



UNIVERSITÀ DEGLI STUDI DI MACERATA

*Dipartimento di Istituzioni Economiche e Finanziarie*

## Roots and Effects of Investments' Misperception

R. Castellano, R. Cerqueti

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## Abstract

This work deals with the problem of investors' irrational behavior and financial products' misperception. The theoretical analysis of the mechanisms driving wrong evaluations of investment performances is explored. The study is supported by the application of Monte Carlo simulations to the remarkable case of structured financial products. Some motivations explaining the popularity among retail investors of these complex financial instruments are also provided. Investors are assumed to compare the performances of different projects through stochastic dominance rules and, to pursue our scopes, a new definition of this decision criteria is introduced.

*Rosella Castellano*, Università degli Studi di Macerata.

E-mail: [castellano@unimc.it](mailto:castellano@unimc.it).

*Roy Cerqueti*, Università degli Studi di Macerata.

E-mail: [roy.cerqueti@unimc.it](mailto:roy.cerqueti@unimc.it).

# 1 Introduction

The need to make forecasting on future performances of economic and financial variables is out of discussion and the way to do this represents the focus of an endless scientific debate. In this respect, *decision theory* can be seen as the field that describes and formalizes the process of making a choice among several possible uncertain alternatives. In a rational financial market, given an identical set of opportunities, two different individuals should operate the same choices, once the decision rule is fixed. Surprisingly, the decision rule is often violated and choices toward the objectively-measured less attractive situations are often registered. This evidence plays a central role in financial theories grounded on the psychology and the human judgment of investors. In this respect, two different questions arise: why random amounts are misperceived? How does the misperception mechanism work? To give possible answers to such questions represents the focus of *behavioral finance*. In particular, the latter question is tackled by *prospect theory*.

In this paper we provide theoretical results, supported by numerical analysis, on how investors distort the stochastic returns and, unreasonably, prefer complicated financial derivatives to standard ones. We apply our arguments to the analysis of structured financial products which seems to be appropriate for our purposes. A possible motivation for the misrepresentation of such instruments is provided through a behavioral-finance type argument. In particular, we show that the presentation by investment banks (or insurance companies) to investors of optimistic prospectuses may represent the ground of the wrong perception mechanism.

The decision tool we deal with is the stochastic dominance, since it undoubtedly represents one of the most general ways to compare financial products. The stochastic dominance criteria rely on the distribution functions of the random amounts and take into account the whole set of their characteristics (i.e.: fat tails, kurtosis, asymmetry, etc.).

This paper contributes to the existing literature in some directions. First, we deal with the problem of misperception of structured financial products, which is quite neglected in financial studies. Despite the scarce attention paid to this topic, it represents a really important issue, since structured products are very popular mainly among retail investors and households. Second, a behavioral finance-type discussion on our results, based on the main features of structured financial products, is addressed. In particular, the misperception concerning structured products can be interpreted as a consequence of the reticent prospectuses proposed by financial institutions to investors. In a more general context, we argue that investment banks and insurance companies are morally responsible of some irrational investors' behaviors and related dramatic consequences. Third, we analyze some peculiar types of distortion of random amounts. In particular, in accordance with some classical models of time series analysis, we assume that random sums may be perturbed via a deterministic trend or a lump sum. The former case concerns a global rereading of random amounts, while the latter relies on an investors' local misunderstanding of some realizations. Fourth, we refer to stochastic dominance decision rules and, in this respect, we propose a weak definition of stochastic dominance by considering the investors' misperception of random amounts. This new definition can be compared with the ones already existing in literature.

The rest of the paper is organized as follows: next section presents a review of the literature on behavioral finance and prospect theory, with a focus on stochastic dominance as decision criteria. Section 3 is devoted to the statement of the theoretical results regarding the violation of stochastic dominance and provides a new definition of stochastic dominance. Section 4 is related with the main features of structured financial products and presents the numerical analysis methodology and results. In section 5 a discussion on our findings is carried on, together with some concluding remarks.

## 2 Literature review

This section is mainly a survey on behavioral finance, prospect theory and stochastic dominance, with a guided tour among their connections. The contributions of some remarkable academic papers are rationalized, even if these topics are still the focus of a great number of researches. The interest in behavioral finance is witnessed by the surveys of Shleifer (2000), Hirshleifer (2001) and Barberis and Thaler (2003), some devoted special issues of academic journals, and a dedicated journal. Prob-

lems related with prospect theory have been studied by uncountable followers of the Nobel prize Daniel Kahneman and his colleague Amos Tversky. A review on prospect theory can be found in Edwards (1996).

Stochastic dominance is commonly accepted as one of the most relevant decision rule. To the best of our knowledge, the most recent monograph on stochastic dominance is provided by Sriboonchitta et al. (2010), who present also several applications to economics and finance, but we mention also Levy (2006).

The analysis of some irrational behaviors of the investors is the focus of an endless and intriguing scientific debate. It is commonly accepted that financial markets are populated by different types of investors. Among them, irrational traders play a key role, since they take positions out of rationality which are able to influence market's dynamics. In this respect, two questions arise: how does the mechanism driving the unreasonable choices works? Why the misperception mechanism is implemented by investors? As already stressed, these questions are tackled by behavioral finance and, in particular, the former question represents the target of prospect theory.

The presence of not fully rational traders is responsible for the occurrence of discrepancies between prices and fundamental values. The persistence of such deviations is analyzed from a theoretical point of view in Friedman (1953). The Author argues that the deviation implies a good investment opportunity and rational traders balance the discrepancy created by irrational traders by exploiting this attractive situation. The empirical evidence shows results that are partially in disagreement with Friedman's point. Indeed, even though it is accepted that deviation is synonymous of attractive opportunity, the strategy implemented by investors to exploit such a situation is sometimes risky and costly, and mispricing does not disappear. Substantially, the irrationality is often nested in the rationality and it is more popular than what one expects.

Ritter (2003) highlights the causes of investors' irrationality through some specific *effects*. The *heuristic effect*, explored in Benartzi and Thaler (2001), is grounded on the attitude of the investors to allocate money in a simple way. In particular, empirical evidence shows that the capital is often equally distributed among available funds, even if the funds experienced very different past performances. The *overconfidence effect*, related to psychology, describes the optimistic investors' vision about their ability to make exact forecasts. Barber and Odean (2001) discuss this point and conclude that this effect is registered mainly when investors are men or entrepreneurs. The *mental accounting effect* takes into account the paradoxes caused by the *separation* made by investors of those decisions that should be taken jointly or, conversely, by the improper *aggregation* of different aspects related to a single decision. In this respect, the evaluation of the same good is different when the motivations for owning it are different (*separation*), or the evaluation of an investment improperly takes into account the amounts already corresponded (*aggregation*). The formalization of this theory begins with Thaler (1980) and it is further developed in Thaler (1985, 1990, 1999). The *framing effect* explains how decisions depend on the way the available alternatives are proposed. In Tversky and Kahneman (1981) and Bazerman (1983) the same experiment, in a slightly different fashion, is presented. People are grouped in two similar populations,  $\mathcal{P}_1$  and  $\mathcal{P}_2$ , and a choice between a gamble  $A$  or a secure plan  $B$  is proposed to each individual. A change in the frame of  $A$  and  $B$  implies a reversal of preferences, in terms of percentages of choices in  $\mathcal{P}_1$  and  $\mathcal{P}_2$ . The *representativeness effect* is first hypothesized by Reichenbach (1934) who describes the attitude to overweight the importance of the most recent experiences. In a larger sense, people influenced by this effect, guess that the salient global properties of a financial or economic variable can be explored by performing a local analysis of the related phenomenon. In contrast with the latter effect, the *conservation effect* can be introduced in order to formalize the slow adjustment to the changes in the financial environment. The *disposition effect* is the tendency of the investors to sell gaining assets and keep losing assets. Its formalization is due to Shefrin and Statman (1985). In Weber and Camerer (1998) this effect is experimentally evidenced with the observation of time-varying positions taken in a market by individuals that buy and sell shares of six risky assets.

