

Takeover Contests, Toeholds and Deterrence ^{*}

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Abstract

We consider a setting in which two potential buyers, one with a prior toehold and one without, compete in a takeover modelled as an ascending auction with participating costs. The toeholder is more aggressive during the takeover process because she is also a seller of her own shares. The non-toeholder anticipates this extra-aggressiveness of the toeholder. Thus, he is deterred from participating unless he has a high valuation for the target company. This leads to large inefficiency losses. For many configurations, expected target returns are first increasing then decreasing in the size of the toehold.

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1 Introduction

In many takeover cases, one bidder owns, prior to the first offer, a *toehold* in the target company. This is a well-documented fact, Betton and Eckbo (2000), for instance, observed in a sample of 1353 tender offer contests over the period 1971-1990 that in 36% of the contests, a bidder owns a toehold greater than or equal to 10%. In the sample, the average toehold size is equal to 14.57%.¹ Now, the existence of such a toehold indisputably affects buyers' motivations in the different outcomes of the takeover. When a *toeholder* wins the takeover, she needs to buy less shares than a standard bidder since she already owns a fraction of the target company. In a way, winning is less costly for her than it is for a *non-toeholder*. When she loses a contested takeover, she can sell her shares to the winner. She is no longer a buyer of shares but rather a seller of her own shares. As any other shareholder, she prefers the winning offer to be as high as possible. In both cases, the profit function of the toeholder differs from the one of a non-toeholder. Therefore, the toehold affects both her motivation to make a takeover offer and her behavior during a (contested) takeover. We are interested in understanding these two effects of the presence of a toehold and their impact on the outcome of the takeover.²

We consider a framework with two potential bidders, one with a toehold and one without, in which, as in Burkart (1995) and Singh (1998), a takeover is modeled as an ascending auction with independent and private valuations. In this setting, Burkart (1995) and Singh (1998) both observe an extra aggressiveness of the toeholder. She stays active (i.e. makes counteroffers) for prices strictly higher than her valuation for the target firm. This extra aggressiveness is explained by the motivations we mentioned. In contrast, the toehold has no effect on the strategy of the other bidder. He stays active as long as the price is below his valuation for the firm and leaves when the price is equal to his valuation. Thus, the toehold smoothly increases the selling price of shares and deteriorates the efficiency of the takeover

¹For other empirical data on the presence of toeholds in a takeover context, see, for instance, Bradley, Desai and Kim (1988), Franks and Harris (1989) or Jarrell and Poulsen (1989).

²As in a large part of the theoretical literature on the takeover issue (see Bulow et al (1999), Burkart (1995), Singh (1998), Hirshleifer and Titman (1990)), we consider the forming of the toehold as an exogenous phenomenon. Firms know that owning a toehold in a firm that you want to buy is an advantage during the takeover process. However, they cannot buy shares in the days or the weeks preceding the beginning of the takeover because such a strategy is identified by the market. Potential competitors may react immediately and the price of the shares rises so much that it is not worth buying this toehold (see Mikelson and Ruback (1985) or Choi (1991)). This explains why, in most cases, firms do not buy toeholds before making a takeover offer. Nevertheless, firms do often own toeholds prior to make a takeover offer. We explain the existence of these toeholds with an extremely simplified two-periods representation. First, firms who have access to a limited amount of money buy shares of several firms of their business sector (for instance, during IPOs or at the time they build partnerships). Later, after several modifications of the structure of the business sector, a firm who bought shares may be interested in buying a company she invested in. Then, firms are precisely in the situation we are interested in. Such situations are often observed in the high tech business where major companies invest in start-ups that they seldom buy later on or in countries in which the firms' capital is structured by these industrial investments : Germany, Italy, France, Spain...

process.

Compared to this literature that only considers the bidding process, we intend to go upstream and understand the impact of shareholdings both on bidding strategies and on the decision to take part in the takeover process. To do that, we endogenize the participation decision in the takeover contest by taking into account participating costs.³

We observe that even with very low participating costs, the toeholder may deter partially or completely the non-toeholder from making any takeover offer. The deterrence relies on the extra-aggressiveness of the toeholder. This extra aggressiveness reduces the expected profit of the non-toeholder. Unless the non-toeholder has a high valuation for the target company, his expected profit becomes lower than the participating cost.⁴ Then, in many cases, the non-toeholder prefers not to participate to avoid incurring the participating cost.

The deterrence phenomenon relies on the toeholder aggressiveness and on the high probability for the toeholder to participate in the takeover process. The higher the probability that the toeholder participates in the takeover is, the more she can deter the non-toeholder from participating. Now, a limit to her participation is the existence of a *de facto* minimum premium required by shareholders to sell their shares. This minimum premium reduces the probability that the toeholder takes part in the takeover process and consequently reduces the deterrence phenomenon. We observe that, for sensitive values of the participating cost and the minimum premium, the deterrence is partial and progressive.

The impact of a toehold on target returns is dual. On the one side, if both bidders participate in the takeover, the toeholder is more aggressive because of her toehold which has a clear-cut positive impact on target returns. On the other side, precisely because of this extra aggressiveness, the presence of a toehold deters other bidders from participating in the takeover battle which reduces target returns. Eventually, there is no general and univocal positive or negative effects of toeholds on shareholders revenue. Expected target returns may be a non monotonic function of the size of the toehold. We can show that, in many configurations, they are first increasing then decreasing in the size of the toehold. These results provide an explanation for the difficulties in finding an empirically robust positive or negative effect of toeholds on target returns. The non monotonicity obtained in our model may explain the diverging results of empirical studies who intend to identify monotonic effects. As a matter of fact, Betton and Eckbo (2000), Eckbo and Langohr (1989) and Jarell and Poulsen (1989) observe negative impact of toeholds on target returns. However, Franks and Harris observed that toeholds increase target returns and Stulz et al (1990) observed no effect of toeholds on target returns.

³The cost of making a takeover attempt includes both opportunity costs and direct expenses paid to bankers, lawyers and so on.

⁴Hirshleifer (1995, section 4.5) shows results close to ours. In particular, he also observes a deterrence phenomenon. However, his results strongly rely on his perfect information assumption.

Apart from Burkart (1995) and Singh (1998), toeholds in a takeover context have also been considered as a way to alleviate the free rider problem in a single bidder context. In the seminal paper of Grossman and Hart (1980) which first introduced the free-rider problem, a bidder, even though she is the only one making an offer, cannot derive any profit from the takeover process. Shareholders refuse to sell their shares below their values after the success of the takeover. Thus, the raider does not make any profit and too few takeovers are realized. In contrast, if a bidder owns a toehold, she can, at least, make a profit on the shares she owned before the takeover. Thus, a larger toehold increases the probability of success of the takeover attempt. Toeholds decrease the amount paid by the winner to shareholders. These results first appeared in Shleifer and Vishny (1986), then, with some modifications, in Hirshleifer and Titman (1990) or Chowdry and Jeegadesh (1994).

Neither the Burkart-Singh model which describes a clear positive impact of toeholds on target returns nor the free-rider model which says that toeholds have a negative effect on target returns provides an explanation for the undecided empirical results that we mentioned.

Bulow Huang and Klemperer (1999) have a different approach. They consider contested takeovers among *financial* bidders. For this kind of bidders, they found that the common value paradigm is more appropriate since financial bidders do not have private motivations (synergies, modifications of the market structure) for buying the target company. In this framework, toeholds become more important because of effects related to the winner's curse issue. They observe that an asymmetric distribution of toeholds induces low bids from bidders with small toeholds and high bids from bidders with large toeholds. If toeholds are distributed nearly symmetrically among bidders, they increase expected target returns. In contrast, if they are distributed very asymmetrically, they decrease expected target returns.⁵ We believe that the limitation to financial bidders is too restrictive. In most cases, bidders are not purely financial actors, they are *strategic*⁶ and valuations derive from motivations specific to each bidder.⁷ That is why we choose to represent the takeover process with the private value paradigm.

The remainder of the paper is organized as follows. Section 2 presents the model. In section 3, we study a simple representation of the takeover process, no participating cost, no minimum premium. In section 4, we enrich the model by taking into account participating costs. In section 5, we consider the impact of the minimum premium required by shareholders. At last, section 6 develops implications of our results and evokes their limits.

⁵There is no efficiency issue since the valuation is the same for all the bidders.

⁶In the Bulow et al (1999) meaning of the word.

⁷Berkovich and Narayanan (1993), Gupta et al (1997) and Goergen and Reneboog (2002) all find strong evidence that synergy is the prime motive for mergers and acquisitions.

