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Stochasitc Monitoring, Optimal Capital Structure and Executive Compensation*

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We consider a one period version of our model of financial contracting in which we allow for stochastic monitoring. We characterize the efficient contract and show that it shares many similar features to the characterization with deterministic monitoring in Cole and Atkeson (1). Because of the nature of the court-based enforcement system, we then add the additional requirement that the monitoring decision be self-enforcing. We show that with self-enforcement, it becomes optimal to separate the debt and equity contracts into separate construct with different holders of these claims. We show that the efficient contract with commitment and stochastic monitoring is not self-enforcing, while the efficient contract with deterministic monitoring can be if the expect payment to debt holders is large enough to cover their monitoring costs. Finally, we provide a partial characterization the efficient contract with self-enforcing monitoring.

The self-enforcing condition is an incentive condition on the principal, and implies that the revelation principal does not hold. In fact, we show that the minimally revealing message space - given the outcome being support - is efficient. Moreover, this message space must involve a nondegenerate partition of the type space for this message space to be admissible in the sense of having the possibility of satisfying the self-enforcing condition. We characterize

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the efficient contract for any admissible partition of the type space. We also show by numerical example that the partition of the state space under self-enforcement may not be minimal. That is, it may be efficient to increase the size of partition beyond that which is necessary for admissibility.

1. Model

There is a collection of risk neutral outside investors who are endowed with a production technology that transforms the labor of a manager into output. There are a large number of identical risk averse managers. These managers have an outside opportunity that offers them utility U_0.

The production process takes place over the course of three sub-periods within the period. In the first sub-period, a manager is chosen to operate the production technology. In the second sub-period, this production technology yields output $y = \theta$, where θ is a productivity shock that is idiosyncratic to this technology. In this sub-period, this productivity shock θ and hence, output as well, is private information to the manager. The set of possible shocks is an interval given by Θ and the distribution of these shocks has c.d.f. P with density p and an expected value of one.

In the second sub-period, a payment can be made out of the output of the firm to the outside investors. The outside investors have the option of monitoring the output of the project to learn the realization of the shock θ in order to condition the payment on the level of output rather than on level of output reported by the manager. This monitoring comes at the cost of γ units of output. At the end of the second sub-period, the manager has the option of spending on perquisites that he alone enjoys up to fraction τ of whatever output of the firm that he has not paid out to the outside investors during this sub-period. The output that the manager does not spend on perquisites is productively reinvested in the firm. For simplicity, we assume that the gross return on this productive reinvestment in the firm between the second and third sub-periods is one.[1]

[1] An alternative interpretation is that the manager has become essential to maintaining the value of the residual output in the third sub-period. Without his cooperation the value of this output is reduced by the factor $(1 - \tau)$ and that based upon this, the manager can, in the third subperiod, renegociate his contract. For simplicity, we assume that the manager has all the bargaining power in this renegociation, and hence he is able to demand that the fraction τ of the residual output be given to him.

In the third sub-period, the outside investors can freely observe both the output of the firm θ and the division of this output between spending on perks for the manager and productive reinvestment.

The contracting problem between the outside investors and the manager can be described as follows. A contract between these parties specifies a probability of monitoring by the outside investors m and a payment from the manager to the outside investors in the second sub-period v, and a payment from the outside investors to the manager in the third sub-period. Both the payment to the investors in the second sub-period and the payment to the manager in the third sub-period are conditioned upon whether monitoring has taken place. We denote these payments by v_0 and x_0 if monitoring doesn't take place and by v_1 and x_1 if it does.

We assume that the monitoring probability is a function of the manager's announcement $\hat{\theta}$ of the output of the project in the second sub-period, and denote it by $m(\hat{\theta})$. Initially, we assume that the outside investors can commit to a probabilistic strategy for paying the cost to monitor the output of the project in the second sub-period as a function of the manager's announcement. Later we will relax this assumption.

