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SUSTAINABILITY IN A MULTIPRODUCT AND MULTIPLE AGENT CONTESTABLE MARKET

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ABSTRACT

We prove that a natural monopoly can set subsidy free pricing and sustainable pricing schedules in general economic environment. The setting is a multiproduct and multiple agent contestable market where demands are elastic and where rivals can enter the sub-markets composed by a set of the products line and a set of agents. Our results suggest that the existence results of the extant literature admit analogues even in an environment where rivals have enlarged possibilities to enter the market and where demands react to prices. The approach makes use of cooperative games to deduce the main results under conditions of fair sharing cost, threshold in the consumption and regularity of the profit function.

Keywords: cooperative game, existence result, natural monopoly, subsidy free pricing, sustainability.

Journal of Economic Literature Classification Numbers: C71, L11, L12.

I. INTRODUCTION

The paper considers a mutiproduct and multiple agent contestable market. The environment is described by an incumbent firm who is in position of natural monopolist, and free entry on the market which is composed by several customer classes, identified by their multidimensional elastic demand functions. Note however that we consider all along the paper an environment where prices are uniform, though personalized prices are possible.

Our economic environment gathers several common typologies of markets that can be found in the contestable market theory. That literature considers usually a restricted environment that

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excludes either elastic demand functions, multiple agents, multidimensional demand functions, or price discrimination. These restrictions can be justified by considering for instance that firms observe a global demand function on the market, have no ability to set personalized prices, or by considering that the natural monopolist provides primary goods that are not sensitive to price changes.

Whatever the specifications of the contestable market, a well-known line of analysis addresses the possibility of setting pricing schedules that preclude entry on the incumbent market. This stable situation is precisely captured by the notion of sustainability: it guarantees a non negative profit to the incumbent and no rival has an incentive to enter the market, given the incumbent pricing schedule. Sustainability yields zero-profit pricing schedules. In addition, it has been shown, under certain conditions, that the sustainable pricing also exhibits other optimality properties. If there exists such a sustainable pricing, the analysis suggests therefore that, in term of public policy, the regulator needs only to guarantee the free entry on the market to obtain a desirable pricing, since the threat of entry forces the monopolist to set a sustainable pricing. This is the essence of the theory of contestable markets (we refer the reader to Baumol et al. (1988) or Sharkey (1984) for a more exhaustive presentation of the theory).

A preliminary requirement to sustainability is captured by the notion of subsidy free pricing. A pricing is said to be subsidy free if the incumbent achieves the budget balance condition and the expenditures of any sub-market is lower than the cost induced by that sub-market. Subsidy free pricing has become of great importance in regulation issues and is a widespread criterion prescribed to monopolists. For instance in the past years, the increasing competition between american and european industries has led regulators to put a very careful attention on irregular or unfair pricing practice, including internal cross subsidization. In addition, the subsidy free pricing is a necessary condition for the existence of a sustainable pricing, therefore both pricing schedules are related, at least from an analytic point of view.

The paper provides existence results for both subsidy free pricing, and sustainable pricing schedules in a general framework. There is already an important literature that precisely deals with the existence of either subsidy free pricing or sustainable pricing schedules. This stream of literature on contestable markets can be originated in the seminal contributions of Baumol, Bailey and Willig (1977), Faulhaber (1975) and Panzar and Willig (1977), other existence results also include the contributions of Bendali, Mailfert and Quillot (2000), Mirman, Tauman and Zang (1985), Sharkey (1981), Spulber(1984), Ten Raa (1983,1984). Despite that rich literature dealing with the existence issue, our analysis differs from the existing analysis in several directions.

First, we allow here for complete elastic demands in the extent that consumption is above a threshold, and for price discrimination. The model differs thus from the initial analysis of Baumol et al. (1977, 1988) on multiproduct monopoly where, first, the demands are taken as inelastic, and second, no price discrimination is allowed. In the first case the existence of subsidy free pricing turns out to be reduced to a simple core non-emptiness result of a cooperative game. Whereas in the second case, Faulhaber (1975) has provided a counter-example to show that subsidy free pricing needs not to exist.

