The Organization of Supply: a Vertical Equilibrium Analysis*

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Abstract

In this paper I study how the make-or-buy decision of a firm depends on the organization of its peers. I consider a multi-firm framework in which firms choose whether to integrate into the supply of an intermediate input or to outsource its production, and choose the size of their supplier network if outsourcing. Firms find it optimal to share the same set of suppliers, as there are economies of scope in investment to suppliers taking multiple designs. These economies are due to spillovers of technical or operational know-how between projects and to savings in the setup costs on physical capital. The model admits multiple vertical equilibria that are Pareto-ranked, the one with the highest level of outsourcing being most efficient. Outsourcing is more likely in larger markets and when the economies of scope are stronger. The size of the optimal supplier network however typically decreases when the spillovers are stronger. These findings provide insight into the patterns of reorganization of vertical supply relations observed over the last two decades.

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1 Introduction

Ample evidence suggests that the way firms organize their supplies of intermediate inputs is influenced by the business environment they operate in. For example, Chinitz (1961) documents that input outsourcing is more prevalent in larger markets, and Holmes (1999) finds that firms located in concentrations of same industry plants are more likely to buy inputs on the outside. Indeed, a firm's decision to outsource the production of an input is likely to hinge upon factors such as the number and quality of potential suppliers, which are determined, to a great extent, by the demand for outsourcing services by other firms. Presumably, the thicker the market for outsourcing services, the more numerous are the potential suppliers, and the higher the level of available expertise.

Somewhat surprisingly, the mechanisms at work are rarely discussed in the literature on vertical organization of firms. This paper aims to fill this gap. The analysis also sheds light on the dramatic reorganization of vertical supply relations that has taken place over the last two decades. A substantial increase in outsourcing of services (examples include accounting, legal and financial services, logistic management, call centers and many more) has been documented across many industries in the US and worldwide.[1] Several authors[2] have argued that the increase in outsourcing was accompanied by an adoption of Japanese-like system of supplier relations by US manufacturers, of which one of the main attributes is a reduction in the number of suppliers. This paper is unique in studying both the extent of outsourcing and the determination of supplier networks within a unified framework. As a result, I am able to explore how these two organizational features depend on the same characteristics of the environment.

Below I develop a theoretical framework in which a firm's make-or-buy trade-off is affected by the organization of its peers. I consider a multi-firm model where each firm requires a similar intermediate input. A firm can either manufacture the input in-house or outsource its production, but outsourcing is not limited to a single supplier. Each outsourcing firm can establish a supplier network and divide its orders of the input among its members. The interdependency between different firms' decisions stems from the fact that it is beneficial for firms to share suppliers with their peers. The reason is that suppliers achieve *economies of scope* by taking designs from several firms. Such economies are either due to spillovers of technical and operational know-how between projects for different buyers, or to the amortization of one-time setup costs of physical capital over a larger group of clients.

[1] See Abraham and Taylor (1996), Feenstra (1998) and numerous references in the business press.

[2] For example McMillan (1995).

Firms do not however interact in the product market, and strategic considerations, such as those explored in the literature on vertical foreclosure, are therefore absent.[3]

In this framework, a simple and intuitive relation between the outsourcing decisions of different buyers is established; the efficiency of outsourcing increases with the number of outsourcing firms. Positive externalities between firms' outsourcing decisions result in a multiplicity of organizational structures in equilibrium. Equilibria differ in the share of firms that outsource, the size of the supplier networks, and in the level of relationship-specific investments made by each of the parties. Admitting multiple organizational equilibria, our multi-firm model is capable of capturing differences between industrial systems that cannot be easily attributed to the underlying characteristics of different bilateral relationships,[4] and hence are not likely to be present in single-firm models of the make-or-buy decision. In line with the empirical findings cited above, we show that outsourcing is more prevalent in larger markets. Within our framework, both an increase in outsourcing and a reduction in the size of supplier networks can be attributed to stronger spillovers between designs, possibly due to improved codification of organizational know-how using information technology. Last I address the relation between the organization of supply and the design of intermediate inputs and final outputs. Anecdotal evidence (discussed below) suggests that outsourcing is often associated with an increase in the standardization of intermediate inputs. Such standardization potentially diminishes the value of the final product, and can constrain firms ability to differentiate themselves. I show how such considerations fit within the general framework.

My model is in the tradition of the Property Rights Theory of the firm (Grossman and Hart (1986), Hart and Moore (1990), Hart (1995)), emphasizing the importance of relationship-specific investments to the organization of supply relations. Firms are unable to write and enforce explicit outsourcing delivery contracts for the input, and the organizational mode determines the ex-ante investment incentives. The emphasis here is however not on assets' ownership as in the papers mentioned above but on access as in Rajan and Zingales (1998). Firms control the access to their design's blueprint, and can decide on the size of their supplier network. Granting access to more suppliers mitigates the firm's fears of being held up and strengthens its incentives to make relationship-specific investments but at the same time dilutes the incentives of the suppliers. There are only a few other

[3]This assumption allows us to focus solely on the interaction between firms' supply decisions. In an extension to this paper, Levy (2003), I explore the implications of introducing product market competition between the firms.

[4]Comparative studies of industrial structure have revealed stark differences in organization of supply between similar industries in relatively similar countries. See McLaren (2000) for several examples.

papers in this literature that go beyond the bilateral framework (single buyer and a single supplier) and those do not consider the same interdependency among firms examined here. Hart and Moore (1990) analyze a general multiparty environment, but they take all parties' investments to be complements to each other. Such an assumption is not plausible for the description of competing suppliers. Substitute investments are discussed in Bolton and Whinston (1993) and Rajan and Zingales (1998). Unlike here however, in both of these papers the number of suppliers is exogenously fixed.

Two recent papers consider research questions similar to those pursued here. McLaren (2000) explore why an increased openness to trade can increase the extent of domestic outsourcing. Grossman and Helpman (2002) develop a general equilibrium model of industrial vertical structure that can be used to explain the growth in international outsourcing. There are significant differences between these papers and the current one. First, while both papers employ a multi-firm equilibrium framework, they limit the organization of supply to bilateral arrangements. Second, there are no bilateral ex-ante investments in the sense the Property Rights literature. Finally, in both of these papers firms do not share suppliers so the effects of spillovers analyzed here are absent. The factors determining the ranking of alternative organizations are therefore very different than here, and my results can be viewed as complementary to theirs.

The remainder of this paper is organized as follows. Section 2 introduces general notation. Section 3 considers the organization of supply of a single firm in isolation, ignoring suppliers' economies of scope from taking the designs of multiple firms. This analysis is of independent interest, as most discussions of the make-or-buy decision do not address the determination of the optimal size of the supplier network of outsourcing firms. Section 4 introduces the full fledged multi-firm setup, with economies of scope in suppliers investment. It is shown that integration provides better incentives for firms' investment than outsourcing, but the gap is reduced the more suppliers are employed under outsourcing. An outsourcing arrangement is better in promoting suppliers' investment, and its advantages are magnified due to economies of scope in investment, the more outsourcing firms there are. Section 5 characterizes the properties of vertical equilibria, and discusses the multiplicity of organizational equilibria. Equilibria are shown to be Pareto-ranked, the one with the highest level of outsourcing being most efficient. Section 6 presents the results of several comparative statics exercises. It is shown that outsourcing is more likely in bigger markets, and that it is more pervasive when the share of spillovers between designs and the level of setup cost savings is higher. The two types of savings have opposite effects on the optimal size of the supplier network. It expands when there are larger savings in setup costs but typically shrinks when the spillovers are stronger.

In section 7, I look at a case where positive spillovers occur only if the input is partially standardized. Standardization diminishes the value of the final product of the buyer by a fixed amount, but can lead to significant cost savings. I show that this formulation fits easily into my framework. Standardization and outsourcing are pervasive in some equilibria and scant in others. Finally in section 8 I discuss alternatives to some of the main modeling assumptions.

2 Notation and General Structure

M downstream buyers (firms) $B_1, B_2, ..., B_M$, each require exactly one unit of an intermediate upstream input. The values of the buyers' final outputs are independent of each other (buyers do not compete with one another in the product market).[5] The process through which buyers decide how to organize their supply of the input is described by a multistage game, whose sequence of events is outlined below:

Integration/Outsourcing decisions: Each buyer decides whether to integrate into the supply of the input or to outsource it. Denote by $m \leq M$ the number of buyers who choose to outsource in equilibrium. Each integrating buyer pays a setup cost $K > 0$.

Access: The access stage consists of a single round of offers in which each of the m outsourcing buyers simultaneously approaches a subset of all potential suppliers with *access offers*. Access offers are publicly observed. An access offer to supplier j specifies a fee F_i^j to be paid to the buyer, and its acceptance grants the supplier access to the blueprint of the input required by the buyer.[6] Let S_i denote the set of suppliers accepting B_i's access offer and $n_i = |S_i|$ denote their number. Each supplier has to invest an identical setup cost K per offer accepted. Suppliers may work with several buyers.

Investment: At the beginning of the investment stage, the identity of all suppliers accepting offers becomes public. Managers of the upstream and downstream units then make

[5] While this assumption mainly serves to focus on the interaction between decisions on the organization of supply, there are nevertheless certain circumstances in which it seems reasonable. For example when firms' territories are geographically segmented, or when the firms are in different lines of business but use some similar inputs.

[6] It is therefore implicitly assumed that suppliers are not cash-constrained and that access is contractible. A similar assumption is made for example in Rajan and Zingales (1998).

design-specific investments. Downstream investment increases the value of the intermediate input to the buyer, whereas upstream investment lowers the cost of producing it. Denote by b_i the downstream investment, by s_i the upstream investment if B_i is integrated and by $\underline{s}_i = \left\{s_i^j\right\}_{j\in S_i}$ the investment profile of the n_i independent suppliers if B_i outsources. Investments are non-monetary and bear a disutility $\psi(x)$ to the managers who undertake them, where x is the investment level and $\psi' > 0$, $\psi'' > 0$ everywhere.[7] Investment levels are observed by the buyer and all of the suppliers, but are non-contractible in the sense of Hart (1995).

