{12}$ is at .0033, where $\delta = 0$, and the maximum value is at 607, where $\mu = 0$.

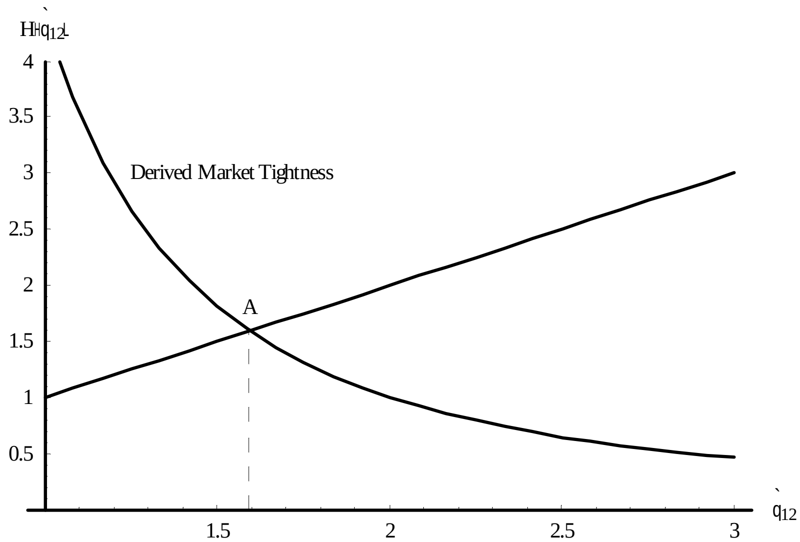

Figure 1: Determination of Equilibrium Market Tightness, θ_{12}

Using 18, it is possible to obtain the marginal products of k_1 jobs or s_2 workers through differentiation:

$$
\begin{aligned}
MP_{k1} &= \frac{dy}{dN_{k1}} = -N_{s1}(1-\delta)a_{11}\frac{\partial u_{11}}{\partial N_{k1}} - N_{s1}\delta a_{12}\frac{\partial u_{12}}{\partial N_{k1}} - N_{s2}a_{22}\frac{\partial u_{22}}{\partial N_{k1}} \quad (19)\\
MP_{s2} &= \frac{dy}{dN_{s2}} = (1-u_{22})a_{22} - N_{s1}(1-\delta)a_{11}\frac{\partial u_{11}}{\partial N_{s2}} \\
&\quad -N_{s1}\delta a_{12}\frac{\partial u_{12}}{\partial N_{s2}} - N_{s2}a_{22}\frac{\partial u_{22}}{\partial N_{s2}} \quad (20)
\end{aligned}
$$

For s_1 workers, separate marginal products can be calculated depending on whether the workers match with k_1 or k_2 jobs. These marginal products are combinations of the derivatives of y with respect to N_{s1} and δ. Changes in N_{s1} and δ affect both the number of workers with k_1 jobs and the number with k_2 jobs. A combination that increases the number with k_1 jobs by one and leaves the number with k_2 jobs the same solves

$$
\begin{aligned}
(1-\delta)\triangle N_{s1} - N_{s1}\triangle\delta &= 1 \\
\delta\triangle N_{s1} + N_{s1}\triangle\delta &= 0 \quad (21)
\end{aligned}
$$

The solution is $\triangle N_{s1} = 1$, $\triangle\delta = -\delta/N_{s1}$. Then

$$
MP_{s11} = \frac{dy}{dN_{s1}}\triangle N_{s1} + \frac{dy}{d\delta}\triangle\delta = \frac{dy}{dN_{s1}} - \frac{\delta}{N_{s1}}\frac{dy}{d\delta} \quad (22)
$$

Similarly,

$$MP_{s12} = \frac{dy}{dN_{s1}} + \frac{1-\delta}{N_{s1}}\frac{dy}{d\delta} \quad (23)$$

$$MP_{k12} = \frac{dy}{dN_{k2}} - \frac{\mu}{N_{k2}}\frac{dy}{d\mu} \quad (24)$$

$$MP_{k22} = \frac{dy}{dN_{k2}} + \frac{1-\mu}{N_{k2}}\frac{dy}{d\mu} \quad (25)$$

These marginal products are used in the next section to establish conditions for efficiency in an OLM.

