Multiple Equilibria and Endogenous Persistence in a Dynamic Model of Employment

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Abstract. In this paper, I consider whether: (1) a dynamic forward-looking model with multiple equilibria can generate persistent fluctuations without persistent sunspots; and (2) indeterminacy is important for these persistent fluctuations. The answer to the first question is a tentative no. The answer to the second question is yes. Extending the approach of Howitt and McAfee (1988, 1992), I work with a dynamic model of long-term employment. In this framework, search externalities allow both hiring and not hiring to comprise symmetric Nash equilibria for some values of the i.i.d. hiring cost. Following Cooper (1994), firms implement the hiring strategy of the previous period unless the realized hiring cost makes a change in strategy the dominant strategy. Calibrating the model, I find that with plausible functional forms, the selection rule can lead to persistent economic episodes only if one uses counterfactual parameters. Turning to the second question, I estimate that the economy has multiple equilibria, in the sense that the current hiring decision depends on the previous hiring decision, around 41 percent of the time. Moreover, I find that without some indeterminacy, the model can not generate expansions and recessions that are both persistent.

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I. INTRODUCTION

In this paper, I consider two questions: (1) can a dynamic forward-looking model with multiple equilibria generate persistent fluctuations without persistent sunspots?; and (2) is indeterminacy important for these persistent fluctuations? The answer to the first question is a tentative no. The answer to the second questions is yes.

The motivation behind these two questions is the following. Macroeconomic models with multiple equilibria are now quite familiar.[1] In many of these models, the economy can follow sunspots: as agents coordinate their behavior around some stochastic process—the sunspot—the economy will fluctuate as the process moves it from equilibrium to equilibrium.[2] It is less clear, however, whether models with sunspots can replicate salient features of the business cycle. One key feature, discussed by Cogley and Nason (1995), is that output growth has a positive serial correlation.[3] In the absence of deterministic cycling, this persistence can come from two sources. The first is persistence in the processes, in this case sunspots, that drive the economy. The second is that the economy can propagate shocks in such a way that even transient shocks have long-lived effects. The two sources of persistence are intimately related, in that if one wants to match some set of output dynamics, the choice of propagation mechanism will restrict the set of admissible driving processes.[4]

Models with sunspots often have problems generating persistence. One class of sunspot models, exemplified by Farmer and Guo (1994), contains variants of the real business cycle model. In these models, sunspots are forecast errors that are serially uncorrelated by definition. Schmitt-Grohe (1997) argues that the ability of these models to deliver persistence, namely, the strength of their propagation mechanisms, is sensitive

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[1]Examples include Azariadis (1981), Diamond (1982), Bryant (1983), Durlauf (1993) and Farmer and Guo (1994). Cooper and John (1988) provide a unifying framework for most of these models. Cooper and Haltiwanger (1996) give a literature review.

[2]While the literature has focused on sunspots that have no intrinsic economic meaning (see, for example, Woodford, 1990), sunspots can in fact be "fundamental" processes. Benhabib and Farmer (1996), Perli (1996) and Jones (1998) explore models where the sunspots include innovations to technology.

[3]While I will be looking just as closely at employment as output, employment growth is also serially correlated.

to underlying parameters. In this paper, I work within another class of sunspot models, where sunspots can be serially correlated. But even here it is hard to independently identify real-world sunspots—as opposed to inferring them from the data—much less identify sunspots that have the necessary serial correlation. (This criticism also applies to the Farmer-Guo-type models.)

These concerns suggest that sunspots might not be the most plausible equilibrium selection device. One appealing alternative is the selection mechanism proposed by Cooper (1994). In contrast to the sunspot approach, where agents coordinate their behavior around some exogenous variable, in Cooper's framework agents coordinate their behavior on the economic outcomes that they observed in the preceding period. Agents continue with the same behavior until they receive a shock that makes switching the dominant strategy. Cooper's selection mechanism thus uses a form of adaptive expectations to generate cycles and persistence in a quite plausible way.

Cooper, however, illustrates his selection mechanism with a model that is, by his own admission, simply a series of repeated static games. Actions taken in one period's stage game do not directly affect payoffs in another. In this paper, I apply Cooper's approach to the framework developed by Howitt and McAfee (1988, 1992), where payoffs are directly linked over time. In particular, agents base their decisions on the expected sequence of employment, the path of which they can shift. In addition to being more realistic, the model I develop allows the Howitt-McAfee framework to produce much richer dynamics.

The rest of the paper is organized as follows. In **Section II**, I construct the theoretical model, and show that it can deliver persistent fluctuations without persistent sunspots. I also show that indeterminacy, in the sense that the current hiring decision depends on the previous hiring decision, is usually a key to persistent fluctuations. These results, however, are only qualitative. In **Section III**, I study persistence quantitatively with simulations. I find that with plausible functional forms, indeterminacy and history-based selection rarely apply, and thus cannot generate persistence. In **Section IV**, I study indeterminacy with econometrics. In particular, the model implies that the probability of

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[4]See Fuerst (1995) or Cogley and Nason (1995) for more discussion of the impulse-propagation distinction.

indeterminacy is a function of the transition probabilities between periods of increasing and decreasing employment. With a sufficiently flexible form for the probabilities, one can measure them in reduced form without having to parameterize and solve the model. Using this measure with quarterly data, I find that the economy is indeterminate over 41 percent of the time. In addition, the econometric estimates are not inconsistent with the hypothesis that duration dependence is caused by changes in labor markets. The estimates also suggest that the model in its current form is too stylized. I conclude in **Section V**.

II. THE MODEL

The model is a direct extension of Howitt and McAfee (1992), who develop the model under perfect foresight, and show how it can support sunspots.

A. Basic Structure

The economy consists of $N \gg 0$ identical price-taking firms, which produce the non-storable consumption good y. The firms' only input is workers, each one of which provides a fixed amount of labor. The pool of potential workers is a mass of size N. Let $N_{i,t}$ be firm i's employment at time t, which can be normalized as $n_{i,t} = N_{i,t}/N$ and let $n_t = \sum_i n_{it}$ be the aggregate employment rate, which individual firms take as given. Each firm's output follows

$$(1) \quad y_{i,t} = fg(n_t)n_{i,t},$$

where $f > 0$ is the net output of each worker, and $g(\cdot)$: $[0, 1] \rightarrow [0, 1]$ is the fraction of output not absorbed by selling costs. Search externalities associated with "thick" *output* markets make $g'(n) > 0$, which raises the possibility of multiple equilibria. The price of output is normalized to 1.

Firms and workers live forever in discrete time. Each period, firms search the pool of unemployed workers for new hires. Firms and new workers then bargain over a lifetime contract, under which workers supply their labor every period until the match dissolves. In return, at each period $t+j$, $j \geq 0$, of the match workers receive the fraction w of their output. All firm-worker matches (including new ones) dissolve exogenously at the rate $\delta \in (0, 1)$.

Each period, firm i hires the fraction $h_{i,t}/N$ of the $(1 - n_t)$ unemployed workers at a cost of $c_t\phi(n_t)$ per worker. c_t is an i.i.d. cost shock with expectation $\bar{c} > 0$. $\phi(\cdot)$ is a continuous function, which in general will be increasing, so as to reflect the difficulty of finding workers in a "thin" *labor* market. For simplicity, $h_{i,t}$ is restricted to the pair $\{0, h_1\}$, with $0 < h_1 < 1$.[5] Workers hired at time t cannot work until time t+1.

[5]A weaker condition would be to restrict h to lie in $[0, h_1]$. Then, because the hiring costs and payoffs faced by individual firms are linear in the number of new employees, firms would always find it weakly optimal to set h to either 0 or h_1.

Firms seek to maximize the expected discounted value of their dividends, which, with a non-storable good, is equivalent to the value of their profits. Discounting at the rate β, the firm's problem boils down to finding a contingent plan for h_{it}:

$$(4) \quad \max_{\left\{h_i\left(\varpi^{t+j}\right)\right\}_{j=0}^{\infty}} E\left\{\sum_{j=0}^{\infty} \beta^j \left[pg\left(n_{t+j}\right) \cdot n_{i,t+j} - c_{t+j}\phi\left(n_{t+j}\right) \frac{h_i\left(\varpi^{t+j}\right)}{N}\left(1-n_{t+j}\right)\right] \middle| \varpi^t \right\},$$

$$s.t.$$

$$(5) \quad n_{i,t+j+1} = (1-\delta)\left[n_{i,t+j} + \frac{h_i\left(\varpi^{t+j}\right)}{N}\left(1-n_{t+j}\right)\right], \quad t = 0,1,2\ldots$$

$$(6) \quad h_i\left(\varpi^{t+j}\right) \in \left\{0, h_1\right\}, \quad \forall \varpi^{t+j} \in \Omega^{t+j}, \quad j = 0,1,2\ldots$$

$$(7) \quad n_0 \ given,$$

with $E\{\cdot \mid \varpi^t\}$ denoting mathematical expectations conditional on the information set $\varpi^t \in \Omega^t$, and $p \equiv f(1 - w)$.

