Demand Dispersion, Metonymy and Ideal Panel Data

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Abstract. In a generic competitive distribution economy with "increasing dispersion," market demand satisfies the weak axiom of revealed preference and equilibrium is unique. Increasing dispersion requires, roughly, that when the households' incomes rise slightly their demand vectors move apart. We show how to test for it using panel data with fixed relative prices under a "structural stability" hypothesis [10]. We also show how to test for it using cross section data if the households' demand functions and incomes are independently distributed, or under a much weaker assumption that is implied by "metonymy." Metonymy, introduced in [6], relates the demand distribution of households with income x to the demand distribution that slightly poorer households would have if their incomes rose to x. We show that metonymy for a single population is untestable—even with ideal panel data that allow a direct test of increasing dispersion. Thus, cross section tests of increasing dispersion rely on an assumption that is not potentially falsifiable.

Keywords: aggregation, weak axiom, increasing dispersion, cross section, structural stability

1 Introduction

In a generic competitive production economy, equilibrium is unique if mean consumer demand satisfies the weak axiom of revealed preference. Moreover, the weak axiom is the weakest demand-side restriction that ensures uniqueness of equilibrium. In a distribution economy (in which the households' shares of aggregate income do not vary with prices) mean consumer demand satisfies the weak axiom if the consumption sector has *increasing dispersion*. Increasing dispersion means, roughly, that a slight increase in the households' incomes makes their demand vectors move apart on average. Considering the importance of the weak axiom for mean consumer demand, it is natural to ask whether the hypothesis of increasing dispersion is consistent with demand data. This question is addressed in [6,8,9].

In the present paper we present a self-contained review the implications of increasing dispersion, following the approach of [12,13]. We present new ways of testing for increasing dispersion using panel and cross section data. The tests depend on auxiliary hypotheses, "structural stability" or "metonymy," that are commonly assumed (at least implicitly) in demand analysis. We will show that these auxiliary hypotheses are too weak to be testable. In particular, metonymy (the hypothesis introduced in [6] that links income effects to cross section data) is untestable even with ideal panel data that permit direct tests of increasing dispersion.

Increasing dispersion and a weak form of individual consistency, which together imply the weak axiom in the aggregate, are satisfied in consumption sectors with sufficient heterogeneity in the sense of Grandmont [5] and Kneip [17,18]. These hypotheses do not restrict the parametric forms for the households' demand functions. They also do not require that the households' demands are generated by competitive optimizing behavior. This contrasts with the sufficient conditions for uniqueness in [19,22,23], which place restrictions on individual utility functions.

Increasing dispersion is a restriction on the effect of raising the households' incomes. With ideal panel data it can be tested directly. The test can be performed if the households' incomes rise in fixed proportion while relative prices remain fixed. Data for such a test are, in principle, obtainable from consumption experiments, as discussed below. Alternatively, increasing dispersion might be tested using time series data from periods of nearly constant relative prices and nearly constant relative income distribution. The latter requirement is not compatible with the equal income changes used in [8]. In order to test for increasing dispersion using time series data it is also necessary for the distribution of household demand functions to evolve in a special way. It is not necessary for the households' preferences to remain unchanged over time, but something like the structural stability hypothesis introduced by Hildenbrand and Kneip [10] seems to be required. We will show how to test for increasing dispersion under a version of structural stability. However, the structural stability hypothesis itself is not testable.

In practice, it is difficult to obtain the data required for a direct test of increasing dispersion—data from periods with structural stability and constant relative prices and relative income distribution. An alternative approach is to estimate the effect of changes in income using cross section data, [6,8,13]. Cross section data can be used to estimate income effects if the households' demand functions and their incomes are independently distributed. But independence is unnecessarily strong. A much weaker restriction, metonymy [6], is sufficient in order to test for increasing dispersion using cross section data. The idea is to treat a measure of the dispersion of the demand vectors of households with income x as an estimate of the dispersion that slightly poorer households would have if their incomes rose to x. Metonymy ensures that the estimation errors are uncorrelated with income, and in that case, the effects of increasing household incomes can be estimated using the method of average derivatives from nonparametric statistics.

The metonymy hypothesis applies to a hypothetical "large" population represented by a continuous distribution of demand functions and incomes. Real consumption sectors are viewed as random samples drawn from the hypothetical population. What is observed in a single period cross section is a set of household incomes and demand vectors. Evstigneev, et. al. [4] shows that metonymy is not testable using a single cross section even if the entire hypothetical continuous distribution of household demand vectors and incomes can be observed. The present paper extends this result by showing that metonymy is not testable with ideal panel data from a finite number of periods. By ideal panel data we mean data on the entire continuous distribution of household demand vectors and incomes in each of several periods. The households' incomes rise from one period to the next, but prices and the households' demand functions and shares of aggregate income remain fixed. With such ideal panel data it is possible to test directly for increasing dispersion, but it is not possible to test for metonymy.

In cross section tests in [8], increasing dispersion is accepted and its negation is rejected. However the tests are potentially sensitive to the treatment of the tails of the income distribution, and the results are affected by the number of high and low-income "outliers" that are thrown out. We present a simpler cross section test based on [13] that might have more stable statistical properties.

In the next section we present the model of a consumption sector in a distribution economy. In section 3, we review the definition of increasing dispersion and show that it implies the weak axiom in the aggregate. In section 4, we show how increasing dispersion can be tested directly using panel data under a modified version of structural stability. In section 5 we show how to test for increasing dispersion using a single cross section, under a weakened version of metonymy. In section 6, we show that metonymy for a single population is not falsifiable, even using ideal panel data. In section 7 we discuss open questions raised by these results.

2 Notation

We consider economies with many heterogeneous households. The consuming units are called "households" because it is usually not possible to obtain data on the division of consumption within real households. We consider only "distribution economies" in which the households' incomes do not vary with prices and are treated as exogenous. A distribution economy can be viewed as a special case of a private ownership economy in which there is a good (income) that each household offers in fixed supply. The households' demands in the distribution economy are their excess demands for goods other than "income" in the corresponding private ownership economy. The sufficient conditions for the weak axiom in the aggregate and the cross section test of increasing dispersion presented below can be modified to apply to general private ownership economies, [15].

