Nonlinearities, Co-Trending and Budget Balance Sustainability

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Abstract: The study provides new evidence on the sustainability debate concerning the US budget balance. Existing studies are often only able to reject non-cointegration between government expenditure and revenue once structural breaks or regime changes are allowed for. Using a testing procedure advocated by Bierens, this study provides a different perspective on this issue. Government expenditure and revenue are both found to be stationary around a nonlinear deterministic trend, and they are co-trended insofar as they share a common nonlinear deterministic trend.

I. INTRODUCTION

The sustainability of fiscal policy has been judged by many in terms of whether or not the intertemporal budget constraint (IBC) holds in present value terms [see, for example, Hamilton and Flavin 1986]. Since the IBC is based on the equality of current debt with the sum of expected future discounted primary surpluses, sustainability is related to satisfaction of the IBC and whether or not fiscal policy is under control. Unsustainable fiscal policy, on the other hand, is characterized by violation of the IBC. This implies that at some time in the future, such policies will have to be changed; otherwise they will lead to the government becoming insolvent or to a collapse of the policy regime.

For many studies, it is important to view fiscal sustainability as a long-run concept insofar as the literature on budget deficit sustainability is primarily concerned with whether or not government’s IBC is violated. This approach relies on the underlying stability of past data processes. The empirical work in this area, however, offers many conflicting results. Whereas studies such as Hamilton and Flavin (1986) and Ahmed and Rogers (1995) find in favour of sustainability, Wilcox (1989) and Hakkio and Rush (1991) find against. Studies such as Tanner and Liu (1994) and Bajo-Rubio et al. (2008) point to the presence of sustainability where the US deficit process has undergone at least one structural shift during recent decades. Other studies such as Arestis et al. (2004) and Payne and Mohammadi (2006) point to possibility of threshold effects in the behaviour of the US budget deficit.

The study reconsiders the relationship between government expenditures (EXP) and receipts (REV). As argued below, the incorporation of breaks, regimes and thresholds has
enabled the existing literature to identify a long-run equilibrium or cointegrating relationship between $EXP$ and $REV$. This highlights the importance of nonlinearities. In sharp contrast to earlier cointegration-based work, this study provides an alternative assessment of the relationship between $EXP$ and $REV$ based on a testing procedure advocated by Bierens (1997a, 1997b, 2000). This procedure considers whether nonlinear trend stationarity is present in the behaviour of these two series and if so, whether they are co-trended sharing the same nonlinear deterministic trend.

The paper is structured as follows. The following section discusses relevant literature and sets out the Bierens approach. The third section reports and discusses the results. The findings here indicate that both $EXP$ and $REV$ are both nonlinear trend-stationary sharing the same non-linear deterministic trend. The final section concludes.

II. LITERATURE AND METHODOLOGY


The abovementioned studies of the US budget highlights the implausibility of arguing that the parameters of the relevant data generation processes for $EXP$ and $REV$ are unchanged over time. This study investigates the relationship between $EXP$ and $REV$ from a different perspective insofar as these series may both be stationary around a common nonlinear deterministic trend. The Bierens (1997a, 1997b) nonlinear augmented Dickey-Fuller (NLADF) test allows the trend to be an almost arbitrary deterministic function of time. The test is based on an ADF-
type auxiliary regression model that sees a nonlinear deterministic trend approximated by a linear function of Chebyshev polynomials. These offer substantial advantages over regular time polynomials because they are orthogonal (with a closed form) and bounded and allow the researcher to distinguish stationarity around a linear trend from stationarity around a nonlinear deterministic trend under the alternative hypothesis.

