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Key words: Adjustment of inputs, dynamic panel data models, stochastic production frontier, time-varying technical efficiency

JEL Classification: C23, D24, L60

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1. Introduction

Estimation of technical efficiency of production units using a stochastic frontier approach and panel data has been a popular area of applied research for the last couple of decades. The advantage of using panel data in the stochastic production frontier analysis is that it enables one to estimate efficiency of production units without imposing too many restrictive assumptions on them. Earlier research on measuring time-invariant technical efficiency (Schmidt and Sickles (1984)) has been further developed by Cornwell, Schmidt and Sickles (1990); Kumbhakar (1990); and Battese and Coelli (1992) to incorporate time-variation in technical efficiency of a production unit. They assume the technical efficiency of production units to be a parametric function of time. Lee and Schmidt (1993) capture temporal variation in efficiency in a more flexible fashion. They consider the temporal pattern of efficiency to be the same for all production units without assuming any functional form. According to Lee and Schmidt (1993), the producer specific effect and its time pattern are unknown parameters to be estimated. A recent research by Ahn, Lee, and Schmidt (2007) further extends this idea and discusses estimation of time-varying technical efficiency from a stochastic frontier model with multiple time-varying individual effects.

Most of the existing studies on stochastic frontiers with time-varying technical efficiency focus on the static analysis of a producer's behavior, in the sense, that these studies assume that inputs are instantaneously adjusted within a production system. However, in the presence of short-run adjustment of inputs, the actual output is likely to be generated by a dynamic process (See Lucas (1967a, 1967b); Treadway (1971); and Hamermesh and Pfann (1996) for discussion on the importance of the process of adjustment of inputs in a production system). The idea behind such a dynamic production process is that inputs require time to adjust within a production process before contributing to their full capacity, and it may not be possible for a producer to produce at the maximum possible level during the period of adjustment of inputs, even in the absence of any other inefficiency in the production system. Further, the suboptimal production plan can be a conscious choice of the producer facing short-run fixity of inputs, changes in the demand for output and expectation about the future economic conditions (Berndt and Fuss (1986), Morrison (1986)).

It has also been established in the literature that if the market is sufficiently competitive or if we study production units for sufficiently long period of time, the technical efficiency of the units are likely to vary with time. However, the speed of adjustment of inputs is also likely to change over time for similar reasons, and this issue has not been investigated in the existing literature. More specifically, as the inputs get familiar with a production system, their speed of adjustment is likely to improve as well. For example, a worker hired in the past is likely to learn faster than a newly hired worker. Therefore, in the presence of short-run adjustment of inputs, a static production model that assumes instantaneous adjustment of all inputs, misspecifies the production model and is likely to generate a biased estimate of technical efficiency of the production units. A static model identifies a producer's failure to produce at the maximum possible level as the effect of (1) random shocks to the production system, and (2) the presence of inefficiency in the system that can be controlled by the producer. However, as mentioned earlier, the short-run adjustment of inputs is an inherent phenomenon of a production process, and hence a part of the short fall in production that occurs during the process of adjustment, does not represent inefficiency of the production system. Thus, a static model is likely to underestimate technical efficiency of production units when inputs require time to adjust and the true process of output generation is dynamic.

Among the preceding studies, Ahn, Good, and Sickles (2000) allow for the sluggish adoption of new technologies to explain the autoregressive nature of the technical efficiency component that varies with time. They also measure the speed of sluggish adoption of technological innovations, but the speed is assumed to be constant over time. In reality, the sluggish adjustment of inputs not only affects the adoption of technological innovations, but can also affect the whole production process by restricting output from reaching its maximum possible level. Moreover, with time, as the inputs get more familiar with a production system, their speed of adjustment is likely to improve as well. As a result, the deviation of actual change in output from the desire change is also likely to vary over time. More specifically, it is likely that the gap between the actual change and desired change in output falls over time. Further, if a production system is studied for substantially long period of time, and the economic structure is sufficiently competitive, the inefficiency effect of a production unit is also likely to change over time (Kumbhakar and Lovell, 2000).

Therefore, in this paper, I present a dynamic production model with time-varying speed of adjustment of output and technical efficiency. Measuring efficiency from such a dynamic model is also not straightforward. Particularly, consistent and efficient estimation of dynamic panel data model with time-varying individual effects[1] is an attractive area of research even in the current time. The first and widely known paper in

[1] The econometric dynamic panel data model with time-varying individual effects corresponds to the dynamic production model with time-varying technical efficiency.

this area is by Holtz-Eakin, Newey, and Rosen (1988), who discuss the estimation method for a dynamic panel data model with time-varying individual effects, but do not discuss estimation of the time-varying individual effects from such a model. By adapting their method, I extend it to suit my purpose of technical efficiency estimation and apply it in this paper.

The objective of this paper is thus, to present a dynamic stochastic production frontier that allows for short-run quasi-fixity of inputs and provide estimation methods to measure the speed of adjustment of output and technical efficiency of production units, both of which vary over time. The paper also compares the estimates of time-varying technical efficiency of production units from such a dynamic model with the estimates from a static production model that assumes instantaneous adjustment of all inputs. For this purpose, I use a panel dataset on private manufacturing establishments in Egypt from the Industrial Production Statistics of the Central Agency for Public Mobilization and Statistics (CAPMAS).

The remainder of this paper is organized as follows. The model specifications are discussed in section 2. Section 3 and section 4 elaborate on the estimation methods and empirical analysis, respectively. Finally, section 5 presents concluding remarks.

2. Model Specification

In the presence of short run adjustment of inputs, the change in actual output between any two periods is a combined result of contribution of a part of new inputs that is adjusted during the period, and contribution of a part of the old inputs that adjusts in that period. Let y_{it}^{*} be the maximum possible production level of firm *i* that uses a vector of inputs X_{it} at time *t*, and let y_{it} be the actual output produced by firm *i* at time *t*. During the adjustment process of inputs, the current output y_{it} is likely to be higher than $y_{i(t-1)}$ but lower than y_{it}^{*}, when y_{it}^{*} is increasing over time. More specifically, the actual change in output is likely to be a fraction of the change in output that is needed to catch up with the potential output at any given time period. Let us refer to the change in output that is needed in any period to catch up with the potential output, as the 'desired change' in output. Further, the difference between the actual and the desired change in output is likely to depend on the speed of adjustment of inputs. In other words, the dynamic production model showing the relationship between actual change and the desired change in output between two periods can be represented as a partial adjustment scheme[2] -

$$\ln y_{it} - \ln y_{i(t-1)} = \lambda_t (y_{it}^{*} - \ln y_{i(t-1)}),\ 0 \le \lambda_t \le 1 \tag{2.1}$$

[2] It can be shown that a partial adjustment scheme for output generation implies that output in any period depends on the current and past inputs, and the speed of adjustment of inputs. For example, if the Cobb-Dauglas production function represents the generation of potential output i.e., $\ln y_{it}^{*} = \beta_0 + \sum_{m=1}^{M} \beta_m \ln x_{mit}$, then the partial adjustment scheme of production $y_{it} - y_{i(t-1)} = \lambda(y_{it}^{*} - y_{i(t-1)})$ is equivalent to

$$\ln y_{it} = \lambda\beta_0 + \lambda(1-\lambda)\beta_0 + \lambda(1-\lambda)^2\beta_0 + \lambda(1-\lambda)^3\beta_0 + ... + \lambda \sum_{m=1}^{M} \beta_m \ln x_{mit} + \lambda(1-\lambda) \sum_{m=1}^{M} \beta_m \ln x_{mi(t-1)} + \lambda(1-\lambda)^2 \sum_{m=1}^{M} \beta_m \ln x_{mi(t-2)} + \lambda(1-\lambda)^3 \sum_{m=1}^{M} \beta_m \ln x_{mi(t-3)} + ...$$

where, λ $(0 \le \lambda \le 1)$ is the speed of adjustment of inputs, x_{mit} is the *m*th input used by producer *i* at time *t* for *m = 1,…,M* , β_m is the elasticity of the *m*th input, and β_0 is the intercept of the potential production frontier. Therefore, the partial adjustment scheme demonstrates that a fraction, λ, of an input $x_{mi(t-k)}$ that is introduced by firm *i* in the period $t - k$ $(0 < k < t)$, contributes to the output in that period. In period $t - k + 1$, λ fraction of the remaining (1-λ) $x_{mi(t-k)}$ contributes to the output, and again λ fraction of the unadjusted $(1-\lambda)^2 x_{mi(t-k)}$ contributes to output in $t - k + 2$. Following this process, λ fraction of $(1-\lambda)^k x_{mi(t-k)}$ contributes to output at time *t*. In this paper, I further generalize the output generation process and incorporate λ_t**,** the speed of adjustment that varies with time.

where, , $i = 1,\ldots,N$ denotes the production unit, $t = 1,\ldots,T$ represents the time periods, y_{it} is the actual output of producer i at time t, y_{it}^{*} denotes the maximum possible output of producer i at time t, and λ_t is the fraction of desired change in output that is realized in time t. If the speed of adjustment is lower than unity, then the change in actual output will be lower than the desired change. Moreover, the higher is the speed of adjustment of inputs, the lower is the deviation of the desired change in output from the actual change, and the desired change in output is exactly similar to the actual change when the speed of adjustment is unity, i.e., when inputs are instantaneously adjusted in the production system. I assume the gap between the actual change ($y_{it} - y_{i(t-1)}$) and the desired change $(y_{it}^{*} - y_{i(t-1)})$ in output is similar for all producers. If the maximum possible output is generated by a Cobb Douglas production function, then (2.1) can be represented as

$$\ln y_{it} = (1-\lambda_t)\ln y_{i(t-1)} + \lambda_t(\beta_0 + \partial t + \sum_{m=1}^{M} \beta_m \ln x_{mit}) \tag{2.2}$$

where x_{mit} is the mth input used by producer i at time t for $m = 1,\ldots,M$, β_m is the elasticity of the mth input, and β_0 is the intercept of the potential production frontier. δ captures the effect of technological changes on the potential output[3].

The stochastic version of the dynamic production model allows for the presence of inefficiency in a production system and also accounts for the random shocks. Thus the stochastic dynamic production frontier corresponding to (2.2) is given by

$$\ln y_{it} = (1-\lambda_t)\ln y_{i(t-1)} + \lambda_t \ln y_{it}^{*} + e_{it} \tag{2.3}$$

[3] Since all the parameters in (2.2) vary with time and the sample size is not very large for this analysis, I consider only time trend instead of time dummies to reduce the number of parameter estimates from (2.2).

or, $\ln y_{it} = (1-\lambda_t)\ln y_{i(t-1)} + \lambda_t(\beta_0 + \partial t + \sum_{m=1}^{M}\beta_m \ln x_{mit}) + e_{it}$ (2.4)

The composed error term e_{it} can be decomposed into the technical inefficiency term, $\theta_t f_i$ $(\theta_t f_i \geq 0)$, that varies with time and the symmetric random shock, τ_{it}, i.e., $e_{it} = -\theta_t f_i + \tau_{it}$, where $\tau_{it} \sim iid(0,\sigma_\tau^2)$. θ_t captures the time-varying influence of the producer specific inefficiency f_i on the current output. In this formulation, the temporal pattern of technical inefficiency is the same for all production units. However, as discussed by Lee and Schmidt (1993), this structure is less restricted than the structures proposed by Cornwell, Schmidt, and Sickles (1990) and Kumbhakar (1990).

To measure the time-varying technical efficiency and speed of adjustment of output, I consider a Cobb-Douglas production function with constant elasticity of inputs for the potential output. Since I do not expect the elasticities to vary when the inputs are producing at their maximum possible level, the assumption of constant elasticities of inputs for the potential output is reasonable. This assumption also assures a considerable reduction in the number of parameter estimates from a small sample. The estimation method for (2.4) is discussed in the next section. However, once the parameters θ_t, and the firm specific effect f_i are estimated, the technical efficiency is measured as –

$TE_{it} = \exp - \{\max_j(-\hat{\theta}_t \hat{f}_j) - (-\hat{\theta}_t \hat{f}_i)\}$

(2.5)

If the speed of adjustment of all inputs is assumed to be unity, as in the static specification of equation (2.4), the following represents the static stochastic frontier model -

$$\ln y_{it} = \beta_0 + \partial t + \sum_{m=1}^{M} \beta_m \ln x_{mit} + \pi_{it} \quad (2.6)$$

where $\pi_{it} = -\rho_t \kappa_i + w_{it}$, $\rho_t \kappa_i \geq 0$ represent the technical inefficiency of producer i at time t, ρ_t is the time-varying influence of the producer specific effect κ_i, and the symmetric statistical noise $w_{it} \sim iid(0, \sigma_w^2)$. The static stochastic frontier (2.6) is estimated following the methods suggested by Lee and Schmidt (1993). Since inputs are likely to be correlated with the producer specific effects, (2.6) is estimated as fixed effects model and ρ_t and κ_i are estimated accordingly. The estimation procedure is discussed in detail in the next section. Then the technical efficiency from (2.6) is calculated as -

$$T\tilde{E}_{it} = \exp - \{\max_j(-\hat{\rho}_t \hat{\kappa}_j) - (-\hat{\rho}_t \hat{\kappa}_j)\}$$

(2.7)

In the presence of short-run adjustment of inputs, the technical efficiency estimates using the static model (2.7) is expected to be biased as compared to those obtained from the dynamic model (2.4).