The above described effects provide a set of motivations for the investors' irrationality and the introduction of these aspects in quantitative models based on decision criteria is necessary. The introduction of subjective *utility functions* and *risk aversion* allows to explain some investors' behaviors. In this respect, several seminal researches concerning decision-making under uncertainty use

increasing and concave utility functions (see, for example, Arrow, 1971; Lintner, 1965; Markowitz, 1952a; Pratt, 1964; Ramsey, 1928; Samuelson, 1958; Sharpe, 1964). However, the classical risk-averse utility theory is rather unsatisfactory for explaining some irrational investors' choices, since the axioms underlying the risk-averse utility framework are systematically violated in a wide variety of contexts. The most famous paradoxes dealing with this problem are discussed in Allais (1953) and Ellsberg (1961). As a further supporting argument, the disposition effect let understand the occurrence of a different perception between gains and losses in investors' mind. Friedman and Savage (1948) and Markowitz (1952b) theorize the existence of such a switch in investors' mind between risk-aversion and risk-seeking, in correspondence of some critical amounts. This remark is formalized through the assumption according to which the utility function can show one or more risk-seeking segments. In particular, Markowitz (1952b) speculates on utility functions that are first concave, then convex, then concave, and finally convex, to address Friedman and Savage's concerns (1948) on the reasons why investors buy insurance (risk-aversion) and lotteries tickets (risk-seeking). Kahneman and Tversky (1979) conduct psychological experimental researches to provide an empirical validation of Friedman, Savage and Markowitz speculations. The experimental results of their analysis give that the preferences of the 87 – 88% of subjects depend on the changes of the wealth,  $x \in \mathbb{R}$ , through an increasing S-shaped *value function*,  $V(x)$ , which is concave for  $x > 0$  and convex for  $x < 0$ . Moreover, Kahneman and Tversky (1979) show that investors distort subjectively probabilities and overweight the rare events. They provide the constitutive framework of *prospect theory*, for which Daniel Kahneman is one of the Nobel laureates in 2002<sup>1</sup>. Despite the relevance of prospect theory, Kahneman and Tversky start from some restrictive assumptions. The most relevant weakness of their setting concerns the introduction of a riskless outcome among the prospects to be evaluated. Certainty may affect investors' decisions with the so-called *certainty effect*. Further details on this point can be found in Battalio et al. (1990) and Tversky and Kahneman (1981). In particular, Schneider and Lopes (1986) argue that the value function cannot be S-shaped when there is not a riskless outcome.

A further important consequence of Kahneman and Tversky (1979) relies on the formalization of the mechanism driving the irrational choices of investors. Once again, we stress that irrationality is grounded on the psychological attitudes of each subject, which is able to distort the evaluation of the involved random amounts. In this context, the topic of performance measures is of some importance. Among the available tools used to compare the admissible financial projects, the stochastic dominance criteria, developed in Fishburn (1964); Hadar and Russel (1969); Hanoch and Levy (1969); Rothschild and Stiglitz (1970), are undoubtedly the most popular, since they allow to explore the whole set of characteristics of the random amounts<sup>2</sup>. Despite this, as we will see, stochastic dominance can be *violated* in a *direct* or *indirect* way. In the former case, investors prefer project  $A$  instead of project  $B$ , even if  $A$  is dominated by  $B$ . In the latter case, project  $B$  is underpriced with respect to a dominated project  $A$  or, alternatively, the transitivity property fails, i.e.  $C$  dominates  $D$  and  $D$  dominates  $E$ , but  $E$  is preferred to  $C$ . Tversky and Kahneman (1986); Birnbaum and Navarette (1998); Leland (1998) deal with the problem of direct violations and provide also an explanation of the reason why violations take place. In particular, they propose a behavioral-finance argument grounded on the scarce transparency of how the stochastic dominance rules should be used to order a set of financial projects. The indirect violations are more frequent than the direct ones, as argued in Birnbaum (1997) and references therein.

Needless to say that stochastic dominance violations can be viewed as base of misperception processes and related irrational choices of investors. In this sense, prospect theory can appropriately describe the biasing mechanisms because of the first order stochastic dominance violation as in the original Kahneman and Tversky's setting. To solve the problem related with first order stochastic dominance violation the *cumulative prospect theory* is introduced (Tversky and Kahneman, 1992). In the new setting, the Authors extend the previous theory to the case of random amounts with a continuous set of realizations. In the new framework, the probability distortion is interpreted as acting on the cumulative probability distribution rather than on the probabilities. Cumulative prospect theory improves prospect theory for what concerns the wider range of analysis. Never-

<sup>1</sup>Amos Tversky passed away in 1996, but his valuable contribution is acknowledged in Kahneman's Nobel lecture.

<sup>2</sup>For a survey on stochastic dominance we refer to Levy (2006).

theless, in a behavioral finance perspective, the violation of stochastic dominance is not necessarily a weakness for describing decision mechanisms, because people often choose dominated lotteries. On the other hand, empirical evidence shows that investors usually violate higher order stochastic dominance rather than the first order one, because of their sensitivity to risk.

The subjective preferences of investors and human intervention in comparing amounts are responsible of the violation of the stochastic dominance. The cumulative prospect theory does not admit first order stochastic dominance violation, but higher orders may continue to be violated. In order to take into account this important point, some new concepts of subjective stochastic dominance rules are necessary.

According to the original framework of prospect theory, related to changes of wealth, risk-aversion for gains and risk-seeking for losses, Levy and Wiener (1998) and Levy and Levy (2002, 2004) develop the *prospect stochastic dominance* criteria to determine the dominance of one project over another, for all S-shaped utility functions. Furthermore, Levy and Levy (2002) introduce also the *Markowitz stochastic dominance* to determine investors' preferences of one investment alternative over another, for all reverse S-shaped functions, in agreement with the original formulation of Markowitz (1952b). Leshno and Levy (2002) deal with the existence of the so-called *pathological preferences*, which are utility functions violating the expected utility criterion. In this context, they define the *almost stochastic dominance* rules to determine preferences of a project over another for all utility functions that are not pathological preferences.