I restrict attention to equilibria with symmetric pure strategies, so that $h_{it} = h_t$, $\forall i$. Aggregate employment then follows

$$(8) \quad n_{t+1} = \begin{cases} n^H\left(n_t\right) \equiv (1-\delta)\left[n_t + h_1(1-n_t)\right] = \alpha + \gamma n_t, & if \ all \ firms \ hire\left(h_i\left(\varpi^t\right) = h_1\right) \\ n^L\left(n_t\right) \equiv (1-\delta)n_t, & if \ no \ firms \ hire\left(h_i\left(\varpi^t\right) = 0\right) \end{cases},$$

$$\alpha \equiv \left(1-\delta\right)h_1 \in \left(0,1\right),$$
$$\gamma \equiv \left(1-\delta\right)\left(1-h_1\right) \in \left(0,1\right),$$

with a maximum sustainable employment, n^H, of

$$(9) \quad n^H = \frac{\alpha}{1-\gamma} \in \left(0,1\right).$$

Equations (8) and (9) imply that $[0, n^H]$ is the absorbing set for aggregate employment.

Finally, I restrict firms to time-invariant decision rules of the form $h_{it} = h(\omega_t)$, where $\omega_t \in \Omega_t$ consists of random variables dated t or $t - 1$. Such a rule would arise if firms operated under perfect foresight, coordinated around some Markov sunspot process, or followed simple history-based selection.

Workers receive wages from their employer and dividends from every firm. Under the assumptions of non-satiable preferences and a perishable good, workers simply consume their income.[6] Since w is non-negative, the labor supply decision is made similarly trivial by assuming that there is no disutility to work.

I turn now to defining an equilibrium. Since the market clearing conditions follow trivially from the worker's problem, it is sufficient to establish that firms are behaving optimally.

Definition. A symmetric equilibrium is a collection of a:

(*Ai*) *hiring rule*: $h(\omega_t)$;

and the stochastic processes for:

(*Aii*) *aggregate employment*: $\{n_t\}_{t=0}^{\infty}$;

(*Aiii*) *coordinating variables*: $\{\omega_t\}_{t=0}^{\infty}$;

such that:

(*Bi*) given $\{\omega_t\}_{t=0}^{\infty}$, $h_{it} = h(\omega_t)$ solves the firm's problem for t = 0,1,2...

(*Bii*) given $h(\omega_t)$ and n_0, n_t follows equation (8) for t = 0,1,2...

(*Biii*) the coordinating vector ω_t contains, but is not limited to, $\{n_j, h_j, c_j\}_{j=t-1}^{t}$.

Note that the definition of ω_t in (*Biii*) is broad enough to include sunspots.

B. History-based Selection

Howitt and McAfee (1992) show that the economy can have multiple equilibria under perfect foresight and can fluctuate as it follows an extrinsic sunspot process. They further show that a process can become a coordinating sunspot when agents engage in adaptive learning.[7] But showing that the economy can follow sunspots provides little guidance as to what these sunspots might be, and there is little independent evidence that any processes are serving as sunspots. In this section, I turn to a more plausible coordinating device, namely the use of adaptive expectations as suggested by Cooper (1994).

[6] While employed and unemployed workers clearly have different incomes, adding insurance markets changes none of the results that follow.

[7] In deriving these results, Howitt and McAfee assume that firms must make their hiring decisions *before* they observe c_t.

The simplest history-based selection rule is to use only the history of the previous period. It is possible that over certain regions of the support of c_t, the equilibrium hiring decision is unique, but that over others, either hiring decision can be supported as an equilibrium. In particular, let firms adopt the following decision rule:

1. If firms did not hire in the preceding period ($h_{t-1} = 0$), firms will not hire in the current period ($h_t = 0$), unless c_t is so low that a firm would find it rational to hire even if every other firm did not hire in the current period, but in future periods did follow the decision rule developed here.
2. If firms did hire in the preceding period ($h_{t-1} = h_1$), firms will hire in the current period ($h_t = h_1$), unless c_t is so high that no firm would find it rational to hire, even if every other firm hired and then followed the decision rule in all future periods.

Adapting the language of Howitt and McAfee, when the economy is following the first branch of the rule, I will say it is on the passive path, and when it follows the second branch, I will say it is on the aggressive path. Behind the rule is the implicit assumption, verified in the numerical exercises, that if hiring is the superior strategy when no other firms hire, it will be if all other firms hire, and that if not hiring is the superior strategy when all other firms hire, it will be if no other firms hire. With this assumption, the decision rule is consistent with symmetric equilibrium.

To state this more formally, rewrite $h(\omega_t)$ as $h(n_t, c_t, h_{t-1})$. Recalling that $\{c_t\}$ is i.i.d., one can define the transition function

$$(10) \quad w(n_t, h_t) = \Pr\{h_{t+1} = h_t \mid h_t, n_t\} = \Pr\{h_{t+1} = h_t \mid h_t, n_t, c_t\},$$

which gives the probability at time t that $h_{t+1} = h_t$.

Let $\lambda(n_t, h_t)$ be the expected present value of an additional hire, *gross* of hiring costs, when the economy follows the history-based selection rule. Taking (10) as given, $\lambda(\cdot)$ follows

$$(11a) \quad \lambda(n_t, 0) = \tilde{\beta}\Big[pg\big(n^L(n_t)\big) + w(n_t, 0)\cdot\lambda\big(n^L(n_t), 0\big) + \big(1 - w(n_t, 0)\big)\cdot\lambda\big(n^L(n_t), h_1\big)\Big],$$

$$(11b)\quad \lambda(n_t,h_1) = \tilde{\beta}\Big[pg\big(n^H(n_t)\big)+\big(1-w(n_t,h_1)\big)\cdot\lambda\big(n^H(n_t),0\big) + w(n_t,h_1)\cdot\lambda\big(n^H(n_t),h_1\big)\Big],$$

$$\tilde{\beta} \equiv \beta(1-\delta).$$

Since λ is well-defined for *any* well-defined $w(\cdot)$, it should not be interpreted as the solution to a maximization problem, but simply as a return function. Let Λ denote the space of all such return functions. Similarly, let W denote the space of all transition probability functions:

$$(12)\quad \Lambda = \{\lambda\colon [0,1]\times\{0,h_1\}\to\mathbb{R}_+\},$$

$$(13)\quad \mathrm{W} = \{w\colon [0,1]\times\{0,h_1\}\to[0,1]\}.$$

Given $w(\cdot)$, equation (11) forms a functional equation with a unique continuous solution.[8] Let T_1: $\mathrm{W}\to\Lambda$ denote the operator that gives the solution to (11) generated by $w(\cdot)$.

Firms will hire, however, only if $\lambda(n_t, h_t) \geq c_t\phi(n_t)$. This means that under the history-based selection rule

$$(14a)\quad h(n_t,c_t,0)=0 \;\; \mathit{iff} \;\; c_t \geq c(n_t,0),$$

$$(14b)\quad h(n_t,c_t,h_1)=h_1 \;\; \mathit{iff} \;\; c_t \leq c(n_t,h_1),$$

$$(15)\quad c(n_t,h)\equiv\frac{\lambda(n_t,h)}{\phi(n_t)},$$

so that

$$(16a)\quad w(n_t,0)=\Pr\{c_{t+1}\geq c\big(n^L(n_t),0\big)\},$$

$$(16b)\quad w(n_t,h_1)=\Pr\{c_{t+1}\leq c\big(n^H(n_t),h_1\big)\}.$$

Let T_2: $\Lambda\to\mathrm{W}$ denote the operator that gives the transition function generated by $\lambda(\cdot)$ under history-based selection.

Equations (11), (14), (15) and (16) provide a formal definition of equilibrium.

