

A Quantitative Study of the Replacement Problem in Frictional Economies*

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Abstract

The question of how technological change affects labor markets is a classical one in macroeconomics. A standard framework for addressing this question is the search/matching model with vintage capital and exogenous technical progress. Within this framework, it has been argued in the literature that the impact of new technology change on labor market outcomes can be qualitatively very different according to the mechanism through which the new technology enters the economy. In particular, it matters whether: (1) new capital replaces old capital by destroying the job and displacing the worker (Schumpeterian creative-destruction) or old capital can be “upgraded” to the frontier technology (Solowian upgrading); (2) firms make the technology adoption decision unilaterally (hold-up), or the investment decision is surplus-maximizing (efficient investment). Our main finding is that, for all parameter values that are quantitatively reasonable, the specific details of the model for how technology is introduced and who decides on investments does not matter for the equilibrium outcomes of our main variables: unemployment, wage inequality, and labor share. The intuition for this “equivalence result” is that these models will yield significantly different implications only if the matching process is very costly and time-consuming, but our calibration shows that this meeting friction is minor.

Keywords: Creative destruction, Hold-up, Inequality, Technical Change, Upgrading.

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1 Introduction

Technological change is gradual and a large part of technological change is embodied in capital equipment. New machines tend to be much more productive than older machines, but not everybody is working with the most advanced equipment. Given that capital-embodied technical change creates substantial productivity inequality among machines, what are its effects on the inequality among labor that uses that equipment? Put differently, what is the economic mechanism linking technical progress to inequality through labor markets?

If we view the labor market as a frictionless environment where workers are paid according to their marginal product, then the impact of technological change on inequality is limited to the extent that workers differ in their ability to use capital. Jovanovic (1998) showed that when capital embodies technological progress and machines are indivisible, faster growth raises wage inequality, as long as skills complement capital: the most skilled workers will be the ones who are efficiently assigned to work on the most productive machines. However, workers who use different vintages of capital but are otherwise identical will be paid the same wage.

On the other hand, if frictions that prevent the free and timeless reallocation of labor among alternative uses are an essential part of the labor market the situation is more complex. First, wages of workers with the same abilities may now reflect the relative productivity difference of the equipment they are working with. Second, frictions generate another key dimension of inequality across workers, namely employment status: since technological change may require the reallocation of labor, but reallocation requires time, not all workers will be employed at any time.

The objective of this paper is to analyze quantitatively how technical progress affects inequality in economies with frictional labor markets. More specifically, we study a matching model à la Diamond-Mortensen-Pissarides (DMP) with vintage capital à la Solow (1960), where technological advancement comes through the introduction of new capital goods. The DMP model over the years has established itself as a standard framework of analysis of the labor market (see Pissarides 2000 for an overview of the approach). Capital-embodied technical change is, arguably, the key driving force of productivity growth in developed economies in the past three decades (e.g., Jorgenson 2001). We maintain, throughout our analysis, a quantitative-theory focus: we tightly parameterize our model economies in order

to match some micro-estimates and certain stable, long-run facts. In particular, within our model we can consistently use data on equipment prices, adjusted for quality, to measure the speed at which the new equipment investment carries technology improvements.¹ Our statements are therefore mostly quantitative, but we always provide an intuition based on the mechanisms operating in the model.

The existing literature has been mainly concerned with the qualitative characterization of how equilibrium unemployment reacts to changes in the speed of technology. Several distinct approaches emerge from the literature. Aghion and Howitt (1994) argue that when new and more productive equipment enters the economy exclusively through the creation of new matches—because existing matches cannot be “upgraded”—it has a Schumpeterian “creative-destruction” effect: new capital always competes with old capital by making it more obsolete and tends to destroy existing matches, because workers are better off separating from their old matches to search for the new firms endowed with the most productive technology. Unemployment tends to go up as growth accelerates, due to a higher job-separation rate. The models of Caballero and Hammour (1998) and Cohen and Saint-Paul (1994) bear similarities to the Aghion-Howitt approach.

Mortensen and Pissarides (1998) propose an alternative approach whereby the new technologies enter into existing firms through a costly “upgrading” process of old capital. Thus, in this latter case, but not in the former, one could say that technology is “match-augmenting”, because it augments the value of existing matches. In the extreme case where upgrading can proceed at no cost, we have the Solowian model of disembodied technological change, even though the carrier of technology is equipment. The separation rate is unaffected by faster growth and all the effects work through job creation. For small values of the upgrading cost, unemployment falls with faster growth, thanks to the familiar “capitalization effect”: investors are encouraged to create more vacancies, knowing that they will be able to incorporate (and hence benefit from) future technological advances.²

The distinction between these two ways in which productivity improvements are introduced into the economy is quite important for inequality: the two forms of technological

¹Hornstein and Krusell (1996), Greenwood, Hercowitz, and Krusell (1997), and Cummins and Violante (2002) provide theoretical justifications and implementations of the price-based approach to the measurement of capital-embodied technical change.

²An interesting qualification to this result is provided by King and Welling (1995): if, unlike what is customarily assumed in this family of models, workers bear the full fixed search cost, then the capitalization effect leads to an increase in the number of searchers and to longer unemployment durations.

change have very different direct effects on the flows into and out of the pools of vacant and matched firms, and thus on unemployment. As a consequence, there are also indirect effects on wage inequality, because unemployment and wage inequality interact when there are frictions: the unemployment rate is a determinant of the labor share and of wage inequality when, as typically assumed, wages are set by Nash bargaining. Conversely, the unemployment rate depends on the share of output accruing to labor via its effect on firm entry (job creation) and on which matches are profitable to maintain (job destruction).³

Caballero and Hammour (1998) and Acemoglu and Shimer (1999) argue that the hold-up problem is pervasive in the labor market and that it influences both unemployment and the division of output between labor and capital, and thus inequality between the owners of the two factors (see Grout 1984 for an early theoretical model of hold-up in the labor market). Within a relationship between two parties, one party is held up if she has to pay the cost, while they both share the revenues. The new equipment investment potentially poses a severe hold-up problem. If, as one might assume, the firm pays for the upgrading/purchase and becomes the sole owner of the new machine, then because the wage is set ex-post according to Nash bargaining, the new investment benefits the worker too. This “contract incompleteness” encourages the firm to underinvest and create fewer jobs, which could hurt the worker as well: in other words, the firm-worker joint surplus would not be maximized, and there would be “money left on the table”. The key question is whether there exist appropriate contractual arrangements allowing firms and workers to co-own capital and whether workers have enough resources to pay firms for the equipment purchase (e.g., workers may be credit-constrained) or, as an alternative, whether workers can commit to accept lower future wages.

From this discussion of the literature it is clear that to understand the role of capital-embodied growth on labor market inequality it becomes necessary to be specific about the way in which the new and more productive capital is introduced into the worker-firm relationship. The two key dimensions of this “capital replacement” problem in frictional economies are: (1) Does the new capital entering the economy benefit only new firms/activities that compete with old ones, or rather enhance old ones as well? and (2) Who pays for the introduction of the new equipment?

³Incidentally, the two approaches have different implications for employment protection policies: in a world where the introduction of more productive capital requires a re-organization of production and a firm-worker separation, employment-protection policies can have a large impact on average productivity, whereas in a world where capital can be upgraded without shedding labor, the effects of these policies will be minor.

In our model, we analyze both dimensions. First, we nest the two ways in which new equipment can enter—one is match-augmenting and one is not—by assuming that (i) new entrant firms can buy new equipment at price I_0 and (ii) existing firms, whether matched or not, can upgrade their equipment to the latest vintage at price I_u . Thus, if $I_u = 0$, technological change is fully match-augmenting and disembodied, whereas if $I_u = \infty$, it has no match-augmenting or disembodied feature at all. In equilibrium, depending on parameter values, new equipment may enter through either channel. Second, we study two possible contractual arrangements for the capital upgrading problem with different degrees of hold-up: one where equipment purchases maximize firm/worker surplus, and one where they only maximize firms’ profits but the worker does not share the investment cost, and we examine to what extent the two models have different implications for labor market inequalities.

We find that the employment effects of a rise in the rate of capital-embodied technological change are closer to those emphasized by Aghion and Howitt (1994) than by the match-augmenting view: unemployment rises. The reason is quantitative. Even when the new equipment enters through upgrading of existing capital, in order to match the data on the average age of capital, which is quite high, this upgrading cost has to be substantial enough that the match-embodied aspect of new technology become *quantitatively unimportant*. Thus, a model without upgrading—where capital only enters through new firms—produces very similar results to one where (quantitatively restricted) upgrading is the main channel. With respect to the hold-up problem, we find again, somewhat surprisingly, that whether there are hold-up problems or not has a very marginal quantitative impact on any results. The reason is that, at the point of upgrading, again because of the restriction that the upgrading cost be quantitatively reasonable, the total surplus in a match is small in relative terms, so whether or not the worker participates in the decision is not important.

Overall the conclusion of the paper is stark: a properly parameterized (i.e., restricted) DMP model has the same quantitative implications for the link between capital-embodied growth and labor market inequalities, independently of the seemingly important details regarding who benefits from (new matches or all matches) and who pays for (workers and firms or firms only) the technological advancement. The intuition for this “equivalence result” is that upgrading can be a lot better than creative destruction only if it is very costly for firms to meet workers, but our calibration shows that this meeting friction is minor.

The outline of the paper is as follows. In Section 2 we describe the vintage-capital

economy without frictions and characterize the competitive equilibrium. This is a useful benchmark, since our general matching model converges to the competitive model as the frictions vanish. In Section 3 we introduce frictions through a standard matching-function setup. We consider various alternatives for capital replacement: one where replacement involves the destruction of the match (Schumpeterian model) and two models where it simply involves upgrading of capital without separation (Solowian model): one where investment is pairwise-efficient and one where the firm is held up by the worker. In Section 4 we calibrate the frictional model to the U.S. economy, we draw a quantitative comparison of the various economies, and we provide an intuitive explanation for the main “equivalence results”. Section 5 concludes.

2 The frictionless economy

We now present a version of the Solow (1960) frictionless vintage capital model where production is decentralized into worker-machine pairs operating Leontief technologies: this decentralized production structure is typical in frictional economies. The competitive economy displays neither wage nor employment inequality, however, it is a useful starting point for our analysis since it embeds many of the economic forces present in the richer (and more complex) frictional model. In particular, we can see how embodied technical change is reflected in the relative price of equipment capital.

Environment— Time is continuous. The economy is populated by a stationary measure 1 of ex-ante equal, infinitely lived workers who supply one unit of labor inelastically. The workers are risk-neutral and discount the future at rate r . Production requires pairing one machine and one worker. Machines (or jobs, or firms, or production units) are characterized by the amount of efficiency units of capital k they embody. A matched worker-machine pair produces a homogeneous output good.

There is embodied and disembodied technical change. The economy-wide disembodied productivity level $z(t)$ grows at a constant rate $\psi > 0$. Technological progress is also embodied in capital and the amount of efficiency units embodied in new machines grows at the rate $\gamma > 0$. Once capital is installed in a machine it is subject to physical depreciation at the rate $\delta > 0$. A machine may also be destroyed, according to a Poisson process with arrival rate $\sigma > 0$. A production unit that at time t has age a and is paired with a worker

has output

$$y(t, a) = z(t) k(t, a)^\omega = z_0 e^{\psi t} \left[k_0 e^{\gamma(t-a)} e^{-\delta a} \right]^\omega, \quad (1)$$

where $\omega > 0$. In what follows we set, without loss of generality, $z(0) = k(0) = 1$.

At any time t firms can freely enter the market upon payment of the initial installation cost $I_0(t)$ for a machine of vintage t . The cost of new vintages grows at the rate g . Existing firms with older machines have the opportunity to upgrade their machine and bring its productivity up to par with the newest vintage. The cost of upgrading $I_u(t, a) \geq 0$ grows at the rate g but it is independent of age for $a \leq \hat{a}$. We assume that once a machine is too old, it becomes infinitely costly to upgrade: $I_u(t, a) = \infty$ for $a > \hat{a}$.

Rendering the growth model stationary— We will focus on the steady state of the normalized economy; this corresponds to a balanced growth path of the actual economy. It is immediate that for a balanced growth path to exist, we need $g = \psi + \omega\gamma$. In order to make the model stationary we normalize all variables dividing by the growth factor e^{gt} . The normalized cost of a new production unit is then constant at I_0 and I_u , and the normalized output of a production unit of age a which is paired with a worker is $e^{-\phi a}$, where $\phi \equiv \omega(\gamma + \delta)$, thus output is defined relative to the newest production unit. Note that the parameter ϕ represents the effective depreciation rate of capital obtained as the sum of physical depreciation δ and technological obsolescence γ . In Hornstein, Krusell and Violante (2003, Appendix A.1), we describe the normalization procedure in detail.

