STRUCTURE, CLEARINGHOUSES AND SYMMETRY

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Abstract

We introduce and justify a taxonomy for the structure of markets and minimal institutions which appear in constructing minimally complex trading structures to perform the functions of price formation, settlement and payments. Each structure is presented as a playable strategic market game and is examined for its efficiency, the number of degrees of freedom and the symmetry properties of the structure.

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Keywords: Strategic market games, clearinghouses, credit

Contents

1 The Properties of Money

In conventional economic theory there are three well-known properties of money. They are:

1. A means of payment,
2. A store of value
3. A numeraire.

There is a fourth property which can be seen when exchange is modeled as a playable game. That is:

4. Money as a way to distinguish agents.

The nature of the strategic actions an agent can take with respect to money may differentiate one agent from another. In particular government, private financial institutions such as banks and private natural persons have different actions they can take with respect to money. Agents are thus distinguished by the differences in their strategy sets with respect to their actions involving the creation and utilization of money.

2 The Approach Adopted

Why do the economies of societies utilize a money? Why is value attached to nearly worthless paper or to an individually valueless abstraction? Is it merely the force of the state as Knapp (1905) suggests? How important is the technology of trade and the role of the settlement mechanism? How are the two networks of trade and of clearance and financial settlement related?

Barter has been more or less replaced in modern economies by a system of trade involving the existence of markets for the exchange of goods and services for a symbolic good called money or for some form of money substitute. The goods and service markets are highly decentralized and, in general, clear individually. The money and credit instruments which are accepted in payment in the goods and service markets are for the most part accepted as appropriate stores of value by law, custom and credit check and, for the most part are sent by the markets (retailers, wholesalers and other trading posts such as auction houses) to the clearinghouses, banks, and other financial agencies which make up the credit evaluation and financial clearance system for the economy.

In the approach adopted here we attempt to define playable games portraying competitive markets together with their payment and clearance systems. In adopting a physical and financial engineering approach our aim is to be able to offer a way to make precise distinctions in monetary and financial payments mechanisms that are otherwise difficult to treat.

In this essay the stress is on the economics and institutional aspects of trade. In a companion essay (Smith and Shubik, 2003) we relate these observations and the solutions to the strategic market games to phenomena encountered in physics. In particular we show that an important distinction in physics, between locally and globally defined symmetries, arises in the solution of market games in the same mathematical form. In all cases symmetries reflect the presence of apparent strategic degrees of freedom, which actually have no effect on allocations. When these are locally defined, it is possible for agents individually to factor them out of decision making, and arrive at the noncooperative equilibria of the game. When they are globally defined, they are under the control of noone, and the uncertainty they create for each agent can make it impossible for the economy to arrive at any pure-strategy equilibrium.

2.1 Minimal Mechanisms

We are concerned with the minimal level of complexity at which a phenomenon of interest appears. This relatively imprecise statement will be clarified in Section 4 in examining the simplest mechanism for producing a market price.

2.2 Playable Games

The phenomena of money and financial institutions are essentially associated with dynamics. Trade is carried out by two interlinked networks that are usually considered as one; but are fruitfully worth considering as separate, but interlinked. They are the market network and the payments network. The first deals with the sale and purchase of goods and services and the second deals with the credit evaluation and the nature of the means of payment and settlement of accounts that accompany the trade.

In this work we represent the activities of trade and settlement by the means of fully defined playable games. Our concern is with games that can be played in a laboratory or classroom. This provides a useful debugging criterion to make sure that features that appear in the economic dynamics, are fully defined. These include items such as the specification of what constitutes a bid or offer, when and why bankruptcy or default conditions must be specified or how to specify what happens in a market where there are bids but no offers (mathematically, division by zero).

2.3 Dimensional Analysis, Conservation and Degrees of Freedom

Because it is so easy to become confused in attempting to model situations where all individuals can issue their own IOU notes we purposely adopt an approach whereby every financial instrument may be regarded as a poker chip of a different color. We are concerned with how and where they are created and destroyed; what is conserved and what bounds behavior in the system. A useful tool in checking the validity of the equations in the system and in counting degrees of freedom is dimensional analysis. This will help to clarify the role of apparently innocent assumptions concerning the selection of a numeraire and means of payment.

2.4 The Games within the Game

The complexity or simplicity of the model being examined depends considerably on the question being answered. If we are concerned with economic anthropology or history our concerns may be with the emergence of governmental and economic institutions. If we are concerned with shorter horizons, institutions may be assumed to be given.

The economy lies within the polity and the polity lies within its society. The time frame of the phenomena being studied is often critical in determining what aspects of the setting of the context for the game are stressed. In particular, if (as is implicit in our analysis here), the time frame is short, it is reasonable to treat institutions as given and the role of government parametrically here rather than treating government as an active strategic player.

3 The Jevons Failure of the Double Coincidence of Wants

Jevons (1875) provided a critical example illustrating the problems with barter. He considered three individuals trading in three commodities. He was able to select conditions to show that barter with utility improvement after each exchange was not sufficient to bring about optimal trade.

A simple illustration of his observation can be shown by considering 3 individuals, each with a utility function of the form $f(xyz)$ with initial endowments given by $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$. It is easy to observe that when we consider only pairwise trade (as is envisioned in barter) there is no motivation for a single round of trade. If there were three markets, where in each market individuals could bid IOU notes in exchange for one of the three commodities, optimal trade could be achieved with one simultaneous action by all. This however would also require a centralized clearinghouse which would perform the credit evaluation of the notes and the netting of the bids and receipts. The appropriate default laws together with a courthouse are needed to resolve default, if the clearinghouse is unable to clear. These institutions are needed to provide the functions of credit evaluation, and settlement of contract in the case of failure. There may be many different institutional arrangements that provide these functions. The provision of the functions is a logical necessity; the form of the specific institutions required may depend on fine details and institutional happenstance. We explore the structure and properties of several financial mechanisms and consider how they might influence the strategic freedom of the individuals and the intrinsic symmetry of the roles of the individuals in the underlying economy.