3.2 Conditions for Efficiency

Definition 3 *An OLM is efficient if the allocation of workers and jobs among submarkets maximizes production.*

A necessary condition for efficiency is that an s_1 worker must have the same marginal product whether seeking a k_1 job or a k_2 job, i.e. $MP_{s11} = MP_{s12}$. Otherwise movement of an s_1 worker from one submarket to another would raise production. Similarly, $MP_{k12} = MP_{k22}$ is also a necessary condition for efficiency. By inspection of 22 and 23, $MP_{s11} = MP_{s12}$ if and only if $dy/d\delta = 0$. By inspection of 24 and 25, $MP_{k12} = MP_{k22}$ if and only if $dy/d\mu = 0$.[6]

Now consider whether decisions of workers and employers would satisfy the necessary conditions for efficiency in an OLM. Workers of type s_1 seek k_1 or k_2 jobs depending on which offers a higher expected income, Y_{s11} or Y_{s12}. Their behavior brings the two expected incomes into equality, a condition for equilibrium in 17. Similarly, behavior of employers with type k_2 jobs brings expected incomes Y_{k12} and Y_{k22} into equality. From Section 2.1, the expected incomes equal the reservation wages and profits that workers and employers would choose when searching for matches. If the reservation wages and profits equal the corresponding marginal products, then worker and employer behavior would lead to the satisfaction of the necessary conditions for efficiency.

The conditions for reservation wages and profits to equal the corresponding marginal products have been established in the literature on search congestion for ordinary markets with homogeneous agents. Search congestion arises when an agent's reservation wage or profit differs from the agent's marginal product, leading to overentry or underentry in the market. Diamond (1982) established conditions for the absence of search congestion under the assumption that the surplus from a match is shared equally (see also Mortensen, 1982, and Lockwood, 1986, 1999). Hosios (1990, p. 286) demonstrated that search congestion is absent if the share of the match surplus going to a worker equals the elasticity of the matching function with respect to the number of unemployed workers. In Section 2.1, the match surplus share is given by β,and in the OLM the share in the submarket with worker type s_i and job type k_j is β_{ij}.

[6] The conditions $dy/d\delta = 0$ and $dy/d\mu = 0$ are sufficient for efficiency if they uniquely determine δ and μ.

Market mechanisms exist that would bring about the match surplus shares satisfying the Hosios condition. Sattinger (1990, pp. 366-369) describes a non-bargaining mechanism where employers can attract more applicants to a vacancy by offering a higher wage prior to a match. The employer first-order condition in equilibrium requires that expected incomes be proportional to marginal contributions to interviews, which is equivalent to the Hosios condition. Moen (1997) describes a more rigorous model in which firms advertise the wages they pay. Other market mechanisms, such as brokers, may also bring about efficiency. Efficiency conditions for markets with heterogeneous agents have been derived by Moen (1997), Shi (2001) and Mortensen and Wright (2002). In these models, agents of a given type on one side of the market only trade with agents of a single type on the other side. All trade therefore takes place in ordinary markets with homogeneous agents.

Efficiency in an OLM can be directly related to these results. If the match surplus share in each submarket satisfies the Hosios condition, then reservation wages and profits in each submarket equal the corresponding marginal products, yielding efficiency. Specifically, when the β_{ij} satisfy the Hosios condition for the submarket with s_i workers and k_j jobs, $Y_{sij} = MP_{sij}$ and $Y_{kij} = MP_{kij}$. Then $dy/d\delta = dy/d\mu = 0$, so that the OLM will be efficient.

3.3 Irreducible Overlapping Labour Markets

The previous section indicates that if wage determination takes place through bargaining and if the match surplus shares do not satisfy the Hosios condition, then inefficiency results. The consequences for the submarkets in an OLM are the same as for ordinary markets with homogeneous workers and are already described in the literature.

This section considers a second potential source of inefficiency that is particular to an OLM. In the derivation of equilibrium in Section 2.2, unemployed s_1 workers can distinguish k_1 jobs from k_2 jobs before they seek employment, and choose between the two submarkets. This is a form of directed search, applying to submarkets rather than individual employers. The same assumption is applied to employers with k_2 jobs, allowing them to choose whether to interview s_1 or s_2 workers. This generates two submarkets, one where employers with k_2 jobs interview and hire s_1 workers, and one where they interview and hire s_2 workers.

Now consider an alternative OLM in which employers with k_2 jobs cannot distinguish between s_1 and s_2 workers prior to job interviews. For example, employers with clerk jobs may not require a college degree, so that applicants with and without a college degree would apply. This would occur if employers passively await job applicants, employers lack information about applicants prior to interviews, or there are legal restrictions preventing unequal treatment at the interview stage. As a result, separate submarkets for s_1 and s_2 workers at k_2 jobs do not form, and employers with k_2 jobs face a labor market with pooled s_1 and s_2 workers. If no pooling occured in actual labour markets, employers would know everything relevant about prospective workers before interviews, and interviews would serve no role in selecting workers. Some degree of pooling is therefore likely in actual labour markets. The following definition formalizes the distinction between the two forms of an OLM.