Definition. A symmetric history-based equilibrium is the collection of a:

(*Ai*) *hiring rule*: $h(n_t, c_t, h_{t-1})$;

(*Aii*) *transition function*: $w(n_t, h_t)$;

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[8]See Howitt and McAfee (1992) and Sargent (1987).

($Aiii$) *return function*: $\lambda(n_t, h_t)$;

such that:

(Bi) *equations* (11), (14), (15) *and* (16) *are satisfied*;

(Bii) *the return function obeys*

$$(17) \quad \lambda(n,0) \leq \lambda(n,h_1) \;\; \forall n \in [0,1].$$

Alternatively, one can define an equilibrium as a return function $\lambda(\cdot)$ that satisfies

$$(18) \quad \lambda(\cdot) = T_1 T_2 \lambda(\cdot)$$

and equation (14) and (17). Since $w(\cdot)$ depends on $\lambda(\cdot)$, the fixed-point properties of T_1T_2 are difficult to characterize.[9] In the numerical exercises, however, I do find fixed points—not necessarily unique—all of which satisfy equation (17).[10] One can define equilibrium in yet another way, by replacing equation (18) with

$$(19) \quad w(\cdot) = T_2 T_1 w(\cdot).$$

Equation (17) makes it possible to define a "range of indeterminacy." Whenever c lies within this range, agents will not hire if they are on the passive path ($h(n, c, 0) = 0$), but will hire if they are on the aggressive path ($h(n, c, h_1) = h_1$). Let $\chi(n)$ denote the probability that c_t lies in the range of indeterminacy, given that $n_t = n$. It follows from equations (16) and (17) that

$$(20) \quad \begin{aligned} \chi(n) &= \Pr\{c(n,0) \leq c \leq c(n,h_1)\} \\ &= w\left(n^{-L}(n),0\right) + w\left(n^{-H}(n),h_1\right) - 1, \end{aligned}$$

where $n^{-L}(\cdot)$ and $n^{-H}(\cdot)$ are the inverses of the employment transition functions $n^L(\cdot)$ and $n^H(\cdot)$, respectively. Equation (20) reveals that indeterminacy and persistence are closely related. In particular, h_t is most persistent when $w(n_t, h_{t-1})$ is large; when $w(\cdot)$ is large, next period's hiring decision will probably be the same as this period's. But $w(n, 0)$ and $w(n, h_1)$ are simultaneously large only when both aggressive and passive behavior are feasible over a large range of costs, namely, when there is a large range of indeterminacy.

[9]Note that when the transition probability $w(\cdot)$ depends on $\lambda(\cdot,\cdot)$, small differences between the functions $\lambda^1(\cdot)$ and $\lambda^2(\cdot)$ can lead to large differences between $w^1(\cdot,\cdot)$ and $w^2(\cdot,\cdot)$, and thus large differences between the expectations in $T_1T_2\lambda^1(\cdot)$ and $T_1T_2\lambda^2(\cdot)$.

[10]Since $g'(n) > 0$ and $\theta \in \{0, h_1\}$ create non-convexities, it is not even immediate that an equilibrium exists. Such a problem, though, has never occurred in the numerical exercises.

At the opposite extreme, if $\chi(n)$ equals 0, the probability that the economy expands (or contracts) next period does not depend at all on whether it expanded (or contracted) this period, once employment is taken into account.

A similar conclusion applies to output. If one interprets search costs as investment, final output is given by $g(n)n$, and the growth rate of output is

$$(21) \quad g_y = \left[\ln\left(g\left(n_t\right)\right) - \ln\left(g\left(n_{t-1}\right)\right)\right] + \left[\ln\left(n_t\right) - \ln\left(n_{t-1}\right)\right].$$

Since $g(\cdot)$ is increasing in n, the two components of output growth have the same sign. This suggests that with enough indeterminacy, the model can yield persistent output growth without persistent sunspots.[11] In the most extreme case $\chi(n)$ equals 1 and expansions (or recessions) never end.

It remains to be seen what "enough" indeterminacy actually is, and whether the model can deliver "enough" indeterminacy once it is reasonably parameterized. In the next section, I address these questions with simulation exercises.

[11]It is difficult to make a more formal claim, for the following two reasons. First, changes in $w(\cdot)$ affect the distribution of n and h; by Simpson's paradox, it is possible that $w_1(n, h) \geq w_2(n, h)$ at all values of n and h, while $E\{w_1\} < E\{w_2\}$. Second, $\chi(n)$ is the sum of $w(n^{-L}(n), 0)$ and $w(n^{-H}(n), h_1)$ which in general will affect persistence in different ways.

III. SIMULATIONS

In the preceding section, it was shown that the model can deliver persistent output growth without persistent sunspots. In this section, I consider whether that qualitative result is also a quantitative one, by calibrating the model and using it to generate artificial time series. In addition, it is illuminating to discuss how one finds the return function $\lambda(n, h)$ and the probability function $w(n, h)$.

A. Simulation Methodology

I estimate $\lambda(n, h)$ and $w(n, h)$ by working recursively with the relationships given by equations (11) and (16). The procedure for a single iteration, say iteration i, is:

1. Begin with $\lambda_i(n, h)$ and $w_i(n, h)$.
2. Find λ_{i+1} by inserting λ_i and w_i into the right-hand side of equation (11).
3. Find w_{i+1} by inserting λ_{i+1} into the right-hand-side of equation (16).

To find $\lambda_0(n, 0)$ and $\lambda_0(n, h_1)$, I solve equation (11) with $w(n, 0) = w(n, h_1) = 1$. I pick $w_0(n, 0)$ and $w_0(n, h_1)$ arbitrarily. I iterate until $\sup|\lambda_{i+1}(n, h) - \lambda_i(n, h)|$ is less than 1.0×10^{-13} over the set $[0, 0.99] \times \{0, h_1\}$.[12] There is no guarantee that this procedure will converge, but in practice I have had little difficulty.

Calculating $\lambda(n, h)$ immediately yields $w(n, h)$. Then given n_0 and s_0, it is straightforward to simulate the employment path of an economy. Given the current hiring decision and current employment, namely h_t and n_t, $w(n_t, h_t)$ provides probabilities for randomly selecting h_{t+1}. Then a random draw from this distribution gives h_{t+1}, and h_t and n_t give n_{t+1}, through equation (8). Choosing the initial state arbitrarily, I simulate an employment time series of 500 periods, discard the first 300 observations, and calculate various summary statistics for the last 200 observations. I repeat this exercise 3600 times, and find the mean and standard deviation of the Monte Carlo results.

B. Simulation Parameters

I present here two parameterizations, with the parameters listed below. In both cases, I assume that the cost shock c_t lies in $C = [c_{min}, c_{max}]$, with C symmetric around 1.

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[12]The simulations were done in GAUSS. To approximate a continuous function, I divided the interval [0,1] into subintervals, calculated $\lambda(n, h)$ and $w(n, h)$ for each discrete value of n, and linearly iterpolated between the discrete points.

I also assume that the distribution of c_t consists of a mass point at 1, and a uniform distribution elsewhere, so that $\bar{c} = 1$, mainly for its computational ease.

Simulation Parameters		
	Baseline Case	High Indeterminacy Alternative
Discount Factor (β)	0.990	0.950
Depreciation (δ)	0.0046	0.450
Hiring Fraction (h_1)	0.089	0.500
Maximum Employment (n^H)	0.951	0.600
Sales Price (p)	1.000	1.000
Flow Return Function ($g(n)$):	$2.44\times10^{-3} + 1.788n^{0.25}$	$0.1824 + 2.88n^2$
Minimum Cost Shock (c_{min})	0.500	0.000
Maximum Cost Shock (c_{max})	1.500	2.000
$\Pr\{c_t = E(c)\}$	0.200	0.550
Hiring Cost ($\phi(n)$)	$20.81 + 134.2n + 0.01n^2$	$1/(1 - n)$

The first parameterization is the baseline case, which is meant to replicate the actual economy. The parameters are set as follows.

δ and h_1 are based on quarterly postwar U.S. employment data, with employment measured as the ratio of employed workers to population.[13] When employment is decreasing, it falls by about 0.42 percentage points per quarter, and when it is increasing, the average increase is about 0.32 percentage points. Since average employment is about 91.67 percent, during both periods of increasing and decreasing employment, it follows from equation (8) that δ is about 0.46 percent and h_1 is about 8.9 percent. Inserting these values into equation (9) implies that $n^H = 95.1$ percent, which is reasonable; upon removing a linear trend, the maximum observed employment rate is roughly 96.6 percent.