Suppose $\text{EXP}$ and $\text{REV}$ are each modelled as $\mu_t = \omega + \phi \mu_{t-1} + v_t$ where $\omega$ is a constant drift parameter and $v_t$ is a stationary autoregressive process. The usual test for linear adjustment towards mean is based assessing the unit root properties of $\mu_t$ through the OLS estimation of ADF regressions such as

$$\Delta \mu_t = \omega + \zeta \mu_{t-1} + \sum_{i=1}^{k} \psi_i \Delta \mu_{t-i} + v_t$$

(1)

where $-2 < \zeta < 0$ indicates stationarity of $\mu_t$. The test of the null hypothesis $\phi = 1$ proposed by Bierens is against the alternative of nonlinear trend stationarity:

$$\mu_t = g(t) + v_t$$

(2)

where $g(t)$ is a possibly nonlinear trend function. The NLADF regression is written as:

$$\Delta \mu_t = \omega + \zeta \mu_{t-1} + \sum_{i=1}^{p} \psi_i \Delta \mu_{t-i} + \theta^T P_{t,n}^{(m)} + v_t$$

(3)

where $P_{t,n}^{(m)} = (P_{0,n}^*(t), P_{1,n}^*(t), ..., P_{m,n}^*(t))^T$ is a vector of orthogonal Chebyshev polynomials such that $P_{0,n}^*(t) = 1$, $P_{1,n}^*(t)$ is equivalent to a linear trend and $P_{2,n}^*(t)$ through to $P_{m,n}^*(t)$ are cosine functions. Under the null hypothesis of a unit root, $\zeta = 0$ and $\theta^T = 0$. The unit root hypothesis can be tested on the basis of the $t$-statistic on $\zeta$. This is a two-tailed test so if the non-stationary null is rejected, the proper alternative hypothesis will depend on the whether there is left- or right-side rejection. A left-rejection favours the alternative of either mean stationarity, linear trend stationarity or nonlinear trend stationarity; whereas a right-rejection favours the alternative of nonlinear trend stationarity alone.

Although some macroeconomic time-series are not unit root processes, they might still behave as if they are cointegrated. This could be accounted for by the presence of a common nonlinear deterministic time trend. Bierens (2000) proposes a nonparametric test for nonlinear co-trending based on the eigenvalues of matrices constructed from the partial sums of the variables. The test is nonparametric in the sense that the nonlinear trends and any serial correlation process do not have to be specified. The generalized eigenvalues of the matrices $M_1$ and $M_2$ are:

$$\hat{M}_1 = \left( \frac{1}{n} \left[ F(1/n)F(1/n) + .... + F(1)F(1) \right] \right)$$

(4)

$$\hat{M}_2 = \left( \frac{1}{n} \left[ dF(1/n)dF(1/n) + .... + dF(1)dF(1) \right] \right)$$

1 Bierens (2000) considers nonlinear co-trending in the context of inflation and interest rates in the US. Further applications include Camarero and Ordonez (2006) who consider European unemployment rates.
with $F(t/n) = (1/n) [\chi_1 + \ldots + \chi_t]$, where $\chi_t$ is the de-trended or demeaned $\mu_t$, and $m = n^a$ with $n$ equal to the number of usable observations. Solving $|\hat{\lambda}_1 - \lambda_2| = 0$ for $\lambda$, the test statistics are calculated as $n^{1-a} \hat{\lambda}$, where $r$ is the number of co-trending vectors under the null. The existence of $r$ co-trending vectors among $r + 1$ series indicates the presence of $r$ linear combinations that are stationary around a linear trend where these series share a single $((r + 1) - r)$ common nonlinear deterministic time trend. This is indicative of a strong degree of co-movement across the $r + 1$ series.

### III. DATA AND RESULTS

Quarterly data for Government current receipts and current expenditures for the study period 1947Q1-2009Q4 are obtained from the Bureau of Economic Analysis. REV and EXP are both expressed as a percentage of GDP. The two series are plotted in Figure 1 which points to periods where they have moved together as well as drifted apart. While both REV and EXP have generally increased in size over the study period, the episodes of sharp swings in their relationship highlights the potential for regime switches and structural breaks. Figure 1 also plots the budget balance ($\text{EXP}$ minus $\text{REV}$) expressed as a percentage of GDP and shows the contrasting experiences of surplus in the early parts of the study period and late 1990s-2001 as well as general deficits in during much of the later years.

