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Households' Portfolio Diversification

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Abstract

This paper performs an efficiency analysis of households portfolios based on the comparison of observed portfolios with the mean-variance frontier of assets returns. Data on household portfolios are drawn from the 2001 Centro Einaudi survey, a representative sample of the Italian population with at least a bank account. We find that most households' portfolios are extremely close to the efficient frontier once we explicitly take into account no short-selling constraints, while the null hypothesis of efficiency is rejected for all portfolios if we don't consider these constraints.

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1. Introduction

This paper performs an efficiency analysis of the portfolios of Italian households based on the comparison of observed portfolios with the *mean-variance frontier* (or *efficient frontier*) of assets returns. Given a set of assets, the mean-variance frontier is the boundary of the set of mean and variances of the returns on all possible portfolios, and is found by minimizing the portfolio variance for any given mean return.¹ Portfolios laying on this frontier are considered efficient in view of the fact that they have the highest expected return among all portfolios with an equal or lower variance.

The efficient frontier is computed using the time series of the main financial assets available to households. We then use expected returns and risk profiles of the assets to locate observed individual portfolios in the mean-variance space. Data on household portfolios are drawn from the 2001 Centro Einaudi Survey on households' savings, a representative sample of the Italian population with at least a bank account. We can therefore measure the distances between households' portfolios and the efficient ones, and analyze the relation between these distances and household characteristics (education, background risk, income and wealth levels, demographic variables).

In Section 2 we briefly summarize the Italian macroeconomic background in which households portfolio choice took place in the year 2001, describe our dataset and estimate household direct and indirect stock market participation. Section 3 presents the analysis of portfolio diversification. We describe the degree of diversification of households' portfolios and the role of individual characteristic. We then construct different mean-variance frontiers and analyze the distance between efficient and households portfolios. The final section summarizes our results.

2. Stockholding in Italy

The portfolio composition of Italian households has experienced dramatic changes over the past decade. In the early 1990s, bank accounts and Italian Treasury Bills represented two thirds of households' financial wealth whereas in 2001 their share was less than one third. Contemporaneously the decade has shown a sharp increase in stockholding, funds and other forms of managed investment (like life insurance and retirement funds): nowadays these assets represents more than 60% of households' financial wealth. The portfolio shift toward direct and indirect stockholding has its origins in the reduction of the Italian T-Bills rate of return, in the changes of the social security system that lowered the expected future income of workers, in the public offer of shares by formerly state owned companies, in the Italian stock

¹ See Merton (1972) and Roll (1977) for detailed treatments.

market growth.² The increasing role of stock market investment is due not only to the increase in the participation rate, but also (and in equal proportion) to a sharp increase in the shares of wealth invested (Guiso and Jappelli, 2003). Moreover, the participation wide spreading represents one of the most interesting phenomenon in the recent history of the Italian financial market.

The increased popularity of the stock market has been favored by several major changes: (1) the increase in competition among companies offering managed investment services, private banking, brokerage and financial advice (the increased competition has caused a significant reduction in entry costs and fees); (2) the reduction in financial information costs; (3) the availability of new financial products; (4) more generally, a real cultural shift among savers.

2.1. Stock market participation

The 2001 Centro Einaudi dataset consists of a sample of 1080 households that reported the ownership of at least a bank account or any kind of financial asset. The dataset contains detailed information on individual participation to financial markets, on portfolio shares and on amounts invested. In particular, the interviewed households were asked to report:

- ownerships of 23 different categories of real and financial assets;
- shares of total household wealth invested in each asset (divided in the same 23 categories);
- amounts invested and, when the amount was not reported, the ratio between total wealth and yearly income (in 12 brackets);
- monthly family income asking both the amount and 16 different income brackets (if none of the previous question was answered, two additional information were available: whether the monthly income was above or below the equivalent of 2500 and 1500 euro.

All households in the sample (1080) reported information on income and financial assets ownership. Data on the portfolio shares are available for 858 observations. Information on the amounts invested is available for 753 observations. The difficulty of obtaining good microeconomic data on private wealth, and the households' unwillingness to report information on their financial investment, are well known phenomena not specific to the Italian context (Miniaci and Weber, 2003). In this concern the Centro Einaudi dataset is not an exception. Overall, the analysis of the unreported answers and a comparison with the Bank of Italy dataset reveals a greater reliability of the ownership data than the data on portfolio shares and amounts invested.

It has to be mentioned that the dataset is not representative of the Italian population but only of the fraction that owns at least a bank accounts or any kind of financial asset.

² The recent years have been characterized by several changes in the pension policy that will reduce retirement benefits. As an outcome, households have now a strong incentive to increase savings and investment to counterbalance the reduction of their expected future income.

According to the 2000 Bank of Italy Survey of Income and Wealth (SHIW), only 73% of Italian households owned at least one of these forms of investment. Moreover, the average yearly income of these households was 28,000 euro, while the average income of households that held neither financial assets nor banks accounts was 12,400 euro. Since the first quartile of the income distribution in the Bank of Italy dataset is 13,500 euro, our dataset disregards households in the first quartile of the income distribution. This is not a weakness but a strength of the Centro Einaudi dataset, since it allows us to produce a detailed analysis of the investment behavior of the relatively richer, potentially more interested in portfolio diversification, and better informed group.

In what follows, assets are split in real and financial. Financial assets are categorized in seven groups: deposits, short term fixed income, long term fixed income, Italian stocks, foreign stocks, managed investment with equities, life insurance and retirement funds. With respect to other microeconomic datasets, the most important information are the ones on managed investment accounts and mutual funds since the high level of disaggregation allows us to ascribe them to the seven categories considered.

Table 1 (last column) reports the shares of households that: invest directly on the stock market in Italy or abroad (19%); put part of their wealth in managed investment with stock market participation (17.4%); participate directly or indirectly to the stock market (28.7%).

Table 1. Stock market participation rates by education level of the household head

	Less than high school	High school degree	College degree or more	All sample
Stocks	0.1762	0.1964	0.2222	0.1898
Managed investment with stock market participation	0.1669	0.1889	0.1597	0.1740
Stocks or managed investment with stock market participation	0.2745	0.2972	0.3055	0.2870
Sample share	0.4990	0.3675	0.1333	1.0000

As mentioned, to compute the participation rate in the total Italian population, the estimated rates are multiplied by a correction factor of 0.73. Therefore, the year 2001 estimates of the Italian households' stock market participation rates 14% for direct participation, 13% for indirect participation and 21% for both direct and indirect participation.

Due to the cited improvements of the nineties, the Italian stock market participation rate has remarkably increased. Even though the Italian participation rate is still lower than the US or UK ones, it is nowadays in line with the major European countries. Guiso, Haliassos and Jappelli (2003) estimate that the direct and indirect participation rate are 23% in France, 19% in Germany, 33% in Netherlands, 49% in U.S., and 45% in U.K.

Under a set of hypothesis (among which the existence of a risk free asset, the possibility of short selling, and the absence of transaction costs, entry costs, information asymmetry, and minimum investment size) finance theory predicts that the portfolio of each investor should be fully diversified. The two funds separation principle applies: each portfolio should be a combination of a risk free asset and a set of risky assets. Moreover, the share of risky assets should depend on the coefficient of risk aversion. The empirical evidence that only a limited

fraction of households hold stocks in their portfolios, points to the lack of realism of one or more of the cited assumptions. The analysis of market participation, portfolio composition and efficiency frontier, helps shading light on this *stockholding puzzle*. The analysis reported in this paper is able to reconcile part of the puzzle, and to indicate further directions of research.

