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# Forward Exchange Rate Puzzle:
# Joining the Missing Pieces in the Rand-US Dollar
# Exchange Market

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# Forward Exchange Rate Puzzle: Joining the Missing Pieces in the Rand-US Dollar Exchange Market

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### Abstract

The Unbiased Forward Rate Hypothesis (UFRH) stipulates that the forward rates should be a perfect predictor for the future spot rates. A number of studies have tested the UFRH and foreign market efficiency and concluded that the hypothesis does not hold. This phenomenon is known as the UFRH puzzle. A number of studies that reject the UFRH have made use of ordinary least square (OLS) methods and support a linear adjustment between spot and forward exchange rates. This paper establishes that the use of a linear model in testing the UFRH can lead to a misspecification problem if indeed there is a nonlinear adjustment between the forward and spot exchange rates. In order to overcome the problem of model misspecification, this paper applies the nonlinear method of the class of the Smooth Transition Regression (STR) model in assessing the relationship between the Rand-US Dollar future spot and forward exchange rates. With the aid of a series of diagnostic tests, the paper shows that there is indeed a nonlinear adjustment process between the Rand-US Dollar spot and forward exchange rates and that there exists a regime in the STR model where the UFRH eventually holds. Furthermore, the out-of-sample forecast results show that the STR forecasting method outperforms the OLS and random walk methods in forecasting the future spot exchange rate.

## 1    Introduction

The Efficient Market Hypothesis (EMH), also known as the informational efficient market, relies on the efficient exploitation of information by economic actors in predicting prices of securities or assets (Reilly and Brown, 2000). For example, foreign exchange market participants are efficient if they use information embedded in the forward exchange rate to predict the future spot rate. Thus, in an efficient forward exchange market, the forward rate is the unbiased predictor of the future spot rate. This is known as the Unbiased Forward Rate Hypothesis (UFRH). A number of studies have been conducted to test whether the forward exchange rate is indeed an unbiased forecast of the future spot rate. While some of the studies support the existence of the unbiased forward rate hypothesis, a number of studies have contested the validity of such a hypothesis (see Engel, 1996 for a comprehensive review of literature on UFRH). The rejection of the UFRH is also known as the forward exchange rate puzzle. In attempting to explain why the UFRH is rejected, Fama (1984) and Nieuwland *et al* (2000) consider the possibility of risk premium in the foreign exchange market. They contend that risk-averse investors require compensation to assume risk and therefore the forward exchange rate is not a rational predictor of the future spot exchange rate. A number of reasons for the failure of the UFRH have been identified; these include irrational expectations by foreign exchange market participants, the assumption of adaptive learning by economic agents and the measurement error or models misspecification (Razzak, 2002; Psaradakis *et al*, 2005).

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This paper focuses on the cause related to the misspecification of the model for the failure of the UFRH and shows that a linear model is inadequate in characterising the relationship between the Rand-US Dollar future spot and forward exchange rates. Wesso (1999) investigated the empirical issue of market efficiency for the South African currency from January 1987 to November 1998. The results of his study rejected the UFRH in the Rand-US Dollar exchange market. Wesso's empirical study was based on the constant coefficients assumption. Nonetheless, the author suggested that further research be devoted to the analysis of the time-variant coefficients considering a number of structural breaks or regime shifts present in the relationship between the Rand-US Dollar spot and forward exchange rates. Sarno, Valente and Leon (2004) conducted a number of Monte Carlo experiments to demonstrate that there is strong evidence that the relationship between spot and forward exchange rates is characterised by significant nonlinearities. In addition, Baillie and Kiliç (2006) support the proposition that nonlinearity is an important aspect of the uncovered interest parity (UIP) and apply the technique of the logistic smooth transition dynamic regression (LSTR) to model the relationship between the rate of appreciation of nine currencies and interest rate differentials. The results of their study indicate that the UIP has a high probability to hold in the outer regime of the LSTR model. As explained in Section two, the UIP is a pre-condition for the UFRH to hold. Expanding on the work of Ballie and Kiliç (2006), this paper aims to test whether the Rand-US Dollar forward exchange rates are unbiased predictors of future spot rates. Firstly, this paper aims to determine whether the relationship between the Rand-US Dollar forward and spot exchange rate is linear or nonlinear. Secondly, the paper selects and estimates a suitable class of smooth transition regression models to illustrate the relationship between the future spot and forward exchange rates in the Rand-US Dollar exchange market. Thirdly, this paper compares the out-of-sample forecasting performance of the STR, linear and random walk methods in predicting the Rand-US Dollar future spot rates.

The remainder of the paper is structured as follows. Section two provides a brief review of related literature. Section three describes the nonlinear model of the type of the smooth transition autoregressive (STR) model. An empirical analysis and the relevant discussion of the reported results are provided in section four. Lastly, section five presents the conclusion and a summary of the main findings of this paper.

## 1.1 Literature Review

In an efficient speculative market, prices should fully reflect information available to market participants and it should be impossible for a trader to earn excess return on speculation (Sarno and Taylor, 2003). Any arbitrage opportunity that presents itself in the foreign exchange market will rapidly cancel out with the change in the conditions of supply and demand. In other words, in the long term, the forward exchange rate should be a perfect predictor of the spot exchange rate.