Definition 4 *In a reducible Overlapping Labour Market, agents distinguish among types of agents with whom they match, so that the Overlapping Labour Market resolves into submarkets with one type of worker and one type of job.*

The OLM considered in this section is irreducible since separate submarkets for s_1 and s_2 workers seeking k_2 jobs will not form. Equilibrium in this irreducible OLM is determined as follows. As before, s_1 workers choose between seeking k_1 and k_2 jobs. The proportion seeking k_2 jobs, δ, then satisfies $Y_{s11} = Y_{s12}$ in equilibrium. Employers with k_2 jobs will hire both s_1 and s_2 workers as they arrive for interviews, rather than some employers hiring s_1 workers and some employers hiring s_2 workers. Employers will then face a common vacancy rate in the market, v_2. Since the s_1 and s_2 workers seeking k_2 jobs are pooled in a single labor market, they will also experience a common unemployment rate, u_2. The number of s_1 and s_2 workers seeking k_2 jobs is $\delta N_{s1} + N_{s2}$. Assume the matching function in this submarket, with pooled s_1 and s_2 workers, is the same as in the other submarkets. Substituting the market ratio $\eta_{12} = N_{k2}/(\delta N_{s1} + N_{s2})$ into 4 determines θ_{12}, and then u_2 and v_2 are found from 3. Although μ is no longer determined by employers with k_2 jobs, the proportion of k_2 jobs going to s_1 workers can be calculated as

$$\phi = \frac{(1-u_2)\delta N_{s1}}{(1-u_2)\delta N_{s1} + (1-u_2)N_{s2}} = \frac{\delta N_{s1}}{\delta N_{s1} + N_{s2}} \tag{26}$$

The wages for s_1 and s_2 workers at k_2 jobs are calculated somewhat differently from Section 2.1. Because of differences in outputs a_{12} and a_{22}, employers with k_2 jobs will pay different wages to s_1 and s_2 workers after bargaining. As before, let w_{12} and w_{22} be the wages for s_1 and s_2 workers, respectively. Then $Y_{s12} = (1-u_2)w_{12}$ and $Y_{s22} = (1-u_2)w_{22}$. The expected profit rate for an employer with a filled k_2 job is $\phi(a_{12} - w_{12}) + (1-\phi)(a_{22} - w_{22})$. Under the simplifying assumption that the interest rate is zero, the reservation profit level used by the employer in bargaining is

$$Y_{k2} = (1-v_2)\left(\phi(a_{12} - w_{12}) + (1-\phi)(a_{22} - w_{22})\right) \tag{27}$$

As before, let β_{12} and β_{22} be the match surplus shares going to s_1 and s_2 workers at k_2 jobs, respectively. Then in equilibrium the wages must satisfy

$$w_{12} = (1-u_2)w_{12} + \beta_{12}(a_{12} - (1-u_2)w_{12} - Y_{k2}) \tag{28}$$

$$w_{22} = (1-u_2)w_{22} + \beta_{22}(a_{22} - (1-u_2)w_{22} - Y_{k2}) \tag{29}$$

Equilibrium in an irreducible OLM can be determined as follows. Consider an arbitrary value of δ, so that $\eta_{11} = N_{k1}/((1-\delta)N_{s1})$. Then conditions in Section 2.1 for the ordinary market with homogeneous agents determine u_{11},

v_{11}, w_{11}, and Y_{s11}. In the pooled second submarket, δ determines θ_{12} as before, ϕ from 26, u_2 and v_2 from 3, and w_{12} and w_{22} from 28 and 29 (with Y_{k2} given by 27). Then Y_{s12} can be calculated as $(1-u_2)w_{12}$. As δ goes from 0 to 1, s_1 workers move from the first submarket to the second pooled submarket, increasing Y_{s11} and decreasing Y_{s12}. Equilibrium in the irreducible OLM occurs at the value of δ such that $Y_{s11} = Y_{s12}$.

3.4 Inefficiency in an Irreducible Overlapping Labour Market

Inefficiency in an irreducible OLM will be analyzed by comparison with an efficient reducible OLM. The following theorem provides a benchmark case in which the equilibrium solutions to a reducible OLM and an irreducible OLM with the same parameters will be identical.

Theorem 5 *Assume that a reducible OLM has an equilibrium with*

$$\mu = \frac{\delta N_{s1}}{\delta N_{s1} + N_{s2}} \tag{30}$$

Then i. the reducible OLM has $\eta_{12} = \eta_{22,}$ $u_{12} = u_{22}$, and $v_{12} = v_{22}$; ii. the equilibrium solution for the reducible OLM will also be the equilibrium solution for the irreducible OLM.