3. Estimation Methods

To estimate the dynamic panel data model with time-varying technical efficiency, as given in equation (2.4), I adapt the method described by Holtz-Eakin, Newey, and Rosen (1988). For identification purposes, I assume that

$$E[\ln y_{is} \tau_{it}] = E[\ln x_{mis} \tau_{it}] = E[f_i \tau_{it}] = 0, \ (s < t), \ (m = 1, ..., M) \quad (3.1)$$

The error term in (2.4) does not have a mean value zero. Therefore, I transform equation (2.4) to eliminate the individual effects in the following way. Let $r_t = \frac{\theta_t}{\theta_{t-1}}$. I consider (2.4) for period (*t*-1), multiply it by r_t, and take the difference of the derived equation from (2.4) for period *t*. This gives us the following quasi-transformed equation-

$$\ln y_{it} = \lambda_t \beta_0 - r_t \lambda_{t-1} \beta_0 + r_t \lambda_{t-1} \partial + (1 + r_t - \lambda_t) \ln y_{it-1} - r_t (1 - \lambda_{t-1}) \ln y_{it-2} + \lambda_t \sum_{m=1}^{M} \beta_m \ln x_{mit}$$

$$+ \lambda_t \partial t - r_t \lambda_{t-1} \partial t - r_t \lambda_{t-1} \sum_{m=1}^{M} \beta_m \ln x_{mit-1} + e_{it} - r_t e_{it-1}$$

(3.2)

The regressors in (3.2) involve one period lagged dependent variable that is correlated with the error term. However, the orthogonality conditions in (3.1) imply that the error term in (3.2) satisfies the following conditions -

$$E[\ln y_{is} \varepsilon_{it}] = E[\ln x_{mis} \varepsilon_{it}] = E[f_i e_{it}] = 0 \text{ for } s < t - 1,\ m = 1,..,M$$

where, $\varepsilon_{it} = e_{it} - r_t e_{it-1}$. Therefore, the vector of instrumental variables that is available to identify the parameters of (3.2) is $Z_{it} = [\ln y_{it-3},..., \ln y_{i1}, \ln x_{mit-2}, \ln x_{mit-3},..., \ln x_{mi1}]$.

The vectors of observation on $i = 1,..., N$ for a given time period are given by

$$Y_t = [\ln y_{1t},..., \ln y_{Nt}]'$$

$$X_t = [\ln x_{m1t},..., \ln x_{mNt}]', m = 1,.., M$$

The vectors of the right hand side variables, error term, and coefficients of (3.2) for a given time period are given by W_t, V_t and B_t respectively, where

$$W_t = [e, Y_{t-1}, Y_{t-2}, t, X_{mt}, X_{mt-1}] \text{ for } m = 1,..,M$$

$$V_t = [v_{1t}, ..., v_{Nt}]' \text{ and}$$

$$B_t = \begin{bmatrix} (\lambda_t - r_t\lambda_{t-1})\beta_0 + r_t\lambda_{t-1}\partial \\ 1 + r_t - \lambda_t \\ -r_t(1-\lambda_{t-1}) \\ (\lambda_t - r_t\lambda_{t-1})\partial \\ \lambda_t\beta_1 \\ \vdots \\ \lambda_t\beta_M \\ r_t\lambda_{t-1}\beta_1 \\ \vdots \\ r_t\lambda_{t-1}\beta_M \end{bmatrix}$$

Therefore, equation (3.2) can be written as

$$Y_t = W_t B_t + V_t \text{ for } t = 4,...,T \quad (3.3)$$

Further, combining observations for each time period, (3.3) can be written as

$$Y = WB + V$$

(3.4)

where,

$Y = [Y_4', ..., Y_T']'$

$B = [B_4', ..., B_T']'$

$V = [V_4', ..., V_T']'$

$W = diag[W_4', ..., W_T']'$

and *diag*[] denotes a block diagonal matrix with the given entries along the diagonal. Thus, the matrix of instrumental variable for period *t* is $Z_t = [Y_{t-3}, ..., Y_1, X_{mt-2}, ..., X_{m1}]$ for $m = 1, .., M$. Consider $Z = diag[Z_4, ..., Z_T]$.

The covariance matrix Ω of the transformed disturbances is $\Omega = E\{Z'VV'Z\}$. To estimate Ω, I use the two-stage least squares (2SLS) estimator of B_t, given by $\tilde{B}_t$, as the preliminary consistent estimator where

$$\tilde{B}_t = [W_t' Z_t (Z_t' Z_t)^{-1} W_t' Z_t (Z_t' Z_t)^{-1} Z_t' Y_t] \tag{3.5}$$

Then, the vector of residuals for period *t* is given by -

$$\tilde{V}_t = Y_t - W_t \tilde{B}_t \tag{3.6}$$

A consistent estimator of ($\tilde{\Omega} / N$) is then formed by -

$$(\tilde{\Omega} / N)_{rs} = \sum_{i=1}^{N} (v_{ir} v_{is} Z_{ir}' Z_{is}) / N \tag{3.7}$$

where v_{it} $(t = r, s)$ is the *i*th element of V_t and Z_{it} is the *i*th row of Z_t .

For the empirical analysis, (3.2) is estimated by the method of GLS (generalized least squares) with $\ln y_{it-3}$ as the instrumental variable. Since *N* is not large (28) for the sample used in this paper, I do not use all the available instruments, in order to avoid the problem of too many instruments. Given the choice of instrumental variable, (3.2) is estimated for $t \geq 4$.

Holtz-Eakin, Newey, and Rosen (1988) do not discuss about estimation of the individual specific effects that vary with time. However, the main objective of this paper is to estimate the time-varying technical efficiency of a production unit, which is a part of the composite error term. For this purpose, I estimate (3.2) following the method discussed above and get estimates for (2M + 4) parameters, where each of these parameters is a nonlinear function of (M + 5) distinct parameters given by r_t, λ_t, λ_{t-1}, β_0, ∂, and $\beta_1,...,\beta_M$. Thus, once (3.2) is estimated, I have an over identified system of (2M + 4) equations to identify M+5 parameters, for $M \geq 1$. I denote the vector of (M + 5) parameters by φ_t and the system of equations by $g(\varphi_t)$. The (2M + 4) estimates from (3.2) are given by a_t, b_t, c_t, d_t, $f_{1t},..., f_{Mt}$, , and $h_{1t},...,h_{Mt}$, and hence, $g(\varphi_t)$ is given by

$$g(\varphi_t) = \begin{bmatrix} (\lambda_t - r_t\lambda_{t-1})\beta_0 + r_t\lambda_{t-1}\partial - a_t \\ 1 + r_t - \lambda_t - b_t \\ -r_t(1-\lambda_{t-1}) - c_t \\ \lambda_t - r_t\lambda_{t-1} - d_t \\ \lambda_t\beta_1 - f_{1t} \\ \vdots \\ \lambda_t\beta_M - f_{Mt} \\ r_t\lambda_{t-1}\beta_1 - h_{1t} \\ \vdots \\ r_t\lambda_{t-1}\beta_M - h_{Mt} \end{bmatrix}$$

To identify the parameters of the original dynamic production model (2.4), I solve the following optimization problem[4] subject to the condition that the speed of adjustment of output in each period $\lambda_t \in [0,1]$ and the input elasticities (β_m) are non-negative. Thus, I get unique estimates for the parameters in the original model as given in (2.4) and also for $r_t = \frac{\theta_t}{\theta_{t-1}}$ by the following -

$$\underset{\varphi_t}{Min}\, g(\varphi_t)' g(\varphi_t) \text{ subject to } 0 \le \lambda_t \le 1 \text{, and } \beta_m \ge 0 \text{ for } m = 1,\ldots,M .$$

Further, to identify θ_t, I normalize[5] $\theta_T = 1$ and accordingly identify θ_t for the periods for which (3.2) is estimated. Finally, I estimate the individual specific effect f_i by the ordinary least squares method for each sector using the following equation

$$\hat{\phi}_{it} = -\hat{\theta}_t f_i + \tau_{it} \tag{3.8}$$

where, $\hat{\phi}_{it} = \ln y_{it} - (1-\hat{\lambda}_t)\ln y_{i(t-1)} - \hat{\lambda}_t(\hat{\beta}_0 + \hat{\partial}t + \sum_{m=1}^{M} \hat{\beta}_m \ln x_{mit})$ (3.9)

Then the time-varying technical efficiency is estimated following equation (2.5)

To compare the technical efficiency estimates from (2.4) with those from the static version of the model that assumes the speed of adjustment is constant and equals unity, I estimate equation (2.6), following the method suggested by Lee and Schmidt

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[4] $\underset{\varphi_t}{Min}\, g(\varphi_t)' V(\varphi_t) g(\varphi_t)$, where V(.) represents variance, makes no considerable changes in the results.

[5] Lee and Schmidt (1993) suggest the normalization $\theta_1 = 1$ for the static model with similar time-varying technical efficiency structure. However, our model being a dynamic one, the parameters cannot be estimated for the initial period and we choose the normalization with respect to the last period.

(1993). Relying on the results of Hausman's specification test (1978), I estimate (2.6) as a fixed effects model such that the producer specific effects are treated as parameters to be estimated. In a general notation the model can be summarized as

$$\ln y_{it} = X_{it}'\beta + \pi_{it} \tag{3.10}$$

where, $\pi_{it} = -\rho_t \kappa_i + w_{it}$, X_{it} is the vector of regressors including a constant term, time trend, and M inputs in logarithmic term. β is the vector of input elasticities, and w_{it} are assumed to be independently and identically distributed with mean zero and variance σ_w^2. The T observations for production unit i can be written as

$$y_i = X_i\beta + \xi k_i + w_i \tag{3.11}$$

where,

$y_i = (\ln y_{i1}, ..., \ln y_{iT})'$, $X_i = (X_{i1}, ..., X_{iT})'$, $w_i = (w_{i1}, ..., w_{iT})'$, and $\xi' = (\rho_1, ..., \rho_{T-1}, 1)$.

The estimator of β is given by

$$\hat{\beta} = \left(\sum_i X_i' \hat{M}_\xi X_i \right)^{-1} \sum_i X_i' \hat{M}_\xi y_i \tag{3.12}$$

Where $\hat{M}_\xi = I_T - \hat{\xi}(\hat{\xi}'\hat{\xi})^{-1}\hat{\xi}'$, and $\hat{\xi}$ is the eigenvector of $\sum_i (y_i - X_i\hat{\beta})(y_i - X_i\hat{\beta})'$ corresponding to the largest eigenvalue[6]. To implement the fixed effects estimator of Lee and Schmidt (1993), first, I estimate β by the ordinary least squares method (OLS) as

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[6] M_ξ is a $T \times T$ idempotent matrix such that $M_\xi \xi = 0$.

$$\tilde{\beta} = \left[\sum_i (X_i - \bar{X})'(X_i - \bar{X}) \right]^{-1} \sum_i (X_i - \bar{X})'(y_i - \bar{y})$$

where, $\bar{X} = \sum_i X_i / N$, and $\bar{y} = \sum_i y_i / N$. Using this initial estimate of β, I iterate the estimation process till it converges. Finally, the producer specific effects are estimates as $\hat{k}_i = \hat{\xi}'(y_i - X_i\hat{\beta}) / \hat{\xi}'\hat{\xi}$. Then, the time-varying technical efficiency is estimated from (2.7).

4. Empirical Analysis

4.1. Data

The dynamic production frontier and the estimation method, as discussed in section 2 and 3, respectively, are applied on a panel data set for nine years (1987/88 – 1995/96) on the private sector manufacturing establishments in Egypt, obtained from the Industrial Production Statistics of the Central Agency for Public Mobilization and Statistics (CAPMAS). The data is in three-digit ISIC (International Standard Industrial Classification) level and for 28 sectors with the total number of observation being 252. The broader categories of output include food, tobacco, wood, paper, chemicals, non-metallic products, metallic product, engineering products, and other manufacturing products. Table 1 in the appendix presents the description of each sector.

This data set is directly taken from a study by Getachew and Sickles (2007) and details about the data can be found in their paper. They use the superlative index number approach to aggregate the data to the three-digit level, such that the establishments in each sector can be viewed as homogeneous in terms of production technology. To get a single aggregate measure of output from heterogeneous and multi-product firms, they consider total revenue from these firms for goods sold, industrial services provided to others, and so on. Finally, they obtain the quantity indices for output and inputs by deflating the total value of output and inputs by the relevant price indices.

Capital, labor, energy, and material are the inputs for the manufacturing sectors' output. As found by Getachew and Sickles (2007), the quantity indices for output and inputs grew over the period under consideration. The summary statistics of the indices are presented in Table 2 in the appendix. Getachew and Sickles (2007) use this data set to analyze relative price efficiency of the Egyptian manufacturing sectors, but they do not measure technical efficiency of these sectors, particularly, in a dynamic framework.