The consumption side of a distribution economy is represented by a joint distribution of household demand types and incomes. Let $\mathbf{R}^n_+$ and $\mathbf{R}^n_{++}$ be the nonnegative and strictly positive orthants of $\mathbf{R}^n$ respectively. We consider a measurable space $(A, \mathcal{A})$ of demand types, and, for each type $a \in A$ and each good $j = 1, \ldots, n$, a nonnegative C^1 demand function $f_j(a, \cdot, \cdot)$ that is homogeneous of degree zero. The vector-valued demand function $f = (f_j)_{j=1}^n$ is assumed to satisfy the budget identity $p \cdot f(a, p, x) = x$ for every price vector $p \in \mathbf{R}^n_{++}$ and income level $x \geq 0$. A *consumption sector* is a triple (η, λ, f) with the following properties: η is a probability measure on $\mathbf{R}$ representing the income distribution (with $\eta(X) = 0$ for every measurable $X \subset (-\infty, 0)$); $\lambda(x, \cdot)$ is the conditional distribution of demand types for the households with income x; $\lambda(\cdot, B)$ is Lebesgue measurable for each $B \in \mathcal{A}$; and $f(\cdot, p, x)$ is measurable on $(A, \mathcal{A})$ for each price and income vector (p, x). We let μ be the corresponding joint distribution of demand types and incomes, the product of η and λ on $A \times \mathbf{R}$.

In the consumption sector (η, λ, f), the *marginal propensity to consume function* is $m = (m_j)_{j=1}^n$, where $m(a, p, x) \equiv \partial_x f(a, p, x)$ is the vector of income derivatives for demand type a. The *Slutsky* (*substitution*) *matrix* of type a at (p, x) is $S(a, p, x) \equiv [\partial_p f(a, p, x)] + m(a, p, x) f(a, p, x)^T$, where $\partial_p f(a, p, x)$ is the Jacobian matrix of $f(a, \cdot, x)$ at p. (Vectors in Euclidean space are treated as columns, and superscript T denotes the transpose.) The Slutsky matrix is a matrix of derivatives of the demands with respect to the prices when the income is adjusted so that the initial demand vector remains barely affordable, [20].

We restrict attention to consumption sectors in which the *mean income*, $\bar{x} \equiv \int x \eta(dx)$, is well defined, and in which, for every p, the functions $m(\cdot, p, \cdot)$ and $\partial_p f(\cdot, p, \cdot)$ are integrable with respect to μ. Then the *mean demand function*

$$F(p, y) \equiv \int f(a, p, yx/\bar{x}) \mu(da, dx) \tag{1}$$

is well-defined. The income argument of f in (1) is chosen so that F satisfies the budget identity and is homogeneous of degree 0.

We will consider hypotheses under which the mean demand function satisfies the weak axiom of revealed preference. A demand function $\hat{F} : \mathbf{R}^n_{++} \times \mathbf{R}_+ \to \mathbf{R}^n_+$ satisfies the *weak axiom* if $p \cdot \hat{F}(q, z) \leq x$ and $q \cdot \hat{F}(p, x) \leq z$ imply $\hat{F}(p, x) = \hat{F}(q, z)$. The demand function $\hat{F}$ satisfies the *weak weak axiom* if $p \cdot \hat{F}(q, z) \leq x$ implies $q \cdot \hat{F}(p, x) \geq z$. The weak axioms are consistency requirements. They are satisfied if the demand function is generated by utility maximization; but they do not imply that the demand function is generated that way. The weak weak axiom has a simple differential characterization that will be used below. A C^1 demand function satisfies the weak weak axiom if and only if at each element of its domain its Slutsky matrix is negative semidefinite, [16]. If at each (p, x) the Slutsky matrix is negative definite on the set $\{v : v \cdot p = 0\}$ then the demand function satisfies the weak axiom. (An $n \times n$ matrix M (not necessarily symmetric) is *positive* [respectively, *negative*] *semidefinite* if $x^T M x \geq [\leq] 0$ for every n-vector x. M is *positive* [resp., *negative*] *definite* on a set U if $x^T M x > [<] 0$ for every nonzero $x \in U$.)

Mean excess demand does not generally satisfy the weak axiom even if all the individual households do. In fact, if there are at least as many household types as goods, then utility maximizing behavior by the households does not imply any useful structure for mean demand beyond homogeneity and the budget identity [3,24].

3 Increasing Dispersion and the Aggregate Weak Axiom

We now introduce hypotheses leading to the weak axiom in the aggregate. The main hypothesis, increasing dispersion, requires that a small proportional increase in all the households' incomes increases the dispersion of their "normalized" demand vectors. The normalized demand vectors are the demand vectors rescaled so as to lie in the frontier of the budget set determined by the mean income. With the subsidy, this budget set expands, so there is more room for the normalized demand vectors to spread out—more possibility for diversity in the households' demands. The hypothesis is especially plausible at the lower end of the income distribution. With additional income, poorer households are less restricted to buying a limited number of necessities. Increasing dispersion can hold even if the rise in income does not lead any households to change the set of goods that they buy. For example, a sufficient condition for increasing dispersion is that the households have homothetic preferences (not necessarily identical). On the other hand, increasing dispersion is a much weaker requirement than homotheticity. It places no restrictions on the forms of the households' demand functions; it only restricts the way in which the demand functions are related to each other.

In order to define increasing dispersion, we let $\bar{x}$ be the mean income and we consider the *normalized demand vector* $(\bar{x}/x)f(a, p, \kappa x)$ of a household of demand type a and initial income $x > 0$ when the income is multiplied by κ. Note that the value of this vector at the prices p is $\kappa\bar{x}$. Increasing dispersion requires that for each price vector p, the variance of the households' normalized demand vectors increases in all directions orthogonal to p when the households are weighted by their shares of aggregate income. When the households' incomes are multiplied by κ, the mean of their normalized demand vectors is $\int(x/\bar{x})[(\bar{x}/x)f(a, p, \kappa x)]\mu(dx, da) = F(p, \kappa\bar{x})$. The variance of their normalized demand vectors in the direction v is

$$\begin{aligned} V^\mu(\kappa, p, v) &\equiv Var\{v \cdot (\bar{x}/x)f(a, p, \kappa x)\} \\ &\equiv \int (x/\bar{x})\{v \cdot [(\bar{x}/x)f(a, p, \kappa x) - F(p, \kappa\bar{x})]\}^2 \mu(da, dx). \quad (2) \end{aligned}$$

The integration with respect to x is over $\mathbf{R}_{++}$. The consumption sector has

Increasing Dispersion (ID) *at* p if for each nonzero v with $v \cdot p = 0$

$$V_1^{\mu}(1, p, v) = \partial_{\kappa} Var\{v \cdot (\bar{x}/x) f(a, p, \kappa x)\}|_{\kappa=1} > 0, \tag{3}$$

where the subscript on V^{μ} denotes the partial derivative with respect to the first argument. The consumption sector has *increasing dispersion* (ID) if it has ID at every $p \in \mathbf{R}^n_{++}$.