2.2 Stock holders' characteristics

The relation between stock market participation and households' socioeconomic characteristics provides a first check of the mentioned assumptions and resulting implications. Table 1 reports stock market participation rates by level of education of the household head. Participation rates, even though not always statistically different, show a positive correlation between level of education and participation. This is a common empirical finding not peculiar to the Italian case. This evidence supports the thesis of an important role for the information treatment process, and its opportunity costs, in consumption and portfolio choices (King and Leape, 1987; Sims, 2003). Moreover, the table shows that to higher levels of education correspond significantly higher levels of direct investment that, by nature, requires a more frequent and intense information treatment, and more complex decisions, than the indirect investment through mutual funds and managed investment.

Table 2. Stock market participation rates by financial wealth quintiles

	Quartile I	Quartile II	Quartile III	Quartile IV	Top 5%
Stocks	0.0274	0.2527	0.2365	0.2303	0.2777
Managed investment with stock market participation	0.0439	0.2087	0.1989	0.2359	0.3333
Stocks or managed investment with stock market participation	0.0549	0.3681	0.3548	0.3595	0.4722

Table 2 looks at the correlation between participation and financial wealth. The first four columns report the participation rates by financial wealth quartiles and the last column reports the participation rates of the richer 5% of the sample. While the last three quartiles do not present significant differences, the first one appears to be strikingly dissimilar. In the first quartile, both direct and indirect participation rates are under the threshold of 5%, supporting the hypothesis of an important role for fixed participation costs and minimum investment size in limiting the access to both the stock market and the managed investment. At the other extreme, the participation of the richer 5% of the sample is extremely high: 47.2% versus the 35.9% of the fourth quartile if we consider both direct and indirect participation. The result seems to be coherent with the theories that underline the peculiarity of the portfolio choice of the richest (Carroll, 2001). The participation rates by income level (not reported) present similar evidences.

Obviously, portfolio choice is only one of the many sources of uncertainty for households. The literature on consumption and saving under uncertainty suggests that economic agents, when facing several sources of risk among which some cannot be hedged (like unemployment risk), should have the tendency to reduce the exposure to avoidable risks (like portfolio risk) even if the risks are independent (Gollier, 2003). Therefore, households with higher levels of

uninsurable income risk (like self-employed and professionals) should reduce their exposure to financial risk. Moreover, this effect should be stronger when the income risk is positively correlated with the portfolio risk. For example, because many aggregate shocks have symmetric effects on the marginal productivity of labor (and therefore salaries) and on the marginal productivity of capital (and therefore on the stock market returns).³ Some of these hypotheses have received empirical support from the work of Guiso, Jappelli and Terlizzese (1996) that uses individual expectation of the income risk available in the 1989 Bank of Italy dataset.

Table 3. Stock market participation rates by occupation of the household head

	Entrepreneurs and professionals	Self-employed	Employed	Retired	Entrepreneurs, professionals and self-employed	Employed and retired
Stocks	0.1527	0.1122	0.2217	0.2151	0.1294	0.2196
Managed investment with stock market participation	0.1944	0.0816	0.1915	0.1940	0.1294	0.1923
Stocks or managed investment with stock market participation	0.2847	0.1530	0.3306	0.3122	0.2088	0.3246
Sample share	0.1333	0.1814	0.4592	0.2194	0.3148	0.6787

Table 3 reports the participation rates for the following work category: entrepreneurs and professionals, self-employed, employed, retired. The chosen categories correspond to different levels of income risk: higher for entrepreneurs, professionals and self-employed; lower for employed and retired (for the last category the risk is virtually null but, given the mechanic of the Italian pension system, there is a “political” risk on future streams of income). In most cases, the participation rates are statistically different and in line with the theory of consumption and saving under uncertainty with multiple risks: the data show a strong inverse correlation between income risk and undertaken portfolio risk. For example, direct and indirect participation is of the order of 20.9% among entrepreneurs, professionals and self-employed, while it rises to 32.5% for employed and retired.

King and Leape (1987) claim that, since the learning process of financial market mechanisms is correlated with age, stock market participation and portfolio diversification should increase over the life cycle. Furthermore, because of liquidity constraints, more likely to bind in the earlier stages of the life cycle, younger households should likely invest in more liquid and less risky assets (Paxson, 1990). The opposite argument is presented in Bodie,

³ The empirical analysis of Baxter and Jermann (1997) claims that labor income risk is highly correlated with returns on domestic assets and much less with foreign assets. Julliard (2002) points out that Baxter and Jermann results are driven by an econometric misspecification rejected by the data. Once the misspecification is corrected, the country level correlations are strongly reduced while the cross-country correlations increase sharply. The paper argues that this could be due to a high level of international technological integration and to redistributive shocks at a country level.

Merton and Samuelson (1998). They underline that the higher labor flexibility of younger households should naturally provide greater diversification opportunity and higher margin to sustain negative shocks. Consequently, younger investors should be able to undertake more risky investment. Moreover, holder investors, having a shorter life horizon, should have fewer opportunities to diversify shocks over time (see also Gollier and Zeckhauser, 1997).

Table 4. Stock market participation rates by age group of the household head

	<30	30-39	40-49	50-59	60-69	>70	Total
Stocks	0.1951	0.1746	0.1866	0.1865	0.2101	0.2258	0.1898
Managed investment with stock market participation	0.2195	0.1746	0.17	0.1641	0.1910	0.1612	0.1740
Stocks or managed investment with stock market participation	0.2926	0.2976	0.2766	0.2686	0.3057	0.3225	0.2870
Sample share	0.0583	0.2129	0.2777	0.2481	0.1453	0.0574	1.0000

Table 4 presents participation rates by age category of the household head. In most of the cases rates are not statistically different from one category to the other. Therefore, the data do not seem to offer empirical support to any of the cited theories.

The analysis reported up to now can be summarized by a probit regression for direct and indirect stock market participation. The variables used as regressors are age, education, household size, number of individuals receiving labor income and area of residence. In order to avoid losing almost one third of the observations, we use income instead of wealth as explanatory variable. Among the regressors we have also a direct measure of individual relevance of the income risk: the variable “unemployment risk” takes value 1 if the interviewed household head stated that he/she “thinks often or sometimes to the possibility of becoming unemployed”, and 0 otherwise. Means of the regressors are reported in Table 5 and the probit regressions are reported in Table 6.

Even though few coefficients are statistically different from zero, the sign of the coefficients in Table 6 is generally coherent with the descriptive analysis. In particular, the dummy variables for the last two income quartiles are positive and statistically different from zero, suggesting that entry costs do matter. Self-employed, professionals and entrepreneurs have roughly a 10% lower probability of investing in the stock market than employed and retired, coherently with the literature on the background risk. Even if not statistically significant at a 5% level, the dummy for self-reported unemployment risk is negative in all the regressions (and significant at a 10% level).