[13]As discussed more thoroughly in the next section, the employment ratios have been rescaled so that the largest value of the employment ratio equals the largest value of $1 - UR$, where UR is the standard unemployment rate.

Setting δ to 0.46 percent implies that the expected duration of employment exceeds 50 years.[14] Similarly, $1/h_1$, which serves as a lower bound on the duration of unemployment (since $h_t = 0$ over 40 percent of the time) exceeds 11 quarters, which is well in excess of the observed mean unemployment duration of 13 weeks.[15] These long durations would seem to be a strike against the model, but what is really happening is that most unemployment spells begin and end within the same quarter; the median unemployment duration is under 7 weeks. These implied durations, then, are for net, rather than single, job tenure.

The functional form for $g(n)$ ensures that output ($g(n)\cdot n$) has returns to scale between 1 and 1.25, the latter being a relatively mild form of increasing returns; since the model lacks capital, total returns are the relevant benchmark.[16] I assumed that $\phi(n)$ is quadratic because it is a simple and flexible functional form.

The coefficients for the return and cost functions were picked to make the model generate time series with moments similar to those in the data. In particular, I picked the coefficients so that the model matched the data on: the mean of employment; 100×(the standard deviation of employment); 100×(the standard deviation of output growth); and the serial correlation of output growth.[17] (The criterion function was the sum of squared differences.[18]) Effectively, this is estimation by simulated method of moments in a calibration context. To keep the search feasible, the distribution parameters for c_t were set informally, but they too were picked to help match these four moments.[19]

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[14]Recall that if the probability of leaving a particular state is q, the expected duration in that state is $1/q$.

[15]The underlying data are monthly averages compiled from the Current Population Survey data by the Bureau of Labor Statistics. The maximum observed average duration is less than 22 weeks.

[16]Several studies, such as Farmer and Guo (1994), utilize returns in excess of 1.5, which many view as too high. See Benhabib and Farmer (1996) or Schmitt-Grohe (1997) on this point.

[17]Multiplying the standard deviations by 100 ensures that all of the moments are of a similar magnitude.

[18]The code to select the parameters used a Simplex algorithm written by Honore and Kyriazidou and downloaded from the GAUSS library at American University. (Gradient-based algorithms yielded no improvement.) To speed up the search, simulated moments were taken over 1600, rather than 3600 repetitions. In addition, the same random numbers were used for each Monte Carlo exercise in the search.

[19]The alert reader will have noticed that $g(n)$ and $\phi(n)$ together have 5 parameters, while I am matching 4 moments. One can rescale $g(n)$ and $\phi(n)$, however, without changing their implications for any moment of employment or output growth. I thus normalize the last coefficient on $\phi(n)$ to 0.01.

The second, or high-indeterminacy, case is far more stylized, and developed mainly as a contrast to the baseline case. As such, the parameters of the high-indeterminacy case are selected mainly to ensure that $\chi(n)$ is high.

C. Simulation Results

As discussed above, the first step in the simulation routine is find $\lambda(n,h)$ and $w(n,h)$. **Figure 1** presents $\lambda(n,h)$. Perhaps the most striking difference between the two simulations is that in the baseline case, $\lambda(n,0)$ and $\lambda(n,h_1)$ are virtually identical, while in the high indeterminacy case, $\lambda(n,h_1)$ is much larger than $\lambda(n,0)$. It follows that in the baseline case, hiring decisions are based almost purely on "fundamentals", while in the high indeterminacy case, the payoff to hiring also depends on whether the economy is on the aggressive or passive path. There are two reasons for this difference: (1) the production function in the high indeterminacy case has higher returns to scale; and (2) δ and h_1 are much higher in the high indeterminacy case, which allows the decision to hire have a greater effect on n. The difference in $\lambda(\cdot)$ also appears as a difference in $w(\cdot)$. In particular, **Figure 2** shows that in the high indeterminacy case $w(n,0)$ and $w(n,h_1)$ are both high, while in the baseline case, the two functions are virtually mirror images of each other.

One interesting aspect of the high indeterminacy case is that even with the history-based selection rule, there are multiple equilibria. In particular, starting the iterative routine with different versions of the initial probability function $w_0(n,h)$ led to different versions of $w(n,h)$ and $\lambda(n,h)$. This illustrates the claim that the joint operator T_1T_2, as used in equation (18), need not have a unique fixed point. Some sense of how this multiplicity might arise can be found by studying Panel B in **Figures 1 and 2**: the sudden drop in $\lambda(n,h_1)$ just after n^H is accompanied by a sudden drop in $w(n,h_1)$, with the drop in each function making the drop in the other one more plausible. I did not find any such multiplicity in the baseline case, although I clearly cannot rule it out.

Table 1 presents the simulated moments. In addition, the first two columns of **Table 1** show estimates for postwar U.S. data (1952II - 1997IV), which were found jointly by Generalized Method of Moments.[20] The data show that employment averages just under 91.7 percent, and has a serial correlation of 97.9 percent. The economy is hiring about 55.7 percent of the time, and the state variable has a correlation of almost 41 percent.[21] **Table 1** also includes the average value of $\chi(n)$, the probability of indeterminacy. The way in which I find this probability is described in some detail in the next section.[22] Using this measure, the data show that about 41 percent of the time the decision to hire varies not only with the cost shock and current employment, but with previous employment as well.

For each of the cases, **Table 1** shows the mean of the sample moment across 3600 simulations, and the standard deviation across the 3600 simulations. In the baseline case, the model does fairly well in matching the mean, standard deviation and serial correlation of employment, as well as the standard deviation of output growth. The model fails, however, to generate persistent output growth: the serial correlation of output is -0.03, while in the data the correlation is 0.33. The baseline case also has too little indeterminacy (0.00, as opposed to 0.44) and too little correlation in the state variable. All of these results follow immediately from **Figure 1**. When $\lambda(n,0)$ and $\lambda(n,h_1)$ differ so little, the decision to hire depends only on employment and the i.i.d. cost shock c_t. Having hired (or not hired) in the previous period has no direct effect on the probability that firms will hire this period, and the "fundamentals" do not provide much persistence on their own.

The opposite occurs in the high indeterminacy case. Here $\lambda(n,0)$ and $\lambda(n,h_1)$ differ enough to let history matter, and to let economic episodes become persistent. The state variable becomes quite correlated, and indeterminacy rises to 0.69. But even here, with highly unrealistic parameters, the model fails to deliver enough persistence in output growth (0.12, as opposed to 0.33). Moreover, it generates far too much volatility in employment and output.

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[20]These estimates were found in RATS, with the standard errors corrected for heteroskedasticity and autocorrelation.

[21]One can find the standard deviation of the state variable as $\sqrt{m(1-m)}$, where m is the sample mean.

[22]In deriving standard errors, I took these quantities as fixed. In addition, I take as fixed the trend that is removed from the employment data.

To sum, without counterfactually high returns to scale, the model generates neither indeterminacy nor persistence. The model thus suffers from the curse of many models of sunspot-driven fluctuations, which is the inability to deliver indeterminacy when production externalities are small.[23] And in the high-return case considered here, the model generates far too much volatility (and still not enough persistence).

Although the assumptions used here are plausible, it is always possible, of course, that the model would succeed with different functional forms for $\phi(n)$ and $g(n)$, or a different distribution for c_t. A promising, if perhaps implausible, approach would be to allow the distribution of c_t to vary with n. In particular, if $w(n, h)$ were known, and $g(n)$ were given, then $\lambda(n, h)$ follows immediately from equation (11). Then once $\lambda(n, h)$ was found, $\phi(n)$ and the employment-specific distributions of c_t could be set so that the assumed $w(n, h)$ obeyed equation (16).

While such an exercise would lead me too far astray, it does point out that the value of an empirical measure of $w(n, h)$. In addition, the simulations suggest that indeterminacy and persistence are closely linked. Keeping these points in mind, I move on to the next section, where I estimate $w(n, h)$ as a step in measuring the amount of indeterminacy in the U.S. economy.

[23]Two notable exceptions are Benhabib and Farmer (1996) and Perli (1996).