*Figure 1: US Government Expenditure, Revenue and Budget Balance, 1947Q1-2009Q4*

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2 Estimation is conducted using the EasyReg International software made available by Herman Bierens.

3 Bierens (1997a) argues there is no definitive method for choosing $m$. If $m$ is too low, it may be insufficient to approximate the nonlinearity under the alternative. If $m$ is too high, it may cause the test to lack power.
Table 1 reports Augmented Dickey Fuller (ADF) and DF-GLS unit root tests conducted on the REV and EXP. In each case, the non-stationary null cannot be rejected at the 10% significance level. Table 2 reports results based on the Perron (1997) unit root tests that allow for a single (unknown) structural break. Three models are considered for both series, where the non-stationary null can only be rejected once in three cases at the 5% significance level in the case of REV. The estimation of equation (1) as a potential cointegrating relationship by fully modified OLS (FMOLS) provides the estimate \( \text{EXP}_t = -8.837 + 1.360\text{REV}_t + \mu_t \). While \( \beta > 0 \) might lend support to the notion of a long-run relationship involving EXP and REV, Table 3 reports tests for non-cointegration based on equation (1) using the procedures advocated by Engle and Granger (1987) and Phillips and Ouliaris (1990). At best, one is able to reject the non-cointegration null at the 10% significance level in only one of the two tests.

One possibility is that potential structural breaks have not been allowed for and this is contributing to the presence of low test power in the tests for non-cointegration. Table 4 reports three Gregory and Hansen (1996) non-cointegration tests based on structural breaks in the constant and/or linear trend. Although mixed, the results here offer some evidence in support of cointegration with rejections of the null in one of the three models considered. In contrast to the abovementioned studies by Tanner and Liu (1994), Payne and Mohammadi (2006) and others, break dates of 1975Q4 and 1996Q2 appear to be of more importance than dates from the 1980s.

Table 5 presents NLADF test results based on the auxiliary regression in equation (3). The lag length \( p \) is chosen using the SIC and the Chebishev time polynomial is set from \( m=20 \). This test can potentially present substantial size distortion so relevant critical values are simulated using a wild bootstrap based on 5000 replications of a Gaussian AR\(_m\) process for \( \Delta \mu_t \) with parameters and error variance equal to the estimated AR\(_m\) null model. According to the \( t\)-stat test, there is a right hand side rejection of the unit root hypothesis in favor of nonlinear trend stationarity at the 90% significance level or better in both cases. The estimated values for \( \zeta \) are -0.296 and -0.300 provide approximated half-lives associated with a deviation from the long-run nonlinear deterministic trend value of about 2 quarters for each series.4

So far, the results indicate that both EXP and REV are non-linear trend stationary. The co-trending test results presented in Table 6 point to the existence of one co-trending vector \( (r=1) \). Whereas the existing literature has struggled to find a long-run equilibrium relationship between EXP and REV unless breaks are allowed for, the evidence here of a single linear combination of the EXP and REV that is stationary around a nonlinear trend suggests that these two series share a common nonlinear deterministic time trend where common trending behavior would appear to be a reasonable statistical characterization. While this should not necessarily be interpreted as causality, the co-trending vector can be written in terms of REV where a positive coefficient of 1.733 confirms the two series moving together over time with a tendency for EXP to exceed REV as demonstrated by a general deficit on the budget balance over the period of study.

4 Approximated as \( (\ln 0.5)/\ln(1+\zeta) \)
IV. SUMMARY AND CONCLUSIONS

This study has provided an alternative perspective on the sustainability of the US budget balance. In contrast to tests based on the null of non-cointegration, we find that government expenditure and revenue are stationary around a nonlinear trend and they can be regarded as related insofar as they share a common nonlinear deterministic time trend. These characteristics are consistent with earlier work that has highlighted the role played by structural breaks and regime change.