Table 5. Demographic characteristics by stockholding categories

	Stocks	Managed investment with stock market participation	Stocks or managed investment with stock market participation	Do not invest in stocks
Age	48.88	48.37	48.51	48.21
Male	0.65	0.61	0.62	0.66
Single	0.22	0.25	0.23	0.23
Married	0.66	0.62	0.65	0.66
Family size	2.88	2.78	2.83	2.90
Labor income recipients	1.71	1.71	1.72	1.79
Employed	0.54	0.52	0.53	0.43
Self-employed	0.21	0.23	0.23	0.35
Retired	0.25	0.25	0.24	0.21
Resident in the North	0.53	0.60	0.58	0.57
Resident in the Center	0.25	0.22	0.23	0.20
Resident in the South	0.22	0.18	0.19	0.23
Disposable income (monthly)	4626	4190	4404	4407
Financial wealth	76496	84281	79182	57537
Real wealth	76311	65308	70880	112636
Sample share	0.19	0.17	0.29	0.71

Table 6. Stock market participation decision

	<i>Direct participation</i>	<i>Indirect participation</i>	<i>Direct or indirect participation</i>
Male	-0.00069 (0.02586)	-0.01807 (0.024927)	-0.02097 (0.030308)
Married	-0.00366 (0.029068)	-0.02201 (0.028222)	0.000697 (0.033708)
35 < Age < 60	-0.0027 (0.029154)	-0.01384 (0.028005)	-0.03455 (0.033909)
Age ≥ 60	0.015272 (0.03802)	-0.01903 (0.034208)	-0.02495 (0.042254)
Family size	0.010766 (0.013472)	-0.00114 (0.013198)	0.004442 (0.01589)
Labor income recipients	-0.04297 (0.019544)*	-0.02492 (0.018791)	-0.04459 (0.02268)*
High school degree	0.006129 (0.027375)	-0.00291 (0.026043)	-0.0058 (0.031738)
College degree	0.027894 (0.042889)	-0.05282 (0.03415)	-0.01686 (0.046872)
Second income quartile	0.001241 (0.035195)	0.034349 (0.035801)	0.033177 (0.041688)
Third income quartile	0.077608 (0.039833)*	0.075057 (0.039181)*	0.116502 (0.045064)**
Forth income quartile	0.100329 (0.044671)*	0.072876 (0.043274)	0.109453 (0.049419)*
Entrepreneurs and professionals	-0.08739 (0.029841)*	-0.00076 (0.035903)	-0.05805 (0.040924)
Self-employed	-0.1004 (0.026649)**	-0.11457 (0.024321)**	-0.17094 (0.031125)*
Resident in the Center	0.049744 (0.03234)	-0.00404 (0.028796)	0.022985 (0.035909)
Resident in the South	0.012797 (0.032869)	-0.02128 (0.03024)	-0.02471 (0.037047)
Unemployment Risk	-0.04433 (0.024618)	-0.01728 (0.024033)	-0.04719 (0.029046)
Number of observations	1080	1080	1080

The table reports probit regressions for stock market participation. Standard errors in parenthesis.

(*) Statistically significant variable at 5% level. (**) Statistically significant variable at 1% level.

3. Portfolio diversification

The last column of Table 7 reports sample averages of financial portfolios' shares. More than half of the financial wealth (56.9%) is invested in checking and saving accounts. Fixed income assets represent one fifth of the assets and the total share invested in stocks is 19.1%, while life insurances and complementary retirements plans sum up to only 3.7% of the financial wealth.

Table 7. Portfolio shares by education level of the household head

	Less than high school	High school degree	College degree or more	All sample
Deposits	0.5929	0.5501	0.5284	0.5689
Short term fixed income	0.1282	0.1571	0.1437	0.1406
Long term fixed income	0.0605	0.0594	0.0725	0.0617
Italian stocks	0.0765	0.0770	0.1129	0.0816
Managed investment with stock market participation	0.1010	0.1059	0.0900	0.1013
Foreign stocks	0.0072	0.0097	0.0077	0.0082
Life insurance and complementary retirement plans	0.0337	0.0407	0.0448	0.0377
Total	1.0000	1.0000	1.0000	1.0000

The portfolio share of foreign stocks deserves a special mention. In all the conditional distribution considered in the paper, it never exceeds the threshold of 2% and the sample mean is less than 1%. This finding, familiar to many others (not only Italian) empirical analyses, contradicts the theory of international portfolio diversification and is commonly indicated as the *international diversification puzzle*. International finance emphasizes the effectiveness of global diversification strategies for cash-flow stabilization and consumption risk sharing.⁴ Therefore, the theory suggests that households, in order to reduce the overall level of risk of their portfolios, should undertake international portfolio diversification. However, empirical evidences on international portfolio positions conclude in favor of a widespread lack of diversification across countries. Our finding is therefore coherent with this empirical literature in showing a strong home country bias in stockholding (in our sample, the mean ratio of foreign stocks to total stockholding is only 4.3%). It has to be mentioned that, since mutual funds may invest in foreign stocks, our analysis tends to underestimate the

⁴ Nevertheless, the size of gains from international risk sharing continues to be a debated issue. Grauer and Hakansson (1987) suggest that an individual's gains from international stock portfolio diversification are large. Cole and Obstfeld (1991) find small gains from perfect pooling of output risks. Obstfeld's (1994) calibration exercises imply that most countries reap large steady-state welfare gains from global financial integration.

investment in foreign stocks because we don't have a sufficient disaggregation of these indirect investment channels. This is due to the fact that the Centro Einaudi dataset differentiates between mutual funds that invests in stocks and mutual funds that do not invest in stocks, but in the former category does not distinguish between funds that invest in domestic and funds that invest in foreign stocks. For the same reason, we likely overestimate the portfolio share invested in domestic stocks through the indirect channel since we consider all mutual funds that invest on the stock market as indirect investment in domestic stocks.

Even if we conjecture that self-reporting gives a downward bias to our estimate of the portfolio share of stock market investment, the share of risky assets (given by the sum of direct and indirect investment in stocks) seems too low to be reconciled with portfolio theory. In order to justify such a portfolio composition, a level of relative risk aversion of at least 10 is needed, and this is often considered too high to sound reasonable. Gollier (2003) outlines that an economic agent with a relative risk aversion of 10 would prefer to lose with certainty 24.4% of his wealth instead of undertaking the risk of winning or losing, with equal probability, a 30% share of his wealth! This evidence is just another aspect of the *stockholding puzzle* already discussed in the section on stock market participation.⁵

Table 7 also reports the portfolio shares by education level of the household head. The data show a positive correlation between level of education and degree of portfolio diversification. The portfolios of households characterized by higher education levels have a larger share of risky assets and stocks. Moreover, more educated households seem to prefer direct to indirect stock market participation. These results are in line with the ones on participation and education reported in Table 1 and with what we mentioned about information treatment.

Gollier (2003) outlines that the relation between the portfolio share of risky assets and the level of households wealth gives indirect information on the households preferences. If the level of relative risk aversion is independent of the wealth level, the portfolio share of risky assets has to be independent of wealth too. Instead, if the relative risk aversion decreases with wealth, the portfolio share of risky assets should increase with wealth.

