CEPS INSTEAD

WORKING PAPERS

Remittances as pure or precautionary investment? Risk, savings and return migration

Matthieu DELPIERRE[1]
Bertrand VERHEYDEN[2]

FNRS, Earth and Life Institute, University of Louvain, Belgium[1]
CEPS/INSTEAD-ERDI, Luxembourg[2]

Working Paper No 2010-39
December 2010

Remittances as pure or precautionary investment? Risk, savings and return migration*

Matthieu Delpierre
FNRS, Earth and Life Institute, University of Louvain, Belgium
Bertrand Verheyden
CEPS/INSTEAD, G.D. Luxembourg

November 2010

Abstract

This paper provides a theory of migrants' decisions to remit and save under uncertainty in connection with future location decisions. We show that the impact of remittances on the risk faced by the migrant is more complex than usually acknowledged. On the one hand, their effect on aggregate risk is non-monotonic. On the other hand, their impact on the geographical location of risk might be counter-intuitive, as remittances increase the migrant's exposure to risk in the origin country. Also, marginal returns to remittances may be increasing, at least locally, due to the endogeneity of the future location. Interior solutions are therefore not guaranteed, and liquidity constraints faced by migrants may be binding. Finally, undocumented migrants are shown to be more likely to remit than legal migrants.

Keywords: return migration; remittances; risk; investment

JEL classification codes: D13, D80, O12, O15

*This research has benefited from the core funding for CEPS/INSTEAD from the Ministry of Higher Education and Research of Luxembourg and from the Belgian National Fund for Scientific Research. The authors would like to thank Oded Stark, Alberto Zazzaro and Vincent Hildebrand for helpful comments.

1 Introduction

The remittances of international migrants provide developing countries with important monetary flows. In contexts of imperfect capital markets, this additional source of income allows recipient households to relax liquidity constraints and to develop their members' education (Edwards & Ureta (2003); Yang (2008); Yang (2009) and Calero et al. (2009)), productive assets (Adams (1998)), and/or social capital (Gallego & Mendola (2009)).

In this paper, we provide a theory of migrants' decisions to remit and save under uncertainty in connection with future location decisions. The model captures two crucial aspects of remittances, namely that they can be considered as pure investments and/or as precautionary investments. As far as pure investments are concerned, Adams (1998) finds that the marginal propensity to invest is significantly higher for households with a migrant member. Investment by the family, or in certain circumstances by the migrant him/herself, is therefore a fundamental motive for remittances. It is also often argued that remittances may serve the purpose of insuring against income shocks by improving the option of returning to the origin country.

We show that the distribution of risks between the host and the origin country plays a crucial rule in the remitting behavior of migrants, as documented by recent empirical contributions. Indeed, Amuedo-Dorantes & Pozo (2006) show that remittances are significantly impacted by the risk faced by the migrant in the host labor market, whereas Dustmann et al. (2010) shows that undocumented migrants are more likely to remit. The model developed below takes account of the initial distribution of risk between host and origin countries and reproduces these stylized facts. For instance, undocumented migrants are likely to face most of their uncertainty in the host country, while documented migrants' risk is mainly located in their origin region where their career prospects may be less clear.

One of the contributions of the paper is to show that remittances have indeed an important impact on the risk faced by the migrant, but in a more complex way than usually acknowledged. Since the migrant is uncertain about her future location, she faces risk in both host and origin countries. Two important insights can be drawn from the model regarding the effect of remittances on risk. On the one hand, their effect on aggregate risk is non-monotonic. On the other hand, their impact on the geographical location of risk might be counter-intuitive, as remittances increase the migrant's exposure to risk in the origin country.

Our model also provides some insights about the interaction between remittances and the migrant's future location. In a pure investment perspective, we show that, due to the uncertainty regarding migrant's future location, marginal returns to remittances may be increasing, at least locally. The intuition is that return prospects interact with remittances. Indeed, on the one hand, the incentive to remit increases with the probability to return to the origin country. On the other hand, the probability to return to the origin country is naturally positively influenced by remittances in the sense that investments at home increase the value of return.

The model also provides insights about the role played by the migrant's characteristics. For instance, the wealth of the family left behind has a negative effect on remittances. However, migrants may face liquidity constraints, and these constraints have an impact on the link between remittances and the migrant's wage at the time of arrival in the host country. Under interior savings, the migrant's initial wage has no impact on remittances, whereas this impact is positive for constrained migrants.

The economic literature on migrants' remittances and return migration

Apart from altruistic purposes, remittances are often seen as an element of a contract between the sender and the recipients.[1] First, this contract can be considered as a mutual insurance contract: since migration contributes to the diversification of the family's income sources (Stark & Levhari (1982)), remittances can play the role of insurance transfers between the migrant and the parent (de la Briere et al. (2002)). Second, a loan contract in which remittances act as a repayment on the investment initially made by parents in migration transaction costs or even in schooling. Third, according to Cox (1987), remittances may be a form of payment for services offered by the family, for instance, taking care of the migrant's cattle or her household members left behind. Another explanation of remittances pertains to the strategic bequest motive (Bernheim et al. (1985)): if the parents can credibly commit to disinherit their child, they are able to attract care and transfers by designing an appropriate reward or bequest function. In this framework, the share of the inheritance that is captured by each heir is increasing in the relative amounts remitted. The empirical relevance of this explanation has been tested and confirmed by Hoddinott (1994) and de la Briere et al. (2002). Finally, the investment motive appears in some empirical contributions: Osili (2007) provides an investigation of Nigerian

[1] Lucas & Stark (1985) are the first to study the various motives for remittances. Rapoport & Docquier (2006) provide an extensive survey of this topic.

migrants' investments in their origin country, whereas Osili (2004) focuses more specifically on the benefits of housing investments. However, on the whole, the investment motive seems overlooked, at least in the theoretical literature, and yet the validity of this motive seems highly relevant.

Regarding the literature on return migration, two theories about the length of stay in the host country are competing. The first one is a life cycle theory, in which households choose the length of stay in order to find a trade-off between the benefit from higher earnings or returns to education obtained in the host country and the cost of living abroad, such as differences in purchasing powers across countries (Stark et al. (1997), Dustmann (2003)). The second theory emphasizes the importance of "target earnings" which must be reached in order to start investments in origin countries in the presence of borrowing constraints (Dustmann & Kirchkamp (2002), Mesnard (2004), Djajic (2010)).[2] Yang (2006) uses exchange rate shocks to empirically oppose both theories and finds a larger support for the life cycle theory.

Our modeling strategy differs from the existing literature and tries to gather both explanations.

First, to the best of our knowledge, our model is the first to treat endogenous return migration with both savings and remittances. The existing models mostly study the interaction between savings and the length of stay before return migration. The introduction of remittances allows us two compare two technologies of transferring resources to the future which differ in several aspects. Whereas savings can be used in both locations, the return to remittances is mostly enjoyed in case of return. The return to remittances may be large, but even if it is lower than that of savings, we show that remittances may be used as a means of mitigating risk. We then say that remittances have a precautionary motive. Furthermore, remittances may be subject to agency costs, which we describe below. Finally, the introduction of remittances is particularly relevant since recent contributions (Dustmann & Mestres (2009) and Sinning (2009)) show that migrant's remitting behavior is intrinsically related to her return intentions.

Second, whereas most theoretical contributions model savings and migration duration as simultaneous decisions, our model introduces a sequence between both decisions. The reason thereof is that future wages in both locations are uncertain, and we argue that there is no reason

[2]Both theories may be affected by location-specific preferences (Hill (1987), Djajic & Milbourne (1988)).

for the ex ante return decision to be sunk when wages become observable. In other words, the migrant should make a decision on a potential return at a particular moment in time only when wages in both locations at this moment are observed.

These two aspects of the model generate both life cycle and target earnings considerations. On the one hand, the migrant's location choice is made when wages are observed. In this sense, return migration is dictated by a life-cycle approach, since location is chosen on the basis of income maximization. On the other hand, the probability to return is endogenously determined by previous decisions on savings and remittances. Whether this investment reaches a level sufficient to trigger a return depends on the realization of wages.

Investment in an agency relationship, the example of social capital

Since the migrant is physically absent from her origin country, she needs to hire an intermediary. Naturally, her family can play this role. As in a classical agency problem, however, information asymmetries prevent the migrant from specifying the investment effort in an enforceable contract. In order to integrate this restriction, our framework assumes that the migrant has zero discretion over the allocation of remittances between the agent's consumption and investment. In such a context, the agent (which we will call the parent) will have a higher incentive to invest if she also derives some benefits. In this paper, we take the example of a household public good. This modelling strategy is interesting for two reasons: first, it solves part of the problems inherent to imperfect monitoring by the migrant and, second, it is highly relevant empirically. Let us take the example of social capital. Once abroad, the migrant is indeed unable to fuel or invest in his social network by himself. And the parents or relatives in the origin country can indeed serve as agents and invest resources on his behalf (see Demonsant (2007) and Gallego & Mendola (2009) on the link between social capital and remittances). Moreover, social capital is fundamental in developing countries. In a context of market imperfections, information and reputation play important roles in economic transactions. For instance, social capital could secure access to credit or positively impact on success in small-scale enterprises such as agricultural trade (Fafchamps (2002)). Osili (2004) argues that remittances are a way for migrants to keep access to community resources. As already stressed, in a context of weakness of formal institutions, access to informal local public goods is a prerequisite for returning to communities of origin. Such an argument can be interpreted as a lower bound to investments in social capital. At higher levels, some migrants are reported to finance schools, churches or mosques in their village of origin. Another form of investment is also covered by our framework, namely housing. Indeed, as argued by Osili (2004), first, it benefits to the whole household. Second, in some areas, housing can be considered as an irreversible investment given the low liquidity level of the renting or secondary market. It is therefore, at the same time, illiquid and immobile, just as social capital. The same can be said of investments in land improvements if land markets are imperfect. It follows that the migrant can only draw the benefits in case of return.

The paper is organized as follows. The general setting is introduced in Section 2. Since the papers aims at analyzing two types of motives for remittances, namely pure investment and precautionary investment, two cases are studied. The case of risk neutral migrants is presented in Section 3 in order to focus on the pure investment motive. In Section 4, the case of risk averse migrants is analyzed, thereby introducing the precautionary investment motive. Comparative statics of both cases are presented in Section 5. Concluding remarks are provided in Section 6.

2 General setting

There are two periods $t \in \{1, 2\}$, and two locations, namely the country of origin o and the host country h. There are two agents, namely the migrant m, and the parent p. The parent lives in both periods in the country of origin. The migrant lives in the host country in the first period and has the possibility to save s and to send remittances r to the household in the host country. Given the remittances she receives, the parent in the origin country invests x in an asset which provides a fixed return R. Since the migrant has potentially contributed to this investment, the returns of the latter are considered as a public good which benefits to the migrant in case of return.[3] In the second period, the migrant observes the wage gap between the labor markets in the host and the origin countries and gets to decide whether to return to the home country or to stay in the host country.