Second, in our contestable market model, potential sub-markets are enlarged to subsets of the product line and/or subsets of the agents. Thus, the formulation of contestable markets is sufficiently general to take into account, as particular cases, contestable markets where there is: no discrimination and a global demand (one agent) as in Baumol et al. (1988) and Faulhaber and Levinson (1981); no price discrimination and a full product line supply as in Faulhaber and Levinson (1981); price discrimination and single output firms as in Bendali et al. (2000).

In order to state the results, a so called fair sharing property of the cost function will play a central role. The property ensures that for any outputs levels, there exists a sharing of the cost such that no sub-market will be charged more than the cost of serving that sub-market alone. Roughly, the property states a subsidy free condition for any constant demand functions. Then, under a threshold condition on the demand functions, we establish that there exists a subsidy free pricing schedule if the cost function is fair sharing. To obtain the existence a sustainable pricing schedule, we extend the standard strategy used in the literature. Briefly, there exists a sustainable pricing if the market satisfies a regularity condition that states that the profit remains positive above the zero-profit curve on a given domain.

The main contribution of the paper is to relax the restrictions of the standard model (except the uniform price assumption) and establish existence results for sustainable pricing in that context. Our results suggest that the main conclusions of the theory of contestable markets are similar in a context where the possibilities of entrants are enlarged and demands are elastic, though our results are not directly comparable to the previous ones.

Our results differ however from the conclusions of the analysis that precisely consider a non-uniform price environment. In that framework, the existing results address mainly two aspects: the "size of the set of sustainable pricing schedules and their existence. These results are somehow more paradoxical. Sharkey and Sibley (1993) consider a contestable market model where the incumbent and the entrants do not fully observe the demands. In that asymmetric information setting, they show that the set of sustainable pricing schedules can be very large since, under structural assumptions, any Pareto optimal two-part tariff schedule can be sustainable.[1] This result gives room for redistributive policies and question therefore the role of regulators. Our results also differ from those of Perry (1984) that shows that monopolists can set a pricing schedule that yields positive profit and prevents entry, under the possibility of multiple price strategies. Thus the existence of zero profit pricing schedule as a result of entrant's threat is invalidated in that model where the incumbent's inability to react to entrants is relaxed.

Finally, it worths noticing that the paper follows the stream of cooperative game modeling of subsidy free pricing. Faulhaber (1975) has first demonstrated existence results for subsidy free pricing within this formalism of TU game (transferable utility game), by considering cost games. Here, the treatment is going further and consists of the construction of a parameterized cost game, where the game is precisely parameterized by prices. The subsidy free pricing is restated as an equilibrium-core of the parameterized game.[2] The equilibrium-core provides thus a solid ground for cooperative game modeling of contestable markets.

In Section 2, we define the model of contestable market with discrimination and the two concepts of pricing schedules. In Section 3, we state the existence results, Theorem 4 and Theorem 5, and before that we show how the pricing existence problem can be deduced from the existence of an equilibrium-core allocation of a well defined cooperative game, Theorem 3. A final remark on the possibility of refined entrants' strategies concludes the paper. The proofs are all gathered in Appendix.

II. THE MODEL

We consider a multiproduct and multiple agent contestable market with price discrimination.[3]

- **(C1)** There are $\ell < \infty$ goods. Goods are denoted by script $b \in L := \{1, ..., \ell\}$. The commodities space is $\mathbb{R}^L_+$. There are $n < \infty$ customer classes (agents in the remainder). Agents are denoted by script $a \in N := \{1, ..., n\}$.
- **(C2)** Price space is $\mathbb{R}^{L\times N}_+$, a typical element is $(p_a)_{a\in N} \in \mathbb{R}^{L\times N}_+$. Each agent $a \in N$ is identified by a positive and continuous demand function $D_a : \mathbb{R}^{L\times N}_+ \to \mathbb{R}^L_{++}$

[1] See also Bernard and Wittwer (2004).

[2] The concept of equilibrium-core has been introduced in Bonnisseau and Iehlé (2007). It has been initially applied to economies where the outcomes depend on an exogenous environment.

[3] Let X be a finite set, we denote by $\mathbb{R}^X$ the Euclidean space whose components are indexed by the elements of X. For each $y, z \in \mathbb{R}^X$, we denote $y < z$ if $y_i \leq z_i$ for each $i \in X$ and $y \neq z$. For any set-valued mapping Γ from Θ into Ω, denote its graph by $\mathrm{Gr}\ \Gamma := \{(x, y) \in \Theta \times \Omega \mid y \in \Gamma(x)\}$

whose maximum is reached in 0.[4]

(C3) Incumbent has productive techniques given by an increasing and continuous cost function $C : \mathbb{R}^L_+ \to \mathbb{R}_+$.