Embodied technical change and the relative price of new capital— Since the cost of new vintage machines in terms of the output good, I_0 and I_u , is growing at the rate g but the number of efficiency units embodied in new vintages is growing at the rate γ , the price of quality-adjusted capital (efficiency units) in terms of output is changing at the rate $g - \gamma = \psi - (1 - \omega)\gamma$. In the quantitative analysis of our model we will use this relationship to obtain a measure of the rate of embodied technical change from the rate at which the observed relative price of equipment capital changes.

Competitive equilibrium— Assume that the labor market is frictionless and competitive so that there is a unique market-clearing wage. In the steady state the wage rate w , now measured relative to the output of the newest vintage, is constant. Consider a price-taker firm with the newest vintage machine. The firm optimally chooses the age \bar{a} that maximizes

the present value of the current machine lifetime profits

$$\Pi(w) = \max_{\bar{a}} \int_0^{\bar{a}} e^{-(r-g+\sigma)a} [e^{-\phi a} - w] da,$$

where $\Pi(w)$ is the profit function, and $r - g + \sigma$ is the effective discount factor. Since flow profits are monotonically declining in age and eventually become negative, there is a unique age at which the machine will be discarded or upgraded. The intuition is that while the wage w is the same for all firms, output falls compared to the new machines because of depreciation and obsolescence. Profit maximization leads to the condition

$$w = e^{-\phi \bar{a}}, \tag{2}$$

stating that the price of labor has to equal the relative productivity of the oldest machine, which is also the marginal productivity of labor. The higher the wage, the shorter the life-length of capital since (normalized) profits per period fall and thus reach zero sooner.

Free entry of firms with new machines or upgrading of existing machines requires that in equilibrium $\Pi(w) = I_0$.⁴ This is the key condition that determines exit/upgrading age \bar{a} , and hence wages. Using the profit-maximization condition (2), the optimal investment condition can be written as

$$I_0 = \int_0^{\bar{a}} e^{-(r-g+\sigma+\phi)a} [1 - e^{-\phi(\bar{a}-a)}] da. \tag{3}$$

It is obvious that in an equilibrium there will always be entry, and there will be upgrading only if $I_u < I_0$. In other words, there is no interesting trade-off in this model given the competitive nature of the labor market. It is only with matching frictions that a firm could, for example, choose upgrading over “creative-destruction” even if $I_u > I_0$, as it could save on search costs.⁵

Existence and uniqueness—Equation (3) allows us to discuss existence and uniqueness of the equilibrium as well as comparative statics.⁶ The right-hand side of this equilibrium

⁴There is always a positive inflow of new machines since machines fail at a constant rate.

⁵Note that we could have allowed firms to replace their capital with machines of any vintage, not just the frontier technology without any change in the equilibrium conditions: profit-maximizing firms always choose the newest capital vintage. The key behind this result is that the labor required to operate new machines is constant over time, which is why new technologies are better: in fact, technological change allows firms to pair their worker with more and more efficiency units of capital over time by using newer and newer equipment. A firm choosing to invest in old capital would, once in operation, generate lower output at the same wage cost. The lower initial installation cost of the old machine would compensate these losses only partially. This argument is easy to verify mathematically, so we omit its proof in the text.

⁶It is straightforward to solve for efficient allocations and show that a stationary solution to the planner’s problem reproduces the competitive allocations (see Hornstein, Krusell, and Violante 2003, Appendix A.2).

condition is strictly increasing in the upgrading/exit age \bar{a} for two reasons. First, in an equilibrium with older firms, the relative productivity of the marginal operating firm is lower and therefore wages have to be lower and profits higher. Second, a longer machine life increases the time-span for which profits are accumulated. Define $\bar{r} \equiv r - g + \sigma + \phi = r + \sigma - \psi + \omega\delta$. The right-hand side of (3) increases from 0 to $1/\bar{r}$ as \bar{a} goes from 0 to infinity. Taken together, these facts mean that there exists a unique steady state exit age \bar{a}^{CE} whenever $I_0 < 1/\bar{r}$. This condition is natural: unless you can recover the initial capital investment at zero wages using an infinite lifetime ($\int_0^\infty e^{-\bar{r}a} da = 1/\bar{r}$ being the net profit from such an operation), it is not profitable to start *any* firm. Finally, with a unit mass of workers, all employed, the firm distribution is uniform with density $1/\bar{a}^{CE}$, which is also the measure of entering/upgrading firms.

Comparative statics— A larger interest rate r decreases present-value profits, thus lowering entry and increasing the life span of the machine \bar{a} . A higher rate of disembodied technical change ψ acts just like a reduced interest rate. An increase in the cost of purchasing a new machine I_0 will raise the life span: fewer machines enter and they stay in operation longer to recover the fixed cost. Conversely, a higher rate of embodied technical change γ lowers the cost of hiring labor because it reduces the relative productivity of the least productive firm. Higher profits imply an increase in entry at the expense of older machines that are forced to exit earlier.⁷

3 The economies with matching frictions

Consider an economy with same preferences, demographics, and technology, but where the labor market is frictional in the sense of Pissarides (2000): an aggregate matching function governs job creation. The nature of the firm’s decision process remains the same as in the frictionless economy: there is free entry of firms which buy a new piece of capital, participate in the search process, start producing upon matching with a worker, and finally either upgrade their capital once it becomes too old or exit if upgrading is too costly. Searching is costless: it only takes time. Existing matches dissolve exogenously at the rate σ : upon dissolution, the worker and the firm are thrown into the pool of searchers.⁸

⁷More formally, the right-hand side of equation (3) is increasing in the growth rate γ since \bar{r} is independent of γ and ϕ is increasing in γ .

⁸Note that σ now only denotes the separation of a machine from a worker, and not the destruction of the machine as in the frictionless economy. We omitted separations from the description of the competitive

In this environment vacant firms are heterogeneous in the vintage of their capital for two reasons: first, newly created firms do not match instantaneously, so they remain idle until luck makes them meet a worker; second, firms hit by exogenous separation will also become idle. Note here a key difference with the traditional search-matching framework (Mortensen and Pissarides, 1998): the traditional models assume that a new machine can be purchased at no cost and that only posting a vacancy entails a flow cost. This assumption implies that the pool of vacancies consists of the newest machines only, and that only matched machines age over time.⁹ Our setup is built on the opposite assumption: purchasing and installing capital is costly—an expense which is sunk when the vacant firm start searching—whereas posting a vacancy is costless.¹⁰ As a result, it can be optimal even for firms with old capital to remain idle.

This class of economic environments is a hybrid between vintage models and matching models. The traditional assumption emphasizes the matching features of the environment, while the explicit distinction between a “large” purchase/setup cost for the machine and a “smaller” search/recruiting flow cost (zero in our model) fits more naturally with its vintage-capital aspects, whose emphasis is on capital investment expenditures as a way of improving productivity. In actual economies, new and old vacancies coexist, as in our setup. Moreover, in Hornstein, Krusell and Violante (2003) we show that the frictionless vintage capital model is obtained as the limit of our model as frictions vanish, whereas the standard model converges to an economy with a degenerate vintage structure (i.e. only the newest vintage of capital is in operation).

3.1 The environment

Random matching—The rate at which a worker meets a firm with capital of age a is $\lambda_w(a)$ and the rate at which she meets any firm is $\lambda_w \equiv \int_0^\infty \lambda_w(a) da$, where we assume that the integral is finite. A firm meets a worker at the rate λ_f . Let $\nu(a)$ denote the measure of vacant firms with machines of age a . We assume that the number of matches in any moment

equilibrium because, without frictions, it is immaterial whether the match dissolves exogenously or not as the worker can be replaced instantaneously by the firm at no cost.

⁹Aghion and Howitt (1994) also describe a vintage capital model with initial setup costs for capital, but they assume that matching is “deterministic”: at the time a new machine is set up, a worker queues up for the machine, and after a fixed amount of time the worker and firm start operations. Hence, in the matching process, all vacant firms are equal (although they do not embody the leading-edge technology).

¹⁰Vacancy heterogeneity will survive the addition of a flow search cost, as long as this cost is strictly less than the initial set-up cost I_0 .

is determined by a constant returns to scale matching function $m(v, u)$, where $v \equiv \int_0^\infty \nu(a) da$ is the total number of vacancies and u is the total number of unemployed workers. We also assume that $m(v, u)$ is strictly increasing in both arguments and satisfies some standard regularity conditions.¹¹ Using the notation $\theta = v/u$ to denote labor market tightness, we then have that

$$\lambda_f = \frac{m(\theta, 1)}{\theta}, \quad (4)$$

$$\lambda_w(a) = \frac{\nu(a)}{v} m(\theta, 1). \quad (5)$$

The expression for the meeting probability in (4) provides a one-to-one (strictly decreasing) mapping between λ_f and θ . Thereafter, when we discuss changes in λ_f , we imagine changes in θ . The measure of worker-firm matches with an a machine is denoted $\mu(a)$ and total employment μ .

Scrapping versus upgrading— Values for the market participants are $J(a)$ and $W(a)$ for matched firms and workers, respectively, $V(a)$ for vacant firms, and U for unemployed workers. We consider two forms of capital replacement and the value functions will differ in the two cases. First, we study an economy where replacement takes place through creative destruction: the nature of the firm's decision process is such that it buys a piece of capital at cost I_0 , then matches with a worker, and exits the economy once the machine reaches the optimal scrapping age \bar{a} . Scrapping implies the destruction of the match. Second, we introduce the option that a firm may upgrade its machine anytime it would like to at the upgrading cost I_u , without separation from the worker. The optimal replacement age is also denoted \bar{a} . Due to the exogenous separations and the matching frictions, some firms will also be vacant. It is immediate to see that vacant machines do not upgrade until they are matched with a worker, and they exit once upgrading becomes impossible, which occurs at the exogenous age $a = \hat{a}$. Replacing capital before meeting a worker is sub-optimal as the machine would get obsolete without being used productively.

¹¹In particular,

$$\begin{aligned} m(0, u) &= m(v, 0) = 0, \\ \lim_{u \rightarrow \infty} m_u(v, u) &= \lim_{v \rightarrow \infty} m_v(v, u) = 0, \\ \lim_{u \rightarrow 0} m_u(v, u) &= \lim_{v \rightarrow 0} m_v(v, u) = +\infty. \end{aligned}$$

The values of matched workers and firms– Under both replacement models, the flow value of a job for firm and worker for $a \leq \bar{a}$ is given respectively by

$$(r - g)J(a) = \max\{e^{-\phi a} - w(a) - \sigma [J(a) - V(a)] + J'(a), (r - g)V(a)\} \quad (6)$$

$$(r - g)W(a) = \max\{w(a) - \sigma [W(a) - U] + W'(a), (r - g)U\} \quad (7)$$

The return of a matched firm with an age a machine is equal to profit, i.e., production less wages $w(a)$ paid to the worker, minus the flow rate of capital losses from separation plus the flow losses/gains due to the aging of machines.¹² Analogously for a worker, the return on being in a match with an age a machine is the wage minus the flow rate of capital losses from separation plus the flow losses/gains due to the aging of machines. For the match to be maintained the flow return from staying in the match must be at least as high as the flow return from leaving the match, i.e., from the firm becoming vacant and the worker becoming unemployed. Note that the capital value equations for matched workers and firms are defined only for matches with machines not older than \bar{a} , since all machines are either scrapped or upgraded at age \bar{a} .

Wage determination– In the presence of frictions, a bilateral monopoly problem between the firm and the worker arises and, thus, wages are not competitive. As is standard in the literature, we choose a cooperative Nash bargaining solution for wages. In particular, we assume that the parameter defining the relative bargaining power is the same when the pair negotiates over how to split output and over how to share the upgrading cost I_u in case of joint maximization. With outside options as in the above equations, the wage is such that at every instant a fraction β of the total surplus $S(a) \equiv J(a) + W(a) - V(a) - U$ of a type a match goes to the worker and a fraction $(1 - \beta)$ goes to the firm:

$$W(a) = U + \beta S(a) \quad \text{and} \quad J(a) = V(a) + (1 - \beta)S(a). \quad (8)$$

Using the surplus-based definition (8) of the value of an employed worker $W(a)$ in equation (7) and rearranging terms, we obtain the wage rate as

$$w(a) = (r - g)U + \beta [(r - g + \sigma)S(a) - S'(a)]. \quad (9)$$

¹²In Appendix A.2 we describe a typical derivation of the differential equations above.

The values of vacant workers and firms— In the model with creative destruction, the values of idle workers and firms are

$$(r - g)V(a) = \max \{ \lambda_f [J(a) - V(a)] + V'(a), 0 \} \quad (10)$$

$$(r - g)U = b + \int_0^{\bar{a}} \lambda_w(a) [W(a) - U] da \quad (11)$$

The return on a vacant firm is equal to the capital gain rate from meeting a worker plus the flow losses/gains due to the aging of capital. Vacant machines that are older than the critical age \bar{a} exit. The return for unemployed workers is equal to their benefits b plus the capital gain rate from meeting vacant firms.