2 The model

A takeover consists in alternate increasing offers by potential buyers. When a bidder makes an offer that no other bidder overbids, he buys all the shares on the market for that price. Besides, we assume that it is not possible to renege on a previous offer or to organize a negotiation between bidders or between a bidder and the board of the target company. With two bidders, as the size of the minimum increment between each offer goes to zero, this process becomes equivalent to an ascending auction. Therefore, we choose to model the takeover process as an ascending auction with two potential buyers.⁸

We make the following assumptions regarding the takeover. (i) The aim of a takeover attempt is, after taking control of the firm, to benefit from synergies between the target firm and the bidder. (ii) These synergies are bidder specific and privately known. (iii) In order to participate in the takeover process, bidders incur a commonly known sunk cost. (iv) Shareholders do not accept to tender if they are not offered a minimum premium.

Formally, we have the following representation: firm A is a potential target for two risk-neutral bidders, 1 and 2. Each possible bidder i has a valuation v_i for the target firm which is private information. It is common knowledge that valuations are independently drawn from a uniform distribution on the interval $[0, 1]$.⁹ It is also common knowledge that bidder 1 owns a fraction α of firm A's capital with $0 < \alpha < \frac{1}{2}$.¹⁰

The takeover is represented by a two-stages game:

- Stage 1: both bidders decide simultaneously if they want to participate in the takeover process.

Between stage 1 and stage 2, bidders who decided in stage 1 to participate pay a sunk cost $c \geq 0$. Participation decisions are observed by all the bidders.

- Stage 2: bidders who paid the sunk cost compete in an ascending auction defined as follows. From an initial value $R \geq 0$, the price gradually increases. At any moment, bidders can quit the auction. The auction stops when there is only one bidder left. With two possible bidders, this is equivalent to the moment when the first bidder quits. All the shares of firm A are bought by the remaining bidder at the current price. If both bidders quit for the same price, bidder 2 wins the takeover and buys the shares for the common price.¹¹

⁸There hardly exist any contested takeovers with more than two bidders.

⁹We normalized the Stock Exchange valorization of company A before the takeover process at zero. We choose a uniform distribution for the sake of simplicity. The spirit of our results remains true for a wider range of distribution functions.

¹⁰Most takeover regulations rules require bidders to disclose their initial stakes.

¹¹For the existence of an equilibrium in cases considered in sections 4 and 5, this tie-breaking rule is required. For more details on the link between tie-breaking rules and equilibrium, see Jackson et al (2002).

Bidder 1 owns, before the takeover process, a fraction α of the target firm. If she is the winner of the auction, she only buys a fraction $(1 - \alpha)$ of firm A's capital. Conversely, if she loses the auction, she sells her toehold to her opponent at the price defined by the auction.¹² On the other hand, bidder 2, if he does not win the auction, derives no profit and, if he wins the auction, buys the whole capital of firm A. We can therefore define utility functions as follows.

- If bidder 1 wins the takeover for a price p

$$U_1 = v_1 - (1 - \alpha)p \quad \text{and} \quad U_2 = 0$$

- If bidder 2 wins the takeover for a price p

$$U_1 = \alpha p \quad \text{and} \quad U_2 = v_2 - p$$

- If firm A remains independent

$$U_1 = U_2 = 0$$

A strategy for bidder i is a couple (d_i, b_i) with $d_i : [0, 1] \rightarrow \{0, 1\}$ ¹³ and $b_i : [0, 1] \rightarrow [R, \infty)$. $d_i(v_i)$ is the participation function of bidder i . If $d_i(v_i) = 1$, bidder i participates in the takeover process when his valuation is v_i . If $d_i(v_i) = 0$, bidder i does not participate in the takeover process when his valuation is v_i . $b_i(v_i)$ is a bidding function, it is the price for which bidder i , if his valuation is v_i , quits the takeover process if he participates and bidder j is still active.

In the remainder of the paper, we only consider equilibria with undominated strategies. All the proofs are in the Appendix.

3 A simple representation of the takeover process

In this section, we focus on a simplified representation of the takeover. Participating to the takeover is free of charge ($c = 0$). Stockholders accept to sell their shares for any price over the quotation before the beginning of the takeover process ($R = 0$). We do not study in details this configuration, we only need it as a benchmark. Besides, Burkart (1995) and Singh (1998) already developed a complete analysis of this issue.

¹²The toeholder, if she loses, prefers selling her shares. If the non-toeholder takes control of the target company, he will divert the synergies he can create. Thus, the toeholder is better off selling her shares before this dilution. Reciprocally, the winning company cannot refuse to buy his adversary's toehold at the price of the winning tender.

¹³We rule out the possibility that a company participates with a probability q with $0 < q < 1$ in order to simplify the proofs we present. Our results would remain unchanged if we allowed buyers to randomize. At the equilibrium, a bidder cannot randomize for an interval of valuations of positive measures

Participating is free of charge, therefore, for $i = 1, 2$, for any $v_i \in [0, 1]$, $d_i(v_i) = 1$ is a dominant strategy. Bidders always participate in the takeover process. Thus, we can focus on the second stage of the game.

Proposition 1 *There is a unique equilibrium. Bidders always participate. Bidder 2 bids his valuation and bidder 1 bids according to b_1 defined as follows:*

$$b_1(v_1) = \frac{v_1 + \alpha}{1 + \alpha} = v_1 + \frac{\alpha}{1 + \alpha}(1 - v_1)$$

For bidder 2 who does not own a toehold, it is a weakly dominant strategy to leave the takeover process for a price equal to his valuation. Leaving for a lower price, he could miss an opportunity to make a strictly positive profit. If he stays active while the price is higher than his valuation and wins the takeover, he derives a strictly negative profit.

This reasoning cannot be applied to bidder 1. Contrary to bidder 2, conditional on losing the auction, she is not indifferent to the price paid by her opponent. If she loses the auction, she sells her shares and thus prefers the price to be the highest possible. Besides, in that case, the final price is equal to the price for which she quits the takeover process. Thus, she has incentives to quit for a price higher than her valuation in order to increase her revenue conditional on losing the takeover. The toeholder submits her valuation plus a strictly positive overbidding, $\frac{\alpha}{1+\alpha}(1 - v_1)$.

This overbidding is an increasing function of α . The higher the fraction of the target company bidder 1 holds, the more important it is for her to raise the price, conditional on her losing the takeover. Besides, if she wins the takeover, she buys a fraction $(1 - \alpha)$ of the target company which is smaller. The risk to pay a price higher than her valuation when winning concerns a lower number of shares.

This overbidding is also decreasing in v_1 . More precisely, it is proportional to $(1 - v_1)$, the probability that bidder 2 has a higher valuation than bidder 1. Suppose that bidder 1 increases her bid by ε . It increases her probability to win the takeover for a price higher than her valuation by ε . Moreover, she will sell her shares to bidder 2 for an ε higher price with a probability $(1 - v_1 - \varepsilon)$. The risk is fixed but the probability to sell for a higher price is increasing in $(1 - v_1)$. A high overbidding is thus more profitable for higher values of $(1 - v_1)$, i.e. for low values of v_1 .¹⁴ The toeholder overbids to make bidder 2 pay a higher price. Thus, the overbidding rises with the probability to increase the price paid by bidder 2.

¹⁴In a wider framework, with a non uniform distribution function, the elements to consider would be $(1 - F(v_1))$ and the probability that bidder 2 leaves for a price in the interval $[v_1, v_1 + \varepsilon]$.

3.1 Efficiency

Bidder 2 quits when the price is equal to his valuation. Bidder 1 quits for a price strictly higher than her valuation.¹⁵ As a result, if $v_1 < v_2 < b_1(v_1)$, bidder 1 wins the takeover while bidder 2 has a higher valuation for the target company. Because of the toehold, the takeover battle is not an efficient way to allocate the target firm. The initial distribution of property rights does have an influence on the efficiency of the allocation procedure.

The overbidding is increasing in α . As a result, the probability of an inefficient allocation, equal to $E[b(v_1) - v_1] = \frac{\alpha}{2+2\alpha}$, is also increasing in α .

3.2 The owner's curse

Bidder 1's overbidding has another consequence. She quits the takeover process for a price exceeding her valuation. Thus, if bidder 2 quits when the price is in the interval $(v_1, b_1(v_1))$, bidder 1 wins the takeover and pays a price for which she would have strictly preferred losing. Because of her toehold, bidder 1 bids more aggressively. Eventually, she may be victim of her aggressiveness. Burkart (1995) and Singh (1998) described in details this issue named *owner's curse* by Burkart. Despite this owner's curse, the expected utility of bidder 1, $\frac{(v_1+\alpha)^2}{2+2\alpha}$, is increasing in α . The increased probability of buying shares for a price exceeding their means value is more than compensated by the advantage of owning an ex-ante toehold.