(C4) Potential sub-markets are elements of the set $\mathcal{M} = \{(A, B) \mid A \subset N$ and $B \subset L\}$. A typical sub-market is denoted $(A, B) \in \mathcal{M}$ where A is a group of agents and B is a subset of the product line. See Figure 1.

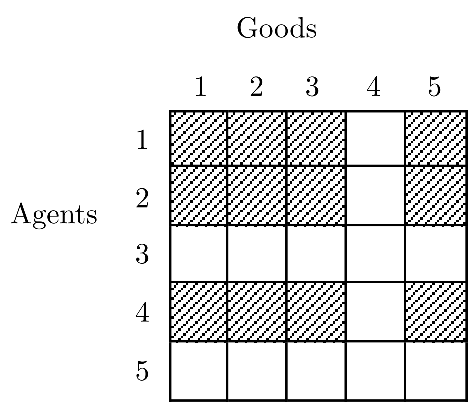

Figure 1: Example of a sub-market (cross-hatched boxes)

In that contestable market, there exists an incumbent that provides the entire product line L to each of the agents $a \in N$ with a technology described by the cost function C. For each $d \in \mathbb{R}^L_+$, $C(d)$ is the induced cost to produce the commodity bundle d. The incumbent is supposed to be full supplier, that is, it provides fully the demand induced by the pricing (set by the incumbent). Therefore, it always holds that $d_a := D_a(p)$ where d_a is the quantity of outputs produced for agent a, and, $D_a(p) \in \mathbb{R}^L_{++}$ is the demand of agent a given the pricing p.[5]

We are now in position to define the standard notion of subsidy free pricing. The main difference with the existing literature is that in addition to the possibility of price discrimination the no subsidy condition (2.) must hold on the extended sub-markets, as defined in (C4) that

[4]Note that demands are inter-dependent across goods but also between agents since they are defined on the whole price space.

[5]See Mirman et al. (1985) or Baumol et al. (1988) for partial supplier analysis, i.e. markets where the incumbent can provide $d_a \leq D_a(p)$.

capture demands of subsets of agents on the restricted product lines.[6]

Definition 1 (Subsidy free) *A pricing schedule* $(p_a)_{a \in N} \in \mathbb{R}_+^{L \times N}$ *is subsidy free if:*

1. $\sum_{a \in N} p_a \cdot D_a(p) = C(\sum_{a \in N} D_a(p))$.

2. $\sum_{a \in A} p_a \cdot D_a(p)^B \leq C(\sum_{a \in A} D_a(p)^B)$ *for each* $(A, B) \in \mathcal{M}$.

Where $D_a(p)^B \in \mathbb{R}_+^L$ *is the demand vector of a restricted to the product line* B*:* $D_a(p)_b^B = D_a(p)_b$ *if* $b \in B$ *and* $D_a(p)_b^B = 0$ *otherwise.*

From the point of view of the incumbent, the subsidy free conditon (2.) and the budget balance condition (1) imply altogether that there is no sub-market achieving a positive profit and subsidizing another that possibly face losses. Subsidy free pricing is somehow an internal criterion of stability since no sub-market is entitled to claim a reward for its participation. As such, that class of pricing can stand for a first criterion to be prescribed by a regulator to a natural monopoly. In addition, it is usually viewed as a first step toward the sustainable pricing that gathers additional requirements and fulfills interesting optimality properties.[7] Let us turn precisely to the notion of sustainable pricing which is the central solution in the framework of contestable markets. Roughly, the solution captures situations where there exists a pricing that guarantees normal profit for the incumbent and does not yield an incentive for potential entrant. Given our context of multiproduct and multiple agent market, the question is even more relevant since the entrant has also the possibility to enter any subset of the market.

Definition 2 (Sustainability) *A pricing schedule* $(p_a)_{a \in N} \in \mathbb{R}_+^{L \times N}$ *is sustainable if:*

1. $\sum_{a \in N} p_a \cdot D_a(p) = C(\sum_{a \in N} D_a(p))$.