IV. ECONOMETRIC EVIDENCE

Under Cooper's history-based selection rule, economic activity in the model falls naturally into two regimes, hiring or not hiring. In particular, it follows from equation (17) that the regime variable s_t (or, equivalently, the hiring variable h_{t-1}—the reason for the lag is given below) is a Markov process with transition probabilities given by

$$(22) \quad \begin{array}{c|c} & \underline{s_{t+1}} \\ & \begin{array}{cc} 0 & 1 \end{array} \\ s_t \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} w(n_{t-1},0) & 1-w(n_{t-1},0) \\ 1-w(n_{t-1},h_1) & w(n_{t-1},h_1) \end{bmatrix}. \end{array}$$

One goal of this paper is to see whether indeterminacy is essential for persistence. The theoretical model and simulations suggest that it is. In this section, I evaluate the importance of indeterminacy by estimating the average value of $\chi(n)$, as defined in equation (20). I find that in quarterly U.S. data the average level of indeterminacy, as measured by $\chi(n)$, is around 41 percent. This figure has already played a prominent role in the previous section. The estimates are also of interest in and of themselves, for in constructing them, I am able to further test the model, and they provide some insight into how the economy might exhibit duration dependence.

Measuring $\chi(n)$ requires an estimate of equation (22), which I find by analyzing the transition probabilities between periods of increasing and decreasing employment. Constructing these estimates allows me to test some of the implications of the model. One of these implications is that the transition probability $w(\cdot)$ is a function only of h_t and n_t. While I do not conduct an exhaustive search, I find that the implication fails to hold when the alternative predictor is capacity utilization.

One advantage of the approach I take here is that it allows me to estimate $w(\cdot)$ in reduced form, by assuming it is logistic, so that I need not make any assumptions about the underlying parameters of the model. This section thus complements the previous one without relying on the parametric assumptions I make there. Indeed, as discussed above, one could use an estimate of $w(\cdot)$ to calibrate the model, i.e., use the reduced form to recover the structural model.

Measuring $\chi(n)$ also requires one to estimate the law of motion for employment:

(23) $n_{t+1} = (1-\delta)h_t + (1-\delta)(1-h_t)n_t$.

(When n_t is measured as a percentage, the first term becomes $100(1 - \delta)h_t$.) In the simulations, I found the parameters of (23) through calibration. In this section, I estimate them econometrically. It turns out that the two sets of estimates are economically, if not statistically, similar.

Using the latent variable approach developed by Hamilton (1989), Boldin (1990) argues that unemployment follows multiple regimes in way consistent with the Howitt-McAfee approach, and estimates regime-dependent mean unemployment rates, and time-invariant transition probabilities. Diebold and Rudebusch (1994) make a similar point, and also point out that Cooper's selection rule maps into the regime-switching structure. In this section, I push the connection further, by estimating employment-varying transition probabilities that relate directly to the theoretical model.

This work also ties into the literature on duration dependence, which studies whether the probability of exiting an expansion (or recession) changes as the expansion (recession) persists. In particular, the model suggests that the employment, rather than duration *per se*, affects transition probabilities. The data are not inconsistent with this hypothesis.

A. Transition Probabilities

The theoretical model suggests that for most—although not all—configurations of $g(\cdot)$ and $\phi(\cdot)$, the probability of leaving an expansion or a contraction will vary with employment. There is some evidence to support this. Filardo (1994) finds that a Markov-switching model of GNP growth better matches the NBER business cycle chronology when the transition matrix is changed from a constant to a function of various economic indices, all of which include a measure of employment. Kim and Yoo (1995) find endogenous transition probabilities in a multivariate model with a regime-switching common trend.

Employment-dependent transition probabilities should also give rise to duration dependence: since employment changes as an economic episode continues, so should the probability that the episode will end. Using the NBER business cycle chronology, Diebold and Rudebusch (1990), Sichel (1991), Diebold, Rudebusch and Sichel (1993) find evidence of positive duration dependence in U.S. prewar expansions and postwar contractions. Durland and McCurdy (1994) look for duration dependence in a Markov-switching model of postwar GNP growth. They too discover strong positive duration dependence in contractions and little dependence of any sort in expansions. Lam (1997) extends Durland and McCurdy's analysis by including a richer statistical model and using prewar data. He finds that the effects of duration are non-linear, but that contractions roughly show positive duration dependence and that expansions roughly show *negative* duration dependence. Lam also finds that mean growth—as opposed the probability of a switch in mean growth—declines as expansions continues.

It is also plausible, however, that the relationship actually works in reverse: employment-dependent transition probabilities are just a manifestation of duration dependence. In a similar vein, one could argue that capacity utilization is as an important a determinant of hiring as labor market tightness. This suggests that when one estimates $w(\cdot)$, all of these effects need to be considered jointly. To do this, I assume that the diagonal elements of (22) are

$$(24)\quad w(n_{t-1}, DUR_t, CAP_t, 0) = \frac{\exp(a_0 + a_1 n_{t-1} + a_2 DUR_t + a_3 CAP_t)}{1+\exp(a_0 + a_1 n_{t-1} + a_2 DUR_t + a_3 CAP_t)},$$

$$w(n_{t-1}, DUR_t, CAP_t, h_1) = \frac{\exp(b_0 + b_1 n_{t-1} + b_2 DUR_t + b_3 CAP_t)}{1+\exp(b_0 + b_1 n_{t-1} + b_2 DUR_t + b_3 CAP_t)},$$

where n_t is the employment rate at time t, DUR_t is the duration of the regime in effect at time t, and CAP_t is the Federal Reserve's measure of capacity utilization.[24] n_t and CAP_t are both measured as deviations from trend.[25]

In contrast to the studies reviewed above, I measure s_t with the first difference of employment—a positive change is labeled an expansion. In this respect, I build upon work by Neftçi (1984) and Falk (1986).[26] Recall from the model that hiring decisions at

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[24]DUR_t is capped at a maximum of 10 quarters. This approach is consistent with the Markov-switching literature, and adds an element of non-linearity to the duration's effect. (Durland and McCurdy (1994) use a grid search to pick a duration ceiling of 9. Lam (1997) uses a ceiling of 40.) In any event, the truncation has little effect on the estimates.

[25]Falk (1986) argues that such an adjustment is necessary when studying transitions between periods of increasing and decreasing unemployment.

[26]A nice discussion of these and related papers can be found in Granger and Teräsvirta (1993).

time t affect employment at time $t + 1$, so that s_t is a function of h_{t-1}. This is the reason why n_{t-1}, rather than n_t, appears in equations (22) and (24). Since s_t is observed, one gets the following conditional likelihood function:

$$(25)\quad L \equiv \sum_{t=2}^{T} \ln f\left(s_t \mid n_{t-2}, DUR_{t-1}, CAP_{t-1}, s_{t-1}\right)$$

$$= \sum_{t=2}^{T} \ln \begin{Bmatrix} s_t \cdot \left[s_{t-1} \cdot w(\cdot,1) + (1 - s_{t-1}) \cdot (1 - w(\cdot,0))\right] \; + \\ (1 - s_t) \cdot \left[s_{t-1} \cdot (1 - w(\cdot,1)) + (1 - s_{t-1}) \cdot w(\cdot,0)\right] \end{Bmatrix},$$

with $w(\cdot, s_{t-1})$ following equation (24). If the theoretical model is true, a_2, a_3, b_2 and b_3 all equal 0, so that $w(n, DUR, CAP, h) = w(n, h)$.[27]

I estimate equation (25) with two measures of employment (n): (1) the ratio of employment to population for civilians; and (2) 1 - UR, where UR is the standard unemployment rate. For my main results, I use the employment ratio, which captures movements in and out of the labor force. I rescale this ratio to reflect that large fraction of the population is permanently out of the labor force.[28] Upon removing a linear trend, the maximum observed employment ratio is around 62 percent. But even if 62 percent constitutes "full employment," it does not follow that frictional employment is 38 percent—the true amount of frictional employment is almost surely much lower. I thus rescale the employment ratio so that both measures of employment show the same maximum employment. This rescaling also facilitates comparison between the two measures. The first two columns of **Table 1** include summary statistics for n_t and s_t with the rescaled employment ratio.

I estimate equation (25) with quarterly data from 1952II to 1997IV;[29] extending the data back to 1948 does not change the general results. I also estimate equation (25) with monthly data.[30] While the signs of the coefficients are similar across frequencies,

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[27]In the interest of brevity, I consider only duration and capacity utilization as alternative predictors—one could find any number of other predictors.

[28]Note that in the model, wages are bounded. In this context, then, it is reasonable to assume that some people's reservation wages are always too high, so that they are permanently out of the labor force.

[29]Lam (1997) points out that observations around the Korean War are usually dropped in the regime-switching literature.