3.3 The price

The other stockholders of the target firm are not concerned by efficiency considerations as long as they sell their shares for the highest possible price. The final price is their only concern. For them, the existence of the toehold has a positive impact. Bidder 2's strategy is not affected by the toehold while the price for which bidder 1 quits the takeover increases with the size of her toehold. As far as revenue is concerned, it seems that the impact of the toehold is unambiguously positive. In the following sections, we will see that this result is questionable.

4 Taking participating costs into account

In this section, we take into account the positive cost to participate in a takeover. The participation decision becomes endogenous. As a result, we observe a dramatic deterrence effect.

In the previous section, we assumed that participating in a takeover is free of charge. This assumption is not credible. Participating in a takeover is costly. First, it requires the mobi-

¹⁵Except when $v_1 = 1$.

lization of the finance direction and the high management which represents an opportunity cost for the firm. Besides, no major takeover is launched without calling on the expensive services of experts such as consulting firms, bankers, lawyers.¹⁶ Whatever the result of the auction is, a bidder that participates in the takeover process has to pay the type of expenses that we have mentioned. Thus, these are sunk costs.

We will show that these sunk costs dramatically change our analysis of the effects of the toehold. But first, note that the participation decision becomes endogenous because of the existence of these sunk costs. Since participating is costly, the participation decision becomes an issue.

Proposition 2 $\forall c, \alpha > 0$, in any equilibrium, bidders' behaviors are as follows: bidder 2 never participates, bidder 1 participates when her valuation is at least equal to c and for $c \leq v_1$, $b_1(v_1) \geq 1$.

Bidder 2 never participates in the takeover process. He is fully deterred by bidder 1. This is true whatever the values of α and c are as long as they are strictly positive.

The intuition is as follows. At the equilibrium, there is a minimum valuation \underline{v} for which bidder 2 participates in the takeover process. Besides, if he participates in the takeover process, it is a dominant strategy for him to quit when the current price is equal to his valuation. Bidder 1, if she loses the contested takeover, prefers the final price to be as high as possible since, in that case, she sells her shares for this price. As a result, if both bidders participate, bidder 1 never quits the auction for a price below \underline{v} . Thus, if bidder 1 participates, bidder 2, when he has a valuation of \underline{v} , does not derive any profit from his participation in the takeover while he has to pay a participation cost $c > 0$. He would have been better off, not participating. As he can anticipate that such an event will occur, he prefers not to participate. This argument remains true for any value of \underline{v} since bidder 1 participates with a high probability, i.e. whenever $v_1 \geq c$. As a result, the deterrence phenomenon is complete even if the participating cost and the toehold are small.

By taking into consideration the participating costs, we endogenized the decision to participate in the takeover. It dramatically changes our previous analysis of the impact of the toehold. Bidder 1 fully deters bidder 2 from participating. The price paid to the remaining shareholders is always zero, i.e. the Stock Exchange quotation before the takeover. The allocation is inefficient with a probability higher than $\frac{1}{2}$ and these results are true for any strictly positive value of the sunk cost.

These results rely on bidder 1's caring about the price paid even when she loses. She can credibly commit to bid up in case bidder 2 would participate in the auction. This commitment

¹⁶When Vodafone bought Mannesman, it paid more than a billion dollars to its lawyers and bankers. It was not even a contested takeover.

is so strong that it completely deters bidder 2 from making any takeover offer whatever his valuation for the target firm is.

Regarding the robustness of this result, we can observe that the complete deterrence does not rely on the limitation to two competitors. Suppose that we have $n \geq 2$ competitors without toehold and one with a toehold. Then, the following strategies are constitutive of an equilibrium: bidder 1, the bidder with a toehold, participates whenever her valuation for the target firm is at least c . If any other bidder participates, bidder 1 quits the takeover process for a price equal to 1. The n other competitors never participate. All the bidders without toehold are fully deterred.

However, the choice of a uniform function is not neutral. It affects the extent of the deterrence phenomenon. Complete deterrence would not hold with any distribution function. However, there is a deterrence effect whatever the distribution function that we could consider. The deterrence phenomenon would exist for any distribution function. But, it would be less pronounced for some other distribution functions. However, we observe that the strategies introduced in proposition 2 that leads to complete deterrence are constitutive of an equilibrium with any distribution F provided that $F(c) \leq c$. The necessary condition for the existence of such an equilibrium only concerns the lower part of the distribution. The lower the probability that bidder 1 have a valuation below c , the participating cost, the stronger the deterrence phenomenon is.

This deterrence phenomenon dramatically changes our analysis of the effect of toeholds. Unlike what we tended to infer from the results of the previous section, the impact of a toehold for the remaining stockholders is not always positive. On the contrary, it can be extremely negative if we integrate its influence on the decision to participate in the takeover process or not. Even a relatively small toehold may have a dramatic deterrence effect and an extremely negative impact on the expected revenue of remaining stockholders.

5 Analysis of the takeover with shareholders requiring a minimum premium

5.1 The context

In this section, we assume that shareholders may refuse to sell their shares if they are not offered a minimum premium. We obtain more balanced results with such a setting.

In the previous section, we observed that, if participating in the takeover is costly, the final price is always zero, that is the stock exchange quotation before the beginning of the takeover process. However, empirical studies show that even when the takeover is not contested, bidders almost always offer a bonus to shareholders.

Several theoretical reasons have been given for justifying this bonus. We mention here the explanation given in Stulz (1988) and Stulz and al (1990) which is compatible with the other assumptions of our model.

We assume that shareholders have specific attributes such as liquidity or tax considerations that affect their tendering decisions. Therefore, the supply of shares tendered is an increasing function of the price per share offered by the bidder. For the sake of simplicity, we assume that shareholders are distributed in two groups. All the members of a group have in common a minimum price for which they agree to sell their shares. Shareholders of the first group accept to sale their shares for a price equal to the current Stock Exchange valorization, zero. Shareholders of the second group because of these tax and liquidity considerations refuse to sell their shares for a price below $R > 0$. Shareholders of the second group are numerous enough so that, in order to be successful, bidders must propose a price for the shares at least equal to R , the commonly known minimum premium.

In practice, integrating these elements in our model is equivalent to the addition of a reserve price. As a matter of fact, if a bidder proposes a price below this minimum price, too many stockholders refuse to sell their shares and the takeover attempt fails. This minimum price may be a function of the size of the toehold and may vary across bidders. Nevertheless, as a first approximation, we consider a unique given reserve price for both bidders.

5.2 The general analysis

To rule out trivial cases, we assume that $c + R < 1$.¹⁷ We obtain the results stated in the following lemma and proposition.

Lemma 1 *For any equilibrium $((d_1^*, b_1^*), (d_2^*, b_2^*))$, there exists a couple (v_1^*, v_2^*) such that for $i = 1, 2$*

$$\begin{aligned} d_i^*(v_i) &= 0 \quad \text{if } v_i < v_i^* \\ d_i^*(v_i) &= 1 \quad \text{if } v_i > v_i^* \\ b_1^*(v_1) &= \max(v_2^*, \frac{v_1 + \alpha}{1 + \alpha}) \\ b_2^*(v_2) &= v_2. \end{aligned}$$

with $v_2^* \geq R + c$ and $v_1^* \geq c$.

Thus, an equilibrium can be defined by a couple $(v_1^*, v_2^*) \in [c, 1] \times [c + R, 1]$.

¹⁷If $c + R \geq 1$, at the equilibrium, bidder 2 never participates while bidder 1 participates if and only if $v_1 > c + (1 - \alpha)R$.

Proposition 3 (v_1^*, v_2^*) is an equilibrium if and only if one of the following series of conditions holds.

- $v_2^*(v_1^* - (1 - \alpha)R) + \alpha(v_2^* - R)(1 - v_2^*) = c$, $v_1^*(v_2^* - R) = c$ and $v_2^* > \frac{v_1^* + \alpha}{1 + \alpha}$
- $v_2^*(2\alpha R - R) + \frac{1 - \alpha}{2}(v_2^*)^2 + \frac{(v_1^* + \alpha)^2}{2 + 2\alpha} - \alpha R = c$, $v_1^*(v_2^* - R) = c$ and $v_2^* \leq \frac{v_1^* + \alpha}{1 + \alpha}$
- $(v_1^*, v_2^*) = (c + (1 - \alpha)R, 1)$ and $(1 - \alpha)(1 + R) \leq c$

In an equilibrium in which the third series of conditions are satisfied, bidder 1 fully deters bidder 2 from participating. In an equilibrium in which one of the two first series of conditions is satisfied, both bidders participate with a strictly positive probability.¹⁸

Complete deterrence is no longer the general rule. It only occurs if $(1 - \alpha)(1 + R) < c$, that is for extreme values of c and α . If bidder 1 wants to deter bidder 2 from participating, she has to participate in the takeover with a high probability. However, if bidder 2 is completely deterred and never participates, bidder 1 only derives a positive profit from her participation if $v_1 - (1 - \alpha)R \geq c$. Thus, if $v_1 < (1 - \alpha)R + c$, she is strictly better if she does not participate. Bidder 1 does not participate with a probability $(1 - \alpha)R + c$. If R is high enough, this probability is non negligible. Bidder 1 is absent from the takeover process often enough and if bidder 2 has a high valuation, he derives a strictly positive expected revenue from a participation in the takeover. Thus, the deterrence is no longer complete. For high values of v_2 , bidder 2 is better off participating in the takeover battle.