Prospect theory and stochastic dominance rules are widely used to analyze and compare different investments alternatives. When a violation is detected, its roots are explained through behavioral finance arguments. It is out of the scope of this review to provide an extensive description of the continuously growing literature on these fields. As already preannounced in the Introduction, our contribution focuses on the misperception of structured products motivated by the framing effect. Therefore, we conclude this section just giving two remarkable references addressing similar questions. Berger and Smith (1998) focus on the prospect theory based on framing effect. In particular, the Author propose an experiment on three simultaneous framing tactics and develop a discussion on their individual main effects and their interactions. The problem is presented in a rather qualitative fashion and perhaps it can be well inserted in the field of marketing. Bernard et al. (2009) discuss on the overpricing of some structural products due to too optimistic framing of the prospectuses proposed by investment banks to retail investors.

### 3 Theoretical framework

In this section the statement of theoretical results regarding the violation of stochastic dominance in a cumulative prospect theory framework is presented. In doing so, a new definition of stochastic dominance rules will be provided.

Consider a random variable  $X$  on a probability space  $(\Omega, \mathcal{F}, P)$  with a cumulative function  $F_X$ . Fix  $r \in \mathbf{R}$  and define

$$A_X^{(n)}(r) = \begin{cases} F_X(r), & \text{if } n = 1; \\ \int_{-\infty}^r A_X^{(n-1)}(t)dt, & \text{if } n \geq 2. \end{cases} \quad (1)$$

In order to be self-contained, it is worth to recall the definition of stochastic dominance.

**Definition 1.** Consider  $n \in \mathbf{N}$  and two random amounts  $X$  and  $Y$ .  $X$  dominates stochastically of order  $n$   $Y$  ( $X >_n Y$ , hereafter) if and only if

$$\begin{cases} A_X^{(n)}(r) \leq A_Y^{(n)}(r), & \forall r \in \mathbf{R} \\ \text{and} \\ \exists r^* \in \mathbf{R} \text{ such that } A_X^{(n)}(r^*) < A_Y^{(n)}(r^*). \end{cases} \quad (2)$$

Fix  $n \in \mathbf{N}$  and consider two random amounts  $X$  and  $Y$ , such that  $X >_n Y$ . If an investor has a particular subjective perception of profit and losses related to  $Y$ , then the  $n$ -th order stochastic

dominance may be violated and the investor may prefer  $Y$  instead of  $X$ . Substantially, the random amount  $Y$  is perceived as a random amount  $Y_p$ , that can be viewed as a perturbation of  $Y$  and such that  $Y_p >_n X$ . The distortion of  $Y$  is attained by introducing a rule  $\mathcal{R}$  which transforms  $Y$  in  $Y_p$ . More formally,  $\mathcal{R}$  is a two variable function which transforms the support and density function of  $Y$  into the support and density function of a different random variable, named  $Y_p$ .

To our purposes, the definition of a subjective concept of stochastic dominance criteria, based on the perturbing rule  $\mathcal{R}$ , is introduced.

**Definition 2.** Consider  $n \in \mathbf{N}$ , two random amounts  $X$  and  $Y$  and a perturbing rule  $\mathcal{R}$ .  $Y$   $\mathcal{R}$ -dominates stochastically of order  $n$   $X$  ( $Y >_{n\mathcal{R}} X$ , hereafter) if and only if  $Y_p >_n X$ , where  $Y_p$  is the perturbation of the random amount  $Y$  generated by the rule  $\mathcal{R}$ . Conversely,  $X >_{n\mathcal{R}} Y$  if and only if  $X >_n Y_p$ .

It is worth noting that Definition 2 is based on the misperception of just one of the two projects to be compared. This asymmetry represents the main difference between the proposed new concept and the definitions given by prospect and Markowitz stochastic dominance that are based on particular transformations of both the investments to be compared. The proposed approach is motivated by the necessity to take into account decision rules which are based on the misperception of only one of the investments under scrutiny. As we will see, this is suitable with the structured products case. This paper, in accord with the classical literature on time series analysis, treats two remarkable cases:  $Y_p$  is obtained by perturbing  $Y$  with a lump sum or a deterministic trend. The related rules will be denoted hereafter as  $\mathcal{R}_1$  and  $\mathcal{R}_2$ , respectively.

### 3.1 Lump sum

Define a random mass  $Z$  as follows:

$$Z = \begin{cases} \bar{z}, & \text{with probability } \pi; \\ 0, & \text{with probability } 1 - \pi. \end{cases} \quad (3)$$

where  $\bar{z} \in \mathbf{R}^+$  and  $\pi \in (0, 1)$ .

The random sum  $Y_p$  is defined through the rule  $\mathcal{R}_1$  as follows:  $Y_p = Y + Z$ .

The following result holds true.

**Proposition 3.** For each  $n \in \mathbf{N}$ , we have

$$A_{Y_p}^{(n)}(r) = \begin{cases} (1 - \pi)A_Y^{(n)}(r), & \text{if } r < \bar{z}; \\ \pi + (1 - \pi)A_Y^{(n)}(r), & \text{if } r \geq \bar{z} \text{ and } n = 1; \\ \frac{\pi(r - \bar{z})^{n-1}}{n-1} + (1 - \pi)A_Y^{(n)}(r), & \text{if } r \geq \bar{z} \text{ and } n \geq 2. \end{cases} \quad (4)$$

*Proof.* The result is straightforward, by invoking the induction principle and by definition of  $Y_p$ .  $\square$

**Proposition 4.** Fix  $n \in \mathbf{N}$  and assume that  $X >_n Y$ . For each  $\bar{z} \in \mathbf{R}^+$  and  $\pi \in (0, 1)$ , we have  $X >_{n\mathcal{R}_1} Y$ .

*Proof.* The cases  $n = 1$  and  $n \geq 2$  are separately treated.

- Assume  $n = 1$ .

Since  $X >_1 Y$ , we have

$$A_X^{(1)}(r) \leq A_Y^{(1)}(r), \quad \forall r \in \mathbf{R} \quad \text{and} \quad \exists r^* \in \mathbf{R} \mid A_X^{(1)}(r^*) < A_Y^{(1)}(r^*).$$

By definition of  $Y_p$  through the lump sum  $Z$ , we can write:

$$A_{Y_p}^{(1)}(r) = \begin{cases} (1 - \pi)A_Y^{(1)}(r), & \text{if } r < \bar{z}; \\ \pi + (1 - \pi)A_Y^{(1)}(r), & \text{if } r \geq \bar{z}. \end{cases} \quad (5)$$

When  $Y >_{1\mathcal{R}_1} X$ , then the following system is satisfied:

$$\begin{cases} (1 - \pi)A_Y^{(1)}(r) \leq A_X^{(1)}(r), & \text{if } r < \bar{z}; \\ \pi + (1 - \pi)A_Y^{(1)}(r) \leq A_X^{(1)}(r), & \text{if } r \geq \bar{z}. \end{cases} \quad (6)$$

Second equation of system (6) brings to

$$p \leq \frac{A_X^{(1)}(r) - A_Y^{(1)}(r)}{1 - A_Y^{(1)}(r)}, \quad \text{with } r \geq \bar{z}, \quad (7)$$

that is satisfied only when  $p = 0$ , since the second term of (7) is not positive. We can conclude that  $X >_{1\mathcal{R}_1} Y$ .