2. *There is no sub-market* $(A, B) \in \mathcal{M}$ *and a pricingschedule* $(p'_a)_{a \in A} \in \mathbb{R}_+^{B \times A}$ *such that*

$$\sum_{a \in A} p'_a \cdot D_a(p|p')^B = C(\sum_{a \in A} D_a(p|p')^B) \text{ for each } (A, B) \in \mathcal{M}.$$

and

$$p'_a < p_a \text{ for each } a \in A$$

[6] In Faulhaber and Levinson (1981) for instance there is no price discrimination and the full product line is supplied. In that case the subsidy free pricing is given by: $p \in \mathbb{R}_+^L$ satisfying:

$$\begin{cases} p \cdot \sum_{a \in N} D_a(p) = C(\sum_{a \in N} D_a(p)) \\ p \cdot \sum_{a \in A} D_a(p) \leq C(\sum_{a \in A} D_a(p)) \text{ for each } A \in \mathcal{N}. \end{cases}$$

Where $\mathcal{N}$ is the set of non-empty subsets of N.

[7] We refer the reader to the numerous literature, see for instance Baumol et al. (1988) and Sharkey (1984) for a presentation of the main insights of the sustainability notion.

The first condition guarantees the budget balance. The second formalizes the stability concept: no firm can enter a sub-market (A, B), by setting a preferred pricing on the product line B for each agent in A and achieving the budget balance condition.

The mechanism of entry in Definition 2 deserves further comments. At first glance, it is an admissible criterion to consider that at the new price $(p|p')$, where $p'_a < p_a$ for each $a \in A$, agents in A are willing to be supplied by the entrant for the product line B, and no more by the incumbent. This is implied implicitly by the incumbent's inability to react to entry in a contestable market, at least in the short run, i.e. the agent in A are still supplied at price p for the remaining goods when the rival firm enters the market. This criterion for preferred prices is however disputable. Indeed we consider for a matter of generality demand functions defined over the whole price space, it means that the demands take into account possible externalities in agent's behavior such as altruism, envy or social norms. Thus, it might be the case that a lower personalized price makes agents worse off if they incorporate altruism in their preferences. Taking into account such a possibility goes beyond the scope of the paper, thus we adopt nevertheless the rough criterion that a lower price is preferred. Otherwise, the reader can still consider the particular class of demand functions that are independent of the prices of other agents: $D_i(p_i, p_{-i}) = D_i(p_i, p'_{-i})$ for each p_{-i} and p'_{-i}. For this particular class of demand functions the notion of entry that we consider in the current paper is no more ambiguous.

Before turning to the existence problem, one can state a simple proposition that relates the two concepts and we define the notion of fair sharing cost function.

Proposition 1 *Any sustainable pricing is subsidy free.*

The next condition is crucial to obtain the existence results of the paper.

Definition 3 *The cost function C is fair sharing if for all $(d_a)_{a \in N} \in \mathbb{R}^{L \times N}_{++}$, there exists $(q_a)_{a \in N} \in \mathbb{R}^{L \times N}_{+}$ such that:*

$$\begin{cases} \sum_{(a,b) \in N \times L} q_{ab} = C(\sum_{a \in N} d_a) \\ \sum_{(a,b) \in A \times B} q_{ab} \leq C(\sum_{a \in A} d_a^B) \text{ for all } (A, B) \in \mathcal{M}. \end{cases} \tag{1}$$

Where d_a^B is the vector in $\mathbb{R}^L_+$ satisfying $d_{ab}^B = d_{ab}$ if $b \in B$ and $d_{ab}^B = 0$ otherwise.

The above condition states, for any structure of outputs $(d_a)_{a \in N}$, the existence of a sharing $(q_a)_{a \in N}$ among all the agents such that the amount paid in each sub-market cannot cover the cost induced by the outputs in the submarket. Roughly the condition is no more than a statement of subsidy free for inelastic demands.

We will obtain the existence of subsidy free pricing within an environment of elastic demands using that fair sharing property. At first glance, the condition seems to be strong but it is a weak requirement compared to the notion of subsidy free pricing. Indeed, in the above definition the shares of the agents $(q_a)_{a\in A}$ induced by the level of outputs do not necessarily coincide with the exact level of their own expenditures $(p_a \cdot d_a)_{a\in A}$,, where $d_a = D_a(p)$, as it is precisely the case in the definition of subsidy free pricing.