The deterrence phenomenon becomes more progressive when taking into account the minimum premium. It becomes a function of the size of the toehold, the participating costs and the minimum premium. Let us consider their effects on the deterrence phenomenon.

- The participating cost, c . As in the standard case, without toehold, it has a negative impact on bidders' participation since it deters bidders from participating when they have low valuations and do not expect to make a profit higher than the participating cost. For the non-toeholder, this phenomenon is reinforced by the presence of the toehold. Since the toeholder is more aggressive, the expected profit of the non-toeholder is lower. Thus, he is more likely to make a profit lower than the participating cost. That is why he participates with a lower probability than in the absence of toehold.
- The minimum premium, R . As in a standard case, it deters bidders with valuations below it from participating in the takeover. Therefore, the higher R is, the lower is the probability for the toeholder to participate. Thus, the non-toeholder fears less the extra aggressiveness of the toeholder. This may more than counterbalance the

¹⁸The difference between equilibria satisfying the first or the second series of conditions are negligible. In equilibria satisfying the first series of conditions, there is a strictly positive probability that, conditional on participating, bidder 1 submits the lowest valuation for which bidder 2 participates in the takeover process. This is not the case in an equilibrium satisfying the second series of conditions.

negative impact that the minimum premium has on his incentives to participate. The non-toehold probability to participate may be locally increasing in R .

- The toehold, α . It has a non ambiguous effect on participation decisions. For higher values of α , the toehold participates more often in the takeover process since he needs to buy a lower fraction of the target firm to win. Besides, it is even more important for him that, conditional on his losing the takeover contest, the final price be high. The effect on the non-toehold incentives to participate in the takeover process is opposite. Since the toehold participates more often and she bids more aggressively for higher values of α , the non-toehold makes a lower expected profit conditional on participating in the takeover process. The higher is α , the lower is the probability for the non-toehold to participate in the takeover.

A rise in α has a clear-cut effect on participation decision. This is not the case as far as target returns are concerned. A rise in α has both a positive and a negative effect on the price reached by the takeover process. It raises the probability that bidder 1 takes part in the takeover process and increases the value of her bid but it also deters bidder 2 from participating for a higher range of valuations and thus decreases his probability to participate.

In general, expected target returns are not a monotonic function of the size of the toehold. Depending on the values of the other parameters, the effect of an increase of α on the expected revenue of shareholders may be positive or negative. Even if we fix c and R , this effect may vary locally. The following example illustrates this issue.

Example 1 *We consider the following setting: participating cost, $c = 0.02$ and minimum premium, $R = 0.15$. Figure 1 represents shareholders' expected target returns depending on the size of the toehold, α . Expected target returns are first increasing then decreasing in the size of the toehold.*

5.3 Numerical applications, descriptions and commentaries

For technical reasons, we are unable to obtain the necessary conditions for the expected revenue of the shareholders to follow the hump shaped pattern that we identified in example 1. Nevertheless, this shape can be observed for many other values of c and R . In order to illustrate this statement, we exhibit a series of graphics of the expected revenue of the takeover contest depending on the size of the toeholds for different values of R and c . Of course, these examples do not have the strength of an analytical resolution. However, this is an applied issue and we believe that it is instructive to use these numerical applications. They show that the observations made in the example 1 can be extended to a broader range of configurations.

Figures 2, 3, 4 and 5 represent expected target returns depending on α for different values of c and R . These graphics indicate that, in many configurations, the expected revenue of shareholders is a non monotonic, hump-shaped, function of α . Besides, the observation of these graphics provides an intuition regarding the conditions for which this property holds. It holds when R is not too high and c is not too small.

If R is large, the main issue is whether any bidder will participate. Deterrence of bidder 2 by bidder 1 is a second order effect. As bidder 1 only needs to buy a fraction $(1 - \alpha)$ of the target firm, her probability to participate increases with α . If R is large, this probability is still low and the deterrence effect is weak. Thus, it only marginally affects bidder 2's participation decision and the major effect of the toehold is an increase in bidder 1's probability to participate. The global effect of a rise in α on the expected price is always positive.

If c is very small, the deterrence effect becomes weaker since bidder 2 participates even if his expected profit is low. Thus, the major consequence of the toehold is bidder 1's overbidding. This overbidding is increasing in α and always has a positive impact on the final price.

In contrast, if R is not too large and c is not extremely small, the expected final price is a hump-shaped function of the size of the toehold. For low values of α , bidder 1 when she participates in the auction, is more aggressive, but not enough to notably deter bidder 2 from participating. The extra aggressiveness, during the auction, overrules the deterrence effect, at the time of the entry decision. The expected price is an increasing function of α . For higher values of α , the deterrence effect dominates. Bidder 2 is often deterred and participates with a low probability. Bidder 1 is more aggressive in case of contested takeovers. However, since bidder 2 seldom participates, this aggressiveness is not useful for the remaining shareholders. Therefore, expected target returns become a decreasing function of α .

Estimating the values of c and R is a difficult task and this is not the object of this paper. Nevertheless, in the considered framework, considering the motivations that we gave for the existence of the minimum premium, not related to the free-rider issue, intermediary values of R seem to be appropriate. As regards the participating cost, we already explained why we believe that it is not negligible.¹⁹ Therefore, it is reasonable to consider that c and R are such that toeholds have a non monotonic effect on the price.²⁰ Below a threshold α^* , the expected price increases in α . Over this threshold, because of the deterrence effect, the expected price decreases in α .

These results may explain the difficulties for empirical studies to find a clear-cut effect of

¹⁹Remember that, with our normalization, c should not be compared to the final price paid to the shareholders but to the bonus paid to them, or more precisely, to the distribution interval of this bonus.

²⁰Besides, even though we chose not to take this element into account, R is likely to be decreasing in α . This reinforces the non monotonic trend.

toeholds on the final price that we mentioned in the introduction. Jarrell and Poulsen (1989) and Betton and Eckbo (2000) observe that toeholds decrease target returns, according to Frank and Harris (1989), they have a positive effect and Stulz and al (1990) observe no effect of toeholds on target returns. Our model provides an explanation for these conflicting results. As a matter of fact, we derive from our model that target returns may be a non monotonic function of the size of the toehold. These studies only consider the possible existence of a positive or a negative effect. If our model captures some aspects of actual phenomena, it may explain why they obtain so diverging results. Depending on the distribution of toeholds sizes in their sample, they may observe different effect of toeholds on target returns.

6 Applications and predictions

In this section, we apply our results to several aspects of the questions of the control and the taking of control of firms and propose testable predictions and suggest limits to the model.

6.1 The optimal size of a toehold

Let us consider the minimum premium, R and the participating cost, c as fixed, elements of the market structure which is common knowledge and assume that these parameters are such that expected target returns follow a hump-shaped function of the toehold size. It is then possible to compute an optimal toehold α^* , α^* being such that the expected revenue of the remaining stockholders is maximized when a possible buyer owns a toehold of size α^* .

For $\alpha < \alpha^*$, the deterrence phenomenon is weak. However, as α is small, bidder 1 does not bid much more aggressively than a bidder without toehold. Then, the expected price of the remaining shares is not maximized. For $\alpha > \alpha^*$, the deterrence effect prevails. Other possible bidders stay out of the takeover process too often. The expected revenue of the remaining stockholders tends to decrease with the size of the toehold.

Thus, a board of administrators concerned about the revenue of all its stockholders should make sure that no possible purchaser buys progressively a toehold substantially higher than α^* . Otherwise, this potential buyer would deter too much other competitors from trying to buy the firm. Eventually, it would have a negative effect on the expected revenue of the other shareholders.

Some legal dispositions seem to be related to this observation. For example, in many legislations, any firm who buys shares of another firm, beyond some well-defined thresholds, must officially declare its intentions. In some cases, the firm is even forced to make a takeover offer. One of the aim of these measures is precisely to protect the remaining shareholders from a too powerful toehold who would deter any other potential buyer from making a takeover offer.

6.2 How to fight the deterrence effect

Suppose that a bidder yet holds a fraction α of firm A such that the deterrence phenomenon exceeds the positive impact of the toehold. What can the board of firm A do in order to reduce the deterrence effect?