Finally, in the model with upgrading, the values of idle workers and firms are

$$(r - g)V(a) = \begin{cases} \max \{ \lambda_f [J(a) - V(a)] + V'(a), 0 \} & \text{for } a \leq \bar{a} \\ \max \{ \lambda_f [J(0) - I_u^J(a) - V(a)] + V'(a), 0 \} & \text{for } \bar{a} < a \leq \hat{a} \end{cases} \quad (12)$$

$$(r - g)U = b + \int_0^{\bar{a}} \lambda_w(a) [W(a) - U] da + \int_{\bar{a}}^{\hat{a}} \lambda_w(a) [W(0) - I_u^W(a) - U] da. \quad (13)$$

The return on a vacant firm now differs if the idle machine is older than \bar{a} . Vacant machines that are older than the critical age \bar{a} do not exit, but wait until they meet a worker and then upgrade. At the time the machine is upgraded, the firm pays its share of the upgrading cost $I_u^J(a)$. When age \hat{a} is reached, upgrading becomes infinitely costly, so the firm exits. The return for unemployed workers contains an additional term that gives the value of meeting vacant machines older than \bar{a} . These firms will upgrade upon meeting a worker, and the worker contributes $I_u^W(a)$ to the upgrading cost and starts working with brand new capital.

Replacement decision— When new capital enters the economy through new firms, these firms are not matched yet, so it is natural to assume that they make unilaterally the entry/adoption decision. When new technologies enter through upgrading in existing matches, there are two alternatives. First we view upgrading as a joint maximization problem and we assume that both firms and workers contribute to the cost of upgrading I_u . The relative contributions are such that firms of age a and workers split the gain from upgrading

$$G(a) \equiv J(0) + W(0) - I_u - J(a) - W(a), \quad (14)$$

according to the surplus sharing rule with parameter β . Thus, they solve jointly

$$\begin{aligned} \max_{I_u^W(a)} & \left[J(0) - J(a) - I_u^J(a) \right]^\beta \left[W(0) - W(a) - I_u^W(a) \right]^{1-\beta} \\ \text{s.t.} & \quad I_u^J(a) + I_u^W(a) = I_u \end{aligned}$$

The upgrading cost is then distributed according to

$$\begin{aligned} I_u^W(a) &= [W(0) - W(a)] - \beta G(a), \\ I_u^J(a) &= [J(0) - J(a)] - (1 - \beta) G(a), \end{aligned} \tag{15}$$

and we can define the gains from upgrading for firm and worker as

$$G_W(a) = \beta G(a) \quad \text{and} \quad G_J(a) = (1 - \beta) G(a). \tag{16}$$

Alternatively, we assume that the firm makes the upgrading decision unilaterally, the worker in a match does not contribute to the upgrading investment and that wages cannot be preset before the upgrading investment is undertaken. The gains from upgrading are then

$$G_W(a) = W(0) - W(a) \quad \text{and} \quad G_J(a) = J(0) - I_u - J(a) \tag{17}$$

and the upgrading cost shares become $I_u^W(a) = 0$ and $I_u^J(a) = I_u$.

3.2 The stationary equilibrium

We characterize the equilibrium of the matching model in terms of two variables: the age at which a firm exits the market or upgrades its machine and the rate at which vacant firms meet workers: (\bar{a}, λ_f) . The two variables are jointly determined by two key conditions. The first condition is labelled the job destruction or job upgrading condition. In the economy with creative destruction (upgrading) this condition expresses the indifference between carrying on and scrapping (upgrading) the machine for a match with capital of age \bar{a} . The second condition, labelled the job creation condition, expresses the indifference for outside firms between creating a vacancy with the newest vintage and not entering. This characterization is conditional on the steady state employment and vacancy distributions. In Section 3.2.4 we show how these distributions can be characterized in terms of the two unknowns (\bar{a}, λ_f) .

3.2.1 The economy with creative destruction

The surplus function– It is useful to start by stating the (flow version of the) surplus equation. Using the definition of the surplus $S(a)$ we arrive at

$$(r - g)S(a) = \max\{e^{-\phi a} - \sigma S(a) - \lambda_f(1 - \beta)S(a) - (r - g)U + S'(a), 0\}. \quad (18)$$

This asset-pricing-like equation is obtained by combining equations (6), (7), (8), and (10): the growth-adjusted return on surplus on the left-hand side equals the flow gain on the right-hand side, where the flow gain is the maximum of zero and the difference between total inside minus total outside flow values. The inside value includes: a production flow $e^{-\phi a}$, a flow loss due to the probability of a separation of the match $\sigma S(a)$, and changes in the value for the matched parties, $J'(a) + W'(a)$. The outside option flows are: the flow gain from the chance that a vacant firm matches $\lambda_f(1 - \beta)S(a)$, the change in the value for the vacant firm $V'(a)$, and the flow value of unemployment $(r - g)U$. Note a key difference with the traditional model: the value of a vacancy is positive, and it contributes towards a reduction of the rents created by the match.

The solution of the first-order linear differential equation (18) is the function

$$S(a) = \int_a^{\bar{a}} e^{-(\bar{r} + \sigma + (1 - \beta)\lambda_f)(\bar{a} - a)} \left[e^{-\phi \bar{a}} - (r - g)U \right] d\bar{a}. \quad (19)$$

We have used the boundary condition associated with the fact that the surplus-maximizing decision is to keep the match alive until an age \bar{a} when there is no longer any surplus to the match, $S(\bar{a}) = 0$, and there is no gain from a marginal delay of the separation, $S'(\bar{a}) = 0$. For lower a 's the match will have strictly positive surplus, and for values of a above \bar{a} the surplus will be equal to zero.¹³ Intuitively, the surplus is decreasing in age a for two reasons: first, the time-horizon over which the flow surplus accrues to the pair shortens with a ; second, the value of a job's output declines with age relative to that of the new vacant jobs.

Equation (19) contains a non-standard term associated with the non-degenerate distribution of vacancies: the non-zero firm's outside option of remaining vacant with its machine reduces the surplus by increasing the “effective” discount rate through the term $(1 - \beta)\lambda_f$. Everything else being equal, the quasi-rents in the match are decreasing as the bargaining power of the idle firm or its meeting rate is increasing.

¹³Straightforward integration of the right-hand side in (19) and further differentiation shows that, over the range $[0, \bar{a})$, the function $S(a)$ is strictly decreasing and convex; moreover, $S(a)$ will approach 0 for $a \rightarrow \bar{a}$.

The job-destruction condition— The optimal separation rule $S'(\bar{a}) = 0$ together with equation (19) implies that the exit age \bar{a} satisfies

$$e^{-\phi\bar{a}} = (r - g)U, \quad (20)$$

for a given value of unemployment U . The left hand side of (20) is the output of the oldest match in operation, whereas the right hand side is the flow-value of an idle worker. The idea is simple: firms with old enough capital shut down because workers have become too expensive, since the average productivity of vacancies and, therefore, the workers' outside option of searching, is growing at a constant rate. Note that this equation resembles the profit-maximization condition in the frictionless economy, with the worker's flow outside option, $(r - g)U$, playing the role of the competitive wage rate.¹⁴

We can now rewrite the surplus function (19) in terms of the two endogenous variables (\bar{a}, λ_f) only, by substituting for $(r - g)U$ from (20):

$$S(a; \bar{a}, \lambda_f) = \int_a^{\bar{a}} e^{-(\bar{r} + \sigma + (1 - \beta)\lambda_f)(\bar{a} - a)} \left[e^{-\phi\bar{a}} - e^{-\phi a} \right] d\tilde{a}. \quad (21)$$

In this equation, and occasionally below, we use a notation of values (the surplus in this case) that shows an explicit dependence of \bar{a} and λ_f . From (21) it is immediately clear that $S(a; \bar{a}, \lambda_f)$ is strictly increasing in \bar{a} and decreasing in λ_f . A longer life-span of capital \bar{a} increases the surplus at each age because it lowers the flow value of the worker's outside option, as evident from (20). A higher rate at which firms, when idle, meet workers reduces the surplus because it increases the outside option for a firm and shrinks the rents accruing to the matched pair.

Using (11) and (8) we obtain the optimal separation (or *job destruction*) condition

$$e^{-\phi\bar{a}} = b + \beta \int_0^{\bar{a}} \lambda_w(a; \bar{a}, \lambda_f) S(a; \bar{a}, \lambda_f) da, \quad (JD)$$

which is an equation in the two unknowns (\bar{a}, λ_f) . The rates $\lambda_w(a)$ at which unemployed workers are matched with firms also depend on the two endogenous variables.

The job-creation condition— We define the value of a vacancy of age a using the new expression (21) for the surplus of a match $S(a; \bar{a}, \lambda_f)$ together with (8). The differential equation for a vacant firm (10) then implies that the net-present-value of a vacant firm equals

$$V(a; \bar{a}, \lambda_f) = \lambda_f(1 - \beta) \int_a^{\bar{a}} e^{-(r-g)(\bar{a}-a)} S(\tilde{a}; \bar{a}, \lambda_f) d\tilde{a}, \quad (22)$$

¹⁴In fact, from the wage equation (9) it follows that the lowest wage paid in the economy (on machines of age \bar{a}) exactly equals the flow value of unemployment.

where \bar{a} equals the age at which the vacant firm exits. Since vacant firms do not incur in any direct search cost, they will exit the market at an age such that this expression equals zero, from which it follows immediately that they will exit at the same age \bar{a} at which matched capital get destroyed and replaced by new firms. Since in equilibrium there are no profits from entry, we must have that $V(0; \bar{a}, \lambda_f) = I_0$, and we thus have the free-entry (or *job creation*) condition

$$I_0 = \lambda_f(1 - \beta) \int_0^{\bar{a}} e^{-(r-g)a} S(a; \bar{a}, \lambda_f) da. \quad (JC)$$

This condition requires that the cost of creating a new job I_0 equals the value of a vacant firm at age zero, which is the expected present value of the profits it will generate: a share $(1 - \beta)$ of the discounted future surpluses produced by a match occurring at the instantaneous rate λ_f . The job creation condition is the second equation in the two unknowns (\bar{a}, λ_f) .

In Hornstein, Krusell, and Violante (2003) we demonstrate that a solution to the two equations (*JC*) and (*JD*) in the pair (\bar{a}, λ_f) exists and is unique, under very general conditions: in particular, the (*JC*) condition traces a strictly decreasing curve in the (\bar{a}, λ_f) space, whereas the (*JD*) condition traces a strictly increasing relationship.

3.2.2 The economy with upgrading and joint decision making

We now consider an economy where the firm and worker jointly decide on when the machine should be upgraded and both parties share in the cost of the project.

Optimal upgrading—A worker-firm pair will not upgrade its machine as long as the gains from upgrading are negative. The match values, being the discounted expected present values of future returns, are continuous functions of the age. Since upgrading is instantaneous, at the time a machine is upgraded the gain from upgrading is then zero:

$$G(\bar{a}) = J(0) + W(0) - I_u - J(\bar{a}) - W(\bar{a}) = 0. \quad (23)$$

At the optimal upgrading age not only is the gain from upgrading zero, but so is the marginal gain from a delay of the upgrading decision. This means that at the upgrading age the derivative of the gain function is zero, that is $J'(\bar{a}) + W'(\bar{a}) = 0$. We can use this condition when we add the value function definitions of matched workers, (7), and firms, (6),

$$(r - g)[J(\bar{a}) + W(\bar{a})] = e^{-\phi\bar{a}} - \sigma[J(\bar{a}) + W(\bar{a}) - V(\bar{a}) - U] + J'(\bar{a}) + W'(\bar{a}).$$

Together the two no-gains conditions then imply

$$(r - g) [J(0) + W(0) - I_u] = e^{-\phi\bar{a}} - \sigma [J(0) + W(0) - I_u - V(\bar{a}) - U]. \quad (24)$$

This condition states that at the optimal upgrading age \bar{a} the firm-worker pair is indifferent between an upgrade and a marginal delay of the upgrade. The left-hand side of this expression gives the return on an upgraded machine at \bar{a} for the matched pair, and the right-hand side is the flow return from a marginal delay of the upgrading decision: the production flow minus the surplus capital loss from delay due to separation.¹⁵

Optimal upgrading depends on the value of a vacancy at the upgrading age, which in turn depends on the expected present value of a vacant firm's gain from upgrading upon meeting a worker (12). From the rule (15) which determines how the gains from upgrading (14) are shared we get

$$J(0) - I_u^J(a) - V(a) = (1 - \beta) [S(0) + I_0 - I_u - V(a)],$$

where we have used the free-entry condition $V(0) = I_0$. Substituting this expression in the definition of the value of a vacancy (12) for $\bar{a} \leq a \leq \hat{a}$, collecting terms, and solving the differential equation subject to the terminal condition $V(\hat{a}) = 0$, we obtain an expression for the vacancy value for $a \geq \bar{a}$

$$V(a) = \kappa_{VJ}(a, \lambda_f) [S(0) + I_0 - I_u], \quad (25)$$

with $\kappa_{VJ}(a, \lambda_f) = \frac{(1-\beta)\lambda_f}{\rho_{VJ}} [1 - e^{-\rho_{VJ}(\hat{a}-a)}]$ and $\rho_{VJ} = (1 - \beta) \lambda_f + r - g$. In an equilibrium the value of a vacancy is non-negative at the upgrading age.