The following analytical developments are divided into two parts with the aim of distin-

[3]For example, the parent can invest in housing, in family businesses or in social capital inside the community. The returns to these activities can easily be considered as public among family members. Investments in social capital in the country of origin can take the following forms: financing of ceremonies and parties, contribution to the provision of public goods, provision of credit to other community members, creation of economic activities generating employment at the local level,...

guishing between the pure investment and the precautionary investment motives. In the first part, in order to focus on the pure investment motive, the migrant is assumed to be risk neutral. This allows us to focus on consumption smoothing over time and on the incentives to invest in the origin country under the above mentioned agency problem. In the second part, the impact of remittances on the migrant's risk composition is analyzed in details.

3 The pure investment motive

3.1 Parent's investment decision

Let us start by describing the parent's situation. Credit markets in origin country are imperfect and the parent faces borrowing constraints. These constraints are modeled by a borrowing ceiling, which is assumed to be positively related to the household's initial wealth w_i since the latter can serve the purpose of a collateral. Moreover, as argued by Osili (2004), remittances may help the recipients to improve their access to credit. The borrowing ceiling is therefore defined as a fixed proportion of the aggregate amount of money the parent is endowed with in the first period. We write the borrowing constraint as

$$b \leq \bar{b} = \pi \left(w_i + r \right),$$

where b denotes borrowing and where $\pi \in (0, 1)$. We normalize without loss of generality the interest rate to one. The parent's utility writes

$$V = v \left(w_i + r - x + b \right) + \delta v \left(w_p + Rx - b \right),$$

where $v(.)$ is a strictly concave increasing function, δ is a discount factor and w_p is the parent's second-period income. One can easily show that, as soon as the (constant) rate of return to family investment is larger than the opportunity cost of funds, that is, the interest rate $(R > 1)$, borrowing is at its upper bound, namely $\pi \left(w_i + r \right)$.[4] The parent's objective function can thus be rewritten as

$$V = v \left(\left(1 + \pi \right) \left(w_i + r \right) - x \right) + \delta v \left(w_p + Rx - \pi \left(w_i + r \right) \right).$$

[4]See Appendix 0.

Given the remittances received in period 1, the parent's objective is to maximize utility with respect to his investment effort x. The first order condition is given by

$$V_x = -v_1' + \delta v_2' R = 0. \tag{1}$$

Therefore, parent's response to remittances (and to initial endowment) is

$$\begin{aligned} \frac{\partial x^*}{\partial w_i} &= \frac{\partial x^*}{\partial r} = \frac{v_1''(1+\pi) + \pi R \delta v_2''}{v_1'' + R^2 \delta v_2''} \\ &\in (0, 1+\pi) \iff R > \frac{\pi}{1+\pi}. \end{aligned} \tag{2}$$

The latter condition always holds since, by assumption, $R > 1$. This term is the marginal propensity to invest of remittances. Notice that one additional unit of remittances provides the parent with $(1+\pi)$ monetary units, that is, remittances themselves plus an extension of the loan. Equation (2) therefore shows that some fraction of this additional income is not invested by the parent ; instead it is consumed. This eviction effect is inherent in the agency relationship and reduces the returns to remittances for the migrant as compared to a situation in which she would invest by herself. Let us take the example of an exponential utility function: $v(y) = -e^{-\eta y}$. Parent's first order condition with respect to the investment level (1) becomes

$$\begin{aligned} V_x &= -\eta e^{-\eta c_1} + \eta R \delta e^{-\eta c_2} = 0 \\ \iff & x^* = (1+R)^{-1} \left[\frac{1}{\eta} \ln(R\delta) + (1+2\pi)(w_i + r) - w_p \right], \end{aligned}$$

and marginal propensity to invest in this case is constant and writes

$$\frac{\partial x^*}{\partial r} = \frac{1+2\pi}{1+R}. \tag{3}$$

3.2 The migrant's choice of location

Let us now turn to the migrant's problem. She faces uncertainty about wages in period 2. Two alternative locations are to be considered. We therefore need to define wages in both host and origin countries:

$$\begin{aligned} w_h &= w + \alpha(Z - \mu), \\ w_o &= w + \alpha(Z - \mu) - Z. \end{aligned}$$

The wage gap $(w_h - w_o)$ is embodied by the random variable $Z \sim F(Z)$, with $E(Z) = \mu$ and $Var(Z) = \sigma^2$.[5] We assume that $\mu > 0$, that is, income prospects are on average better in the host economy. It follows that

$$\begin{aligned} E(w_h) &= w, \\ E(w_o) &= w - \mu. \end{aligned}$$

$\alpha \in (0,1)$ is a parameter defining the allocation of the income risk between both locations. Indeed,

$$\begin{aligned} Var(w_h) &= \alpha^2\sigma^2, \\ Var(w_o) &= (1-\alpha)^2\sigma^2. \end{aligned}$$

In order to understand this specification, consider the case where $\alpha = 0$. Then, w_h becomes a constant, while w_o has variance σ^2: all the income risk is located in the origin country. One can indeed consider that different types of migrants may face different values of α. For instance, illegal migrants are more likely to face risk in the host country, in which case α is close to one. On the other hand, well established and integrated migrants should face very little risk in the host country, while uncertainty in case of return is relatively higher.[6]

Once the wage gap has been observed in period 2, the migrant gets to decide whether to return to the origin country or to stay in the host country. This decision is made by comparing the utility levels in both locations. The migrant decides to return provided the utility she derives from consumption in the origin country is higher than the utility level achieved in the host country. The consumption levels are given by

$$\begin{aligned} c_{2o} &= \tau s + Rx^* + w + \alpha(Z-\mu) - Z, \\ c_{2h} &= \tau s + w + \alpha(Z-\mu), \end{aligned}$$

[5]Differences in purchazing powers or location-specific preferences are neglected in this version of the model, but the implications of such aspects are discussed in the concluding remarks. If location-specific preferences are modeled as an additive utility premium in the origin country, such a premium would be incorporated by the parameter $\mu = E(Z)$, that is a translation of the distribution of Z. In other words, if the migrant has an intrinsic willingness to return to the origin country, then she is ready to accept some income drop and therefore a higher wage gap.

[6]The composition of the income risk is not going to play any role under risk neutrality. But since we make use of the same income specification under risk aversion, we already introduce it in this section.

where τs is the return to the migrant's savings in period 1. Note that we assume for simplicity of the exposition that the migrant benefits from her family investment in case of return only. This simplification does not alter qualitatively our results, as will be discussed in the conclusion. The migrant chooses to return to the origin country if and only if

$$c_{2o} - c_{2h} > 0 \iff Z < Rx^*.$$

This condition simply states that the wage gap must not be larger than the returns to family investment, which depends on the amount remitted in period 1.

3.3 Modeling the migrant's preferences

Under the problem under study, the migrant's preferences have two relevant aspects, namely risk aversion and consumption smoothing over time. The standard representation of preferences based on expected utility does not allow us to disentangle these two aspects. Using a standard expected utility specification and assuming risk neutrality would impose the utility function to be linear, in which case consumption smoothing disappears and consumption is always at a corner in one of the two periods.

We therefore make use of the representation of preferences based on the so-called concept of non-expected utility developed by Selden (1978) and Kreps & Porteus (1978), which allows us to disentangle risk aversion and consumption smoothing.[7] Before introducing this general concept, let us first define y as the component of the migrant's income which is conditional on location. This component is the following random variable:

$$\begin{aligned} y &= Rx^* + w + \alpha(Z-\mu) - Z, \text{ if } Z \in (-\infty, Rx^*] \qquad (4) \\ y &= w + \alpha(Z-\mu), \text{ otherwise.} \end{aligned}$$

Using the general formulation of Kreps & Porteus (1978), the migrant's utility writes

$$U = u(w_1 - s - r) + \delta u\left\{g^{-1}E_y\left(g\left(\tau s + y\right)\right)\right\}, \qquad (5)$$

where both $u(.)$ and $g(.)$ are increasing, concave functions and $g^{-1}E_y(g(\tau s + y)) = \tau s + \tilde{y}$

[7]The models of Kreps and Porteus (1978) and Selden (1978) are equivalent but we use the formulation of Kreps and Porteus which is more intuitive. See Chapter 20 of Gollier (2004) for an introduction to non-expected utility. An interesting recent contribution in this field is due to Kimball & Weil (2009).

is the certainty equivalent functional of period-2 consumption. While the form of $u(.)$ captures the migrant's preferences in terms of consumption smoothing, the concavity of $g(.)$ captures its degree of risk aversion.

Some particular cases of Kreps-Porteus preferences are worth mentioning. First, if $u\left(.\right)$ and $g\left(.\right)$ are identical, non-expected utility and standard expected utility are identical. In other words, expected utility is a particular case of non-expected utility. Second, and particularly useful in the next section, assuming $g(.)$ to be linear represents the preferences of a risk-neutral migrant's while being able to account for his need for consumption smoothing. This benchmark case allows us to focus on the pure investment motive while ignoring motives related to risk.

3.4 Optimal savings and remittances under risk neutrality

Taking into account the optimal location choice, the migrant's preferences are, following equation (5) and assuming linearity of $g(.)$,

$$\begin{aligned} U &= u\left(w_1 - s - r\right) \\ &\quad + \delta u \left\{ \int_{-\infty}^{Rx^*} \left[\tau s + w + Rx^* + \alpha\left(Z-\mu\right) - Z\right] dF(Z) + \int_{Rx^*}^{+\infty} \left[\tau s + w + \alpha\left(Z-\mu\right)\right] dF(Z) \right\}, \end{aligned} \tag{6}$$

where w_1 stands for migrant's first period income and where savings s and remittances r are the choice variables. This simple version of the model does not account for the migrant's altruism towards the parent. We discuss the implication of altruism in the concluding remarks.

Uncertainty about future location may prevent the migrant from financing remittances by borrowing. Savings therefore have to take non-negative values and the migrant's problem writes

$$\underset{\{s,r\}}{Max}\, U = u\left(w_1 - s - r\right) + \delta u\left(\tau s + E\left(y\right)\right) \text{ s.t. } s \geq 0; r \geq 0,$$

where

$$\begin{aligned} E\left(y\right) &= \int_{-\infty}^{Rx^*} \left(Rx^* + w - \alpha\mu - \left(1-\alpha\right) Z\right) dF(Z) + \int_{Rx^*}^{+\infty} \left(w - \alpha\mu + \alpha Z\right) dF(Z) \\ &= w + F\left(Rx^*\right)\left[Rx^* - E(Z \mid Z < Rx^*)\right]. \end{aligned}$$

The first order condition with respect to savings writes

$$U_s = -u_1' + \delta u_2' \tau \leq 0. \tag{7}$$

The first order condition with respect to remittances writes

$$U_r = -u_1' + \delta u_2' \frac{\partial E\,(y)}{\partial r} \leq 0, \tag{8}$$

where

$$\begin{aligned} \frac{\partial E(y)}{\partial r} &= \frac{\partial E\,(y)}{\partial x}\frac{\partial x^*}{\partial r} \\ &= F(Rx^*)R\frac{\partial x^*}{\partial r} > 0. \end{aligned} \tag{9}$$

First notice that $E(y)$ increases with x. This is intuitive since, as soon as there is some probability to return, the migrant's expected second period consumption should increase with amounts invested by the family. More precisely, as can be seen in equation (9), marginal returns to remittances are given by the combination of three factors, namely the intrinsic return to family investment R, parent's marginal propensity to invest and, finally, the probability to return to the origin country and hence to benefit from the household public good. Making use of (3), marginal returns to remittances rewrite

$$\frac{\partial E(y)}{\partial r} = F(Rx^*)\,(1+2\pi)\,\frac{R}{1+R}.$$

Since both $F(Rx^*)$ and $\frac{R}{1+R}$ are strictly lower than 1 and given that the migrant can save at a constant interest rate $\tau > 1$, remittances are only profitable if π is sufficiently large. In other words, in an agency relationship, the eviction effect through which part of the remittances are consumed rather than invested can only be outweighed by an important positive impact on the parent's access to credit.