Define $\alpha(a, p, x) \equiv f(a, p, x)/x$, the *average propensity to consume vector* of a household of demand type a and income $x > 0$. When the households are weighted by their shares of aggregate income, the mean of their average propensities to consume is $\bar{\alpha} \equiv \int (x/\bar{x}) \alpha(a, p, x) \mu(da, dx)$, and the mean of their marginal propensities is $\bar{m} \equiv \int (x/\bar{x}) m(a, p, x) \mu(da, dx)$.

It is easy to verify that the derivative in (3) equals $2\bar{x}^2 v^T C v$, where

$$\begin{aligned} C &\equiv \int_{\mathbf{R}_{++}} \int_A (x/\bar{x})(m(a, p, x) - \bar{m})(\alpha(a, p, x) - \bar{\alpha})^T \lambda(x, da) \eta(dx) \\ &= \frac{1}{\bar{x}} \int [m(a, p, x) f(a, p, x)^T - (x/\bar{x}) m(a, p, x) F(p, \bar{x})^T] \mu(da, dx) \end{aligned} \tag{4}$$

is the covariance matrix of the households' marginal and average propensities to consume. Increasing dispersion is equivalent to positive definiteness of this covariance matrix on the space orthogonal to p. When it holds, households with higher than average budget shares for a good tend to have higher than average marginal propensities to consume that good. If the households' demands are generated by homothetic preferences then the marginal and average propensity to consume vectors are equal and C is the positive semidefinite variance-covariance matrix of the households' average propensities to consume. It follows from theorems of Kneip [17,18] that NAS and ID are satisfied if the households' demands are sufficiently heterogeneous in the sense of Grandmont [5] or Kneip [17,18].

Increasing dispersion is defined in [8] as requiring

$$\partial_{\delta} \int [v \cdot f(a, p, x + \delta) - F(p, \bar{x} + \delta)]^2 \mu(da, dx)|_{\delta=0} \geq 0 \tag{5}$$

whenever $v \cdot F(p, \bar{x}) = 0$. This is a statement about the effect on the variance of the households' demand vectors when their incomes rise by the same small amount. Condition (5) is equivalent to positive semidefiniteness of C and also to a slightly weakened form of increasing dispersion with the strict inequality in (3) replaced by a weak inequality, cf. [14]. However unlike (3), the version of increasing dispersion in (5) cannot be tested using ideal panel data in the manner described in the next section. The problem is that (5) applies to v in a subspace that changes with the aggregate demand vector as the household incomes change.

In order for mean demand to satisfy the weak axiom it is neither necessary nor sufficient that the households satisfy the axiom. It helps if the households do satisfy the axiom, but a weaker restriction is sufficient when ID holds. We say that the consumption sector has

Nonpositive Average Substitution (NAS) *at p* if the mean of the households' Slutsky matrices, $\int S(a,p,x)\mu(da,dx)$, is negative semidefinite. The sector has NAS if it has NAS at every $p \in \mathbf{R}^n_{++}$.

The sector has NAS if all the households satisfy the weak weak axiom. It follows that the sector has NAS if the households are competitive utility maximizers. However, experimental evidence suggests that consumers often violate even the weak weak axiom, cf. [2,21,25]. NAS allows for such violations. The households' Slutsky matrices do not have to be negative semidefinite. If some households have positive substitution effects for price changes in a particular direction, the negative substitution effects of other households can make up for it.

Proposition 1. *In a consumption sector with NAS and ID, the mean demand function satisfies the weak axiom.*

Proof. It is sufficient to show that for each $p \in \mathbf{R}^n_{++}$ the Slutsky matrix of the mean demand function $S^*(p,\bar{x})$ is negative definite on the set of v orthogonal to p.

$$\begin{aligned} S^*(p,\bar{x}) &= [\partial_p F(p,\bar{x})] + [\partial_y F(p,y)|_{y=\bar{x}}]F(p,\bar{x})^T \\ &= \int [\partial_p f(a,p,x)\mu(da,dx) + \int (x/\bar{x})m(a,p,x)\mu(da,dx)F(p,\bar{x})^T \\ &= \int S(a,p,x)\mu(da,dx) - \int m(a,p,x)f(a,p,x)^T\mu(da,dx) \\ &\quad + \int (x/\bar{x})m(a,p,x)\mu(da,dx)F(p,\bar{x})^T \\ &= \int S(a,p,x)\mu(da,dx) - \bar{x}C \end{aligned} \tag{6}$$

where C is defined in (4). NAS implies that $\int S(a,p,x)\mu(da,dx)$ is negative semidefinite, and ID implies that C is positive definite on $\{v : v \cdot p = 0\}$. Together they imply that $S^*(p,\bar{x})$ is negative definite on $\{v : v \cdot p = 0\}$, hence that mean demand satisfies the weak axiom. ♢

It is clear from the decomposition in (6) that increasing dispersion is not necessary in order for mean demand to satisfy the weak weak axiom. Violations of increasing dispersion can be compensated for by sufficient substitutability among the goods in consumption, which implies negativity of the quadratic form of the mean of the households' Slutsky matrices.

4 A Direct Test of Increasing Dispersion Using Panel Data

Increasing dispersion can be tested directly if the households' demands can be observed when their incomes rise proportionally with no change in relative prices. The comparison can also be made under more general conditions described in the next proposition. It is not necessary for the households' shares of aggregate income to remain fixed. Some households can move up while others move down in the relative income distribution. We assume, however, that there is no change in the distribution of characteristics of the households with a given set of observable attributes and a given income share in that attribute class. The fraction of the population with a particular set of observable attributes is allowed to change over time.