The first four columns of Table 8 show the portfolio composition by financial wealth quartiles and the last column reports the portfolio composition of the richest 5% of the sample. Like in Table 2, there are not significant differences among second, third and fourth quartiles. The first quartile is instead characterized by a strong preference for more liquid and less risky forms of investment: almost two third of the financial wealth is invested in deposits (while deposits represent only 53.9% of the financial wealth for the second quartile and

⁵ In the static model of portfolio choice, if we assume that the stock market returns are lognormally distributed, the portfolio share invested in risk assets (ω) is given by:

$$\omega = \frac{1}{\rho} \frac{E(z-r)}{\text{var}(z-r)} (1+r)$$

where ρ is the relative risk aversion coefficient, $E(z-r)$ is the equity premium and $\text{var}(z-r)$ is the variance of the risk premium. During the period 1984-2000 the equity premium has been 0.03, $\text{Var}(z-r) = 0.019$ e $(1+r) = 1.04$, where r is the real risk free rate (the short-term Italian T-bill rate). This equation, given the share of risky assets observed in households portfolios in our dataset, delivers an implicit measure of relative risk aversion of 10.

41.6% for the top 5% of the wealth distribution). The total portfolio share of stocks in this quartile is only 12.2% (20.2% for the second quartile). The top 5% of the wealth distribution present a much more diversified portfolio and a higher propensity to stockholding (27.6% versus the 20.9% of the fourth quartile as a all).

Table 8. Portfolio shares by financial wealth quintiles

	Quartile I	Quartile II	Quartile III	Quartile IV	Top 5%
Deposits	0.6586	0.5394	0.5649	0.5159	0.4165
Short term fixed income	0.1148	0.1544	0.1261	0.1648	0.2142
Long term fixed income	0.0585	0.0725	0.0563	0.0825	0.0646
Italian stocks	0.0453	0.0956	0.0932	0.0844	0.1211
Managed investment with stock market participation	0.0761	0.0999	0.1074	0.1178	0.1507
Foreign stocks	0.0009	0.0069	0.0101	0.0064	0.0040
Life insurance and complementary retirement plans	0.0458	0.0312	0.0421	0.0280	0.0289
Total	1.0000	1.0000	1.0000	1.0000	1.0000

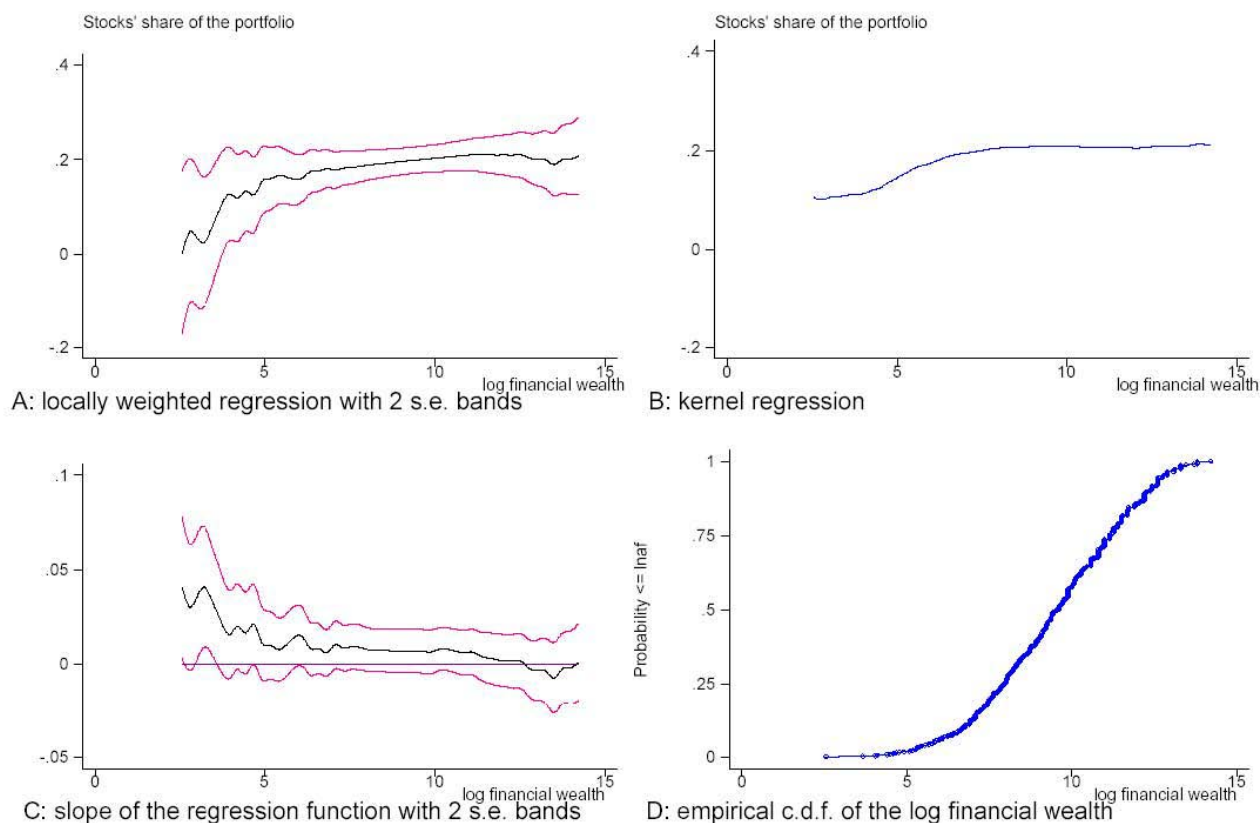
Even though few coefficients are statistically different from zero, the sign of the coefficients in Table 6 is generally coherent with the descriptive analysis. In particular, the dummy variables for the last two income quartiles are positive and statistically different from zero, suggesting that entry costs do matter. Self-employed, professionals and entrepreneurs have roughly a 10% lower probability of investing in the stock market than employed and retired, coherently with the literature on the background risk. Even if not statistically significant at a 5% level, the dummy for self-reported unemployment risk is negative in all the regressions (and significant at a 10% level).

Figure 1 presents a nonparametric analysis of the relation between total portfolio share of stocks and total financial wealth. Panel A reports a locally weighted regression of the stocks shares on the natural logarithm of financial wealth with a 95% confidence interval computed by bootstrap method. Panel D reports the empirical cumulative distribution function of log financial wealth. Considering together panel A and D it seems evident that the portfolio share of stocks increases significantly only in the first part of the wealth distribution while it is roughly constant in the higher three quartiles of the distribution.

Panel B reports a kernel regression of stocks shares on the log financial wealth. The kernel regression confirms the results of the locally weighted regression except for the first part of the distribution, but this is due to the well know bias of the kernel regression on the extremes of the support of the regressor. Panel C formally tests the hypothesis that the portfolio share of risky assets is constant over wealth levels. The figure reports the slope of the regression function in Panel A with a 95% confidence interval computed by bootstrap method. Under the null of constant share of risky assets, the slope should be zero at each wealth level. The null hypothesis cannot be rejected for the higher three quartiles of the wealth distribution. The null hypothesis cannot be rejected for the higher 99% of the wealth distribution. These evidences suggest that the hypothesis of constant relative risk aversion can be rejected only for the

lowest 1% of the income distribution (but this rejection may be due to other factors like liquidity constraints).

Figure 1. Stocks' share of the portfolios and total financial wealth



Note. Panel A reports a locally weighted regression of stocks' shares on the natural logarithm of financial wealth (quadratic kernel with bandwidth = 3.3) and two standard error confidence bands computed by bootstrap method. Panel B reports a kernel regression of stocks' share on log financial wealth (Epanechnikov kernel with bandwidth = 2.3). Panel C reports the slopes of the regression function in Panel A with two standard error confidence bands, and the line correspondent to the null hypothesis of constant relative risk aversion. Panel D reports the empirical cumulative distribution function of the log of the financial wealth.