Note finally that the above condition is also a restatement of a standard condition used in the literature in a more simple setting where either the demands are not elastic or there is a single agent on the market. In the initial analysis of Baumol et al. (1988), such a property has been thoroughly studied in a more simple framework, the authors show especially that the property is satisfied whenever the cost function exhibits weak complementarities. Less specifically, the property can be met as soon as the costs exhibit a form of increasing returns. In Faulhaber and Levinson (1981), the condition is called stand alone test and used to establish the existence of subsidy free pricing in a contestable market with one single agent and independent demands.

III. GAME MODELING AND EXISTENCE RESULTS

Using cooperative game modeling, this section establishes the existence of subsidy free pricing for general elastic demands. It shall be shown that subsidy free pricing can be restated as allocations of equilibrium-core of an associated parameterized cost game.

Let us turn to the cooperative game modeling of the contestable market and define a TU cooperative game. Consider a finite set of players X, $\mathcal{X}$ the set of non-empty subsets of X. The characteristic function is $v : \mathcal{X} \to \mathbb{R}$. Let $(v_S, S \in \mathcal{X})$ denote a TU game.

An imputation in S is a vector $q = (q_i)_{i\in S}$ such that $\sum_{i\in S} q_i \leq v_S$. Consider an imputation q in N, it is dominated by an imputation q' in S if $q'_i > q_i$ for each $i \in S$. The core of a game is the set of imputations in N that are undominated in any coalition. It is well known that the core can be restated as the set of imputations q in X such that:

$$\sum_{i\in S} q_i \geq v_S \text{ for each S } \in \mathcal{X} \tag{2}$$

A family of coalitions $\mathcal{B} \subset \mathcal{X}$ is balanced if there exists $\lambda_S \in \mathbb{R}_+$ for each $S \in \mathcal{B}$ such that: $\sum_{S\in\mathcal{B}} \lambda_S \mathbf{1}^S = \mathbf{1}^X$.[8] A TU game is said to be balanced if for any balanced family of coalitions $\mathcal{B}$ with balancing coefficients $(\lambda_S)_{S\in\mathcal{B}}$, it holds that: $\sum_{S\in\mathcal{B}} \lambda_S v_S \leq v_N$.

We need to consider now a more general model of TU games called parameterized TU games.[9] The parameter set is Θ, the characteristic function is now defined over Θ, $v_S : \Theta \to \mathbb{R}$ for each

[8] $\mathbf{1}^S$ is the vector with coordinates equal to 1 in S and equal to 0 outside S.

[9] The reader is referred to Bonnisseau and Iehlé (2007) for further details.

coalition S, and an equilibrium condition is represented by a mapping $G : \text{Gr } V \to \Theta$, where $V : \Theta \to \mathbb{R}^X$ is a set-valued mapping such that, for $\theta \in \Theta$, $V(\theta) := \{q \in \mathbb{R}^X \mid \sum_{i \in X} q_i = v_X(\theta),\ q_i \geq v_{\{i\}}(\theta),\ \forall i \in X\}$.[10] Let $((v_S)_{S\in\mathcal{X}}, \Theta, G)$ denote a parameterized TU game.

Definition 4 *Let $((v_S)_{S\in\mathcal{X}}, \Theta, G)$ be a parameterized game. An equilibrium-core allocation is a pair $(\theta, q) \in \Theta \times \mathbb{R}^X$ such that q belongs to the core of the game $(v_S(\theta), S \in \mathcal{X})$ and $\theta = G(\theta, q)$.*

We can state now a weak version of the abstract result of Bonnisseau and Iehlé (2007). The original version holds in NTU games.

Theorem 1 *(Bonnisseau-Iehlé (2007)) Let $((v_S)_{S\in\mathcal{X}}, \Theta, G)$ be a parameterized TU game such that, for each $\theta \in \Theta$, the TU game $(v_S(\theta), S \in \mathcal{X})$ is balanced, and: (i) Θ is a non-empty, convex, compact subset of $\mathbb{R}^X$ and G is continuous on Gr V; (ii) For each $S \in \mathcal{X}$, v_S is continuous on Θ.*

Then there exists an equilibrium-core allocation.