The private sector has always been important for the economic growth and development in Egypt. However, the Egyptian government adopted rigorous privatization policies in the early 1990 that were followed by increased growth of the private manufacturing sectors, and as a result, Egypt's manufacturing sector became the highest contributor to the value-added at the national level. Several sub-sectors of the private manufacturing sector (like food and textile) generated good opportunities of employment for unskilled and semi-skilled labors, particularly in a labor abundant country like Egypt. Moreover, during the 1990s, the activities that contributed higher value-added at the national level got more priorities and as a result the input ratios were changing within different sectors. Since frequent or rapid changes in the input ratios and use of unskilled and semi-skilled labor are potential source of sluggish adjustment of inputs, I expect the production process and technical efficiency of the Egyptian private manufacturing sectors to be affected by the adjustment of inputs, and the speed of adjustment to change over time.

4.2. Results

As discussed before, the technical efficiency as well as the speed of adjustment of output may vary over time. More specifically, it is likely that the rate of adjustment of an input improves over time by the process of learning and doing, and as a result, the speed of adjustment of output increases as well. Consequently, the technical efficiency of a production unit is likely to increase with time. Using a Cobb-Douglas production function to specify production of the potential output of the manufacturing sectors[7], the dynamic specification as given in equation (2.4) is estimated as -

$$\ln y_{it} = (1-\lambda_t)\ln y_{i(t-1)} + \lambda_t(\beta_0 + \partial t + \sum_{m=1}^{4}\beta_m \ln x_{mit}) + e_{it} \qquad (4.1)$$

where $e_{it} = -\theta_t f_i + \tau_{it}$. The inputs are capital, labor, energy and material with m = 1 for capital, m = 2 for labor, m = 3 for energy, and m = 4 for material.

Following the method described in section 3, we estimate the speed of adjustment of output for time periods $t =$ 4,…,9, and the time varying technical efficiency for each sector i = 1,…,28. I use the two-stage least squares results that are consistent and find that the coefficient of the lagged dependent variable in the transformed equation (3.2) is

[7] I compare a Cobb-Douglas and a more general translog production function for the data. Based on the information criterion (AIC and BIC) from these two models, I select the Cobb-Douglas production function.

positive and significant for every period, implying that the true process of output generation is dynamic. The coefficient estimates[8] of the lagged dependent variable are given in Table 3 along with their *t*-ratios that use heteroskedasticity corrected standard errors[9]. Finally, I recover the parameter estimates from the original model for each period by minimizing an over identified system of equation, as discussed in the previous section.

The estimation results show that the speed of adjustment of output ranges from 43% to 55% for the sample over the period under consideration (given in Figure 1), with an average of 49%. Thus, on average, the actual change in output in any period is 49% of the change in output that is needed to catch up with the potential output. Moreover, the gap between the change in actual output and the desires change reduces over time as the inputs gets more time to learn and adjust within the production system.

The average time-varying technical efficiency as measured from (4.1) is given in Table 4, which shows that during the period, the private manufacturing sectors of Egypt were approximately 90% technically efficient on average. To compare these results with the estimates from a static stochastic frontier, I also estimate the time-varying technical efficiency from the static model –

$$\ln y_{it} = \beta_0 + \partial t + \sum_{m=1}^{4} \beta_m \ln x_{mit} + \pi_{it} \tag{4.2}$$

where $\pi_{it} = -\rho_t \kappa_i + w_{it}$. The average technical efficiency for a sector during the period under consideration is found to be only 79% when measured from a static model that assumes instantaneous adjustment of all inputs. Thus, in the presence of sluggish

[8] The coefficient of the lagged dependent variable in the transformed equation (3.2) is given by $(1 + r_t - \lambda_t)$ for each *t*.

[9] The total number of parameter estimates from the quasi-transformed model is 72 and I present only the relevant ones.

adjustment of inputs, a static model misspecifies the production process and underestimates the true technical efficiency of a sector on average which is likely to be the result of attributing the shortfall in output that occurs during the short-run adjustment of inputs to inefficiency of the production unit.

Further, I find that the absolute difference between the efficiency estimates from the static and the dynamic model is 17 percentage points on average, and can be as high as 54 percentage points for a sector in a period. Since the static model seems to underestimate the technical efficiency, I present the magnitude of this underestimation in Table 4 as well. I find that the static model underestimates the technical efficiency of production units by 11 percentage points on average, i.e., the static model underestimates technical efficiency of a sector in a period by 12%, on average.

Instead of presenting the technical efficiency for all observations I present the average for each sector in Table 5. Figure 2 further illustrates the contents of this table. From column (1) and (2) of Table 5, that show the average technical efficiency estimates for each sector respectively, it is evident that by ignoring the adjustment process of inputs, the static model underestimates the technical efficiency for most of the sectors on average. However, due to the fact that only relative efficiency has been measured using the stochastic frontier approach, the technical efficiency estimates from the static model can be either higher or lower than the estimates from the dynamic model, for a particular sector. Though the direction of bias may not be uniquely identified for all sectors while comparing relative technical efficiency measures from the static and the dynamic model, I find that the static model underestimates technical efficiency of a sector in a period by 12% on average. This underestimation can be attributed to the fact that the static production model considers the natural process of input adjustment as a source of inefficiency of production units.

I also find that the ranking of sectors from the dynamic and the static model are markedly different, and the best performing sector is also not the same according to these two production models. The ranking of sectors according to the dynamic and the static production model are given in column (3) and (4) of Table 5, respectively. Further investigation on the ranks of sectors, as assigned by the dynamic and static production model, reveals that the Spearman’s correlation coefficient is 0.34 for them, and I cannot reject the hypothesis that the ranks from the static and dynamic model are independent at the 5% significance level (*p*-value for the test statistic is 0.08).

Finally, I look into the pattern of variation of technical efficiency over time for each sector, and compare them as obtained from a dynamic and a static production model. The time-varying technical efficiency estimates from both models are presented in Figure 3, separately for each of the 28 sectors in the sample (Figure 3(i) – 3(xxviii)). Figure 3 reveals that the dynamic production model identifies more variation in the time pattern of technical efficiency, for each sector, when compared to the pattern of time variation of technical efficiency as estimated from a static production frontier. Thus, by ignoring the lagged adjustment of inputs, the static model not only provides biased estimates of technical efficiency, but it also fails to capture the temporal variation in the efficiency measures.

A closer look at the economic conditions of Egypt during the period under consideration reveals that the Egyptian government adopted rigorous privatization policies in the early 1990. Since then, there have been substantial changes in the structure of the private manufacturing activities. The new economic policies enhanced competition and opened up possibilities for further privatization through international investment banking. Consequently, it tended to attract investment for high technology and managerial and marketing skills that was likely to foster higher level of productivity and efficiency. From Figure 3, it is visible that starting with 1991/1992, which is the 5th year in Figure, technical efficiency of each sectors improved substantially as shown by the efficiency estimates from the dynamic production model. Every sector followed an upward rising trend in the technical efficiency after 1991/1992, signifying the effects of new economic policies implemented by the Egyptian government in early 1990s. As a result of these new economic policies, production resources were geared more toward the sectors, that were likely to promote growth, and the private manufacturing sectors were the prominent ones among them. Thus the production in the private manufacturing sectors experienced significant change in the input structure. Moreover, the private manufacturing sector was also a source of employment for the unskilled and semi-skilled labor. Therefore, it is very plausible that the inputs of production exhibited substantial adjustment process during 1990s, supporting a dynamic production model, and efficiency of sectors markedly improved in the 1990s.

However, the pattern of time variation in technical efficiency for each sector as estimated by the static production model fails to capture this phenomenon as shown in the Figure 3. By assuming instantaneous adjustment of inputs, the static model estimates a steady but slow improvement in efficiency for all the sectors, and thus do not show the marked improvements in efficiency of sectors after implementation of the privatization policies. Therefore, it is clear from the Figure 3 that the dynamic production model captures more variation in the time pattern of technical efficiency than the static model, by allowing for sluggish adjustment of inputs.

5. Conclusion

This paper discussed estimation methods for the speed of adjustment of output and technical efficiency of production units that vary over time from a dynamic stochastic production frontier, which described the process of output generation in the presence of lagged adjustment of inputs. The dynamic production model acknowledged the fact that output could be lower in the short-run when the inputs were adjusted within a production system, and accordingly measured technical efficiency of production units. The paper further illustrated the methods of estimation using data from the private manufacturing sectors in Egypt, and found that the speed of adjustment of output was significantly lower than unity for the period under consideration. The dynamic model also identified that the gap between the actual change in output and the desired change reduced slowly over the period under consideration. This, in turn, suggests that the conventional static model that assumes instantaneous adjustment of inputs is missspecified, and provides biased estimates of technical efficiency. Comparing the technical efficiency estimates from the dynamic model with those from a static model, I found that the static model underestimated technical efficiency of different sectors by 11 percentage points on average that could be as high as 54 percentage points.

Further, I found that the dynamic production model captured more variation in the time-pattern of technical efficiency of a production unit as compared to a static production model, and provided an internal ranking of production units considering their short-run adjustment of production plans. Particularly, for the private manufacturing sectors of Egypt, I found that efficiency of the sectors significantly increased after implementing privatization policies in the early 1990s that was captured by the dynamic production model but not by the static production model.

To conclude, this paper has provided a more realistic and rigorous approach for capturing the dynamics of a production system, and measuring the speed of adjustment of output and technical efficiency both of which may vary over time. The dynamic production frontier, as discussed in this paper is particularly suitable for country like Egypt where sluggish adjustment of inputs is a very plausible phenomenon in light of the facts that during the period under consideration, Egypt employed unskilled and semi-skilled labor in the manufacturing sectors and also underwent through several changes in those sectors. Since producers often take important production decisions based on the efficiency of the units, a dynamic frontier that incorporates the short-run quasi-fixity of inputs is a reasonable one to use for this purpose.

The theoretical and econometric models, as discussed in this chapter, are based on the simplifying assumption that the speed of adjustment of inputs is similar for all inputs, and every production unit. However, different production units and inputs may have different speeds of adjustment. The econometric method for estimating such a dynamic production frontier with time-varying individual effects with large N (number of production units) and fixed T (time period under consideration) is an open research area till now. While this paper does not discuss methods to estimate technical efficiency under less restrictive assumptions, these should be interesting areas of exploration for future research in this field. Moreover, instead of measuring relative efficiency of production units and thus failing to generally specify a direction of bias of efficiency estimates from a misspecified production model, using bootstrapping techniques to compare the efficiency estimates from a static and a dynamic model with time varying technical efficiency and speed of adjustment would be another area to explore in the future.

Appendix : Tables and Figures

Table 1: Sectors and the Industrial Activities at the Three-digit ISIC Level

Sector Number	Industrial activity
1	Food manufacturing
2	Other food manufacturing
3	Beverage and liquor
4	Tobacco
5	Manufacture of textile
6	Manufacture of wearing apparels
7	Manufacture of leather products
8	Manufacture of footwear
9	Manufacture of wood products
10	Manufacture of furniture & fixture
11	Manufacture of paper products
12	Printing and publishing industries
13	Manufacture of industrial chemicals
14	Manufacture of other chemical products
15	Other petroleum and coal
16	Manufacture of rubber products
17	Manufacture of plastic products
18	Manufacture of pottery and china
19	Manufacture of glass and glass products
20	Manufacture of other non metallic products
21	Iron and steel basic industries
22	Non-ferrous basic industries
23	Manufacture of fabricated metal products
24	Manufacture of machinery except electrical
25	Manufacture of electrical machinery
26	Manufacture of transport equipment
27	Manufacture of professional equipment
28	Other manufacture industries

Table 2: Variable Descriptions and Summary Statistics

Variable	Description	Observation	Mean	Standard deviation	Minimum	Maximum
Yearid	id number for 9 years of data for each sector	252	5	2.59	1	9
Sectorid	id numbers for the 28 three digit manufacturing sectors	252	14.5	8.09	1	28
Output	Output quantity index	252	2888.90	3333.39	67	19236
Capital	Capital quantity index	252	288.84	475.29	1	3437
Labor	Labor quantity index	252	273.34	344.06	10.50	1689.2
Energy	Energy quantity index	252	61.97	116.56	0.20	860.1
Material	Material quantity index	252	1823.44	2168.83	44.8	11853.8

Source: Getachew and Sickles (2007).

Table 3: Coefficient of the Lagged Dependent Variable in the Time-Varying Technical Efficiency Model

Year	Coefficient of lagged dependent variable	t-ratio
4	0.069	5.61
5	0.073	8.43
6	0.084	7.83
7	0.095	10.77
8	0.087	12.26
9	0.082	20.41

Note: The results are from the two-stage least squares analysis. The standard errors are corrected for heteroskedasticity.

Table 4: Difference in the Time-Varying Technical Efficiency Estimates from Dynamic and Static Specifications

Variables	Mean	Maximum
Technical Efficiency_Dynamic	90.26	100
Technical Efficiency_Static	79.48	100
Difference in Efficiency Estimates	17.12	54.47
Underestimation by the Static Model	10.77	54.47

Note: The technical efficiency estimates from the dynamic and the static models are presented in percentage terms. These estimates show the efficiency level of a production unit relative to the most efficient unit in the sample.