The simple exchange economy described above is completely symmetric and should have a symmetric outcome. In this essay we examine many variations of trading mechanisms applied to an extension of this simple model where there may be n goods and n types of trader and where there may be r traders of each type. We can then observe the influence of competition on the solution as r becomes large.

In our analysis we contrast the generalized Jevons example with the situation in which all individuals own a general supply of all commodities, i.e., we consider both the extended Jevons' example aI,which we denote by a_{spec}. (A five good example is shown in Table 1 and a more general endowment (a_i^j) denoted by a_{gen}.)[1]

a	0	0	0	0
0	a	0	0	0
0	0	a	0	0
0	0	0	a	0
0	0	0	0	a

Table 1:

It is worth noting that in a modern economy the model where an individual has only one good or service for sale is probably closer to reality than the model where she has endowments of all of the goods and services for sale.

4 Strategic Market Games and Payment Systems

In the past few years, one of us together with several colleagues (Shubik, 1999a,b) developed a set of games called strategic market games. They are fully formulated games which can be played, simulated and analyzed. They serve to contrast a game theory and gaming approach to the functioning of competitive markets for the exchange of individually owned economic goods and services with the general equilibrium analysis (see Debreu, 1959) The stress in the formulation of strategic market games is

[1]If all $a_i^j = a$ and utilities are the same there will be no trade, but a shadow price can be established.

upon the explicit formulation of the mechanisms for trade: in other words, upon the complete definition of the state space or the outcome set.

A strategic market game for traders $i \in N$, with endowments $a_i \in \Omega^m$ and utility functions $u_i : \Omega^m \to R$, is defined by specifying the market structure M and the strategy sets S_i for each $i \in N_i$.

Under fairly weak assumptions indicated below, it can be shown that noncooperative equilibria exist for all of the class of games described above (Dubey and Shubik, 1978), where the participants in the markets use only simple messages. The conditions required are:

(A) Each u_i is concave and nondecreasing in each variable.

(B) $\sum_{i \in N} a_i > 0$.

(C) For any $(j, k) \in M$, there exist at least two traders who have positive endowments of j and desire k, as well as two traders who have positive endowments of k and desire j. (A trader i is said to desire commodity j if u_i is a strictly increasing function of the jth variable.)

4.1 The Simplest Price Formation Mechanism and the Sell-all Game.

In Section 2.1. we noted the concept of a minimal mechanism. Here we provide an example of the minimal price formation mechanism. The dimensions of price are the amounts of Good A offered in exchange for the amounts of Good B. The simplest game we can construct that will produce a price is one that has a single simultaneous move by all traders. In particular this guarantees that there are no strategic contingencies. A move and a strategy by all participants are identical.

Assume that there are n traders trading in $m + 1$ goods, where the $m + 1$st good has a special operational role, in addition to its possible utility in consumption. We attribute to each trader an initial bundle of goods

$$a_i = (a_i^1 ..., a_i^m, a_i^{m+1}),$$

where $a_i^j \geq 0$ for all $j = 1, ..., m+1$ and $a^j > 0$ for at least two traders and a concave utility function

$$u_i = (x_i^1, ..., x_i^m, x_i^{m+1}),$$

We emphasize that u_i need not actually depend on x_i^{m+1}; the possibility of a fiat money is not excluded.

The general procedure is for the traders to put up quantities of the first m goods to be sold, and simultaneously to put up quantities of the $m + 1$st good to buy them, all at prices determined by the market-wide supply and demand for each good.

Price Formation. The prices now emerge in a natural way, as a result of the simultaneous bids of all buyers; we define

$$p^j = \sum_{i=1}^{n} b_i^j / \sum_{i=1}^{n} a_i^j.$$

Bids precede prices. Traders allocate their budgets fiscally, committing quantities of their means of payment to the purchase of each good without definite knowledge of what the per-unit price will be. At an equilibrium this will not matter, as prices will be what the traders expect them to be. In a multi-period context, moreover, the traders will know the previous prices and may expect that fluctuations in individual behavior in a mass market will not change prices by much. But any deviation from expectations will result in changing the quantity of goods received, rather than the quantities of cash spent. In practice, if one allocates a portion of one's budget for purchase of a certain good in a mass market, this will be different — but not too different — from a decision to buy a specific amount at an unspecified price.

The prices are so determined that they will exactly balance the books at each trading post. The amount of the jth good that the ith trader receives in return for his bid is: $x_i^j = b_i^j / p^j$.

The traders are required to offer for sale all of their holdings of the first m goods, though they need not spend all of their $m+1$st good. A trader may (and usually will) buy some of his own goods back, but they must go through the market. In other words, in this version of the model, the trader does not own his initial bundle outright; he merely owns a claim on the proceeds when the bundle is sold.

Let us imagine m separate "trading posts," one for each of the first m commodities, where the total supplies $(1, ..., m)$ have been deposited for sale "on consignment." Each trader i makes bids by allocating amounts b_i^j of his $m+1$st commodity among the m trading posts, $j = 1, ..., m$. We denote his strategy, in the game-theoretic sense, by the vector $b_i = (b_i^1, ..., b_i^m)$.

There are a number of possible rules governing the permitted range of bids. In the simplest case, with no credit of any kind, the limits on b_i are given by $\sum_{i=1}^m b_i^j \leq a_i^{m+1}$ and $b_i^j \geq 0$, for $j = 1, 2, ..., m$.