According to the terms and results of our model, the board should try to change the minimum premium or to reduce participating costs. A priori, the board cannot affect the value of the minimum premium. Thus, an adequate policy for the board would be to minimize the participating costs. For this purpose, it could, for instance, destroy existing poison-pills. However, it is doubtful that the high management have the right incentives to do so. If a takeover succeeds, whoever the winner is, members of the high management are likely to lose their jobs. Therefore, they prefer maintaining the probability of a successful takeover at a low level.

In contrast, the high management could make the following statement: “If this stockholder sets off a takeover process, we will help any other competitor who could be interested in a takeover. We will reduce his participating cost through any kind of alliance or by providing the help of *shareholder-friends*”. This type of commitment reminds of the white knight searching process. The management has the right incentives to do so. Once a bidder first announced his takeover attempt, the high management has incentives to find another possible bidder. There is a common belief that the white knight will be more likely than the other bidder to keep the high management, or a fraction of it, in place, out of gratitude. This strategy is also in the stockholders interest, as it increases the expected final price.²¹ In the model, this behavior is equivalent to the creation of differentiated entry costs, c_1 and c_2 with $c_2 < c_1$. This kind of attitude is often observed although, in numerous cases, the board attempts are fruitless and no other competitor makes an offer.

6.3 How to control safely a firm with less than half of its shares

Let us consider the results through a different angle. The toeholder point of view. We often observe that a firm has, *de facto*, the control of another firm without owning 50% of its capital and without even trying to buy these 50%. Our model could provide an explanation for this kind of situation.

R and c are given, parts of the market structure with c not extremely low and R not extremely high. Then, there exists an $\tilde{\alpha} < \frac{1}{2}$ such that if a firm owns a fraction α of firm A, it almost completely deters any other competitor from attempting to buy firm A through a takeover. If shareholdings are scattered, as a major shareholder, the firm owning the toehold

²¹It is positive for the shareholders only as long as it does not deter too much bidder 1 from making an offer. However, since bidder 1 has more reasons than bidder 2 to participate, she is unlikely to be seriously deterred.

$\tilde{\alpha}$ has the effective control of firm A without the need to buy the majority of firm A's capital. Besides, the probability that any other firm tries to buy firm A is extremely low. Therefore, it may be useless while costly to buy the remaining necessary shares to own the 50%. Because of its high toehold, the firm can deter almost any other competitor from making an attempt to buy firm A. As long as having the effective control is the main objective, buying the majority of the shares is almost superfluous.

6.4 Limits

We chose to restrain our study to a specific distribution function, the uniform distribution function. We already mentioned the impact of that choice. With a distribution function which puts more weight on lower valuations, the deterrence phenomenon would be stronger. With distribution functions which puts more weight on intermediary and high valuations the deterrence phenomenon would be weaker.

We considered cases in which only one possible buyer owns a toehold. A more general model would represent situations in which both bidders may own a fraction of the target company. In such a case, results would crucially depend on the distribution of toeholds. Bidders may own toeholds of different sizes. If one bidder owned a large toehold and the other a small one, we would obtain results close²² to the one we have presented. Now, if bidders had toeholds of almost identical size, we would obtain extremely different results. The deterrence phenomenon that we highlighted would almost disappear. There would not be any more a *favoured* bidder who deters a *less favoured* bidder. In that case, toeholds have a positive effect on the final price and efficiency (see intuitions of these results in Engelbrecht-Wiggans (1994), Maasland and Onderstal (2002) and Ettinger (2002)).

The assumption that bidders perfectly know their valuations for the target company at the time they decide to participate and incur the sunk cost is also a key element of the model. If bidders were to discover their valuations after deciding to take part in the takeover and to pay the cost, the equilibrium would be different.²³ Nevertheless, for some configurations, we could still observe a non monotonic effect of toeholds on target returns. However, such a representation seems less accurate than the one we considered.

²²However, even without a minimum premium, bidder 1, the bidder with the highest toehold, would not completely deter bidder 2 from participating. Bidder 2 would participate with a positive probability, would it only be to rise the final price. Since he owns a toehold, if he loses the takeover process, he also prefers the final price to be high.

²³The equilibrium of this game is as follows. For c below a threshold \underline{c} , both bidders always participate in the takeover process. For intermediary values of c , between \underline{c} and \bar{c} , bidder 1 always participate and bidder 2 never participates. For c higher than \bar{c} , no bidder ever participates. \underline{c} is decreasing in α and \bar{c} is increasing in α . Besides, bidder 2's participation decisions is not informative. Thus, bidder 1 submits $b_1(v_1)$ (b_1 being the bidding function that we defined in proposition 1) if her valuation is v_1 .

A Proof of proposition 1

For bidder 2, it is a weakly dominant strategy to bid his valuation. Bidding that way, he wins the auction when bidder 1 bids less than v_2 , in these cases, it is worth winning for bidder 2, and never wins the auction when bidder 1 bids strictly more than v_2 , in these cases bidder 2 prefers losing.

These arguments are not valid for bidder 1 since she cares about the selling price when bidder 2 obtains the good. If she loses the auction, she becomes the seller of her toehold and prefers the price to be high. Bidding her valuation is not a dominant strategy.

However, as bidder 2's strategy is known, bidder 1 faces a simple optimization program

$$\max_{b_1} (v_1 - (1 - \alpha) \frac{b_1}{2}) b_1 + \alpha (1 - b_1) b_1$$

There is a unique solution to this program : $b_1 = \frac{v_1 + \alpha}{1 + \alpha}$. Q.E.D.

B Proof of proposition 2

First, we show that if $((d_1, b_1), (d_2, b_2))$ is an equilibrium, then for $\forall i = 1, 2, \exists \widehat{v}_i$ such that $d_i(v) = 0$ if $v < \widehat{v}_i$ and $d_i(v) = 1$ if $v > \widehat{v}_i$.

Suppose that $\exists i, \underline{v}, \bar{v}$ such that $\underline{v} < \bar{v}$, $d_i(\underline{v}) = 1$ and $d_i(\bar{v}) = 0$. As $d_i(\underline{v}) = 1$, the expected revenue of bidder i with valuation \underline{v} if he participates in the takeover must be at least c , otherwise he would be strictly better off not participating. We write it $EU_i(\underline{v}, b_i(\underline{v})) \geq c^{24}$. If bidding $b_i(\underline{v})$, bidder i never wins the auction then $EU_i(\bar{v}, b_i(\underline{v})) = EU_i(\underline{v}, b_i(\underline{v}))$, otherwise $EU_i(\bar{v}, b_i(\underline{v})) > EU_i(\underline{v}, b_i(\underline{v}))$. Suppose $i = 2$, then as $EU_2(\underline{v}, b_2(\underline{v})) \geq c$, bidder 2 bidding $b_2(\underline{v})$ must win the takeover with a strictly positive probability as otherwise his utility, gross of the payment of the sunk cost, is equal to zero. If $i = 1$, because it is a dominated strategy for bidder 2 with a valuation below c to participate, if bidder 1 participates in the takeover, she wins it with a strictly positive probability. Then, $\forall i = 1, 2$, if bidder i participates, he wins the auction with a strictly positive probability. Then, $\forall i = 1, 2, EU_i(\bar{v}, b_i(\underline{v})) > EU_i(\underline{v}, b_i(\underline{v})) > c$ and if bidder i has a valuation \bar{v} , he is strictly better off participating and bidding $b_i(\underline{v})$ than non participating. Hence, $d_i(\bar{v}) = 0$ is impossible.

$b_2 = Id$ is a dominant strategy since the arguments we gave in section 3 are still valid. The inference that bidder 2 could make on bidder 1's valuation because bidder 1 participates does not alter this result.

Bidder 1 has no dominant strategy. Besides, bidder 1 can infer information from bidder 2's decision to participate. At the equilibrium, if bidder 2 participates in the takeover, it

²⁴ $EU_i(\widehat{v}, \widehat{b})$ is the expected utility of bidder i with valuation \widehat{v} and bidding \widehat{b} , if he participates in the takeover process.

means that $v_2 \geq \widehat{v}_2$. Besides, if bidder 1 loses the auction, her utility is strictly increasing in the price paid by bidder 2. Then, at the equilibrium, if both bidders participate in the auction, bidder 1 cannot bid strictly less than \widehat{v}_2 . Therefore, bidder 2, if his valuation is \widehat{v}_2 , only makes a profit when bidder 1 does not participate. At the equilibrium, we must have

$$\widehat{v}_1 \widehat{v}_2 = c \quad \text{or} \quad \widehat{v}_2 = 1$$

Suppose that, if bidder 1 participates, whatever her valuation v_1 is, she always quits the takeover process when the price is \widehat{v}_2 . She derives an expected utility, $v_1 \widehat{v}_2 + \alpha(1 - \widehat{v}_2) \widehat{v}_2 - c$. Consequently, $\widehat{v}_1 \widehat{v}_2 + \alpha(1 - \widehat{v}_2) \widehat{v}_2 - c$ is a lower bound on bidder 1's expected utility when her valuation is \widehat{v}_1 .