Multilateral bargaining and production: Employing the unit of intermediate input, B_i produces and sells a final product, bearing revenues

$$v(b_i) = \alpha_B b_i,$$

where $\alpha_B \in \mathbb{R}_+$. A supplier $j \in S_i$ can produce $x_i^j \in \mathbb{R}_+$ units of intermediate input at a variable cost $c\left(x_i^j, s_i^j\right)$ provided it made an earlier cost-reducing investment of s_i^j. Variable costs are strictly convex in x_i^j. Specifically we assume

$$c\left(x_i^j, s_i^j\right) = \widehat{c}\left(x_i^j\right) - \alpha_S s_i^j x_i^j \ ,$$

where $\alpha_S \in \mathbb{R}_+$ and $\widehat{c}(0) = 0$, $\widehat{c}' > 0$, and $\widehat{c}''$ is bounded above zero on $[0, 1]$.[8]

The production of the unit of intermediate input required by the buyer can be divided between suppliers. If, conditional on investments $\underline{s}_i$, *production is efficiently allocated* among a non-empty subset P of S_i, then the aggregate cost of production of the unit of intermediate output is

$$c^*\left(\underline{s}_i, P\right) = \min_{\left\{x_i^j\right\}_{j\in P}} \left\{ \sum_{j\in P} c\left(x_i^j, s_i^j\right) \ \middle| \sum_{j\in P} x_i^j = 1,\ x_i^j \geq 0 \right\}. \tag{1}$$

By the Theorem of the Maximum (Berge 1959), $c^*\left(\underline{s}_i, P\right)$ is continuous in $\underline{s}_i$, and $\underline{x}_i\left(\underline{s}_i, P\right)$ is upper hemi-continuous. Furthermore as $\sum_{j\in P} c\left(x_i^j, s_i^j\right)$ is strictly convex in x_i^j, the maximizer $\underline{x}_i\left(\underline{s}_i, P\right)$ is continuous and its partial derivatives with respect to the elements of $\underline{s}_i$ exist.[9]. Finally, suppliers' production costs are additive across buyers and hence if j produces for several buyers its production cost is $\sum_{\{k|j\in S_k\}} c\left(x_k^j, s_k^j\right)$.

[7] The specification is somewhat different under integration. See section 3.2.

[8] I discuss the implications of working with alternative specifications of the cost function in footnote 11 and in section 8 below.

[9] A necessary and sufficient condition for the partial derivatives of $\underline{x}_i\left(\underline{s}_i, P\right)$ to exist is that the bordered Hessian matrix,

The mutual surplus of *vertical structure* i (consisting of a buyer B_i and its supplier network, S_i), taking investment costs as sunk, is given by:

$$v(b_i) - c^*(\underline{s}_i, S_i).$$

This surplus is always positive, for all levels of investments.

Outsourcing buyers bargain with their pool of suppliers over the division of these rents from trade. We take an axiomatic approach to the determination of the bargaining outcome: the share of this surplus each party receives is determined by its respective Shapley value. We outline these shares explicitly below.

3 Single Buyer/Many Suppliers

We begin by considering the manner in which a single buyer B_i chooses to organize its supply in isolation. We analyze and compare two modes of organization: *outsourcing* to multiple suppliers versus *vertical integration* and internal supply. We defer to the next section the analysis of interdependence between organizational choices made by different buyers.

3.1 Outsourcing

The game is analyzed backwards, starting from the bargaining stage.

3.1.1 Bargaining and Production

Assume that B_i has invested b_i in its design, that a set S_i of suppliers has been given access to it, and that each $j \in S_i$ has invested s_i^j in cost-reduction. Production is then allocated efficiently between the suppliers and the mutual surplus, $v(b_i) - c^*(\underline{s}_i, S_i)$, is divided by the parties according to their respective Shapley values. The buyer's share is defined as the sum of its marginal contributions to all possible coalitions that include any non-empty subset P of the potential suppliers:

$$\phi_i(b_i, \underline{s}_i) \equiv \frac{\sum_{P \subseteq S_i, P \neq \varnothing} [v(b_i) - c^*(\underline{s}_i, P)] \cdot (|P|)!\,(n_i - |P|)!}{(n_i + 1)!}. \tag{2}$$

$$\begin{bmatrix} 1 & 1 & \dots & 1 & 0 \\ \widehat{c}''(x_i^1) & 0 & \dots & 0 & 1 \\ 0 & \widehat{c}''(x_i^2) & \dots & . & . \\ . & . & \dots & . & . \\ 0 & 0 & & \widehat{c}''(x_i^n) & 1 \end{bmatrix}$$

has full rank, where $n = |P|$. This condition is clearly satisfied as $\widehat{c}'' > 0$ on $[0, 1]$.

The marginal contribution of supplier j is positive only for coalitions that include B_i as well. Define $m_j(\underline{s}_i, P)$ as the marginal contribution of j to a coalition that includes B_i and a subset of the suppliers $P \subseteq S_i$ such that $j \notin P$. Then:

$$m_j(\underline{s}_i, P) \equiv \begin{cases} v(b_i) - c\left(1, s_i^j\right) & if \quad P = \phi, \\ c^*(\underline{s}_i, P) - c^*(\underline{s}_i, P \cup j) & otherwise. \end{cases} \tag{3}$$

The Shapley value of each supplier $j \in S_i$ is given by

$$\phi_i^j(b_i, \underline{s}_i) \equiv \frac{\sum_{P \subseteq S_i \setminus j} m_j(\underline{s}_i, P) \cdot (|P|+1)!\,(n_i - |P| - 1)!}{(n_i+1)!}. \tag{4}$$

3.1.2 Investments

Anticipating the bargaining outcomes outlined above, each party decides on its investment level. We turn now to characterize equilibrium investments in this subgame.[10]

A buyer B_i investment is a solution to

$$\max_{b_i} \phi_i(b_i, \underline{s}_i) - \psi(b_i). \tag{5}$$

Differentiating (2) above, the marginal return to a buyer's investment is then:

$$\begin{aligned} \frac{\partial\left[\phi_i(b_i, \underline{s}_i)\right]}{\partial b_i} &= \frac{\sum\limits_{P \subseteq S_i, P \neq \varnothing} (|P|)!\,(n_i - |P|)!}{(n_i+1)!} v'(b_i) \\ &= \frac{\sum_{k=1}^{n_i} \binom{n_i}{k} (k)!\,(n_i - k)!}{(n_i+1)!} v'(b_i) \\ &= \frac{n_i}{n_i+1} \alpha_B. \end{aligned} \tag{6}$$

where the second line follows from the fact that there are $\binom{n_i}{k}$ coalitions with $|P| = k$ suppliers.

For a supplier $j \in S_i$, investment s_i^j maximizes

$$\max_{s_i^j} \phi_i^j\left(b_i, \left(s_i^j, s_i^{-j}\right)\right) - \psi\left(s_i^j\right). \tag{7}$$

Differentiating (4), the marginal return to the supplier j's investment is then:

$$\frac{\partial\left[\phi_i^j(b_i, \underline{s}_i)\right]}{\partial s_i^j} = \frac{\sum_{P \subseteq S_i \setminus j} \frac{\partial\left[m_j(\underline{s}_i, P)\right]}{\partial s_i^j} (|P|+1)!\,(n_i - |P| - 1)!}{(n_i+1)!},$$

[10]The strict quasi-concavity of the payoff functions in own investments and their continuity in $(b_i, \underline{s}_i)$ guarantee the existence of equilibrium in pure strategies.

where following (3),

$$\frac{\partial\left[m_j\left(\underline{s}_i,P\right)\right]}{\partial s_i^j}=\begin{cases}-c_s\left(1,s_i^j\right) & if \quad P=\phi,\\ -\frac{\partial\left[c^*\left(\underline{s}_i,P\cup j\right)\right]}{\partial s_i^j} & otherwise.\end{cases}$$

As the constraint set of the production allocation problem (1) does not depend on the investment profile, $\underline{s}_i$, the envelope theorem implies

$$\frac{\partial\left[c^*\left(\underline{s}_i,P\cup j\right)\right]}{\partial s_i^j}=c_s\left(x_i^j\left(\underline{s}_i,P\cup j\right),s_i^j\right),$$

and therefore given the functional form used here, for all P,

$$\frac{\partial\left[m_j\left(\underline{s}_i,P\right)\right]}{\partial s_i^j}=\alpha_S x_i^j\left(\underline{s}_i,P\cup j\right).$$

The marginal return to supplier j's investment is then

$$\frac{\partial\left[\phi_i^j\left(b_i,\underline{s}_i\right)\right]}{\partial s_i^j}=\alpha_S\sum_{P\subseteq S_i\backslash j}\left\{\frac{(|P|+1)!\,(n_i-|P|-1)!}{(n_i+1)!}x_i^j\left(\underline{s}_i,P\cup j\right)\right\}. \tag{8}$$

As suppliers are symmetric, the equilibrium investments are symmetric as well , $s_i^j=s^O$ for all $j\in S_i$. Denote the equilibrium investment profile by $\left(b^O,\underline{s}^O\right)$ where $\underline{s}^O=\left(s^O,...,s^O\right)$. Given symmetric investments, the cost-minimizing allocation of input production between suppliers, for every subset $P\subseteq S_i$, is symmetric as well, $x_i^j\left(\underline{s}^O,P\right)=\frac{1}{|P|}$, $\forall j\in P$. Substituting back into (8) we get

$$\frac{\partial\left[\phi_i^j\left(b^O,\underline{s}^O\right)\right]}{\partial s_i^j}=\alpha_S\sum_{P\subseteq S_i\backslash j}\left\{\frac{(|P|+1)!\,(n_i-|P|-1)!}{(n_i+1)!}\cdot\frac{1}{|P|+1}\right\}.$$

The marginal contribution of an additional *identical* supplier to a coalition containing the buyer and a subset P of size k of S_i is a function of the number of suppliers k only. Collecting all terms pertaining to coalitions P of size k and noting that there are $\binom{n_i-1}{k}$

such coalitions, we write:

$$\begin{aligned}
\frac{\partial\left[\phi_i^j\left(b^O,\underline{s}^O\right)\right]}{\partial s_i^j} &= \alpha_S\frac{\sum_{k=0}^{n_i-1}\left\{\binom{n_i-1}{k}(k+1)!\,(n_i-k-1)!\frac{1}{k+1}\right\}}{(n_i+1)!} \\
&= \alpha_S\frac{1}{n_i\left(n_i+1\right)}\sum_{k=0}^{n_i-1}\left\{(k+1)\frac{1}{k+1}\right\} \\
&= \alpha_S\frac{1}{n_i\left(n_i+1\right)}\sum_{k=1}^{n_i}\left\{k\cdot\frac{1}{k}\right\} \\
&= \frac{1}{(n_i+1)}\alpha_S.
\end{aligned}$$

Consider next the second-order conditions for the investment problems. The global concavity of the buyer's program is assured as $\psi'' > 0$ everywhere. In the appendix we derive the second-order condition for the supplier's problem and show that the following condition is sufficient for that program to be locally concave in the neighborhood of the equilibrium investments' profile $\left(b^O,\underline{s}^O\right)$.