The difference in efficiency estimates is calculated by taking the absolute difference in the technical efficiency estimates from the dynamic and the static model. The difference in efficiency estimates and the underestimation by static model are presented in terms of percentage points.

Table 5: Average Time-Varying Technical Efficiency Estimates and Ranking of Sectors under Dynamic and Static Specifications

Sectorid	Technical Efficiency from Dynamic Specification (%) (1)	Technical Efficiency from Static Specification (%) (2)	Rank_Dynamic Specification (3)	Rank_Static Specification (4)
1	90.07	81.19	13	14
2	89.18	67.03	21	25
3	92.19	86.81	3	7
4	89.09	71.25	22	22
5	89.62	84.95	18	8
6	88.45	82.72	28	12
7	90.22	99.55	12	2
8	88.52	88.55	24	6
9	88.91	91.85	23	5
10	89.67	76.13	16	18
11	88.50	47.61	26	28
12	90.36	84.18	10	10
13	89.95	80.61	15	15
14	89.97	69.99	14	23
15	92.06	100.00	4	1
16	94.77	97.04	1	4
17	90.59	82.81	9	11
18	94.30	82.17	2	13
19	90.78	74.78	7	21
20	90.78	69.28	8	24
21	89.47	56.08	19	27
22	90.26	98.02	11	3
23	91.83	75.53	5	20
24	89.63	84.61	17	9
25	91.72	77.94	6	17
26	88.47	58.61	27	26
27	88.52	76.07	25	19
28	89.41	80.23	20	16

Note: Technical efficiency of a sector is measured relative to the most efficient sector.

Figure 1: Speed of Adjustment from Dynamic and Static Specifications

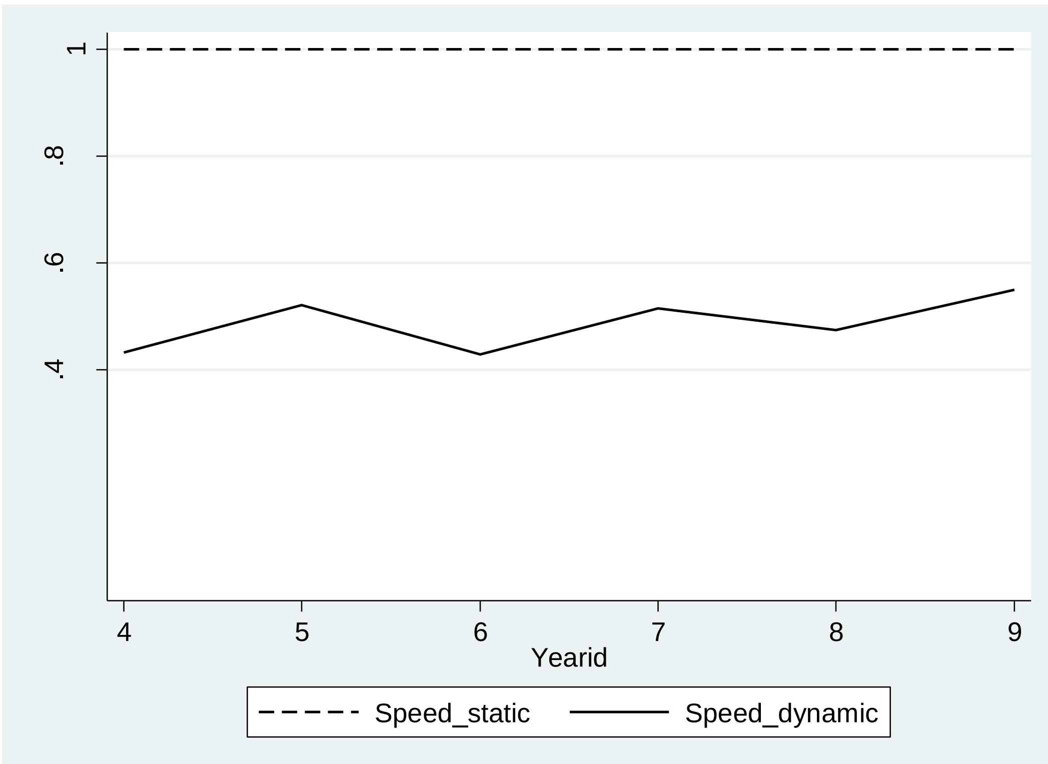


Figure 2: Average Time-Varying Technical Efficiency for Sectors from Dynamic and Static Specifications

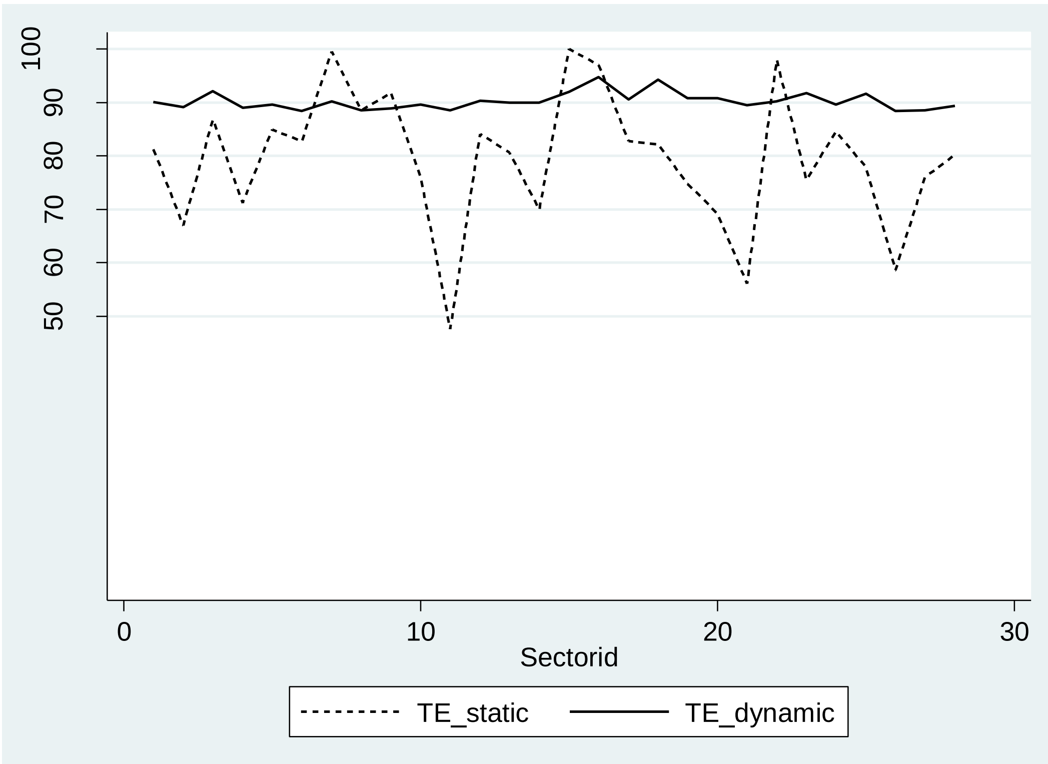
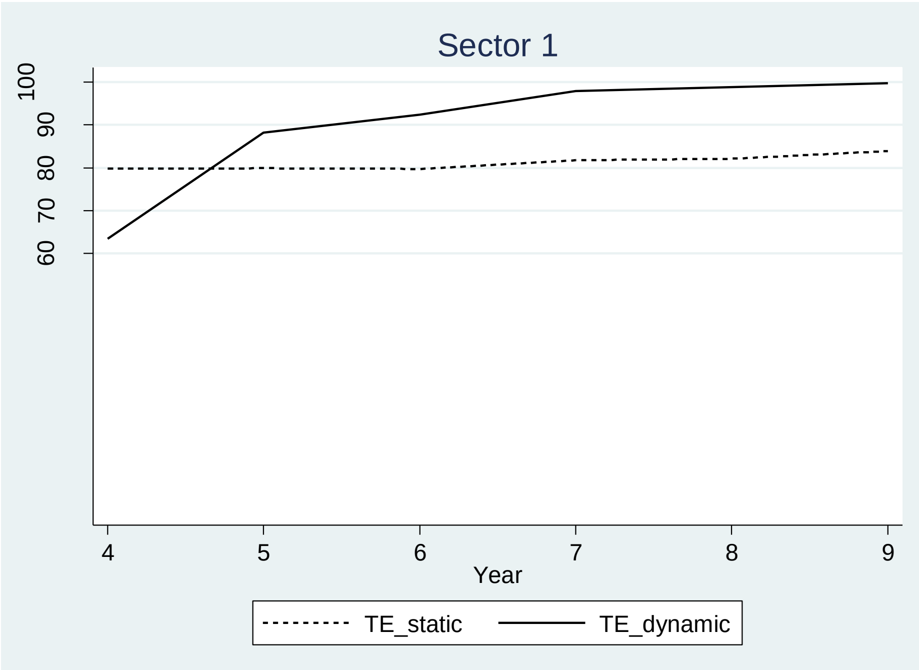


Figure 3: Time-Varying Technical Efficiency for Different Sectors



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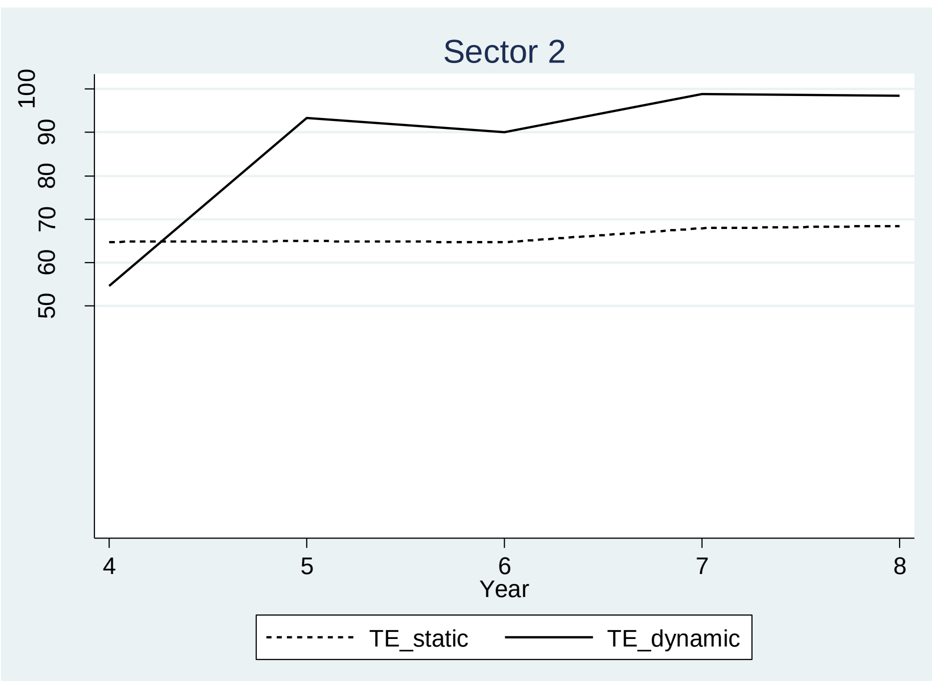

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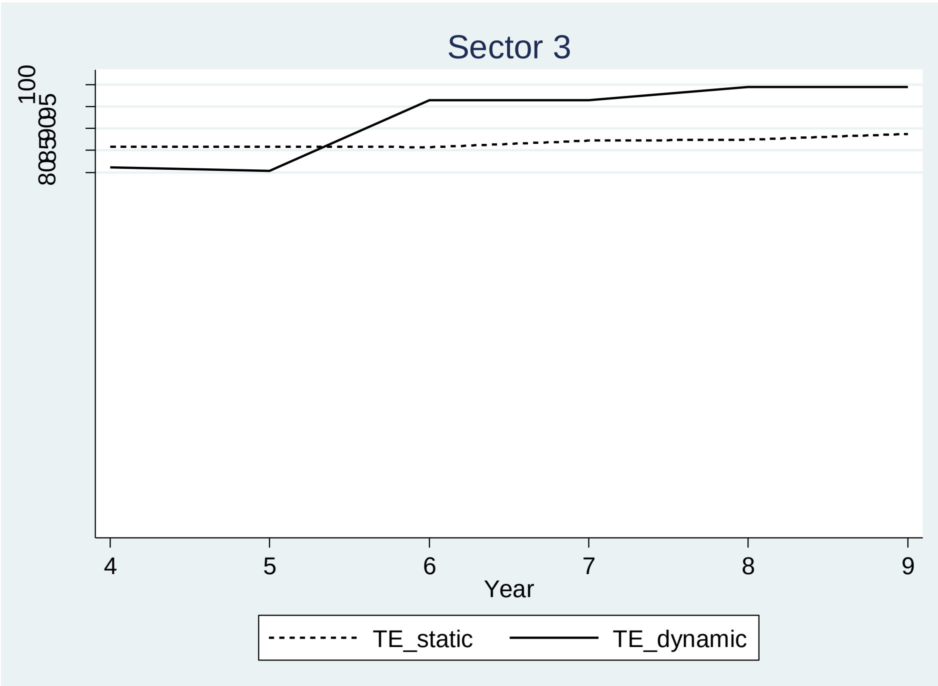