Consistently with the evidences on stock market participation, Table 9 shows that the households with a higher income risk (entrepreneurs, professionals and self-employed) invest in stocks a significantly smaller share of their wealth than employed and retired (16.7% versus 20.3%). Table 10 reports the portfolio shares by age category of the household head. Most of the differences are not statistically significant. In particular, the share of direct and indirect stockholding is roughly constant (19%). This is in line with the empirical evidence brought by the SHIW dataset (Guiso and Jappelli, 2001).

The main evidence brought by the analysis of stock market participation and portfolio diversification is that portfolio choices are strongly influenced by the household's socio-economic characteristics. In particular, education level, work category and wealth seem to play a key role. Moreover, background risk, entry costs and participation costs seem to discourage stockholding.

Table 9. Portfolio shares by work category of the household head

	Employed and retired	Entrepreneurs, professionals and self-employed
Deposits	0.5622	0.5819
Short term fixed income	0.1400	0.1398
Long term fixed income	0.0611	0.0630
Italian stocks	0.0890	0.0663
Managed investment with stock market participation	0.1059	0.0932
Foreign stocks	0.0085	0.0076
Life insurance and complementary retirement plans	0.0333	0.0483
Total	1.0000	1.0000

Table 10. Portfolio shares by age category of the household head

	<30	30-39	40-49	50-59	60-69	>70	Total
Deposits	0.4886	0.5350	0.5740	0.5910	0.5570	0.6293	0.5689
Short term fixed income	0.2269	0.1347	0.1536	0.1178	0.1529	0.1094	0.1406
Long term fixed income	0.0225	0.0846	0.0548	0.0717	0.0439	0.0386	0.0617
Italian stocks	0.0643	0.0855	0.0840	0.0774	0.0803	0.0884	0.0816
Managed investment with stock market participation	0.1356	0.1278	0.0839	0.0792	0.1213	0.0958	0.1013
Foreign stocks	0.0295	0.0083	0.0029	0.0089	0.0080	0.0136	0.0082
Life insurance and complementary retirement plans	0.0325	0.0242	0.0468	0.0450	0.0366	0.0248	0.0377
Total	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

The level of detail on portfolio's composition available in the Centro Einaudi dataset allows us to compute the expected risk and return of each individual portfolio. Using this feature, we ask in the next paragraphs whether the households portfolios are "efficient" or not.

3.1. The efficient frontier

The efficient frontier is given by the set of portfolios that, for any given expected return, have the minimum level of risk (measured by the variance of the returns).⁶ From the space of possible investments, we decide to exclude the real activities (that have as main component the home where the household lives). The reasons for this choice are the followings. First, residential housing is not only an investment choice that allows transferring purchasing power through time. In most of the cases it is a direct source of utility. Therefore, considering only the financial return, underestimates the impact on the household's utility of this kind of investment. Second, differently from financial assets, measuring the return on real assets is not simple since it often depends on the local condition of the real estate market. Third, often owners do not consider the value of the real estate as a part of their disposable wealth (for example because of the transaction and search costs connected with selling and buying residential housing).⁷ The first step of this part of the analysis will be the construction of the efficient frontier using time series of financial assets available to Italian investors. Then, we will compare the efficient portfolios with the ones chosen by the households in our sample.⁸ The construction of the efficient frontier is based on the hypothesis, used to estimate expected returns and their covariance matrix, that the time series of returns have been generated by a stationary stochastic process (see Hansen and Jagannathan, 1985). The time series are monthly data over the period October 1978 - December 2000. It has to be mentioned that none of the results is qualitatively different using only data after 1990 or after 1996. The details on the construction of the efficient frontier and on the series used are reported in the appendix.

In order to match the analysis to the available macroeconomics time series on assets returns, we have chosen to aggregate further the information on portfolio holdings available in the Centro Einaudi dataset into five categories: deposits, short term fixed income, long term fixed income (including life insurances and complementary retirement plans), Italian stocks (direct and indirect holding), foreign stocks.⁹ These aggregated categories are treated as macro

⁶ See, among many others, Cochrane (2001).

⁷ See Cocco (2000) on portfolio choice with real activities.

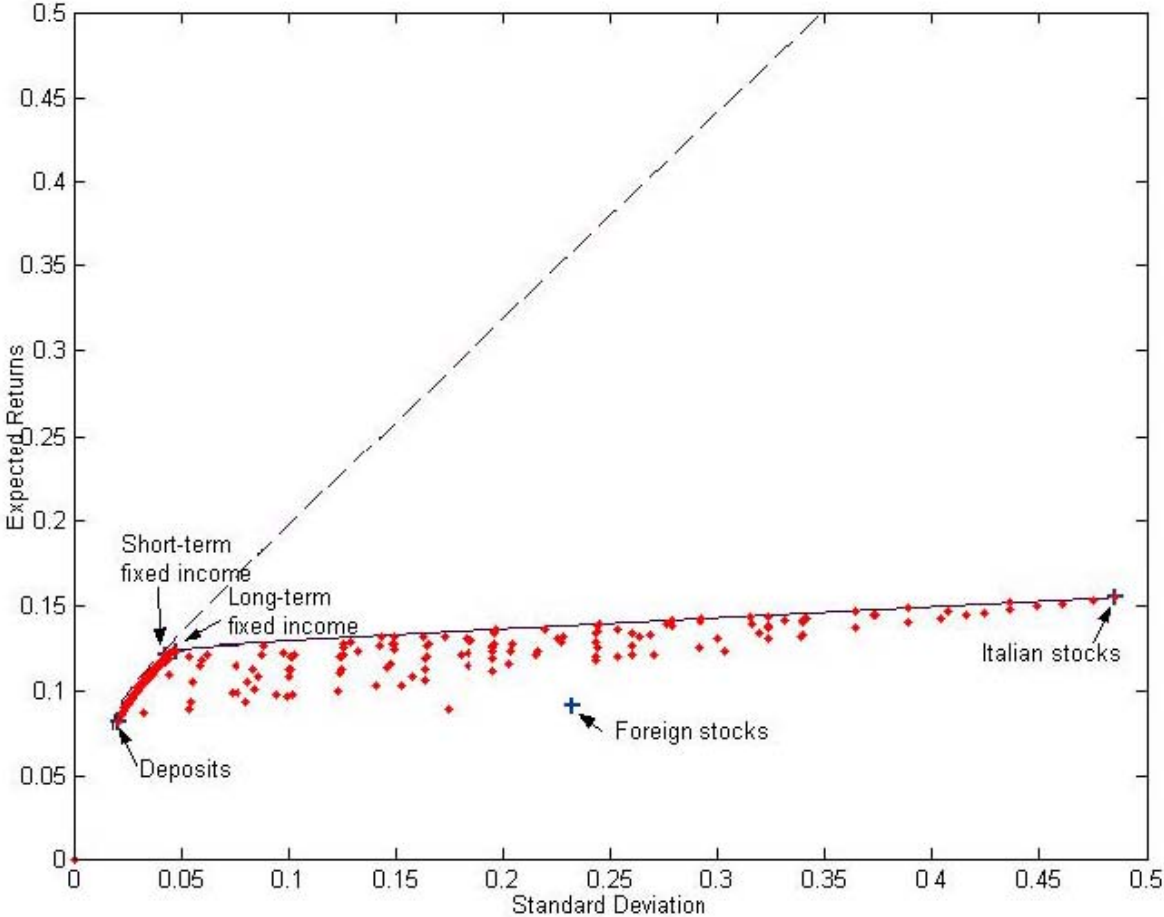
⁸ Blake (1996), with UK data, constructs the efficient frontiers with 3 assets (deposits, treasury bills and stocks), and each asset is considered risky. After computing the risk and return profiles of 6 portfolios (one for each income category), he concludes that the portfolios are efficient. The strength of our analysis, compared to Blake's one, is that we use microeconomic data on household portfolios. Blake's approach does not consider that not all the family invest in each asset. Moreover, our approach allows us to study the characteristics of the efficient households and of the inefficient ones.