Making use of (9), combining both first order conditions provides the following arbitrage condition between remittances and savings, which imposes that the returns to both activities must equalize at a potential interior solution:

$$F(Rx^*)R\frac{\partial x^*}{\partial r} = \tau. \tag{10}$$

However, we precisely show that such an interior solution is unlikely to obtain with a risk neutral migrant. Lemma 1 establishes the condition under which an interior solution for both savings and remittances can exist. This condition relies on how the migrant's marginal returns to remittances evolve with remittances, that is, the sign of $\frac{\partial^2 E(y)}{\partial r^2}$.

Definition 1. *Marginal returns to remittances under risk neutrality are decreasing if and only if*

$$\frac{\partial^2 E(y)}{\partial r^2} = f(Rx^*)\left(R\frac{\partial x^*}{\partial r}\right)^2 + RF(Rx^*)\frac{\partial^2 x^*}{\partial r^2} \leq 0. \tag{11}$$

Lemma 1. *The migrant's objective admits an interior maximum if and only if, at this point, marginal returns to remittances are decreasing.*

Proof. See Appendix. □

An implication of Lemma 1 is that a necessary condition for savings and remittances to be simultaneously interior is decreasing marginal propensity to invest for the parent: $\frac{\partial^2 x^*}{\partial r^2} \leq 0$:

Proposition 1. *If the migrant is risk-neutral and if the parent's marginal propensity to invest does not decrease with income, interior savings and remittances cannot be obtained simultaneously.*

Proof. This result is a direct corollary of Lemma 1. □

Under risk neutrality, the migrant simply compares marginal returns to both instruments. Since the interest rate τ is constant, the migrant starts saving as soon as returns to remittances become lower that τ. However, marginal returns to remittances are unlikely to be decreasing and, as a consequence, the migrant either saves or remits, but does not jointly use both technologies. Indeed, let us analyze in details the second derivative of $E(y)$ with respect to remittances presented in equation (11). The first term on the right hand side of (11) is positive and reflects the fact that remittances increase the probability that the migrant returns to the origin country. Indeed, the incentive to locate in the origin country in the second period is higher if the family investment is large. Given that the latter is positively influenced by the amount of money remitted, the increase in the probability to benefit from the family investment therefore leads to the possibility of convex returns to remittances. In other words, if second period location is endogenous, the more the migrant invests in his country of origin, the higher the benefits of return and the higher the probability to return. To the best of our knowledge, this effect, which is sensible and robust to various modeling strategies, has not been studied in the literature so far. The second term on the right hand side of (11) pertains to parent's marginal propensity to invest. According to Adams (1998), marginal propensity to invest is higher for households with a migrant member. This suggests that this term could be positive, reinforcing convexity. This makes sense since poor families could first allocate remittances to meeting their basic needs. Besides, note that decreasing marginal returns to remittances would be more likely if marginal returns to family investment R were themselves decreasing in the amount invested. We have assumed it constant. The purpose of this simplification is to highlight, later on, the impact of risk aversion on the pure investment motive for remittances. Moreover, the partial effect of remittances on the location choice would nevertheless remain under decreasing marginal returns to family investment and would be a source of ambiguity for the shape of returns to remittances.

Suppose that the parent's preferences are described by an exponential utility function. We have shown earlier that marginal propensity to invest is, in this case, constant: $\frac{\partial^2 x^*}{\partial r^2} = 0$ and, hence, returns to remittances convex. From Proposition 1, we know that an interior solution for both technologies (savings and remittances) is impossible in this case. This does not imply however that the migrant remits an infinite amount of money since she is prevented from borrowing. If savings are at a corner, $s = 0$, the conditions for having a maximum interior in remittances are the first and second order conditions solely on r:

$$\begin{aligned} U_r &= -u_1' + \delta u_2' \frac{\partial E(y)}{\partial r} = 0, \\ U_{rr} &= u_1'' + \delta u_2'' \left(\frac{\partial E(y)}{\partial r} \right)^2 + \delta u_2' \frac{\partial^2 E(y)}{\partial r^2} \leq 0. \end{aligned}$$

The latter condition might be fulfilled even with convex returns to remittances. Indeed, the more the migrant remits, the more her consumption path is distorted at the expense of first period utility. This cost is convex, while the benefit in terms of second period consumption is concave. In addition, the partial convex effect on returns to remittances is naturally bounded since $F(Rx)$ cannot be larger than 1. Once the migrant returns to the home country with certainty, returns to remittances become constant. This allows to see that, under the sufficient condition that $R\frac{\partial x^*}{\partial r} < \tau$, the migrant does not remit at all. This remark concludes the resolution of migrant's optimization under risk neutrality.

As highlighted in the subsequent analysis, a final implication of the fact that a risk neutral migrant either saves or remits is that the strategy of sending remittances cum savings is mainly driven by risk-coping considerations.

4 The precautionary investment motive

Let us now introduce risk aversion in order to discuss the precautionary investment motive for remittances. We make use of the same set of assumptions regarding the timing of the game and the income composition. The migrant's preferences write

$$\begin{aligned} U &= u\left(w_1 - s - r\right) \qquad (12) \\ &+\delta \left\{ \int_{-\infty}^{Rx^*} u\left(\tau s + w + \alpha\left(Z-\mu\right) + Rx^* - Z\right) dF(Z) + \int_{Rx^*}^{+\infty} u\left(\tau s + w + \alpha\left(Z-\mu\right)\right) dF(Z) \right\}, \end{aligned}$$

where $u(.)$ is an increasing, strictly concave function. Note that the representation of the migrant's preferences used in this section is based on the standard concept of expected utility, which does not explicitly disentangle consumption smoothing and risk aversion, but which is used here to keep the exposition as simple as possible. It is important to remember however that this representation is another particular case of the Kreps-Porteus preferences defined in equation (5) where $g(.) = u(.)$. We start this section by a discussion about the impact of remittances on the risk composition, which provides new surprising insights.

4.1 The impact of remittances on the risk composition

We first analyze how remittances affect the migrant's risk structure. The usual intuition regarding this link states that investing in the country of origin in a context of uncertainty allows the migrant to improve her exit option and hence to reduce the risk she faces. However, as is shown below, the impact of remittances on migrant's aggregate risk and on its composition is more subtle than that. In particular, the impact on the exit option is ambiguous. In order to proceed, it is useful to divide the random wage gap Z into two truncated variables determining wages in the two locations. The migrant's utility is then rewritten as

$$\begin{aligned} U &= u\left(w_1 - s - r\right) + \delta F\left(Rx^*\right) \int_{-\infty}^{Rx^*} u\left(\tau s + w - \alpha\mu + Rx^* - (1-\alpha) Z\right) \frac{f(Z)}{F\left(Rx^*\right)} dZ \\ &+\delta\left[1 - F\left(Rx^*\right)\right] \int_{Rx^*}^{+\infty} u\left(\tau s + w - \alpha\mu + \alpha Z\right) \frac{f(Z)}{1-F\left(Rx^*\right)} dZ, \\ U &= u\left(w_1 - s - r\right) + \delta F\left(Rx^*\right) E_{Z_1} u\left(\tau s + w - \alpha\mu + Rx^* - (1-\alpha) Z_1\right) \\ &+\delta\left[1 - F\left(Rx^*\right)\right] E_{Z_2} u\left(\tau s + w - \alpha\mu + \alpha Z_2\right), \end{aligned}$$

where $Z_1 \sim \frac{f(Z_1)}{F(Rx^*)}$ is distributed over the support $(-\infty, Rx^*]$ and $Z_2 \sim \frac{f(Z_2)}{1-F(Rx^*)}$ over the support $(Rx^*, +\infty)$, with

$$
\begin{aligned}
E\left(Z_1\right) &= E(Z \mid Z < Rx^*) = \mu_1\left(Rx^*\right), \\
E\left(Z_2\right) &= E(Z \mid Z > Rx^*) = \mu_2\left(Rx^*\right), \\
Var\left(Z_1\right) &= \sigma_1^2\left(Rx^*\right), \\
Var\left(Z_2\right) &= \sigma_2^2\left(Rx^*\right).
\end{aligned}
$$

Given that both random variables are defined by the truncation level which itself depends on family investment and thereby on remittances, their characteristics in terms of mean and variance are endogenous. In addition, let us define η as the migrant's degree of absolute risk aversion. Utility rewrites

$$
U = u\left(w_1 - s - r\right) + \delta F\left(Rx^*\right) u\left(\tilde{c}_{2o}\right) + \delta\left[1 - F\left(Rx^*\right)\right] u\left(\tilde{c}_{2h}\right),
$$

where

$$
\begin{aligned}
\tilde{c}_{2o} &\approx \tau s + w - \alpha\mu + Rx^* - (1-\alpha)\frac{1}{2}_1 - (1-\alpha)^2\frac{\eta}{2}\sigma_1^2, \\
\tilde{c}_{2h} &\approx \tau s + w - \alpha\mu + \alpha\frac{1}{2}_2 - \alpha^2\frac{\eta}{2}\sigma_2^2.
\end{aligned}
$$

$\tilde{c}_{2o}$ and $\tilde{c}_{2h}$ are the certainty equivalents of the second period consumption levels in origin and host countries, respectively.[8] In order to decompose the impact of remittances on expected utility, it is instructive to calculate ex ante marginal utility of remittances:

$$
\begin{aligned}
U_r &= -u_1' \\
&\quad + \delta\frac{\partial x^*}{\partial r}\left\{f\left(Rx^*\right)R\left[u\left(\tilde{c}_{2o}\right) - u\left(\tilde{c}_{2h}\right)\right] + F\left(Rx^*\right)u'\left(\tilde{c}_{2o}\right)\frac{\partial \tilde{c}_{2o}}{\partial x} + \left[1 - F\left(Rx^*\right)\right]u'\left(\tilde{c}_{2h}\right)\frac{\partial \tilde{c}_{2h}}{\partial x}\right\}.
\end{aligned} \tag{13}
$$