Let us construct now explicitly the parameterized game $((v_{S)})_{(S)\in\mathcal{X}}, \Theta, G)$ derived from the initial contestable markets. Let $X = N \times L$ be the set of players. Let Θ be the $n\ell$-dimensional cube with vertex $[0; \frac{C(D(0))}{\epsilon}]$ for each $(a, b) \in N \times L$. For each $S \in \mathcal{X}$ and $\theta \in \Theta$ we define $v_S(\theta) = -C(\sum_{a\in A} D_a(\theta)^B)$ if S is such that there exist $A \in N$ and $B \in L$ such that $(a, b) \in S$ iff $a \in A$ and $b \in B$. Otherwise, set $v_S(\theta) = 0$. And for each $(\theta, q) \in \text{Gr } V$,

$$G(\theta, q) = -(\frac{q_{ab}}{D_a(\theta)_b})_{(a,b)\in N\times L}$$

The cooperative game approach of the pricing problem can be now summarized in the two following theorems. We show how the subsidy free pricing existence problem can be deduced from Theorem 1.

Theorem 2 *Let (θ, q) be an equilibrium-core allocation of the parameterized game $((v_S)_{S\in\mathcal{X}}, \Theta, G)$ defined above then θ is a subsidy free pricing in the initial contestable market.*

We impose an additional assumption on the demand functions:

(D) There exists a minimal threshold $\epsilon > 0$ in the consumption, i.e. $D_a \geq \epsilon\mathbf{1}$ for each $a \in N$.[11]

Theorem 3 *Under (D), any contestable market with a fair sharing cost function admits a parameterized game representation $((v_S)_{S\in\mathcal{X}}, \Theta, G)$ that meets the assumptions of Theorem 1.*

[10] Given $\theta \in \Theta$, the set $V(\theta)$ is actually called the set of individual rational payoffs in cooperative game theory.

[11] In Ten Raa (1983), the author also uses the condition of threshold in the consumption.

We can deduce then the main result of the paper from Theorems $1, 2, 3$:

Theorem 4 *Under (D), any contestable market with fair sharing property admits a subsidy free pricing $p \in \Theta$.*

To obtain the existence of a sustainable pricing on the contestable market, one needs an additional requirement on the profit function.

For any sub-market $(A, B) \in \mathcal{M}$, the profit function is given by:

$$\Pi_{(A,B)} : p \longrightarrow \sum_{a \in A} p_a \cdot D_a(p)^B - C(\sum_{a \in A} D_a(p)^B)$$

The market is said to be regular if it satisfies the following condition:

(R) $\Theta \cap \{\Pi_{(A,B)} > 0\} = \Theta \cap \{\{\Pi_{(A,B)} = 0\} + \mathbb{R}^{L \times N}_{++}\}$,'
where Θ is the cube in $\mathbb{R}^{L \times N}$ defined after Condition (D).

Condition (R) simply says that the profit remains positive within Θ above the zero profit level (in the price space). This type of condition is usual in the literature of contestable markets, though we do not assume the profit to be non decreasing, as it is the case in Mirman et al. (1984). Our requirement is weaker since it is also compatible with a bell curve of the profit function for instance. Nevertheless, our condition still exhibits the main drawback of this class of conditions on the profit functions, namely it involves altogether the cost function and the demand functions. The two sides of the market cannot be not considered any more independently.

We can now establish the last existence result on sustainability.

Theorem 5 *Under (D) and (R), any contestable market with a fair sharing cost function admits a sustainable pricing schedule.*

This strategy to deduce the existence of a sustainable pricing schedule from a subsidy free pricing schedule is rather standard. The idea behind the assumption is the following: if it is not true that Condition (R) does hold, it might be the case that there exists a sub-market such that at a lower pricing than the subsidy free pricing where profit is non positive, entry is sufficiently profitable to capture the associated demand. Roughly we want to avoid the pathological situations where the subsidy free pricing, besides the requirements on the sub-markets, is dominated by a pricing that leads to a non-negative profit. This pitfall is precisely excluded under (R). See Figure 2 for an illustration of Assumption (R).