Assumption 1 $\psi''(s) > \frac{(\alpha_S)^2}{2*\inf\{\tilde{c}''(x)|x\in[0,1]\}}$ *everywhere.*

The next Proposition summarizes the results above and characterizes the equilibrium investments:

Proposition 1 *There exists a symmetric equilibrium for the investment subgame for which investments under outsourcing are* $b_i = b^O(n_i)$ *and* $s_i^j = s^O(n_i)$ *for all* $j \in S_i$*, characterized by the first-order conditions:*

$$\frac{n_i}{n_i+1}\alpha_B - \psi'\left(b^O\right) = 0. \tag{9}$$

and

$$\frac{1}{(n_i+1)}\alpha_S - \psi'\left(s^O\right) = 0, \tag{10}$$

The equilibrium investments satisfy the following properties:

1. *A buyer's investment is increasing in the number of suppliers* (n_i).

2. *Each supplier's investment is decreasing in the number of suppliers* (n_i) *.*

Proof. Given the concavity of the objectives, the equilibrium is characterized by the first-order conditions of (5) and (7) respectively.

Treating n_i as a continuous variable, differentiating (9) and applying the Implicit Function Theorem:

$$\frac{db^O}{dn_i} = \frac{\alpha_B}{\psi''\left(b^O\right)\left(n_i+1\right)^2} > 0.$$

Similarly differentiating (10),

$$\frac{ds^O}{dn_i} = -\frac{\alpha_S}{\psi''\left(s^O\right)\left(n_i+1\right)^2} < 0.$$

■

It is instructive to compare the equilibrium investments to the *first-best* investments, $b_i = b^{FB}, s_i^j = s^{FB}$ for all $j \in S_i$. These are solutions to

$$\begin{aligned} \alpha_B - \psi'\left(b^{FB}\right) &= 0, \\ \frac{1}{n_i}\alpha_S - \psi'\left(s^{FB}\right) &= 0. \end{aligned}$$

As in most Property Rights models (see for example Hart (1995)), both parties *under-invest* in relationship-specific capital compared with the first-best levels. With regard to the suppliers' investments however, this result depends on the functional form of the cost function $c\left(x_i^j, s_i^j\right)$ used.[11] We choose to work here with a specification that yields under-investment mainly for resemblance to the bulk of the literature.

Next consider the *total surplus,* $S^O(n)$, net of investment and setup costs, generated by a vertical structure in the symmetric equilibrium above. We have:

$$S^O(n) = \alpha_B b^O - n\widehat{c}\left(\frac{1}{n}\right) + \alpha_S s^O - \psi\left(b^O\right) - n\psi\left(s^O\right) - nK. \tag{11}$$

In the appendix, we derive expressions for $\frac{dS^O(n)}{dn}$ and $\frac{d^2S^O(n)}{dn^2}$. It is easy to see that $\frac{dS^O(n)}{dn} < 0$ for $n > \widetilde{n} > 0$. While it is hard to characterize $S^O(n)$ further without additional assumptions, we show that for a parametrization of the model with a quadratic disutility of effort, $\psi(x) = \frac{x^2}{2}$, the surplus $S^O(n)$ is single-peaked in n. While single-peakedness is likely to hold under plausible restrictions on the model's primitives, we do not provide a full characterization of these conditions here. Rather we posit

Assumption 2 *$S^O(n)$ is single-peaked in n.*

[11] Over-investment is possible when alternative specifications of the cost function are employed. Supplier's gross payoff, (4), is a weighted sum of its marginal contributions to coalitions smaller than the grand one, in which the share of production allocated to each supplier is higher than in the symmetric social optimum. For this reason the private return to investment may end up higher than the social return, though not in the case studied here. It can be shown for example that over-investment would result if $c(x,s) = a(s)x^d$ where $a' < 0$ and $d \geq 2$.

While the single-peakedness assumption greatly simplifies the exposition, it is not essential to what follows.

3.1.3 Access

We now consider the optimal choice of access by B_i.[12] As access offers are, by assumption, non-negotiable, B_i can extracts the entire transactional surplus[13] using the access fee. If n^O suppliers are given access then an access fee F^O such that

$$F^O = \phi_i^j \left(b^O \left(n^O\right), \underline{s}^O \left(n^O\right)\right) - \psi \left(s^O \left(n^O\right)\right) - K,$$

leaves suppliers exactly indifferent between accepting the offer and rejecting access. The optimal number of suppliers to be given access to B_i's design, n^O, therefore maximizes the total surplus, S^O.[14] Provided that $S^O(n)$ is single-peaked as assumed, we define n^O as follows:

Definition 1 $n^O = \max \left\{n \in \mathbb{N} \mid S^O(n) > S^O(n-1)\right\}$

3.2 Integration

In the event that B_i integrated upstream into the production of the intermediate input, B_i's owner can make investments pertaining to both the downstream and upstream units. Denote by b_i the downstream investment and by s_i the upstream one. We assume that the *owner's upstream investment is less efficient than it is in the downstream business*, due either to managerial overload or simply poor understanding of the upstream business.[15]

[12]The optimal choice of access in an "incomplete contracts" setup was first studied by Rajan and Zingales (1998). In their model, in which only suppliers but not the buyer invest ex-ante, it is never optimal for a buyer grant access to more than one supplier, whenever suppliers' investments are *perfect substitutes*. There are two main differences between their analysis and the current one. First, both upstream and downstream parties invest here, and an increase in the number of suppliers has therefore a positive effect on the investment incentives of the downstream unit. Second, due to the decreasing returns nature of the input production technology, suppliers are not perfect substitutes to each other.

[13]This serves mainly to simplify the exposition.

[14]If suppliers are cash-constrained, access fees may not be used, and the number of suppliers would be chosen to maximize the buyer's bargaining payoff. Another possibility is that access is not contractible, as the design's blueprint is readily available to all suppliers and buyers are unable to pre-commit to exclude suppliers at the bargaining stage. In that case, suppliers may enter freely and their number is determined by a zero-profit condition.

[15]Our assumptions here depart somewhat from the norm of Property Rights models. There it is typically assumed that under upstream integration the supply unit is run by a non-owner manager. If such manager does not enjoy any quasi-rents at the production stage and may be costlessly replaced, he invests nothing. Such an assumption is made in for example in Bolton and Whinston (1993). Here we implicitly maintain a similar assumption while allowing the owner to invest (inefficiently) by himself.

Assumption 3 (Inefficiency of upstream investment under integration) *The disutility of an upstream investment s_i equals $\lambda\psi(s_i)$ for some $\lambda > 1$.*

λ measures the relative "inefficiency" in upstream investment of the common owner, B_i. The disutility from downstream investment is $\psi(b_i)$ as before.[16]

The integrated firm can set multiple production lines for the input, whose number we denote by l_i, at cost of K each. Setting up multiple lines may be efficient given the decreasing returns nature of the input production technology.[17] For a given number of production lines l_i, investments b_i and s_i maximize

$$\max_{b_i,s_i} \alpha_B b_i - l_i \widehat{c}\left(\frac{1}{l_i}\right) + \alpha_S s_i - \psi(b_i) - \lambda\psi(s_i) - l_i K$$

The optimal investments, b^I, s^I then satisfy the following necessary and sufficient conditions:

$$\alpha_B - \psi'\left(b^I\right) = 0, \tag{12}$$

$$\frac{1}{\lambda}\alpha_S - \psi'\left(s^I\right) = 0. \tag{13}$$

The bottom first-order condition is rearranged to facilitate the comparison to the outsourcing first-order conditions, (9)-(10). Under integration, downstream investment is at the first-best level and hence above its outsourcing level. On the other hand, for an equal number of production facilities under integration and outsourcing, $l_i = n_i$, the incentives to invest in the upstream unit are diminished if

$$\frac{1}{\lambda} < \frac{1}{n_i + 1},$$

or

$$n_i + 1 < \lambda.$$

The following assumption implies that the incentives for upstream investment are lower under integration, at least in comparison with the case of outsourcing to a single supplier.

Assumption 4 $\lambda > 2$*.*

Finally, the optimal number of lines, l^I, then satisfies

$$l^I = \arg\max_{l} \alpha_B b^I - l\widehat{c}(1/l) + \alpha_S s^I - \psi\left(b^I\right) - \lambda\psi\left(s^I\right) - lK$$

and $S^I(\lambda)$ is the value of the objective above at b^I, $s^I = s\left(l^I\right)$.

[16]An additional implication of the assumption is that any integrated buyer B_i is inferior compared with an independent supplier in supplying the needs of any other buyer B_k. This rules out the possibility that an integrated firm supplies several of its peers.

[17]This assumption is made to assure that an integrated organization is not inferior to one in which the production is outsourced to multiple suppliers simply due to technological considerations.

3.3 Comparison Between Organizations

As in the standard bilateral model discussed in the literature (a canonical example is Hart (1995)), downstream integration in our setup promotes downstream-specific investment while outsourcing favors upstream-specific investment. Which organization would prevail depends on the magnitude of the different effects.

4 Many Buyers/Many Suppliers

We now turn to analyze the full model where M buyers simultaneously choose the organization of their supply for the upstream input. The main difference with the single-buyer case is that the efficiency of outsourcing is determined by the number of buyers that outsource. The central assumption driving the multi-buyer model is that there are economies of scope in investment when a supplier takes several designs. We envision two possible types of savings:

Assumption 5 (Economies of scope of taking multiple designs)

1. *The per-design (or average) setup cost $K(d) \geq 0$ is decreasing in the number of designs taken: $K'(d) \leq 0$.*
2. *Supplier's investment in one buyer's design spills over to other designs that it undertakes. Specifically we denote the share of investment in one design that spills over by $\gamma \in [0, 1]$.*

An example of the first type is the installation of an information technology platform (e.g.: inventory management system) that can be used by a supplier in its transactions with multiple clients. As an example of the second type, consider a "process innovation" of either technological or organizational nature, that lowers the cost of producing the input. Know-how and experience that has been acquired while working with one buyer may be applicable to some degree in jobs performed for others.

4.1 Outsourcing

We focus attention on the case where all m outsourcing buyers give access to the same set of suppliers, $S_i = \{1, ..., n\}$ for all i. Below we argue that such a result would indeed emerge in equilibrium.

4.1.1 Multilateral Bargaining

Given our assumptions, the bargaining between each buyer and members of its supplier network is independent of other buyers actions. The analysis here is therefore identical to that of the single buyer case (section 3.1.1), where the investments are interpreted as including a spillover component. This component is explicitly described in the next section.