Let us interpret this expression in details. The first term on the right hand side of (13) measures marginal disutility associated to a reduction of first period consumption. The second term pertains to the second period and indicates a marginal increase in the probability to return to the origin country and to consume $\tilde{c}_{2o}$ rather than $\tilde{c}_{2h}$. The third and fourth terms form together the expected marginal utility of second period consumption. Assuming constant absolute risk

[8]We have made use of Pratt's approximation of the risk premium.

aversion, the effects of remittances (through an increase of family investment) on the certainty equivalents of consumption in both locations are given by

$$\begin{aligned}\frac{\partial \tilde{c}_{2o}}{\partial x} &= R\left[1-(1-\alpha)\frac{\partial \frac{1}{2}_1}{\partial Rx}-(1-\alpha)^2\frac{\eta}{2}\frac{\partial \sigma_1^2}{\partial Rx}\right],\\ \frac{\partial \tilde{c}_{2h}}{\partial x} &= R\left[\alpha\frac{\partial \frac{1}{2}_2}{\partial Rx}-\alpha^2\frac{\eta}{2}\frac{\partial \sigma_2^2}{\partial Rx}\right].\end{aligned}$$

On the one hand, taking any type of density for the wage gap Z, if the truncation level increases, it is easy to see that the means of both random variables Z_1 and Z_2 increase: $\frac{\partial \frac{1}{2}_1}{\partial Rx} > 0$; $\frac{\partial \frac{1}{2}_2}{\partial Rx} > 0$. On the other hand, we have that the variance of Z_1 increases through a larger support, while the variance of Z_2 is reduced due to the opposite effect: $\frac{\partial \sigma_1^2}{\partial Rx} > 0$; $\frac{\partial \sigma_2^2}{\partial Rx} < 0$. With those elements, it is easier to see how migrant's income prospects are affected by remittances. Let us first discuss the effects on host country income prospects. On the one hand, the expected wage in the host country increases. Recall that the second period location is a decision variable. It follows that the migrant chooses the place where her income level is the highest once wages are observed. Therefore, if returns to family investment Rx increase, the migrant will require a higher host country wage in order to stay. On the other hand, since the ex ante probability to return to the origin country increases, exposure to the uncertainty of the host labor market is reduced. On the whole, anticipating her optimal location decision, ex ante, the migrant's income prospects in the host economy are unambiguously improved. This is in this sense that intuition usually suggests that remittances protect the migrant against risk. However, the effect is exactly opposite regarding income prospects in the origin country. As far as the mean is concerned, the migrant is ready to accept lower wages in order to benefit from the household public good at home since her threshold on the wage gap increases. As regards uncertainty, by increasing the probability of returning to the origin country, the migrant increases her exposure to risk in the origin country. Whereas the most intuitive effect is that remittances are a self insurance mechanism, the way the risk structure is impacted does not guarantee that the aggregate risk level decreases with remittances (see infra). Remittances only allow the migrant to transfer some risk from the host to the origin country. By doing so, she can adjust the proportions of both labor markets risks she faces ex ante. Finally, it can be noted that this mechanism only makes sense if the future location remains uncertain. If the migrant returns (or stays) with certainty, remittances have no impact on risk, except as pure precautionary savings. In other words, they have no impact on the objective risk faced by the migrant.

4.2 The impact of remittances on aggregate risk

Let us turn to the analysis of the impact of remittances on aggregate risk. Using the random variable y defined in equation (4), the risk-averse migrant's utility defined in equation (12) can be rewritten as

$$\begin{aligned} U &= u\left(w_1 - s - r\right) + \delta E_y u\left(\tau s + y\right) \\ &= u\left(w_1 - s - r\right) + \delta u\left(\tau s + \tilde{y}\right), \end{aligned} \tag{14}$$

where $\widetilde{y} \approx E\left(y\right) - \frac{\eta}{2}\sigma_y^2$, where $\sigma_y^2 \equiv Var(y)$ and η is the migrant's degree of absolute risk aversion. The variance of y writes

$$\sigma_y^2 = \int_{-\infty}^{Rx^*} \left[Rx^* - (1-\alpha)\, Z\right]^2 dF(Z) + \int_{Rx^*}^{+\infty} (\alpha Z)^2\, dF(Z) - \left[E\left(y\right) - w + \alpha\frac{1}{2}\right]^2.$$

Proposition 2 describes the effect of remittances on the aggregate income risk faced by the migrant and highlights the role played by the distribution of wage variances across the two locations.

Proposition 2. *Remittances are risk-reducing provided that α is sufficiently high, that is, the variance of period-2 wages in the host country is sufficiently high compared to the variance of wages in the origin country. Formally,*

$$\frac{\partial \sigma_y^2}{\partial r} = 2R\frac{\partial x^*}{\partial r}F(Rx^*)\left[E(y \mid Z < Rx^*) - E(y)\right] \tag{15}$$

$$< 0 \iff E(y \mid Z < Rx^*) < E(y \mid Z > Rx^*)$$

$$\iff \alpha > \alpha_r \equiv \frac{Rx^* - E(Z \mid Z < Rx^*)}{E(Z \mid Z > Rx^*) - E(Z \mid Z < Rx^*))} \in [0, 1]\,. \tag{16}$$

Proof. See Appendix. □

Proposition 2 indicates, first, that remittances decrease the aggregate income risk faced by the migrant if her initial risk structure is such that uncertainty is mainly located in the host country (α sufficiently high). This is in accordance with our preceding result about the impact of remittances on the risk composition. Remittances tend to improve host country income prospects at the expense of an increase in the origin country income risk. A transfer of risk from host to origin country can only be profitable if the former was high enough as compared to the latter. Second, it can be seen that the variance of y is minimized if

$$E(y \mid Z < Rx^*) = E(y \mid Z > Rx^*).$$

In other words, aggregate risk is at a minimum if the migrant's expected wage, conditional on location, is the same across the two possible locations.

An important implication of Proposition 2 is that α_r is itself a function of the level of remittances, so that the latter's effect on aggregate risk may be non-monotonic. In order to illustrate that, let us assume that the wage gap Z is uniformly distributed with density f. The next proposition states that the impact of remittances on risk is non-monotonic.

Proposition 3. *If Z is uniformly distributed, α_r increases with remittances. Therefore, remittances are likely to have a non-monotonic impact on aggregate risk, being first risk-decreasing and for larger values risk-increasing.*

Proof. Under the uniform distribution, $\alpha_r = F(Rx)$, so that $\frac{\partial \alpha_r}{\partial r} = fR\frac{\partial x}{\partial r} > 0$. Therefore, for levels of remittances, α_r is likely to be smaller than α, in which case, by Proposition 2, remittances are decreasing risk. As remittances increase, α_r may become larger than α, and remittances become risk-increasing. □

The impact of remittances on aggregate risk is largely driven by the fact that our framework accounts for the location choice. More precisely, it can be argued that, in this setting, the genuine self insurance device is the migrant's ability to choose her location after information about wages is revealed. Indeed, for any initial risk composition, the migrant is protected against low wages by the ability to select the maximum of the two final incomes. Moreover, as we show in the following example, this instrument is most powerful if risk is evenly distributed between locations. Example 1 provides the intuition behind Propositions 2 and 3, namely that remittances are risk-reducing if risk is mainly located in the host economy and that transferring too much risk towards the origin country might, at the end, increase the aggregate risk faced by the migrant. In order to illustrate these points, we use a simplified framework where the only decision variable is the location:

Example 1 Suppose an agent can choose between two locations $\{a, b\}$, where incomes are random and given by

$$\begin{aligned} w_a &= w - (1-\alpha)\,\theta, \\ w_b &= w + \alpha\theta, \end{aligned}$$

where $\theta \sim F(\theta)$. Assume $E(\theta) = 0$. Hence,

$$\begin{aligned}
E(w_a) &= E(w_b) = w \\
Var(w_a) &= (1-\alpha)^2 Var(\theta) \\
Var(w_b) &= \alpha^2 Var(\theta).
\end{aligned}$$

Ex ante, taking into account the optimal location choice, the agent's income is given by a random variable with the following definition

$$\begin{aligned}
\omega &\equiv Max\{w_a, w_b\} \\
&= \begin{cases} w - (1-\alpha)\theta, & \text{if } \theta < 0 \\ w + \alpha\theta, & \text{otherwise.} \end{cases}
\end{aligned}$$

Let us first mention that, in such a setting, the ability to choose a location after the risk is revealed allows the agent to earn a higher income, on average, than if she was locked in one of the two locations. Indeed,

$$\begin{aligned}
E(\omega) &= \int_{-\infty}^{0} [w - (1-\alpha)\theta]\, dF(\theta) + \int_{0}^{+\infty} (w + \alpha\theta)\, dF(\theta) \\
&= w - F(0)E(\theta \mid \theta < 0) \\
&> w.
\end{aligned}$$

This result obviously applies to the more general framework of this paper. Second, let us discuss the impact of the initial risk composition α on the agent's aggregate risk. To this end, we calculate the variance of the agent's income:

$$Var(\omega) = \int_{-\infty}^{0} (1-\alpha)^2 \theta^2 dF(\theta) + \int_{0}^{+\infty} \alpha^2\theta^2 dF(\theta) - \left[\int_{-\infty}^{0} \theta dF(\theta)\right]^2.$$

Note that the third term does not depend on α. Hence, $Var(\omega)$ is minimized for

$$\alpha^* = \int_{-\infty}^{0} \theta^2 dF(\theta) \left[\int_{-\infty}^{+\infty} \theta^2 dF(\theta)\right]^{-1} \in [0,1].$$

which, under the sufficient condition that the distribution of θ is symmetric, implies that $\alpha^* =$

$1/2$.[9] In other words, the risk that an agent who can choose her location faces is minimized when the initial income risk is identical in both locations. ■

As this simplified example illustrates, on the one hand, the location choice allows the migrant to improve her expected income. On the other hand, it shows that aggregate risk depends on the initial risk composition. More precisely, the more even the ex ante distribution of risks between both locations, the higher the migrant's ability to reduce her income risk by moving. If risk is not evenly spread, remittances can then be used to decrease aggregate risk. Indeed, remittances are a way to balance the risk composition provided initial risk is disproportionately located in the host economy. Undocumented migrants are therefore more likely to remit than documented ones (Dustmann et al. (2010))[10]. On the contrary, as stated in Proposition 3, high levels of remittances may increase aggregate risk as soon as the distribution of risk becomes biased towards the origin country.