The UFRH is derived from two important parity conditions, namely the uncovered interest parity (UIP) and the covered interest parity (CIP). The UIP is given as follow:

$$\frac{E_t S_{t+k}}{S_t} = \frac{1+r}{1+r_f} \tag{1}$$

Where $r$ and $r_f$ represent the domestic and foreign interest rate respectively prevailing at time $t$. $S_t$ is the spot exchange rate and $E_t S_{t+k}$ is the expectation at time $t$ of the spot price prevailing at time $t+k$. Equation (1) states that the risk-free return from a local investment is equal to the comparable return in a foreign instrument plus an expected appreciation rate of the foreign currency (Marston, 1997). Since the outcome of the future spot rate is uncertain, investors with risk aversion prefer to sell the total proceeds of their investment in the forward market. As a result, a covered version of interest rate parity is expressed as follow:

$$\frac{F_t}{S_t} = \frac{1+r}{1+r_f} \tag{2}$$

Where $F_t$ is the $k$ period's forward rate at time $t$.

Linking expression (1) and (2) and applying the natural logarithm expression, one would then arrive at the conclusion that:

$$f_t = E_t\left(s_{t+k}\right) \tag{3}$$

Where $f_t$ and $s_{t+k}$ are the natural logarithm of $F_t$ and $S_{t+k}$, respectively. Expression (3) implies that the forwad rate at time $t$ should be equal to the market expectations of the future spot rate, given the information at time $t$. This expresion provides a basis for the testing of an unbiased forward rate hypothesis.

According to Siegel (1972) a paradox arises when Equation (3) is formulated in levels rather than logarithm. Siegel's paradox entails at least one currency to be biased for purely mathematical reasons. It can be indicated that if Expression (3) is expressed in level, changing the numeraire will result in the following:

$$\frac{1}{F_t} = E_t\left(\frac{1}{S_{t+k}}\right) \tag{4}$$

Equation (4) cannot imply to be the inverse of the level Equation (3). However, Mcculloch (1975) indicated that the Siegel paradox is unimportant for empirical work. Thus, most empirical work defines the variables in logarithm form and the UFRH is mostly estimated with the use of the following equation:

$$s_{t+k} = \alpha + \beta f_t + \eta_{t+k} \tag{5}$$

$\eta_{t+k}$ is the rational expectations forecast error with $E\left[\eta_{t+k}/I_t\right] = 0$. In order to test the hypothesis that the forward rate is an unbiased predictor of the spot rate, the restriction $\beta{=}1$ is tested. A strong form of an unbiased market efficiency hypothesis and no risk premium implies testing $\alpha{=}0$ (a constant risk premium equals zero) and $\beta{=}1$. Moreover, for the UFRH test the error terms, $\eta t + k$ with $k = 0...n$, should be serially uncorrelated and homoskedastic.

A number of estimations that have tested the unbiased forward rate hypothesis with the aid of Equation (5) have rejected the hypothesis and many reasons have been advanced. For example, Fama (1984) argues that the failure of the UFRH is due to the fact that the risk premium on foreign exchange rates is extremely variable. The OLS method is not designed to model time varying parameters. However, McCallum (1994) provides a different explanation. McCallum explains the failure of the UFRH as neglect to take into account the fact that monetary authorities pursue interest rate smoothing and avoid exchange rate changes. Therefore, there is a missing variable to account for monetary policy behaviour in Equation (5). Several other reasons have been identified for the rejection of the efficient market hypothesis and UFRH. These reasons include measurement misspecification, rational bubbles, learning about regime shifts or about fundamentals (Lewis, 1989), the 'peso problem' originally suggested by Rogoff (1979) and the inefficiency in processing information (Bilson, 1981). Wesso (1999) suggested that the analysis of the time–variant coefficients be considered in testing for the efficiency of the exchange rate market in South Africa considering a number of structural breaks in the Rand-US Dollar spot and forward exchange rates. Specifically, casual inspection of data on exchange rates from January 1987 and November 1998 by Wesso reveals a marked increase in the variability of the rand, particularly during 1996 and 1998. Moreover, the author has noted that the weakness of the Rand during 1996 was a combination of several factors, such as large-scale speculation triggered by unfounded rumours about the health of the then president Mandela, and negative views about the South African socio-political situation. Furthermore, according to Wesso, the major depreciation of the Rand in 1998 was due to emerging market contagion from the Asian crisis as well as the increase in the South African Reserve Bank's net open forward position that inspired important speculation against the Rand.

With regards to the application of nonlinear models in predicting the future spot rate from the forward rate, Clarida *et al* (2003) apply a multivariate Markov-switching framework using weekly data on major spot and forward Dollar exchange rates over the period 1979 to 1995. The results

of their analysis support the presence of nonlinearity in the relationship between spot and forward exchange rates. Moreover, the results suggest that the Markov-switching framework forecasts are strongly superior to the random walk forecasts at a range of up to 52 weeks ahead.