IV. A CONCLUDING REMARK

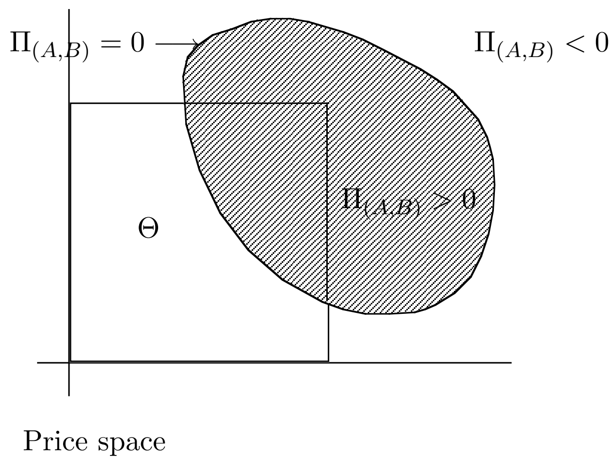

Figure 2: Assumption (R)

To conclude we point out that one can go a step further to describe the strategies of the entrant. In the present paper we have considered that any potential submarket for entrants was defined as a subset A of agents and a subset B of the product line. That is to say that the entrant commit to providing the same product line B to all agents in A. However a possible extension considers that the entrant can refine its supply by providing a specific product line $B(a)$ to each agent in A. This possibility is better described in Figure 3.

To take into account that possibility of extended submarkets, the assumption $(C4)$ that describes the contestable market should be replaced by the following:

(C5) Potential sub-markets are elements of the set $\mathcal{M}$ defined as the set of all nonempty subsets of $N \times L$. A typical sub-market is denoted $S \in \mathcal{M}$. Equivalently a submarket $S \in \mathcal{M}$ can be then described by a list $(a, (S(a))_{a \in S_N}$ where $S_N = \{a \in N \mid \exists b \in L, \text{ s.t. } (a,b) \in S\}$ is the set of agents that are supplied in the submarket S and $S(a) := \{b \in L \mid (a,b) \in S\}$ is the product line provided to agent a in the coalition S.

In the present paper, we gave favor the case where entrants commits to providing the same restricted product line to agents. Therefore we do not proceed to the analysis of the contestable

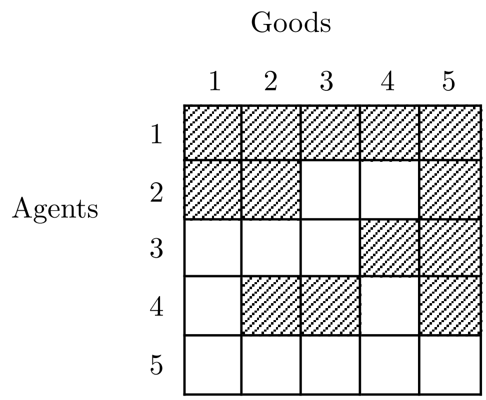

Figure 3: Example of a sub-market where each agent is supplied in a different product line

market setting associated to ($C5$). But it is an easy matter to check that the previous analysis can be easily extended to establish parallel results in the spirit of Theorem 4 and Theorem 5.

APPENDIX

Proof of Proposition 1. Suppose that condition (1) holds for some p and that there exists $(A, B) \in \mathcal{M}$, such that $\sum_{a \in A} p_a \cdot D_a(p)^B > C(\sum_{a \in A} D_a(p)^B)$. The profit of the firm on the submarket (A, B) is non positive at the price $(p|0_{(A,B)})$, where $0_{(A,B)}$ is the vector of $\mathbb{R}^{B \times A}$ whose components are equal to zero. From a continuity argument (the cost and demand function are continuous), there exists $p'' \in \mathbb{R}^{B \times A}$ such that $(p|p'') \in [(p|0_{(A,B)}), p[$ and such that $\sum_{a \in A} p''_a \cdot D_a(p|p'')^B = C(\sum_{a \in A} D_a(p|p'')^B)$. The new price p'' satisfies therefore Condition 2 of Definition 2, and it follows that the price system p cannot be sustainable since the sub-market (A, B) is potentially vulnerable against entrant. □

Proof of Theorem 2. Let (θ, q) be an equilibrium-core allocation of the parameterized game $((v_S)_{S \in \mathcal{X}}, \Theta, G)$. Then q belongs to the core of the TU game $(v_S(\theta), S \in \mathcal{X})$ and $\theta \in G(\theta, q)$.