4.3 Optimal savings and remittances under risk aversion

Migrant maximizes expected utility (14) with respect to savings and remittances. The first order conditions are given by

$$U_s = -u_1' + \delta u_2' \tau \leq 0, \tag{17}$$

$$U_r = -u_1' + \delta u_2' \frac{\partial \tilde{y}}{\partial r} \leq 0, \tag{18}$$

where, under constant absolute risk aversion,

$$\frac{\partial \tilde{y}}{\partial r} \approx \frac{\partial E(y)}{\partial r} - \frac{1}{2}\eta \frac{\partial \sigma_y^2}{\partial r} = F\left(Rx^*\right) R \left\{1 - \eta \left[E(y \mid Z < Rx^*) - E(y)\right]\right\} \frac{\partial x^*}{\partial r},$$

[9]To see why, let us compute the first and second derivatives of $Var\left(\omega\right)$ with respect to α:

$$\frac{\partial Var\left(\omega\right)}{\partial \alpha} = -2\int_{-\infty}^{0} \theta^2 dF\left(\theta\right) + 2\alpha^* \int_{-\infty}^{+\infty} \theta^2 dF\left(\theta\right),$$

so that $\frac{\partial Var(\omega)}{\partial \alpha} = 0$ for $\alpha = \alpha^*$ and

$$\frac{\partial^2 Var\left(\omega\right)}{\partial \alpha^2} = 2\int_{-\infty}^{+\infty} \theta^2 dF\left(\theta\right) > 0.$$

[10]This result is formally derived in the section devoted to comparative statics.

We are now able to formally distinguish the precautionary investment motive from the pure investment motive. Marginal returns to remittances in a pure investment perspective are given by $\frac{\partial E(y)}{\partial r}$. As we have seen earlier, the magnitude of this term is unlikely to be large for at least two reasons. On the one hand, the migrant only benefits from the family investment with some probability (that is, in case of return migration). On the other hand, there is an eviction effect through which part of the amount remitted is allocated to immediate family consumption. If the interest rate in the host country is larger, the migrant should not remit at all, unless remittances are risk-reducing. In this sense, they can be said to be sent for a precautionary motive.

The condition for the migrant to be simultaneously interior in both savings and remittances under risk aversion is also given by Lemma 1, except that here the definition of decreasing marginal returns to remittances takes risk aversion into account:

Definition 2. *Under constant absolute risk aversion, the marginal returns to remittances under risk aversion are decreasing if and only if*

$$\frac{\partial^2 \tilde{y}}{\partial r^2} = \frac{\partial^2 E\left(y\right)}{\partial r^2} - \frac{\eta}{2}\frac{\partial^2 \sigma_y^2}{\partial r^2} \leq 0.$$

It is important to highlight that the possibility of locally increasing returns remains under risk aversion. However, as we have shown earlier, at some point, transferring risk from host to origin country starts to have adverse consequences on the migrant's aggregate exposure to risk. It follows that, through this effect, marginal returns to remittances tend to decrease for large amounts, namely when the probability of return migration becomes too important. Moreover, if the migrant's borrowing constraint (non-negativity constraint for savings) becomes binding, the willingness to smooth consumption across time enters the picture and incentives to remit are reduced.

To conclude this section, we have seen that the condition under which both savings and remittances are strictly positive at the optimum is not always satisfied. The reason behind the potential corner solutions, which are inherent to the subject under study, are intimately linked to the potential convexity of returns to one of the migrant's technologies, namely remittances. There are therefore cases where corner solutions prevail, in particular when savings are nil, that is, when the migrant would like to increase remittances but cannot do it because of liquidity constraints. The main difference between the two cases is related to considerations pertaining to intertemporal preferences. Indeed, under interior savings, the migrant's consumption path is smooth, while, under corner savings, it is distorted at the expense of first period consumption. In the former case, the remitting behavior should only be affected by expected returns and risk. In the latter case, however, remittances are the only instrument to simultaneously affect expected returns and consumption smoothing across states and across time. The last section presents a comparative statics analysis which distinguishes the cases of interior and corner savings, but prior to that, let us try to provide a taxonomy of the motives for remittances.

4.4 Remittances as pure or precautionary investment motives?

Figure 1 describes the type of motive for remittances depending on how those remittances affect at the margin the aggregate risk and the average return of y.

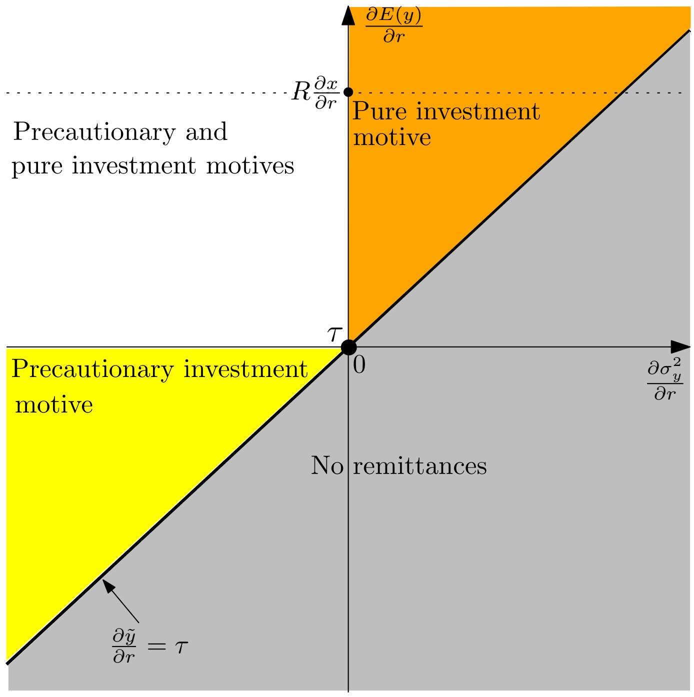

Figure 1: Taxonomy of the motives for remittances

From the first order conditions presented in equations (17) and (18), we know that a lower bound on the returns to remittances $\frac{\partial \tilde{y}}{\partial r}$ is defined by the returns to savings τ. Indeed, if both variables are at an interior solution,

$$\begin{aligned}\frac{\partial \tilde{y}}{\partial r} &\equiv \frac{\partial E(y)}{\partial r} - \frac{\eta}{2}\frac{\partial \sigma_y^2}{\partial r} = \tau \\ &\iff \frac{\partial E(y)}{\partial r} = \tau + \frac{\eta}{2}\frac{\partial \sigma_y^2}{\partial r}.\end{aligned}$$

In other words, if $\frac{\partial \tilde{y}}{\partial r} < \tau$, the migrant has no motive for remittances at all and would better only save. This case is captured by the lower right half of the figure, since the locus $\frac{\partial \tilde{y}}{\partial r} = \tau$ is represented by the bold straight line of slope $\frac{\eta}{2}$ in the space $\left(\frac{\partial \sigma_y^2}{\partial r}; \frac{\partial E(y)}{\partial r}\right)$.

As soon as $\frac{\partial \tilde{y}}{\partial r} > \tau$ (the upper left part of the figure), remittances become attractive for one reason or another. If $\frac{\partial E(y)}{\partial r} < \tau$, the investment motive is irrelevant since savings provide a better return. However, the fact that $\frac{\partial \tilde{y}}{\partial r} > \tau$ implies in this case that remittances are risk reducing at the margin. Therefore, remittances are spent on a precautionary motive in this case, which is represented by the yellow triangle. The opposite case is captured by the orange triangle, in which only the investment motive is relevant, since $\frac{\partial E(y)}{\partial r} > \tau$ whereas remittances increase risk: $\frac{\partial \sigma_y^2}{\partial r} > 0$. Finally, the white square represents the area where both motives are met since remittances have a higher average return than savings and decrease the aggregate risk.

5 Comparative statics

This section is devoted to a comparative statics analysis of the case of risk averse migrants. Because of the potential convexities in returns to remittances, we have to take into consideration the case of corner savings. In this section, in order to compute comparative statics of remittances with respect to the different parameters of the model, we therefore distinguish the cases of interior and corner savings.

5.1 Comparative statics of remittances under interior savings

The comparative statics rule is given by Lemma 2.

Lemma 2. *The comparative statics rule under interior savings:*

1. *The sign of the impact of any parameter ϕ on optimal remittances is determined by the sign of $U_{r\phi} - U_{s\phi}$. Formally, $\frac{\partial r^*}{\partial \phi} > 0 \iff U_{r\phi} - U_{s\phi} > 0$.*
2. *The sign of the impact of any parameter ϕ on optimal savings is determined by the sign of $\Omega U_{r\phi} - U_{s\phi}U_{rr}$, where $\Omega \equiv U_{ss} = U_{sr} = U_{rs}$. Formally, $\frac{\partial s^*}{\partial \phi} > 0 \iff \Omega U_{r\phi} - U_{s\phi}U_{rr} > 0$.*

Proof. See Appendix. □

Proposition 4. *Comparative statics with respect to migrant's first period income (w_1) under interior savings:*

1. *Under interior savings, remittances do not depend on the migrant's first period income (w_1).*

2. *Savings are increasing in* w_1.

Proof. See Appendix. □

This prediction may appear at first in contradiction with the empirical evidence, which establishes a positive link between the migrant's income and remittances.[11] This apparent discrepancy does not invalidate the model for several reasons. First, introducing altruism of the migrant would modify this prediction (Lucas & Stark (1985)). Second, we will show in the next subsection that the migrant's first period income does affect the amounts remitted when savings are nil. Therefore, if these empirical contributions did not control for liquidity constraints, the average estimated effect of the migrant's wage on remittances could be significantly positive. Finally, the result obtained in this model under interior savings is perfectly sensible. Indeed, a migrant with interior savings should have a similar behavior to a migrant in the hypothetical context of perfect capital markets. In the absence of liquidity constraints, the investment level, and remittances in particular, should be optimal. Since the optimal level must be unique, there is no reason for remittances to be affected by the migrant's first-period income.

Besides, in order to maintain a smooth consumption path, savings increase.

Proposition 5. *Comparative statics with respect to the household's initial wealth* (w_i) *under interior savings:*

1. *Under interior savings, remittances are decreasing in the household's initial wealth* (w_i).
2. *Savings increase with* w_i.

Proof. See Appendix. □

As a preliminary step in the interpretation of this result, let us rule out two sources of explanation. On the one hand, notice that this result is obtained with strictly selfish migrants. In other words, it is not altruism which induces the migrant from poorer families to remit more ceteris paribus. On the other hand, one could raise an argument based on liquidity constraints and decreasing marginal propensity to invest. Put differently, remittances, by relaxing liquidity constraints, would first allow the household to seize remunerative investment opportunities

[11] See Lucas & Stark (1985), Hoddinott (1994), Funkhouser (1995), Brown (1997), Sinning (2009) and Osili (2004) or Osili (2007) in the specific case of remittances motivated by investment.

such as education, small businesses, etc. Following this line of arguments, the fraction of remittances invested could be larger for poorer families and hence returns to remittances from the migrant's viewpoint. However, on the one hand, our result holds for a constant marginal propensity to invest per monetary unit of remittances (see equation (3)). On the other hand, as already mentioned, decreasing marginal propensity to invest would be in contradiction with the results of Adams (1998). Rather, the explanation relies on a perfect substitution between remittances and household wealth for financing the family investment. In other words, this is due to fungibility, that is the migrant's inability to monitor the allocation of funds. If remittances are interior, the migrant equalizes the marginal return to remittances to the interest rate. There is a unique investment x of the migrant's family which satisfies this condition. It follows that the migrant remits up to a point where the amount invested fulfils this condition. Hence, the richer the household, the more the latter invests by himself, and the less the migrant has an incentive to remit.