[30]The quarterly employment rate is a simple average of the monthly rates.

the magnitudes (and t-statistics) change a bit. I focus on the quarterly data here, as it is the standard business cycle frequency, although I will briefly discuss how changing the frequency affects the probability of indeterminacy.

Table 2 presents the transition probability functions. Panel A of this table displays the results that arise when n_t is measured with the employment ratio, and Panel B displays the results that arise when employment is measured with the standard unemployment rate. **Table 2** reveals that in most cases, as the labor market loosens (employment falls), recessions become more likely to end ($a_1 > 0$). Similarly, as the labor market tightens, expansions become more likely to end ($b_1 < 0$). These results also hold when one considers unemployment in combination with duration and capacity utilization. In Panel A, where the employment ratio is used, a_1 and b_1 always have p-values below 9 percent. In contrast, in much of Panel B one cannot reject the possibility that $g(n)$ and $\phi(n)$ have a configuration that makes the transition probabilities constant across employment. Consistent with Neftçi (1984), the transition probabilities do not appear to be symmetric. This is not surprising; in the underlying data, employment is increasing ($s_t = 1$) over 55 percent of the time.

Figure 3 shows the transition probabilities implied by the first column of Panel A, along with 95 percent confidence intervals.[31] **Figure 3** also shows the transition probabilities generated by the both simulations. Although the empirical and simulated transition functions have similar shapes, both simulations generate transition functions that lie well outside the confidence intervals.

Recall that the model suggests that once unemployment is accounted for, duration and capacity utilization should not affect the transition probabilities.[32] Using a standard likelihood ratio test, one cannot reject the hypothesis that duration has no effect.[33] This

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[31]To find the confidence intervals for $w(n, 0)$, I first found the confidence intervals for $a_0 + a_1[n - E\{n\}]$ at each value of n. Inserting these values into the logistic function yielded the final confidence intervals. The confidence intervals for $w(n, h_1)$ were found in a similar fashion.

[32]Lam (1997) argues that the effect of duration is best modeled as a quadratic function. Preliminary estimates that included such second-order effects failed to converge.

[33]The test statistic is calculated as twice the difference in the log-likelihood. For two restrictions, the likelihood test statistic is distributed $\chi^2(2)$, with critical values of 5.99 at the 5 percent level and 9.21 at the 1 percent level. (See Hamilton, 1994, pp. 144 and 754.) The observed statistics are 0.800 and 0.796 for the employment-ratio and unemployment chronologies, respectively.

suggests that the underlying cause of duration dependence, should it exist, is tightening or loosening labor markets. On the other hand, capacity utilization is a significant predictor of the probability of exiting a recession. This implies that the model in its current form is too stylized.

C. Employment Dynamics

In addition to estimating the transition probabilities, one can also estimate the law of motion for employment. In doing this, I use a slightly more general specification than the one suggested by the theoretical model. First, I assume that $h_t \in \{h_0, h_1\}$, with h_0 no longer constrained to equal 0.[34] Second, I consider the possibility that the current model might be too stylized, and thus modify equation (23) to so that it no longer holds exactly, but instead includes an autocorrelated unobservable term. This residual can be interpreted as effects (e.g., capacity) that are not included in the theoretical model. Adding these changes, equation (23) becomes

$$(26) \quad n_t = 100 \cdot (1-\delta)\big(h_0 + s_t(h_1 - h_0)\big) + (1-\delta)\big(1 - h_0 - s_t(h_1 - h_0)\big)n_{t-1} + \nu_t,$$

$$(27) \quad \nu_t = \sum_{j=1}^{4} \phi_{sj} \nu_{t-j} + \varepsilon_t, \quad s = s_t \in \{0,1\},$$

$$(28) \quad E\{\varepsilon_t \varepsilon_r\} = 0, \quad \forall\, r \neq t,$$

Boldin (1990) estimates a system similar to equations (26) - (28) as part of a Markov-switching model. In this section, I proceed more directly, and use the employment chronologies to identify s_t. This allows me to estimate (26) - (28) by non-linear least squares. **Table 3** presents some results. Panel A of this table shows the results that arise when the ϕ_{sj}s are restricted to equal 0. Panel B shows the results that arise when these coefficients are not restricted. In both panels, h_0 is constrained to equal 0 in the first two columns, as suggested by the model, and is unrestricted in the second two. When unrestricted, h_0 has a p-value of under 9 percent in Panel A, and over 80 percent in Panel B.

In all specifications δ is less than or equal to 0.5 percent, so that employment nearly follows a random walk, and h_1 is less than or equal to 8 percent. In all cases, one

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[34]Given the wage scheme, the theoretical model implies that firms never have any incentive to lay off workers.

can not reject the hypothesis, used in the baseline simulation, that $\delta = 0.46$ percent. On the other hand, the simulation hypothesis that $h_1 = 8.9$ percent can be rejected. This statistically significant difference, however, has an economically insignificant effect on the simulations.

With the monthly data, the upper bounds on δ and h_1 are 0.22 percent and 5.29 percent respectively.

D. Indeterminacy

At this point, the estimates of δ, h_1 and $w(n, h)$ can be combined, through equation (20), to estimate $\chi(n)$. In particular, I use the function $w(n, h)$ shown in Figure 3, and the estimates of δ and h_1 in the restricted case of Panel B of **Table 3**. **Table 1** reveals that with these inputs, $\chi(n)$ has an average value of 41.4 percent,[35] which suggests that indeterminacy is an important, although not prevailing, feature of the economy.

It is revealing to redo these estimates with monthly data. With monthly data, $\chi(n)$ has a mean value of -0.07.[36] With this low level of indeterminacy comes low persistence: monthly employment growth ($n_t - n_{t-1}$) has a serial correlation of -0.11. (Monthly output growth is not readily available.) In contrast, with quarterly data, $\chi(n)$ has a mean value of 0.41. With this high level of indeterminacy comes high persistence: quarterly employment growth has a serial correlation of 0.49. The data thus reinforce what the theory and the simulations suggest; indeterminacy is essential for persistence.

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[35]In **Table 1**, the standard error for the mean of $\chi(n)$ does not reflect that δ, h_1 and $w(n, h)$ are estimates.

[36]This is at odds with the theoretical model, where $\chi(n)$ must be non-negative.

V. CONCLUSION

In this paper, I considered whether: (1) a dynamic forward-looking model with multiple equilibria can generate persistent fluctuations without persistent sunspots; and (2) indeterminacy is important for these persistent fluctuations. The answer to the first question was a tentative no. The answer to the second question was yes.

My approach was to apply a history-based selection rule to a dynamic model with multiple equilibria. Each period, firms adopted the hiring strategy of the previous period, unless the stochastic hiring cost made a change in hiring policy the dominant strategy. In theory, an economy that followed this history-based selection rule could exhibit persistent fluctuations without persistent sunspots. But the simulations showed that with plausible parameters, indeterminacy and history-based selection rarely came into play, and so were unable to generate persistence.

The theoretical model and the simulations suggested that the model, with one i.i.d. shock, can not deliver persistence without indeterminacy. A persistent recession requires that the cost shocks that induce a hiring switch are rare, and so does a persistent expansion. But in such a case, there will almost always be cost shocks where the hiring decision depends on the previous hiring decision, i.e., a high probability of indeterminacy. To get a sense of how much indeterminacy is "enough," I estimated the empirical transition function between periods of increasing and decreasing employment. If the model holds, these transition functions yield the probability of indeterminacy. With quarterly postwar data, I found the economy to be indeterminate a little over 41 percent of the time. This empirical results thus suggest that indeterminacy is essential to persistence as well.

Two future lines of research seem especially promising. One is to study the model at hand with more flexible functional forms, perhaps as described at the end of Section III. The other is to consider models such as those developed by Benhabib and Farmer (1996) or Perli (1996), where indeterminacy occurs under relatively low returns to scale, to see if history-based selection could be feasible there.