The direct test of increasing dispersion will be formalized as a comparison of different consumption sectors even though the goal is to test for increasing dispersion in a single sector. The sectors to be compared are interpreted as belonging to the same economy at different dates. An alternative interpretation applying to experimental data from a single period will be discussed below. If the households' demand functions change over time, then only in special cases can we use information about the distribution of demands in one period to draw conclusions about effects of income changes in another period. The main hypothesis that allows such conclusions to be drawn is a modification of the "structural stability" hypothesis introduced by Hildenbrand and Kneip [10]. Structural stability places no restrictions on the evolution of any particular household's characteristics. It restricts only the evolution of the *distribution* of household characteristics.

In order to formulate our version of the structural stability hypothesis we need new notation representing information about observable household attributes. We consider a population of households consisting of a finite number of attribute classes observed over a number of periods. The households in each attribute class $c \in \mathcal{C}$ in period $t \in \mathcal{T}$ form their own consumption sector $\mathcal{D}_t^c \equiv (\eta_t^c, \lambda_t^c, f)$, with μ_t^c the joint distribution of demand types and incomes. The consumption sector for the whole population at date t is the aggregate of all the consumption sectors for the different attribute classes. It is denoted by $(\mathcal{D}_t^c, \pi_t^c)_{c \in \mathcal{C}}$, where $\pi_t^c > 0$ is the fraction of the population in attribute class c in period t. The mean income of attribute class c in period t is $x_t^c \equiv \int x \eta_t^c(dx)$, and the mean demand function is

$$F_t^c(p, y) \equiv \int f(a, p, yx/x_t^c) \lambda_t^c(x, da) \eta_t^c(dx).$$

The mean income and mean demand for the entire population are respectively $x_t = \sum_c \pi_t^c x_t^c$ and $\sum_c \pi_t^c F_t^c(p, x_t^c)$.

Our structural stability hypothesis requires that for households with the same observable attributes who stay in the same percentile in the income distribution of their attribute class, the conditional distribution of demand types remains the same over time. Formally, the population has *structural stability for attributes* over $\mathcal{T}$ if $\eta_t^c((-\infty, \xi_t]) = \eta_s^c((-\infty, \xi_s])$ implies $\lambda_t^c(\xi_t, \cdot) = \lambda_s^c(\xi_s, \cdot)$ for each attribute class c and all periods t and s in $\mathcal{T}$. Hildenbrand and Kneip [10] use a slightly different version of this hypothesis to derive a simple formula for changes in the ratio of aggregate consumption to aggregate income. Our version of the hypothesis holds under the standard assumption that the households' budget shares are functions of prices, household income or total expenditure and observable household attributes, plus a random variable with a distribution that does not change over time. Since structural stability for attributes is a hypothesis about unobservables it cannot be tested directly.

Structural stability for attributes is not strong enough to permit time series testing of increasing dispersion. We will assume in addition that (a) each attribute class has *fixed relative income* (relative to the mean income of the whole population), so that x_t^c / x_t is constant in $t \in \mathcal{T}$, and (b) each attribute class c has *invariant relative income distribution*, so that for each $\kappa \in \mathbf{R}_+$ and t and s in $\mathcal{T}$, $\eta_t^c((-\infty, \kappa x_t^c]) = \eta_s^c((-\infty, \kappa x_s^c])$. Under (b), within each attribute class the relative distribution of income is constant over time. Hypotheses (a) and (b) can be tested using time series data. If all of the above hypotheses are satisfied during a period of constant relative prices, then it is possible to test for increasing dispersion using time series data. The next proposition shows how.

For each pair of periods t and s, define

$$x_{ts} \equiv \sum_c \pi_s^c x_t^c \qquad \text{and} \qquad F_{ts}(p) \equiv \sum_c \pi_s^c F_t^c(p, x_t^c).$$

These are respectively what the mean income and mean demand for the entire population would be in period t if the sizes of the attribute classes remained the same as in period s. The variance-covariance matrix of the households' normalized demands would then be

$$V_{ts} \equiv \sum_c \pi_s^c \int \frac{x}{x_{ts}} \Big[\frac{x_{ts}}{x} f(a, p_t, x) - F_{ts}(p_t)\Big]\Big[\frac{x_{ts}}{x} f(a, p_t, x) - F_{ts}(p_t)\Big]^T \mu_t^c(da, dx). \tag{7}$$

Proposition 2. *In the notation above, suppose that the consumption sectors* $(\mathcal{D}_t^c, \pi_t^c)_{c,t}$ *have structural stability for attributes, and that each attribute class* c *has fixed relative income and invariant relative income distribution over* $\mathcal{T}$. *If ID holds in period* $s \in \mathcal{T}$ *then* $V_{ts} - V_{ss}$ *is positive* [*resp. negative*] *definite on* $\{v : p_s \cdot v = 0\}$ *whenever* $p_t = \zeta p_s$, *with* $x_t > [<] \zeta x_s$.

In order to test for ID, we need to compare the variance-covariance matrix of the households' normalized demands in period s with what the variance-covariance matrix would be in period t if the sizes of the attribute classes were the same as in period s. The latter matrix, V_{ts}, can be estimated by replacing the terms on the right side of (7) by their values in samples of households in each class c from periods t and s.

Proof. Under the hypotheses, $F_t^c = F_s^c$ for each attribute class c and each t and s in $\mathcal{T}$. To see this, note that invariance of the relative income distribution implies that for each $x \in \mathbf{R}_+$, $\eta_s^c((-\infty, x]) = \eta_t^c((-\infty, x_t^c x/x_s^c])$. Therefore, by structural stability for attributes, for each $p \in \mathbf{R}^n_{++}$,

$$\begin{aligned} F_s^c(p, x_s^c) &= \int f(a,p,x)\lambda_s^c(x,da)\eta_s^c(dx) \\ &= \int (x_t^c/x_s^c) f(a,p,x)\lambda_t^c(x_t^c x/x_s^c, da)\eta_t^c((x_t^c/x_s^c)dx) \\ &= \int f(a,p,(x_s^c/x_t^c)z)\lambda_t^c(z,da)\eta_t^c(dz) \\ &= F_t^c(p, x_s^c). \end{aligned} \tag{8}$$

Let $V^s(\kappa, p, v)$ be the variance defined in (2) when μ is replaced by μ_s, the joint distribution of demand types and incomes in the whole population in period s. The homogeneity of $f(a,\cdot,\cdot)$ implies that $V^s(\cdot,\cdot,v)$ is homogeneous of degree zero, and that its partial derivative with respect to the first argument, $V_1^s(\cdot,\cdot,v)$ is homogeneous of degree -1. Suppose that the consumption sector for the entire population in period s has ID. If $v \cdot p = 0$ and $v \neq 0$ then for every $\zeta > 0$ we have $V_1^s(1, \zeta p, v) > 0$ and $V_1^s(1/\zeta, p, v) = \zeta V_1^s(1, \zeta p, v) > 0$. Therefore, $V^s(\kappa, p, v) > V^s(1, p, v)$ when $\kappa > 1$.