4.1.2 Investments

The *effective investment of* supplier $j \in \{1, ..., n\}$'s in B_i's design includes spillovers from all other designs it undertakes and is given by $\widetilde{s}_i^j \equiv s_i^j + \gamma \sum_{k \neq i} s_k^j$. Denote by $\widetilde{\underline{s}}_i$ the profile of effective investments and by $\widetilde{\underline{s}}_i^{-j}$ the profile of investments by all suppliers except j. Supplier j's investments in all m designs, $\left\{s_i^j\right\}_{i=1}^m$, therefore maximizes:

$$\max_{\left\{s_i^j\right\}_{i=1}^m} \sum_{i=1}^m \left\{\phi_i^j \left(b_i, \left(s_i^j + \gamma \sum_{k \neq i} s_k^j,\ \widetilde{\underline{s}}_i^{-j}\right)\right) - \psi\left(s_i^j\right)\right\}. \tag{14}$$

The buyer B_i solves

$$\max_{b_i} \phi_i\left(b_i, \widetilde{\underline{s}}_i\right) - \psi\left(b_i\right). \tag{15}$$

Analogous to Proposition 1, one can show the following:

Proposition 2 *Given that $m \leq M$ buyers outsource and all give access to an identical set of suppliers $\{1, ..., n\}$, there exists a symmetric equilibrium for the investment subgame for which investments are $b_i = b^O(n)$ and $s_i^j = s^O(m, n)$ $\forall i \in \{1, ..., m\}$, $\forall j \in \{1, ..., n\}$, characterized by the following first-order conditions:*

$$\frac{n}{n+1}\alpha_B - \psi'\left(b^O\right) = 0, \tag{16}$$

and

$$\frac{1}{(n+1)}\alpha_S\left(1 + \gamma\left(m-1\right)\right) - \psi'\left(s^O\right) = 0. \tag{17}$$

The equilibrium investments satisfy the following properties:

1. *Buyer's investment is increasing in the number of suppliers (n)*
2. *Suppliers' investments are decreasing in the number of suppliers (n)*
3. *Suppliers' investments are increasing in the spillover share (γ)*
4. *Suppliers' investments are increasing in the number of outsourcing buyers (m)*

Let $\widetilde{s}^O(m,n) = s^O(m,n)[1+\gamma(m-1)]$ denote the equilibrium effective investment per buyer and $\underline{\widetilde{s}}^O(m,n)$ the profile of symmetric effective investments.[18] Define $S^O(m,n)$ as the equilibrium surplus of a vertical structure comprised of a single buyer and n suppliers all of which take $m-1$ additional designs. Then:

$$S^O(m,n) = v\left(b^O\right) - n\widehat{c}\left(\frac{1}{n}\right) + \alpha_S\widetilde{s}^O - \psi\left(b^O\right) - n\psi\left(s^O\right) - nK(m). \qquad (18)$$

As in the single buyer case, we make the following assumption:

Assumption 6 *$S^O(m,n)$ is single-peaked in n for every m.*

In addition, $S^O(m,n)$ is increasing in m. To see that, write

$$\begin{aligned}\frac{\partial\left[S^O(m,n)\right]}{\partial m} &= \alpha_S\frac{\partial\widetilde{s}^O}{\partial m} - n\psi'\frac{\partial s^O}{\partial m} - nK'(m)\\ &= \alpha_S\gamma\widetilde{s}^O + \left[\alpha_S(1+\gamma(m-1)) - n\psi'\right]\frac{\partial s^O}{\partial m} - nK'(m).\end{aligned}$$

The first and third terms above are always positive. To sign the second term, note that one can rewrite the first-order condition, (17), as

$$\alpha_S(1+\gamma(m-1)) - (n+1)\psi'\left(s^O\right) = 0,$$

the sign of the second term equals that of $\psi'\left(s^O\right)$ which is positive. Therefore

$$\frac{\partial\left[S^O(m,n)\right]}{\partial m} > 0. \qquad (19)$$

4.1.3 Access

In the access stage, the m outsourcing buyers simultaneously approach suppliers with access offers. The access game generally admits a multiplicity of equilibria. We focus attention on one in which the m outsourcing buyers offer access to the same set of $n^O(m)$ suppliers, where

$$n^O(m) = \max\left\{n \in \mathbb{N} \mid S^O(m,n) > S^O(m,n-1)\right\}.$$

Each buyer sets an identical access fee, $F_i^j = F^O$, to all suppliers where

$$F^O = \phi_i^j\left(b^*, \underline{\widetilde{s}}^*\right) - \psi(s^*) - K(m),$$

$b^* = b^O\left(n^O(m)\right)$, $s^* = s^O\left(n^O(m), m\right)$ and $\underline{\widetilde{s}}^* = \underline{\widetilde{s}}\left(n^O(m), m\right)$. The access fee stipulated leaves suppliers exactly indifferent between accepting the offer or rejecting it. Their expected

[18]For brevity, I omit n and m when referring to the equilibrium investments if appropriate.

net revenues from working for B_i in the continuation equilibrium, if they take m designs altogether are

$$\phi_i^j\left(b^*, \underset{\sim}{\widetilde{s}}^*\right) - \psi\left(s^*\right) - K\left(m\right) - F^O = 0.$$

The buyers therefore extract the entire transactional surplus.

By the single-peakedness assumption, $n^O\left(m\right)$ is the number of suppliers, each with $m-1$ additional designs, that maximizes the surplus per vertical structure. As buyers extract the entire surplus, and if restricted to choose only such suppliers, each buyer would give access to $n^O\left(m\right)$ of them if possible. In the proposed equilibrium above, in which all firms choose identical suppliers, exactly $n^O\left(m\right)$ such suppliers are indeed available.

In the most general setting however, it is not straightforward that the buyers would indeed find it optimal to give access to the same set of suppliers as their peers. Two offsetting effects are at work. First, as a supplier's average setup cost per design is decreasing in the overall number of designs undertaken, and as suppliers' investment is increasing (by proposition 2), a supplier's *direct* contribution to the surplus is increasing in the number of designs it undertakes. If there are no spillovers, $\gamma = 0$, this is the only effect. However, when $\gamma > 0$, because suppliers' investments are strategic substitutes to each other, the increase in supplier's incentives to invest due to taking more designs has an adverse effect on investments by all other suppliers. This *indirect* effect favors giving access to suppliers with fewer designs.

In the appendix we develop a necessary and sufficient condition under which the direct effect dominates the indirect effect everywhere. In a nutshell this requires the suppliers' best responses in the investment stage not to be too sensitive. In what follows we assume that this is the case.[19] Then, it is easy to verify that *in all equilibria,* suppliers offer access to the same set of suppliers, and that the number of suppliers given access is no more than $n^O\left(m\right)$.[20] The equilibrium we focus on Pareto-dominates all other possible equilibria with as $S^O\left(m, n\right)$ is single-peaked in n.

The next point regards the way the $n^O\left(m\right)$, the size of the outsourcing buyers' supplier network changes with the number of outsourcing firms, m. Again there are two offsetting effects. This may be seen by looking at how $\partial S^O\left(m, n\right)/\partial n$ changes with m. As is shown

[19] Also in section 8 we outline an alternative modelling approach that overcomes these difficulties assuming that access offers are hidden and only observed by the recipients.

[20] Suppose B_i employs a supplier j that is not employed by B_k. There are two possible cases: If B_k employs another supplier l that is not employed by B_i, then either B_i is better off switching from j to l or B_k better off switching from l to j. Otherwise if , without loss of generality, the set of suppliers employed (optimally) by B_i is strict superset of that employed by B_k, then as buyers are identical, B_k should give access to the additional suppliers as well.

in the appendix:

$$\frac{\partial^2 \left[S^O(m,n)\right]}{\partial m \partial n} = \frac{1}{(n+1)}\alpha_S \left[\frac{\partial^2 s^O}{\partial m \partial n}(1+\gamma(m-1)) + \gamma \frac{\partial s^O}{\partial n}\right] - \psi'\left(s^O\right)\frac{\partial s^O}{\partial m} - K'(m).$$

As the average setup cost per design decreases with the number of designs, $K'(m) < 0$, this effect favors additional suppliers as m increases. On the other hand, provided there are positive spillovers ($\gamma > 0$), an increase in m may favor a smaller number of suppliers per buyer. As is shown in the appendix, a sufficient (but not necessary) condition for the effect due to spillovers (the first term, in brackets) to be negative is $\psi''' \leq 0$. The intuition for this latter effect is that as m increases, the marginal decrease in investment by each of the infra-marginal suppliers due to an additional one is higher.

For the two polar cases in which only one of these effects is present we get the following result

Lemma 1

1. *If there are only spillover effects* $(K'(m) = 0)$ *and* $\psi''' \leq 0$ *then*
$$\frac{dn^O(m)}{dm} \leq 0.$$
2. *If there are only setup cost savings* $(\gamma = 0)$ *then*
$$\frac{dn^O(m)}{dm} \geq 0.$$

Proof. Appendix. ■

Finally we denote the surplus per outsourcing buyer in the m-buyers' symmetric equilibrium outlined above by

$$S^O(m) \equiv S^O\left(m, n^O(m)\right). \tag{20}$$

By our assumptions this surplus is also the profit per buyer.

Lemma 2 $S^O(m)$ *is increasing in* m.

Proof. For every m,

$$\begin{aligned}
& S^O(m+1) - S^O(m) \\
= \ & S^O\left(m+1, n^O(m+1)\right) - S^O\left(m, n^O(m)\right) \\
= \ & \left[S^O\left(m+1, n^O(m+1)\right) - S^O\left(m+1, n^O(m)\right)\right] + \\
& \left[S^O\left(m+1, n^O(m)\right) - S^O\left(m, n^O(m)\right)\right] \\
> \ & 0
\end{aligned}$$

The first term in brackets above is non-negative by the definition of $n^O(m+1)$. That the second term is positive as well follows from the fact established above that $\frac{\partial S^O(m,n)}{\partial m} > 0$ for all m, n. ■

The equilibrium surplus per buyer from outsourcing is increasing in the number of outsourcing buyers. The significance this result becomes clear in the next section where we characterize the equilibria of the entire game and show that equilibria differ in the number of outsourcing firms m and hence in their overall efficiency.

4.2 Integration

As buyers do not interact in the downstream market, the analysis of integration in the many buyers case is completely analogous to that of the single buyer model (section 3.2). It is worth emphasizing however that in comparison with outsourcing, integration is further disadvantaged due to the existence of economies of scope to suppliers under outsourcing. This can be seen by comparing the first-order conditions characterizing upstream investment under integration, (13), and under outsourcing, (17).