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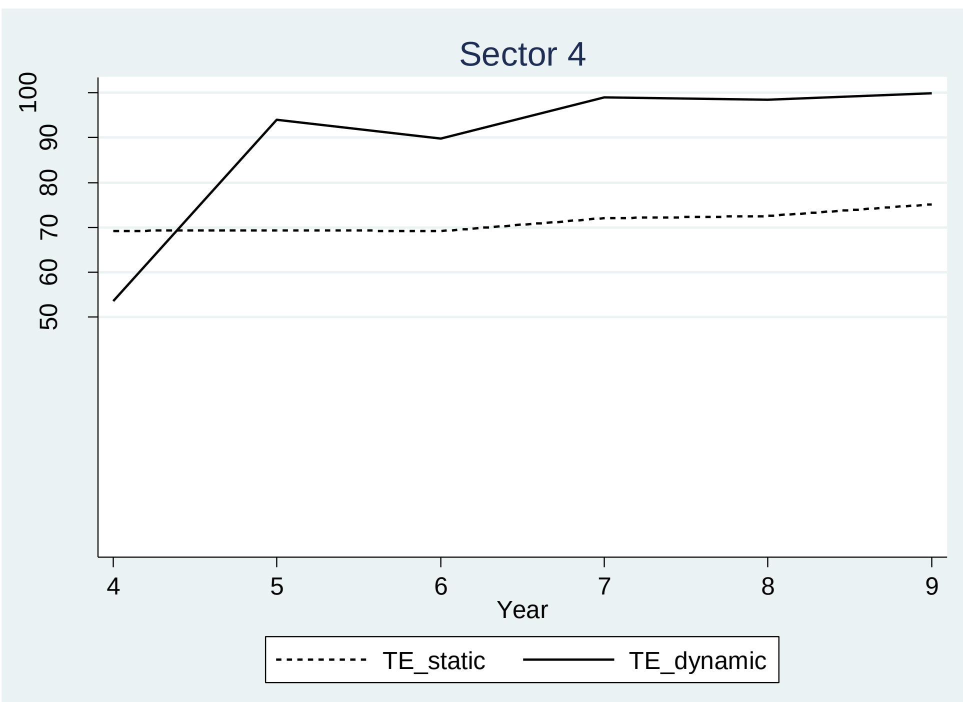

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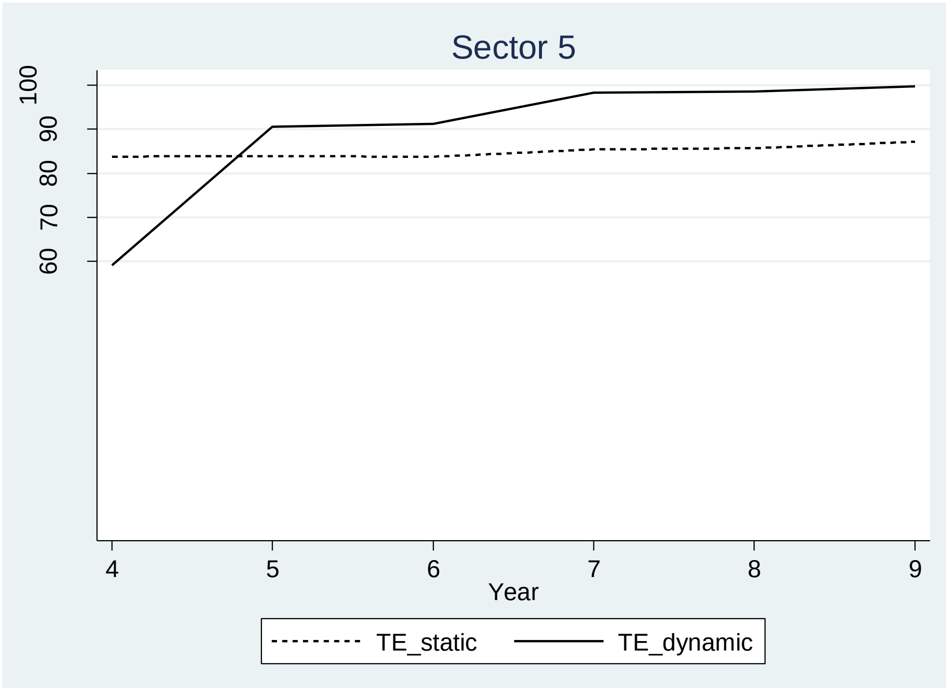

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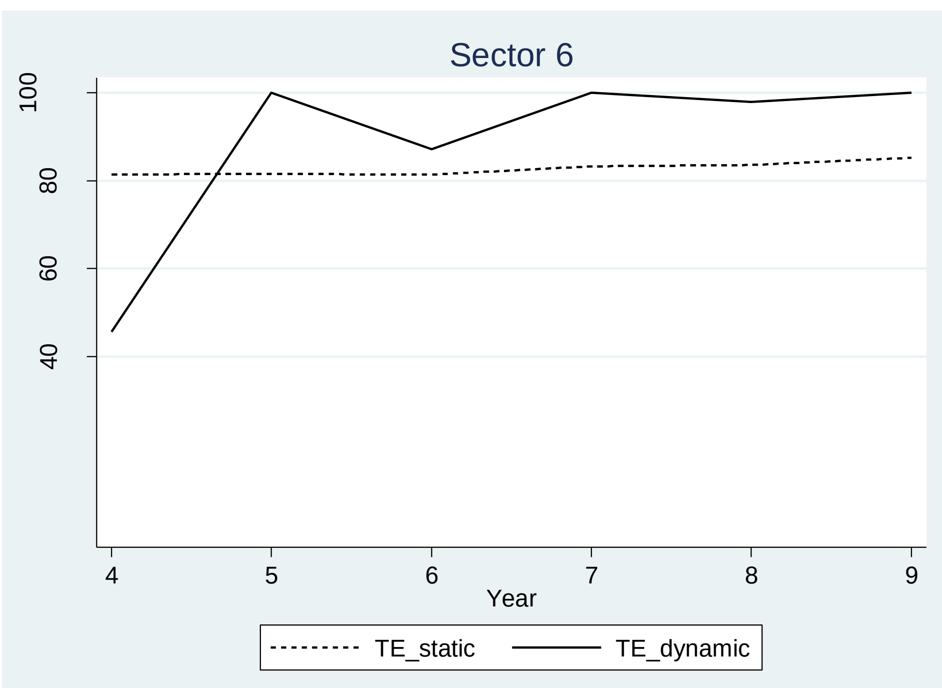

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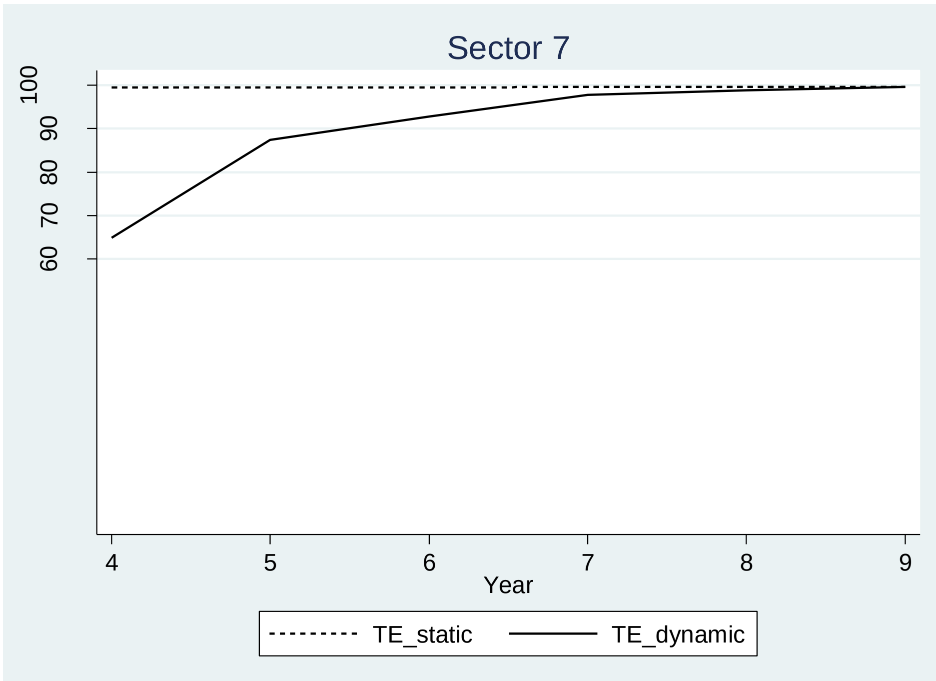

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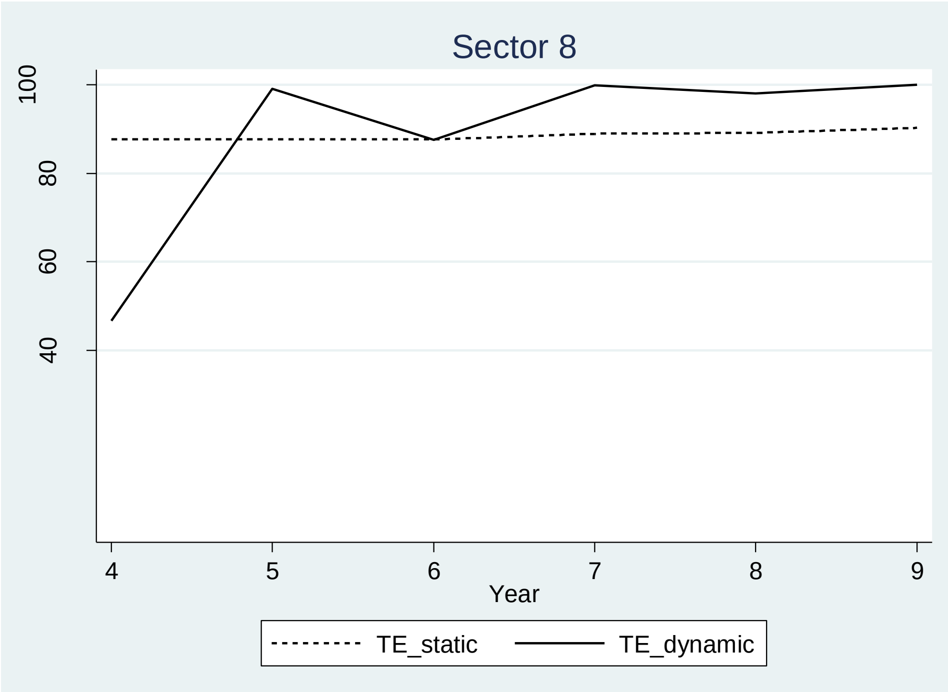

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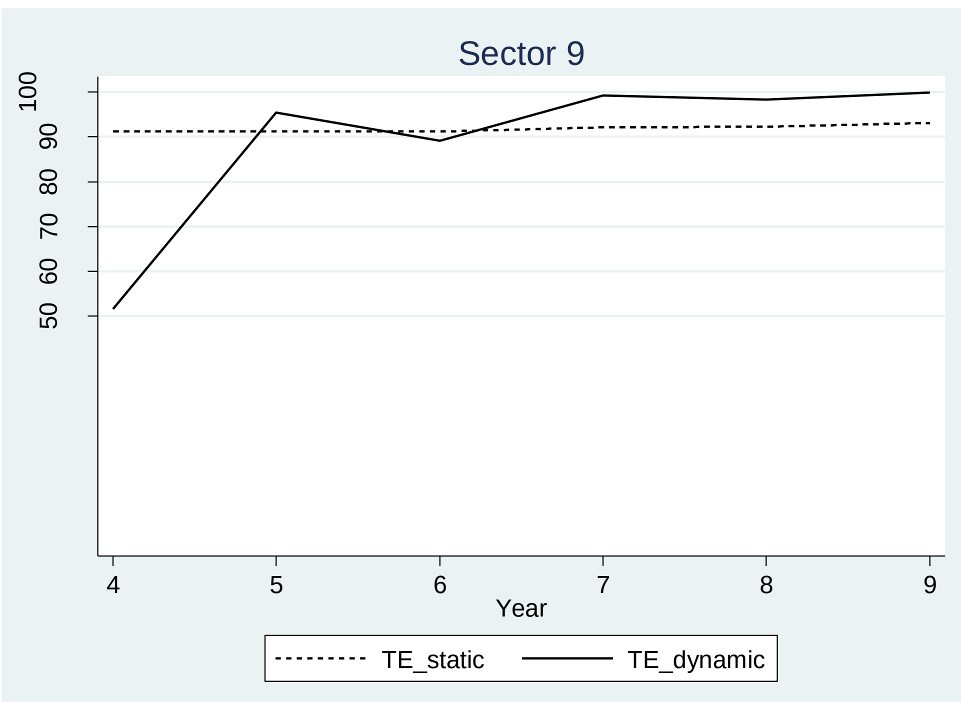