The interpretation of this spending limit is that the traders are required to pay cash in advance. More generally, we might allow them to defer payment, either in anticipation of receipts or under some other credit arrangement that would have to be made explicit in the model.

The prices determined exactly balance the books at each trading post. The amount of the jth good that the ith trader receives in return for his bid b_i^j is

$$x_i^j = \begin{cases} b_i^j / p^j, & \text{if } p^j > 0,\ j = 1, 2, ..., m \\ 0, & \text{if } p^j = 0,\ j = 1, 2, ..., m \end{cases} .$$

His final amount of the $m+1$st good, taking into account his sales as well as his purchases, is

$$x_i^{m+1} = a_i^{m+1} - \sum_{j=1}^{m} b_i^j + \sum_{j=1}^{m} p^j a_i^j$$

His payoff must be expressed as a function of all the traders' strategies; accordingly we write

$$\Pi_i(b_1, ..., b_i, ..., b_n) = u_i(x_i^1, ...x_i^{m+1}).$$

where the x's depend on the b's as indicated in the equations above.

Because of the mechanism for price formation and the anonymous allocation of sales, all traders pay the same price at the same time for the same good.

Suppose that there are n types of individuals and $m+1$ goods which must be sold and purchased through markets. Suppose that the $m+1$st good is utilized as a means of payment and that all other goods must be offered up for sale at the m markets where good j ($j = 1, m$) is exchanged for the $m+1$st good. In our following analysis we utilize the buy-sell game rather than the sell-all game even though it is somewhat more complex because it permits freedom in both demand and supply. But the sell-all model has been noted in order to stress the simplest mechanism (see Shapley, 1976; Shapley and Shubik, 1977).

Although we observe that it is unreasonable to require that individuals sell all of their assets in the market each period, such an arrangement would be a tax collector's dream as all income would be monetized every period and the problems of measuring income would be greatly simplified.

4.2 The Buy-sell Game

The buy-sell game gives each individual the strategic freedom to be on either side of every market. We assume that a trader will enter each market as a seller or buyer, as both, or neither.

A strategy for a trader i is vector (q_i, b_i) of dimension $2m$ such that:

$$b_i^j \geq 0, \ 0 \leq q_i^j \leq a_i^j \text{ for } j = 1, ..., m \text{ and } \sum_{j=1}^{m} b_i^j \leq a_i^{m+1}$$

where b_i^j is the bid for and q_i^j is the offer of good j by i and the amount of commodity j in the final possession of trader i is:

$$\begin{cases} x_i^j = a_i^j - q_i^j + b_i^j \sum_{i=1}^{n} q_i^j / \sum_{i=1}^{n} b_i^j \\ a_i^j - q_i^j \text{ if } b_i^j = 0 \text{ for } j = 1, 2, ...m \end{cases}$$

and

$$x_i^{m+1} = a_i^{m+1} - \sum_{j=1}^{m} b_i^j + \sum_{j=1}^{m} q_i^j p^j$$

where $p^j = b^j / q^j$ and $p^j = 0$ if $q^j = 0$.

The distinction in the size of the strategies for the sell-all game and the buy-sell is that the former have a size of m and the second of $2m$. Neither involves any contingencies.

4.3 The Simultaneous Double-auction Market

A more complex model where the strategy set of an individual has dimension $4m$ is the simultaneous double auction market where now the bids and offers are no longer automatically self-clearing at each post, but involve contingent statements which must be reconciled and cleared by a financial clearing system.

Here we consider a somewhat more complex strategy involving quantities of goods bid for or offered together with prices bid or asked. In this instance a strategy involves $4m$ numbers and both prices and quantities appear as strategic variables.

A mechanism is described for a single market and it generalizes without difficulty for m markets. Suppose that n traders have endowments of two commodities, the second of which serves as a means of payment and a numeraire. We fix the price of a unit of numeraire at 1.

All traders are required to move simultaneously without knowledge of each other's actions by bidding and offering in a market. The endowment of trader i is given by (a_i^1, a_i^2); a move by trader i (which is also his strategy) is described by four numbers $(p_i, q_i, \widetilde{p}_i, \widetilde{q}_i)$ which are interpreted as follows:

$p_i =$ the maximum price (in terms of the numeraire) that i will pay to buy q_i or fewer units of the first good. We require that $p_i q_{i \leq} a_i$, in words, he cannot bid more money than he has on hand. We assume $q_i \geq 0$.

$\tilde{p}_i =$ the minimum price that i will accept to sell $\widetilde{q}_i$ or fewer units of the first good. We require that $0 \leq \tilde{q}_i \leq a_i$, i.e., he cannot offer for sale that which he does not possess.

Although it may be unlikely that a trader will wish both to sell and buy the same item at the same time, there is no a priori reason to rule out this behavior, hence the strategies employed here permit an individual to be on both sides of the market if he so chooses. The details of the simultaneous double auction market are provided in Shubik (1999a, Chapter 7)

4.4 A Disclaimer on Velocity

In macroeconomic analysis and business the concept of the velocity of money plays an important role. How many times one's dollars turn over in a single period is of concern. How fast are payments made? How does financial activity slow down or quicken? These are all questions of some applied concern. In the specification of the simplest models here the formal structure of the one-period games is not designed to deal with velocity. At most the velocity of any money as defined here is one.

4.5 A Disclaimer on Durables and Perishables

Because we have limited our investigation to a one period game we are unable to make three strategic distinctions among goods being traded. They are: (1) perishable consumables which must be consumed during the period or they are lost, like ripe tomatoes; (2) storable consumables, like a can of beans, which can be consumed or stored at the will of the owner, or (3) durables, such as jewelry which may last for a considerable time providing a stream of services, where for any period the service is used or lost.