5 Vertical Equilibria

In this section we turn to analyze the ex-ante choice of organizational mode by buyers and characterize the ensuing equilibria of the complete game. A choice between outsourcing and upstream integration foresees a play of the continuation equilibrium outlined above, and the tradeoff between the different modes of organization can be summarized as follows: An integrated organization provides better incentives for downstream investment than outsourcing. However the gap is reduced with an increase in the buyer's network of suppliers. An outsourcing arrangement is advantageous at promoting upstream investment, but the advantage is dampened by increasing the size of the supplier network. The advantages of outsourcing are also magnified when suppliers takes the designs of multiple buyers, due to economies of scope in investment.

Recall from section 3.2 that a buyer B_i's relative inefficiency in upstream investment is denoted by λ_i. To assess the implications of buyers' heterogeneity, we assume that buyers may differ in the inefficiency of their upstream investment.[21] Formally:

Assumption 7 (Heterogeneity in cost of integration) $\lambda_1 \geq \lambda_2 \geq .... \geq \lambda_M > 2$.

[21] Heterogeneity between firms is possible along other dimensions as well, e.g.: the relative importance of a firm's investment compared to its suppliers' and so on.

B_i's choice of organizational mode is then determined by the difference in surplus between integration and outsourcing,

$$\Delta(m, \lambda_i) \equiv S^I(\lambda_i) - S^O(m). \qquad (21)$$

Buyer B_i integrates if $\Delta(m, \lambda_i) > 0$ and outsources otherwise.

Lemma 3 *$\Delta(m, \lambda_i)$ is decreasing in m and decreasing in λ_i.*

Proof. That $\Delta(m, \lambda_i)$ is decreasing in m follows immediately as $S^O(m)$ was shown increasing in m (lemma 2). As $S^I(\lambda_i)$ was shown decreasing in λ_i in section 3.2, $\Delta(m, \lambda_i)$ is decreasing in λ_i. ■

The marginal buyer, $\lambda(m)$, implicitly defined by $\Delta(m, \lambda(m)) = 0$, is indifferent between integration and outsourcing given that $m-1$ other buyers outsource. A buyer B_i outsources if and only if it is relatively inefficient under integration, that is if $\lambda_i \geq \lambda(m)$. Clearly given lemma 3, $\lambda(m)$ is decreasing in m.

The next definition characterizes the stable organizations of the set of buyers:

Definition 2 (Vertical equilibrium) *A partition of the set of buyers into $O \subseteq \{B_1, ..., B_M\}$, the set of outsourcing buyers, and $I = \{B_1, ..., B_M\} \backslash O$, the set of integrating buyers, is a vertical equilibrium if*

$$\Delta(|O|, \lambda_i) \leq 0 \qquad \forall i \in O,$$

and

$$\Delta(|O| + 1, \lambda_i) \geq 0 \qquad \forall i \in I.$$

Proposition 3 *All vertical equilibria of the game can be characterized as follows:*

For some $m \in \{1, ..., M\}$, buyers $\{B_1, ..., B_m\}$ outsource and buyers $\{B_{m+1}, ..., B_M\}$ integrate into the supply of the input.

Proof. We need only to prove that if B_j outsources then every B_i, $i < j$ outsources as well. Now suppose to the contrary that $\exists i, j$ such that $\lambda_i > \lambda_j$, with $j \in O$ but $i \in I$. This implies that

$$\begin{aligned} \Delta(|O|, \lambda_j) &\leq 0, \\ \Delta(|O| + 1, \lambda_i) &> 0. \end{aligned}$$

But applying the results of lemma 3:

$$\Delta(|O| + 1, \lambda_i) \leq \Delta(|O|, \lambda_i) \leq \Delta(|O|, \lambda_j) \leq 0,$$

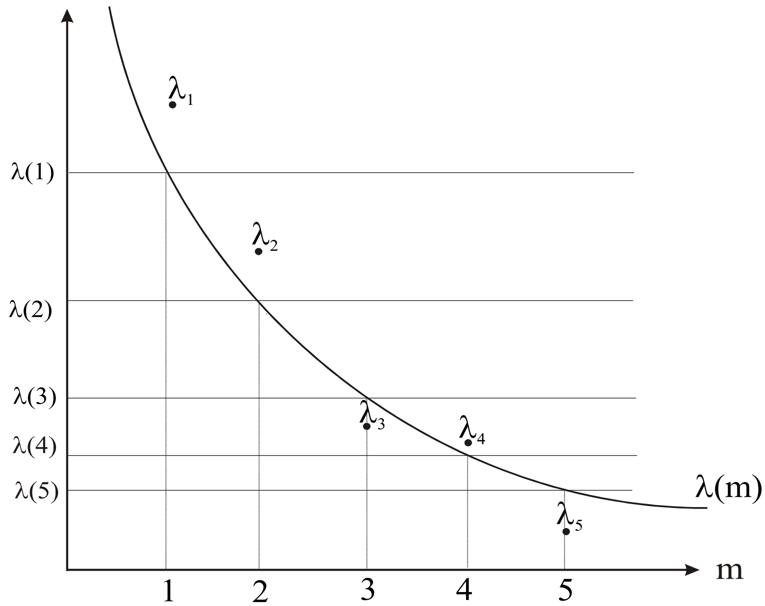

Figure 1: Multiplicity of vertical equilibria - Illustration

where the first inequality follows from part one of the lemma and the second inequality from the second part, as $\lambda_i > \lambda_j$. Hence a contradiction. ■

As $\lambda(m)$ is decreasing in m, a configuration $O = \{B_1, ..., B_m\}$, $I = \{B_{m+1}, ..., B_M\}$ is an equilibrium organization if and only if $\lambda_m \geq \lambda(m)$ and $\lambda_{m+1} < \lambda(m+1)$. With heterogeneous buyers the model therefore potentially admits a *multiplicity of vertical equilibria*. An illustration of this possibility is given in Figure 1. In this example there are two equilibria: one with two outsourcing firms, $O = \{B_1, B_2\}$, and a second with four outsourcing firms, $O = \{B_1, B_2, B_3, B_4\}$.
Mixed equilibria with both integrated and non-integrated firms appears however only if there is a strict heterogeneity across firms, that is if $\lambda_1 \neq \lambda_M$. If the buyers are homogenous ($\lambda_1 = \lambda_M$), the only possible equilibria are: *all integration* and *all outsourcing*.

For future use, it is useful to define the following:

Definition 3 *Let*

$$\overline{m} \equiv \begin{cases} M & \text{if } \lambda_M \geq \lambda(M), \\ \max\limits_{m} \{m \in 1..M-1 \mid \lambda_m \geq \lambda(m) \text{ and } \lambda_{m+1} < \lambda(m+1)\} & \text{otherwise.} \end{cases}$$

The ***maximal outsourcing equilibrium*** *configuration is then*

$$O = \{B_1, ..., B_{\overline{m}}\},\ I = \{B_1, ..., B_M\} \backslash O$$

In lemma 2 we established that the surplus under outsourcing is increasing in the number of outsourcing firms, m. As the surplus under integration is not affected by other buyers' organizational choices, equilibria are therefore Pareto-ranked in m. Equilibria with a higher number of outsourcing firms are superior to those with lower number. As is argued in the previous section, the effect of m on the equilibrium size of the suppliers network of outsourcing buyers is ambiguous in general.

Finally, in order to make some comparisons between the levels of investments in different equilibria, we proceed by looking at the two polar cases separately.

Lemma 4

1. *If there are only spillover effects* $(K'(m) = 0)$ *and* $\psi''' \leq 0$, *the equilibrium downstream investment under outsourcing* $b^O(m)$ *is decreasing with* m *and the upstream investment* $s^O(m)$ *per supplier is increasing with* m.

2. *If there are only setup cost savings* $(\gamma = 0)$, *the downstream investment under outsourcing* $b^O(m)$ *is increasing with* m.

Proof.

$$\begin{aligned} \frac{db^o(n^o(m))}{dm} &= \frac{\partial b^o}{\partial n}\frac{dn^o(m)}{dm}, \\ \frac{ds^o(n^o(m), m)}{dm} &= \frac{\partial s^o}{\partial n}\frac{dn^o(m)}{dm} + \frac{\partial s^o}{\partial m}. \end{aligned}$$

1. Whenever $K'(m) = 0$ and $\psi''' \leq 0$, it is shown in lemma 1 that $n^o(m)$ is decreasing in m. Applying the results of proposition 2, we get $\frac{db^o}{dm} < 0$ and $\frac{ds^o}{dm} > 0$.

2. When $\gamma = 0$, it was shown in lemma 1 that $n^o(m)$ is increasing in m. Thus by a similar logic to the above, $\frac{db^o}{dm} > 0$. It is impossible to sign $\frac{ds^o}{dm}$ in the same way as the first and second effects work in opposite directions.

■

The multiplicity of equilibria demonstrates that very different patterns of industry vertical organization can arise from similar starting conditions. Indeed, comparative studies of industrial structure have revealed stark differences in organization between similar industries in relatively similar countries.[22] While these differences may also be attributed to other institutional details, the results here suggest that this prevalence may be an implication of the externalities between firms' decisions outlined above.

6 Comparative Statics

In this section we present comparative statics results with respect to several of the main parameters of the model. At times, we analyze the changes in the maximal outsourcing equilibrium described above. This equilibrium was shown to be Pareto-superior to all other equilibria of the game.

6.1 An Increase in the "Size of the Market" (M)

Lemma 5 *Consider two markets (sets of buyers) characterized by* $\Lambda_1 = \{\lambda_1, ..., \lambda_M\}$ *and* Λ_2 *such that* $\Lambda_1 \subseteq \Lambda_2$*. Then for every equilibrium of market* 1 *in which* B_i *outsources, there exists an equilibrium in market* 2 *in which* B_i *outsources as well.*

Proof. Buyers are unrelated if integrated and as the value of outsourcing for each buyer, $S^O(m)$, is shown in lemma 2 to depend positively on the number of outsourcing buyers, m, then for each equilibrium of market 1, a similar equilibrium with at least the same set of buyers outsourcing exists for market 2. ■

One implication of the lemma is that a particular buyer may integrate in *all* equilibria of the smaller market, but outsource in *some* equilibria of the bigger one. Grossman and Helpman (2002) derive a comparable result in their model, and relate it to anecdotal evidence suggesting that outsourcing is more prevalent in large economies. The result is also reminiscent of Stigler's (1951) celebrated hypothesis that industries would disintegrate as they expand in size (and integrate again when in decline).