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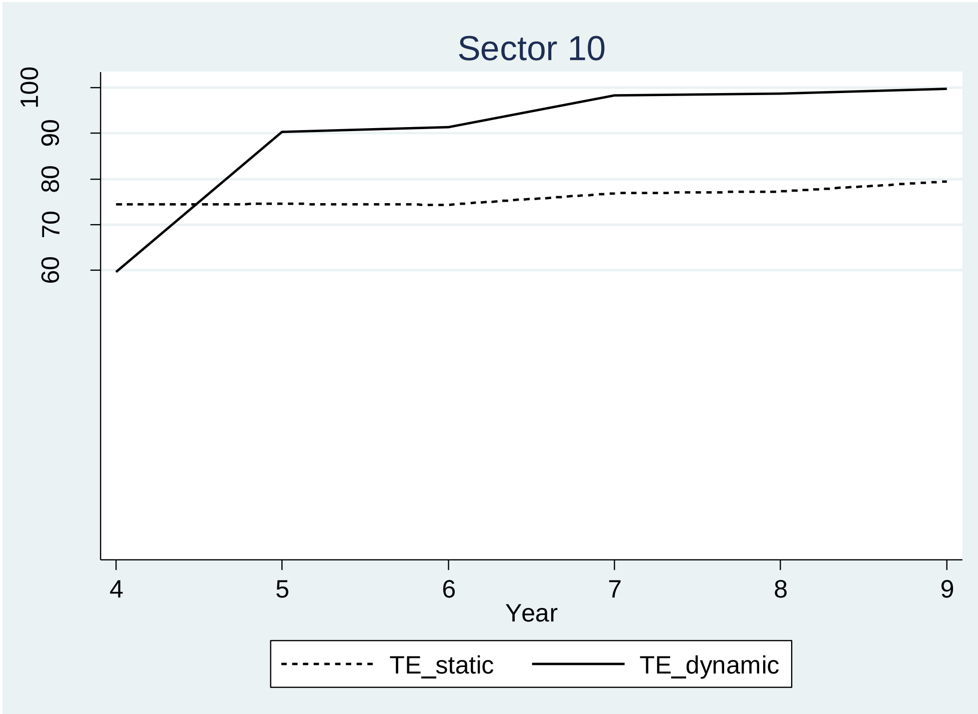

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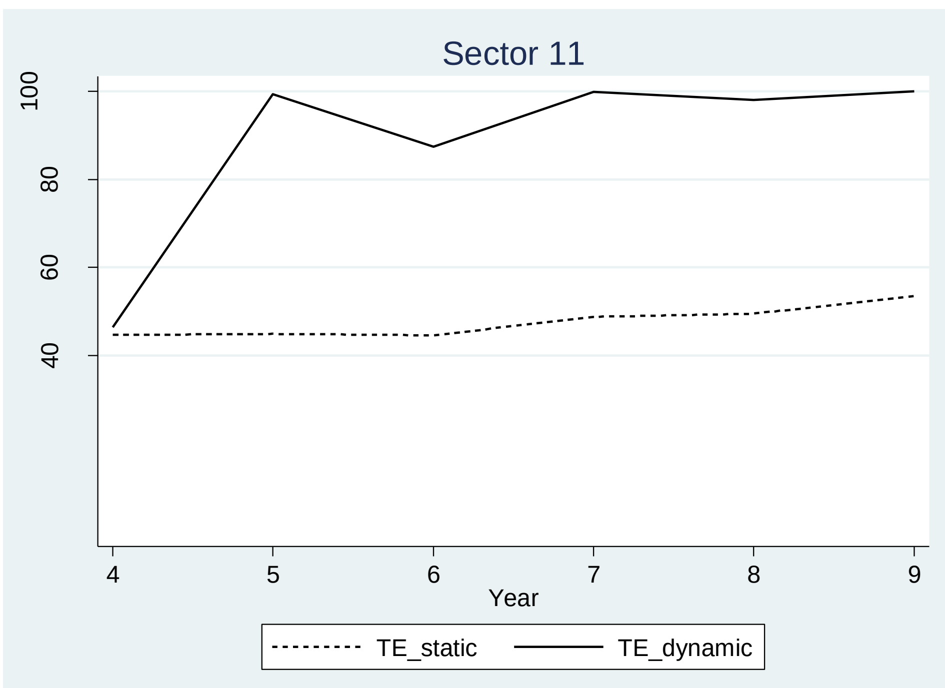

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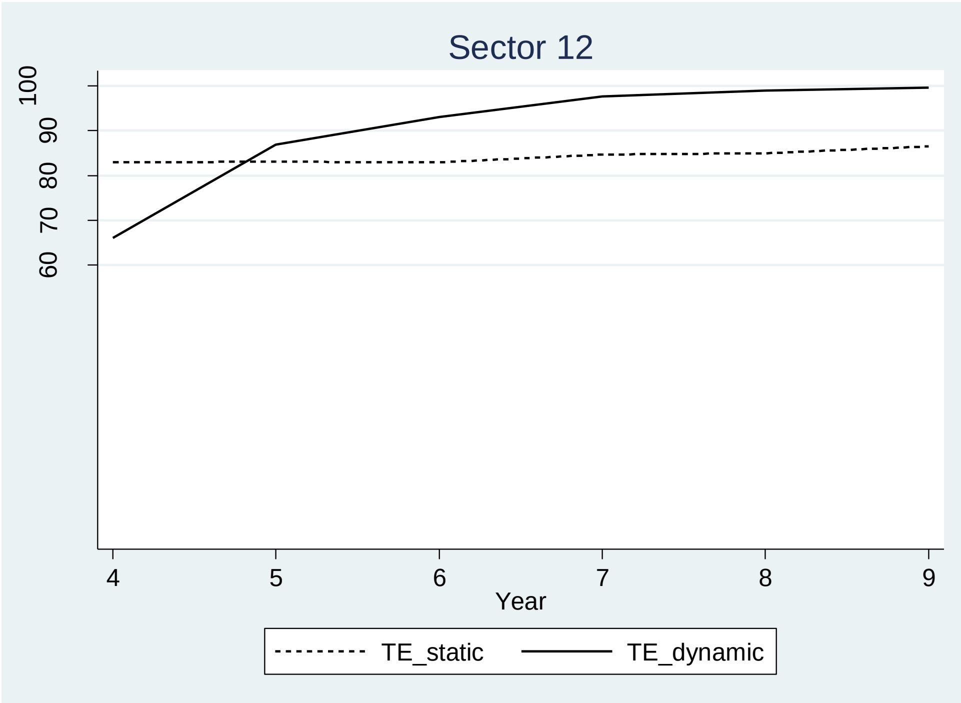

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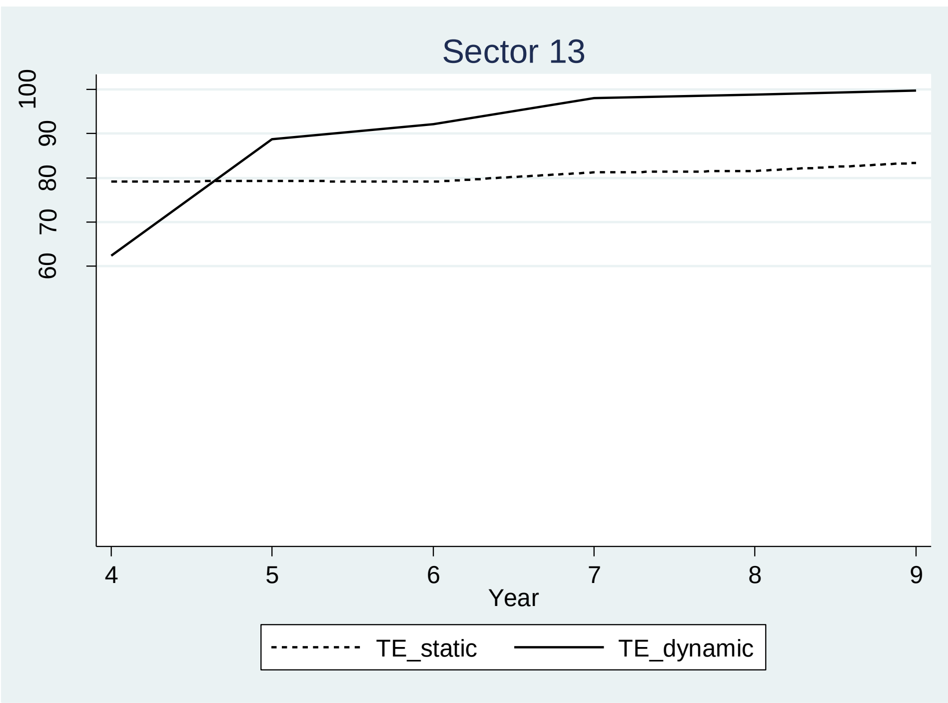

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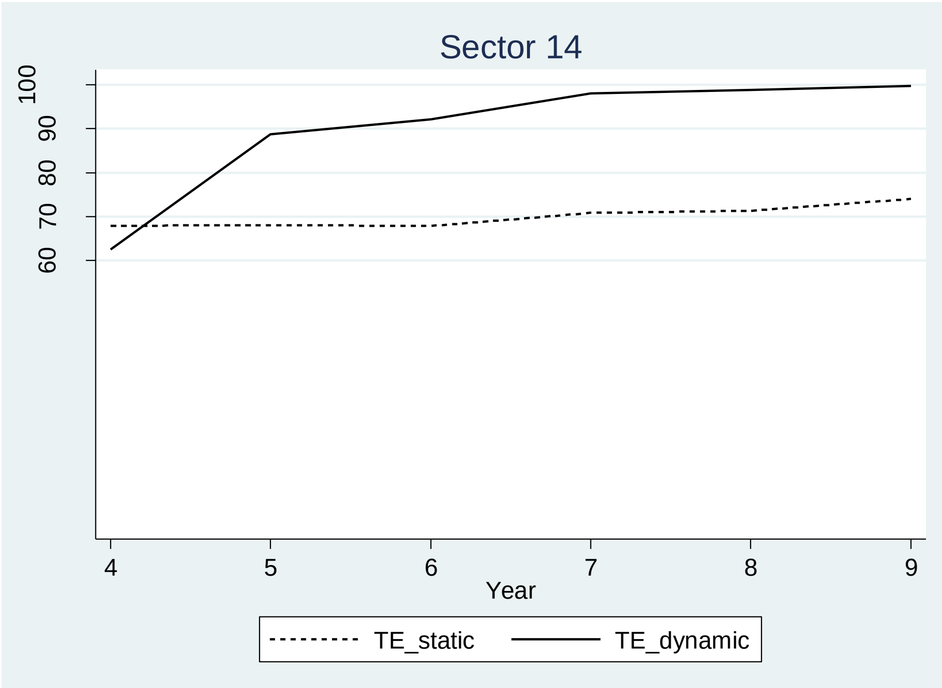

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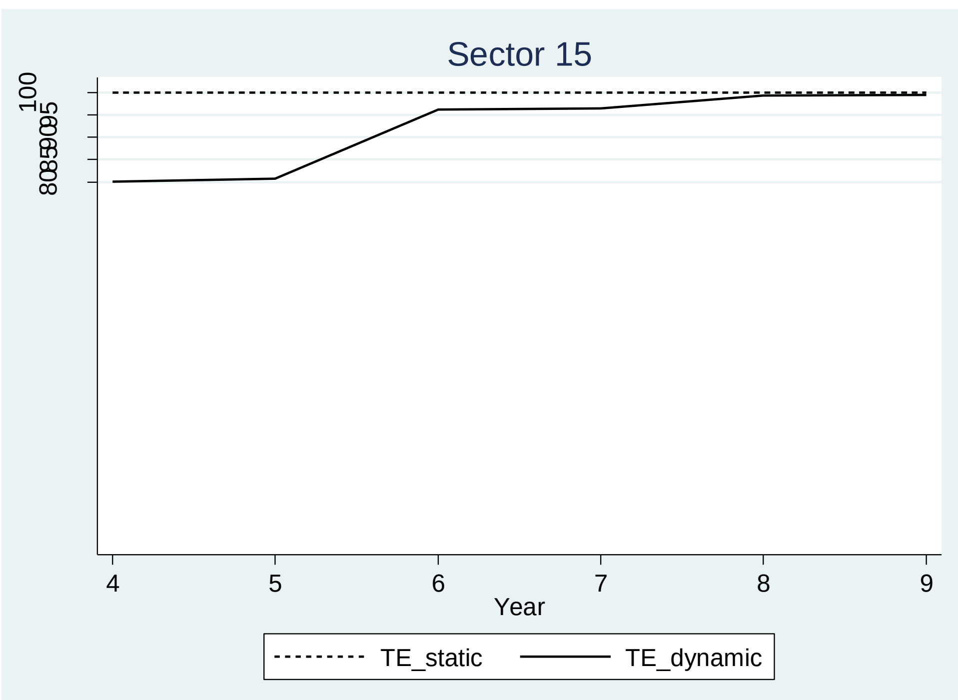