From the first conclusion and restating the core requirements as in (2), one deduces that $\sum_{(a,b) \in N \times L} q_{ab} = -C(\sum_{a \in N} D_a(\theta))$ and $\sum_{(a,b) \in A \times B} q_{ab} \geq -C(\sum_{a \in A} D_a(\theta)^B)$ for each $(A, B) \in \mathcal{M}$.

From the second conclusion, one deduces that $\theta_{ab} D_a(\theta)_b = -q_{ab}$, for all $(a, b) \in N \times L$. Hence, one gets the result. □

Proof of Theorem 3. We show that the game $(v_S(\theta), S \in \mathcal{X})$ is balanced for each $\theta \in \Theta$, using fair sharing property. Indeed, let $\mathcal{B}$ be a family of balanced coalitions of $\mathcal{X}$ with balancing weights λ_S for each $S \in \mathcal{B}$. Then $\sum_{S\in\mathcal{B}} \lambda_S v_S(\theta) = -\sum_{(A,B)\in\mathcal{B}\cap\mathcal{K}} \lambda_{(A,B)} C(\sum_{a\in A} D_a(\theta)^B)$ from the construction of v and the fact $v_S = 0$ on the coalitions that do not belong to $\mathcal{K}$ and identifying without lost of generality the coalitions $S \in \mathcal{K}$ by (A, B). From (1), there exists $q \in \mathbb{R}_+^{L\times N}$ such that:

$$\begin{cases} \sum_{(a,b)\in N\times L} q_{ab} = C(\sum_{a\in N} D_a(\theta)) \\ \sum_{(a,b)\in A\times B} q_{ab} \leq C(\sum_{a\in A} D_a(\theta)^B) \text{ for all } (A,B) \in \mathcal{B}\cap\mathcal{K}. \end{cases}$$

We deduce that:

$-\sum_{(A,B)\in\mathcal{B}\cap\mathcal{K}} \lambda_{(A,B)} C(\sum_{a\in A} D_a(\theta)^B) \leq -\sum_{(A,B)\in\mathcal{B}} \lambda_{(A,B)} \sum_{(a,b)\in A\times B} q_{ab}$
$= -\sum_{(a,b)\in N\times L} q_{ab} \sum_{(A,B)\in\mathcal{B}, a\in A, b\in B} \lambda_{(A,B)} \leq -\sum_{(a,b)\in N\times L} q_{ab}1$
$= -C(\sum_{a\in N} D_a(\theta))$

The last inequality follows from the definition of the balancing weights while the last equality stems from the definition of the fair sharing cost function.

Hence, $\sum_{(A,B)\in\mathcal{B}} \lambda_{(A,B)} v_{(A,B)}(\theta) \leq v_{(N,L)}(\theta)$.

It remains to check now that $(i)-(ii)$ are fulfilled. For (i), the continuity of the mapping G is trivially satisfied. Now let us verify that G is well defined from Gr V to Θ. Given $(p, q) \in$ Gr V, clearly, from positivity of the demand functions, there exists θ such that $\theta_{ab} D_a(p)_b = -q_{ab}$ for all $(a, b) \in N \times L$. Furthermore since $(p, q) \in$ Gr V, one has $-q_{ab} \leq C(D_a(p)_b)$ for all $(a, b) \in N \times L$. Using $(C2 - 3)$ one also deduces that $C(D_a(p)_b) \leq C(D(0))$ and $\theta_{ab} D_a(p)_b \geq \theta_{ab}\epsilon$. Hence, one gets $\theta_{ab} \leq \frac{C(D(0))}{\epsilon}$ for each $(a, b) \in N \times L$. Thus, $\theta \in \Theta$ as was to be proved.

(ii) is satisfied from the continuity of the demand and cost functions. □

Proof of Theorem 5. The proof is obvious since it suffices to remark that if the following holds for some $p \in \Theta$: $\sum_{a\in A} p_a \cdot D_a(p)^B \leq C(\sum_{a\in A} D_a(p)^B)$, then under (R) it must be true that $\sum_{a\in A} p'_a \cdot D_a(p|p')^B < C(\sum_{a\in A} D_a(p|p')^B)$ for any $p' \in \mathbb{R}_+^{B\times A}$ such that $p'_a < p_a$ for each $a \in A$. From Theorem 4, we know that there exists at least one subsidy free pricing that belongs to Θ, therefore p is also sustainable from the above lines of arguments. □

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