Define $F_{ts} \equiv \sum_c \pi_s^c F_t^c(p_t, x_t^c)$. Under the hypotheses, $x_{ts} = \sum_c \pi_s^c x_t^c = \sum \pi_s^c x_s^c x_t/x_s = x_t$. Let

$$G(a,p,x,b) \equiv (b/x) f(a,p,x) - F_{ts}.$$

In the notation above, using (7) and the change of variables $z = x_s^c x/x_t^c = x_s x/x_t$,

$$\begin{aligned} V_{ts} &= \sum_c \int \frac{x}{x_t} G(a,p_t,x,x_t) G(a,p_t,x,x_t)^T \lambda_t^c(x,da)\eta_t^c(dx) \\ &= \sum_c \int \frac{z}{x_s} G(a,p_t,x_t z/x_s,x_t) G(a,p_t,x_t z/x_s,x_t)^T \lambda_s^c(z,da)\eta_s^c(dz). \end{aligned} \tag{9}$$

It follows that if $p_t = \zeta p_s$ then $v^T V_{ts} v = V^s(x_t/\zeta, p_s, v)$. Also, $v^T V_{ss} v = V^s(1, p_s, v)$, so ID implies that $v^T[V_{ts} - V_{ss}]v > 0$ when $x_t/\zeta > x_s$. ◊

We have referred to t and s as labels for time periods, but time series are not the only way of obtaining ideal panel data for a test of increasing dispersion. In principle, it should be possible to obtain ideal panel data from consumption experiments using the design due to Sippel [25]. Each subject is asked to choose consumption vectors in a number of different budget sets, then one of the sets is selected at random and the subject consumes the vector chosen for the selected budget set. In this way the subjects have the incentive to report accurately what they would buy in several different budget situations, and several demand choices from a single period are observed.

As noted above, the alternative definition of increasing dispersion in (5) cannot be tested directly even using idea panel data. Another test, using cross section data under a weak version of metonymy, will be described in the next section.

5 A Test of Increasing Dispersion Using Cross Section Data

In this section we show how increasing dispersion can be tested using a single cross section. We will need a hypothesis, called *dispersion metonymy* that links the effects of increasing the households' incomes to the effects of moving up the income distribution in the cross section. We will use a measure of the dispersion of the demands of households with income x as an estimate of the dispersion that slightly poorer households would have if their incomes rose to x. Dispersion metonymy is essentially the hypothesis that the estimation error is uncorrelated with household income. The estimation error is zero if the households' demand functions and incomes are independently distributed. Dispersion metonymy is a substantially weaker hypothesis. It is similar to, but different from "average covariance metonymy" in [8]. We will show in the appendix that dispersion metonymy is weaker than the metonymy hypothesis introduced in [6] and examined in the next section. This implies that, like metonymy, dispersion metonymy is not refutable with ideal panel data.

Let (η, λ, f) be a consumption sector with mean income $\bar{x} \equiv \int x\eta(dx)$ and mean demand vector $F \equiv \int f(a, p, x)\lambda(x, da)\eta(dx)$ at the price vector p. We focus on a single cross section so that p is fixed and will be suppressed as an argument of the functions f and m.

If the households' demand functions and incomes are independently distributed, then $\lambda(x, \cdot)$, the distribution of demand types with income x does not depend on x. In that case,

$$
\begin{aligned}
&\partial_x \int_A (f(a,x) - (x/\bar{x})F)(f(a,x) - (x/\bar{x})F)^T \lambda(x, da) \\
&= \int_A [(m - (1/\bar{x})F)(f - (x/\bar{x})F)^T + (f - (x/\bar{x})F)(m - (1/\bar{x})F)^T]\lambda(x, da) \\
&= \int_A \Big[mf^T - \frac{x}{\bar{x}} mF^T + fm^T - \frac{x}{\bar{x}} Fm^T\Big]\lambda(x, da), \qquad (10)
\end{aligned}
$$

where the omitted argument of m and f is (a, x).

We will only require (10) to hold "on average." This means that we require (10) to hold when all the terms are integrated with respect to income, x. We say that the consumption sector has *dispersion metonymy* at p if

$$\int_{\mathbf{R}} \Big[\partial_x \int_A (f(a,x) - \frac{x}{\bar{x}}F)(f(a,x) - \frac{x}{\bar{x}}F)^T \lambda(x, da)\Big]\eta(dx) = \bar{x}(C + C^T) \quad (11)$$

Increasing dispersion is equivalent to positive definiteness of C on the space orthogonal to p. It can be tested under dispersion metonymy by estimating the left side of (11) and testing whether it is positive semidefinite. The left side of (11) is the average derivative of a regression function, the regression of the matrix of household demand dispersions, $(f(a,x) - (x/\bar{x})F)(f(a,x) - (x/\bar{x})F)^T$ on x. The average derivative matrix can be estimated nonparametrically using a random sample of observations of the household demand vectors $f(a,x)$ from the cross section, as in [9]. The distribution of these average derivatives can be estimated by bootstrap methods to obtain a statistical test of the hypothesis that the left side of (11) is positive definite on the space orthogonal to p. Under dispersion metonymy, this is a test of increasing dispersion.

According to Proposition 3 below, dispersion metonymy is weaker than the metonymy hypothesis of [6], which is described in the next section. Since we show that metonymy is untestable using ideal panel data, dispersion metonymy is also untestable.

6 Unfalsifiable Metonymy

In this section we show that the metonymy hypothesis of [6] is stronger than dispersion metonymy, yet is still too weak to be testable using ideal panel data. This extends the main theorem in [4].

Metonymy was introduced so that the mean of the households' symmetrized income effects matrices $(m_i f_j + m_j f_i)$ could be estimated using cross section data. Here, m_i is the marginal propensity to consume function for good i. As in the previous section, the price vector is fixed and suppressed as an argument of all functions. Metonymy relates averages of income effects $\int_A \partial_x[f_i(a,x)f_j(a,x)^T]\lambda(x,da)$ to slopes of cross section regression functions: $\partial_x \int_A f_i(a,x)f_j(a,x)\lambda(x,da)$. These derivatives are equal if the households' demand functions and incomes are independently distributed. Under metonymy, they need not be equal, but their averages over the income distribution are equal. The average derivatives of the regression functions can be estimated from cross section data as in [6,8,9].