Table 1: Simulation Results

	U.S. Data: 1952II - 1997IV		Baseline Case		High Indeterminacy Alternative	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Average Employment	0.9167	0.0029	0.8494	0.0102	0.3735	0.0372
100×Employment Std. Deviation	1.8804	0.1768	1.9025	0.4490	20.1516	1.3278
Employment Serial Correlation	0.9785	0.0272	0.9367	0.0291	0.8509	0.0285
Average State[a]	0.5574	0.0534	0.2905	0.0254	0.4760	0.0658
State Serial Correlation[b]	0.4103	0.0712	-0.0175	0.0695	0.5824	0.0563
Average Output Growth	0.0041	0.0009	0.0000	0.0002	-0.0001	0.0124
100×Output Growth Std. Deviation	0.9950	0.0728	0.9195	0.0535	92.0872	9.0237
Output Growth Serial Correlation	0.3311	0.0672	-0.0276	0.0686	0.1225	0.0416
Average Indeterminacy (χ)	0.4142	0.0060	0.0007	0.0001	0.6915	0.0090

[a]The standard deviation of the state variable can be found as $\sqrt{m(1-m)}$, where m is the sample mean.
[b]Data are for 1952III - 1997IV.

Note: For the simulations, means are averages of the sample statistic across 3600 simulations and standard deviations are standard deviations of the sample statistic across 3600 simulations. The sample statistics for the U.S. are jointly estimated by Generalized Method of Moments.

Table 2: Transition Probabilities for the Economy's State

Panel A: n_t Measured by Employment/Population						
	Specification 1		Specification 2		Specification 3	
	Coefficient	Std. Error	Coefficient	Std. Error	Coefficient	Std. Error
a_0	0.676	0.241	0.519	0.428	1.000	0.490
a_1	0.328	0.188	0.353	0.207	0.759	0.243
a_2			0.052	0.119	-0.257	0.171
a_3					-0.272	0.107
b_0	1.046	0.232	0.779	0.412	0.770	0.413
b_1	-0.340	0.190	-0.399	0.205	-0.412	0.240
b_2			0.070	0.088	0.068	0.091
b_3					0.012	0.080
Log-Likelihood	-106.283		-105.883		-101.008	

Panel B: n_t Measured by 100 - Unemployment Rate						
	Specification 1		Specification 2		Specification 3	
	Coefficient	Std. Error	Coefficient	Std. Error	Coefficient	Std. Error
a_0	0.499	0.262	0.145	0.475	0.028	0.506
a_1	0.223	0.197	0.309	0.223	0.817	0.391
a_2			0.134	0.156	0.020	0.178
a_3					-0.213	0.118
b_0	1.421	0.241	1.287	0.399	1.351	0.408
b_1	-0.307	0.204	-0.330	0.208	-0.656	0.315
b_2			0.032	0.074	-0.027	0.081
b_3					0.180	0.118
Log-Likelihood	-99.565		-99.167		-95.983	

Table 2: Transition Probabilities for the Economy's State

Note: The coefficients describe the two-state variable s_t, which has the transition probability function:

$$(T1) \quad \Pr\left\{ s_{t+1} = 0 \,\middle|\, s_t = 0 \right\} = \frac{\exp\left(a_0 + a_1 n_{t-1} + a_2 DUR_t + a_3 CAP_t\right)}{1 + \exp\left(a_0 + a_1 n_{t-1} + a_2 DUR_t + a_3 CAP_t\right)},$$

$$(T2) \quad \Pr\left\{ s_{t+1} = 1 \,\middle|\, s_t = 1 \right\} = \frac{\exp\left(b_0 + b_1 n_{t-1} + b_2 DUR_t + b_3 CAP_t\right)}{1 + \exp\left(b_0 + b_1 n_{t-1} + b_2 DUR_t + b_3 CAP_t\right)},$$

where: n_t is employment measured in percent; s_t equals 1 when $n_t - n_{t-1} > 0$, and 0 otherwise; DUR_t is the lesser of 10 or the duration of the current economic episode; and CAP_t is the Federal Reserve's measure of manufacturing capacity utilization. The data are U.S. quarterly data for the period 1952II - 1997IV.

Table 3: Parameters for the Evolution of Employment

Panel A: Uncorrelated Errors				
	h_0 Restricted		h_0 Unrestricted	
	Coefficient	Std. Error	Coefficient	Std. Error
δ	0.0042	0.0004	0.0023	0.0012
h_0	0	NA	-0.0235	0.0138
h_1	0.0803	0.0051	0.0603	0.0130
Sum of Squared Residuals	17.2559		16.8723	

Panel B: Correlated Errors				
	h_0 Restricted		h_0 Unrestricted	
	Coefficient	Std. Error	Coefficient	Std. Error
δ	0.0043	0.0004	0.0046	0.0021
h_0	0	NA	0.0041	0.0225
h_1	0.0771	0.0051	0.0806	0.0224
ϕ_{10}	0.1652	0.1290	0.1699	0.1162
ϕ_{20}	0.1488	0.1279	0.1610	0.1310
ϕ_{30}	0.0958	0.1182	0.0986	0.1226
ϕ_{40}	0.2224	0.0615	0.2287	0.0692
ϕ_{11}	-0.1683	0.1157	-0.1648	0.1156
ϕ_{21}	0.0417	0.0767	0.0465	0.0785
ϕ_{31}	-0.0275	0.1370	-0.0271	0.1494
ϕ_{41}	-0.0016	0.0587	0.0016	0.0622
Sum of Squared Residuals	14.7452		14.7412	

Table 3: Parameters for the Evolution of Employment

Note: The coefficients are for the following model of employment dynamics:

$$(T1) \quad n_t = 100 \cdot (1-\delta)\big(h_0 + s_t(h_1 - h_0)\big) + (1-\delta)\big(1 - h_0 - s_t(h_1 - h_0)\big)n_{t-1} + \nu_{t+1},$$

$$(T2a) \quad \nu_t = \varepsilon_t, \quad \text{in Panel A,}$$

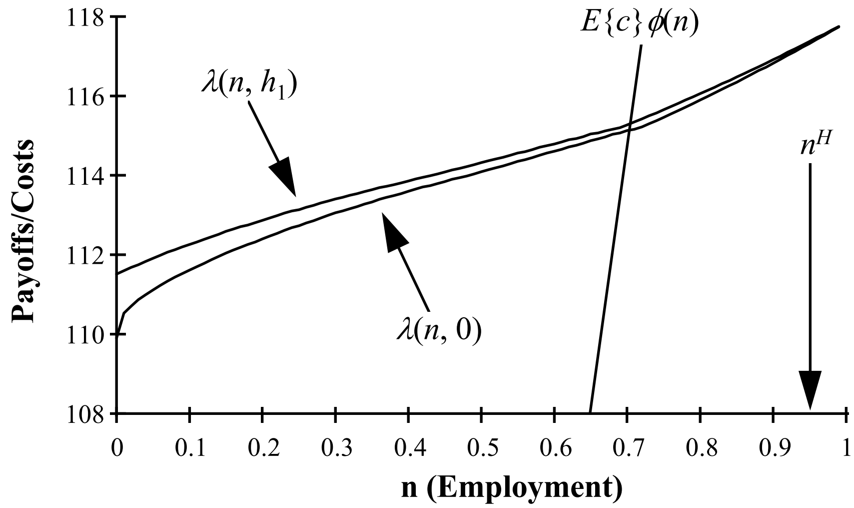
$$(T2b) \quad \nu_t = \textstyle\sum_{j=1}^{4} \phi_{sj}\nu_{t-j} + \varepsilon_t, \quad s = s_t \in \{0,1\}, \quad \text{in Panel B,}$$

$$(T3) \quad E\{\varepsilon_t \varepsilon_r\} = 0, \quad \forall\, r \neq t,$$

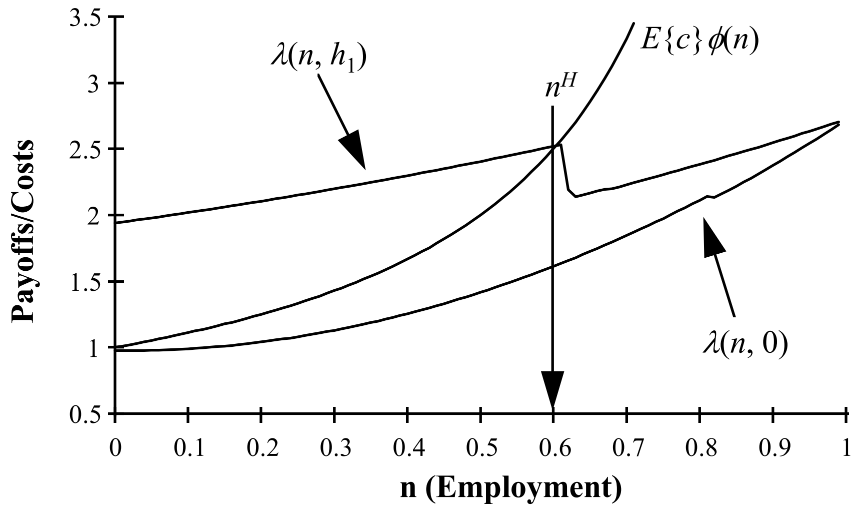
where: n_t is employment measured in percent; and s_t equals 1 when n_t - $n_{t\text{-}1} > 0$, and 0 otherwise. The data are U.S. quarterly data for the period 1952II - 1997IV.