- Assume  $n \geq 2$ .

By Proposition 3 we have that the following system has to be fulfilled, in order to have  $Y >_{n\mathcal{R}_1} X$ :

$$\begin{cases} \pi \geq \max \left\{ \sup_{r \geq \bar{z}} \left[ \frac{A_X^{(n)}(r) - A_Y^{(n)}(r)}{(r - \bar{z})^{n-1} - A_Y^{(n)}(r)} \right]; 1 - \inf_{r < \bar{z}} \left[ \frac{A_X^{(n)}(r)}{A_Y^{(n)}(r)} \right] \right\} \\ \bar{z} > \sup_{r > \bar{z}} \left\{ r - [(n-1)A_Y^{(n)}(r)]^{\frac{1}{n-1}} \right\}. \end{cases} \quad (8)$$

Since

$$\sup_{r > \bar{z}} \left\{ r - [(n-1)A_Y^{(n)}(r)]^{\frac{1}{n-1}} \right\} = +\infty,$$

then the second condition of (8) can not be attained, and this completes the proof.  $\square$

### 3.2 Deterministic trend

The rule  $\mathcal{R}_2$  is introduced by the definition of an increasing function  $t : \mathbf{R} \rightarrow \mathbf{R}$ , that can be viewed as a deterministic trend affecting the realizations of the random sum  $Y$ . The intervention of the function  $t$  on  $Y$  drives the definition of the perturbed random amount  $Y_p$  through the cumulative probability function:

$$F_{Y_p}(r) = F_Y(r - t(r)). \quad (9)$$

The following result is trivial, but it is useful to formalize it in order to pursue a precise analysis of the first order stochastic dominance in this context.

**Proposition 5.** *Assume that  $X >_1 Y$ . If the function  $t$  is such that  $A_Y^{(1)}(r - t(r)) \leq A_X^{(1)}(r)$ , for each  $r \in \mathbf{R}$ , and there exists  $r^* \in \mathbf{R}$  such that  $A_Y^{(1)}(r^* - t(r^*)) < A_X^{(1)}(r^*)$ , then  $Y >_{1\mathcal{R}_2} X$ .*

For the stochastic dominance of order greater than 1 no general results can be explicitly formalized. We then restrict the analysis only to the most frequent cases, well-known in the financial econometric literature: constant trend and linear trend.

- *Constant trend*

In this case  $t(r) = t \in \mathbf{R}$ , for each  $r \in \mathbf{R}$ .

**Proposition 6.** *For each  $n \in \mathbf{N}$ , we have*

$$A_{Y_p}^{(n)}(r) = A_Y^{(n)}(r - t) \quad (10)$$

*Proof.* We use the induction principle.

By definition, the thesis is true when  $n = 1$ .

Now, assume that (10) holds for  $n - 1$ . Then we have:

$$A_{Y_p}^{(n)}(r) = \int_{-\infty}^r A_{Y_p}^{(n-1)}(s) ds = \int_{-\infty}^r A_Y^{(n-1)}(s - t) ds = \int_{-\infty}^{r-t} A_Y^{(n-1)}(s) ds = A_Y^{(n)}(r - t).$$

$\square$

By using Proposition 6 we derive a sufficient condition for the inversion of the stochastic dominance criterion.

**Proposition 7.** Fix  $n \in \mathbf{N}$  and assume that  $X >_n Y$ . Moreover, assume that

$$\begin{cases} A_Y^{(n)}(r-t) \leq A_X^{(n)}(r), & \forall r \in \mathbf{R} \\ \text{and} \\ \exists r^* \in \mathbf{R} \text{ such that } A_Y^{(n)}(r^*-t) < A_X^{(n)}(r^*). \end{cases} \quad (11)$$

Then  $Y >_{n\mathcal{R}_2} X$ .

*Proof.* By Proposition 6 and by Definition 2 we obtain the thesis.  $\square$

- *Linear trend*

In this case there exists  $\alpha \neq 0$  such that  $t(r) = \alpha r$ , for each  $r \in \mathbf{R}$ .

**Proposition 8.** For each  $n \in \mathbf{N}$ , we have

$$A_{Y_p}^{(n)}(r) = \frac{1}{(1-\alpha)^{n-1}} A_Y^{(n)}(r - \alpha r). \quad (12)$$

*Proof.* Also in this case, the result is proved by using the induction principle.

For  $n = 1$ , the result is trivially true.

Assume that (12) holds for  $n - 1$ . Then

$$\begin{aligned} A_{Y_p}^{(n)}(r) &= \int_{-\infty}^r A_{Y_p}^{(n-1)}(s) ds = \frac{1}{(1-\alpha)^{n-2}} \int_{-\infty}^r A_Y^{(n-1)}(s - \alpha s) ds = \\ &= \frac{1}{(1-\alpha)^{n-2}} \int_{-\infty}^{(1-\alpha)r} \frac{A_Y^{(n-1)}(s)}{1-\alpha} ds = \frac{1}{(1-\alpha)^{n-1}} A_Y^{(n)}(r - \alpha r), \end{aligned}$$

and the result is proved.  $\square$

Proposition 8 implies the following sufficient condition for the inversion of the stochastic dominance relation.

**Proposition 9.** Fix  $n \in \mathbf{N}$  and assume that  $X >_n Y$ . Moreover, assume that

$$\begin{cases} \frac{1}{(1-\alpha)^{n-1}} A_Y^{(n)}(r - \alpha r) \leq A_X^{(n)}(r), & \forall r \in \mathbf{R} \\ \text{and} \\ \exists r^* \in \mathbf{R} \text{ such that } \frac{1}{(1-\alpha)^{n-1}} A_Y^{(n)}(r^* - \alpha r^*) < A_X^{(n)}(r^*). \end{cases} \quad (13)$$

Then  $Y >_{n\mathcal{R}_2} X$ .

*Proof.* The proof comes from Definition 2 and Proposition 8.  $\square$

The theoretical results show that misperception is attained when perturbation is given by the introduction of a trend, while no stochastic dominance violation takes place when a lump sum is introduced. This findings provide interesting information since a trend affects the entire set of realizations of a random amount, while a lump sum is related to an impulsive shock. Therefore, when agents have a global misperception irrationally prefer the worst project. Conversely, the presence of an adjunctive isolate gain in the dominated project is not able to invert agents' mind and drive the decision process.

## 4 Structured financial products

Structured products, also known as market-linked products, are generally pre-packaged investment strategies based on derivatives written on a single security, a basket of securities, options, indices, commodities, debt issuances and/or foreign currencies and, to a lesser extent, swaps. The variety of products just described is demonstrative of the fact that there is no single, uniform definition of a structured product.