6.2 An Increase in Spillovers (γ)

In this section we compare two markets: $1, 2$ characterized by an identical set of buyers $\{\lambda_1, ..., \lambda_M\}$, and greater spillovers between designs in market 2 ($\gamma_2 > \gamma_1$). For emphasis, we denote explicitly by $S^O(m;\gamma)$ the equilibrium surplus and by $n^O(m;\gamma)$ the upstream industry size *conditional on* γ.

[22] See McLaren (2000) and the examples discussed therein.

Lemma 6

1. *The number of firms outsourcing in a maximal outsourcing equilibrium is increasing in* γ*:* $\overline{m}_2 \geq \overline{m}_1$.

2. *If* $\psi''' \leq 0$*, the number of suppliers per design is decreasing in* γ *for all* $m : \forall m \in \{1, ..., M\}$, $n^O(m;\gamma_2) \leq n^O(m;\gamma_1)$.

3. *If there are no setup costs savings* ($K'(m) = 0$) *and* $\psi''' \leq 0$*, the (maximal outsourcing) equilibrium number of suppliers per buyer is at least as great in market 1:* $n^O(\overline{m}_2;\gamma_2) \leq n^O(\overline{m}_1;\gamma_1)$

Proof.

1. As can be seen from (18) , $\forall (m, n)$

$$\begin{aligned}\frac{\partial\left[S^O(m,n;\gamma)\right]}{\partial\gamma} &= \alpha_S\frac{\partial\widetilde{s}^{Out}}{\partial\gamma} - n\psi'\frac{\partial s^O}{\partial\gamma} \\ &= \left[\alpha_S(1+\gamma(m-1)) - n\psi'\right]\frac{\partial s^O}{\partial\gamma} + \alpha_S(m-1)s^O \\ &= \psi'\frac{\partial s^O}{\partial\gamma} + \alpha_S(m-1)s^O \\ &> 0,\end{aligned}$$

where the third line follows by substituting in the first-order condition, (17). Then

$$\begin{aligned}S^O(m;\gamma_2) &= S^O\left(m, n^O(m;\gamma_2);\gamma_2\right) \\ &\geq S^O\left(m, n^O(m;\gamma_1);\gamma_2\right) \\ &\geq S^O\left(m, n^O(m;\gamma_1);\gamma_1\right) = S^O(m;\gamma_1)\end{aligned}$$

Thus $S^O(m;\gamma)$ is increasing in γ and consequentially $\Delta(m,\lambda;\gamma)$ is decreasing in γ, $\forall\lambda$. By definition $\forall m \leq \overline{m}_1$, $\Delta(m,\lambda_m;\gamma_2) \leq \Delta(m,\lambda_m;\gamma_1) \leq 0$. If $\Delta(\overline{m}_1+1,\lambda_{\overline{m}_1+1};\gamma_2) > 0$ then $O = \{\lambda_1, ..., \lambda_{\overline{m}_1}\}$ is market 2's equilibrium as well. Otherwise there exists an equilibrium in market 2 with at least $\overline{m}_1 + 1$ outsourcing firms, and hence $\overline{m}_2 \geq \overline{m}_1$.

2. In the appendix, it is shown that

$$\frac{\partial^2\left[S^O(m,n;\gamma)\right]}{\partial n\partial\gamma} = \frac{1}{(n+1)}\alpha_S\left[\frac{\partial^2 s^O}{\partial n\partial\gamma}(1+\gamma(m-1)) + (m-1)\frac{\partial s^O}{\partial n}\right] - \psi'\left(s^O\right)\frac{\partial s^O}{\partial\gamma}$$

and that $\frac{\partial^2\left[S^O(m,n;\gamma)\right]}{\partial\gamma\partial n} < 0$ if $\psi''' \leq 0$. Therefore for every m,

$$\begin{aligned}
& \int_{\gamma_1}^{\gamma_2}\int_{n^O}^{n^O+1} \frac{\partial^2 S^O(m,n;\gamma)}{\partial\gamma\partial n} dnd\gamma \\
= \; & S^O\left(m,n^O+1;\gamma_2\right) - S^O\left(m,n^O;\gamma_2\right) - \\
& \left[S^O\left(m,n^O+1;\gamma_1\right) - S^O\left(m,n^O;\gamma_1\right)\right] \\
\leq \; & 0.
\end{aligned}$$

By definition of n^O then $S^O\left(m,n^O+1;\gamma_1\right) - S^O\left(m,n^O;\gamma_1\right) \leq 0$, implying $S^O\left(m,n^O+1;\gamma_2\right) - S^O\left(m,n^O;\gamma_2\right) \leq 0$ and therefore $n^O(m;\gamma_2) \leq n^O(m;\gamma_1)$ (as as S^O is single-peaked).

3. When there are no setup costs savings ($K'(m) = 0$) and $\psi''' \leq 0$, it is shown in lemma 1 that $\frac{dn^{out}(m)}{dm} < 0$. Therefore by parts 1 and 2,

$$n^O(\overline{m}_2;\gamma_2) \leq n^O(\overline{m}_1;\gamma_2) \leq n^O(\overline{m}_1;\gamma_1).$$

■

McMillan (1995) argues that during the 80's and the 90's firms have increased their outsourcing and subcontracting activities and at the same time also transformed the nature of their supplier relations, with close relationships replacing arms' length dealings. A central characteristic of these new relationships is a significant reduction in the number of suppliers engaged. One rationale for such a decrease in the size of suppliers' pool is given by Bakos and Brynjolfsson (1993). They argue that the diffusion of information technology has increased the importance of non-contractible investments by suppliers, on such dimensions as quality, responsiveness and innovation. As a result firms are likely to employ fewer suppliers, even if the transaction costs of working with additional suppliers decrease.

The results of this section can be interpreted as offering a complementary view to that of Bakos and Brynjolfsson. One effect of the increased use of information technology is an improved ability to codify organizational know-how in the form of decision and support systems. Thus knowledge acquired by a supplier becomes more transferable between different clients' accounts, a fact which we model by an increase in spillovers. The implications according to the lemma above are an increase in outsourcing and, under the conditions of part 3., a decrease in the equilibrium supplier network size.

6.3 A Decrease in Per-Design Setup Costs $(K(m))$

In this section we compare two markets: $1, 2$ characterized by an identical set of buyers $\{\lambda_1, ..., \lambda_M\}$, and lower setup costs per design in market 2 , $K_2(m) \leq K_1(m)$, $\forall m$.

Lemma 7

1. *The number of outsourcing firms in a maximal outsourcing equilibria is higher in market 2 than in market 1:* $\overline{m}_2 \geq \overline{m}_1$.

2. *For any given number of outsourcing buyers, m, the optimal number of suppliers per design is higher in market 2:* $\forall m \in \{1, ..., M\}$, $n^O(m; K_2) \geq n^O(m; K_1)$.

3. *If there are no spillovers* $(\gamma = 0)$, *the (maximal outsourcing) equilibrium number of suppliers per buyer is larger in market 2:* $n^O(\overline{m}_2; K_2) \geq n^O(\overline{m}_1; K_1)$

Proof.

1. Immediate.

2. As $\forall m, n$
$$S^O(m, n; K_2) = S^O(m, n; K_1) + n(K_1(m) - K_2(m))$$
then
$$\begin{aligned} & S^O\left(m, n^O(m; K_1); K_2\right) - S^O\left(m, n^O(m; K1) - 1; K_2\right) \\ = \ & S^O\left(m, n^O(m; K_1); K_1\right) - S^O\left(m, n^O(m; K_1) - 1; K_1\right) + K_1(m) - K_2(m) \\ > \ & 0 \end{aligned}$$
and therefore $n^O(m; K_2) \geq n^O(m; K_1)$.

3. When $\gamma = 0$, it was established that $\frac{dn^O(m)}{dm} \geq 0$. Therefore by parts 1 and 2,
$$n^O(\overline{m}_2; K_2) \geq n^O(\overline{m}_1; K_2) \geq n^O(\overline{m}_1; K_1).$$

■

It is interesting to note that the two types of economies of scope considered have similar implications with respect to outsourcing but may have opposing implications for the size of the suppliers' network for each buyer.

7 Outsourcing and the Standardization of Inputs

Anecdotal evidence suggests that greater benefits of outsourcing are achieved when some standardization of the inputs acquired by different buyers takes place. Japanese automobile and computers manufacturers for example have reportedly embraced of late a degree of "openness" of supply, sharing input designs and suppliers in favor of their traditional "closed" model.[23] Often such standardization of inputs is associated with a "loss of distinction" in the final product. An example comes from the airline food industry.[24] Over the last decade, most major airlines have turned to outsourcing of in-flight meal preparation, while the activity was by and large vertically integrated beforehand. While the airlines have been able to achieve cost-savings through outsourcing, there seems to be a consensus that the variety of food being offered has become rather limited, and pretty much standard, at least for economy class travelers. A second example regards outsourcing of information technology (IT) services, where several of the firms that turned to outsourcing solutions from big IT contractors over the last decades, complained later that many of the services received were minimally tailored "off-the-shelf" products.[25]

We now show that it is very easy to modify our framework to accommodate such a trade-off. Suppose that economies of scale to suppliers of taking several buyers' designs can only be materialized if the specifications of inputs used by different buyers are (at least partially) standardized. Assume that the value of each outsourcing buyer B_i product using a standardized input instead of a fully customized one is lower by $T > 0$. The equilibrium investments by the parties are not affected by these changes and are therefore as described above. In a vertical equilibrium with m outsourcing buyers, the following conditions are met:

$$\begin{aligned} \Delta(m, \lambda_i) + T &< 0 \quad \text{for } i \in \{1, ..., m\}, \\ \Delta(m, \lambda_i) + T &\geq 0 \quad \text{for } i \in \{m+1, ..., M\}, \end{aligned}$$

where $\Delta(m, \lambda_i)$ is defined as in (21).

Some buyers would find the gains from outsourcing large enough to offset the loss in value due to input standardization, while others would not. As in the basic model the gains from outsourcing increase with the number of others firms that outsource (and standardize). The model may also incorporate heterogeneity in the loss T_i to buyers due to the use of a non-customized input. The analysis is similar to the one above: firms with highly customized

[23] See "Japan discovers openness", The Economist, October 16, 1993 and McMillan (1995).
[24] "A pressurized environment", The Economist, March 13, 1999
[25] "The outing of outsourcing", The Economist, November 25, 1995.

input demands are less likely to outsource, and the share of buyers outsourcing changes between different equilibria.

When buyers are product market competitors, the standardization of inputs limits the ability of firms to *differentiate themselves from one another.* In such a framework, the "efficiency" benefits from outsourcing to firms are traded-off against a more intensified competition. To model this tradeoff structurally, one would seek to integrate the model presented here with a framework of product market competition between differentiated producers. A first attempt in this direction is taken in Levy (2003).