It might seem that the income effects could be estimated using ideal panel data in which the households' incomes rise without changes in relative prices. Then metonymy could be tested by comparing the estimated income effects to the cross sectional average derivatives. We will show that this intuition is incorrect. Metonymy is a local, differential condition, and it cannot be tested using data from a finite number of shifts in household incomes.

Without loss of generality we let every price equal 1. We also assume that *demands are positive*, i.e., that each household with a positive income demands a positive quantity of every one of the n goods. This simplifies the proof considerably and does not seem unduly restrictive since the demands are allowed to be arbitrarily close to zero.

We will need to consider variations in the measurable space of household demand types, so we include that space in the notation for a consumption sector. The consumption sector $\Lambda \equiv (A, \mathcal{A}, \eta, \lambda, f)$ satisfies *metonymy* (or is *metonymic*) if for each $i, j = 1, \ldots, n$,

$$\begin{aligned} &\int_{\mathbf{R}} \int_A \{\partial_x [f_i(a,x) f_j(a,x)]\} \lambda(x, da) \eta(dx) \\ &= \int_{\mathbf{R}} [\partial_x \int_A f_i(a,x) f_j(a,x) \lambda(x, da)] \eta(dx). \end{aligned} \tag{12}$$

Our definition of a consumption sector implies that the integral on the left side of (12) exists. We say that the consumption sector has a *smooth cross section* if the integral on the right side of (12) exists. In that case, the right side of (12) can be estimated as an average derivative of the regression function $\int_A f_i(a, \cdot) f_j(a, \cdot) \lambda(\cdot, da)$ using cross section data, treating the observed demand vectors as random draws from the distribution on $\mathbf{R}^n_+$ induced by f.

Proposition 3. *A metonymic consumption sector has dispersion metonymy.*

Proof. In the notation above, a metonymic consumption sector satisfies

$$\int [\partial_x \int f(a,x) f(a,x)^T \lambda(x, da)] \eta(dx) = \int (m f^T + f m^T) \mu(da, dx), \tag{13}$$

where the omitted argument for m and f is (a, x). Post multiplying both sides of (13) by the price vector, we obtain

$$\int [\partial_x \int x f(a,x) \lambda(x, da)] \eta(dx) = \int (xm + f) \mu(da, dx). \tag{14}$$

By (13) and (14),

$$\begin{aligned} &\int [\partial_x \int (f(a,x) - (x/\bar{x})F)(f(a,x) - (x/\bar{x})F)^T \lambda(x, da)] \eta(dx) \\ &= \int [\partial_x \int (f f^T - \frac{x}{\bar{x}} f F^T - \frac{x}{\bar{x}} F f^T + (x^2/\bar{x}^2) F F^T) \lambda(x, da)] \eta(dx) \\ &= \int [m f^T + f m^T - \frac{1}{\bar{x}}(xm + f) F^T - \frac{1}{\bar{x}} F (xm + f)^T + \frac{2x}{\bar{x}^2} F F^T] \mu(da, dx) \\ &= \int (m f^T + f m^T - \frac{x}{\bar{x}} m F^T - \frac{x}{\bar{x}} F m^T) \mu(da, dx) - \frac{2}{\bar{x}} F F^T + \frac{2}{\bar{x}} F F^T \\ &= \bar{x}(C + C^T), \end{aligned}$$

where the missing argument of m and f is (a, x), and where C is defined in (4). Thus the consumption sector has dispersion metonymy. ♢

Metonymy is a restriction on the effects of giving every household the same small income subsidy δ. Suppose that we can give the households such a subsidy and observe the effects. Starting with a consumption sector $\Lambda = (A, \mathcal{A}, \eta, \lambda, f)$ we obtain a new consumption sector called an *income translation* or δ*-translation*) of Λ. The δ*-translation* of Λ is the sector $\Lambda_\delta \equiv (A, \mathcal{A}, \eta_\delta, \lambda_\delta, f)$, where $\eta_\delta(X) \equiv \eta(X - \delta)$ for measurable $X \subset \mathbf{R}$, and where $\lambda_\delta(x, A') \equiv \lambda(x - \delta, A')$ for each $x \in \mathbf{R}$ and $A' \in \mathcal{A}$.

Given the consumption sector $\Lambda \equiv (A, \mathcal{A}, \eta, \lambda, f)$ and $\Delta \subset \mathbf{R}$, define $f^\Delta(a, x) \equiv \{f(a, x + \delta)\}_{\delta \in \Delta}$. A measurable function $G : (A, \mathcal{A}) \to \mathbf{R}^k$ determines a measure $G(\cdot) \circ \lambda(x, \cdot)$, which takes the value $\lambda(x, U)$ at Q, where $U \equiv \{a \in A | G(a) \in Q\}$. The income translations $\{\Lambda_\delta\}_{\delta \in \Delta}$ and $\{\Gamma_\delta\}_{\delta \in \Delta}$ of the consumption sectors Λ and $\Gamma = (B, \mathcal{B}, \eta, \gamma, g)$ are *observationally equivalent* if $f^\Delta(\cdot, x) \circ \lambda(x, \cdot) = g^\Delta(\cdot, x) \circ \gamma(x, \cdot)$ for every $x \in \mathbf{R}$. The last equation states that for each x the two consumption sectors have the same distribution of *Engel curves* $f(a, \cdot)$ restricted to $\{x + \delta\}_{\delta \in \Delta}$ for households with initial income x. In addition the consumption sectors Λ and Γ are required to have the same income distribution η. Note that observational equivalence is stronger than the requirement that $f(\cdot, x) \circ \lambda_\delta(x, \cdot) = g(\cdot, x) \circ \gamma_\delta(x, \cdot)$ for all x and all $\delta \in \Delta$. The latter condition restricts the distributions of households' demand vectors in Λ_δ and Γ_δ for each $\delta \in \Delta$. But observational equivalence restricts the distribution of household *Engel curves* at a variety of income levels for households at each initial income level x.

The following result shows that metonymy for a single population cannot be tested with panel data on a finite number of income translations, even if we have a continuum of data from each cross section.