In supporting the evidence of nonlinear and asymmetric adjustment in the foreign exchange market, Lyons (2001) confirms the existence of a band of inaction where the forward bias does not attract speculative capital and, therefore, does not imply any profitable opportunity. This band of inaction will persist until it generates a Sharpe ratio that is large enough to attract speculative capital. This indicates that the equilibrium correction between the spot and forward exchange rate is not symmetric or linear. Baillie and Kiliç (2006) estimate a nonlinear model of the form of logistic smooth transition autoregression (LSTR) to test whether the UIP hypothesis holds. The authors include different transition variables in their model such as the lagged forward premium, monetary and income fundamentals. The results reported for nine different currencies provide evidence for the existence of an outer regime that is consistent with UIP.

In an attempt to establish which regimes are consistent with the UFRH, this paper selects a transition variable based on the mechanics of arbitrage activities in the forward exchange market. With forward premium selected as a transition variable, this paper shows that a sufficiently large deviation of the forward premium from an estimated threshold value triggers arbitrage opportunity. Active arbitrage behaviours are associated with more rapid convergence between the future spot and the forward exchange rates (Dwyer, Lock and Yu, 1996). Nevertheless, a small deviation of the forward premium from an estimated threshold value entails no-arbitrage bound that allows persistent deviation between the future spot and forward exchange rates and therefore unrelenting failure of the UFRH.

# 2    Smooth transition regression model

Smooth Transition Regression (STR) models have gained significant popularity in the literature of economics and finance as a means to transcend well-known estimation and forecasting limitations of both linear and binary switching models, such as the threshold autoregressive (TAR) and Markov Switching models. Original theoretical contributions of the STR model belong to Tong (1983) and Tong and Chan (1986). Nonetheless, Luukkonen *et al.* (1988) and Teräsvirta (1994) contribute to the STR model by developing a composite theory and evaluation technique for determining whether the STR model suits exponential and logistic transition functions.

According to Granger and Teräsvirta (1993), the nonlinear adjustment process can be characterised in terms of a smooth rather than a sharp transition as in TAR models. The STR models have interesting properties. Firstly, the STR models do not assume a sharp switch from one regime to the other, for instance as found in the TAR or the Hamilton's Markov switching regime models. Teräsvirta (1994) noted that it is unlikely that economic agents change their behaviour simultaneously whatever the circumstances. Thus, the change in regime may be smooth rather than discrete. Secondly, STR models nest linear regression model. They are regimes that can accommodate linear interaction between variables.

Following Teräsvirta (2004), the standard STR model is defined as follows:

$$y_t = \delta' z_t + \theta' z_t G\left(\gamma, c, s_t\right) + \mu_t \varphi \tag{6}$$

Where $z_t = \left(w_t', x_t'\right)'$ is a vector of explanatory variables. $w_t' = (1, y_{t-1}, \ldots, y_{t-p})'$ and $x_t' = (x_{1t}, \ldots, x_{kt})'$ are vectors of exogenous variables. $\delta'$ and $\theta'$ are parameter vectors and $\mu_t \approx iid\left(0, \sigma^2\right)$. $G\left(\gamma, c, s_t\right)$ is the transition function. The transition function is a bounded function of the continuous transition variable $s_t$, continuous everywhere in the parameter space for any value of $s_t$. $\gamma$ is the slope parameter and $c$ is a vector of location parameters. The transition variable $s_t$ is either

stationary or a time trend $(t)$. If the transition function is a general logistic function, LSTR $(k)$ model is expressed as

$$G\left(y,c,s_t\right) = \left(1 + \exp\left\{-\gamma \prod_{k=1}^{k}\left(s_t - c_k\right)\right\}\right)^{-1} \tag{7}$$

Where $\gamma > 0$, $c_1 \leq \ldots \leq c_k$

The transition function is assumed to be twice continuously differentiable in $\gamma$ and c. If the scale parameter $\gamma$ and/or the coefficient $\theta'$ are zero, then the process collapses to a linear regression. A most commonly used specification for transition functions are the case where $k = 1$; this yields the LSTR (1) model. When $k= 2$, the LSTR (2) or Exponential STR (ESTR) are used. The slope parameter $\gamma$ determines the speed of transition between the two extreme regimes and the vector of location parameters $c$ determines the location of the transition. Teräsvirta (1994, 1998) outlined a modelling strategy for building STR models. The modelling cycle consists of three main stages, namely specification, estimation and evaluation. Specification includes testing for linearity, choosing the transition variable and deciding whether the model fit the LSTR (1) or the LSTR (2). Estimation involves finding the appropriate starting values for the nonlinear estimation and estimating the model. Evaluation of the model entails various tests for misspecification, such as error autocorrelation, parameter non-constancy, remaining nonlinearity, ARCH and non-normality.