⁹ The choice of adding life insurance and complementary retirement plans to long term fixed income is due to the institutional constraints that bind Italian insurance companies to invest most of their funds in assets belonging to this category. Nevertheless, the choice would be inappropriate for *unit linked* and *index linked* life insurance (the returns on this kind of insurance is linked to the stock market performances). Unfortunately, the Centro Einaudi dataset does not differentiate between traditional insurance policies, *unit linked* and *index linked*. However, even ascribing life insurance to the stock market assets category, the results are qualitatively unchanged.

assets, each with his own time series of returns, among which investors can chose their portfolio composition.

Figure 1 reports returns and standard deviation of the five macro assets considered. The mean returns are 8% for deposits (0.02 of standard deviation), 9.1% for foreign assets (0.23 of standard deviation), 15.4% for Italian stocks (0.48 of standard deviation), and short and long term fixed income have intermediate figures of expected return and risk. Each of the five assets is considered risky even if the risk is extremely low for deposits and high for Italian stocks.

Figure 2. Efficient frontier and households' portfolio choice



Note. The dotted line represents the efficient frontier estimated without imposing short selling constraints. The solid line is the efficient frontier estimated with short selling constraints. The scatter plot represents Italian households' portfolios. The points corresponding to deposit, short and long term bonds, Italian and foreign stocks, represent the expected returns and standard deviations of portfolio composed only by that single asset.

The dotted line in Figure 1 represents the efficient frontier. It gives the portfolios that, for each level of risk, have the higher expected returns. This efficient frontier is constructed allowing for short selling i.e. negative portfolio shares are allowed by construction. Dealing with households data this hypothesis is not realistic. If for institutional investors it is relatively simple to borrow at low risk to invest in assets with higher expected return (and higher risk), it is hard to think that households could act the same way (for example borrowing from their

banks to invest on the stock market). The solid line in Figure 1 represents the efficient frontier computed imposing no short selling constraints. The effect is remarkable and strongly reduces the space of returns and risks available to the investors. Explicitly considering no short selling constraints has three main effects. First, it helps explaining limited stock market participation (therefore, it helps understanding the *stockholding puzzle*). Second, it partially justifies the low average level of risky assets in households' portfolios. Third, it reconciles the efficient portfolios with the observed ones.

Notice that the no short selling constraint has a clear effect on investors' welfare. According with the theory, investors choose their preferred portfolio on the efficient frontier according with their preferences (aversion) toward risk. Utility is higher the closer the portfolio is to the northwest corner of the return-risk space i.e. given an expected return, less risk is preferred, and given a risk level, higher expected returns are preferred. More risk averse investors will choose safer portfolios while less risk averse investors will choose riskier portfolios with an higher expected return.

Comparing the efficient frontiers in Figure 1, makes it clear that the welfare loss is higher for investors with lower levels of risk aversion (and therefore willing to invest a larger share of their wealth in the stock market). The welfare loss is smaller for investors with high levels of risk aversion since these investors in any case would prefer portfolios mainly composed by fixed income and deposits, like in the lower left side of Figure 1. Therefore, the no short selling constraint substantially reduces the set of available portfolios in the return-risk space, reduces the portfolio share of risky assets and households' welfare, especially for the less risk averse agents.

3.2. Are observed portfolios efficient?

In order to compare efficient portfolios with observed ones, we aggregate households' financial assets in the same categories used in the construction of the efficient frontier: deposits, short term fixed income, long term fixed income (including life insurances and complementary retirement plans), Italian stocks (direct and indirect holding), foreign stocks. As before, these categories are treated as macro assets in households' portfolios and the corresponding shares are computed following the same aggregation scheme.

Using the estimates of the first two moments of assets returns exploited for the construction of the efficient frontier, we then characterize each household's portfolio in the mean variance space. Let w_i be the vector of portfolio shares of household i , μ the vector of expected returns and Σ the assets' covariance matrix. The expected return of household's i portfolio will be given by

$$E[R_i] = w_i' \mu,$$

and the portfolio standard deviation will be

$$\sigma_i = \sqrt{w_i' \Sigma w_i}$$

Therefore, risk and expected return will be different for each household only as a function of w_i , while μ and Σ are considered constant.¹⁰ This procedure gives us a simple way to compare the risk-return profile of 853 households' portfolios (227 households don't provide data on their w_i) with the ones on the efficient frontier.

A first comparison between efficient and observed portfolios is given by Figure 2 where the scatter plot represents households portfolios. By construction, household portfolios lie under, or at most on, the efficient frontier (since the frontier is the locus of higher returns that can be achieved given μ and Σ). From the figure it is clear that household portfolios are far from the efficient frontier without no short selling constraint. Moreover, a large share of households is close to the lower part of the efficient frontier where portfolios are characterized by low risk and low expected returns. These are households that invest mainly in deposits and fixed income assets.

As measure of the distance between efficient and observed portfolios, we compute the difference in expected returns for each individual portfolio risk level i.e. the vertical distance between the points of the scatter plot and the efficient frontiers in Figure 2.¹¹ This is a simple and intuitive measure since it represents the increase in mean returns that investors could have achieved, choosing mean variance efficient portfolios, without increasing the level of riskiness of their holdings.

The median (mean) distance between individual portfolios and efficient frontier without no short selling constraints is 0.9% (9.7%). This means that, if households were allowed to short sell, they could have had a median (mean) 0.9% (9.7%) increase in expected returns without increasing the overall level of risk of their portfolios. If instead we compute the distances with respect to the efficient frontier with no short selling constraints, the median (mean) is only 0.02% (0.3%).

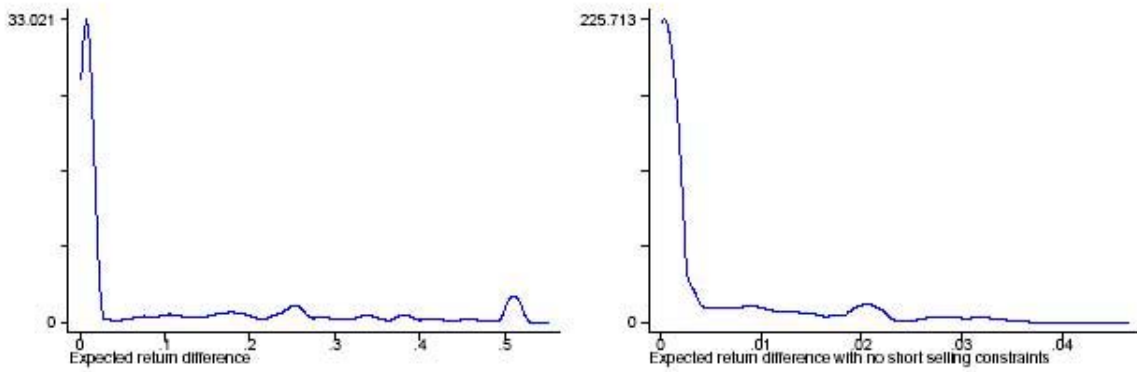
Kernel estimates of the sample distributions of the two measures of the distance are reported in Figure 3. In the case without short selling constraints, 2/3 of the households have a distance from the efficient frontier that is less than 1%. The proportion of households with a distance lower than 1% rises to 85% when we consider no short selling constraints.