The U.S. Securities and Exchange Commission<sup>3</sup> (SEC) defines structured securities as "securities whose cash flow characteristics depend upon one or more indices or that have embedded forwards or options or securities where an investor's investment return and the issuer's payment obligations are contingent on, or highly sensitive to, changes in the value of underlying assets, indices, interest rates or cash flows".

To the definition given by the SEC it may be added that a common feature of some structured products is represented by a *principal guarantee* function, which offers protection of principal if held to maturity.

Structured products, usually issued by investment banks or affiliates, are synthetic securities created to meet specific needs that cannot be satisfied by standardized financial products and are proposed as an alternative to direct investments, as part of the asset allocation process to reduce the risk exposure of a portfolio, or to utilize current market trends. These investment tools are also available at the mass retail level, since one of their attractions is the ability to customize a variety of assumptions into one instrument.

Disadvantages of structured products may include: credit risk (structured products are unsecured debt from investment banks); lack of liquidity (structured products rarely trade after issuance and anyone looking to sell a structured product before maturity should expect to sell it at a significant discount<sup>4</sup>); high complexity (the complexity of the return calculations means that only few investors truly understand how the structured product will perform relative to simply owning the underlying asset). At the latter point we can tie a further consideration related to the lack of pricing transparency. Since there is no uniform standard for pricing, it is hard to compare the net-of-pricing attractiveness of alternative structured products and investor cannot know for sure what are the implicit costs of the instrument.

The market-linked products considered in this paper and representing the subject of the numerical application are *globally-floored*, in the sense that provide a guaranteed minimum return. Hence, absent default risk, the final return will never be less than a prespecified floor applying to the entire life of the contract. Furthermore, using real world examples, *capped* contracts will be considered. These type of structured products are quite popular in the world of equity derivatives, since they offer protection against downside risk, coupled with significant upside potential. Capping the maximum ensures that the payoff is never too extreme and that the value of the contract is not too outrageous. The basic reasons why this type of structured contracts is considered here are essentially due to their popularity among retail investors and their weird characteristics, which are not fully understood and have not been widely studied in literature. In particular, we will show that locally capped contracts are dominated by the globally capped ones but, surprisingly, the formers are more popular than the latters. In this respect, the percentage of globally floored contracts listed in the AMEX, as April 2008, is about 45% for the locally capped contracts and about 10% for the globally capped ones. To explain this irrational behavior, a prospect theory perspective, through the introduction of a constant trend, will be used and a behavioral finance type discussion for this misperception will be provided.

### 4.1 Globally floored - locally/globally capped contracts

In many cases, the redemption amount paid out on guaranteed minimum return product depend exclusively on the periodical performance of the underlying asset (i.e. the *reference portfolio*).

<sup>3</sup>SEC, Rule 434 regarding certain prospectus deliveries

<sup>4</sup>Because of this, structured products tend to be more of a buy-and-hold investment decision rather than a means of getting in and out of a position with speed and efficiency.

It is not uncommon to lock in gains at specific dates, capping the periodical returns at a given maximum. At maturity, the payoff is given by a combination of periodic returns that are added<sup>5</sup> to the guaranteed redemption amount and paid out to the investor.

The final payoff,  $Y_T$ , for this globally floored - locally capped contract is given by (Boyle and Tset, 1990, Boyle et al., 2009):

$$Y_T = Y_0 \left\{ (1 + F) + \max \left[ 0, \sum_{k=1}^{nT} \min \left( c, \frac{S_{t_k}}{S_{t_{k-1}}} - 1 \right) - F \right] \right\} \quad (14)$$

with  $T$  representing the maturity of the contract;  $Y_0$  the amount paid at the inception date,  $t = 0$ ;  $F$  the global floor (guaranteed minimum return at maturity);  $c$  the local cap (maximum allowed periodical return) and  $S_{t_k} \in [0, T]$  the price of the reference portfolio at the prespecified dates  $t_k$ .

The maximum and minimum pay-offs of this contract are given respectively by  $Y_0(1 + nT \cdot c)$  and  $Y_0(1 + F)$ .

This market-linked product is characterized by high complexity since its final pay-off is highly path-dependent and cannot be easily replicated. From the point of view of the seller, aimed at minimizing market risk, their main exposure is to volatility since this contract is very subtle in its dependence on the assumed volatility model. The classical references to this phenomenon which is not the focus of this paper are Avellaneda et al. (1995) and Lyons (1995) and Wilmott (2000).

Conversely, a particular and simpler case is given when the redemption amount of a contract is made up of a guaranteed minimum return (i.e. the floor rate) and a bonus return which varies according to the performance of the reference portfolio between the issue and maturity dates. The bonus return in this case is calculated as a percentage of the difference by which the price of the reference portfolio on the maturity date exceeds its price on the issue date. If the price falls, no bonus return is paid out and the rate of return of the structured products is given by the floor rate. Thus the investor profits from a rise in the price of the reference portfolio but, if the price drops, does not have to bear the loss, since the bearer only participates in the relative performance of the reference portfolio up to a certain maximum value.

The issuer promises a final payoff proportionate to the change in the reference portfolio's price. In cases where the price of the reference portfolio decreases, the issuer guarantees a minimum redemption amount. At the same time, the issuer limits the investor's participation in the instrument's performance by setting an upper limit (i.e., the global cap).

The standard final payoff,  $X_T$ , for a globally floored - globally capped contract is given by (Boyle and Turnbull, 1989; Bernard et al., 2009):

$$X_T = X_0 \left\{ (1 + F) + \max \left[ 0, \min \left( C, \frac{S_T}{S_0} - 1 \right) - F \right] \right\}, \quad (15)$$

with  $T$  representing the maturity of the contract;  $X_0$  the amount invested at the inception date,  $t = 0$ ;  $F$  the global floor (guaranteed minimum return at maturity);  $C > F$  the global cap (maximum allowed return);  $S_0$  and  $S_T$  the price of the reference portfolio at the inception date and maturity, respectively.

The initial investment of  $X_0$  yields a rate of return in the closed interval  $[F, C]$ . The cap becomes operational only if the return of the reference portfolio on the interval  $[0, T]$ , computed as  $\frac{S_T}{S_0} - 1$ , exceeds the global cap rate  $C$ .

By no arbitrage the current price of this contract can be replicated by a portfolio of three securities: a long position in a zero coupon bond which reaches maturity on  $T$  and has a face value of  $X_0(1 + F)$ ; a long position on  $\frac{X_0}{S_0}$  European call options on the underlying reference portfolio with strike  $S_0(1 + F)$  and maturity  $T$ ; a short position on  $\frac{X_0}{S_0}$  European call options on the underlying reference portfolio with strike  $S_0(1 + C)$  and maturity  $T$ . It is worth noting that the maximum pay-off of this contract is given by  $X_0(1 + C)$ , while the minimum pay-off is  $X_0(1 + F)$ . Hence, to avoid arbitrage must be:

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<sup>5</sup>It is worth to notice that there are two common types of globally-floored locally-capped contracts, the compound contracts, where returns in each year are compounded, and the simple contracts where the returns are periodically added together to give the final payoff.

$$(1 + C) > e^{rT} > (1 + F), \quad (16)$$

with  $r$  representing the risk-free rate.