Table 1: Unit Root Tests on US Government Total Expenditure and Revenue

<table>
<thead>
<tr>
<th></th>
<th>ADF</th>
<th>DF-GLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>REV</td>
<td>-1.819</td>
<td>-0.896</td>
</tr>
<tr>
<td>EXP</td>
<td>-1.331</td>
<td>0.593</td>
</tr>
</tbody>
</table>

Notes: These are ADF and Elliot et al. (1996) DF-GLS unit root tests conducted on the US government total expenditure and revenue (excluding a deterministic trend). The lag lengths are based on the SIC where the respective 10% critical values are -2.57 and -1.62.

Table 2: Perron (1997) Unit Root Tests

<table>
<thead>
<tr>
<th></th>
<th>IO1</th>
<th>IO2</th>
<th>AO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_b$</td>
<td>$t_{\alpha}$</td>
<td>$T_b$</td>
</tr>
<tr>
<td>EXP</td>
<td>1995Q1</td>
<td>-4.229</td>
<td>1979Q2</td>
</tr>
<tr>
<td>REV</td>
<td>2001Q1</td>
<td>-5.091*</td>
<td>2001Q1</td>
</tr>
</tbody>
</table>

The models are the Innovational Outlier model (IO1) incorporating a change in the intercept, the Innovational Outlier model (IO2) incorporating a change in the intercept and the slope, and the Additive Outlier (AO) model incorporating a change in the slope only, but both segments of the trend function are joined at the time break. $T_b$ denotes the time of the break and $t_{\alpha}$ denotes the test statistic for a unit root. The respective 5 then 10% critical values are -5.10, -5.55, -4.65 then -4.82, -5.25, -4.38 for each model in turn. * and ** respectively denote rejection of the non-stationary null at the 10 and 5% significance levels.

Table 3: Cointegration Tests on US Government Total Expenditure and Revenue

<table>
<thead>
<tr>
<th></th>
<th>$\tau$ (Engle-Granger)</th>
<th>$\tau$ (Phillips-Ouliaris)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXP</td>
<td>-2.349 (0.351)</td>
<td>-3.121 (0.088)</td>
</tr>
</tbody>
</table>

Notes: $\tau$ (Engle-Granger) and $\tau$ (Phillips-Ouliaris) refer to the non-cointegration tests advocated by Engle and Granger (1987) and Phillips and Ouliaris (1990). In each case, $p$-values are reported in parentheses.
Table 4: Gregory and Hansen (1996) Cointegration Tests

<table>
<thead>
<tr>
<th>Level break, no trend</th>
<th>Level break, trend</th>
<th>Full structural break</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_b$</td>
<td>$t_{\hat{\alpha}}$</td>
<td>$T_b$</td>
</tr>
<tr>
<td>1975Q4</td>
<td>-3.774</td>
<td>1996Q2</td>
</tr>
</tbody>
</table>

The 1 and 5% critical values are respectively -5.13 and -4.61 for the level break model with no trend, -5.45 and -4.99 for the level break model with trend, and -5.47 and -4.95 for the full structural break model. ** denotes rejection of the non-cointegration null at the 5% significance level. $T_b$ denotes the time of the break and $t_{\hat{\alpha}}$ denotes the minimum test statistic for a unit root. In each case, the lag length is determined by the SIC.

Table 5: NLADF Tests

<table>
<thead>
<tr>
<th>EXP</th>
<th>REV</th>
</tr>
</thead>
<tbody>
<tr>
<td>ζ</td>
<td>-0.296</td>
</tr>
<tr>
<td>t-stat</td>
<td>-6.384 (0.909)</td>
</tr>
</tbody>
</table>

Note: the simulated p-values are based on 5000 replications and are given in parentheses.

Table 6: Nonlinear Co-Trending Analysis

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Test statistic</th>
<th>10% critical value</th>
<th>5% critical value</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r = 1$</td>
<td>$r = 0$</td>
<td>0.076</td>
<td>0.352</td>
<td>0.466</td>
<td>Accept</td>
</tr>
<tr>
<td>$r = 2$</td>
<td>$r = 1$</td>
<td>1.612</td>
<td>0.536</td>
<td>0.674</td>
<td>Reject</td>
</tr>
</tbody>
</table>

Note: $r$ denotes the number of co-trending vectors.

REFERENCES