In the history of the evolution of money, gold, silver and copper have played prominent roles. In a one period model we cannot pick up the important properties of durability and other time dependent properties. Thus when we use the name gold for a money it is more or less a metaphor as, in a one period model, we are unable to distinguish operationally, perishable commodities from durables.

5 Structure and Solutions

In all of the comments above the concept of what is a solution has not been discussed explicitly. When we formulate economic exchange as a game the first task is to fully define the strategic opportunities available to each agent. After that has been done we need to define preferences or utilities for each outcome. After this has been done we may wish to define what is to be considered as a solution to the game that has been defined.

5.1 Financial Instruments, Mass Markets and Optimality

The specification of the mechanisms of trade may constrain the feasible set of outcomes attainable by the traders. In particular individuals constrained to utilize a specific means of payments and credit system may be able to improve their economic outcomes by utilizing financial instruments and clearing arrangements which increase their strategic flexibility. In the models in Section 4, the first two clear all markets under all bids and offers, but they involve no credit arrangements or netting. Each post or market is able to clear its own trade fully. They both are variants of cash-in-advance trading. No centralized clearinghouse is required.

The formal introduction of markets provides a device that produces a single market price at any point in time for any extant set of moves by the traders. A market is an aggregating-disaggregating device over a mass of anonymous individuals. This contrasts with barter arrangements where trade is between individually identified pairs; i.e., trade is between dyads where A knows that she is trading with B.

5.2 Price Formation, Information and Dynamics

In institutional reality there are many ways in which prices are formed . They depend upon information, many transactions costs and time lags determined by technology and societal considerations. We avoid the important empirical aspects of price formation in many different markets by limiting our concern to the one period simultaneous move market with the most elementary of price formation mechanisms.

We also avoid introducing explicit transactions costs at this time. Although we believe that the institutional form of many of our financial institutions depends considerably on the details of transactions costs, we suggest here that the need for many of these institutions would still be there even if they were run without cost.

5.3 Payoffs and Noncooperative Equilibria

In our discussion up to this point we have concentrated on specifying the full state space or set of outcomes to a game resulting from all combinations of strategies by all agents. The interpretation of what constitutes a reasonable solution to a one period game is somewhat tricky. We will be utilizing the concept of a noncooperative equilibrium which presupposes consistent expectations by all agents, but we concentrate on a single period. One may adopt the view that the justification for the noncooperative equilibrium is normative and that all individuals will think their way through to consistent expectations. Another viewpoint may regard the noncooperative solution as essentially process oriented and the one period game must be interpreted as being embedded in time to motivate reasons for picking a strategy. The initial conditions for playing the game will require at least one period of the pre-history together with the assumptions made about the expected choices of others. Thus a Bayesian description is natural.

In the models considered here, because of the clear economic structure, it is desirable to point out the relationship between the noncooperative equilibrium solution and the general equilibrium solution often applied to economic models.

We characterize the noncooperative equilibrium outcomes as being sets of strategies with consistent expectations of all agents. In general, especially when multiperiod economic models are considered there is a proliferation of noncooperative equilibria and special solutions such as perfect equilibria may be considered; or solutions involving relatively simple local best response behavior may be considered as an appropriate solution concept. Or one may resort to utilizing a continuum of agents to make information conditions irrelevant (Dubey and Shubik, 1981).

In our discussion here we limit our concern to simple one period Cournot-Nash equilibria and observe how some of the set of the noncooperative equilibria may approach the competitive equilibria when the number of small agents becomes large.

5.4 The No-trade Equilibrium

Underlying all of the variations of the buy-sell market given in Section 6 is the presence of an inactive equilibrium where no one trades. This is easily seen by observing that if all but one individual stay out of the market then the last individual has no motivation to enter. This is a highly implausible state, but formally fits the definition of a non-cooperative equilibrium.

6 Degrees of Freedom and Symmetry in Cash and Credit Markets

In the economics of money and financial institutions there have been several problems that have for the most part eluded careful modeling and have been treated primarily by verbal methods and historical commentary. They include, with some exceptions, items such as what difference does it make to the economy if different goods or instruments are selected as the numeraire; can a system operate with everyone being a banker and using their own IOU notes as a means of payment (see Black, 1970); are the bankruptcy and default laws institutional curiosities or do they represent logical necessities in a system designed to promote trade; how do the roles of a central bank differ or overlap with those of a money market; what is a merchant banker?

We approach these problems by considering a set of different models and considering the differences in the symmetry of their strategy sets and in the degrees of freedom present in their decision structure.

The models are differentiated by the assumptions we make concerning the four properties of money noted in Section 1. In Section 4 our exposition of the basic models was in terms of a commodity money without offering an argument for the choice. Here we are more specific. In the following analysis we, in general consider an exchange economy with m consumable goods of value to all. We assume that there are rm individuals where there are r traders of each of the m types where any individual i of type j has the same initial endowment as any other. For most purposes we may consider $r = 1$, but, as noted in Section 5 when we wish to consider the attenuation of individual market power over price we consider r to be large.

6.1 A Problem in Dimensions and Taxonomy

We have suggested above a basic model for examining different monetary structures. We utilize variants of this model to demonstrate how the degrees of freedom vary with apparently slightly different models, and how intrinsically symmetric trade may become asymmetric by changes in the monetary structure. In our basic models there are m individuals and m goods. There is little difficulty in defining what is an individual economic agent, but there is somewhat more difficulty in providing a useful taxonomy of goods and services. There are of the order of 300 million individuals in the United States. How many goods and services are there? How many markets are there?[2] There is considerable leeway in operationally defining both goods and services. Any result which depends delicately on whether the number of types of individuals is precisely equal to the number of goods must be suspect unless, as we suggest here, it illuminates particular properties of the system being studied.