## 4.2 Simulation results

The complex (globally floored - locally capped) and simple (globally floored - globally capped) contracts described in the previous section show quite different properties. In particular, the first one is characterized by a highly path-dependent final pay-off for which there is no closed form solution to compute its initial price. On the contrary, the simpler contract can be decomposed into a portfolio consisting of a zero coupon bond and two standard call options. Thus it may even be priced using Black and Scholes' formula.

Since the application of this work is based on a comparison of the two above described structured products, we believe the two contracts should be priced and analyzed using the same numerical methodology, in order to achieve more reliable results.

As per usual, in case of path dependency property required to determine the final payoff (i.e. Asian options, barrier options, cliquet options and many other exotics), one of the most commonly used approach is Monte-Carlo (MC) method since, as opposed to other numerical approaches, offers a greater flexibility and becomes increasingly attractive compared to other methods of numerical integration as the dimension of the problem increases.

The first building block of this application based on the globally floored - globally capped and globally floored - locally capped (from now on simple contract and complex contract, respectively) is represented by the fair determination of the global and local cap,  $C$  and  $c$  respectively. To find these quantities, using Black-Scholes-Merton's assumptions<sup>6</sup>, we take the expectation under the risk neutral distribution of the payoff, discounted at the risk free rate of return, assuming the price and other contract parameters as given, and solving for the global and local caps that make the price of the two contracts equal to an assigned level  $\widehat{P}$ .

We use here the standard lognormal model for the reference portfolio,  $S_t$ , so that the accumulation factors  $\frac{S_t}{S_{t-1}}$  are independent and identically lognormally distributed. The complex contract involves the sum of lognormal random variables that, with a sufficiently large number of simulations, can provide arbitrarily close to the true value of the contract (Boyle et al., 1997).

Using the assumption of no arbitrage, the expectation is taken with respect to a transformation of the original probability measure (i.e. the risk-neutral measure; see Duffie, 2001, for a survey on the subject).

The applied approach consists of the following steps.

- Select a given vector of parameters,  $\xi = \left[ T \quad X_0 = Y_0 \quad F \quad r \quad \sigma \quad \delta \quad S_0 \quad \widehat{P} \right]$ , where  $S_0$  is the current stock price,  $r$  is the riskless interest rate,  $\delta$  and  $\sigma$  are respectively the dividend yield and the volatility of the reference portfolio,  $T$  is the contract's maturity,  $F$  is the minimum guaranteed return,  $X_0$  is the amount invested,  $\widehat{P}$  is the price of the contract at time zero.
- Select an integer  $q$  and a  $q$ -dimensional vectors,  $\Gamma$ , whose elements are the values to be assigned to the unknown global cap,  $C$ .
- Simulate sample paths of the underlying state variables (e.g., underlying reference portfolio price) over the relevant time horizon,  $T$ , according to the risk-neutral measure. As in Black-Scholes-Merton's model, we assume the reference portfolio price follows a log-normal diffusion. Independent replications of the terminal stock price under the risk-neutral measure can be generated from the formula:

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<sup>6</sup>Despite the stringent assumptions of the Black-Scholes-Merton framework, Stoimenov and Wilkens (2005) and Wilkens and Stoimenov (2007) used the celebrated approach to evaluate structured products issued in Germany. More realistic assumptions (i.e. stochastic volatility and interest rate; jumps in the stochastic process describing the underlying dynamics, credit risk, etc.) could be considered, but this is not the focus of this work.

$$S_T^{(i)} = S_0 e^{(r-\delta-\frac{1}{2}\sigma^2)T+\sigma\sqrt{T}Z_i} \quad i = 1, \dots, m \quad (17)$$

where the  $Z_i$  are independent samples from the standard normal distribution and  $m = 10^4$ .

- Evaluate, for each element  $j^{th}$  of the  $q$ -dimensional vector  $\Gamma = (\Gamma^{(j)})_{j=1, \dots, q}$ , the discounted cash flows of the option component on each sample path, as determined by the structures of the simple contracts:

$$\widehat{P}_{simple}^{(j)} = \frac{1}{m} \sum_{i=1}^m e^{-rT} X_0 \left( (1+F) + \max \left( 0, \min \left( \Gamma^{(j)}, \frac{S_T^{(i)}}{S_0} - 1 \right) - F \right) \right), \quad j = 1, \dots, q. \quad (18)$$

- Average the discounted cash flows over sample paths and find  $j^* \in \{1, \dots, q\}$  such that  $\Gamma^{(j^*)}$  minimize the distance of the risk neutral price  $\widehat{P}_{simple}^{(j^*)}$  from the given price  $\widehat{P}$ . Lastly, denote  $\widehat{P}_{simple} = \widehat{P}_{simple}^{(j^*)}$ .

Similarly, we find the local cap level,  $c$ , of the complex contract, implementing an appropriate discretization for (17). Using the given vector of parameters,  $\xi$ , and the  $q$ -dimensional vectors,  $\gamma = (\gamma^{(j)})_{j=1, \dots, q}$ , whose elements are the values to be assigned to the unknown local cap,  $c$ , we find  $j_* \in \{1, \dots, q\}$  such that  $\gamma^{(j_*)}$  minimize the distance of the risk neutral price  $\widehat{P}_{complex}$  from the given price  $\widehat{P}$ :

$$\widehat{P}_{complex} = \widehat{P}_{complex}^{(j_*)} = \frac{1}{m} \sum_{i=1}^m e^{-rT} Y_0 \left\{ (1+F) + \max \left[ 0, \sum_{k=1}^{nT} \min \left( \gamma^{(j_*)}, \frac{S_{t_k}^{(i)}}{S_{t_{k-1}}^{(i)}} - 1 \right) - F \right] \right\}. \quad (19)$$

To perform the numerical analysis we choose the vector of parameters,

$\xi = \left[ T \quad X_0 = Y_0 \quad F \quad r \quad \sigma \quad \delta \quad S_0 \quad \widehat{P} \right]$ , for both simple and complex contracts. The contracts share the same underlying,  $S$ , whose dynamics is described by (17), with parameters  $r = 0.05$ ,  $\delta = 0.02$ ,  $\sigma = 0.15$  and  $S_0 = 10$ . Simple and complex contracts have the same maturity ( $T = 5$  years), same initial investment ( $X_0 = Y_0 = 1,000$ ) and same guaranteed minimum return ( $F = 0.1$ ). In both cases the initial price of the contract is  $\widehat{P}_{simple} = \widehat{P}_{complex} = 920$ . The complex contract is based on a quarterly sum cap ( $n = 4$ ) and the estimated fair cap that produces an initial value  $\widehat{P}_{complex} = 920$  is  $\widehat{c} = 0.0867$  each quarter. The fair global cap level, producing an initial value  $\widehat{P}_{simple} = 920$ , is  $\widehat{C} = 0.3053$ . The parameters selected for the application correspond to standard market assumptions. However, the results hold also for different vectors of parameters,  $\xi$ .