## 2.1 *Choosing the transition variable and testing linearity*

When testing for linearity, the different variables are defined as potential transition variables. The test is then executed for each of the candidates and the variable with the strongest test rejection (in terms of the p-value) is retained as a candidate for the transition variable. Mostly, the initial choice of a set of potential transition variables is based on economic theory. For the purpose of this paper, the choice of the transition variable is based on the motivation for arbitrage activities in the forward exchange market. Active arbitrage behaviour in the forward exchange market guarantees a long-run equilibrium between the future spot and forward exchange rate (Hull, 2003). In particular, arbitrage activities are effective when arbitrageurs believe that there is a possibility of realising profit from taking an open position in the exchange market. Thus, the candidate transition variable is the forward premium. The deviation of the forward premium from the transaction cost (threshold value), has been cited as the major cause for active arbitrage behaviour as well as for nonlinearity in the foreign exchange market (Dumas, 1992). The deviation of the forward premium from the transaction costs creates a band of inaction within which the long-run equilibrium between the future spot and the forward exchange rates are left uncorrected as arbitrageurs do not find it profitable to actively participate in the exchange market.

The test of linearity is performed by using the third-order Taylor expansion around $\gamma = 0$. The approximation yields the following regression:

$$Yt = \beta_0' z_t + \sum_{j=1}^{3}\beta_j' \bar{z}_t s_t^j + \mu_t^*, t = 1, T \tag{8}$$

Then, it can be assumed that the transition variable is an element of $z_t$ and $z_t = \left(1, \bar{z}_t\right)$, where

$\bar{z}_t$ is a $(m \times 1)$ vector. In testing for linearity, the null hypothesis is $\text{Ho:}\beta_1 = \beta_2 = \beta_3 = 0$. The rejection of the null hypothesis implies that the model is nonlinear. When linearity is rejected, the next step will consist of selecting the model type. The choice is between LSTR (1) and LSTR (2).

The following tests are performed, based on Equation (8), for selecting the STR model type:

1. Test of the null hypothesis H04: $\beta_3 = 0$

2. Test of the null hypothesis H03: $\beta_2 = 0/\beta_3 = 0$

3. Test of the null hypothesis H02: $\beta_1 = 0/\beta_2 = \beta_3 = 0$

LSTR (2) is selected if the test of H03 yields the strongest rejection measure suggested by the p-value. Otherwise the selection of LSTR (1) is recommended.

## 2.2 *Model Estimation and Evaluation*

The parameters of the STR model are estimated using conditional maximum likelihood. Generally, estimation involves finding the appropriate starting values for the nonlinear estimation and estimating the model. In addition, evaluation of the model entails various tests for misspecification or diagnostic tests. Misspecification tests are important tools for checking the quality of an estimated model. This paper applies different misspecification tests to evaluate whether the UFRH condition can be represented by a linear or a nonlinear model. These tests range from the test of error autocorrelation to the ARCH and no additive nonlinearity tests.

# 3 Data Analysis and Empirical Results

This paper utilises monthly data on Rand-US Dollar spot exchange rates ($spot_t$) and one-month forward exchange rates ($f_t$) to test for the UFRH and foreign exchange market efficiency. The series are transformed in logarithm form. The sample observation covers the period from January 1994 to August 2008. The empirical analysis is divided into two parts. In the first part, the paper compares the validity of a linear and a nonlinear specification in modelling the UFRH condition in the period between January 1994 and November 2006. The criterion for comparison of the two models during that period is based on misspecification or diagnostic tests. In the second part, the out-of-sample forecasting performance of the LSTR (nonlinear) model specification is compared with the OLS and random walk models in predicting the future spot rates. The period between December 2006 and August 2008 is used for this aim.

Table A1 presents the results of the unit root test for the two series using the augmented Dickey-Fuller (ADF) test. The results reveal that the two series have a unit root. The Engle-Granger two-step procedure was used to test for cointegration between ($spot_t$) and ($f_t$). The residuals from the regression of ($spot_{t+1}$) on $f_t$ are I (0). This confirms that the two series are cointegrated. The results of the linear estimation of the UFRH condition as in Equation (5), where $k = 1$, are provided in Table A2. The results indicate that the coefficient of ($f_t$) is different to unity. A Wald test is applied to test the restriction as to whether the coefficient of $\alpha$ (the constant coefficient) is equal to zero and $\beta$ (the coefficient of $f_t$) is equal to unity. This restriction is necessary for the UFRH to hold. The results reported in Table A3 show that this restriction is rejected at 1% level of significance, thus the UFRH does not hold. Furthermore, the CUSUM (cumulative sum of the recursive residuals) test shows that the null hypothesis of constant parameters in the relationship between ($spot_{t+1}$) and ($f_t$) is rejected (the results are reported in figure A1). The findings of the CUSUM test support parameter instability as the cumulative sum of the recursive residuals goes outside the area between the two critical lines in Figure A1. This indicates that coefficients $\alpha$ and $\beta$ obtained from a linear estimation are not stable. Moreover, a linearity test is carried out to assess whether there is indeed a nonlinear relationship between the future spot and forward exchange rates. As stated earlier, the chosen transition variable is the lagged forward premium, $pre(-1)$. The choice of the forward premium as a transition variable is supported by the fact that the size of the forward premium, compared to an estimated threshold, triggers asymmetric behaviour in the forward exchange market. While the large size of the forward premium, compared to the threshold value, may trigger arbitrage activity in the forward exchange market and guarantees a long-term equilibrium between future spot and forward exchange rates, a small forward premium may leave arbitrageur indifferent to participating in the forward exchange market.