Proof. Given 30, the market ratios in the reducible OLM are

$$\eta_{12} = \frac{\mu N_{k2}}{\delta N_{s1}} = \frac{\delta N_{s1}}{\delta N_{s1} + N_{s2}} \frac{N_{k2}}{\delta N_{s1}} = \frac{N_{k2}}{\delta N_{s1} + N_{s2}} \tag{31}$$

$$\eta_{22} = \frac{(1-\mu) N_{k2}}{N_{s2}} = \left(1 - \frac{\delta N_{s1}}{\delta N_{s1} + N_{s2}}\right) \frac{N_{k2}}{N_{s2}} = \frac{N_{k2}}{\delta N_{s1} + N_{s2}} \tag{32}$$

Since $\eta_{12} = \eta_{22,}$ the unemployment and vacancy rate solutions will be the same, establishing i. Let δ_R be the equilibrium value of δ in the reducible OLM and let η_2 be the market ratio for the pooled submarket with both s_1 and s_2 workers. Then at $\delta = \delta_R$,

$$\eta_2 = \frac{N_{k2}}{\delta_R N_{s1} + N_{s2}} = \eta_{12} = \eta_{22} \tag{33}$$

Since the market ratios are the same, the pooled submarket has the same unemployment and vacancy rates as the submarkets in the reducible OLM. Because the reducible OLM was in equilibrium, $Y_{k12} = Y_{k22}$ or

$$(1 - v_{12})(a_{12} - w_{12}) = (1 - v_{22})(a_{22} - w_{22}) \tag{34}$$

Since $v_{12} = v_{22}$ from part i, $a_{12} - w_{12} = a_{22} - w_{22}$. Then the expected income for an employer with a k_2 job in the irreducible OLM is

$$\begin{aligned} Y_{k2} &= (1 - v_2) \frac{(1-u_2)(a_{12} - w_{12})\delta_R N_{s1} + (1-u_2)(a_{22} - w_{22}) N_{s2}}{(1-u_2)\delta_R N_{s1} + (1-u_2) N_{s2}} \\ &= (1 - v_2)(a_{12} - w_{12}) = (1 - v_2)(a_{22} - w_{22}) = Y_{k12} = Y_{k22} \end{aligned} \tag{35}$$

Then the wages w_{12} and w_{22} from the reducible OLM will also satisfy the bargaining conditions in the irreducible OLM. At $\delta = \delta_R$, Y_{s12} will then be the same in the irreducible OLM as in the reducible OLM, and Y_{s11} will continue to be the same. Then δ_R will also be the equilibrium value of δ in the irreducible OLM and the unemployment, vacancy and wage rates will also be the same, completing the proof.

Essentially, the irreducible OLM has the same solution as the reducible OLM when 30 holds because employers with k_2 jobs, if they could choose which workers to seek, would seek the same proportion of s_1 workers as generated by the pooled submarket in 26. The next theorem is a companion to theorem 5, identifying a condition in the irreducible OLM (instead of in the reducible OLM) that indicates when the two solutions will be the same.

Theorem 6 *Assume that an irreducible OLM has an equilibrium with $\delta = \delta_I$ and*

$$a_{12} - w_{12} = a_{22} - w_{22} \tag{36}$$

Then i. the reducible OLM with the same parameters has $\mu = \delta_I N_{s1}/(\delta_I N_{s1} + N_{s2})$ and ii. the equilibrium solution for the reducible and irreducible OLM's are the same.

Proof. Since the irreducible OLM is in equilibrium, $Y_{s12} = Y_{s11}$, or $(1 - u_2)w_{12} = (1 - u_{11})w_{11}$. At $\delta = \delta_I$ and $\mu = \delta_I N_{s1}/(\delta_I N_{s1} + N_{s2})$, a reducible OLM with the same parameters will have

$$\begin{aligned} \eta_{12} &= \frac{\mu N_{k2}}{\delta_I N_{s1}} = \frac{\delta_I N_{s1}}{\delta_I N_{s1} + N_{s2}} \frac{N_{k2}}{\delta_I N_{s1}} = \frac{N_{k2}}{\delta_I N_{s1} + N_{s2}} = \eta_2 \\ \eta_{22} &= \frac{(1-\mu) N_{k2}}{N_{s2}} = \frac{N_{s2}}{\delta_I N_{s1} + N_{s2}} \frac{N_{k2}}{N_{s2}} = \frac{N_{k2}}{\delta_I N_{s1} + N_{s2}} = \eta_2 \end{aligned} \tag{37}$$

Thus $\eta_{12} = \eta_{22}$ in the reducible OLM and both market ratios equal η_2 in the irreducible OLM. Then $u_{12} = u_{22} = u_2$ by Theorem 1. Using the values of variables in the irreducible OLM, the expected incomes for employers with k_2 jobs are $Y_{k12} = (1 - u_{12})(a_{12} - w_{12}) = (1 - u_2)(a_{12} - w_{12})$ and $Y_{k22} = (1 - u_2)(a_{22} - w_{22})$. By 36, $Y_{k12} = Y_{k22}$. With $\delta = \delta_I$ and $\mu = \delta_I N_{s1}/(\delta_I N_{s1} + N_{s2})$, the equilibrium conditions for the reducible OLM in 17 are satisfied by the equilibrium solution for the irreducible OLM, completing the proof.