6.2 Functions and Institutional Forms

Mass economies with mass (more or less anonymous) markets require many functions to be performed in facilitating the completion of trade. Among the functions are:

1. Aggregation of bids and offers
2. Identification, verification and auditing
3. Credit evaluation
4. Record keeping
5. Insurance, storage and transportation
6. Credit granting

[2]The BLS uses an eight digit code.

7. Clearance and payment
8. Final settlement and default resolution.

These functions are necessary parts of a financial system designed to facilitate trade in a dynamic economy involving both space and time.

In many economies these functions are supplied by a variety of institutions and individuals. For example there are individual traders, retailers, wholesalers, and non-financial firms. There are markets, credit evaluation agencies, banks, central banks and other government agencies, insurance companies, clearinghouses, accountants, lawyers, notaries and courts. At first glance the names on the list appear to be peculiarly institutional, yet the functions they perform are an integral part of defining the dynamics of trade.

In our analysis here we do not propose to dwell in any detail on all of these institutions, but we show that even with extremely stripped down models considered at a high level of abstraction several institutions are called for. In particular some form of credit evaluation agency, a clearinghouse, goods markets, a money market, the central bank and the courts must appear in relatively simple models.

6.3 Barter, Commodity Money, IOUs and Clearing

In the shade of monetary theory there are a host of questions which from one point of view do not seem to be of central importance but are not quite covered in formal theory. By limiting ourselves to a single move game we are able to provide a formal structure to cover the many institutional differences in monetary systems. Yet, as is shown in Table 2 there are only a small number of structures that cover most of the essentials of monetary institutions found in the history of economics and finance.

In the Table 2 the salient features of 8 market structures are illustrated. They are each considered below in some detail.

6.3.1 Barter

Barter involves bilateral trade between pairs of agents or coalitions. It describes a state before the existence of formal markets. As exchanges are postulated to be value given for value received there is no need for institutions.

The emergence of markets and financial institutions from barter economies poses problems in economic anthropology, history and in the investigation of long term economic dynamics. These are all not dealt with here.

6.3.2 A Commodity Money: Gold

The buy-sell model presented in Section 4.2. is an economy where one consumer good is treated as the money. We may also assume that the means of payment is also selected as the numeraire, thus in this example, gold serves both roles. As was observed in Section 4.2. a money is utilized to bid in all markets, while a non-monetary good is offered for sale only in its own market. Thus, in general, a strategy for an individual i of type g is:

$$(b_i^1, q_i^1; ...; b_i^{m-1}, q_i^{m-1})$$

This has dimension of $2(m-1)$.

In the special extended Jevons example a strategy is of the form

$$(b_i^1, 0; ...; b_i^{m-1}, 0)$$

for the moneyed individual. This has dimension $m-1$, while the individuals without money each have a strategy of dimension of 1.

Even in this simple model a question concerning bidding and offering appears. It involves the possibility of wash sales. A wash sale occurs when an individual both buys and sells the same commodity thus creating the impression that the net market activity is greater than it is. Price formation for commodity j in general is given by:

$$p^j = \sum_{i=1}^{n} b_i^j / \sum_{i=1}^{n} a_i^j.$$

It has been shown elsewhere (Shubik, 1999a, Ch. 7) that the utilization of wash sales creates a continuum of equilibrium points; but as r, the number of individuals of each type increases, the range of wash sales equilibria converges to no wash sales, i.e., the market has become thick enough that one individual has negligible influence.

We could arbitrarily rule out wash sales by imposing the condition that for a trader i of type g trading in commodity j, $b_{ig}^j q_{ig}^j = 0$ for all $j = 1, ..., m-1$. This removes $m-1$ degrees of freedom in general. In the special instance of the extended Jevons model the dimensions of the strategies are unchanged.

The selection of the commodity money radically destroys the intrinsic symmetry of the underlying economic structure. The selection of the numeraire could be other than the gold. For example the means of exchange could be the golden sovereign and the numeraire or unit of account could be the guinea which might not exist. However the rules of the game would be required to fix an exchange rate between the guinea and the sovereign.

A minor item which requires comment is that we have not included the spot market of gold for gold. Such a market is not ruled out by logic, but will be inactive by virtually any type of optimizing solution. If individuals are assumed to act randomly or in error or under misperception it is possible to design games such as the "Dollar auction game" (Shubik, 1971) which never should be played by an optimizing individual.

	Barter	One commodity	Bimetallism	(a) All goods as money	(b) All goods as money	Money market and gold	(a) All IOUs with with default	(b) All IOUs with clearing
Markets	not defined	m–1	m–2	$m(m–1)/2$	$m(m–1)$	m	m	m
Numeraire	free "ideal"	gold	gold linked to silver	free "ideal"	free "ideal"	gold	free "ideal"	free "ideal"
Means of payment	all	gold	gold & silver	all	all	gold	all IOUs	all IOUs
Store of value	all	yes	yes both	all	all	yes	no	no
Agent types	1	2	3	1	1	2	1	1
Financial markets	no	no	no	no	no	yes	no	no
Credit evaluation	no	no	no	no	no	no (variant yes)	no	yes
Clearinghouse	no	no	no	no	no	yes	yes	yes
Courts for default	no	no	no	no	no	yes	yes	no
Efficiency	not always	sometimes	generically no	generically no	yes	yes if enough money	yes	yes
DOF a_{spec}	not defined	$2(m–1)$	$3(m–2)$	$m(m–1)$	$2m(m–1)$	$m^2+m–1$	$m^2–1$	$m^2–1$
DOF a_{gen}	not defined	$2m(m–1)$	$2m(m–1)$	$m^2(m–1)$	$2m^2(m–1)$	$2m^2$	$(2m+1)(m–1)$	$(2m+1)(m–1)$

Table 2: The salient features of one-period exchange. First line is the multiplicity of structures. The next four lines illustrate the four properties of money. The next four lines illustrate the emergence of financial institution function, and the last three cover efficiency and characteristics of exchange.