The surplus and the vacancy value at the upgrading age and the surplus and vacancy value for a firm with a new machine differ only through the cost of upgrading. To see this use the surplus sharing rule (8) and the free-entry condition in the no-gains condition (23) and we get

$$S(\bar{a}) + V(\bar{a}) = S(0) + I_0 - I_u. \quad (26)$$

This means that the surplus at the upgrading age is given by

$$S(\bar{a}) = [1 - \kappa_{VJ}(\bar{a}, \lambda_f)] [S(0) + I_0 - I_u]. \quad (27)$$

¹⁵In Appendix A.2 we provide a heuristic derivation of this indifference condition based on the limit of discrete time approximations.

In an equilibrium, both the surplus and vacancy value at the upgrading age are non-negative. Note that in the economy with upgrading the surplus and vacancy value at the upgrading age can be strictly positive, as opposed to the creative destruction economy where machines are scrapped at the exit age because the surplus of the match is zero. Since $\kappa_{VJ} \in (0, 1)$ either the surplus and vacancy value are both zero or both strictly positive.

In the definition of the job-upgrading and job-creation conditions below we use the surplus function $S(a; \bar{a}, \lambda_f)$, defined for machines which have not yet reached the upgrading age, $0 \leq a \leq \bar{a}$. In Appendix A.3 we show that we can write the surplus as a function of the upgrading age and the worker meeting rate only. As a first step towards that result we derive an expression for the surplus value of a new machine as a function of (\bar{a}, λ_f) only, $S(0; \bar{a}, \lambda_f)$.

The job-upgrading condition– From the condition for the optimal delay of upgrading (24), together with the expression for the vacancy value at the threshold age \bar{a} (25) and the surplus definitions (8), it is easy to derive that

$$e^{-\phi\bar{a}} - \kappa_{JJ}(\bar{a}, \lambda_f) [S(0; \bar{a}, \lambda_f) + I_0 - I_u] = (r - g) U, \quad (28)$$

with $\kappa_{JJ}(\bar{a}, \lambda_f) = r + g + \sigma [1 - \kappa_{VJ}(\bar{a}, \lambda_f)]$. Now consider the flow value of unemployment (13). Using the surplus sharing rules (8) in the first integral term and the upgrading cost sharing rule (15) in the second integral term of the RHS, and substituting the expression (28) above for the flow return on unemployment on the LHS, we obtain an expression for the job upgrading condition entirely as a function of (\bar{a}, λ_f) :

$$e^{-\phi\bar{a}} = b + \beta \int_0^{\bar{a}} \lambda_w(a; \bar{a}, \lambda_f) S(a; \bar{a}, \lambda_f) da \quad (\text{JU-j})$$

$$+ \left\{ \kappa_{JJ}(\bar{a}, \lambda_f) + \beta \int_{\bar{a}}^{\hat{a}} \lambda_w(a; \bar{a}, \lambda_f) [1 - \kappa_{VJ}(a, \lambda_f)] da \right\} [S(0; \bar{a}, \lambda_f) + I_0 - I_u]$$

Comparing this equation with the job destruction condition in the economy with creative destruction (JD), we note an additional term, always positive, implying that the upgrading age \bar{a} is lower than the destruction age –how much lower depends on the size of the extra term. Note however that the expression for the surplus function $S(a; \bar{a}, \lambda_f)$ and the distributions $\lambda_w(a; \bar{a}, \lambda_f)$ are not the same in the two economies.¹⁶

The job-creation condition– The condition that ensures zero profits at entry is always $I_0 = V(0; \bar{a}, \lambda_f)$ but we now have a different expression for the value of a vacant job. From

¹⁶We discuss the invariant distributions below, in section 3.2.4, and as noted above we derive the expression for the surplus in Appendix A.3.

(12) and (8), it is easy to see that for $a \leq \bar{a}$

$$V(a; \bar{a}, \lambda_f) = \lambda_f(1 - \beta) \int_a^{\bar{a}} e^{-(r-g)(\bar{a}-a)} S(\tilde{a}; \bar{a}, \lambda_f) d\tilde{a} + e^{-(r-g)(\bar{a}-a)} V(\bar{a}; \bar{a}, \lambda_f).$$

Since the cost of upgrading a machine is independent of the age as long as the machine is not too old, $a \leq \hat{a}$, vacant machines older than \bar{a} will be upgraded. Vacant machines at the upgrading age \bar{a} therefore tend to have positive value as opposed to vacant machines at the exit age in a creative destruction economy. We can substitute (25) for the vacancy value at \bar{a} and obtain the job-creation equilibrium condition, as a function of the pair of unknowns (\bar{a}, λ_f) only

$$I_0 = \lambda_f(1 - \beta) \int_0^{\bar{a}} e^{-(r-g)a} S(\tilde{a}; \bar{a}, \lambda_f) da + e^{-(r-g)\bar{a}} \kappa_{VJ}(\bar{a}, \lambda_f) [S(0; \bar{a}, \lambda_f) + I_0 - I_u]. \quad (\text{JC-j})$$

3.2.3 The economy with upgrading and firm decision

We now consider the case where firms unilaterally decide whether a machine should be upgraded, and the worker does not contribute to the cost of upgrading the machine. This structure introduces a hold-up problem to the investment decision: ex-ante the worker cannot share in the investment cost even though it is in the worker's interest that the project is undertaken, and ex-post the firm cannot commit to a future wage path that is contingent on the upgrading decision. Relative to the joint upgrading decision, machines are thus upgraded too late.

Optimal upgrading– A firm will postpone upgrading if its own gain from upgrading is negative, and a machine is upgraded the moment the gain to the firm is zero

$$G_J(\bar{a}) = J(0) - I_u - J(\bar{a}) = 0. \quad (29)$$

Note that from the point of view of the total match value upgrading occurs too late. Since the gain to a worker when the firm upgrades the machine is strictly positive, the worker would be willing to pay the firm to upgrade the machine earlier.

For \bar{a} to be optimal the firm also has to be indifferent between upgrading now and a marginal delay of the decision, that is $G'_J(\bar{a}) = -J'(\bar{a}) = 0$. We use both no-gain conditions in the differential equation for a matched firm's value and obtain

$$(r - g) [J(0) - I_u] = e^{-\phi\bar{a}} - w(\bar{a}) - \sigma [J(0) - I_u - V(\bar{a})]. \quad (30)$$

The left-hand side of this expression is the return on an upgraded machine at \bar{a} for a firm, and the right-hand side is the flow return from a marginal delay of the upgrading decision: the profit flow, i.e., production minus wage payments, minus the capital loss from delay due to the possibility of separation.¹⁷ This condition on the optimal marginal delay of upgrading depends on the wage that the firm has to pay. We now use the wage equation (9) and the differential equation for the surplus value (18) to express the optimality condition with respect to the firm value and unemployment value only.¹⁸ After rearranging terms we obtain

$$(r - g) [J(0) - I_u] = (1 - \beta) \left[e^{-\phi \bar{a}} - (r - g)U + \beta \lambda_f S(\bar{a}) \right] - \sigma [J(0) - I_u - V(\bar{a})]. \quad (31)$$

which corresponds to equation (24) for the case of joint upgrading.

Optimal upgrading depends on the value of a vacancy at the upgrading age, which in turn depends on the expected present value of a vacant firm's gain from upgrading upon meeting a worker (12). Since the firm pays for upgrading, $I_u^J = I_u$, we can easily solve the differential equation for the vacancy value (12) for $\bar{a} \leq a \leq \hat{a}$, subject to the terminal condition $V(\hat{a}) = 0$,

$$V(a) = \kappa_{VF}(a, \lambda_f) [(1 - \beta) S(0) + I_0 - I_u], \quad (32)$$

with $\kappa_{VF}(a, \lambda_f) = \frac{\lambda_f}{\rho_{VF}} \left[1 - e^{-\rho_{VF}(\hat{a}-a)} \right]$ and $\rho_{VF} = \lambda_f + r - g$. We have used the free entry condition to substitute for $V(0)$, and the surplus sharing rule (8) to substitute for $J(0)$.

In the economy where the firm makes the upgrading decision, the sum of a firm's vacancy value and its surplus share when the firm has a new machine as opposed to a machine at the upgrading age differ only through the upgrading cost. To see this use the surplus sharing rule (8) in the no-gains condition (29) and we get

$$(1 - \beta) S(\bar{a}) + V(\bar{a}) = (1 - \beta) S(0) + I_0 - I_u. \quad (33)$$

This means that the surplus at the upgrading age is given by

$$S(\bar{a}) = [1 - \kappa_{VF}(\bar{a}, \lambda_f)] [(1 - \beta) S(0) + I_0 - I_u]. \quad (34)$$

In Appendix A.3 we derive the surplus value on the interval $[0, \bar{a}]$ as a function of the upgrading age and the worker meeting rate only (\bar{a}, λ_f) .

¹⁷In Appendix A.2 we provide a heuristic derivation of this indifference condition based on the limit of discrete time approximations.

¹⁸Note that the definition of the surplus equation is the same for the economies with creative destruction and with upgrading.

The job-upgrading condition– We can further simplify the condition for the optimal delay of upgrading (31) by substituting expression (32) for the vacancy value at the threshold age \bar{a} , and by using the surplus definition (8) for the firm value:

$$e^{-\phi\bar{a}} - \kappa_{JF}(\bar{a}, \lambda_f) [(1 - \beta) S(0; \bar{a}, \lambda_f) + I_0 - I_u] = (r - g) U, \quad (35)$$

with $\kappa_{JF}(\bar{a}, \lambda_f) = \{r + g + [1 - \kappa_{VF}(\bar{a}, \lambda_f)](\sigma - \beta\lambda_f)\} / (1 - \beta)$. Now consider the flow value of unemployment (13). Using the surplus sharing rules (8) in the integral terms and substituting the expression (28) above for the flow return on unemployment on the LHS, we obtain an expression for the job-upgrading condition entirely as a function of (\bar{a}, λ_f) :

$$e^{-\phi\bar{a}} = b + \int_0^{\bar{a}} \lambda_w(a; \bar{a}, \lambda_f) S(a; \bar{a}, \lambda_f) da + \beta S(0; \bar{a}, \lambda_f) \int_{\bar{a}}^{\hat{a}} \lambda_w(a; \bar{a}, \lambda_f) da \quad (\text{JU-f}) \\ + \kappa_{JF}(\bar{a}, \lambda_f) [(1 - \beta) S(0; \bar{a}, \lambda_f) + I_0 - I_u]$$

Again, relative to the job-destruction condition in the economy with creative destruction (JD), we note an additional term, always positive, implying that the upgrading age \bar{a} is lower than the destruction age.¹⁹

The job-creation condition– Similar to the previous cases this condition ensures zero profits at entry is always $I_0 = V(0; \bar{a}, \lambda_f)$. Again we solve the differential equation (12) on the interval $[0, \bar{a}]$, but for the terminal condition we use expression (32) for the vacancy value at the upgrading age:

$$I_0 = \lambda_f(1 - \beta) \int_0^{\bar{a}} e^{-(r-g)a} S(\tilde{a}; \bar{a}, \lambda_f) da \quad (\text{JC-f}) \\ + e^{-(r-g)\bar{a}} \kappa_{VF}(\bar{a}, \lambda_f) [(1 - \beta) S(0; \bar{a}, \lambda_f) + I_0 - I_u].$$

3.2.4 Invariant employment and vacancy distributions

The economy with creative destruction– We now complete the characterization of the equilibrium in the economy with creative destruction and derive explicit expressions for the matching probabilities in terms of the endogenous variables (\bar{a}, λ_f) . For this purpose we need to characterize the stationary vacancy and employment distribution of firms. Denote with $\mu(a)$ the measure of matches between an a firm and a worker, and denote total employment with μ .²⁰

¹⁹Note however that the expression for the surplus function $S(a; \bar{a}, \lambda_f)$ and the distributions $\lambda_w(a; \bar{a}, \lambda_f)$ are not the same in the two economies. We discuss the invariant distributions below, in section 3.2.4, and as noted above we derive the expression for the surplus in Appendix A.3.

²⁰In Appendix A.3 we derive the differential equations which characterize the stationary employment dynamics, equations (36) and (37) below, and solve for the stationary measures, (40) and (41).