As long as $0 < c < 1$, we can show that corner solutions, $\widehat{v}_1 = 0$ or $\widehat{v}_1 = 1$, are impossible. $\widehat{v}_1 = 0$ is not an equilibrium strategy since, with $c > 0$, bidder 2's best response would be never to participate and, in that case, bidder 1 would rather not participate when $v_1 < c$. Now, suppose that $\widehat{v}_1 = 1$ with $c < 1$, then bidder 2's best response is $\widehat{v}_2 = c$ and if $v_1 > 1 - \alpha(1 - c)$, bidder 1 can participate and bid c , his expected payoff is $cv_1 + (1 - c)\alpha c - c > c(1 - \alpha(1 - c)) + (1 - c)\alpha c - c = 0$. Bidder 1's expected payoff is higher if he participates than if he stays out. $\widehat{v}_1 = 1$ cannot be part of an equilibrium.

Since bidder 1 must be indifferent between participating and not participating for $v_1 = \widehat{v}_1$, we must have

$$\widehat{v}_1 \widehat{v}_2 + \alpha(1 - \widehat{v}_2) \widehat{v}_2 - c \leq 0$$

We also found that $\widehat{v}_1 \widehat{v}_2 = c$ or $\widehat{v}_2 = 1$. Suppose that $\widehat{v}_1 \widehat{v}_2 = c$ then $\alpha(1 - \widehat{v}_2) \widehat{v}_2 \leq 0$, since $\widehat{v}_2 > 0$, this means that $\widehat{v}_2 = 1$. Hence, either $\widehat{v}_2 = 1$ or $\widehat{v}_1 \widehat{v}_2 = c$ which implies that $\widehat{v}_2 = 1$, then, in any case, $\widehat{v}_2 = 1$. $\widehat{v}_1 = c$ is the unique best response to $\widehat{v}_2 = 1$.

In order to sustain such an entry equilibrium, we must have $\forall v \geq c, b_1(v) \geq 1^{25}$ and $b_2(1) = 1$. Besides, bidder 1 should believe that, if bidder 2 participates, bidder 2's valuation is 1. Q.E.D.

C Proof of Lemma 1

First, we must show that if $((d_1, b_1), (d_2, b_2))$ is an equilibrium, then $\forall i = 1, 2, \exists \widehat{v}_i$ such that $d_i(v) = 0$ if $v < \widehat{v}_i$ and $d_i(v) = 1$ if $v > \widehat{v}_i$. The proof of this point is the same as the one we exhibited in the second paragraph of the proof of proposition 2.

Second, bidder 2's dominant strategy, if he participates, is to bid its valuation for the reasons we gave before.

Bidder 1 faces an optimization problem close to the one we considered in section 3, the

²⁵Such a bid is not dominated since bidder 1 has a strict preference for a higher price if she loses the auction.

only difference is that at the equilibrium, if bidder 1 observes that bidder 2 participates, she knows that bidder 2's valuation is in the interval $[v_2^*, 1]$. Then bidder 1 maximizes the following expression, choosing a $b_1 \geq v_2^*$, since bidding less than v_2^* is a strictly dominated strategy

$$v_2^*(v_1 - (1 - \alpha)R) + (b_1 - v_2^*)(v_1 - (1 - \alpha)\left(\frac{b_1 + v_2^*}{2}\right)) + (1 - b_1)\alpha b_1 \quad (1)$$

$$= v_2^*(v_1 - (1 - \alpha)R) - v_2^*v_1 + (1 - \alpha)\frac{v_2^*}{2} + (v_1 - (1 - \alpha)\frac{b_1}{2})b_1 + \alpha(1 - b_1)b_1 \quad (2)$$

The first part of the second expression does not depend on the choice of b_1 and the second part is maximized when $b_1 = \frac{v_1 + \alpha}{1 + \alpha}$ (see the proof of proposition 1).

Now, we have to consider two cases :

- If $v_2^* \leq \frac{v_1 + \alpha}{1 + \alpha}$, $b_1 = \frac{v_1 + \alpha}{1 + \alpha}$ is the solution of our maximization program (the second derivative is negative in $\frac{v_1 + \alpha}{1 + \alpha}$).
- If $v_2^* > \frac{v_1 + \alpha}{1 + \alpha}$, then $\forall b_1 \geq v_2^*$, the derivative of the expression (2) is negative and the optimal bid for bidder 1 is v_2^* .

D Proof of proposition 3

Let us first eliminate corner solutions.

Suppose that $v_1^* = 0$. As bidder 1 bids at least v_2^* , bidder 2 is fully deterred and participates with probability zero. Then bidder 1 is strictly better off not participating if $v_1 < c + (1 - \alpha)R$. $v_1^* = 0$ cannot be part of an equilibrium. Besides, $v_2^* = 0$ cannot be part of an equilibrium since participating to the takeover is costly.

Suppose that $v_1^* = 1$, then bidder 2's best response is $v_2^* = c + R$. For bidder 1 not to participate with valuation $v_1 < 1$, it must be the case that she does not derive a strictly positive profit if she participates when $v_1 = 1$. Then

$$-c + (c + R)(1 - (1 - \alpha)R) + (1 - c - R)(1 - (1 - \alpha)\frac{1 + c + R}{2}) \leq 0 \quad (3)$$

equivalent to

$$-R^2 + 1 - 2c + c^2 + \alpha + \alpha R - \alpha c^2 \leq 0 \quad (4)$$

$\alpha + \alpha R - \alpha c^2 > 0$ and $-R^2 + 1 - 2c + c^2 = (1 - c - R)(1 - c + R) > 0$. As a result, the inequation (4) cannot be verified and $(v_1^*, v_2^*) = (1, c + R)$ cannot be an equilibrium.

Then any equilibrium must satisfy one of the following conditions:

$$- 0 < v_1^* < 1 \text{ and } v_2^* = 1,$$

- $0 < v_1^*, v_2^* < 1$ and $v_2^* > \frac{v_1^* + \alpha}{1 + \alpha}$
- $0 < v_1^*, v_2^* < 1$ and $v_2^* \leq \frac{v_1^* + \alpha}{1 + \alpha}$

which we study separately.

- $0 < v_1^* < 1$ and $v_2^* = 1$

Necessary and sufficient conditions for such a (v_1^*, v_2^*) to be an equilibrium are as follows. Bidder 1 is indifferent between participating and not participating when her valuation is v_1^* . Bidder 2, if his valuation is 1, is not strictly better off participating than non participating²⁶. These conditions are equivalent to

$$v_1^* = c + (1 - \alpha)R \quad \text{and} \quad -c + v_1^*(1 - R) \leq 0$$

Replacing, in the second equation, v_1^* by its expression in the first equation, we obtain

$$v_1^* = c + (1 - \alpha)R \quad \text{and} \quad (1 - \alpha)(1 - R) \leq c$$

- $0 < v_1^*, v_2^* < 1$ and $v_2^* > \frac{v_1^* + \alpha}{1 + \alpha}$

A sufficient and necessary condition for such a (v_1^*, v_2^*) to be an equilibrium is that for $i = 1, 2$, bidder i be indifferent between participating and not participating if $v_i = v_i^*$. This is equivalent to the following conditions,²⁷ for bidder 1 and bidder 2 respectively

$$v_2^*(v_1^* - (1 - \alpha)R) + \alpha(1 - v_2^*)v_2^* - c = \alpha(1 - v_2^*)R \quad (5)$$

$$v_1^*(v_2^* - R) - c = 0 \quad (6)$$

Formula (5) is equivalent to

$$v_2^*(v_1^* - (1 - \alpha)R) + \alpha(v_2^* - R)(1 - v_2^*) - c = 0$$

- $0 < v_1^*, v_2^* < 1$ and $v_2^* \leq \frac{v_1^* + \alpha}{1 + \alpha}$

We can apply the arguments we have just mentioned. As $v_2^* \leq \frac{v_1^* + \alpha}{1 + \alpha}$, in v_1^* , bidder 1 does not bid v_2^* . Then, the equivalent of expressions (6) and (5) are respectively:

$$v_1^*(v_2^* - R) - c = 0$$

and :

$$\begin{aligned} & v_2^*(v_1^* - (1 - \alpha)R) + \left(\frac{v_1^* + \alpha}{1 + \alpha} - v_2^*\right)(v_1^* - \left(\frac{1 - \alpha}{2}\right)\left(\frac{v_1^* + \alpha}{1 + \alpha} + v_2^*\right)) + \alpha\left(1 - \frac{v_1^* + \alpha}{1 + \alpha}\right)\frac{v_1^* + \alpha}{1 + \alpha} - c \\ = & \alpha(1 - v_2^*)R \end{aligned}$$

²⁶And from lemma 5.1, we can derive that at the equilibrium, bidder 1, whenever he participates, bids 1 so that bidder 2 makes a profit only when bidder 1 does not participate.