Overall, has shown in Figure 3, imposing no short selling constraints reduces the support of the distribution of the distances by one order of magnitude. It is therefore evident that taking explicitly into account the no short selling constraints strongly helps reconciling individual portfolio choices with efficient ones.

¹⁰ In reality μ and Σ may change too for each household, as example because of different tax rates on households' incomes. This is not taken into account by our analysis.

¹¹ Similar results to the ones presented here are obtained using the horizontal difference i.e. the difference in standard deviation for each portfolio expected return.

Figure3. Distance between efficient frontier and households portfolios



Note. The figure reports kernel density estimates of the distance between efficient frontier and households' portfolios (Epanechnikov kernel with optimal bandwidth).

Gibbons, Ross and Shanken (1989) have developed a test of the significance of the distance between the actual portfolio held by an investor and a corresponding efficient portfolio (i.e. a portfolio that has the same expected return or the same variance of the one under test). The test is based on the difference between slopes of lines from the origin passing through the two portfolios in the mean-standard deviation space. If the observed portfolio is efficient, the two slopes will be equal. Instead, if the actual portfolio is inefficient, the slope of the efficient portfolio will be greater. Define the statistic

$$\Lambda = \frac{M(M - N - 1)}{N(M - 2)} \left[\frac{1 + \mu_e / \sigma_e}{1 + \mu_i / \sigma_i} - 1 \right]$$

where (μ_i / σ_i) is the slope of a line from the origin through the observed portfolio i , (μ_e / σ_e) is the slope of a line from the origin through the corresponding efficient portfolio, N is the number of assets in the portfolio and M is the number of time series observations used to estimate expected returns and standard deviations. Gibbons, Ross and Shanken (1989) show that, if asset returns are multivariate normal and under the null hypothesis that the actual portfolio is efficient, Λ is distributed has a F-distribution with N and $(M - N - 1)$ degrees of freedom.¹²

Table 11 summarizes the results of the efficiency test performed on the observed portfolios. If we consider the distance between actual portfolios and corresponding portfolios on the efficient frontier constructed without no short selling constraints, all the observed portfolios have a distance from the frontier that is statistically different from zero i.e. all the

¹² MacKinlay (1985) shows by simulation that this test is fairly robust even when assets returns are not normal but have distributions that are leptokurtic relative to the normal.

portfolios can be considered inefficient. If instead we consider the distances computed with respect to the efficient frontier with no short selling constraints, roughly one third of the observed portfolios have a distance from the frontier that is statistically different from zero i.e. for two thirds of the observed portfolios the null hypothesis of being efficient cannot be rejected.

If we assume that (1) investors maximize their expected utility function of wealth, (2) their utility function is characterized by constant relative risk aversion,¹³ and (3) the first two moments of the returns are well defined, then the indifference curves can be locally approximated by

$$\bar{R} = \bar{U} + \frac{1}{2} \gamma \sigma^2.$$

where \bar{R} and σ^2 are expected return and variance of a portfolio, \bar{U} is an index of expected utility, and γ is the coefficient of relative risk aversion. The households' optimality conditions imply that the indifference curve should be tangent to the efficient frontier.

Table 11. Share of inefficient portfolios

	Distance in expected returns	
	<i>Without constraints</i>	<i>With no short selling constraints</i>
At 5% significance level	100%	33.29%
At 10% significance level	100%	31.89%
Number of observations	853	853
	Distance in standard deviation	
	<i>Without constraints</i>	<i>With no short selling constraints</i>
At 5% significance level	100%	41.03%
At 10% significance level	100%	40.68%
Number of observations	853	853

We can therefore estimate the implied relative risk aversion of the households whose portfolios are not statistically different from the corresponding efficient portfolios. Table 12 reports the implied mean γ for different significance levels and measures of distance from the efficient frontier. The point estimates are all between 4 and 5 i.e. well inside the plausible range. Moreover, 83% of the efficient sub-sample displays a relative risk aversion lower than 6.

¹³ This hypothesis does not seem implausible given the results presented in Section 3.

It is interesting to study households' characteristics by different categories of distance from the efficient frontier. Table 12 reports the means of several households characteristics for the portfolios that have distances over and under 1%. Except the different share of self-employed (and retired, in the case without no short selling constraints), there are no significant differences between categories.

Table 12. Implied coefficient of risk aversion for the efficient households

	Implied mean relative risk aversion	
	<i>Distance in expected returns</i>	<i>Distance in standard deviation</i>
At 5% significance level	4.735068 (10.97054)	4.678438 (10.73108)
At 10% significance level	4.980296 (11.21315)	4.530596 (10.67393)

Note. Standard errors in parenthesis.

Table 13. Demographic characteristics by distance categories

	Distance from the efficient frontier			
	<i>Without constraints</i>		<i>With no short selling constraints</i>	
	<0.01	>0.01	<0.01	>0.01
Age	48.5779	48.7624	48.2867	50.7097
Male	64%	63%	63%	64%
Married	66%	66%	65%	73%
Family size	2.91	2.85	2.87	2.98
Labor income recipients	1.79	1.73	1.78	1.70
Entrepreneurs and professionals	12%	14%	13%	12%
Employed	44%	51%	46%	48%
Self-employed	20%	9%	18%	8%
Retired	23%	26%	22%	32%
Unemployed	1%	0%	1%	0%
Resident in the North	57%	60%	57%	60%
Resident in the Center	19%	20%	20%	19%
Resident in the South	24%	20%	23%	22%
Disposable income (monthly)	4379	4143	4389	3783
Real wealth	87786	72996	81352	91348
Financial wealth	76088	77002	76580	75293
Unemployment Risk	41%	34%	39%	35%
Sample share	67%	33%	85%	15%

These findings are confirmed by the Tobit regressions in Table 13. Most of the coefficients are not statistically different from zero. Among the significant variables, human capital risk and income level seems again to have predictive power for households' portfolio choices. The distance from the efficient frontier generally increases for poorer households and decreases for self-employed.