Given the vector of parameters,  $\xi$ , the second building block of this application relies on the simulation of the final payoffs for the simple contract,  $X_T$ , and complex contract,  $Y_T$ . In addition, here we implement stochastic dominance rules in order to classify, according to their final performances, the two contracts representing the subject of the analysis.

Using (17) and (15), with  $\Gamma^{(j^*)} = \widehat{C}$  and  $m = 10^4$  replications, we obtain the empirical probability distribution for the globally capped contract,  $X_T$ . Using an opportune discretization for (17) and (14), with  $\gamma^{(j_*)} = \widehat{c}$  and  $m = 10^4$ , we get also the empirical probability distribution for  $Y_T$ .

INSERT FIGURE 1 ABOUT HERE.

**Caption of Figure 1:** Simulated PDF of the globally and locally capped contract.

Figure 1 shows the empirical probability distribution functions of the payoffs of the simple contract (globally capped, left side),  $X_T$ , and the complex contract (locally capped, right side),  $Y_T$ . Given the assumptions about the vector of parameters,  $\xi$ , the complex contract has a very high probability of yielding the minimum guaranteed return,  $F = 10\%$ , with a probability mass of 62.32%. The probability of attaining a return greater than 70% is 0.79%, while the probability of the maximum attainable return in the right tail,  $\widehat{c}nT = 173,4\%$ , is practically equal to 0.

It is worth to notice that the sum of 20 quarterly returns will be equal to 173,4% if and only if all consecutive quarterly returns exceed the local cap level,  $\hat{c} = 0.0867$ , so that the probability of this event is virtually quite impossible.

On the other hand, the simple contract has a distribution characterized by two probability masses: one at the minimum guaranteed return  $F = 10\%$ , with probability 51.65%, and the other at the maximum attainable return,  $\hat{C} = 30.53\%$ , with probability 30.56%. Thus, investors have a probability of only 17.79% of obtaining an intermediate return between these two extremes. As it can be seen, the returns are almost uniformly distributed in the central part of the distribution (as shown in the graph on the left side of Figure 1).

In order to evaluate the random amounts of the two considered contracts via the stochastic dominance criteria discussed in previous section, two distinct prospects for the simple and complex contracts, with cumulative density functions respectively given by  $\hat{F}_X$  and  $\hat{F}_Y$ , are determined.

Given (1) and definition 2, we have that  $X$  dominates stochastically  $Y$  of order  $n \in \mathbf{N}$  ( $X >_n Y$ ) if and only if:

$$\begin{cases} \hat{A}_X^{(n)}(r) \leq \hat{A}_Y^{(n)}(r), & \forall r \in \mathbf{R} \\ \text{and} \\ \exists r^* \in \mathbf{R} \text{ such that } \hat{A}_X^{(n)}(r^*) < \hat{A}_Y^{(n)}(r^*). \end{cases} \quad (20)$$

Applying (20) to the simulated cumulative density function of the simple and complex contracts we obtain results that partially contradict what occurs on the real markets and highlight the problem of investors' irrational choices. In particular, even though market data suggest that the complex contract,  $Y$ , is more popular among retail investors than the simpler contract,  $X$ , we find that the stochastic dominance criteria predict that consumers should prefer the simpler contract,  $X$ .

In order to verify (20), we test wheter  $X$  dominates  $Y$  of order  $n$ , running the algorithm described in the first building block to obtain  $j = 1, \dots, K$  ( $K = 10^4$ ) prospects, named  $X^{(j)}$  and  $Y^{(j)}$ . In practice, we compute:

$$\Psi_K^{(n)} = \frac{1}{K} \sum_{j=1}^K 1 \left( \hat{D}_j^{(n)}(r) \leq 0, \forall r \in \mathbf{R} \text{ and } \exists r^* : \hat{D}_j^{(n)}(r^*) < 0 \right) \quad (21)$$

where:

$$\hat{D}_j^{(n)}(r) = \hat{A}_{X^{(j)}}^{(n)}(r) - \hat{A}_{Y^{(j)}}^{(n)}(r) \quad (22)$$

and  $1(\cdot)$  denotes the indicator function.

The simulation results can be summarized as follows: global  $X$  is not dominated neither dominates local  $Y$  of the first order; in 55% of cases we have  $X >_2 Y$  and in 100% of cases we obtain  $X >_3 Y$ . Our simulated results clearly point out that investor having von Neumann-Morgenstern utility functions and expected utility maximizer should prefer contract  $X$  to contract  $Y$ . Therefore, the popularity of the locally capped contracts with respect to the globally capped ones is counterintuitive, and can be explained through investors' irrational behavior. Since first order stochastic dominance requires that investors prefer higher returns to lower ones, implying a utility function with a non negative first derivative, on the ground of the obtained results we can conclude that this type of investor is indifferent in the choice among the two contracts. On the contrary, tests based on second order stochastic dominance, positing diminishing marginal utility (sufficient for risk aversion) and third order stochastic dominance, implying the empirical attractive feature of increasing absolute risk aversion (non-negative third derivative), provide significant results in favor of the simple contract.

The third building block of the application is based on the implementation of the misperception algorithm which describe the mechanism that push retail investors to chose complex products, instead of simple ones. As remarked in the previous section, the misperception algorithm is applied only to one of the two considered projects since we need to take into account decision rules which are based on the misperception of the complex structured product,  $Y$ . Given the obtained theoretical results, we will consider the rule  $\mathcal{R}_2$  in the particular case of constant trend,  $t(r) = t \in R, \forall r \in R$  to perturbate the empirical cumulative distribution function of the contract  $Y$  and to map  $Y$  into  $Y_p$ . The misperception algorithm allows us to find the minimum constant trend,  $t^*$ , such that  $F_Y(r - t^*) \leq$

$F_X(r)$  for each  $r \in R$ . The optimal  $t^*$  will provide an inversion of the investors' preferences according to the first and second order stochastic dominance.

The approach consists of the following steps:

- Given the  $j = 1, \dots, K$  prospects obtained in the second building block, select an integer  $v$  and a  $v$ -dimensional vectors,  $\Delta = (\Delta^{(l)})_{l=1, \dots, v}$ , whose elements are the values to be assigned to the unknown constant trend,  $t$ .
- Perturbate  $Y$  through  $\Delta$  and set the vector  $Y_p = (Y_p^{(l)})_{l=1, \dots, v}$ .
- Run the algorithm described in the first building block to obtain  $X^{(l)}$ , with  $l = 1, \dots, v$ .
- Evaluate, for each element  $l^{th}$  of the  $v$ -dimensional vector  $\Delta$ , the stochastic dominance criteria as determined by the perturbation of the complex contract,  $Y$ , as above. In particular, in order to verify the  $\mathcal{R}_2$ -stochastic dominance, we compute:

$$\Phi_v^{(n)} = \frac{1}{v} \sum_{l=1}^v \mathbf{1} \left( \widehat{\mathcal{R}_2 D_l}^{(n)}(r) \leq 0, \forall r \in \mathbf{R} \text{ and } \exists r^* : \widehat{\mathcal{R}_2 D_l}^{(n)}(r^*) < 0 \right) \quad (23)$$

where:

$$\widehat{\mathcal{R}_2 D_l}^{(n)}(r) = \widehat{A}_{Y_p^{(l)}}^{(n)}(r) - \widehat{A}_{X^{(l)}}^{(n)}(r). \quad (24)$$

- Find the minimum value  $l^* \in \{1, \dots, v\}$  such that  $Y_p^{(l^*)} >_n X^{(l^*)}$ .