²⁷From lemma 1, we know the bidding function of bidder 1 and 2 if they participate.

The second equation is equivalent to

$$v_2^*(2\alpha R - R) + \frac{1 - \alpha}{2}(v_2^*)^2 + \frac{(v_1^* + \alpha)^2}{2 + 2\alpha} - \alpha R = c$$

Q.E.D.

References

- [1] Betton, S and Eckbo, B E (2000) “Toeholds, Bid-Jumps and Expected Payoffs in Takeovers”, *Review of Financial Studies*, 13, pp841-882.
- [2] Berkovitch, E and Narayanan, M P (1993) “Motives for Takeovers: An Empirical Investigation”, *Journal of Financial and Quantitative Analysis*, 28, pp347-362.
- [3] Bradley, M, Desai, A and Kim, E H (1988) “Synergistic Gains from Corporate Acquisitions and Their Division Between the Stockholders of Target and Acquiring Firms”, *Journal of Financial Economics*, 21, pp3-40.
- [4] Bulow, J, Huang, M and Klemperer, P (1999) “Toeholds and Takeovers”, *Journal of Political Economy*, 107, pp427-454.
- [5] Burkart, M (1995) “Initial Shareholding and Overbidding in Takeover Contests”, *Journal of Finance*, 50, pp1491-1515.
- [6] Choi, D (1991), “Toeholds acquisitions , Shareholder Wealth, and the Market for Corporate Control”, *Journal of Financial and Quantitative Analysis*, 26, pp391-408.
- [7] Chowdhry, B and Jegadeesh, N (1994) “Pre-Tender Offer Share Acquisition Strategy in Takeovers”, *Journal of Financial and Quantitative Analysis*, 29, pp117-129.
- [8] Eckbo, B E and Langohr, H (1989) “Information Disclosure, Method of Payment and Takeover Premiums: Public and Private Tender Offers in France”, *Journal of Financial Economics*, 24, pp363-403.
- [9] Engelbrecht-Wiggans, Richard (1994) “Auctions with Price-Proportional Benefits to Bidders”, *Games and Economic Behavior*, 6, pp339-346.
- [10] Ettinger, D (2002) “Auctions and shareholdings”, working papers Ceras.
- [11] Franks, J R and Harris, R S (1989) “Shareholder Wealth Effects of Corporate Takeovers”, *Journal of Financial Economics*, 23, pp225-249.
- [12] Goergen, M and Reneboog, L (2002) “Shareolder Wealth Effects of European Domestic and Cross-border Takeover Bids”, Working paper 2002-50, Center, Tilburg University.
- [13] Grossman S J and Hart, O D (1980) “Takeover Bids, the Free-Rider Problem and the Theory of Corporation”, *Bell J. of Econ.*, 11, pp42-64.
- [14] Gupta, A, LeCompte, R and Misra, L (1997) “Acquisitions of Solvent Thrifts: Wealth Effects and Managerial Motivations”, *Journal of Banking and Finance*, 21, pp1431-1450.

- [15] Hirshleifer, D (1995) “Mergers and Acquisitions: Strategic and Informational Issues” in Finance, edited by R A Jarrow, V Maksimovic and W T Ziemba. 3d ed Handbook in Operations Research and Management Science, vol 9, Amsterdam, North-Holland.
- [16] Hirshleifer, D and Titman, S (1990) “Share Tendering Strategies and the Success of Hostile Takeover Bids”, *Journal of Political Economy*, 98, pp295-324.
- [17] Jarrell, G A and Poulsen, A B (1989) “Stock Trading before the Announcement of Tender Offers: Insider Trading or Market Manipulation?”, *Journal of Law, Economics and Organization*, 5, pp225-248
- [18] Jackson, M, Simon L K, Swinkels J M and Zame W R (2002) “Equilibrium, Communication, Sharing Rules in Discontinuous Games of Incomplete Information”, *Econometrica*, 70, pp1711-1740.
- [19] Maasland, E and Onderstal, S (2002) “Auctions with Financial Externalities”, Working paper 2002-22, Center, Tilburg University.
- [20] Mikelson, W H and Ruback, R S (1985), “An Empirical Analysis of the Interfirm Investment Process”, *Journal of Financial Economics*, 14, pp523-553.
- [21] Shleifer, A and Vishny, R W (1986) “Large Shareholders and Corporate Control”, *Journal of Political Economy*, 94, pp223-249.
- [22] Singh, R (1998) “Takeover Bidding with Toeholds: the Case of the Owner’s Curse”, *Review of Financial Studies*, 11, pp679-704.
- [23] Stulz, R M (1988) “Managerial Control of Voting Rights: Financing policies and the Market for Corporate Control”, *J. of Financial Econ.*, 20, pp25-54.
- [24] Stulz, R M, Walkling, R A and Song, M H (1990) “The Distribution of Target Ownership and the Division of Gains in Successful Takeovers”, *Journal of Finance*, 45, pp817-833.
- [25] Walkling, R A (1985) Predicting Tender Offer Success: A Logistic Analysis”, *Journal of Finance and Quantitative Analysis*, 20, pp461-478.

Graphics

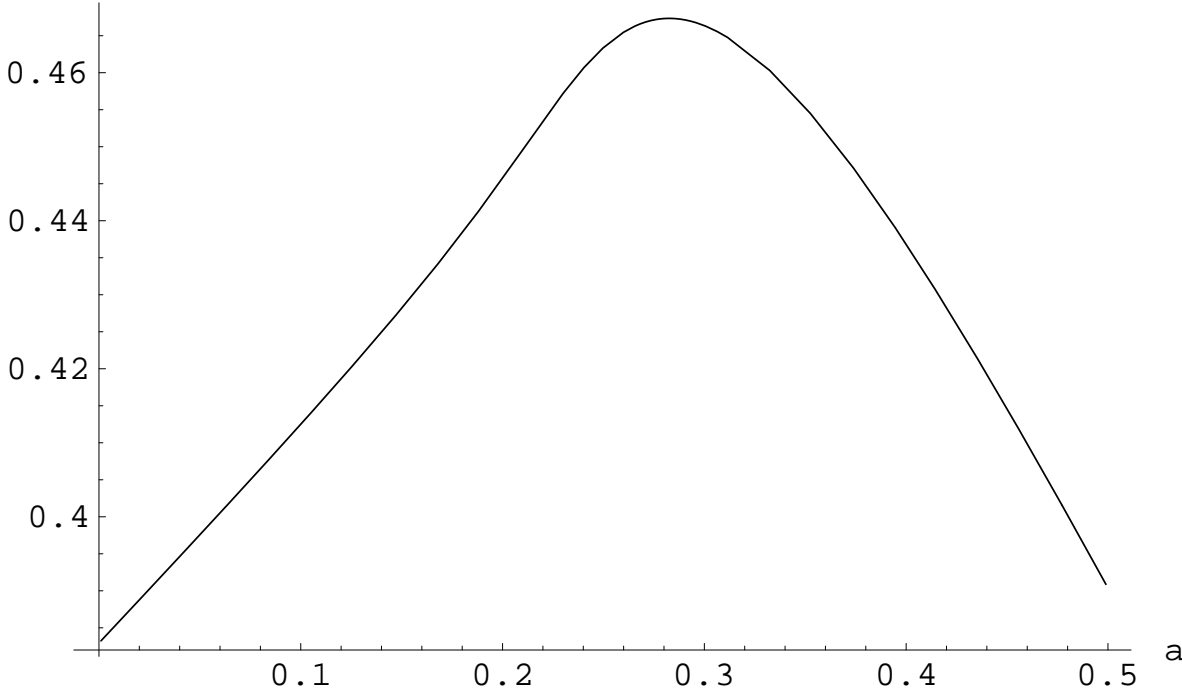


Figure 1: Shareholders expected revenue depending α , for $c = 0.02$ and $R = 0.15$.