Table 14. Regressions for the distance from the efficient frontier

Regressors:	Distance from the efficient frontier	
	Without constraints	With no short selling constraints
Male	-0.00404 (0.011419)	0.000044 (0.00062)
Married	0.000751 (0.012769)	0.000649 (0.000694)
35 < Age < 60	-0.02803 (0.012967)*	-0.00057 (0.000706)
Age ≥ 60	-0.01887 (0.016609)	0.000865 (0.0009)
Family size	0.000558 (0.006069)	0.000684 (0.00033)*
Labor income recipients	-0.0055 (0.00855)	-0.00089 (0.000465)
High school degree	-0.00689 (0.012213)	-0.00029 (0.000662)
College degree	-0.00922 (0.018157)	-0.00113 (0.000989)
Second income quartile	0.015998 (0.015201)	-0.00225 (0.000823)**
Third income quartile	0.029905 (0.015696)	-0.00052 (0.000848)
Forth income quartile	0.032824 (0.017407)	-0.00079 (0.000941)
Entrepreneurs and professionals	0.006664 (0.017201)	-0.00073 (0.00094)
Self-employed	-0.0489 (0.015221)**	-0.00238 (0.000826)**
Resident in the Center	-0.01338 (0.013854)	0.000124 (0.00075)
Resident in the South	-0.00892 (0.014255)	-0.0008 (0.000775)
Unemployment Risk	-0.02485 (0.011239)*	-0.00045 (0.000611)
Constant	0.129547 (0.021755)	0.003954 (0.001174)
Number of observations	853	853

Tobit regression for the vertical distance between efficient frontiers and households' portfolios. Standard errors in parentheses. (*) Statistically significant variable at 5% level. (**) Statistically significant variable at 1% level.

4. Summary

The paper uses the Centro Einaudi to compare households' portfolio holdings with mean-variance efficient portfolios. The measure of the distance between individual portfolios and efficient ones used in the paper is the increase in expected returns that investors could achieve without increasing the riskiness of the portfolios. We find that most households' portfolios are close to the efficient frontier only when we take into account no short-selling constraints, while the null hypothesis of efficiency is rejected for all the portfolios if we do not consider these constraints.

Even though most of the observed portfolios are very close to efficient ones (with no short selling constraints), only a small fraction of households fully diversifies risk. Our analysis therefore suggests that participation costs, information constraints, and human capital risk reduce the propensity to invest in stocks. Moreover, no short selling constraints help explaining the *stockholding puzzle*, delivering an estimated mean relative risk aversion lower than five and offering a justification for both the low stock market participation rate and the low share of risky assets in households' portfolios.¹⁴

¹⁴ The hypothesis of constant relative risk aversion is rejected by the data only for the lowest 1% of the wealth distribution, and this rejection may be due to other factors like liquidity constraints.

References

- Baxter, M. and U.J. Jermann (1997), "The international diversification puzzle is worse than you think," *American Economic Review* 87, 170-180.
- Blake, D. (1996), "Efficiency, risk aversion and portfolio insurance: an analysis of financial asset portfolios held by investors in the United Kingdom," *Economic Journal* 106, 1175-92.
- Bodie, Z., R. Miller and W. Samuelson (1998), "Labour supply flexibility and portfolio choice in a life-cycle model," *Journal of Economic Dynamics and Control* 16, 427-49.
- Carroll, C.D. (2003), "Portfolios of the rich," in *Household Portfolios*, L. Guiso, T. Jappelli and M. Haliassos eds. Cambridge: MIT Press.
- Cochrane, J.H. (2001), *Asset Pricing*. Princeton: Princeton University Press.
- Cole H. and M. Obstfeld (1991), "Commodity trade and international risk sharing: how much do financial markets matter?" *Journal of Monetary Economics* 28, 3-24.
- Gibbons, M. R., Ross, S.A. and J. Shanken (1989), "A test of efficiency of given portfolios," *Econometrica* 57, 1121-52.
- Gollier, C. (2003), "What does the classical theory have to say about household portfolios?" in *Household Portfolios*, L. Guiso, T. Jappelli and M. Haliassos eds. Cambridge: MIT Press.
- Grauer, R.R. and N.H. Hakansson (1987), "Gains From International Diversification: 1968-1985 Returns on Portfolios of Stocks and Bonds." *Journal of Finance* 42, 721-39.
- Guiso, L., and T. Jappelli (2003), "Household portfolios in Italy, in *Household Portfolios*, L. Guiso, T. Jappelli and M. Haliassos eds. Cambridge: MIT Press.
- Guiso, L., T. Jappelli and M. Haliassos (2003), *Stockholding in Europe*. New York: Palgrave Macmillan.
- Guiso, L., T. Jappelli and D. Terlizzese (1996), "Income risk, borrowing constraints and portfolio choice," *American Economic Review* 86, 158-72.
- King, M., and J. Leape (1987), "Asset accumulation, information, and the life-cycle," NBER Working Paper n. 2392.
- Gollier, C., and R. Zeckhauser (1997), "Horizon length and portfolio risk," NBER Technical Working Paper n. 216.
- Julliard, C. (2002), "The international diversification puzzle is *not* worse than you think," mimeo, Princeton University.
- Hansen, L.P. and R. Jagannathan (1991), "Implications of security market data for models of dynamic economies," *Journal of Political Economy* 99, 225-262.
- Lucas, R.E. (1976), "Econometric policy evaluation: a critique," *Carnegie-Rochester Conferences on Public Policy*, Supplement to the *Journal of Monetary Policy*, 19-46
- Merton, R. (1969), "An analytic derivation of the efficient portfolio frontier," *Journal of Financial and Quantitative Analysis* 7, 1851-1872
- Miniaci, R. and G. Weber (2003), "Survey design and estimation of portfolio models," in *Stockholding in Europe*, L. Guiso, T. Jappelli and M. Haliassos eds. New York: Palgrave Macmillan.

- Obstfeld, Maurice (1994), "Risk-taking, global diversification, and growth," *American Economic Review* 84, 1310-29.
- Roll, R. (1977), "A critique of the asset pricing theory's tests: part I," *Journal of Financial Economics* 4, 129-176.
- Sargent, Thomas J. (1984), "Autoregressions, expectations, and advice," *American Economic Review Papers and Proceedings* 74, 408-15.
- Sims, Christopher A. (1984), "Projecting policy effects with statistical models," *Revista de Analisis Economico*, 3-20.
- Sims, Christopher A. (2003), "Implications of rational inattention," *Journal of Monetary Economics* 50.

Appendix

Construction of the efficient frontier

Let Σ be the covariance matrix of financial assets, w_i the vector of portfolio shares of each asset in the portfolio i , μ the vector of assets' expected returns, and $E[R_i]$ the expected return of a portfolio characterized by w_i . Each mean-variance efficient portfolio solves the following static optimization problem:

$$\begin{aligned} \text{Min } \sigma^2 &= w_i' \Sigma w_i \\ \{w_i\} \end{aligned}$$

such that

$$\begin{aligned} w_i' \mu &= E[R_i] \\ w_i' \underline{1} &= 1 \end{aligned}$$

where $\underline{1}$ is a vector of ones of appropriate dimension. The no short selling constraints can be written as:

$$w_{ij} \geq 0 \quad \forall j$$

where w_{ij} is the share of the j -th asset in the i -th portfolio. The solution of the optimization problem gives, for each $E[R_i]$, the portfolio w_i with lower variance (risk).

The time series used to estimate expected returns and covariance matrix are the following:

Deposits: mean rates on retail bank deposit (source: Bank of Italy, July 2001).

Short-term fixed income: value-weighted average of returns of BOT, CCT and CTZ (source: Bank of Italy, July 2001).

Long-term fixed income: value-weighted average of returns on BTP with time to maturity of 3, 5, 10 and 30 years (source: Bank of Italy, July 2001).

Italian stocks: Morgan Stanley Capital International (MSCI) index for the Italian stock market.

Foreign stocks: Morgan Stanley Capital International (MSCI) world market index (source of the time series on exchange rates: DRI).

All series are annualized monthly data over the period 1978-2000 (276 observations).