Figure 1: Hiring Costs and Payoff Functions

Panel A: Baseline Case



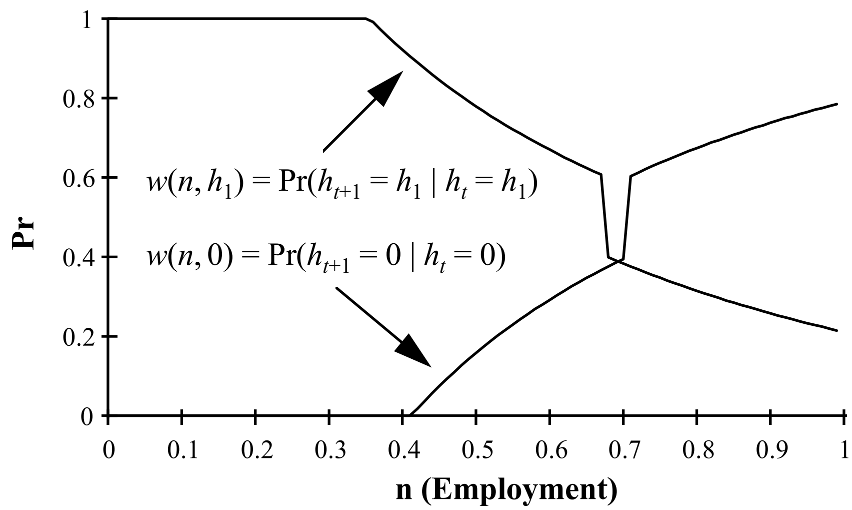
Panel B: High Indeterminacy Case



Note: λ^L and λ^H give the expected present value of an additional worker under Cooper's (1994) history-based equilibrium selection rule. n^H is maximum sustainable employment. $E\{c\}\phi(n)$ gives the cost of finding and hiring an additional worker.

Figure 2: Regime Switching Probabilities as a Function of Employment

Panel A: Baseline Simulation



Panel B: High Indeterminacy Simulation

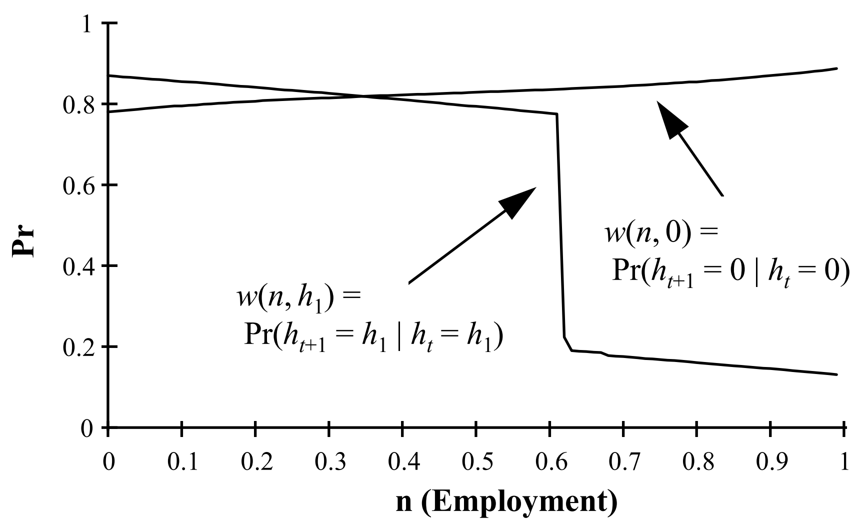
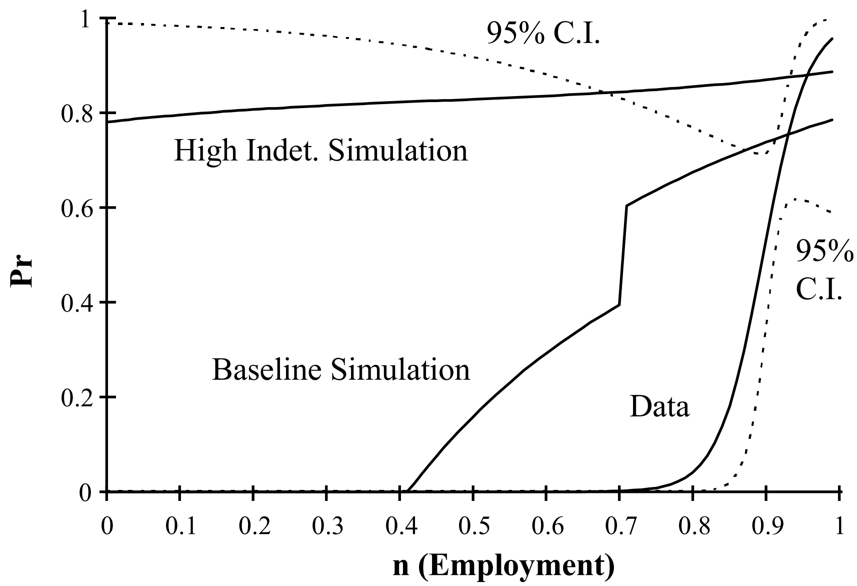


Figure 3: Regime Switching Probabilities as a Function of Employment

Panel A: $w(n, 0) = \Pr(h_{t+1} = 0 \mid h_t = 0)$



Panel B: $w(n, h_1) = \Pr(h_{t+1} = 1 \mid h_t = 1)$

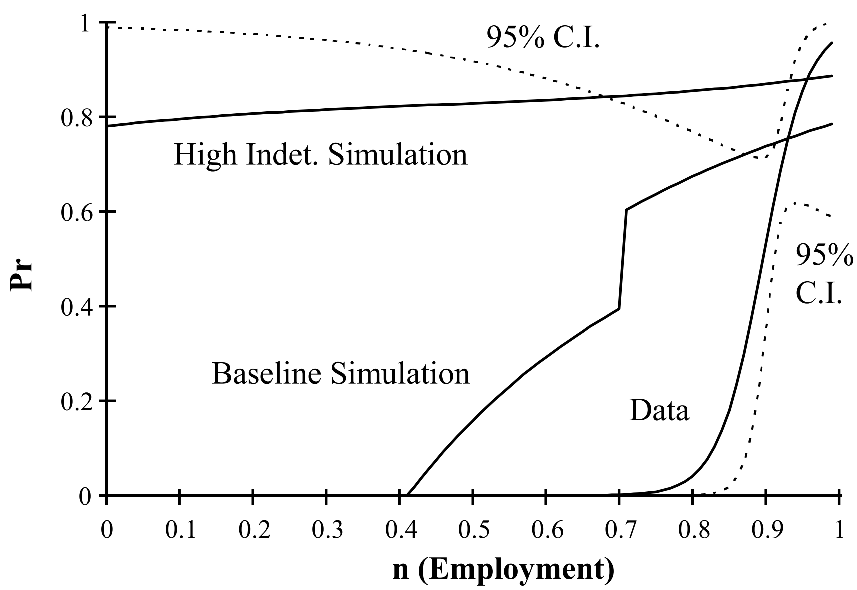


Figure 3: Regime Switching Probabilities as a Function of Employment

Note: "Data" consists of the following two functions:

$$(T1)\quad \Pr\left\{ h_{t+1} = 0 \mid h_t = 0 \right\} = 1 - \frac{1}{1 + \exp\left(0.676 + 0.328 \cdot \left(100n_t - 91.67\right)\right)},$$

$$(T2)\quad \Pr\left\{ h_{t+1} = 1 \mid h_t = 1 \right\} = 1 - \frac{1}{1 + \exp\left(1.046 - 0.340 \cdot \left(100n_t - 91.67\right)\right)},$$

See **Table 2** for the derivation of these numbers. 91.67 is the average employment rate for 1952-1997, estimated by Generalized Method of Moments (see **Table 1**).

See text for description of how confidence intervals are set.

The simulation results are drawn from **Figure 2**.

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