With theorems 5 and 6 in place, conditions for an irreducible OLM to be efficient can now be identified.

Theorem 7 *An irreducible OLM is efficient if 36 holds and the reducible OLM with the same parameters is efficient.*

Proof. Given 36, the solution to the irreducible OLM will be the same as the solution to the reducible OLM with the same parameters, by theorem 6. If the reducible OLM is efficient, $MP_{s11} = MP_{s12}$ and $MP_{k12} = MP_{k22}$. Since the allocation is the same, these conditions also hold for the irreducible OLM with the same solution, establishing efficiency and completing the proof.

The misallocation arising when 30 does not hold can be briefly described. Suppose a reducible OLM is efficient and $\mu > \delta N_{s1}/(\delta N_{s1} + N_{s2})$. Then in the irreducible OLM with the same parameters, fewer k_2 jobs will be allocated to s_1 workers than for efficiency. The misallocation arises because, in the irreducible OLM, employers with k_2 jobs cannot shift interviews to s_1 workers, for whom the returns are higher. The misallocation of k_2 jobs in this case generates a misallocation of s_1 workers. The unemployment rate for s_1 workers in the irreducible OLM will be given by the higher pooled rate, u_2, reducing Y_{s12}. Then for equilibrium in the irreducible OLM, there will be fewer s_1 workers seeking k_2 jobs than for efficiency. In response to these misallocations, it is possible to identify policies that would move the labor market towards an efficient allocation. One policy would be to tax employment of s_1 workers at k_1 jobs. This would reduce Y_{s11} (at a given value of δ) and induce more s_1 workers to seek k_2 jobs, raising δ. While δ may approach its efficient level, there could still be misallocations of k_2 jobs between s_1 and s_2 workers, caused by pooling in the irreducible OLM. Although the policy of taxing s_1 workers at k_1 jobs could increase production, it would have distributional effects on incomes of workers and employers.

4 Distribution

4.1 Supply and Demand Shifts

An OLM differs from an ordinary market with homogeneous agents in that proportional changes in all types of jobs can affect the distribution of earnings among workers. In an ordinary market with homogeneous agents, changes in supply or demand affect the distribution of income between workers and employers. However, questions about the distribution of earnings among workers are absent for the simple reason that all workers are affected uniformly (at least ex ante). This section analyzes the impact of proportional changes in all workers or jobs.

Specifically, assume

$$N_{kj} = \rho N_{kj0}, \quad j = 1, 2 \tag{38}$$

where N_{kj0} is fixed, $j = 1, 2$. An increase in ρ therefore represents a proportional expansion in the numbers of both types of jobs. The effect of ρ on the wage rates can be decomposed as

$$\frac{dw_{ij}/d\rho}{w_{ij}} = \frac{dw_{ij}/d\theta_{ij}}{w_{ij}} \frac{d\theta_{ij}}{d\eta_{ij}} \left(\frac{\partial \eta_{ij}}{\partial \rho} + \frac{\partial \eta_{ij}}{\partial \delta} \frac{d\delta}{d\rho} + \frac{\partial \eta_{ij}}{\partial \mu} \frac{d\mu}{d\rho} \right) \tag{39}$$

From 10,

$$\frac{dw_{ij}/d\theta_{ij}}{w_{ij}} = \frac{1 - \beta_{ij}}{(1 - \beta_{ij})\theta_{ij} + \beta_{ij}\theta_{ij}^2} \tag{40}$$

This result immediately yields the following proposition.

Proposition 8 ***Theorem 9*** *Comparing two ordinary markets with homogeneous labour that experience the same increase in the tightness ratio θ, the labour market with the lower tightness ratio (and higher unemployment rate) will experience the greater proportional wage increase.*

From 5, $d\theta_{ij}/d\eta_{ij} > 0$. The market ratios are proportional to ρ so that $\partial\eta_{ij}/\partial\rho > 0$, $i, j = 1, 2$. The other partial derivatives in the brackets in 39 depend on the specific submarket. From 16,

$$\begin{aligned} \frac{\partial\eta_{11}}{\partial\delta} &= \frac{\eta_{11}}{1-\delta} > 0, \quad \frac{\partial\eta_{12}}{\partial\delta} = \frac{\eta_{12}}{\delta} > 0, \quad \frac{\partial\eta_{22}}{\partial\delta} = 0 \\ \frac{\partial\eta_{11}}{\partial\mu} &= 0, \quad \frac{\partial\eta_{12}}{\partial\mu} = \frac{\eta_{12}}{\mu} > 0, \quad \frac{\partial\eta_{22}}{\partial\mu} = \frac{-\eta_{22}}{1-\mu} < 0 \end{aligned} \tag{41}$$

Now consider how δ and μ change when ρ goes up. In hierarchical assignment models, more productive workers are generally assigned to more productive jobs. With a top-down assignment, an increase in numbers of all jobs results in workers being reassigned to more productive jobs. The same phenomenon is at work in an OLM. Type s_1 workers at k_2 jobs serve as a potential source of workers for k_1 jobs. When numbers of jobs increase, s_1 workers shift from k_2 jobs to k_1 jobs (if the difference in output $a_{11} - a_{12}$ is sufficiently large). Then δ is smaller for larger ρ. The decline in s_1 workers at k_2 jobs leads employers with k_2 jobs to switch to s_2 workers, reducing μ.