8 Discussion

In this section I discuss some of the modelling assumptions and the implications of alternatives ones.

Convex costs of input production

Assuming linear variable costs, a pure strategies equilibrium with symmetric investments by suppliers does not exist under outsourcing.[26] The ex-post division of the surplus according to the respective Shapley values results in discontinuous best-response functions in the investment stage subgame. The intuition is that at any symmetric profile of investment, each supplier can increase its production allocation and its share of the surplus by a discrete amount by increasing its investment infinitesimally. The decreasing returns to scale assumption introduces "smoothness" to the production allocations (as well as some degree of realism), and allows us to maintain a symmetric and therefore more tractable model.

Bargaining procedure

I have also attempted to model the multilateral bargaining between an outsourcing buyer and its supplier network using a variation on an alternating offers game with outside option (such a bargaining procedure is employed for example in Bolton and Whinston (1993)) instead of the Shapley value. One drawback of such a formulation is that the buyer receives (in the margin) the full return on its investment whenever it employs two or more suppliers. Thus increasing the size of the supplier network above two does not have any effect on the buyer's investment. As can be seen from (9), the marginal return is always increasing, but never full when the Shapley value is used.

[26] A result in this spirit is established in Rajan and Zingales (1998), Lemma 1. An asymmetric equilibrium does exist however.

Hidden access offers

In the model above, we have assumed that access offers are publicly observed. An alternative is to assume that offers are observed only by the recipient supplier, and the identity of all accepting suppliers is revealed at the beginning of the investment stage. With such a formulation, it is possible to prove the existence of an equilibrium in which all buyers grant access to the same set of suppliers $S_i = \{1,..,n^O(m)\}$, without any additional assumptions. We outline the argument below.

- Buyers set an identical access fee F^O that leaves suppliers indifferent between accepting the offer and rejecting it.

$$F^O = \phi_i^j\left(b^*, \underline{\widetilde{s}}^*\right) - \psi\left(s^*\right) - K\left(m\right)$$

  where $b^* = b^O\left(n^O\left(m\right)\right)$, $s^* = s^O\left(n^O\left(m\right), m\right)$ and $\underline{\widetilde{s}}^* = \underline{\widetilde{s}}\left(n^O\left(m\right), m\right)$.

- A supplier $j \in \left\{1, \ldots, n^O\left(m\right)\right\}$ accepts any access offer stipulating a fee $F_i^j \geq F^*$ and rejects any other offer.

- A supplier $j \notin \left\{1, \ldots, n^O\left(m\right)\right\}$ rejects access offers stipulating a fee $F_i^j > 0$, maintaining a belief that B_i has given access to a number of suppliers sufficiently high so that they would end up making a loss from the transaction.

Note however that this game admits many other equilibria.

A Appendix

A.1 A second-order condition for the supplier's investment problem

In here we calculate a second-order condition for supplier investment problem, (7), and derive a sufficient condition for concavity to hold locally. As an intermediate step, we begin by calculating the derivative of the supplier's production allocation, x_i^j, with respect to its investment, s_i^j.

Derivation of $\frac{\partial x_i^j}{\partial s_i^j}$

Consider the Lagrangian for the cost minimization problem, (1),

$$\mathcal{L} = \sum_{j \in P} c\left(x_i^j, s_i^j\right) - \lambda\left(\sum x_i^j - 1\right),$$

where λ is the Lagrange multiplier. Then as $c(x,s) = \widehat{c}(x) - \alpha_S xs$, the system of first-order conditions may be written as

$$\begin{aligned}
\widehat{c}'\left(x_i^j\right) - \alpha_S s_i^j - \lambda &= 0, \qquad j = 1,..,n \\
-\sum x_i^j + 1 &= 0,
\end{aligned}$$

where $n = |P|$. One can then apply the Implicit Function Theorem and show, after some derivations that,

$$\frac{\partial x_i^j}{\partial s_i^j} = \frac{\alpha_S \sum\limits_{k=1,k\neq j}^{n} \left\{ \prod\limits_{l=1,l\notin\{k,j\}}^{n} \widehat{c}''\left(x_i^l\right) \right\}}{\sum\limits_{k=1}^{n} \left\{ \prod\limits_{l=1,l\neq k}^{n} \widehat{c}''\left(x_i^l\right) \right\}}.$$

Note that at the symmetric allocation, $x_i^j = \frac{1}{n}$ for all $j = 1,...,n$, the derivative above simplifies to

$$\frac{\partial x_i^j}{\partial s_i^j} = \frac{\alpha_S (n-1)}{n * \widehat{c}''\left(\frac{1}{n}\right)}.$$

Second-order condition

The second-order condition for supplier j's investment around the equilibrium investments' profile $\left(b^O, \underline{s}^O\right)$, can be seen from (8) to equal

$$\alpha_S \sum_{P\subseteq S_i\backslash j} \left\{ \frac{(|P|+1)!\,(n_i - |P| - 1)!}{(n_i+1)!} \cdot \frac{\partial x_i^j\left(\underline{s}^O, P\right)}{\partial s_i^j} \right\} - \psi''\left(s^O\right) \leq 0. \tag{22}$$

Substituting for $\frac{\partial x_i^j\left(\underline{s}^O,P\right)}{\partial s_i^j}$ from above and simplifying, we obtain

$$\begin{aligned}
&\alpha_S \sum_{k=0}^{n_i-1} \left\{ \frac{\binom{n_i-1}{k}(k+1)!\,(n_i-k-1)!\frac{\alpha_S k}{(k+1)\widehat{c}''\left(\frac{1}{k+1}\right)}}{(n_i+1)!} \right\} - \psi''\left(s^O\right) \\
=\; & \frac{(\alpha_S)^2}{n_i(n_i+1)} \sum_{k=0}^{n_i-1} \left\{ \frac{k}{\widehat{c}''\left(\frac{1}{k+1}\right)} \right\} - \psi''\left(s^O\right) \leq 0.
\end{aligned}$$

Define $\underline{c} = \inf\{\widehat{c}''(x)\,|x \in [0,1]\}$. By assumption $\underline{c} > 0$. Hence

$$\begin{aligned}
& \frac{(\alpha_S)^2}{n_i(n_i+1)} \sum_{k=0}^{n_i-1} \left\{ \frac{k}{\widehat{c}''\left(\frac{1}{k+1}\right)} \right\} - \psi''\left(s^O\right) \\
\leq \; & \frac{(\alpha_S)^2}{n_i(n_i+1)} \frac{(n_i-1)\,n_i}{2\underline{c}} - \psi''\left(s^O\right) \\
= \; & \frac{(n_i-1)}{(n_i+1)} \frac{(\alpha_S)^2}{2\underline{c}} - \psi''\left(s^O\right) \\
\leq \; & \frac{(\alpha_S)^2}{2\underline{c}} - \psi''\left(s^O\right).
\end{aligned}$$

Thus a sufficient condition for the second-order condition, (22), to be satisfied is

$$\psi''(s) > \frac{(\alpha_S)^2}{2\underline{c}} \text{ everywhere.}$$

A.2 Derivatives of surplus terms

The derivatives of the surplus function $S^O(m,n)$ calculated here are used in numerous proofs in the main text. All derivations are for the many buyers/many suppliers case. The equations for the single buyer case ($m = 1$) can be obtained by substituting $m = 1$.

From (18),

$$S^O(m,n) = v\left(b^O\right) - \psi\left(b^O\right) - \alpha_S \widehat{s}^{Out} - n\psi\left(s^O\right) - n\widehat{c}\left(\frac{1}{n}\right) - nK(m),$$

and therefore

$$\begin{aligned}
\frac{\partial\left[S^O(m,n)\right]}{\partial n} &= \left[\alpha_B - \psi'\right]\frac{\partial b^O}{\partial n} + \left[\alpha_S * [1+\gamma(m-1)] - n\psi'\right]\frac{\partial s^O}{\partial n} - \psi\left(s^O\right) \qquad (23) \\
&- \left[\widehat{c}\left(\frac{1}{n}\right) - \frac{1}{n}\widehat{c}'\left(\frac{1}{n}\right)\right] - K(m).
\end{aligned}$$

As

$$\left[\alpha_B - \psi'\right] = \left[\frac{n}{n+1}\alpha_B - \psi'\right] + \frac{1}{n+1}\alpha_B = \frac{1}{n+1}\alpha_B,$$

and

$$\begin{aligned}
\left[\alpha_S(1+\gamma(m-1)) - n\psi'\right] &= n\left[\frac{1}{n}\alpha_S[1+\gamma(m-1)] - \psi'\right] \\
&= n\left[\frac{1}{n+1}\alpha_S[1+\gamma(m-1)] - \psi'\right] + \frac{1}{(n+1)}\alpha_S[1+\gamma(m-1)] \\
&= \frac{1}{(n+1)}\alpha_S[1+\gamma(m-1)],
\end{aligned}$$

we can rewrite $\frac{\partial\left[S^{O}(m,n)\right]}{\partial n}$ as follows:

$$\frac{\partial\left[S^{O}(m,n)\right]}{\partial n} = \frac{1}{n+1}\alpha_{B}\frac{\partial b^{O}}{\partial n}+\frac{1}{(n+1)}\alpha_{S}\frac{\partial s^{O}}{\partial n}\left[1+\gamma(m-1)\right]-\psi\left(s^{O}\right) - \left[\widehat{c}\left(\frac{1}{n}\right)-\frac{1}{n}\widehat{c}'\left(\frac{1}{n}\right)\right]-K(m). \tag{24}$$

Therefore

$$\frac{\partial^{2}\left[S^{O}(m,n)\right]}{\partial n^{2}} = -\frac{1}{(n+1)^{2}}\alpha_{B}\frac{\partial b^{O}}{\partial n}+\frac{1}{n+1}\alpha_{B}\frac{\partial^{2}b^{O}}{\partial n^{2}} -\frac{1}{(n+1)^{2}}\alpha_{S}\frac{\partial s^{O}}{\partial n}\left[1+\gamma(m-1)\right] +\frac{1}{n+1}\alpha_{S}\frac{\partial^{2}s^{O}}{\partial n^{2}}\left[1+\gamma(m-1)\right]-\psi'\frac{\partial s^{O}}{\partial n} -\frac{1}{n^{3}}\widehat{c}''\left(\frac{1}{n}\right). \tag{25}$$