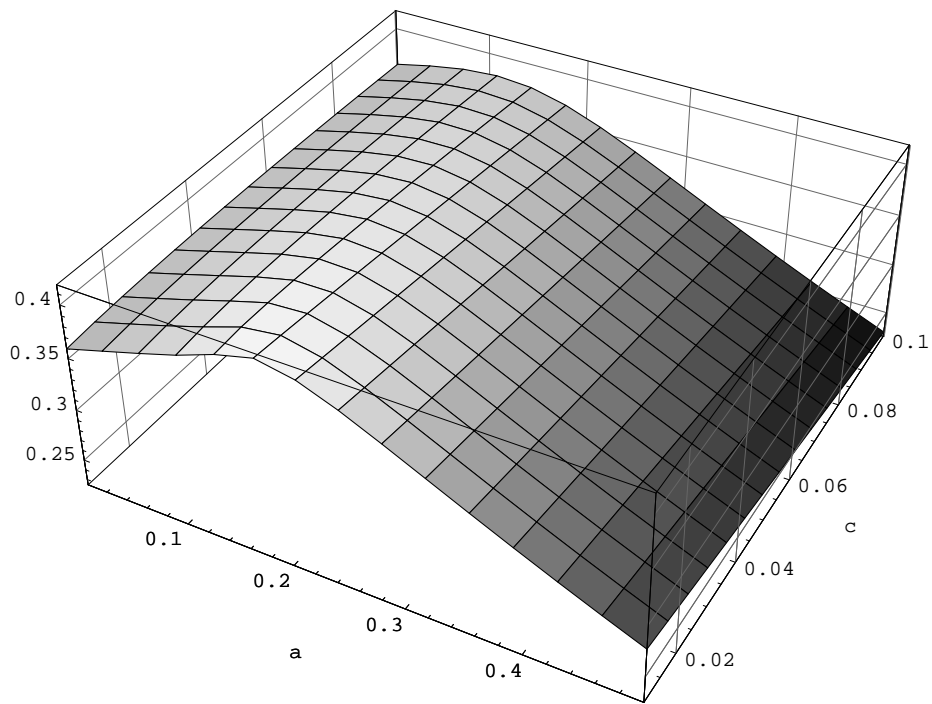


Figure 2: Shareholders expected revenue depending α and c , for $R = 0.1$.

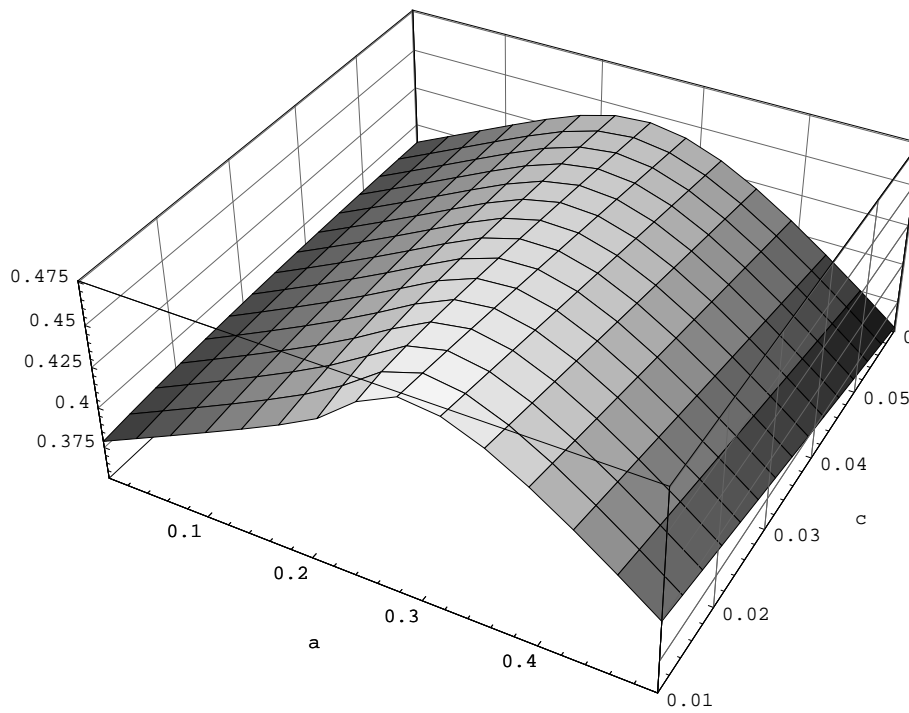


Figure 3: Shareholders expected revenue depending α and c , for $R = 0.15$.

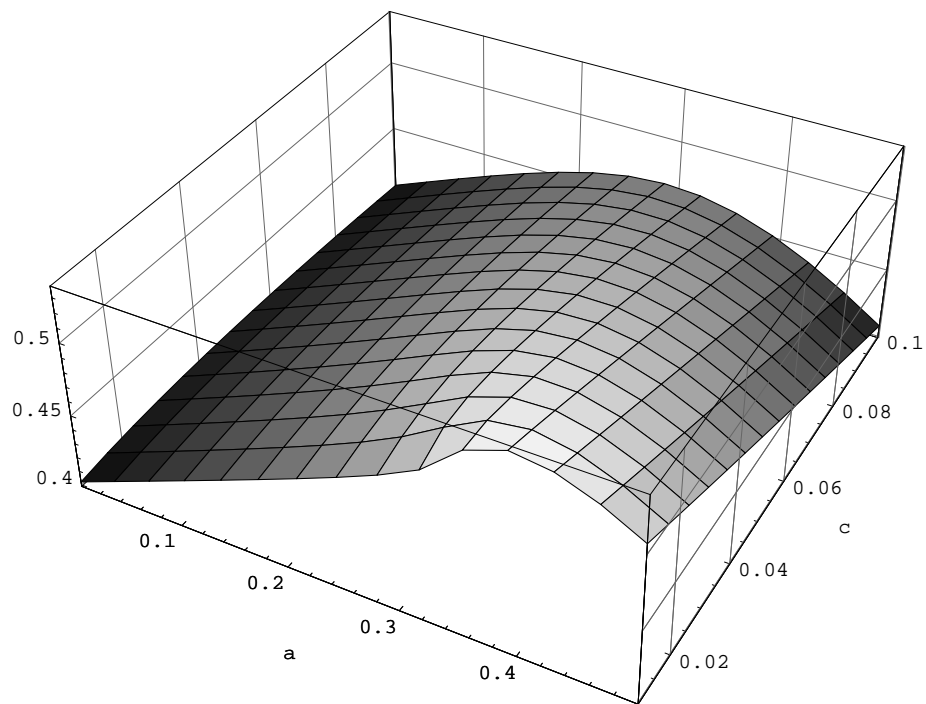


Figure 4: Shareholders expected revenue depending α and c , for $R = 0.2$.

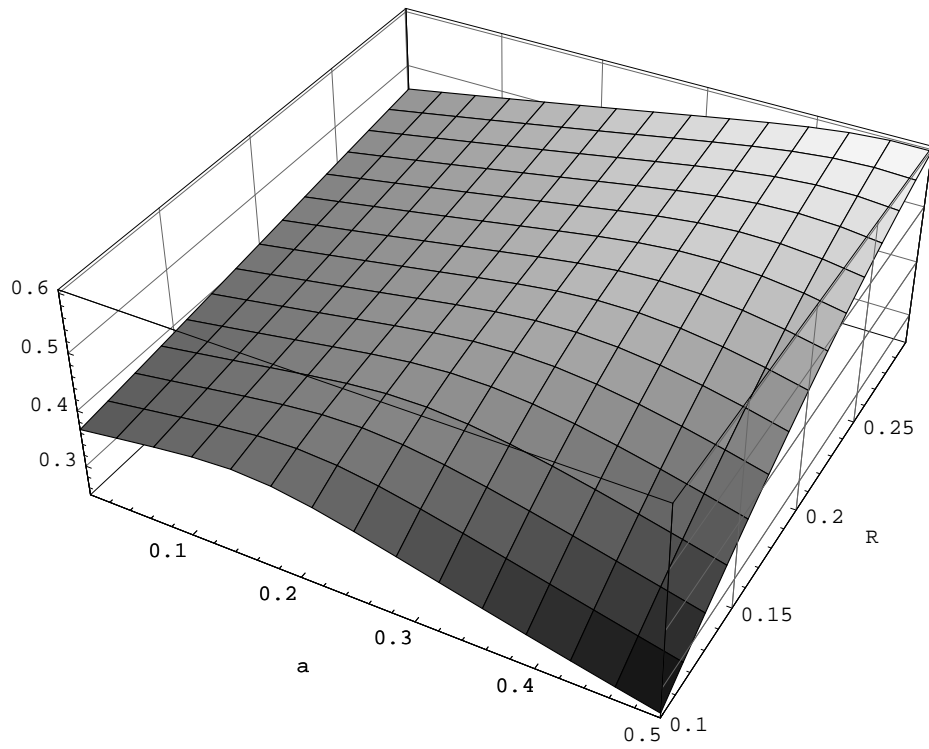


Figure 5: Shareholders expected revenue depending α and R , for $c = 0.05$.