6.3.3 Bimetallism: An Economy with Both Gold and Silver as Money

"The medium-of-exchange function can tolerate more than one money without too much trouble; the unit of account function cannot." (Kindleberger 1984, p. 55).

A well known old question in monetary theory has been the feasibility of using two or even three metals simultaneously as a means of payment. The usual physical argument is that individuals need to make three types of payment. Large payments to buy a house, for instance; middling payments to buy a bicycle and small payments to buy a glass of beer. Gold is too valuable for the last two; silver fits best in the middle range and copper fits at the bottom.

Unfortunately if two or more commodities are used as a means of payment and, there is any change in the endowments of any commodity of value, as there almost certainly will be, the relative prices between the two moneys will change. If a country has open trade with others, silver or gold may flow in or out as a function of relative prices (see Kindleberger 1984, Ch. 4 for a nice summary). A central government will have to adjust the relative prices of silver and gold coinage if it wishes to keep both in circulation. In 1717, Sir Isaac Newton, Master of the Mint observed that a lewidor (louis d'or) was worth 17s and 3f (f = farthing) in France, but 17s 6d in England, which brought a large inflow of gold to London (Kindleberger, 1984, p.54). Newton set the price of gold at £3 17s 10 1/2d in an attempt to adjust the ratio between gold and silver.

If we are concerned only with a one period market with fixed endowments then an all-seeing government could do the appropriate calculations and announce a fixed mint price between gold and silver. Thus the number of free markets in an m commodity world would be reduced to $m-2$. At equilibrium, in this special case law, custom and free markets would coincide and there would be no net inflow or outflow of gold or silver if there were no arbitrage opportunities. The static equilibrium theory shows that it is logically feasible for a country to impose an extra constraint by fixing the price between gold and silver. But the hope for running a dynamic economy without constant readjustment is negligible.

In terms of the buy-sell model in 4.2. the concept of a single money is well-defined strategically. If good m is the money it can be used to purchase all other goods directly. It enters the numerator of the price-formation mechanism for all the goods it purchases. Thus if an individual i of type m has as his initial endowment $(0, 0, ..., a)$ where the mth good is deemed to be the money, his strategy is of the form $(0, b_{im}^1, 0, ..., 0, b_{im}^{m-1})$ where

$$\sum_{j=1}^{m-1} b_{im}^j \leq a$$

and $b_{im}^j \geq 0$. The bid has dimension $m-1$.

Suppose however, that there are two monies, say gold and silver, the mth and the $m-1$th commodities. For simplicity our remarks are confined to our closed one-period economy model. If a money is a means of payment but is not sold as a commodity then we can construct a well-defined playable one period game for the extended Jevons example as follows.

An individual i of type m has as his initial endowment $(0, 0, ..., a)$ where the mth good (gold) is deemed to be the money, his strategy is of the form $(0, b^1_{im}, 0, ..., 0, b^{m-2}_{im})$ where

$$\sum_{j=1}^{m-2} b^j_{im} \leq a$$

The bid has dimension $m - 2$. Similarly an individual i of type $m - 1$ (who owns silver, and has an endowment $(0, 0, ..., a, 0)$) has a bid of dimension $m - 2$.

A strategy of a trader of type g where $g = 1, 2, ...m-2$ is of the form $(0, 0; 0, 0; ..., q^g_{ig}, 0; ...; 0, 0)$ which is of dimension 1.

With the two monies how is price formed? We can specify generally for good j where $j = 1, 2, ..., m - 2$

$$p_j = \frac{f(\sum_{i=1}^k b^j_{i,m-1}, \sum_{i=1}^k b^j_{im})}{\sum_{i=1}^k q^j_{ij}}$$

If by government law a linear relationship in the valuation of gold versus silver has been set (for example around 1700 the gold/silver ratio was in the range of 1 to 15 or 16) then the price formation is specified as:

$$p_j = \frac{\sum_{i=1}^k b^j_{i,m-1} + \alpha \sum_{i=1}^k b^j_{im}}{\sum_{i=1}^k q^j_{ij}}.$$

We now have a completely well-defined game whose solution will depend on the parameter α. It is here that law clashes with custom and free markets and context free mathematical economic models may mislead us away from institutional understanding. If governments rule out by law market structures, black markets will spring up and ways to avoid the laws will be devised. The legal restrictions will become a cost of doing business and not a pure barrier.

In economic history as the relative prices of gold and silver moved, coins or ornaments were melted down and recast as ornaments or coin. This suggests that gold and silver could be considered as both monies and commodities for sale. If we were to consider them as both then a somewhat different model from the above can be considered. We may add in two extra markets. The market where the commodity, silver is sold for the money, gold and the market where the commodity, gold is sold for the money, silver. The strategy of an individual i of type m (those who own gold) now becomes of the form $(0, b^1_{im}, 0, ..., 0, b^{m-1}_{im}, q^m_{im}, 0)$ where

$$\sum_{j=1}^{m-1} b^j_{im} + q^m_{im} \leq a \text{ where } b^j_{im} \geq 0 \text{ and } q^m_{im} \geq 0$$

The bid has dimension m. Similarly an individual i of type $m - 1$ (who owns silver) and has an endowment $(0, 0, ...a, 0)$ has a bid of dimension m.

Can the government enforce the fixed rate between the two numeraires? If the government mint had a large supply of gold and silver and was willing to buy or sell in unlimited quantities at the rate it had set it could control the ratio. In these two models no quantitative intervention is modeled.