The results in Table A4 indicate that the null hypothesis of linearity is strongly rejected when $pre(-1)$ is chosen as the transition variable. The results of the test of linearity suggested that the LSTR (1) is the appropriate model for measuring the relationship between $spot_{t+1}$ and $f_t$. The LSTR (1) model for the UFRH regression to be estimated is expressed as:

$$spot_{t+k} = [\alpha_1 + \beta_1 f_t] + [\alpha_2 + \beta_2 f_t] G(\gamma, c, pre(-1)) + \mu_{t+k} \quad Where \ k = 1 \qquad (9)$$

The transition function, $G(\gamma, c, pre(-1))$, changes monotonically from 0 to 1 as $pre(-1)$ increases. There are different regimes associated with Equation (9). The inner regime corresponds with $pre(-1) = c$. In this inner regime, Equation (9) is represented as:

$$spot_{t+k} = [\alpha_1 + 0.5\alpha_2] + [\beta_1 + 0.5\beta_2] f_t + \mu_{t+k} \qquad (10)$$

The upper regime corresponds for a given $\gamma$ and $c$ to $\lim_{pre(-1)\to+\infty}$. In the upper regime Equation (9) becomes:

$$spot_{t+k} = [\alpha_1 + \alpha_2] + [\beta_1 + \beta_2] f_t + \mu_{t+k} \qquad (11)$$

For the lower regime where $\lim_{pre(-1)\to-\infty}$, Equation (9) is represented as:

$$spot_{t+k} = \alpha_1 + \beta_1 f_t + \mu_{t+k} \qquad (12)$$

Because $G(\gamma, c, pre(-1))$ is a continuous function, Equations (10), (11) and (12) imply that:
$\frac{d(spot_{t+k})}{d(f_t)} = [\beta_1 \quad \beta_1 + \beta_2]$, meaning that the coefficients of $f_t$ is determined in the closed interval between $\beta_1$ and $\beta_1 + \beta_2$.

Table A5 provides details of the estimation of the LSTR (1) model represented in Equation (9). The results presented in Table A5 are reported in an equation form as follows:

$$\begin{aligned} Spot_{t+1} \quad = \quad & 0.02626 + 1.00328 f_t + \\ & (-0.01977 - 0.03635 f_t)(1 + \exp(-3.2[pre(-1) + 0.00834]))^{-1} \end{aligned} \qquad (13)$$

Where, $G(\gamma, c, pre(-1)) = (1 + \exp(-3.2[pre(-1) + 0.00834]))$. In this expression $\gamma = 3.2$ and $pre(-1) = -0.00834$

The results reported in Table A5 indicate that the coefficients of $f_t$ are statistically significant in the linear and nonlinear parts of the LSTR (1) estimation. Furthermore, because $G(\gamma, c, pre(-1))$ is a continuous function and bounded between 0 and 1, the estimation results in Equation (13) imply that $\frac{d(spot_{t+k})}{d(f_t)} = [0.96693 \ 1.00328]$. This suggests that the coefficient of $f_t$ can take any value within the interval comprises of 0.96693 and 1.00328. This is an indication that there exists a regime where the coefficient of $f_t$ may take the value of unity. This finding suggests that there exists a regime within the LSTR (1) model where the UFRH holds.

Figure A2 illustrates the relationship between the transition function and the transition variables for given observations. The figure indicates that the transition function has a steep slope. This is due to high speed of transition between the two extreme regimes owing to the large estimated value of the slope parameter, $\gamma = 3.2$ in Equation (13). The vertical line in Figure A2 represents sample observations in the middle regime where $pre(-1) = c = -0.00834$ and $G(\gamma, c, pre(-1)) = 0.5$. In this regime the estimated value of the coefficient of $f_t$ equals 0.985275. Nonetheless, for observations in the upper regime with $G(\gamma, c, pre(-1)) = 1$, the coefficient of $f_t$ takes the value of 0.9671 and in the lower regime, with the transition variable $G(\gamma, c, pre(-1)) = 0$, the value of the coefficient of $f_t$ equals 1.00345. This finding indicates that the UFRH condition is more likely to hold in the lower regime as the transition variable $G(\gamma, c, pre(-1))$ moves toward zero and $\lim_{pre(-1)\to-\infty}$. This indicates that the UFRH holds for negative forward premium (positive forward discount) on the Rand. In contrast, large forward premia are consistent with the UFRH anomaly.