The inflow of new firms is $v(0)$: new firms acquire the new capital and proceed to the vacancy pool. Thereafter, these firms transit stochastically back and forth between vacancy and match: firms are matched with workers at rate λ_f and they become vacant at rate σ . Finally firms exit at $a = \bar{a}$, whether vacant or matched. The evolution of employment and vacancies in a stationary distribution is then determined by the differential equations

$$v'(a) = \sigma\mu(a) - \lambda_f v(a), \text{ for } 0 \leq a \leq \bar{a} \quad (36)$$

$$\mu'(a) = \lambda_f v(a) - \sigma\mu(a), \text{ for } 0 \leq a \leq \bar{a}. \quad (37)$$

The evolution of matched machines is the mirror image of the evolution of vacancies, i.e. $\mu'(a) = -v'(a)$. This implies that the number of vacant and matched machines of age a less than \bar{a} remains constant:

$$v(a) + \mu(a) = v(0) + \mu(0), \text{ for } 0 \leq a \leq \bar{a}. \quad (38)$$

Because all firms proceed first to the search pool with their new machines $\mu(0) = 0$. For $a \in [0, \bar{a})$, the evolution of $\mu(a)$ therefore follows

$$\mu'(a) = -\sigma\mu(a) + \lambda_f v(a) = \lambda_f v(0) - (\sigma + \lambda_f)\mu(a). \quad (39)$$

We can solve this differential equation subject to the initial condition $\mu(0) = 0$ and get

$$\frac{\mu(a)}{\mu} = \frac{1 - e^{-(\sigma+\lambda_f)a}}{\bar{a} - \frac{1}{\sigma+\lambda_f}(1 - e^{-(\sigma+\lambda_f)\bar{a}})}, \text{ and} \quad (40)$$

$$\frac{v(a)}{v} = \frac{\sigma + \lambda_f e^{-(\sigma+\lambda_f)a}}{\bar{a}\sigma + \frac{\lambda_f}{\sigma+\lambda_f}(1 - e^{-(\sigma+\lambda_f)\bar{a}})}. \quad (41)$$

The employment (vacancy) density is therefore increasing and concave (decreasing and convex) in age a . The reason for this is that for every age $a \in [0, \bar{a})$ there is a constant number of machines, and older machines have a larger cumulative probability of having been matched in the past. This feature distinguishes our model from standard-search vintage models where the distribution of vacant jobs is degenerate at zero and the employment density is decreasing in age a at a rate equal to the exogenous destruction rate σ .

With the vacancy distribution in hand, we now have the explicit expression for the value of $\lambda_w(a)$,

$$\lambda_w(a; \bar{a}, \lambda_f) = \lambda_w \frac{\nu(a)}{v} = m(\theta, 1) \frac{\sigma + \lambda_f e^{-(\sigma+\lambda_f)a}}{\bar{a}\sigma + \frac{\lambda_f}{\sigma+\lambda_f}(1 - e^{-(\sigma+\lambda_f)\bar{a}})}, \quad (42)$$

which depends only on the pair of endogenous variables (\bar{a}, λ_f) , given the strictly decreasing relation between θ and λ_f , equation (4).

The economy with upgrading– In the economy with upgrading, the key difference is that after the machine of a firm reaches age \bar{a} the firm does not exit, but it upgrades the machine if it is matched to a worker and thereby resets the age to 0. Only vacant firms that do not meet a worker by age \hat{a} exit the economy. For firms with relatively young machines, i.e., firms which do not upgrade their machines, the evolution of employment and vacancies in a stationary distribution continues to be determined by the differential equations (37), (36), and equation (38). With the possibility of instantaneous upgrading of machines at age \bar{a} and ongoing upgrading of vacant machines with age $a \geq \bar{a}$, there is however now a strictly positive employment density of new machines

$$\mu(0) = \mu(\bar{a}) + \lambda_f \int_{\bar{a}}^{\hat{a}} v(a) da. \quad (43)$$

This means that in the economy with upgrading the employment density is maximal for the newest vintage – it receives the machines which have just upgraded in addition to all vintages which immediately upgrade upon meeting a worker – whereas in the economy with creative destruction the employment density is minimal for new machines. Finally, the evolution of vacancies that are older than the upgrading age is given by

$$v'(a) = -\lambda_f v(a), \text{ for } \bar{a} < a \leq \hat{a} \quad (44)$$

Since matched machines never reach age $a > \bar{a}$ there are no exogenous separations. Once the machine of a vacant firm reaches age \hat{a} , upgrading becomes infeasible and the firm exits.

In Appendix A.5 we show how to solve the system of equations (36), (37), (38), (43), and (44) for the employment and vacancy distribution, $\mu(a)$ and $v(a)$, conditional on the pair (\bar{a}, λ_f) . With the vacancy distribution in hand, we use (5) to obtain $\lambda_w(a)$, i.e., the rate at which workers meet vacant firms.

4 The quantitative analysis

In the previous sections, we have outlined three different economies that capture some key aspects of the process through which innovations and productivity improvements are introduced into the economy: 1) creative-destruction, 2) (pairwise) efficient upgrading, and 3) upgrading with hold-up.

We want to know if distinguishing among these particular mechanisms has quantitatively important implications for labor market inequality. We do it in a number of ways. First, for each economy we target a common set of steady-state aggregate variables and verify whether the implied parametrization is very different. Second, we reverse the exercise: we fix the parametrization and we verify if the implications for some key equilibrium variables are significantly different. Third, we study the response of the labor market variables of interest in each of the three economies to an empirically plausible increase of 1) the rate of disembodied technical change, and 2) the rate of embodied technical change.

4.1 Common steady-state targets

Overall, we have 13 parameters to calibrate: $(r, A, \alpha, \sigma, \beta, \delta, I_0, I_u, \hat{a}, \psi, \omega, \gamma, b)$. In the calibration procedure, we aim to match the same set of aggregate U.S. variables across the three model economies. Below we explain our calibration strategy.

Parameters calibrated “externally”— We set r to match an annual interest rate of 4%. We normalize the scale parameter A of the matching function to 1 and we set the matching elasticity with respect to vacancies, α , to 0.5, an average of the values reported in the comprehensive survey of empirical estimates of matching functions by Petrongolo and Pissarides (2001, Table 3).

Parameters calibrated “internally” — We simultaneously calibrate $(\sigma, \beta, \delta, I_0, I_u, \hat{a})$ so that the steady state of each model economy generates (1) an unemployment rate of 4% (the U.S. historical average); (2) a labor income share of 0.685 (Cooley and Prescott (1995)); (3) a maximal age of capital of 23 years, which corresponds to an average age of capital of about 11.5 years, as reported by the Bureau of Economic Analysis (2002); and (4) an average unemployment duration of approximately 8–9 weeks, as reported by Abrahams and Shimer (2001). The unemployment rate together with the average unemployment duration imply an annual separation rate for workers from employment to unemployment of 21%, which is in line with the data reported in CEPR (1995, page 10).²¹

²¹In an economy with upgrading the separation rate, i.e., the unconditional probability that a worker separates from a job within a period of length Δ , is $1 - \exp(-\sigma\Delta)$. In an economy without upgrading the separation rate is defined as $\left\{ [1 - \exp(-\sigma\Delta)] \int_0^{\bar{a}-\Delta} \mu(a) da + \int_{\bar{a}-\Delta}^{\bar{a}} \mu(a) da \right\} / \mu$ for the economy. Note that it would be incorrect to match this variable to job destruction rates (i.e., job flows rather than worker flows, as we do) since the event occurring at rate σ involves only a separation of workers and machines, but not the destruction of the job.

Technical change— We have two sources of growth in the model: disembodied productivity change, occurring at rate ψ , and capital-embodied productivity change. Hornstein and Krusell (1996) measured annual disembodied growth in the U.S. for 1954–1993 to be 0.8% per year, whereas more recently Cummins and Violante (2002) compute it to be 0.3% per year from 1965–2000. We set $\psi = 0.5\%$. At least since Greenwood et al. (1997), a number of authors have suggested to measure the speed of embodied technical change through the (inverse of the) rate of decline of the quality-adjusted relative price of capital. In section 2 we have argued that in our environment embodied technical change is directly reflected in the rate at which the relative price of new capital declines, $\gamma - g$. Gordon’s (1990) influential work on quality-adjusted prices for durable goods suggests a value for the annual rate of embodied technical change in the U.S. around 3%. Given the observed average U.S. output growth rate $g = 2\%$, a 3% rate of price decline for capital implies that $\gamma = 5\%$. From the relation $g - \gamma = \psi - (1 - \omega)\gamma$, we obtain a capital elasticity parameter $\omega = 0.3$.

Institutions— The parameter b is supposed to summarize a wide range of benefit policies that vary with unemployment duration and family status (none of which we model). The OECD Employment Outlook (1996) provides replacement rates for unemployment benefits in OECD countries from 1961 to 1995 for two earnings levels, three family types, and three durations of unemployment. The reported average replacement rate for the United States in that period was 10% and we choose b to replicate this number.

The parameter values are summarized in Table 1 below. A clear conclusion emerges: if we take the distance between parameters as a measure of closeness of the three economies, then it appears that the three economies are remarkably similar: very small parametric differences are needed to match the same set of facts.

Table 1. Common steady-state targets

Common parameters			
	$r = 0.04, A = 1, \alpha = 0.5,$		
	$\psi = 0.005, \gamma = 0.05, \omega = 0.3$		
Model specific parameters			
	Economy with upgrading investment		Economy without upgrading investment
	Joint surplus max	Firm value max	
δ	0.060	0.064	0.641
σ	0.238	0.251	0.210
β	0.670	0.662	0.609
b	0.048	0.047	0.047
I_0	4.537	4.308	4.165
I_u	4.182	4.093	NA
\hat{a}	28.00	28.00	NA

4.2 Common parametrization

We now reverse the logic of the previous exercise and we fix the parametrization across the three economies, using the calibration of the economy with efficient (joint-surplus maximizing) upgrading decision. Given this set of parameter values we have then calculated the steady state when the firm decides on upgrading in order to maximize its own value and when there is no upgrading. The results are in Table 2 below.

Given that machines can be upgraded, the nature of the upgrading decision is not very important for the outcome: the numbers in the first and second column of Table 2 are essentially the same.²² The “creative destruction” feature of the economy without upgrading (third column) comes through in a slightly higher separation rate and unemployment duration, leading to a rate of unemployment 1.1% larger. If machines cannot be upgraded, relatively old matches will be destroyed for sure, which also results in a somewhat smaller job creation rate (and unemployment duration). At this higher unemployment rate, however both wage inequality and the labor share are essentially the same for the economies with and without upgrading. Overall, the difference in the key measures of equilibrium labor market inequalities between the economies with and without upgrading does not appear to be quantitatively very important.

²²Notice also that in the calibrated economy with upgrading for all practical purposes there is no entry. In Table 2, the entry rate $v(0)$ is of the magnitude 10^{-10} for the economies with upgrading.

Table 2. Common parametrization

	Economy with upgrading investment		Economy without upgrading investment
	Joint Surplus Max	Firm Value Max	
Unemployment Rate	0.040	0.040	0.051
Unemployment Duration	0.173	0.176	0.195
Separation Rate	0.212	0.212	0.243
Upgrading/Exit Age	23.00	23.00	25.068
Wage Income Share	0.685	0.682	0.677
90-10 log-wage differential	0.079	0.081	0.078

4.3 Comparative statics

We now analyze, for the baseline parametrization, the response of the three economies to accelerations in the rate of embodied technical change, γ , and in the rate of disembodied technical change, ψ , of an empirically plausible magnitude. In our analysis we focus on the behavior of the unemployment rate and unemployment duration, the job separation rate, the critical age at which upgrading/exit occurs, the 90-10 log-wage inequality, and the wage income share.

Embodied technical change – Krusell et al. (2000) and, more recently, Cummins and Violante (2002) have argued that the annual rate of embodied technical change in the U.S. has increased substantially in the past two decades, up to 6.5% over the years 1995-2000.²³ This estimate, together with the assumption that ψ and ω remain constant, means that γ has increased to $\gamma = 10\%$ so as to generate a decline in the relative price of capital of 6.5% per year.

The results of this experiment are reported in Figure 1.a. A faster rate of embodied technical change increases the unemployment rate and wage inequality and lowers the wage income share. The accelerated technical change shortens the useful life-time of a machine, that is machines are either upgraded at a faster rate or they exit the economy at a faster rate. Although wages fall, reflected in the declining wage income share, they do not fall enough to compensate completely for the shortened life-time of machines. In consequence the value of firms declines, but in an equilibrium the value of a new machine has to equal

²³Other authors, using measurement techniques different from quality-adjusted relative prices, arrived at very similar conclusions on the pace of embodied technical change in the postwar era (see for example Hobbijn (2000)) for the United States.

the constant (normalized) cost of a new machines. Therefore the rate at which firms meet workers has to increase and the measure of active firms and employment declines.

Employment in the economy without upgrading declines somewhat more than in the economies with upgrading because of the creative destruction effect: upon firm exit workers enter the unemployment pool. Overall, the differences across the three economies are minor along all dimensions examined.