6.3.4 All Goods a Money? Case 1: Money or Goods?

In some writings (Clower, 1967, for instance), it has been suggested that when all goods can be used as a means of payment then every good as a money is the equivalent of barter. This misses the important distinction concerning the existence or nonexistence of mass markets which produce a single price for many agents trading simultaneously through some form of aggregating/disaggregating mechanism (see Amir et al., 1990).

Constraining ourselves to the buy-sell game when all goods may be utilized as a means of payment we have complete markets. There is a market between every pair of goods. But generalizing from bimetallism, given the definition of a good and a money, in our extended Jevons example, if an individual has only one commodity, say oranges which may be used as a money to make purchases, can it also be sold as a good? If a good selected as a money can only be used for bidding then in the extended Jevons example nothing can be offered for sale as all individuals have only a money.

The game is completely symmetric, but requires one of two breaks with our treatment of all the other markets. If bids and offers retain a unique definition, so that the notation agrees with the other cases, then any individual i of type g has a strategy of the form: $(0, b_{ig}^{1}; ...0, b_{ig}^{g-1}; 0, 0; 0, b_{ig}^{g+1} ..., 0, b_{ig}^{m})$ This has a dimension of $m-1$ and the only equilibrium is inactive. By definition only $m(m-1)/2$ markets can be considered and none of them can go active because there are no goods for sale, there are only monies which can be bid. Alternatively, a new clearing rule with the same symmetry as the markets may be designed (Smith and Shubik, 2003), which produces active and type-symmetric solutions, but without short sales these interior solutions can only be attained by two types.

6.3.5 All Goods a Money? Case 2: Money and Goods

If any money utilized for purchasing can also be used as a good for sale, then it is reasonable, in this instance to consider the existence of all $m(m-1)$ markets not $m(m-1)/2$. The owner of oranges buys apples in the oranges/apples market but can, if she wishes sell oranges in the apples/oranges market. There is no need to specify a numeraire. The game is completely symmetric and each individual has a strategy of $2(m-1)$ dimensions.

6.3.6 A Commodity Money and a Money Market

In an economy which employs a single commodity money such as gold without other money substitutes the equilibrium will be interior if there is enough money which is well distributed (the specific inequalities are given in Shubik, 1999a, Ch. 9). If the distribution of money is inappropriate, even if there is a sufficiency of money a boundary solution will exist. Efficiency can be improved by introducing a money market. A money market is a financial market. It requires a new instrument, the individual IOU note and calls for both a collection agency and the courts to fully specify its functions under all contingencies. The price in the money market can be regarded as an endogenous rate of interest. It is formed by bidding IOU notes for a supply of money offered in the money market. In the extended Jevons example the rate of interest will be:

$$1+\rho = \frac{\sum_{i=1}^{r}\sum_{g=1}^{m-1} z_{ig}^{m}}{\sum_{i=1}^{r} w_{im}^{m}}$$

where the z are the IOU notes for which the money is offered.

We must consider that the IOU notes are redeemed after trade. But it is possible that the economy could reach a state where an individual would not be in a position to redeem her IOUs. The default rules must be specified. The dimensions of the penalty will be utility/money.

The simplest game can still be defined with only one information set per individual. The extensive form has the individual borrow before bidding in the goods market. Realistically he would be informed before he bids, but this information can be finessed if we permit the individual to allocate percentages of his (unknown) buying power. In the extended Jevons example the strategy of a borrower is of the form: $(0, b_{ig}^{1}; 0, ...q_{ig}^{g}, b_{ig}^{g}; 0, b_{ig}^{m-1}; z_{ig}^{m}, 0)$ where $b_{ig}^{j} \geq 0$ and $z_{ig\geq}^{m} 0$.

The strategy has dimension $m+1$.

The strategy of a lender is of the form: $(0, b_{im}^{1}; 0, ...; 0, b_{im}^{m-1}; w_{im}^{m}, 0)$ which has dimension of m.

In this model as all transactions are for cash, all sellers can be paid directly from the market posts in cash, but then a collection agency must solicit each borrower for repayment and the courts must take care of any defaults

A variation of this model can have accurate credit evaluation which attaches a discount to each individual bid in a manner that avoids bankruptcy. We discuss this variation for the model in 6.3.8.

6.3.7 Everyone Their Own Banker (a): All issue IOUs with no credit evaluation but default rules

Suppose that each individual is permitted to issue her own IOU notes as a means of payment. We may impose some arbitrary upper bound M on the amount of notes that any individual can bid. Furthermore we select an imaginary money "the ideal" as the numeriare, thus an individual IOU is a promise to redeem the paper in ideals. The utilitarian value of the ideal in default is established by the laws or rules of default.

We consider the extended Jevons example. A strategy by an individual i of type g, $g = 1, 2, ..., m$ is of the form $(0, b_{ig}^{1}; 0, ...q_{ig}^{g}, b_{ig}^{g}; 0, b_{ig}^{m-1}; b_{ig}^{m}, 0)$. It is of dimension $m+1$.

We develop two models. In this model each market immediately ships all goods to all the bidders, but before it settles with the suppliers it sends all of the IOU notes to a clearinghouse together with its intended payments. The clearinghouse nets all IOUs along with the intended payments. If all net to zero, its work is done. The clearinghouse informs all individuals who are in default and all individuals who are creditors and it turns over these accounts to be settled by the courts.

The symmetry among all agents in this model is obtained by introducing the two societal agencies, the clearinghouse and the courts. In institutional fact the clearinghouse could be an agency of the government such as the Fedwire of the United States or it could be a privately owned institution such as CHIPS (Clearing House Interbank Payment System,). The courts should reflect their society, at any moment of time the laws are part of the rules of the game, but in a longer horizon they will be subject to change. The modeling and analysis of such a system is highly dependent on the time horizon considered. If only a few months or years are under consideration it is reasonable to accept the institutions as given. If several decades or centuries are being considered it makes sense to investigate the trade-off between law and custom.