Now compare the proportional changes in w_{11} and w_{22} in moving to a higher ρ. If $\theta_{11} > \theta_{22}$, theorem 8 implies that

$$\frac{dw_{11}/d\theta_{11}}{w_{11}} < \frac{dw_{22}/d\theta_{22}}{w_{22}} \tag{42}$$

Since $\partial\eta_{11}/\partial\delta > 0$, the decline in δ contributes to a lower proportional change in w_{11}. Since $\partial\eta_{22}/\partial\mu < 0$, the decline in μ contributes to a greater proportional change in w_{22}. On the basis of these comparisons, if $\theta_{11} > \theta_{22}$, a proportional increase in jobs would reduce w_{11}/w_{22}. A proportional increase in numbers of workers, N_{s1} and N_{s2}, would have the same effects as a reduction in ρ.

Table 1 shows the distributional effects of $\rho = 1.02$ versus $\rho = .98$ using the same assumptions as for Figure 1. At the higher level of ρ, w_{11}/w_{22} is lower, consistent with the foregoing analysis and the declines in δ and μ. Using expected earnings to compare s_1 and s_2 workers, Y_{s11}/Y_{s22} is also lower at a higher ρ. Thus proportional changes in numbers of jobs or numbers of workers have distributional impacts in an OLM.

Table 1

Proportional Factor	$\rho = .98$	$\rho = 1.02$
δ	.500	.493
μ	.512	.504
w_{11}	.794	1.060
u_{11}	.059	.045
w_{12}	.783	1.045
u_{12}	.046	.031
w_{22}	.297	.554
u_{22}	.072	.043
w_{11}/w_{22}	2.674	1.913
Y_{s11}/Y_{s22}	2.715	1.909

Table 1 can also be used to describe the adjustment of an OLM to different states in a business cycle.[7] Because of the adjustments in δ and μ, the submarket with more productive s_1 workers and k_1 jobs experiences smaller fluctuations in wages and unemployment rates than the submarket with s_2 workers and k_2 jobs.

It is possible to undertake comparative static analysis of changes in individual parameters in the OLM. For example, an increase in N_{s2}, corresponding to an influx of less-skilled workers, results in $Y_{k22} > Y_{k12}$ at the former levels of δ and μ. Then using the analysis in Figure 1, it can be shown that δ and μ decline, Y_{k22} and Y_{k12} increase, Y_{s11} and Y_{s12} decline, and both w_{11} and w_{22} decline. The influx of less skilled workers results in wage declines for all workers in the OLM.

4.2 Productivity Increases

Changes in the output levels a_{ij} can also have distributional effects. If the output levels a_{ij} all increase by the same proportion, then an increase in all wages by the same proportion will return the OLM to equilibrium with no changes in δ, μ, market ratios or relative incomes. If the output levels change by unequal proportions, then there will in general be distributional impacts. An increase in a_{11}, holding a_{12} and a_{22} constant, can be interpreted as skill biased technological change, which has been proposed as an explanation for observed increases in skill differentials.[8]

In a number of theoretical contexts, skill biased technological change can be shown to yield an increase in skill differentials. Using the OLM, the effects of an increase in a_{11} can be determined and yield an increase in w_{11}/w_{22}. However, this result does not by itself establish that skill biased technological change is present or that it is the cause of observed increases in skill differentials, since other changes could also raise skill differentials. For example, Section 4.1

[7] Equilibrium in an OLM is derived in Section 2.2 assuming workers and empoyers believe conditions will continue indefinitely. Application of the solutions to different stages in a business cycle requires the assumption that expectations correspond to current conditions.

[8] It is also possible to interpret skill biased technological change as an increase in both a_{11} and a_{12}, which would require a different analysis.

shows that a proportional increase in labour supply can raise the skill differential w_{11}/w_{22} while raising the ratio of s_1 workers at k_1 jobs to s_2 workers at k_2 jobs, $(1-u_{11})N_{s1}/((1-u_{22})N_{s2})$.The following theorem provides ancillary consequences that could be used to determine whether an increase in skill differentials is caused by skill biased technological change.