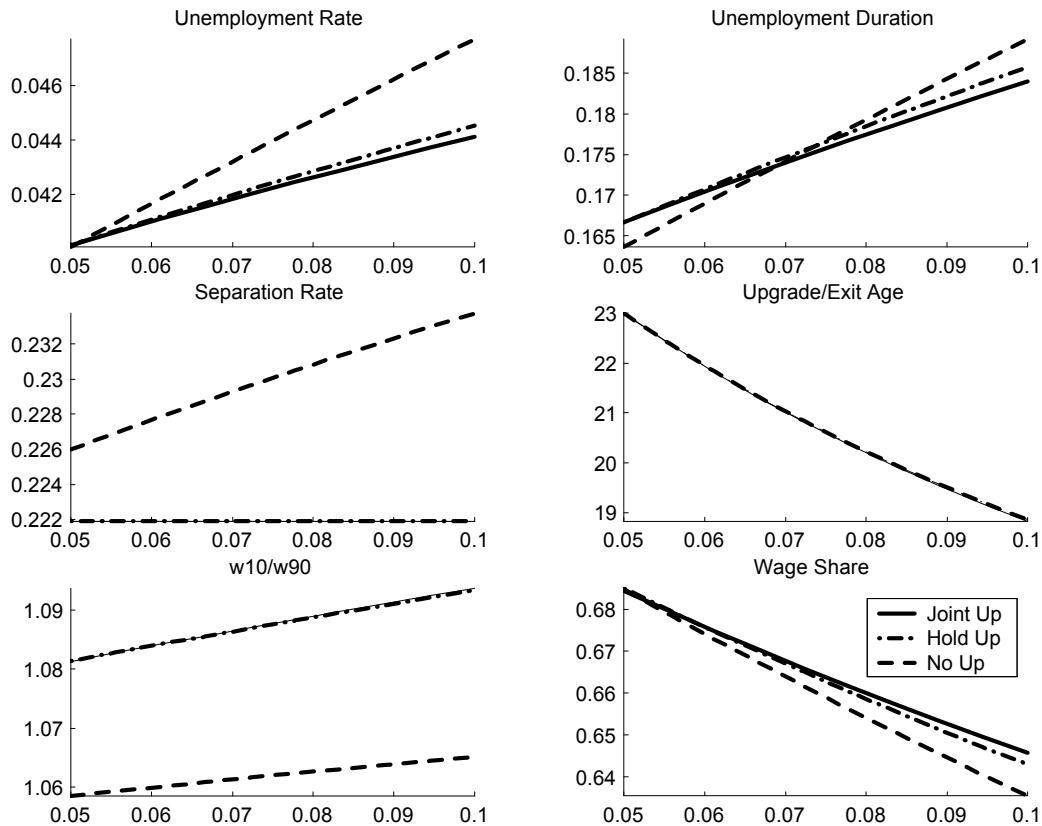


Figure 1.a. Comparative Statics for Embodied Technical Change γ

Disembodied technical change – Mortensen and Pissarides (1998) have pointed out that there is a qualitative difference between embodied and disembodied technical change. Whereas a higher rate of embodied technical change tends to lower the value of existing machines, a higher rate of disembodied technical change increases the value of machines because it increases the output over the life time of a machine. Machines become more valuable, therefore vacancy values increase, more machines seek to match with workers and unemployment declines. Wage inequality declines and the wage income share increases.

From Figure 1.b we see (1) that the economies with and without upgrading essentially

respond in the same way to a change in the rate of disembodied technical change, and (2) that doubling the rate of disembodied technical change has a negligible quantitative effect on labor market variables.

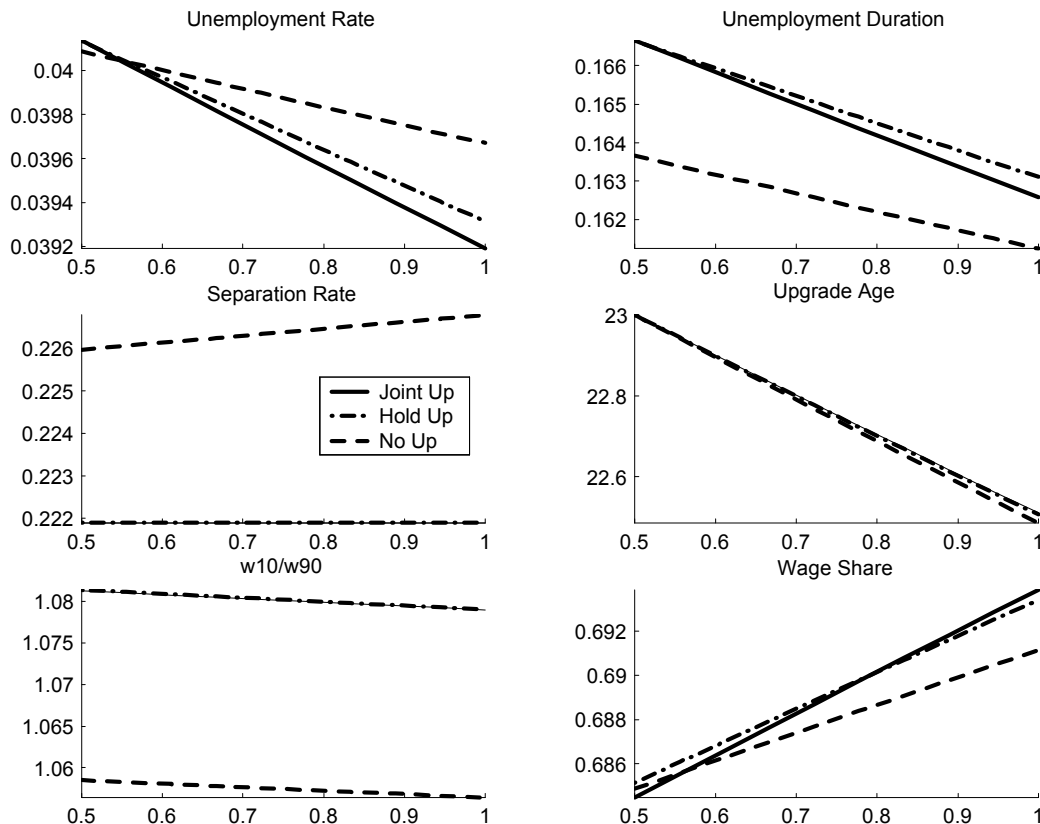


Figure 1.b. Comparative Statics for Disembodied Technical Change ψ

4.4 Equivalence of the replacement problems: an intuitive argument

4.4.1 Creative destruction vs. upgrading

One of the results of our calibration exercise is that the implied meeting rate for firms is relatively large. We use the key equilibrium conditions of the two replacement models (creative-destruction and upgrading) to show through a limiting argument that when the instantaneous meeting rate for firms becomes large enough, the economy with upgrading converges to the economy with creative destruction.²⁴ We use the upgrading model with

²⁴Firms meet workers at a rate $\lambda_f = 5.9$, that corresponds to an average duration of a vacancy of roughly 8 weeks. Thus in our calibration the average duration of a vacancy is of the same magnitude as the average duration of unemployment. Hall's (2003) work suggest that vacancies are filled even faster; he estimates an average vacancy duration of 4 weeks from JOLTS data (Hall 2003, Table 2 page 19).

joint replacement decision and proceed as follows. We have to assume that $I_0 = I_u$. We first show that as $\lambda_f \rightarrow \infty$ the “extra” terms that appear in the conditions (JC-j) and (JU-j) but do not appear in the conditions (JC) and (JD) vanish. Second, we show that the expressions for the surplus function and the distributions in the two economies converge as well.

Consider the extra term in (JC-j) and let $\lambda_f \rightarrow \infty$. The expression $\kappa_{VJ}(\bar{a}, \lambda_f)$ converges to 1 and $S(0; \bar{a}, \lambda_f)$ converges to zero. The latter limit is clear from simple inspection of (47) in Appendix A.3, since both $\Sigma_{1J}(\bar{a}, \lambda_f)$ and $\Sigma_{2J}(\bar{a}, \lambda_f)$ converge to zero as λ_f gets large enough. Hence, the extra term in (JC-j) converges to zero. Consider now the extra term in (JU-j) and let $\lambda_f \rightarrow \infty$. Since $\kappa_{VJ}(a, \lambda_f)$ converges to 1 for all a 's and since $S(0; \bar{a}, \lambda_f)$ converges to zero, this term goes to zero, provided that $\lambda_w(a; \bar{a}, \lambda_f)$ stays finite.

It is easy to see, from (46) in Appendix A.3 that the extra term (the second line) in the surplus function of the economy with upgrading goes to zero as $\lambda_f \rightarrow \infty$, thus the expressions for the surplus in the two economies converge. It only remains to show that the vacancy and employment distributions converge, but this is trivial once it is recognized that as the meeting rate for firms goes to infinity, the measure of vacancies tends to zero and the employment density is simply $\mu(a)/\mu = 1/\bar{a}$.

We conclude that when the labor market frictions are “small” from the firm’s perspective, the firm becomes indifferent between scrapping and upgrading: for a given cost of new machines, the upgrading option is substantially better than scrapping only if the matching process is long and costly.

4.4.2 Hold-up vs. efficient upgrading

In our baseline calibration it does not really matter that much whether or not workers share in the cost of upgrading investment. This might be surprising but a more detailed analysis shows that in the equilibrium of the baseline economy where workers contribute to the upgrading cost, they contribute only a very small share to the total investment cost. Therefore switching to an economy where workers do not contribute at all to the investment cost has minor quantitative implications.

The worker’s share in upgrading investment cost in the economy with joint surplus maximization is defined in equation (15). Using the surplus share rule (8) and the fact that the gain from upgrading is zero at the upgrading age we get

$$I_u^W(\bar{a}) = W(0) - U - \beta S(\bar{a}).$$

Now for the baseline calibration the surplus at upgrading is quite small, close to zero. Recall the expression for the surplus value, (27)

$$S(\bar{a}) = [1 - \kappa_{VJ}(\bar{a}, \lambda_f)] [S(0) + I_0 - I_u]. \quad (45)$$

In the baseline calibration the rate at which firms find workers, λ_f , is quite high, vacancies do not last very long. At least relative to the effective discount rate. Therefore

$$\kappa_{VJ} \approx \frac{(1 - \beta) \lambda_f}{(1 - \beta) \lambda_f + r - g} \approx 1,$$

and the first term in (45) is small. For the second term in (45) we substitute (47) and we get

$$\begin{aligned} S(0; \bar{a}, \lambda_f) + I_0 - I_u &\approx \left\{ \frac{1}{\rho_s} + I_0 - I_u \right\} / \{1 - r + g\} \\ &\approx \frac{1}{\sigma + (1 - \beta) \lambda_f} + I_0 - I_u. \end{aligned}$$

Again for our calibration these terms are quite small, such that the product of the two terms which define the surplus value at the upgrading age is small.

Thus with joint surplus maximization the contribution of the worker to the upgrading cost depends on the surplus of a worker in a match with a new machine, $W(0) - U$. For our baseline economy the capital value of unemployment is relatively large, and the additional gain from being in the best possible job is not that high. Workers get a relatively large share of the output, because they are unemployed only infrequently and if unemployed they find a job relatively fast. Because the gain from being in the best possible match is not that high, workers do not contribute that much to the upgrading cost with joint surplus maximization.

Figures 2.a and 2.b below illustrate this point. In both figures we plot the capital values of (un)matched firms and workers, and the surplus capital value of a match. Figure 2.a plots the value functions for the baseline calibration with a long life-span of machines. We can see that the equilibrium with joint surplus maximization is in a sense better than the equilibrium with the hold-up problem: capital values tend to be lower when the firm alone bears the upgrading cost. Notice that with joint surplus maximization the surplus value at the upgrading age is quite small, although not zero, and that the surplus gain for a worker of being in a match with the best relative to the worst available machine is quite small, about 0.2, whereas the gain for a machine is quite large, about 4. Notice also that these two gains

add up to the cost of upgrading, about 4.2. Thus even with joint surplus maximization the firm bears about 95% of the cost of upgrading.