In order to ascertain whether the STR is a better model specification in testing for the UFRH compared to a linear model during the period between January 1994 and November 2006 (In–sample period) , additional misspecification tests are conducted for the estimated LSTR (1) model. The test of no error autocorrelation in Table A6 shows that the null hypothesis of no error autocorrelation is not rejected at lag 3. This is an indication that there is no autocorrelation in the residual of the LSTR (1) estimation. A test of no-remaining-nonlinearity or no-additive-nonlinearity is conducted to determine whether the LSTR (1) model is a satisfactory characterisation of the nonlinearity detected in the specification stage. The corresponding p-value in Table A7 indicates that the null hypothesis of no remaining linearity is not rejected at the 5% confidence level. This is indicative that the LSTR (1) model, with $pre(-1)$ as a transition variable, is adequate for UFRH representation. The ARCH-LM test with 3 lags in Table A5 indicates that there is no ARCH effect in the LSTR (1) estimation. Lastly, results of parameter constancy tests are reported in Table A8. They indicate that the null hypothesis of parameter constancy is not rejected. In the light of these results the LSTR (1), as a type of STR models, appears to be a reasonable specification for testing the UFRH compare to a linear model during the period between January 1994 and November 2006.

Furthermore, to test whether the LSTR (1) model specification of the UFRH performs better than the OLS model, and the random walk model in predicting the future spot rates, the out-of-sample forecasting performance of the three models is compared. The paper makes use of the root mean square error (RMSE) to evaluate the accuracy of forecasts generated by the three models. In Table A9, the results of the RMSE of the one-month-ahead out-of-sample forecasts generated by the three models, for the period between December 2006 and August 2008, are compared. The results show that the LSTR (1) produces the lowest RMSE, compare to the OLS and random walk models, in predicting the future spot rates. Also, the OLS model outperforms the random walk model in predicting the future spot rates. These results indicate the forward exchange rates provide better forecast of the future spot rates than do the current spot rates in the period between January 1994 and August 2008. These results confirm the findings by Clarida *et al* (2003) that forward rates contain more useful information to forecast spot exchange rate than do conventional fundamentals and random walk model.

Figure A3 compares the actual and predicted future spot rates obtained from the LSTR (1), OLS and random walk model, respectively. As with the results of the RMSE in Table A9, the display in Figure A3 shows how close are the out-of-sample performances of the three models in predicting the future spot rate.

# 4 Conclusion

This paper assessed the validity of the UFRH in the Rand-US Dollar exchange market. A number of studies have attributed the failure of the UFRH and thus the rejection of market efficiency hypothesis to model misspecification. These studies have raised concern for the use of a linear model to represent behaviour in the exchange market. This paper considered a nonlinear model of the LSTR specification to account for asymmetric adjustment and regime change in the relationship between the Rand-US Dollar future spot and forward exchange rates. The LSTR model, represented with the forward premium as a transition variable, shows that the UFRH holds for observations situated in the lower regime of the transition function in the period between January 1994 and November 2006. Furthermore, diagnostic tests were undertaken to assess the validity of a linear and nonlinear model in representing the relationship between the Rand-US Dollar forward and future spot rates. The paper finds that a nonlinear model of the LSTR (1) specification is a more reliable representation when testing for the UFRH in the Rand exchange market. The paper therefore concludes that there exists a regime where the foreign exchange market is efficient in South Africa in the period between January 1994 and November 2006. Nonetheless, this paper recommends that further research be conducted about the issue of why the UFRH holds in the lower regime where the Rand is at high

forward discount and not in the upper regime consistent with high forward premia. In addition, in order to determine whether the LSTR provides better forecasts of the future spot exchange rates compared to the OLS and random walk models, this paper compares the out-of-sample forecasting performance of the three models. The findings contend that the one-month-ahead out-of-sample forecast of the future spot rate obtained from the LSTR outperforms the OLS and the random walk models in the period between December 2006 and August 2008. The results confirm the findings by Clarida *et al* (2003) that forward rates contain more useful information to forecast spot exchange rate than do conventional fundamentals and random walk model.

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## Appendix A

### Table A1 Unit Root test of the series at the level

| Variables | ADF (Adjusted t-statistics) |
|---|---|
| $spot_t$ | -1.667115 |
| $f_t$ | -1.650438 |

Test critical values are: -3.472813, -2.880088 and -2.576739 for 1%, 5% and 10% respectively.

Therefore, the null hypothesis of unit root is not rejected.

### Table A2 Regression Estimation of SPOT on FORW

Dependent Variable: $spot_{t+1}$

| Variables | Coefficient | Std. Error | T-statistic | Prob. |
|---|---|---|---|---|
| Constant | -0.004624 | 0.008566 | -0.539776 | 0.5901 |
| $f_t$ | 0.999871 | 0.010869 | 91.99347 | 0.0000 |

| | | | |
|---|---|---|---|
| Adjusted R-Squared | 0.98221 | Akaike Info Criterion | -50145 |
| S.E of Regression | 0.018354 | Schwartz criterion | -5.105890 |
| Durbin-Watson | 1.918553 | F-Statistic | 8462.798 |
| Sum squared residual | 0.051539 | Prob (F-Statistic) | 0.00000 |

Note: The residuals obtained from this estimation are I (0). This justifies that the two series are cointegrated according to Engle-Granger two-step procedure.