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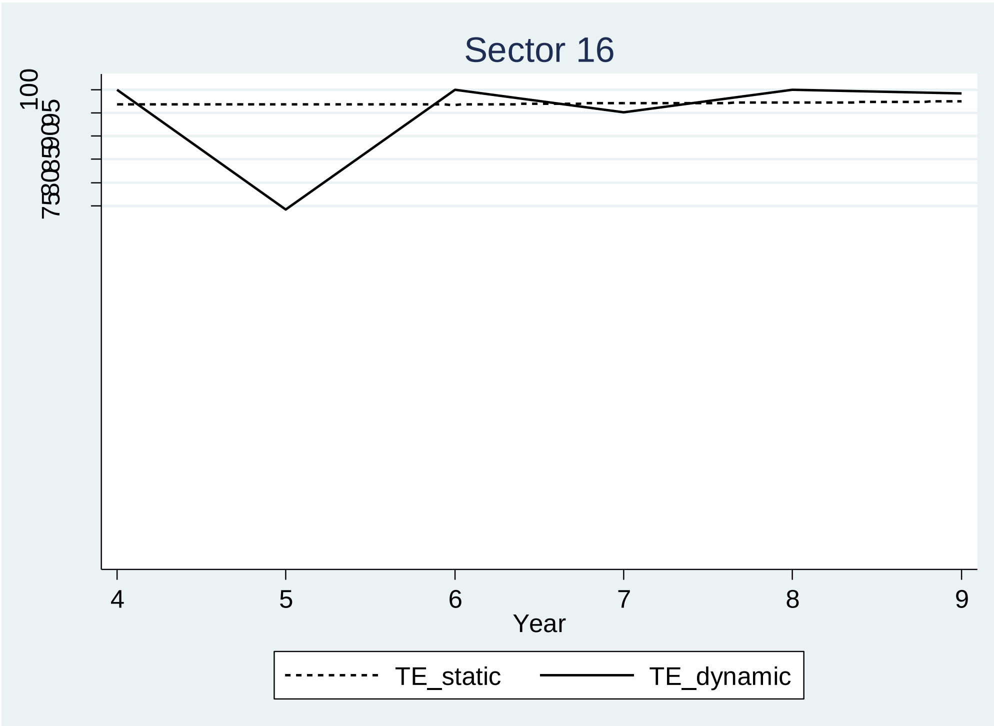

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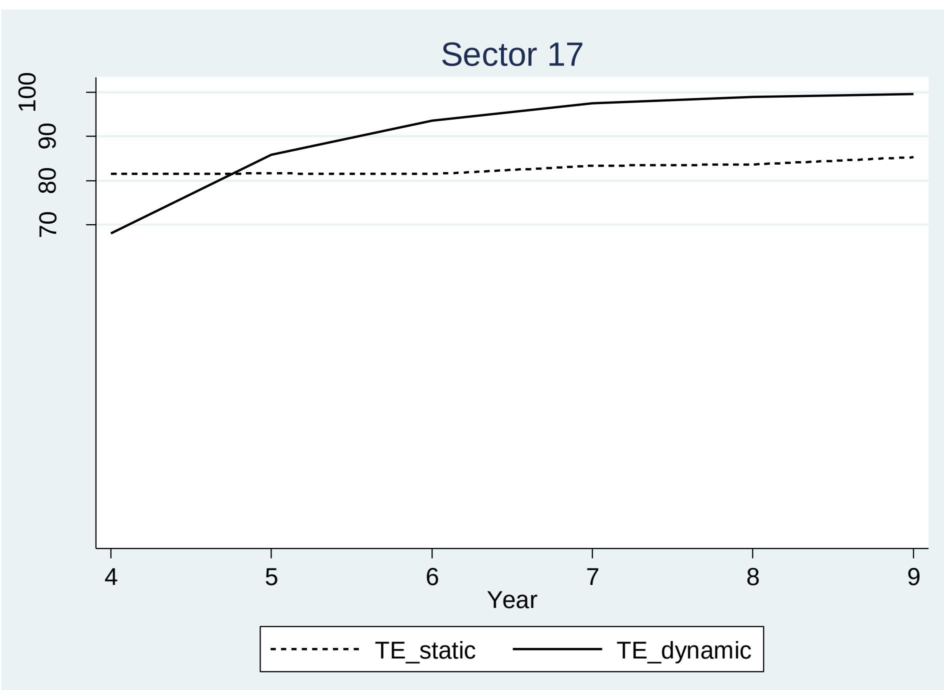

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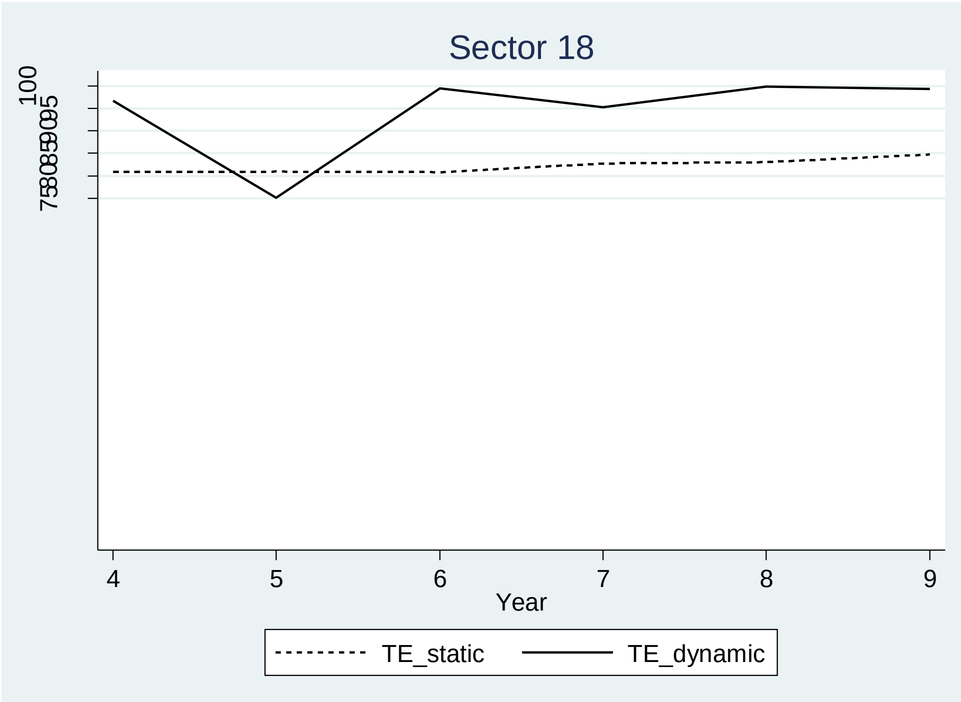

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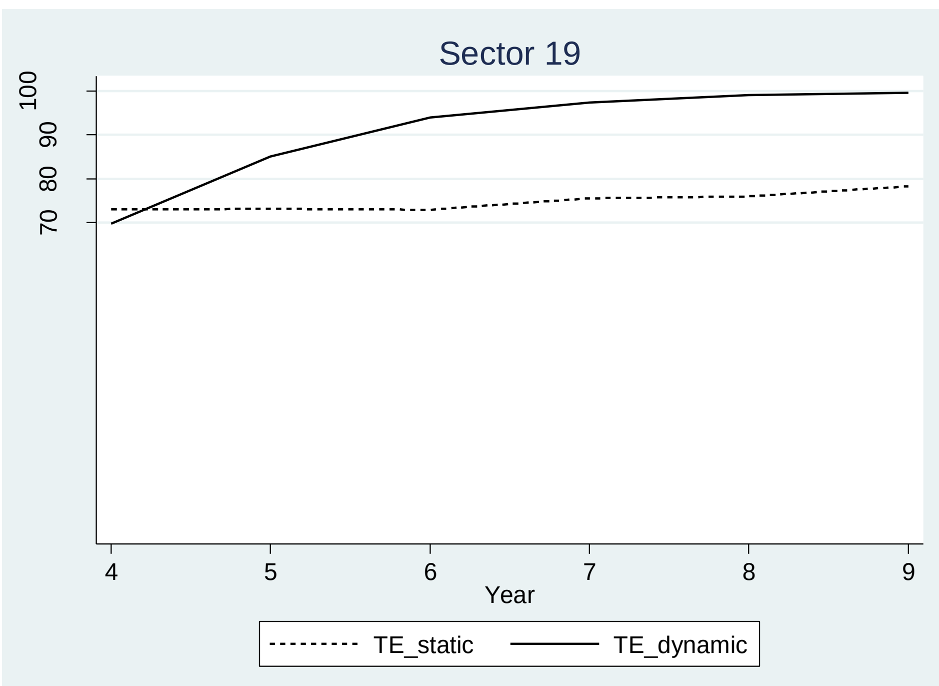

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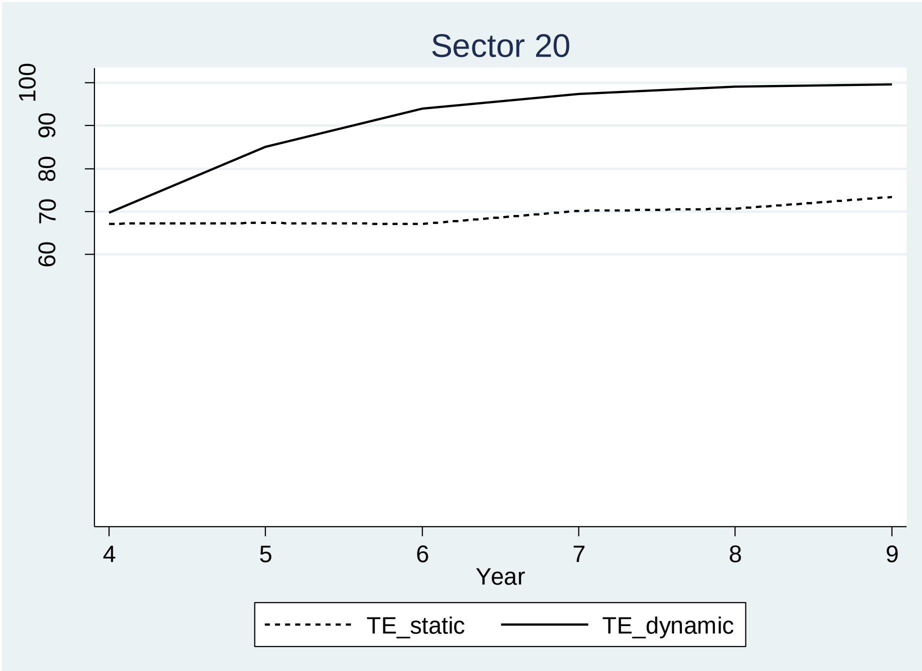

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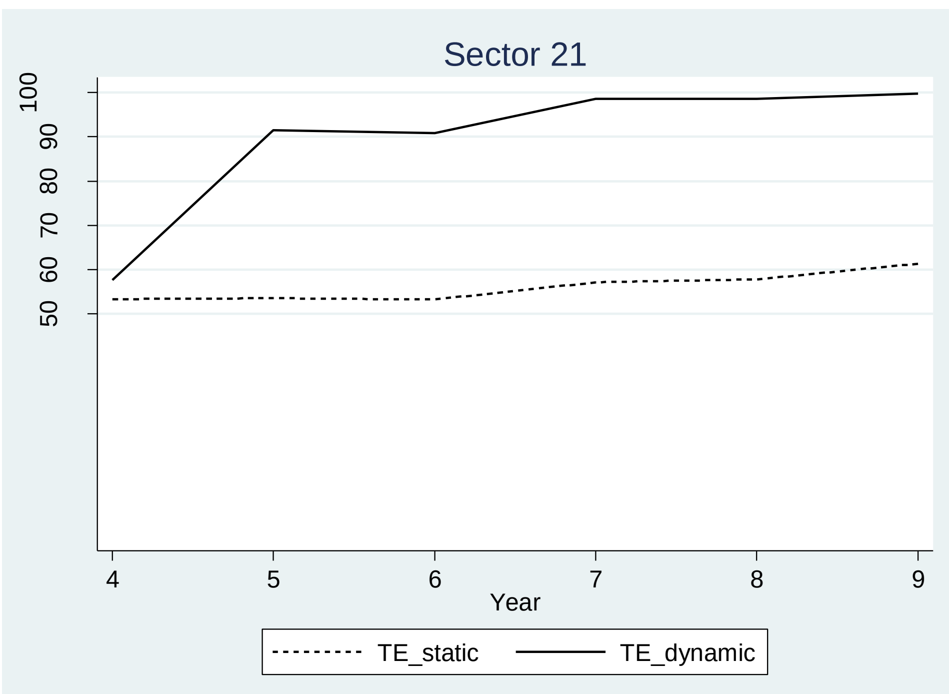