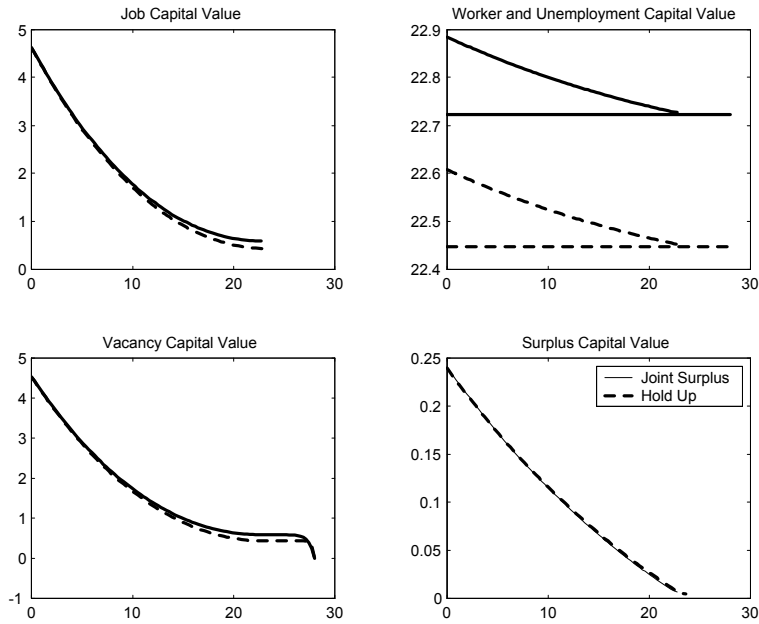


Figure 2.a Capital values for the baseline calibration

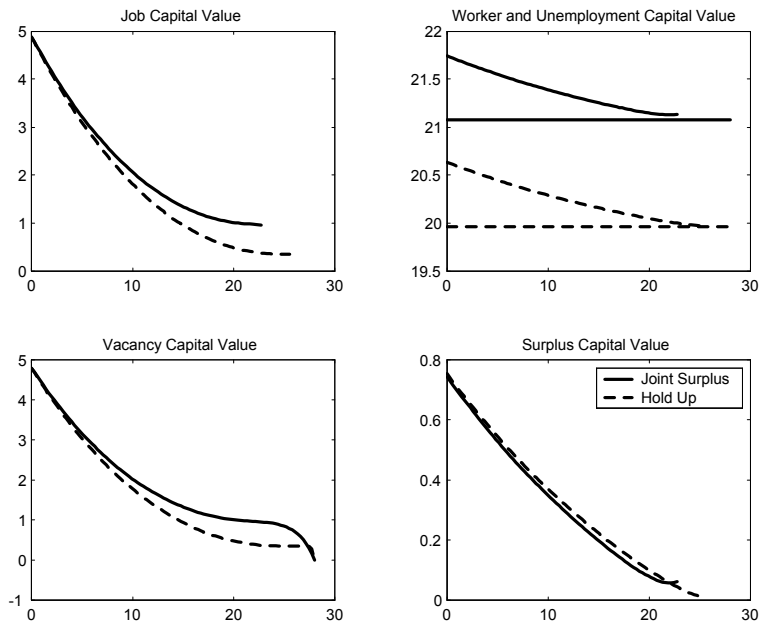


Figure 2.b. Capital values with longer unemployment duration

Figure 2.b plots the same value functions when we calibrate the economy such that the equilibrium unemployment rate is the same as in the baseline economy, but the exit rate from unemployment is much lower: the average duration of unemployment is 40 weeks, and not

8.5 weeks as in the baseline economy. This alternative calibration implies that the gain for a worker of being with the best possible match is now 0.5, whereas the gain to a firm is about 4. Both terms add up roughly to the total upgrading cost. We can now see a noticeable difference in the capital value functions for the two set-ups. The firm now contributes less to total upgrading costs, about 90%, but this is still a large part of total costs, and even with this extreme parameterization of the labor market the steady states of the economy with joint surplus maximization and hold-up are not that different at the same parameter values.²⁵

5 Conclusions

Technological progress and productivity growth, by definition, increase the resources available to an economy, but at the same time they can have substantial reallocative effects across different members of the economy. The labor market is an important channel through which technological developments translate into changes in many dimensions of inequality: share of income accruing to capital and labor owners, employment status among workers, and wage inequality among employed workers.

The literature has pointed out that it matters *qualitatively* for equilibrium unemployment (1) whether technological progress benefits only new matches or also ongoing relationships and (2) how big the hold-up problem is in the technology adoption decision.

In this paper we have shown that, if one takes the view—common in modern macroeconomics—that economic models should be calibrated and tightly parameterized to replicate certain key features of aggregate data, then the qualitative ambiguity of the growth-unemployment relationship is resolved: it gives the stark answer that the various approaches to the capital replacement problem in frictional economies all yield equivalent quantitative results. This conclusion is reinforced once one looks not only at the unemployment rate, but also at the equilibrium income shares and wage inequality.

The driving force behind this result is that quantitatively the labor market frictions

²⁵Our result is reminiscent of Ljungqvist (2002) who shows the equivalence of two alternative wage bargaining mechanisms in a matching environment with lay-off costs. For the first scheme the lay-off costs never affect the outside option in wage bargaining, and for the second scheme the outside option only affects wage bargaining after a match has been formed, but not when a worker and firm meet the first time. Whereas Ljungqvist (2002) can prove equivalence of the two wage bargaining mechanisms, we can only argue that the quantitative differences between the two investment mechanisms are small.

are very small: the average duration of a vacancy and unemployment spell is just 8 weeks—nothing compared to the average life of capital, which is over 11 years. Consequently, upgrading gives only a very small advantage compared to innovating through creative destruction. Moreover, the worker is not willing to contribute much to the upgrading of the old capital, when allowed, since the alternative of unemployment is only a short transient state.

A corollary of what we just said is that in economies where frictions are more severe, like in continental European labor markets where average unemployment duration can reach 6-8 months, our equivalence result could be weaker. Future work should be directed toward evaluating this conjecture.

Finally, our result has useful implications with respect to a recent literature that tries to empirically identify the relative importance in the U.S. economy of disembodied technical change vis-a-vis productivity advances exploiting the different implications these shocks have on job creation, job destruction, and the unemployment rate (see Pissarides and Vallanti 2003, and Lopez-Salido and Michelacci, 2003). In our analysis, all conclusions are based on steady-state comparisons. In other words, our equivalence result holds in the long run, but we have not yet studied the short-run predictions of the different models. In this sense, we provide a cautionary remark and a suggestion: it seems that it will be extremely difficult to disentangle the different sources of technical change from a low-frequency analysis of the data, whereas a high-frequency analysis of the response of labor market flows to technology shocks might prove to be more informative.

Appendix

A.1 Derivations of typical value functions.

The value functions of our continuous-time model can be derived as limits of a discrete time formulation. A typical derivation of the differential equations for value functions (6)-(11) goes as follows. Consider the value of a vacant firm with capital of age $a \leq \bar{a}$ at time t , $\tilde{V}(t, a)$. For a Poisson matching process, the probability that the vacant firm meets a worker over a small finite time interval $[t, t + \Delta]$ is $\Delta\lambda_f$. We can define the vacancy value recursively as

$$\tilde{V}(t, a) = \Delta\lambda_f \left[\tilde{J}(t + \Delta, a + \Delta) - \tilde{V}(t + \Delta, a + \Delta) \right] + e^{-r\Delta}\tilde{V}(t + \Delta, a + \Delta),$$

where the first term is the expected capital gain from becoming a matched firm with value \tilde{J} and the second term is the present value of remaining vacant at the end of the time interval. On a balanced growth path all value functions increase at the rate g over time, i.e., $\tilde{V}(t, a) = e^{gt}V(a)$ and $\tilde{J}(t, a) = e^{gt}J(a)$. Subtracting $\tilde{V}(t + \Delta, a)$ from both sides, substituting the balanced growth path expressions for \tilde{V} and \tilde{J} , and dividing by $\Delta e^{g(t+\Delta)}$, we can rearrange the value equation into

$$\begin{aligned} -e^{-g\Delta}V(a) \frac{e^{g\Delta} - 1}{\Delta} &= \lambda_f [J(a + \Delta) - V(a + \Delta)] + \frac{e^{-r\Delta} - 1}{\Delta}V(a + \Delta) \\ &\quad + \frac{V(a + \Delta) - V(a)}{\Delta}. \end{aligned}$$

As we shorten the length of the time interval and take the limit for $\Delta \rightarrow 0$, we obtain the differential equation (10):

$$-gV(a) = \lambda_f [J(a) - V(a)] - rV(a) + V'(a).$$

A.2 Derivation of optimal upgrading condition.

Consider the following discretization of the investment decision when a worker-firm pair maximizes the joint value of the match.²⁶ The length of a time period is Δ . At \bar{a} the firm and worker prefer to upgrade at \bar{a} rather than delaying it by one period:

$$\underline{W(0) + J(0) - I_u} \geq e^{-\phi\bar{a}}\Delta + e^{-(r-g)\Delta} \{(\sigma\Delta) [V(\bar{a} + \Delta) + U] + (1 - \sigma\Delta) [W(0) + J(0) - I_u]\}$$

²⁶We consider the formulation of the problem after variables have been made stationary, that is normalized.

The left hand side is the joint capital value after upgrading at \bar{a} , and the right hand side is the flow return from production without upgrading plus the expected present value from upgrading in the next period. Note that the match separates with probability $\sigma\Delta$ and loses the upgrading opportunity. Rearranging terms and dividing by Δ we get

$$[W(0) + J(0) - I_u] \frac{1 - (1 - \sigma\Delta) e^{-(r-g)\Delta}}{\Delta} \geq e^{-\phi\bar{a}} + \sigma e^{-(r-g)\Delta} [V(\bar{a} + \Delta) + U].$$

Taking the limit as $\Delta \rightarrow 0$ yields

$$[W(0) + J(0) - I_u] (r - g + \sigma) \geq e^{-\phi\bar{a}} + \sigma [V(\bar{a}) + U].$$

At $\bar{a} - \Delta$ the firm and worker prefer not to upgrade, but to delay until \bar{a} :

$$W(0) + J(0) - I_u \leq e^{-\phi(\bar{a}-\Delta)} \Delta + e^{-(r-g)\Delta} \{(\sigma\Delta) [V(\bar{a}) + U] + (1 - \sigma\Delta) [W(0) + J(0) - I_u]\}$$

Rearranging terms and taking the limit as $\Delta \rightarrow 0$ we get

$$[W(0) + J(0) - I_u] (r - g + \sigma) \leq e^{-\phi\bar{a}} + \sigma [V(\bar{a}) + U]$$

Therefore we must have that

$$[W(0) + J(0) - I_u] (r - g + \sigma) = e^{-\phi\bar{a}} + \sigma [V(\bar{a}) + U]$$

which is equation (24) in the main text.

A similar expression can be obtained for the upgrading problem when only the firm bears the cost of upgrading. At \bar{a} the firm weakly prefers to upgrade, rather than delay the decision one more period:

$$J(0) - I_u \geq [e^{-\phi\bar{a}} - w(\bar{a})] \Delta + e^{-(r-g)\Delta} \{(\sigma\Delta) V(\bar{a} + \Delta) + (1 - \sigma\Delta) [J(0) - I_u]\}$$

Rearranging terms and taking the limit as $\Delta \rightarrow 0$ we get

$$[J(0) - I_u] (r - g + \sigma) \geq e^{-\phi\bar{a}} - w(\bar{a}) + \sigma V(\bar{a}).$$

Analogously, the firm prefers to delay upgrading at age $\bar{a} - \Delta$ by one period, and we get the condition for optimal upgrading (30) in the main text:

$$[J(0) - I_u] (r - g + \sigma) = e^{-\phi\bar{a}} - w(\bar{a}) + \sigma V(\bar{a}).$$

A.3 Derivation of the surplus as a function of (λ_f, \bar{a}) in the economy with upgrading.

In section 3.2.1 we have derived the differential equation for the surplus value of a matched firm-worker pair in a creative-destruction economy. This equation determines the surplus as a function of the age of the firm's machine and it is defined from the time of entry to the time of exit, $0 \leq a \leq \bar{a}$. The surplus value in an economy with upgrading satisfies the same differential equation but the terminal condition for the surplus value differs. In the creative-destruction economy the firm/machine exits at age \bar{a} and the surplus at the time of exit is zero, $S(\bar{a}) = 0$. In the economies with upgrading the machine is upgraded at age \bar{a} and the surplus is defined in equation (27) if the upgrading is jointly done by the firm and worker and in equation (34) if the firm unilaterally makes the upgrading decision.