Theorem 10 *Consider a reducible OLM in equilibrium with $\delta = \delta_0$, $\mu = \mu_0$ and $\theta_{12} = \theta_0$. Suppose a_{11} increases, with all other parameters staying the same. Then in the new equilibrium, i. $\theta_{12} > \theta_0$, ii. u_{12} is lower, v_{12} is higher, w_{12} is higher, Y_{s12} is higher and Y_{k12} is lower; iii. Y_{s11} is higher and Y_{k22} is lower; iv. μ is lower, θ_{22} is higher and u_{22} is lower; v. δ and θ_{11} are lower and u_{11} is higher.*

Proof. The result concerning θ_{12} will be established using the argument for existence and uniqueness in Section 2.2. At $\widehat{\theta}_{12} = \theta_0$, the derived value of Y_{s12} stays the same. Since Y_{s11} at δ_0 increases (from the higher value of a_{11}), the derived value of δ (at which $Y_{s11} = Y_{s12}$) must be lower. The derived value of η_{12} at θ_0, given by $\mu N_{k2}/(\delta N_{s1})$, increases. By 5, the derived value $H(\theta_0)$ also increases. Thus the function $H(\widehat{\theta}_{12})$ in Figure 1 shifts upward, so that the equilibrium value of θ_{12} must be greater, establishing i. The results in ii. follow from i. using the results in Section 2.1. Since $Y_{s11} = Y_{s12}$, Y_{s11} is greater in the new equilibrium; since $Y_{k22} = Y_{k12}$, Y_{k22} is lower in the new equilibrium, establishing iii. The lower value of Y_{k22} implies η_{22} and θ_{22} are higher, from 14. Then μ and u_{22} must be lower, establishing iv. Since θ_{12} is higher from i., η_{12} is also higher. Since μ is lower while $\eta_{12} = \mu N_{k2}/(\delta N_{s1})$ is higher, δ must be lower. Then η_{11} and θ_{11} are lower and u_{11} is higher, establishing v. and completing the proof.

The most notable result in theorem 9 is that the unemployment rate for s_1 workers at k_1 jobs increases, since s_1 workers shift from seeking k_2 to seeking k_1 jobs. At the same time, the unemployment rate for s_1 workers at k_2 jobs goes down.

5 Conclusions

This paper has emphasized differences between OLM's and ordinary markets with homogeneous or heterogeneous workers. Conditions for efficiency in ordinary markets with homogeneous workers are extended to irreducible OLM's in theorems 5 through 7. When employers with k_2 jobs do not distinguish between s_1 and s_2 workers at the interview or meeting stage, a misallocation of interviews arises unless 30 holds.

In an ordinary market with homogeneous workers, supply and demand shifts can have no effects on relative wages of workers since all workers are affected uniformly. However, in an OLM, a proportional increase in all jobs can have distributional impacts. In the example shown in Table 1, a proportional increase in jobs reduces the wage differential w_{11}/w_{22}. Relative wage changes previously attributed to skill biased technological change could then be generated by simple changes in supply and demand. Theorem 9 provides results that can be used to distinguish between skill biased technological change and other causes.

An essential feature of OLM's is that s_1 workers at k_2 jobs serve as a reserve of labour for the more productive k_1 jobs. Then an increase in output at k_1 jobs, or simply a proportional increase in all jobs, can lead s_1 workers to shift to seeking k_1 jobs. Depending on the availability of s_1 workers, employers with k_2 jobs may shift the jobs away from the low-paid s_2 workers.

While a market is a fundamental concept in economics, the boundaries of a market are poorly defined in both product and factor markets. With extensive heterogeneity among workers and among jobs, it is unlikely that knowledge of worker and job types would be sufficient to partition labour markets so finely that each worker was in an ordinary market with homogeneous agents. Alternatively, it can be characterized as an ordinary labour market with heterogeneous agents only if workers and employers have so little knowledge of each other that they waste substantial time in meetings and interviews that could not lead to matches. Neither ordinary markets with homogeneous agents nor ordinary markets with heterogeneous agents can realistically characterize aggregate labour markets. Overlapping Labour Markets arise with partial but incomplete knowledge of the types with which an agent matches and therefore provides a more faithful representation of actual labour markets.

References

[1] Acemoglu, Daron, 1999. "Changes in Unemployment and Wage Inequality: An Alternative Theory and Some Evidence." *American Economic Review* Vol. 89(5), 1259-1278.

[2] Albrecht, James and Vroman, Susan, 2002. "A Matching Model with Endogenous Skill Requirements." *International Economic Review* Vol. 43(1), 283-305.

[3] Diamond, Peter, 1982. "Wage Determination and Efficiency in Search Equilibrium," *Review of Economic Studies,* XLIX, 217-227.