Besides, in order to maintain a smooth consumption path, savings increase as a response to the reduction of remittances.

Proposition 6. *Comparative statics with respect to the initial distribution of risk between host and origin country* (α) *under interior savings:*

1. *Remittances are increasing in* α.
2. *The impact of* α *on savings is indeterminate.*

Proof. See Appendix. □

This result states that the amount remitted is larger if the risk is mainly located in the host country. This result is in accordance with our analysis of the impact of remittances on the risk composition and on aggregate risk and with the empirical findings of Amuedo-Dorantes & Pozo (2006). The effectiveness of the location choice as a self-insurance device is the highest if the risk is evenly distributed between locations. It follows that, abstracting from the pure investment motives, an increase in α reinforces the importance of the precautionary motive.

The impact on savings cannot be signed. However, savings always play the same role, namely intertemporal consumption smoothing. The effect therefore depends on the direct impact of α and the subsequent adjustment of remittances on the distribution of consumption over time.

5.2 Comparative statics of remittances under corner savings

If remittances are interior, first and second order conditions with respect to remittances must be satisfied:

$$\begin{aligned} U_r &= -u_1' + \delta u_2' \frac{\partial \tilde{y}}{\partial r} = 0, \\ U_{rr} &= u_1'' + \delta \left[u_2'' \left(\frac{\partial \widetilde{y}}{\partial r} \right)^2 + u_2' \frac{\partial^2 \widetilde{y}}{\partial r^2} \right] < 0. \end{aligned}$$

However, under corner savings, we cannot exclude the case of convex returns to remittances: $\frac{\partial^2 \widetilde{y}}{\partial r^2} > 0$. As already mentioned, convex returns can be outweighed by the concavity of the utility function and the willingness to smooth consumption over time. As a second element, recall that, if migrant's liquidity constraint is binding, remittances constitute a unique choice variable with the aim of smoothing consumption across states and across time. Further use non-expected utility for this comparative statics exercise is convenient. Starting from the general formulation presented in equation (5), let us suppose constant absolute risk aversion, that is, $-\frac{g''}{g'} = \eta$, in which case $\tilde{y}$ is strictly the same as in the whole analysis of Section 4. In addition, let us suppose that $-\frac{u''}{u'} = \gamma$. We can then proceed to the final part of the comparative statics under corner savings.

Proposition 7. *Under corner savings, remittances increase with migrant's first period income* w_1.

Proof. Applying the implicit function theorem on the first order condition, we know that $\frac{\partial r^*}{\partial w_1}$ has the same sign as $U_{rw_1} = -u_1'' > 0$. □

If the migrant faces liquidity constraints, it is intuitive that an increase in her migration wage will translate in an increase in remittances. As we have discussed earlier, this is accordance with the empirical literature.

Proposition 8. *Under corner savings, remittances are decreasing in household's initial wealth* w_i *if and only if* $\frac{\partial^2 \widetilde{y}}{\partial r^2} \left(\frac{\partial \widetilde{y}}{\partial r} \right)^{-2} < \gamma$.

Proof. The same reasoning applies and the sign of $\frac{\partial r^*}{\partial w_i}$ is given by the sign of

$$\begin{aligned} U_{rw_i} &= \delta \left[u_2'' \frac{\partial \widetilde{y}}{\partial w_i} \frac{\partial \widetilde{y}}{\partial r} + u_2' \frac{\partial^2 \widetilde{y}}{\partial w_i \partial r} \right] \\ &= \delta \left[u_2'' \left(\frac{\partial \widetilde{y}}{\partial r} \right)^2 + u_2' \frac{\partial^2 \widetilde{y}}{\partial r^2} \right], \end{aligned}$$

since the impact on the parent's investment x^* of one additional unit of household wealth is the same as the impact of one additional unit of remittances (see (2)). Finally,

$$U_{rw_i} < 0 \iff \frac{\partial^2 \widetilde{y}}{\partial r^2}\left(\frac{\partial \widetilde{y}}{\partial r}\right)^{-2} < \gamma.$$

□

The intuitions developed earlier in the case of interior savings do not apply in this case. Indeed, under liquidity constraints, the migrant's remittances (and hence the family investment level) are suboptimal from her point of view. Therefore, the increase in the parent's own ability to invest does not automatically result in a decrease in remittances. Rather, the condition states that remittances are going to decrease if γ, the aversion to consumption fluctuations over time, is high enough. In this case, the migrant takes advantage of the increase in family investment to further smooth consumption over time. Notice also that if returns to remittances are not increasing at this point, the condition is automatically satisfied.

Let us finish with the analysis of the α, that is the relative magnitude of the wage risk in the host country compared to the origin country.

Proposition 9. *Under corner savings, remittances increase with α if and only if* $\gamma \frac{\partial \widetilde{y}}{\partial \alpha}\frac{\partial \widetilde{y}}{\partial r}\left[\frac{\partial^2 \widetilde{y}}{\partial \alpha \partial r}\right]^{-1} < 1.$

Proof. The sign of $\frac{\partial r^*}{\partial \alpha}$ is given by the sign of

$$U_{r\alpha} = \delta\left[u_2''\frac{\partial \widetilde{y}}{\partial \alpha}\frac{\partial \widetilde{y}}{\partial r} + u_2'\frac{\partial^2 \widetilde{y}}{\partial \alpha \partial r}\right],$$

where $\frac{\partial^2 \widetilde{y}}{\partial \alpha \partial r} > 0$, as shown in the Appendix (proof Proposition 6). Hence,

$$U_{r\alpha} > 0 \iff \gamma \frac{\partial \widetilde{y}}{\partial \alpha}\frac{\partial \widetilde{y}}{\partial r}\left[\frac{\partial^2 \widetilde{y}}{\partial \alpha \partial r}\right]^{-1} < 1.$$

□

Corollary 1. *Under corner savings, remittances increase with α if $\frac{\partial \sigma_y^2}{\partial \alpha} > 0$.*

Proof. Provided in the Appendix. □

To conclude this corollary about the effect of α under corner savings, let us discuss $\frac{\partial \sigma_y^2}{\partial \alpha}$. It is shown in the Appendix that this condition is satisfied provided that α is sufficiently large. The intuition is the following. If the migrant has interior remittances, then the initial distribution of risk has to be sufficiently biased towards the host region so as to trigger precautionary remittances.

6 Concluding remarks

In this paper, we provide a theory of the migrant's decision to remit under uncertainty and link it to the future location decision. The model captures two crucial aspects of remittances, namely that they can be considered as pure and/or as precautionary investments. To this end, and since we focus on risk considerations, we first analyze the pure investment motive as a benchmark, and then introduce risk aversion, which needs to be disentangled from intertemporal consumption smoothing.

Under the pure investment motive, remittances are only used as long as their marginal return is larger than the return to savings. Interior savings and remittances cannot be obtained at equilibrium under the case where the marginal returns to remittances are increasing. This case is relevant and is notably due to the endogeneity of the location choice.

Once risk aversion is introduced, other considerations enter the picture. We show that remittances affect the risk faced by migrants in a more complex way than usually acknowledged. On the one hand, remittances have a non-monotonic effect on aggregate risk. On the other hand, their impact on the geographical location of risk might be counter-intuitive. In particular, remittances, by increasing the probability of return migration, increase the migrant's exposure to risk in the origin region, which contributes to deteriorate her exit option rather than improve it. In fact, the genuine self-insurance device is the location choice rather than remittances. Furthermore, the more risk is evenly distributed across locations, the more powerful the location decision to mitigate this risk.

The model also provides insights about the role played by the migrant's characteristics. For instance, the initial distribution of risk between host and origin countries affects the remitting behavior of migrants. Undocumented migrants, who are likely to face most of their uncertainty in the host country, are shown to be more likely to remit. Also, the wealth of the family left behind has a negative effect on remittances. However, migrants may face liquidity constraints, and these constraints have an impact on the link between remittances and the migrant's wage at the time of arrival in the host country. Under interior savings, the migrant's initial wage has no impact on remittances, whereas this impact is positive for constrained migrants.

Finally, let us discuss some of the assumptions of the model. First, we have assumed that the return to remittances can only be captured by the migrant in case of return, which makes sense if remittances are invested by relatives in social capital or in housing expenditures for example. One might object that the migrant might be able to capture at least partly the returns to remittances even though he/she remains in the host country. The main changes induced by this modification are to reduce the probability of return and therefore the intensity of the convexity of returns to remittances. However, convexity still holds despite this fact and results are qualitatively unchanged. Second, the migrant is assumed purely selfish. Introducing altruism towards the family left behind clearly provides an additional motive to remit. However, altruism reduces the degree of convexity in the returns to remittances since the parent's utility is concave in remittances. Unlike the first assumption discussed, convexity in the returns to remittances may disappear with the introduction migrant altruism. Third, the paper does not take into account the difference in purchasing powers between both locations. Technically speaking, incorporating this aspect is equivalent to assuming that migrant's marginal utility of consumption is conditional on location. If utility depends on consumption and location in an additive way, that is, under the form of a positive utility premium for returning in period 2, our results are unchanged since this would be captured in the parameter μ. However, if both aspects interact with each other, for instance in the case of different purchasing powers, the specification differs. Let us discuss here some of the implications of this modification. The main impact of this assumption is that the migrant's wealth, and her savings in particular, would have a different value according to the chosen location. It follows that the probability of return migration, which in our model solely depends on remittances, would positively depend on both remittances and savings, given that a higher wealth has a higher value in the origin country. As a result, the intuition behind increasing marginal returns to remittances would apply to savings as well: savings increase the probability of return and therefore have increasing returns. Moreover, in our basic framework, the two instruments serve different purposes: savings are essentially used to smooth consumption over time and remittances can be used as pure and/or precautionary investments. Introducing purchasing power parity would entangle both considerations and both instruments. In this perspective, adopting this assumption and, thereby, reconciling this model with the theory on precautionary savings would be an interesting avenue for future research.

References

Adams, Richard H, J. (1998). Remittances, investment, and rural asset accumulation in pakistan. *Economic Development and Cultural Change*, 47(1), 155–73.

Amuedo-Dorantes, C. & Pozo, S. (2006). Remittances as insurance: evidence from mexican immigrants. *Journal of Population Economics*, 19(2), 227–254.

Bernheim, B. D., Shleifer, A., & Summers, L. H. (1985). The strategic bequest motive. *Journal of Political Economy*, 93(6), 1045–76.

Brown, R. P. C. (1997). Estimating remittance functions for pacific island migrants. *World Development*, 25(4), 613–626.

Calero, C., Bedi, A. S., & Sparrow, R. (2009). Remittances, liquidity constraints and human capital investments in ecuador. *World Development*, 37(6), 1143 – 1154.