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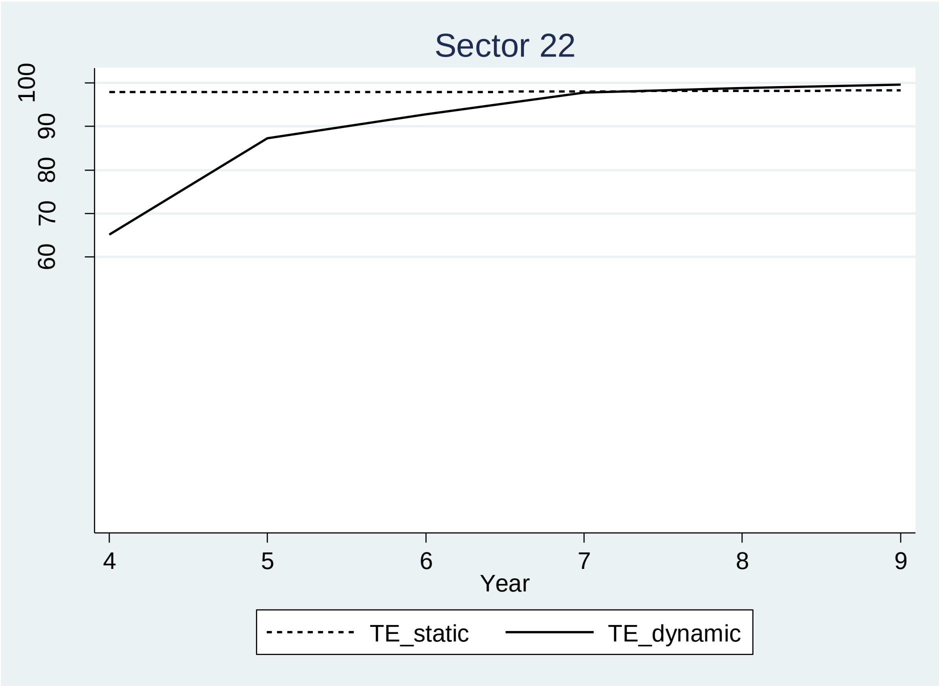

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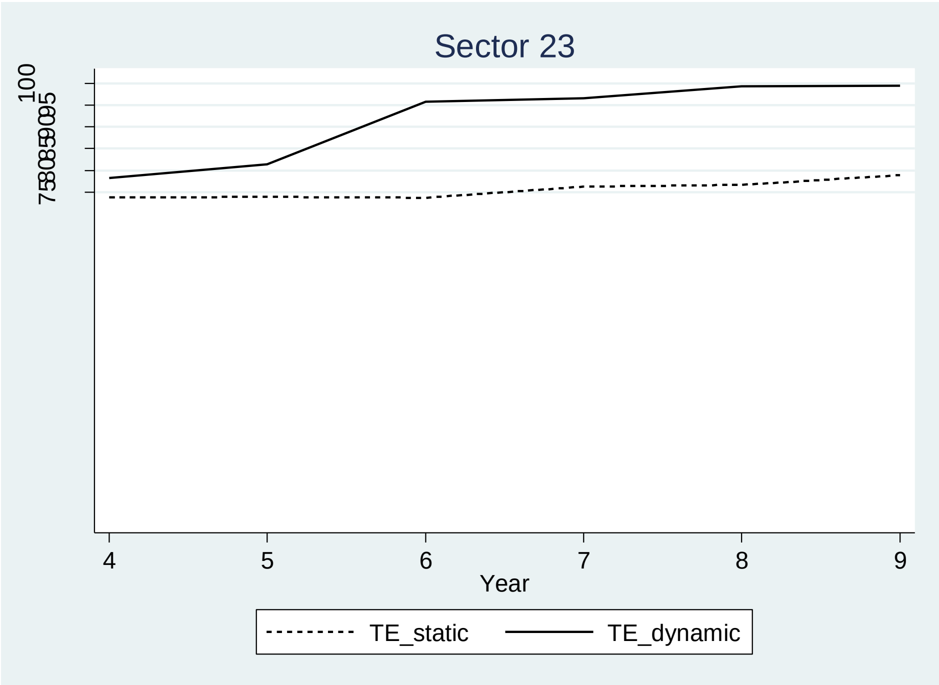

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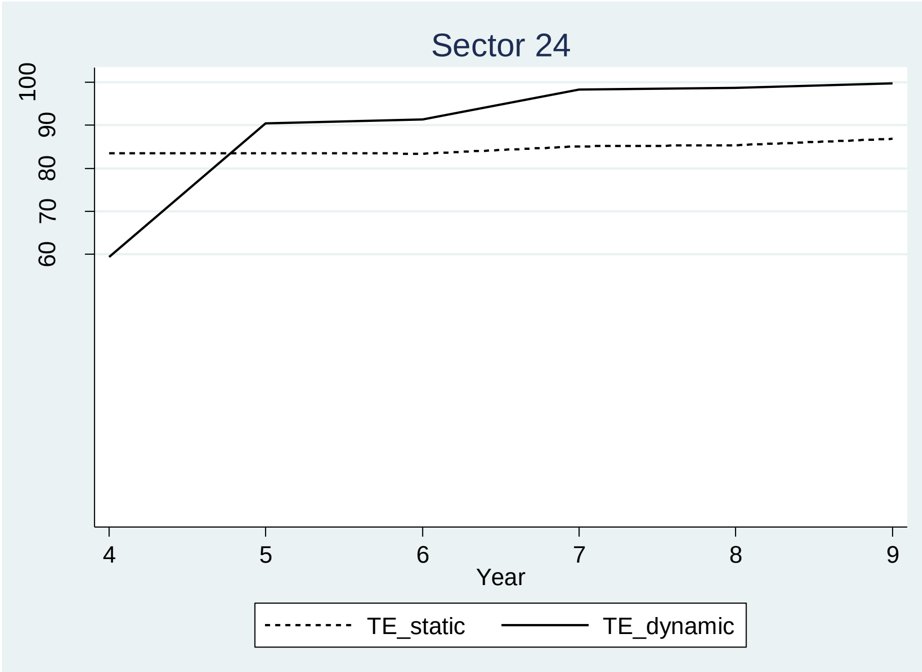

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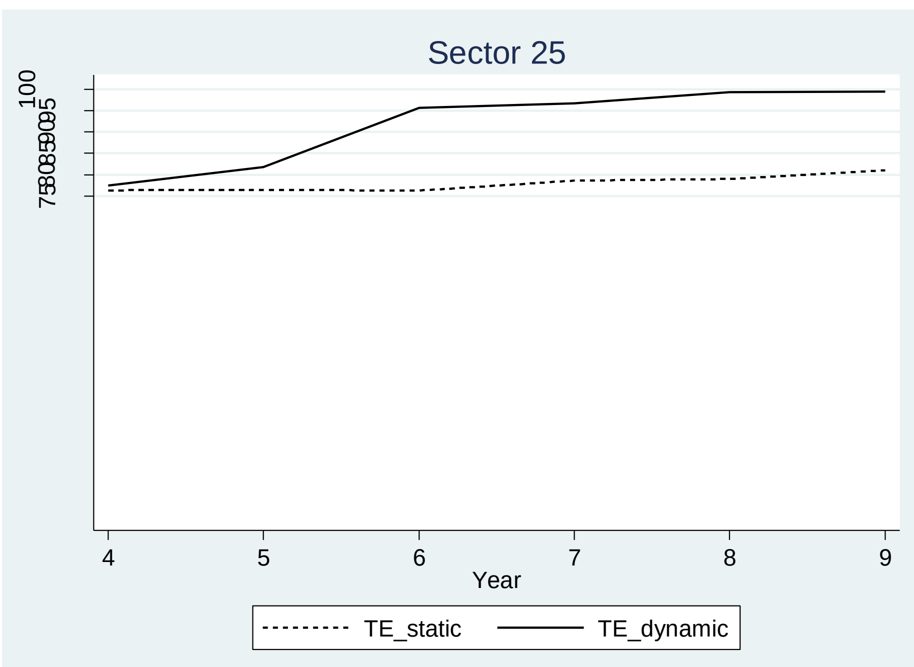

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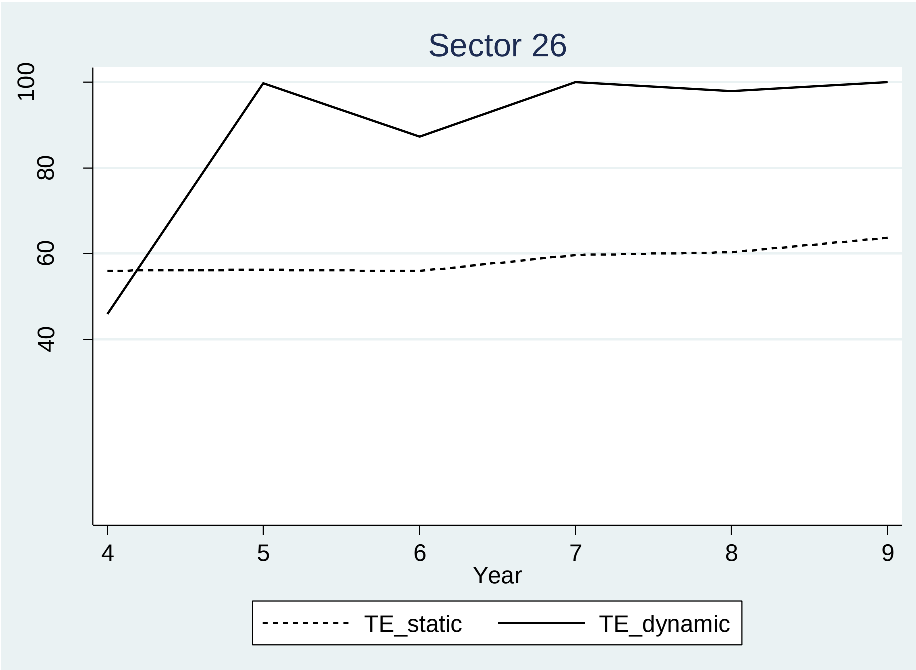

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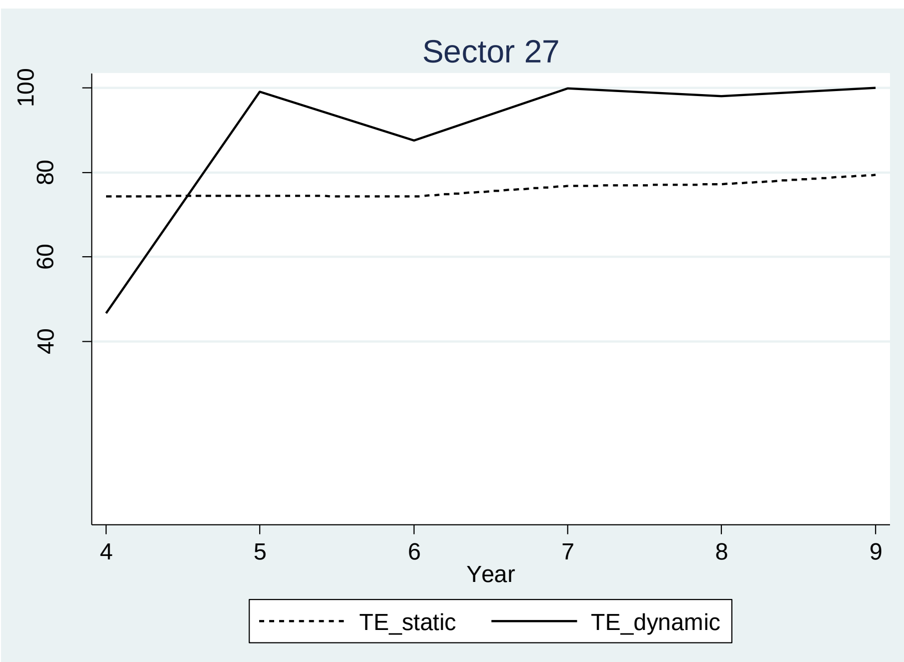

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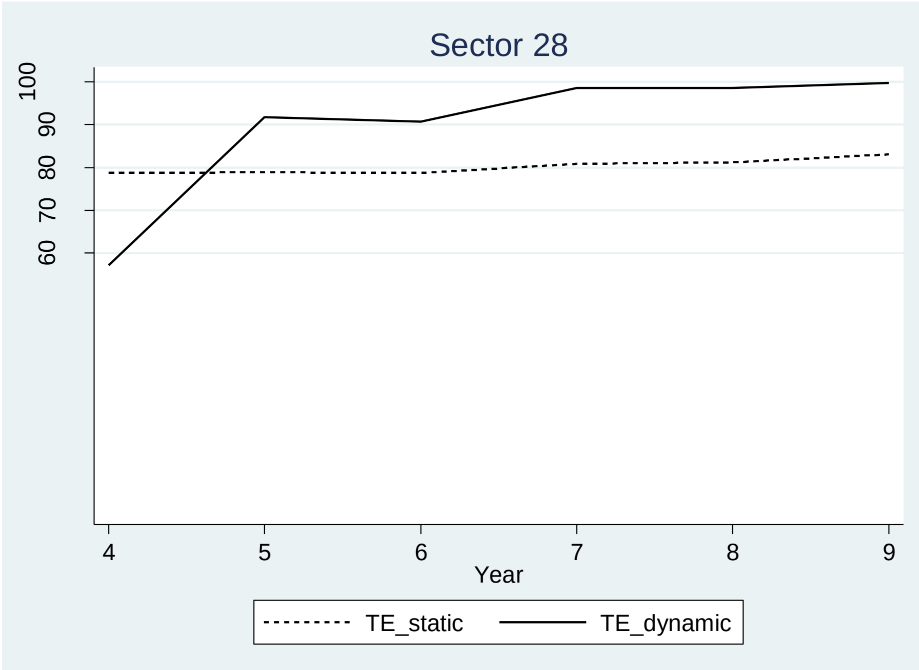

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