6.3.8 Everyone Their Own Banker (b): All issue IOUs with credit evaluation to balance the books

Suppose that, as in Case (a) above each individual is permitted to issue her own IOU notes as a means of payment. We may impose some arbitrary upper bound on bids as was done above. The strategy sets of the individuals are as before; but settlement rules are changed and we are able to introduce a credit evaluation agency to dispense with the courts and default rules by imposing a balancing of all accounts under all circumstances. This is done by utilizing the clearinghouse as both a clearing agency (see Sorin, 1996) and a credit evaluator.

All markets receive rm different IOU notes in bids for the goods for sale. In order to make this game reasonably playable we envision that each market bundles all of the IOU notes and sends them to the central clearinghouse where they are all matched and netted. As they are all denominated in the Ideal which does not have a physical existence, the clearinghouse can impose a settlement rule by solving a simple linear system requiring the full balance of each individual by imposing a relative valuation of the notes. There are some technical problems with division by zero when individuals offer no goods whatsoever for sale, but nevertheless bid. We may evaluate all bids accompanied with a zero offer as of zero value and $0/0$ is interpreted as no trade.

6.4 A comment on the clearinghouse versus cash-in-advance

The explicit introduction of a clearinghouse is consistent with the no transactions costs aspects of general equilibrium theory. The difference being that we present a formal process oriented model where, in essence, the clearinghouse provides a zero interest loan in clearinghouse credit for the small period of time during which accounts are netted (see Summers, 1994, Shubik and Slighton, 1997). The clearinghouse approach contrasts with the cash-in-advance models where a full time period lag is attributed to settlement. In Part II we establish that the existence of fiat money is incompatible with a 100% efficient clearinghouse.

7 Fiat Money Clearing Houses and Credit

Fiat money is operationally a special form of commodity money. It is the only financial instrument that does not necessarily have an offsetting asset or instrument against it. Yet a commodity money such as gold or salt has an intrinsic value as a commodity. A key problem in monetary theory is what is the basis for a value being attached to fiat money. This problem has been approached by Kiyotaki and Wright (1989) and by Bak, Nørrelykke and Shubik (1999) where the valuation is supported not by intrinsic value or trust but in the dynamics which supports the belief by A that it is in the self-interest of B to accept the money.

7.1 The Hahn Paradox Revisited

In the Debreu (1959) treatment of the general equilibrium price system, there is, in essence, no role for money or for the rate of interest. As Koopmans phrased it, the general equilibrium analysis is preinstitutional. Institutions are the carriers of process and the general equilibrium existence proof involved no process analysis. The processes are abstracted out of the analysis.

The payments, credit evaluation and other institutions of the financial system provide the control and evaluation mechanisms over the economy. In equilibrium the financial instruments match the underlying economy so perfectly that they seem to disappear. Out of equilibrium the tension between the economy of the physical goods and services and the financial system becomes visible (in particular, relations that in general equilibrium theory are manifested as equalities enter as inequalities).

The equilibrium points obtained in general equilibrium theory are consistent with the limit equilibrium points of many different completely defined playable games when a continuum of agents is considered (see Dubey, Mas Colell and Shubik, 1980). In particular the game with everyone issuing her own IOU notes could be made consistent with the general equilibrium model including the clearinghouse and the courts.

As was pointed out by Hahn (1965) if one tries to introduce an institutional stuff such as fiat money which is intrinsically worthless into the general equilibrium structure in the context of an economy with a finite horizon, if there is any worthless money left over at the end of time, by backwards induction one can argue that it will always be worthless and hence not accepted in trade. As is noted below a richer model is required to offer an adequate explanation of the Hahn paradox.

7.2 Economies or Games with Fiat Money

In keeping with our basic approach in dividing difficulties here we limit ourselves to one period models which could be played as experimental games. With such a constraint it does not appear that we can offer an adequate institutional explanation that has an easily identifiable counterpart in an economy. Nevertheless we can consider formal economic rules which may serve to support the use of fiat in a one period game. Several ways in which this can be done is to require that any fiat that exists in the system be paid back to the referee with penalties for failure to repay. Taxes may be introduced and fiat may be utilized to pay taxes. An interest rate which removes money from the economy may also be considered. An interest rate may be regarded as an operator which enables individuals to shift consumption backwards or forward through time. Because the introduction of fiat or government money brings in both government as a player controlling its money supply and/or the fiat money interest rate we defer the investigation of fiat money to Part II of this paper.

One needs to consider at least two periods and terminal settlement before one can distinguish the key function of a central bank from that of a money market. Both can redistribute the money supply but the central bank can vary it.

8 Time, Law and Custom

We consider this to be an essay in mathematical institutional economics and econophysics with an emphasis on examining and understanding the meaning of the aspects of symmetry and the degrees of freedom to be found in various models of an exchange economy with a monetary payments system.

One of the basic difficulties to be encountered in considering such a study is to be able to reflect adequately the tensions that exist between law and custom. We believe that there is no single simple explanation of what gives fiat money or gold or salt its value as a means of exchange. It is a blend of law and custom, not one or the other. The structure cannot be understood without considering the relationship of the local and global structures. The economic game is a game within the larger games of the polity and society and the rules dominating the economic game may be changed and modified over time.

The time span being considered in the study of the economy calls for methods that vary in the context of the time and space scales being considered. The shorter the time, the more institutions and agents can be considered as given within an explicit context. In the relatively short period under consideration the interface of the physical economy and its financial control system can be considered in terms of the rules of a reasonably well defined game and can be usefully analyzed by the means suggested here.

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