Running the procedure for  $n = 1, 2$ , we find that: with  $t^* = 0.453$  the perturbed complex contract dominate for the first order stochastic dominance the simple contract,  $Y >_{1\mathcal{R}_2} X$ , in the 71% of the  $K$  cases; with  $t^* = 0.4672$  we have  $Y >_{1\mathcal{R}_2} X$ , in the 100% of the  $K$  cases and with  $t^* = 0.0316$  the perturbed complex contract dominate for the second order stochastic dominance the simple contract,  $Y >_{2\mathcal{R}_2} X$ .

The results imply that if investors erroneously perceive that the probability mass (62.32%) of the complex contract return is concentrated in correspondence of 46.72% in such a way that the most likely final wealth is given buy  $Y_T = 1613.92$ , they select the complex contract in light of the first stochastic dominance criteria.

## 5 Results and concluding remarks

In this paper the problem of misperception is tackled, with a particular reference on why and how it is implemented. A theoretical analysis is presented and supported by some simulation results. This is a key issue to explain the popularity of some complex structured financial products. In particular, the preference accorded by investors to locally-capped contracts with respect to globally-capped ones is motivated by the framing effect, which is commonly exploited by financial institutions to pursue profit targets.

Some points need to be emphasized, in order to highlight the findings of this paper.

- The simulation results derived for the structured products allows us to state an hypothesis to formalize the misperception mechanism for locally-capped contracts. It is undoubtedly true that a violation takes place, since the complex contracts are unreasonably more popular than the simple ones. An explanation of the reasons why global caps are misperceived can be obtained by analyzing the main features of this type of financial product and developing a behavioral finance argument.

We recall that the presence of a guaranteed minimum return and the easy access to these products make them suitable for retail investors and households. The behavior of this type of agents in a market is generally driven by the confidence on the financial institutions proposing the investments. In this respect, the intervention of investment banks and insurance companies

in retail investors' decisions plays a key role. Financial institutions pursue profit targets and therefore may present financial products in a rather fraudulent fashion, exploiting the framing effect on investors' choices. Structured financial products, with a particular focus on the locally-capped contracts, can be seen as a case of misperception due to framing. Financial institutions usually show to potential investors some prospectuses on the future performances of the local cap. Clearly, not all the possible scenarios can be shown, because of the randomness of the evolution of the reference underlying. Scenarios may vary depending on two parameters: the probabilities of their occurrence and the expected returns at the expiration date. A fair and honest proposal should reflect either optimistic and pessimistic outcomes, and the sample scenarios should be opportunely weighted with their related occurrence probabilities. Almost all developed Countries regulates this aspect, by introducing some devoted regulations. For instance, the Federal Act on Collective Investment Schemes (the so-called CISA) entered into force in Switzerland on January 1st, 2007 establishing some transparency and simplification criteria introduced to guarantee a conscious understanding of the structured products to the investors. CISA provides also a guideline to the contents of the prospectuses that financial institutions should submit to investors' attention. A further example can be found in the Prevention of Fraud (Investments) Act, issued in UK in 1959 and aimed at protecting investors by the introduction of penalties for fraudulently inducing investors to invest their money. In a larger sense, this rule induces banks and insurance companies to propose the structured financial products without reticences on their bad admissible future performances. Unfortunately, the reality is quite different and the state securities acts are systematically violated. It is also worth noticing that the mispricing of complex structured products is commonly accepted to be one of the roots of the actual financial crisis since they are subject also to credit risk and their true implicit risk is not fully disclosed to investors. The prospectuses that investment banks propose to their clients are often too optimistic and the framing effect may push investors to purchase these products, even if they are reasonably unsuitable for a wide part of households. Details on this argument are presented in Bernard et al. (2009), where some samples of real prospectuses for locally-capped contracts are reported. Furthermore, in Illinois the lawyers Burke and Stoltmann are preparing arbitration claims to recover losses against Wall Street brokerage firms for the selling of structured products<sup>7</sup>. They deeply investigate this phenomenon and conclude that such investments were not clearly presented to investors and inappropriately pitched as 100% safe and secure.

To conclude, we think that financial institutions are sometimes morally responsible for the negative consequences due to investors' misperception, in the sense that they frame in a unclear way the main characteristics of such investments to pursue profit targets. Bernard et al.(2009) report some real examples extracted from the prospectus proposed by several investment banks.

- The introduction of a new definition of stochastic dominance is needed to capture the selectiveness of the distortions in evaluating different projects. With this respect, we propose stochastic dominance rules based on the misperception of one of the investments to be compared. This choice is grounded on the evidence, discussed partially above, that investment banks may emphasize some positive aspects and be reticent on other remarkable characteristics of the investments. In this case, it is rather obvious that just one of such investments will be misperceived by the client. In this direction, it would be an intriguing development of this work the replication of the original Kahneman and Tversky's experiment. One may subgroup people in two identical populations  $\mathcal{P}_1$  and  $\mathcal{P}_2$  and propose two gambles, say  $G_1$  and  $G_2$ , with a great number of stochastic future performances and with  $G_1$  stochastically dominating by  $G_2$ . Letting  $G_2$  always fairly presented, one could present  $G_1$  fairly to  $\mathcal{P}_1$  and frame  $G_1$  with optimistic scenarios for population  $\mathcal{P}_2$ . By repeating the experiment for a large number of optimistic prospectuses, an empirical analysis of the stochastic dominance inversion by comparing the behaviors of  $\mathcal{P}_1$  and  $\mathcal{P}_2$  may be performed. One can also get information on the risk attitudes of the population  $\mathcal{P}_2$ . Moreover, if the misperception is hypothesized to

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<sup>7</sup>More information on <http://www.structuredproductfraud.com/>

depend on a deterministic trend rule on  $G_1$  as in our approach, it is possible to deduce the minimal levels of trend parameters allowing the violation of  $n$ -th order stochastic dominance.

- The misperception is theoretically attained when the complex contracts are distorted by the introduction of a trend, while no stochastic dominance violation takes place when a lump sum is introduced. This finding provides an interesting information on the nature of the human judgment implemented by investors, when the performance measure is the stochastic dominance. A trend affects the entire set of realizations of a random amount, while a lump sum is related to an impulsive shock. Therefore, investors having a certain global misperception irrationally prefer the worst project. Conversely, the presence of an isolate gain in the dominated investment is not able to invert subjects' mind and drive the decision process. A future theoretical research in this direction concerns the introduction of different misperception rules.

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