### Table A3 Wald test of coefficient restrictions $\alpha$ =0 and $\beta$ =1

| Test Statistic | Value | df | Probability |
|---|---|---|---|
| F-statistic | 5.133996 | (2,153) | 0.0069 |
| Chi-Square | 10.26799 | 2 | 0.0059 |

The null hypothesis of $\alpha$ =0 and $\beta$ =1 is rejected

**Table A4  P-Value of the linearity test model (Pre (-1) is the transition variable)**

| Hypothesis | P- Value |
|---|---|
| H04 | 1.4034e-03 |
| H03 | 4.4919e-02 |
| H02 | 7.4192e-03 |

The p-value of the test of H03 is much larger than the corresponding H02 and H04, therefore the null hypothesis of linearity is rejected and LSTR (1) model is chosen

**Table A5  LSTR (1) regression estimation of Equation (9). Pre (-1) is the transition variable**

Dependent variable: $spot_{t+1}$

| Variable | estimate | t-stat | P-value |
|---|---|---|---|
| **Linear part** | | | |
| Const | 0.02606 | 1.7788* | 0.0773 |
| $f_t$ | 1.00345 | 55.8408*** | 0.0000 |
| | | | |
| **Nonlinear part** | | | |
| Const | -0.02002 | -1.8139* | 0.0667 |
| $f_t$ | -0.03614 | -2.0747** | 0.0261 |
| | | | |
| $\gamma$ | 3.21001 | | |
| C | -0.00831 | | |
| | | | |
| Akaike Info criteria | -9.3528 | pARCH(3) = 0.9987 | |
| Adjusted $R^2$ | 0.9957 | | |
| Variance of the transition var. | 0.0003 | | |
| Variance of the residual | 0.00001 | | |

pARCH (3) shows the p-value of the LM test of no ARCH against ARCH at order 3. *, ** and ***  mean significant at 10%, 5% and 1% level .

## Table A6 Test of no error autocorrelation

| Lag | F-value | p-value |
|-----|---------|---------|
| 1 | 1.7803 | 0.1842 |
| 2 | 1.0603 | 0.3490 |
| 3 | 0.7466 | 0.5260 |

The p-value indicates that the null hypothesis of no error autocorrelation is not rejected at 5% level of confidence.

## Table A7 Test of no remaining nonlinearity conducted with H03 hypothesis

| Transition variable | H03 |
|---------------------|-----|
| Pre (-1) | 0.69976 |

Test conducted with hypothesis H03 that leads to the selection of LSTR (1) model. See Table A4.The null

hypothesis of no remaining nonlinearity is not rejected.

## Table A8 p-value of parameter constancy test of the LSTR (1) model

| Hypothesis | p-value of the LM type test |
|------------|------------------------------|
| All parameters are constants | 0.2323 |

The null hypothesis of constant parameters is not rejected.

## Table A9 RMSE of the one-month-ahead forecasts of the future spot rates

| Model Specification | RMSE |
|---------------------|------|
| LSTR (1) | 0.03825 |
| OLS | 0.03861 |
| Random Walk | 0.03909 |

**Figure A1 Comparison of the cumulative sum of residual (CUSUM) and 5% critical line**
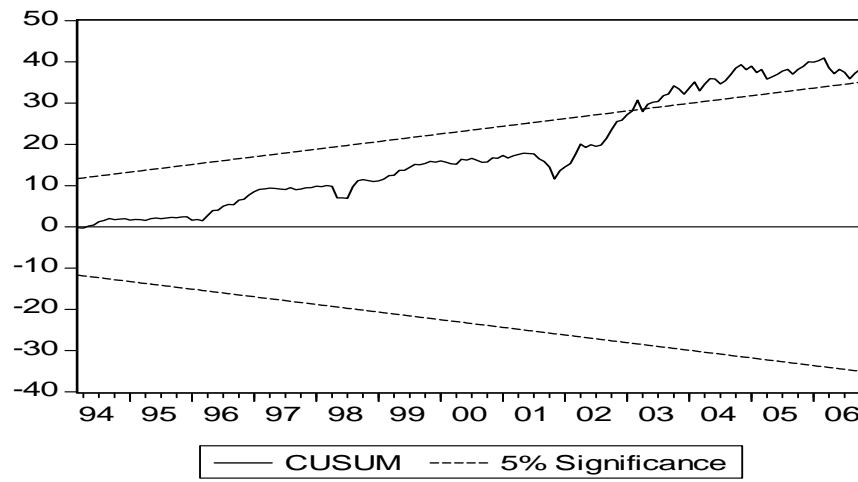


The test finds parameters instability as the CUSUM goes outside the 5% critical lines