Cox, D. (1987). Motives for private income transfers. *Journal of Political Economy*, 95(3), 508–46.

de la Briere, B., Sadoulet, E., de Janvry, A., & Lambert, S. (2002). The roles of destination, gender, and household composition in explaining remittances: an analysis for the dominican sierra. *Journal of Development Economics*, 68(2), 309–328.

Demonsant, J.-L. (2007). Family prestige as old-age security: Evidence from rural senegal. (EC200802).

Djajic, S. (2010). Investment opportunities in the source country and temporary migration. *Canadian Journal of Economics*, 43, 663–682.

Djajic, S. & Milbourne, R. (1988). A general equilibrium model of guest-worker migration : The source-country perspective. *Journal of International Economics*, 25(3-4), 335–351.

Dustmann, C. (2003). Return migration, wage differentials, and the optimal migration duration. *European Economic Review*, 47(2), 353–369.

Dustmann, C., Fasani, F., & Speciale, B. (2010). Remittance behaviour of undocumented migrants. *Unpublished*.

Dustmann, C. & Kirchkamp, O. (2002). The optimal migration duration and activity choice after re-migration. *Journal of Development Economics*, 67(2), 351–372.

Dustmann, C. & Mestres, J. (2009). Remittances and temporary migration. *Journal of Development Economics*, Forthcoming.

Edwards, A. C. & Ureta, M. (2003). International migration, remittances, and schooling: evidence from el salvador. *Journal of Development Economics*, 72(2), 429–461.

Fafchamps, M. (2002). Returns to social network capital among traders. *Oxford Economic Papers*, 54(2), 173–206.

Funkhouser, E. (1995). Remittances from international migration: A comparison of el salvador and nicaragua. *The Review of Economics and Statistics*, 77(1), 137–46.

Gallego, J. M. & Mendola, M. (2009). Labor migration and social networks participation in southern mozambique. *Centro Studi Luca d'Agliano Development Working Papers*, 279.

Gollier, C. (2004). *The Economics of Risk and Time*. MIT Press.

Hill, J. K. (1987). Immigrant decisions concerning duration of stay and migratory frequency. *Journal of Development Economics*, 25(1), 221–234.

Hoddinott, J. (1994). A model of migration and remittances applied to western kenya. *Oxford Economic Papers*, 46(3), 459–76.

Kimball, M. & Weil, P. (2009). Precautionary saving and consumption smoothing across time and possibilities. *Journal of Money, Credit and Banking*, 41(2-3), 245–284.

Kreps, D. M. & Porteus, E. L. (1978). Temporal resolution of uncertainty and dynamic choice theory. *Econometrica*, 46(1), 185–200.

Lucas, R. E. B. & Stark, O. (1985). Motivations to remit: Evidence from botswana. *Journal of Political Economy*, 93(5), 901–18.

Mesnard, A. (2004). Temporary migration and capital market imperfections. *Oxford Economic Papers*, 56(2), 242–262.

Osili, U. O. (2004). Migrants and housing investments: Theory and evidence from nigeria. *Economic Development and Cultural Change*, 52(4), 821–49.

Osili, U. O. (2007). Remittances and savings from international migration: Theory and evidence using a matched sample. *Journal of Development Economics*, 83(2), 446–465.

Rapoport, H. & Docquier, F. (2006). The economics of migrants' remittances. *Handbook on the Economics of Giving, Reciprocity and Altruism*, 1, 1135–1198.

Selden, L. (1978). A new representation of preferences over "certain x uncertain" consumption pairs: The "ordinal certainty equivalent" hypothesis. *Econometrica*, 46(5), 1045–60.

Sinning, M. G. (2009). Determinants of savings and remittances: empirical evidence from immigrants to germany. *Review of Economics of the Household*.

Stark, O., Helmenstein, C., & Yegorov, Y. (1997). Migrants' savings, purchasing power parity, and the optimal duration of migration. *International Tax and Public Finance*, 4(3), 307–324.

Stark, O. & Levhari, D. (1982). On migration and risk in ldcs. *Economic Development and Cultural Change*, 31(1), 191–96.

Yang, D. (2006). Why do migrants return to poor countries? evidence from philippine migrants' responses to exchange rate shocks. *The Review of Economics and Statistics*, 88(4), 715–735.

Yang, D. (2008). International migration, human capital, and entrepreneurship: Evidence from philippine migrants exchange rate shocks. *The Economic Journal 118*, 118(531), 591–630.

Yang, D. (2009). International migration and human development. *Working paper series of the University Library of Munich*, (19212).

Appendix 0

Taking into account the borrowing constraint and the non-negativity constraint for the family investment, the parent's objective can be written in the form of a Lagrangian:

$$L = v\left(w_i + r - x + b\right) + \delta v\left(w_p + Rx - b\right) - \lambda\left(b - \bar{b}\right) + \nu x.$$

Kuhn-Tucker conditions are

$$\begin{aligned} L_x &= -v_1' + \delta v_2' R + \nu = 0, \\ L_b &= v_1' - \delta v_2' - \lambda = 0, \\ KT_b &: \lambda\left(b - \bar{b}\right) = 0, \\ KT_x &: \nu x = 0. \end{aligned}$$

Rearranging, one obtains

$$\begin{aligned} v_1' &= \delta v_2' R + \nu, \\ v_1' &= \delta v_2' + \lambda. \end{aligned}$$

Hence, $\delta v_2' \left(R - 1\right) = \left(\lambda - \nu\right)$, which implies that $\lambda > \nu \iff R > 1$. It follows that the borrowing constraint is always the first to be binding.

Appendix 1: Proof of Lemma 1

For the interior solution to the system of equations determined by the two first order conditions to be a maximum, the Hessian of U has to be definite negative, i.e. second derivatives are negative and the determinant of the Hessian is positive. The Hessian writes

$$H = \begin{bmatrix} U_{ss} & U_{sr} \\ U_{rs} & U_{rr} \end{bmatrix},$$

where

$$
\begin{aligned}
U_{ss} &= u_1'' + \delta u_2'' \tau^2 < 0, \\
U_{sr} &= U_{rs} = u_1'' + \delta u_2'' \tau \frac{\partial E(y)}{\partial r} < 0, \\
U_{rr} &= u_1'' + \delta u_2'' \left(\frac{\partial E(y)}{\partial r} \right)^2 + \delta u_2' \frac{\partial^2 E(y)}{\partial r^2}.
\end{aligned}
$$

By the arbitrage condition (10),

$$
\begin{aligned}
U_{ss} &= U_{sr} = U_{rs} \equiv \Psi, \\
U_{rr} &= \Psi + \delta u_2' \frac{\partial^2 E(y)}{\partial r^2},
\end{aligned}
$$

The determinant of the Hessian is positive if and only if

$$
\begin{aligned}
U_{ss}U_{rr} - U_{sr}U_{rs} &= \Psi \left(\Psi + \delta u_2' \frac{\partial^2 E(y)}{\partial r^2} \right) - \Psi^2 = \Psi \delta u_2' \frac{\partial^2 E(y)}{\partial r^2} > 0 \\
\Longleftrightarrow \quad & \frac{\partial^2 E(y)}{\partial r^2} = f(Rx^*) \left(R \frac{\partial x^*}{\partial r} \right)^2 + RF(Rx^*) \frac{\partial^2 x^*}{\partial r^2} < 0 \\
\Longrightarrow \quad & \frac{\partial^2 x^*}{\partial r^2} < 0.
\end{aligned}
$$

Appendix 2: Proof of Proposition 2

Recalling the expression of the variance of y:

$$
\sigma_y^2 = \int_{-\infty}^{Rx^*} \left[Rx^* - (1-\alpha) Z \right]^2 dF(Z) + \int_{Rx^*}^{+\infty} (\alpha Z)^2 dF(Z) - \left[E(y) - w + \alpha \frac{1}{2} \right]^2 .
$$

The derivative of this expression with respect to remittances writes $\frac{\partial \sigma_y^2}{\partial r} = R \frac{\partial x^*}{\partial r} \frac{\partial \sigma_y^2}{\partial Rx}$, where, making use of the Leibniz's rule and of equation (9),

$$
\frac{\partial \sigma_y^2}{\partial Rx} = 2F(Rx^*) \left\{ Rx^* - (1-\alpha) E(Z \mid Z < Rx^*) - \left[E(y) - w + \alpha \frac{1}{2} \right] \right\},
$$

where

$$
w - \alpha \frac{1}{2} + Rx^* - (1-\alpha) E(Z \mid Z < Rx^*) = E(y \mid Z < Rx^*),
$$

by definition of y. Hence,

$$\begin{aligned}
\frac{\partial \sigma_y^2}{\partial r} &= 2R\frac{\partial x^*}{\partial r}F(Rx^*)\left[E(y \mid Z < Rx^*) - E(y)\right] < 0 \\
&\Longleftrightarrow E(y \mid Z < Rx^*) < E(y) \\
&\Longleftrightarrow E(y \mid Z < Rx^*) < E(y \mid Z > Rx^*) \\
&\Longleftrightarrow w - \alpha\frac{1}{2} + Rx^* - (1-\alpha)\,E(Z \mid Z < Rx^*) < w - \alpha\frac{1}{2} + \alpha E(Z \mid Z > Rx^*) \\
&\Longleftrightarrow \alpha > \frac{Rx - E(Z \mid Z < Rx)}{E(Z \mid Z > Rx) - E(Z \mid Z < Rx))} \in [0,1]\,.
\end{aligned}$$

Appendix 3: Proof of Lemma 2

Applying Cramer's rule to the system of first order conditions, comparative statics of the interior solution with respect to any parameter ϕ are given by

$$\begin{aligned}
\frac{\partial r^*}{\partial \phi} &= \frac{U_{s\phi}U_{rs} - U_{ss}U_{r\phi}}{|H|}, \\
\frac{\partial s^*}{\partial \phi} &= \frac{U_{sr}U_{r\phi} - U_{s\phi}U_{rr}}{|H|}.
\end{aligned}$$

where

$$\begin{aligned}
U_{ss} &= u_1'' + \delta u_2'' \tau^2 < 0, \\
U_{sr} &= U_{rs} = u_1'' + \delta u_2'' \tau \frac{\partial \tilde{y}}{\partial r} < 0, \\
U_{rr} &= u_1'' + \delta u_2'' \left(\frac{\partial \tilde{y}}{\partial r}\right)^2 + \delta u_2' \frac{\partial^2 \tilde{y}}{\partial r^2}.
\end{aligned}$$

By the arbitrage condition, $\frac{\partial \tilde{y}}{\partial r} = \tau$, hence

$$\begin{aligned}
U_{ss} &= U_{sr} = U_{rs} \equiv \Omega, \\
U_{rr} &= \Omega + \delta u_2' \frac{\partial^2 \tilde{y}}{\partial r^2}.
\end{aligned}$$