Proposition 4. *For every finite set of income translations of an arbitrary consumption sector with positive demands and smooth cross section there is an observationally equivalent set of metonymic income translations.*

Proof. Consider the consumption sector $\Lambda = (A, \mathcal{A}, \eta, \lambda, f)$ and δ-translations $\Lambda_\delta = (A, \mathcal{A}, \eta_\delta, \lambda_\delta, f)$ for δ in a finite set Δ. Without loss of generality we can let $\Delta \subset \mathbf{R}_+$. Let L^2_δ be the Hilbert space of real valued functions on $A \times \mathbf{R}_+$ that are square-integrable with respect to the measure μ_δ, the product of η_δ and λ_δ. For ϕ and ψ in L^2_δ define $\langle \phi, \psi \rangle_\delta \equiv \int \phi(a, x)\psi(a, x)\mu_\delta(dx, da)$. Let

$$\Phi^{ij}_\delta \equiv \int_{\mathbf{R}} [\partial_x \int_A f_i(a, x) f_j(a, x) \lambda_\delta(x, da)] \eta_\delta(dx).$$

The consumption sector Λ_δ satisfies metonymy if for each $i, j = 1, \ldots, n$,

$$\Phi^{ij}_\delta = \langle f_i, m_j \rangle_\delta + \langle f_j, m_i \rangle_\delta, \tag{15}$$

where m is the marginal propensity to consume function. The proof of Proposition 1 in [4] implies that for each $\delta \in \Delta$ there exist functions $\psi_{\delta i} \in L^2_\delta$, $i = 1, \ldots, n$, satisfying $\sum_i \psi_{\delta i} = 1$ and $\Phi^{ij}_\delta = \langle f_i, \psi_{\delta j} \rangle_\delta + \langle f_j, \psi_{\delta i} \rangle_\delta$ for

$i, j = 1, \ldots, n$. We will use these functions $\psi_{\delta i}$ to construct a consumption sector and its income translations, which are metonymic and observationally equivalent to the Λ_δ sectors.

Let $B \equiv A \times \mathbf{R}$ and let $\mathcal{B}$ be the product measure of $\mathcal{A}$ and Lebesgue measure on $\mathbf{R}$. Define $\gamma(x, \cdot) = \beta_x \circ \lambda(x, \cdot)$, where $\beta_x : a \in A \mapsto (a, x)$ for each $x \in \mathbf{R}$. The measure $\gamma(x, \cdot)$ is concentrated on the set $B_x = \{(a, y) \in B | y = x\}$, with the same structure on that set as the measure $\lambda(x, \cdot)$ on A.

Let $\Delta = \{\delta_1, \delta_2, \ldots, \delta_k\}$ with $\delta_j < \delta_{j+1}$ for $j = 1, \ldots, k-1$. Fix $(a, y) \in B$ and define $x_j \equiv y + \delta_j$ and $f_{ij} \equiv f_i(a, x_j)$ for each $i, j = 1, \ldots, k$. For each $i = 1, \ldots, n$, consider the piecewise linear function ϕ_i on $[x_1, x_k]$ satisfying $\phi_i(z) = f_{ij} + [(z - x_j)(f_{i,j+1} - f_{ij})/(x_{j+1} - x_j)]$ when $z \in [x_j, x_{j+1}]$, $j = 1, \ldots, k-1$. Then ϕ_i is affine on each segment $[x_j, x_{j+1}]$ and $\phi_i(x_j) = f_{ij}$ for each j. By construction, for $0 < z \in [x_1, x_k]$, $\phi_i(z) > 0$ and $\sum_i \phi_i(z) = z$. For each $i = 1, \ldots, n-1$, let $g_i(a, y, \cdot)$ be a C^1 nonnegative real valued function on $\mathbf{R}$ such that $g_i(a, y, 0) = 0$ and $g_i(a, y, x) > 0$ for $x > 0$, and such that $g_i(a, y, x) = f_i(a, x)$ and $\partial_x g_i(a, y, x) = \psi_{\delta i}(a, x)$ whenever $x = y + \delta$ for some $\delta \in \Delta$. The function $g_i(a, y, \cdot)$ restricted to $[x_1, x_k]$ is a smoothed approximation to ϕ_i, and can be chosen close enough in the sup norm to ϕ_i so that $\sum_{i=1}^{n-1} g_i(a, y, x) < x$ for each $x > 0$. Since $f_i(\cdot, x)$ and $\psi_{\delta i}(\cdot, x)$ are measurable, $g_i(\cdot, x)$ can be chosen to be measurable for each x. Define $g_n(a, y, x) \equiv x - \sum_{i=1}^{n-1} g_i(a, y, x)$. Since $\sum_{i=1}^{n} f_i(a, x) = x$ for $x \geq 0$, and $\sum_{i=1}^{n} \psi_{\delta i}(a, x) = 1$, we have $g_n(a, y, x) = f_n(a, x)$ and $\partial_x g_n(a, y, x) = \psi_{\delta n}(a, x)$ when $x = y + \delta$ for $\delta \in \Delta$.

By construction $g = (g_i)_{i=1}^n$ satisfies $g(a, y, x) = f(a, x)$ and $\partial_x g_i(a, y, x) = \psi_{\delta i}(a, x)$ for every $a \in A$, $y, x \in \mathbf{R}_+$ with $x = y + \delta$ for some $\delta \in \Delta$. In addition, $g(\cdot, x)$ is measurable in B for each $x \in \mathbf{R}$. It follows from the definition of γ that $g^\Delta(\cdot, x) \circ \gamma(x, \cdot) = f^\Delta(\cdot, x) \circ \lambda(x, \cdot)$. Thus letting $\gamma_\delta(x, \cdot) = \gamma(x - \delta, \cdot)$, the income translations $\Gamma_\delta = (B, \mathcal{B}, \eta_\delta, \gamma_\delta, g)$ are observationally equivalent to the Λ_δ sectors.