**Figure A2 Transition function of Equation (4) as a function of transition variable**
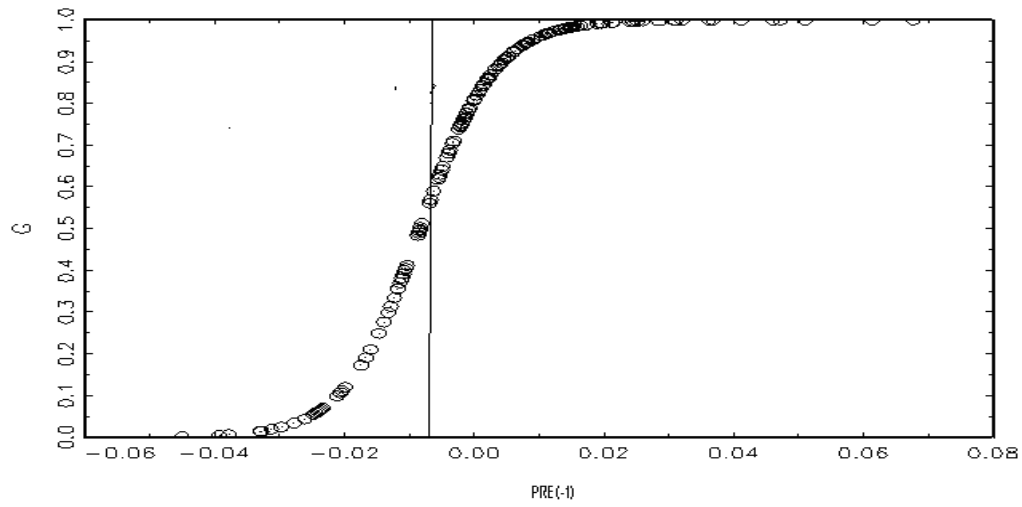


Note: Each dot corresponds to at least one observation. The vertical line shows the observations in the middle regime, where $pre(-1) = c$ and $G(\gamma, c, pre(-1)) = 0.5$. The upper regime is at the right of the vertical line while the lower regime is at the left of the vertical line

**Figure A3 Actual and predicted future spot rates obtained form the LSTR (1), OLS and random walk models (out-of- sample)**
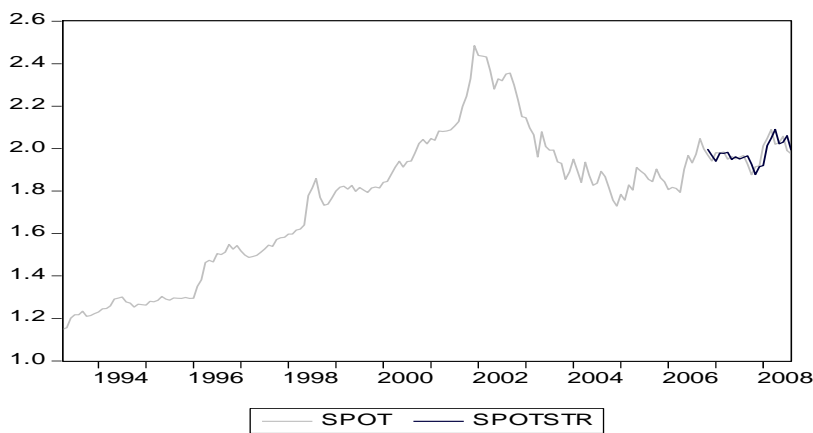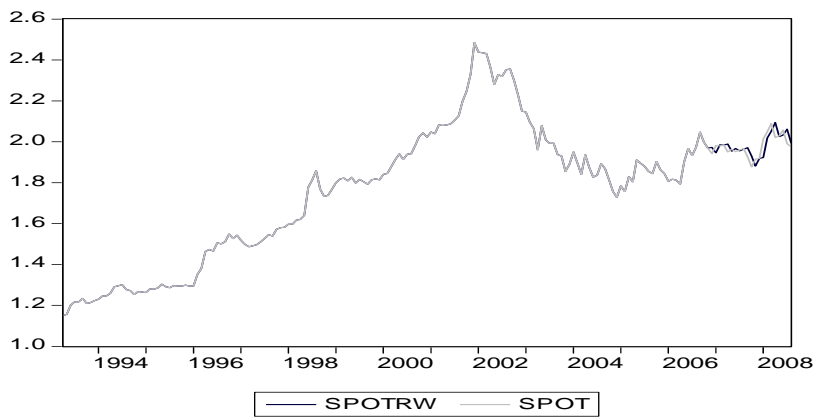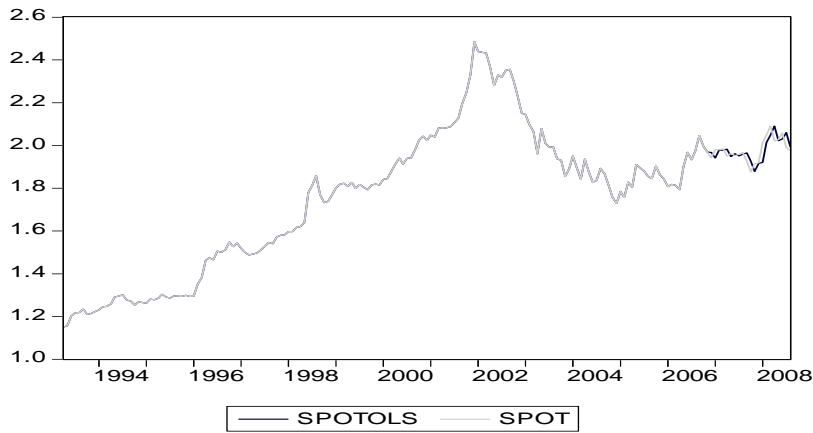


Note: SPOTOLS, SPOTRW and SPOTSTR are the predicted future spot rates generated by the OLS, random walk and LSTR models, respectively. SPOT is the actual future spot rate.