Moreover, given that the determinant of the Hessian is positive at an interior maximum, we have that

$$\begin{aligned}
\frac{\partial r^*}{\partial \phi} &= \frac{\Omega}{|H|}\left(U_{s\phi} - U_{r\phi}\right) > 0 \iff U_{r\phi} - U_{s\phi} > 0, \\
\frac{\partial s^*}{\partial \phi} &= \frac{\Omega U_{r\phi} - U_{s\phi}U_{rr}}{|H|} > 0 \iff \Omega\left(U_{r\phi} - U_{s\phi}\right) - U_{s\phi}\delta u_2' \frac{\partial^2 \tilde{y}}{\partial r^2} > 0.
\end{aligned}$$

Appendix 4: Proof of Proposition 4

In order to applying Lemma 2, we need the following elements:

$$U_{rw_1} = U_{sw_1} = -u_1''.$$

$$\begin{aligned} \frac{\partial r^*}{\partial w_1} &= 0, \text{ since } U_{rw_1} - U_{sw_1} = 0, \\ \frac{\partial s^*}{\partial w_1} &> 0, \text{ since } \Omega \left(U_{rw_1} - U_{sw_1} \right) - U_{sw_1} \delta u_2' \frac{\partial^2 \tilde{y}}{\partial r^2} = \delta u_2' u_1'' \frac{\partial^2 \tilde{y}}{\partial r^2} > 0, \end{aligned}$$

where use has been made of the fact that $|H| > 0 \iff \frac{\partial^2 \tilde{y}}{\partial r^2} < 0$.[12]

Appendix 5: Proof of Proposition 5

$$U_{sw_i} = \delta \tau u_2'' \frac{\partial \widetilde{y}}{\partial x} \frac{\partial x^*}{\partial w_i} = \delta \tau u_2'' \frac{\partial \widetilde{y}}{\partial r} = \Omega - u_1'',$$

since $\frac{\partial x^*}{\partial w_i} = \frac{\partial x^*}{\partial r}$ and by arbitrage. After substituting $\frac{\partial x^*}{\partial r}$ for $\frac{\partial x^*}{\partial w_i}$, U_{rw_i} writes

$$\begin{aligned} U_{rw_i} &= \delta u_2'' \left(\frac{\partial \widetilde{y}}{\partial r} \right)^2 + \delta u_2' \frac{\partial^2 \widetilde{y}}{\partial r^2} \\ &= \Omega + \delta u_2' \frac{\partial^2 \tilde{y}}{\partial r^2} - u_1''. \end{aligned}$$

It follows that

$$\begin{aligned} \frac{\partial r^*}{\partial w_i} &< 0, \text{ since } U_{rw_i} - U_{sw_i} = \delta u_2' \frac{\partial^2 \tilde{y}}{\partial r^2} < 0 \\ \frac{\partial s^*}{\partial w_i} &> 0, \text{ since } \Omega \left(U_{rw_i} - U_{sw_i} \right) - U_{sw_i} \delta u_2' \frac{\partial^2 \tilde{y}}{\partial r^2} \\ &= \Omega \delta u_2' \frac{\partial^2 \tilde{y}}{\partial r^2} - \delta u_2' \frac{\partial^2 \tilde{y}}{\partial r^2} \left(\Omega - u_1'' \right) = \delta u_2' u_1'' \frac{\partial^2 \tilde{y}}{\partial r^2} > 0. \end{aligned}$$

Appendix 6: Proof of Proposition 6

In order to applying Lemma 2, we need the following elements:

$$\begin{aligned} U_{s\alpha} &= \delta \tau u_2'' \frac{\partial \widetilde{y}}{\partial \alpha}, \\ U_{r\alpha} &= \delta u_2'' \frac{\partial \widetilde{y}}{\partial r} \frac{\partial \widetilde{y}}{\partial \alpha} + \delta u_2' \frac{\partial^2 \widetilde{y}}{\partial r \partial \alpha}. \end{aligned}$$

[12] The proof is similar to the one developped in Appendix 1 under risk-neutrality.

By arbitrage, we have that

$$U_{r\alpha} - U_{s\alpha} = \delta u_2' \frac{\partial^2 \widetilde{y}}{\partial r \partial \alpha} > 0 \iff \frac{\partial^2 \widetilde{y}}{\partial x \partial \alpha} > 0.$$

Recalling

$$\frac{\partial \widetilde{y}}{\partial x} = F\left(Rx^*\right) R \left\{1 - \eta \left[E(y \mid Z < Rx^*) - E(y)\right]\right\}.$$

The cross derivative is then given by

$$\begin{aligned} \frac{\partial^2 \widetilde{y}}{\partial x \partial \alpha} &= -F\left(Rx^*\right) R\eta \left[\frac{\partial E(y \mid Z < Rx^*)}{\partial \alpha} - \frac{\partial E(y)}{\partial \alpha}\right] > 0 \\ &\iff \frac{\partial E(y \mid Z < Rx^*)}{\partial \alpha} - \frac{\partial E(y)}{\partial \alpha} < 0. \end{aligned}$$

Recall that

$$E\left(y\right) = w - \alpha \frac{1}{2} + \int_{-\infty}^{Rx^*} \left[Rx^* - (1-\alpha) Z\right] dF(Z) + \int_{Rx^*}^{+\infty} \alpha Z dF(Z).$$

The derivative of this expression with respect to α is

$$\frac{\partial E(y)}{\partial \alpha} = -\frac{1}{2} + \int_{-\infty}^{Rx^*} Z dF(Z) + \int_{Rx^*}^{+\infty} Z dF(Z) = 0.$$

Therefore,

$$\frac{\partial r^*}{\partial \alpha} > 0 \iff \frac{\partial E(y \mid Z < Rx^*)}{\partial \alpha} < 0,$$

where, since

$$\begin{aligned} E(y \mid Z < Rx^*) &= w - \alpha \frac{1}{2} + \frac{1}{F\left(Rx^*\right)} \int_{-\infty}^{Rx^*} \left[Rx^* - (1-\alpha) Z\right] dF(Z), \\ \frac{\partial E\left(y | Z < Rx^*\right)}{\partial \alpha} &= -\frac{1}{2} + \frac{1}{F\left(Rx^*\right)} \int_{-\infty}^{Rx^*} Z dF(Z) \\ &= E(Z \mid Z < Rx^*) - E(Z) < 0. \end{aligned}$$

Proof and discussion of Corollary 1

Since $\frac{\partial \widetilde{y}}{\partial r} > 0$ and $\left[\frac{\partial^2 \widetilde{y}}{\partial \alpha \partial r}\right]^{-1} > 0$,

$$\frac{\partial \widetilde{y}}{\partial \alpha} < 0 \implies \gamma \frac{\partial \widetilde{y}}{\partial \alpha} \frac{\partial \widetilde{y}}{\partial r} \left[\frac{\partial^2 \widetilde{y}}{\partial \alpha \partial r}\right]^{-1} < 0 < 1,$$

which implies that $\frac{\partial r^*}{\partial \alpha} > 0$. Let us now analyze the condition that $\frac{\partial \widetilde{y}}{\partial \alpha} < 0$ and show that it is equivalent to $\frac{\partial \sigma_y^2}{\partial \alpha} > 0$.

$$\begin{aligned} \frac{\partial \widetilde{y}}{\partial \alpha} &= \frac{\partial E(y)}{\partial \alpha} - \frac{1}{2}\eta\frac{\partial \sigma_y^2}{\partial \alpha} \\ &= -\frac{1}{2}\eta\frac{\partial \sigma_y^2}{\partial \alpha}, \end{aligned}$$

since the expected income does not depend on the distribution of the risk between locations:

$$\frac{\partial E(y)}{\partial \alpha} = -\frac{1}{2} + \int_{-\infty}^{Rx} ZdF(Z) + \int_{Rx}^{+\infty} ZdF(Z) = 0.$$

Therefore,

$$\frac{\partial \widetilde{y}}{\partial \alpha} < 0 \iff \frac{\partial \sigma_y^2}{\partial \alpha} > 0 \text{ QED}.$$

To conclude this corollary about the effect of α under corner savings, let us analyze $\frac{\partial \sigma_y^2}{\partial \alpha}$ in details.

$$\begin{aligned} \frac{\partial \sigma_y^2}{\partial \alpha} &= \int_{-\infty}^{Rx} 2\left[Rx - (1-\alpha) Z\right] ZdF(Z) + \int_{Rx}^{+\infty} 2\alpha Z^2 dF(Z) \\ &= 2\int_{-\infty}^{Rx} RxZdF(Z) - 2\int_{-\infty}^{Rx} Z^2 dF(Z) + 2\alpha\int_{-\infty}^{Rx} Z^2 dF(Z) + 2\alpha\int_{Rx}^{+\infty} Z^2 dF(Z) \\ &= 2RxF\left(Rx\right) E\left(Z|Z < Rx\right) - 2F\left(Rx\right) VAR\left(Z|Z < Rx\right) + 2\alpha VAR\left(Z\right) \\ &= 2F\left(Rx\right)\left[RxE\left(Z|Z < Rx\right) - VAR\left(Z|Z < Rx\right)\right] + 2\alpha VAR\left(Z\right) \end{aligned}$$

Therefore,

$$\frac{\partial \sigma_y^2}{\partial \alpha} > 0 \iff \alpha > \widetilde{\alpha} \equiv \frac{F\left(Rx\right)\left[VAR\left(Z|Z < Rx\right) - RxE\left(Z|Z < Rx\right)\right]}{VAR\left(Z\right)}.$$

This condition can be compared to the condition on interior remittances. Indeed, if the migrant has interior remittances, this means that the initial distribution of wage risks is sufficiently biased towards the host region so as to trigger precautionary remittances. Formally, in order to have interior remittances, α has to be larger than α_r which is defined in equation (16). Therefore, under interior remittances, $\frac{\partial \sigma_y^2}{\partial \alpha} > 0$ is always true if $\alpha_r < \widetilde{\alpha}$, that is,

$$\frac{Rx - E(Z \mid Z < Rx)}{E(Z \mid Z > Rx) - E(Z \mid Z < Rx))} < \frac{F\left(Rx\right)\left[VAR\left(Z|Z < Rx\right) - RxE\left(Z|Z < Rx\right)\right]}{VAR\left(Z\right)},$$

or equivalently,

$$\begin{aligned} & \left(E(Z|Z > Rx) - E(Z|Z < Rx)\right) F \left(VAR\left(Z|Z < Rx\right) - RxE(Z|Z < Rx)\right) \\ > \;& V\left(Rx - E(Z|Z < Rx)\right). \end{aligned}$$

A necessary condition for this to be true can be obtained by replacing Rx by $E(Z|Z < Rx)$. This necessary condition is

$$\left(E(Z|Z > Rx) - E(Z|Z < Rx)\right) F \left(VAR\left(Z|Z < Rx\right) - E(Z|Z < Rx)^2\right) > 0,$$

which is equivalent to

$$VAR\left(Z|Z < Rx\right) > E(Z|Z < Rx)^2,$$

or

$$E(Z^2|Z < Rx) > 2E(Z|Z < Rx)^2.$$

CEPS
INSTEAD

B.P. 48
L-4501 Differdange
Tél.: +352 58.58.55-801
www.ceps.lu