[4] Doeringer, Peter B. and Piore, Michael J, 1971. *Internal Labor Markets and Manpower Analysis.* Lexington, MA: D.C. Heath.

[5] Dolado, Juan J., Jansen, Marcel, and Jimeno, Juan F., 2002. "A Matching Model of Crowding-Out and On-the-Job Search (with an Application to Spain)." IZA Discussion Paper No. 612.

[6] Gautier, Pieter A, 2002. "Unemployment and Search Externalities in a Model with Heterogeneous Jobs and Workers." *Economica* Vol. 69, 21-40.

[7] Hartog, Joop, 1992. *Capabilities, Allocation and Earnings*, Dordrecht: Kluwer.

[8] Hosios, Arthur J., 1990. "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, Vol. 57, 279-298.

[9] Lockwood, Ben, 1986. "Transferable Skills, Job Matching, and the Inefficiency of the 'Natural' Rate of Unemployment," *Economic Journal*, Vol. 96, 961-974.

[10] ——————, 1999. "Market Participation and Inefficiency in Matching with One-Sided Heterogeneity," Unpublished manuscript, University of Warwick.

[11] Lu, Xiaohua, and McAfee, R. Preston, 1996. "Matching and Expectations in a Market with Heterogeneous Agents," *Advances in Applied Microeconomics* Vol. 6, JAI Press, 121-156.

[12] Mandal, Kumarjit, 2001. "A Study of Labor Tying Contracts in Overlapping Agricultural Labor Markets in Less Developed Countries," unpublished manuscript, University at Albany, State University of New York.

[13] Moen, Espen R., 1997 "Competitive Search Equilibrium," *Journal of Political Economy*, Vol. 105, No. 2, 385-411.

[14] ——————, 2002. "Do Good Workers Hurt Bad Workers–Or Is It the Other Way Around?" Unpublished paper, Norwegian School of Management.

[15] Morgan, Peter, 1995. "A Model of Search, Coordination, and Market Segmentation," unpublished manuscript, SUNY Buffalo.

[16] Mortensen, Dale, 1982. "Property Rights and Efficiency in Mating, Racing, and Related Games," *American Economic Review* Vol. 72, 968-979.

[17] Mortensen, Dale and Pissarides, Christopher, 1999. "New Developments in Models of Search in the Labor Market," in Orley Ashenfelter and David Card (eds), *Handbook of labor Economics*, Vol. 3B, Amsterdam: Elsevier.

[18] Mortensen, Dale, and Wright, Randall, 2002. "Competitive Pricing and Efficiency in Search Equilibrium." *International Economic Review* Vol. 43(1), 1-20.

[19] Petrongolo, Barbara and Pissarides, Christopher, 2001. "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature*, Vol. 39(2), 390-431.

[20] Pissarides, Christopher, 2000. *Equilibrium Unemployment Theory,* second edition. Cambridge: MIT Press.

[21] Rogerson, Richard and Wright, Randall, 2002. "Search-Theoretic Models of the Labor Market: A Survey." Penn Institute for Economic Research Working Paper 02-041.

[22] Saint-Paul, Gilles, 1996. *Dual Labor Markets: A Macroeconomic Perspective.* Cambridge: MIT Press.

[23] Sattinger, Michael, 1975. "Comparative Advantage and the Distributions of Earnings and Abilities," *Econometrica* Vol. 43, 455-468.

[24] ——————, 1985. *Unemployment, Choice and Inequality.* Berlin: Springer-Verlag.

[25] ——————, 1990. "Unemployment, the Market for Interviews, and Wage Determination," *Journal of Political Economy,* Vol. 98, No. 2, 356-371.

[26] ——————, 1995. "Search and the Efficient Assignment of Workers to Jobs," *International Economic Review* Volume 36(2), 283-302.

[27] Shi, Shouyoung, 2001. "Frictional Assignment, Part I: Efficiency," *Journal of Economic Theory* Vol. 98, 232-260.

[28] ——————, 2002. "A Directed Search Model of Inequality with Heterogeneous Skills and Skill-Biased Technology." *Review of Economic Studies* Vol. 69(2), 467-491.

[29] Shimer, Robert and Smith, Lones, 2000. "Assortative Matching and Search," *Econometrica* Vol. 68(2), 343-369.

[30] Sicherman, Nachum, 1991. "'Overeducation' in the Labor Market," *Journal of Labor Economics* Vol. 9(2), 101-122.

[31] Teulings, Coen, 1995. "The Wage Distribution in a Model of the Assignment of Skills to Jobs," *Journal of Political Economy* Vol. 103, 280-315.

[32] Tinbergen, Jan, 1951. "Some remarks on the Distribution of Labour Incomes," *International Economic Papers*, 195-207.

[33] ——————, 1956. "On the Theory of Income Distribution," *Weltwirtschaftliches Archiv* Vol. 77, 155-173.