Also

$$\frac{\partial^{2}\left[S^{O}(m,n)\right]}{\partial m\partial n}=\frac{1}{(n+1)}\alpha_{S}\left[\frac{\partial^{2}s^{O}}{\partial m\partial n}\left[1+\gamma(m-1)\right]+\gamma\frac{\partial s^{O}}{\partial n}\right]-\psi'\left(s^{O}\right)\frac{\partial s^{O}}{\partial m}-K'(m), \tag{26}$$

and

$$\frac{\partial^{2}\left[S^{O}(m,n)\right]}{\partial\gamma\partial n}=\frac{1}{(n+1)}\alpha_{S}\left[\frac{\partial^{2}s^{O}}{\partial n\partial\gamma}\left[1+\gamma(m-1)\right]+(m-1)\frac{\partial s^{O}}{\partial n}\right]-\psi'\left(s^{O}\right)\frac{\partial s^{O}}{\partial\gamma}. \tag{27}$$

As can be easily verified from (17),

$$\frac{\partial s^{O}}{\partial n}=-\frac{\alpha_{S}\left[1+\gamma(m-1)\right]}{\psi''\left(s^{O}\right)\cdot(n+1)^{2}}<0,$$

and

$$\frac{\partial^{2}s^{O}}{\partial m\partial n} = \frac{\partial s^{O}}{\partial n}\frac{\gamma}{(1+\gamma(m-1))}+\frac{\alpha_{S}\left[1+\gamma(m-1)\right]}{\left[\psi''\left(s^{O}\right)\cdot(n+1)^{2}\right]^{2}}\psi'''\left(s^{O}\right)\frac{\partial s^{O}}{\partial m} \tag{28}$$

$$\frac{\partial^{2}s^{O}}{\partial\gamma\partial n} = \frac{\partial s^{O}}{\partial n}\frac{m-1}{(1+\gamma(m-1))}+\frac{\alpha_{S}\left(1+\gamma(m-1)\right)}{\left[\psi''\left(s^{O}\right)\cdot(n+1)^{2}\right]^{2}}\psi'''\left(s^{O}\right)\frac{\partial s^{O}}{\partial\gamma} \tag{29}$$

Thus $\psi''' \leq 0$ is a sufficient (but not necessary) condition for $\frac{\partial^{2}s^{O}}{\partial m\partial n}, \frac{\partial^{2}s^{O}}{\partial\gamma\partial n}<0$, and therefore also for $\frac{\partial^{2}\left[S^{O}(m,n)\right]}{\partial\gamma\partial n}<0$. Also it is sufficient for the effect on $S^{O}(m,n)$ due to spillovers (the term in brackets in (26)) to be negative. In the case where $K'(m)=0$ then $\frac{\partial^{2}\left[S^{O}(m,n)\right]}{\partial m\partial n}<0$ as well.

A.3 Single-peakedness of $S^O(n)$ for a parametric example

In the following example we consider a parameterization of the basic model of section 3 with a quadratic disutility of effort function, $\psi(x) = \frac{x^2}{2}$.

It can be shown that for the parameterization above:

$$
\begin{aligned}
b^O(n) &= \frac{n}{n+1}\alpha_B \\
s^O(n) &= \frac{1}{n+1}\alpha_S
\end{aligned}
$$

and that

$$S^O(n) = h_1(n) + h_2(n)$$

where

$$
\begin{aligned}
h_1(n) &= \frac{n(n+2)}{2(n+1)^2}(\alpha_B)^2 + \frac{n+2}{2(n+1)^2}(\alpha_S)^2 - nK, \\
h_2(n) &= -n\widehat{c}\left(\frac{1}{n}\right).
\end{aligned}
$$

Now

$$
\begin{aligned}
h_1'(n) &= \frac{(\alpha_B)^2 - (n+3)\frac{(\alpha_S)^2}{2}}{(n+1)^3} - K, \\
h_1''(n) &= \frac{(n+4)(\alpha_S)^2 - 3(\alpha_B)^2}{(n+1)^4},
\end{aligned}
$$

and one can further show that $h_1'(n) < -K$ for all $n \geq n_1$ and that $h_1''(n) > 0$ if and only if $n \geq n_2$, where $n_2 > n_1$. Also $h_2'(n) > 0$, $h_2''(n) < 0$ is easily verified.

Claim 1 *$S^O(n)$ has interior single peak under the following (sufficient) conditions*

1. $h_1'(0) > 0$
2. $h_2'(n_2) < K$

Proof. Note first that $h_1'(n) > 0$ if and only if $(\alpha_B)^2 > (n+3)\frac{(\alpha_S)^2}{2} + K\ (n+1)^3$. As the right-hand side is increasing in n, $h_1(n)$ changes sign at most once and, as long as $h_1'(0) > 0$, has an interior single peak.

We now turn to $S^O(n) = h_1(n) + h_2(n)$. Under the conditions above it is clear that $S^{O\prime}(0) > 0$ and that $S^{O\prime}(n) = h_1'(n) + h_2'(n) < -K + K = 0$ for all $n \geq n_2$. Furthermore $S^{O\prime\prime}(n) < 0$ for all $n < n_2$ and therefore $S^O(n)$ has a single peak, located in $(0, n_2)$. ■

A.4 Access game

In this section we derive sufficient conditions under which a buyer, when making a choice between two suppliers, always prefers giving access to the supplier with the higher number of additional designs. As is argued in the main text, a direct implication is that in all equilibria of the access game, buyers offer access to the same set of suppliers.

Suppose that a buyer B_i gives access to a set S_i of n_i suppliers, and that supplier k is taking $m_k \geq 1$ designs in total. The overall surplus generated by vertical structure i is then

$$S(m_1, ..., m_n) = v(b_i) - c^*(\underline{s}_i, S_i) - \psi(b_i) - \sum_{k=1}^{n_i} \psi\left(s_i^k\right) - \sum_{k=1}^{n_i} K(m_k).$$

where $b_i, \underline{s}_i = \left(s_i^1, ..., s_i^{n_i}\right)$ are the equilibrium investments. While $\{m_k\}_{k=1}^{n_i}$ are discrete variables, we consider next the effect of an infinitesimal change in m_j for some $j \in \{1, .., n_i\}$ on the symmetric equilibrium surplus.

As the buyer investment b_i depends only on the number of suppliers given access, and on $\{m_k\}_{k=1}^{n_i}$ we have

$$\frac{dS(m_1, ..., m_{n_i})}{dm_j} = -\frac{dc^*(\underline{s}_i, S_i)}{dm_j} - \sum_{k=1}^{n_i} \psi'\left(s_i^k\right) \frac{\partial s_i^k}{\partial m_j} - K'(m_j).$$

The last term is positive as $K' < 0$ everywhere. Consider next the first two terms,

$$-\frac{dc^*(\underline{s}_i, S_i)}{dm_j} = -\sum_{k=1}^{n_i} \frac{\partial c^*}{\partial s_i^k} \cdot \frac{\partial s_i^k}{\partial m_j}$$

as we argued in the main text, $\frac{\partial c^*}{\partial s_i^k} = -\alpha_s x_i^k$, and hence

$$-\frac{dc^*(\underline{s}_i, S_i)}{dm_j} = \sum_{k=1}^{n_i} \alpha_s x_i^k \frac{\partial s_i^k}{\partial m_j}$$

where $\sum_{k=1}^{n_i} x_i^k = 1$. The sum of the two first terms is then

$$\sum_{k=1}^{n_i} \left[\alpha_s x_i^k - \psi'\left(s_i^k\right)\right] \frac{\partial s_i^k}{\partial m_j}$$

Denote by $b_k \equiv \alpha_s x_i^k - \psi'\left(s_i^k\right)$. Thus $\frac{dS\left(m_1,...,m_{n_i}\right)}{dm_j} > 0$ if and only if

$$\sum_{k=1}^{n_i} b_k \frac{\partial s_i^k}{\partial m_j} > K'(m_j).$$

It is straightforward to show that $\frac{\partial s_i^j}{\partial m_j} > 0$ and that $\frac{\partial s_i^k}{\partial m_j} < 0$ for all $k \neq j$, and the necessary and sufficient condition then implies that the direct positive effect of an increase in m_k on supplier k's equilibrium investment is strong enough, compared with the indirect effect on all other suppliers investments, so that the weighted sum of the effects is not too negative (compared with the savings in setup costs). Given a parametric formulation of the model, it is possible to derive exact restrictions under which the condition holds.

Finally, if the necessary and sufficient condition holds for all $(m_1, ..., m_{n_i})$ then the effect on overall surplus of a discrete increase in a certain m_k is also positive.

A.5 Proofs

Proof of Lemma 1.

1. Provided that $K'(m) = 0$ and $\psi''' \leq 0$, it can be seen from (26) above that $\frac{\partial^2 \left[S^O(m,n)\right]}{\partial m \partial n} \leq 0$. Then for all $m_2 \geq m_1$

$$\begin{aligned} & \int_{m_1}^{m_2} \int_{n^O(m_1)}^{n^O(m_1)+1} \frac{\partial^2 \left[S^O(m,n)\right]}{\partial m \partial n} dndm \\ = \; & S^O\left(m_2, n^O(m_1)+1\right) - S^O\left(m_2, n^O(m_1)\right) - \\ & \left[S^O\left(m_1, n^O(m_1)+1\right) - S^O\left(m_1, n^O(m_1)\right)\right] \\ \leq \; & 0. \end{aligned}$$

   The second term (in square brackets) is non-positive, by definition of $n^O(m_1)$. Hence

$$S^O\left(m_2, n^O(m_1)+1\right) - S^O\left(m_2, n^O(m_1)\right) \leq 0.$$

   Given the single-peakedness of $S^O(m,n)$ then $n^O(m_2) \leq n^O(m_1)$.

2. If $\gamma = 0$, $s^O(m,n)$ is independent of m implying $\frac{\partial^2 \left[S^O(m,n)\right]}{\partial m \partial n} = -K'(m) \geq 0$. Then

$$\begin{aligned} & \int_{m_1}^{m_2} \int_{n^O(m_1)-1}^{n^O(m_1)} \frac{\partial^2 \left[S^O(m,n)\right]}{\partial m \partial n} dndm \\ = \; & S^O\left(m_2, n^O(m_1)\right) - S^O\left(m_2, n^O(m_1)-1\right) - \\ & \left[S^O\left(m_1, n^O(m_1)\right) - S^O\left(m_1, n^O(m_1)-1\right)\right] \\ \geq \; & 0. \end{aligned}$$

   The second term (in square brackets) is non-negative, by definition of $n^O(m_1)$. Hence

$$S^O\left(m_2, n^O(m_1)\right) - S^O\left(m_2, n^O(m_1)-1\right) \geq 0.$$

The single-peakedness of $S^O(m,n)$ implies $n^O(m_2) \geq n^O(m_1)$.

■

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