Joint upgrading decision— Substitute (28) for $(r - g)U$ into the differential equation for the surplus value (18), and solve that equation subject to the terminal condition (27) for $S(\bar{a})$

$$\begin{aligned}
S(a; \bar{a}, \lambda_f) &= \int_a^{\bar{a}} e^{-\rho_s(s-a)} \{e^{-\phi s} - e^{-\phi \bar{a}}\} ds \dots \\
&+ [S(0) + I_0 - I_u] \left\{ [r - g + \sigma(1 - \kappa_V(\bar{a}, \lambda_f))] \int_0^{\bar{a}-a} e^{-\rho_s s} ds \dots \right. \\
&\quad \left. + (1 - \kappa_V(\bar{a}, \lambda_f)) e^{-\rho_s(\bar{a}-a)} \right\}
\end{aligned} \tag{46}$$

with $\rho_s = r - g + \sigma + (1 - \beta)\lambda_f$. This is an expression for the surplus as a function of the two unknowns (\bar{a}, λ_f) and $S(0)$. Now evaluate this expression (46) at $a = 0$ and solve for the surplus value of a new machine $S(0)$. We obtain

$$S(0; \bar{a}, \lambda_f) = \frac{\Sigma_{1J} + (I_0 - I_u) \Sigma_{2J}}{1 - \Sigma_{2J}} \tag{47}$$

with

$$\begin{aligned}
\Sigma_{1J}(\bar{a}, \lambda_f) &\equiv \int_0^{\bar{a}} e^{-\rho_s a} \{e^{-\phi a} - e^{-\phi \bar{a}}\} da, \\
\Sigma_{2J}(\bar{a}, \lambda_f) &\equiv \{r - g + \sigma[1 - \kappa_{VJ}(\bar{a}, \lambda_f)]\} \int_0^{\bar{a}} e^{-\rho_s a} da + [1 - \kappa_{VJ}(\bar{a}, \lambda_f)] e^{-\rho_s \bar{a}}.
\end{aligned}$$

The firm makes the upgrading decision— Substitute (35) for $(r - g)U$ into the differential equation for the surplus value (18), and solve that equation subject to the terminal

condition (34) for $S(\bar{a})$

$$\begin{aligned}
S(a; \bar{a}, \lambda_f) &= \int_a^{\bar{a}} e^{-\rho_s(s-a)} \{e^{-\phi s} - e^{-\phi \bar{a}}\} ds \dots \\
&+ [(1 - \beta) S(0) + I_0 - I_u] \dots \\
&\left\{ \frac{1 - \kappa_{VF}(\bar{a}, \lambda_f)}{1 - \beta} e^{-\rho_s(\bar{a}-a)} + \kappa_{JF}(\bar{a}, \lambda_f) \int_0^{\bar{a}-a} e^{-\rho_s s} ds \right\}.
\end{aligned} \tag{48}$$

This is an expression for the surplus as a function of the two unknowns (\bar{a}, λ_f) and $S(0)$. Now evaluate this expression (48) at $a = 0$ and solve for the surplus value of a new machine $S(0)$. We obtain

$$S(0; \bar{a}, \lambda_f) = \frac{\Sigma_{1F} + \Sigma_{2F} (I_0 - I_u) / (1 - \beta)}{1 - \Sigma_{2F}} \tag{49}$$

with

$$\begin{aligned}
\Sigma_{1F}(\bar{a}, \lambda_f) &\equiv \int_0^{\bar{a}} e^{-\rho_s a} \{e^{-\phi a} - e^{-\phi \bar{a}}\} da, \\
\Sigma_{2F}(\bar{a}, \lambda_f) &\equiv [1 - \kappa_{VF}(\bar{a}, \lambda_f)] e^{-\rho_s \bar{a}} + (1 - \beta) \kappa_{JF}(\bar{a}, \lambda_f) \int_0^{\bar{a}} e^{-\rho_s a} da.
\end{aligned}$$

A.4 Derivation of the steady state employment dynamics

The equations describing employment dynamics are derived as follows. Consider the measure of matched vintage a firms at time t . Over a short time interval of length Δ , the approximate change in the measure is

$$\mu(t + \Delta, a) = \mu(t, a - \Delta)(1 - \Delta\sigma) + \Delta\lambda_f\nu(t, a - \Delta).$$

Subtracting $\mu(t, a)$ from both sides and dividing by Δ we obtain

$$\frac{\mu(t + \Delta, a) - \mu(t, a)}{\Delta} = -\frac{\mu(t, a) - \mu(t, a - \Delta)}{\Delta} - \sigma\mu(t, a - \Delta) + \lambda_f\nu(t, a - \Delta).$$

Taking the limit for $\Delta \rightarrow 0$ we obtain

$$\mu_t(t, a) = -\mu_a(t, a) - \sigma\mu(t, a) + \lambda_f\nu(t, a).$$

At steady state, these measures do not change with t , and we obtain the result stated in (37).

In the economy with upgrading the initial measure of matched firms with new machines evolves according to

$$\mu(t + \Delta, 0) = \mu(t, \bar{a}) + (\Delta\lambda_f) \cdot v(t, 0) + \sum_{i=\bar{a}/\Delta}^{\hat{a}/\Delta} (\Delta\lambda_f) \cdot v(t, a_i)$$

Taking the limit for $\Delta \rightarrow 0$ we get (43)

$$\mu(t, 0) = \mu(t, \bar{a}) + \lambda_f \int_{\bar{a}}^{\hat{a}} v(t, a) da.$$

A.5 The invariant employment and vacancy distributions as functions of (λ_f, \bar{a})

We solve the differential equation (37) for matched pairs backwards and get

$$\mu(a) = \lambda_f m(0) \int_0^a e^{-(\sigma+\lambda_f)(a-\tilde{a})} d\tilde{a} + \mu(0) e^{-(\sigma+\lambda_f)a}. \quad (50)$$

Evaluating this measure at \bar{a} we have

$$\begin{aligned} \mu(\bar{a}) &= m(0) \lambda_f A_1 + \mu(0) e^{-(\sigma+\lambda_f)\bar{a}} \\ A_1 &\equiv \int_0^{\bar{a}} e^{-(\sigma+\lambda_f)a} da. \end{aligned} \quad (51)$$

We solve the differential equation (44) for vacancies on the interval $[\bar{a}, \hat{a}]$ backwards and get

$$v(a) = e^{-\lambda_f(a-\bar{a})} v(\bar{a}). \quad (52)$$

The total measure of vacancies on $[\bar{a}, \hat{a}]$ is then

$$\begin{aligned} \int_{\bar{a}}^{\hat{a}} v(a) da &= v(\bar{a}) A_2 \\ A_2 &\equiv \int_0^{\hat{a}-\bar{a}} e^{-\lambda_f a} da. \end{aligned} \quad (53)$$

This is enough for the creative-destruction economy since we can use the initial condition $\mu(0) = 0$.

In the economy with upgrading we have to solve for the employment density of new machines. This density satisfies (43) into which we substitute (52) and (51),

$$\begin{aligned} \mu(0) &= \mu(\bar{a}) + \lambda_f A_2 v(\bar{a}) \\ &= \mu(\bar{a}) + \lambda_f A_2 [\mu(0) + v(0) - \mu(\bar{a})] \\ &= [1 - \lambda_f A_2] \left[\{\mu(0) + v(0)\} \lambda_f A_1 + \mu(0) e^{-(\sigma+\lambda_f)\bar{a}} \right] \\ &\quad + \lambda_f A_2 [\mu(0) + v(0)] \end{aligned}$$

We can solve this expression for the density of new employed machines as a function of new vacant machines

$$\begin{aligned}\mu(0) &= Bv(0) \\ B &= \frac{(1 - \lambda_f A_2) \lambda_f A_1 + \lambda_f A_2}{1 - (1 - \lambda_f A_2) \left(\lambda_f A_1 + e^{-(\sigma + \lambda_f) \bar{a}} \right) - \lambda_f A_2}\end{aligned}\tag{54}$$

Note that B can be simplified to

$$B = \frac{1 - e^{-\lambda_f(\hat{a} - \bar{a})} \left\{ 1 - \left[1 - e^{-(\sigma + \lambda_f) \bar{a}} \right] \lambda_f / (\sigma + \lambda_f) \right\}}{e^{-\lambda_f(\hat{a} - \bar{a})} \left[1 - e^{-(\sigma + \lambda_f) \bar{a}} \right] \sigma / (\sigma + \lambda_f)}.$$

For the calibration of our economy B is very large since the denominator is close to zero. This will be important when we obtain numerical solutions of the steady state.

Substituting (54) and (43) into the expression for the density of employed machines at the upgrade age \bar{a} (51) yields $\mu(\bar{a}) = v(0) (1 + B) \lambda_f A_1 + Bv(0) e^{-(\sigma + \lambda_f) \bar{a}}$ or

$$\begin{aligned}\mu(\bar{a}) &= C_1 v(0) \\ C_1 &= (1 + B) \lambda_f A_1 + B e^{-(\sigma + \lambda_f) \bar{a}}\end{aligned}\tag{55}$$

Evaluating (43) at \bar{a} and solving for $v(\bar{a})$ we have $v(\bar{a}) = \mu(0) + v(0) - \mu(\bar{a})$. After we substitute (55) for $\mu(\bar{a})$ and (54) for $\mu(0)$ we have

$$\begin{aligned}v(\bar{a}) &= C_2 v(0) \\ C_2 &\equiv (1 + B) (1 - \lambda_f A_1) - B e^{-(\sigma + \lambda_f) \bar{a}}.\end{aligned}\tag{56}$$

Integrating the employment density (50) over the interval $[0, \bar{a}]$ yields total employment

$$\int_0^{\bar{a}} \mu(a) da = \lambda_f m(0) \int_0^{\bar{a}} \left[\int_0^a e^{-(\sigma + \lambda_f) \tilde{a}} d\tilde{a} \right] da + \mu(0) \int_0^{\bar{a}} e^{-(\sigma + \lambda_f) \tilde{a}} d\tilde{a}$$

Using (43) for $m(0)$, and substituting (54) for $\mu(0)$ yields

$$\begin{aligned}\int_0^{\bar{a}} \mu(a) da &= C_3 v(0) \text{ with} \\ C_3 &= (1 + B) \lambda_f (\bar{a} - A_3) / (\sigma + \lambda_f) + B A_3 \\ A_3 &= \int_0^{\bar{a}} e^{-(\sigma + \lambda_f) a} da\end{aligned}\tag{57}$$

We can now calculate the total measure of vacancies on the interval $[0, \bar{a}]$. Using (43) we get

$$\int_0^{\bar{a}} v(a) da = \int_0^{\bar{a}} [m(0) - \mu(a)] da = m(0) \bar{a} - \int_0^{\bar{a}} \mu(a) da$$

and using equations (54) and (57) we get

$$\begin{aligned} \int_0^{\bar{a}} v(a) da &= C_4 v(0) \\ C_4 &= (1+B) [\bar{a} - (\bar{a} - A_3) \lambda_f / (\sigma + \lambda_f)] - BA_3 \end{aligned} \quad (58)$$

Combining equations (53), (56), and (58) yields total vacancies as

$$\begin{aligned} \int_0^{\hat{a}} v(a) da &= C_5 v(0) \\ C_5 &\equiv (1+B) [\bar{a} - (\bar{a} - A_3) \lambda_f / (\sigma + \lambda_f) + A_2 (1 - \lambda_f A_1)] \\ &\quad - B [A_3 + A_2 e^{-(\sigma + \lambda_f) \bar{a}}] \end{aligned} \quad (59)$$

To get the density of new firms coming into the economy with new machines we use the definition of labor market tightness

$$\theta = \frac{\int_0^{\hat{a}} v(a) da}{1 - \int_0^{\hat{a}} \mu(a) da} = \frac{C_5 v(0)}{1 - C_3 v(0)}$$

and solve for $v(0)$

$$v(0) = \frac{\theta}{\theta C_3 + C_5}.$$

For the calibration of the economy entry $v(0)$ is essentially. Note that both C_3 and C_5 are linear in B , and since B is large entry is essentially zero. A good approximation of the employment and vacancy densities is then obtained by multiplying $v(0)$ with B and dividing all densities with B .

$$\begin{aligned} \tilde{v}(0) &= Bv(0) = \theta / [\theta \tilde{C}_3 + \tilde{C}_5] \\ \tilde{C}_5 &= (1 + 1/B) [\bar{a} - (\bar{a} - A_3) \lambda_f / (\sigma + \lambda_f) + A_2 (1 - \lambda_f A_1)] \\ &\quad - [A_3 + A_2 e^{-(\sigma + \lambda_f) \bar{a}}] \\ &\approx [\bar{a} - (\bar{a} - A_3) \lambda_f / (\sigma + \lambda_f) + A_2 (1 - \lambda_f A_1)] - [A_3 + A_2 e^{-(\sigma + \lambda_f) \bar{a}}] \\ \tilde{C}_4 &= (1 + 1/B) \lambda_f (\bar{a} - A_3) / (\sigma + \lambda_f) + A_3 \\ &\approx \lambda_f (\bar{a} - A_3) / (\sigma + \lambda_f) + A_3 \\ \tilde{C}_3 &= (1 + 1/B) \lambda_f (\bar{a} - A_3) / (\sigma + \lambda_f) + A_3 \\ &\approx \lambda_f (\bar{a} - A_3) / (\sigma + \lambda_f) + A_3 \\ \tilde{C}_2 &= (1 + 1/B) (1 - \lambda_f A_1) - e^{-(\sigma + \lambda_f) \bar{a}} \\ &\approx (1 - \lambda_f A_1) - e^{-(\sigma + \lambda_f) \bar{a}} \end{aligned}$$

$$\begin{aligned}
\tilde{C}_1 &= (1 + 1/B) \lambda_f A_1 + e^{-(\sigma + \lambda_f) \bar{a}} \\
&\approx \lambda_f A_1 + e^{-(\sigma + \lambda_f) \bar{a}} \\
\mu(0) &= \tilde{v}(0), \mu(\bar{a}) = \tilde{C}_1 \tilde{v}(0), v(\bar{a}) = \tilde{C}_2 \tilde{v}(0),
\end{aligned}$$

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