By construction, for each $\delta \in \Delta$ and each nonnegative $x \geq \delta$, we have $\int_B g_i(b, x)[\partial_x g_j(b, x)]\gamma_\delta(x, db) = \int_A f_i(a, x)\psi_{\delta j}(a, x)\lambda_\delta(x, da)$ and therefore

$$\int_{\mathbf{R}_+ \times B} \{\partial_x [g_i(b, x) g_j(b, x)]\} \gamma_\delta(x, db) \eta_\delta(dx) = \langle f_i, \psi_{\delta j} \rangle_\delta + \langle f_j, \psi_{\delta i} \rangle_\delta = \Phi_\delta^{ij},$$

so $\Gamma_\delta \equiv (B, \mathcal{B}, \eta_\delta, \gamma_\delta, g)$ satisfies metonymy. This completes the proof. $\diamondsuit$

Metonymy is also unfalsifiable if one observes finitely many consumption sectors, each obtained by multiplying the incomes of all the households in one sector by a common factor. Given $X \subset \mathbf{R}$, let $\kappa X \equiv \{\kappa x : x \in X\}$ for $\kappa \in \mathbf{R}$. A *multiple* or *κ-multiple* of a consumption sector $\Lambda = (A, \mathcal{A}, \eta, \lambda, f)$ is a sector $\Lambda^\kappa \equiv (A, \mathcal{A}, \eta^\kappa, \lambda^\kappa, f)$ obtained by multiplying every household's income by $\kappa > 0$, where $\eta^\kappa(X) \equiv \eta((1/\kappa)X)$ for measurable $X \subset \mathbf{R}$, and where $\lambda^\kappa(x, A') \equiv \lambda(x/\kappa, A')$ for each $x \in \mathbf{R}$ and $A' \in \mathcal{A}$.

Given $\mathcal{K} \subset \mathbf{R}_{++}$, define $f^{\mathcal{K}}(a,x) \equiv \{f(a,\kappa x)\}_{\kappa\in\mathcal{K}}$. Sectors $\{\Lambda^\kappa\}_{\kappa\in\mathcal{K}}$ and $\{\Gamma^\kappa\}_{\kappa\in\mathcal{K}}$, multiples of $\Lambda = (A, \mathcal{A}, \eta, \lambda, f)$ and $\Gamma = (B, \mathcal{B}, \eta, \gamma, g)$ are *observationally equivalent* if $f^{\mathcal{K}}(\cdot, x) \circ \lambda(x, \cdot) = g^{\mathcal{K}}(\cdot, x) \circ \gamma(x, \cdot)$ for every $x \in \mathbf{R}$.

A slight modification of the proof of Proposition 4 yields the following.

Proposition 5. *For every finite set of multiples of an arbitrary consumption sector with positive demands and smooth cross section there is an observationally equivalent set of metonymic multiples of some consumption sector.*

We showed in section 4 how to test for increasing dispersion in a single consumption sector using data on a finite set of κ-multiples of that sector. But according to Proposition 5, data on the κ-multiples are not enough to test whether a sector Γ is metonymic. Whether it is or not, there is another sector with metonymic κ-multiples that are observationally equivalent to the κ-multiples of Γ.

7 Conclusion

In a competitive distribution economy with increasing dispersion, mean demand satisfies the weak axiom and there is a unique equilibrium allocation. We have described new ways to test for increasing dispersion using suitable data from periods with constant relative prices and a form of structural stability, or using data from a single cross section under metonymy or dispersion metonymy. However, the structural stability used above and metonymy and dispersion metonymy cannot be tested using ideal panel data.

Other sources of information might offer hints about the plausibility of these auxiliary hypotheses. It might be reasonable to apply the structural stability hypothesis to periods in which the consumers report that their preferences and other circumstances did not change significantly. Metonymy seems plausible when applied to narrow attribute classes, defined by characteristics other than prices and income that are the most important determinants of demand. The proof of Proposition 4 suggests another way to evaluate the metonymy hypothesis. The construction of the metonymic consumption sectors in that proof will not necessarily work if there are bounds on the households' marginal propensities to consume. If there are known bounds on the marginal propensities, then metonymy is potentially refutable using data from finitely many income translations or income multiples.

We have not considered tests of the nonpositive average substitution (NAS) assumption in Proposition 1. Experimental evidence suggests that although consumers often violate the weak axiom, the violations are not large in the sense of Afriat [1,2,21,25]. The connection between the sizes of violations of the weak axioms and violations of Slutsky negative semidefiniteness is described in [11] for the case of deterministic smooth demand functions.

Most of the results in the present paper can be modified to apply to general private ownership economies, following the approach in [15]. But many questions remain concerning the relationship between the models and data. For example, how do we allow for discreteness of purchases? At what levels of commodity aggregation is increasing dispersion likely to hold? With finely defined commodities, there are likely to be inferior goods that violate increasing dispersion. When the households' incomes rise enough, the variance of their demands for such goods falls as the demands approach zero.

The treatment of time is also problematic. Consumption data typically come from surveys in which households report their expenditures during a two to four week period. But the duration of applied models is typically longer. This means that part of the demand vector in the models is not observed. However, in that case, the propositions and empirical tests discussed above can be interpreted as applying to models with suitable time separability of consumer demands. For those models, the empirical dispersion analyses based on the Slutsky decomposition (6) have led to a deeper understanding of the structure of market demand and the conditions under which competitive price adjustment is likely to be stable.

Acknowledgments

This paper is an extended version of "Dispersed Consumer Demands and the Weak Axiom in the Aggregate." It was completed with financial support from the Ecole des Hautes Etudes en Sciences Sociales and the Deutsche Forschungsgemeinschaft during visits to Delta and the University of Mainz. I thank both institutions and especially Roger Guesnerie and Alois Kneip for their hospitality. The paper synthesizes and extends results obtained during three years of visits to the University of Bonn. I thank Werner Hildenbrand for inviting me there. The environment was especially stimulating because of his efforts to promote empirical investigation and collaboration among economists and statisticians. He has made lasting contributions to demand analysis with his classic result on the impact of the income distribution [7], his search for properties created (not merely preserved) by aggregation and his insistence on relaxing parametric and "rationality" assumptions. I learned a great deal about demand aggregation from him and from his students, colleagues and other visitors: J. Arns, C. Blackorby, R. Blundell, E. Dierker, H. Dierker, J. Engel, I. Evstigneev, P. Funk, J.-M. Grandmont, B. Grodal, W. Härdle, H. Hennig-Schmidt, T. Hens, K. Hildenbrand, I. Jang, R. John, K. Koch, A. Kneip, M. Lehmann-Waffenschmidt, A. Lewbel, A. Mas-Colell, R. Matzkin, W. Neuefeind, J.-U. Scheer, U. Schweizer, W. Shafer, R. Sippel, T. Stoker, K. Utikal, K. Vind.

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