The payments v from the manager to the outside investors in the second sub-period are contingent on the manager's announcement of the productivity shock $\hat{\theta}$ as well as the outcome of the monitoring decision. Let $v_0(\hat{\theta})$ denote the payment that the manager makes to the outside investors in the second sub-period as a function of the announcement $\hat{\theta}$ in case monitoring does not take place, and let $v_1(\hat{\theta}, \theta)$ denote the payment that the manager makes as a function both of the announcement $\hat{\theta}$ and the true value of θ in case monitoring does take place. Finally, let $x_i(\hat{\theta}, \theta)$ denote the payment from the outside investors to the manager in the third sub-period as a function of his report $\hat{\theta}$ in the second sub-period and the realized production shock θ, where $i = 0$ denotes the case in which monitoring did not take place and $i = 1$ denotes that in which it did.

For reasons of limited liability, we require

$$(1) \quad v_0(\hat{\theta}) \leq \hat{\theta}, \; v_1(\hat{\theta}, \theta) \leq \theta, \; \text{and } x_i(\hat{\theta}, \theta) \geq 0.$$

We assume, without loss of generality, that $x_i(\hat{\theta}, \theta)$ is chosen to ensure that the manager chooses not to take any perks for himself. This assumption implies a constraint on $x_0(\hat{\theta}, \theta)$ and $x_1(\hat{\theta}, \theta)$ that

$$(2) \quad u(x_0(\hat{\theta}, \theta)) \geq u(\tau(\theta - v_0(\hat{\theta}))) \text{ for all } \hat{\theta}, \theta$$
$$u(x_1(\hat{\theta}, \theta)) \geq u(\tau(\theta - v_1(\hat{\theta}, \theta))) \text{ for all } \hat{\theta}, \theta.$$

Given the terms of the contract, m, v_0, v_1, x_0, and x_1 the manager chooses a strategy for reporting θ denoted $\sigma(\theta)$. We say that the report $\sigma(\theta) = \hat{\theta}$ is *feasible* given v_0 and θ if $v_0(\hat{\theta}) \leq \theta$. Note that this definition requires that the manager has the resources to make the payment $v_0(\sigma(\theta))$ in the event that he reports $\sigma(\theta) = \hat{\theta}$. We require the manager to choose a reporting strategies such that $\sigma(\theta)$ is feasible given v_0 for all θ. We interpret this constraint as following from the assumption that there is an optimal contract in which the outside investors choose to monitor if the manager announces $\hat{\theta}$ but then does not pay $v_0(\hat{\theta})$ and that $x(\hat{\theta}, \theta) = 0$ in this event.

We restrict attention to contracts in which the manager truthfully reports θ. Hence, we impose the incentive constraint

$$(3) \quad m(\theta)u(x_1(\theta, \theta)) + (1 - m(\theta))u(x_0(\theta, \theta)) \geq m(\hat{\theta})u(x_1(\hat{\theta}, \theta)) + (1 - m(\hat{\theta}))u(x_0(\hat{\theta}, \theta))$$
$$\text{for all } \theta \in \Theta \text{ and feasible } \hat{\theta} \text{ given } \theta \text{ and } v_0.$$

The managers payoff under the contract is given by the expectation of $u(x_i(\theta, \theta))$. Since managers have an outside opportunity that delivers them utility U_0, we require the individual rationality constraint

$$(4) \quad \int \left[m(\theta)u(x_1(\theta, \theta)) + (1 - m(\theta))u(x_0(\theta, \theta))\right] p(\theta)d\theta \geq U_0.$$

2. Characterizing an efficient contract

We characterize a contract that maximizes the expected payoff to the outside investors

$$(5) \quad \int \left[\theta - \gamma m(\theta) - m(\theta)x_1(\theta,\theta) - (1-m(\theta))x_0(\theta,\theta)\right] p(\theta)d\theta$$

subject to the constraints (1), (2), (3), (4) and

$$(6) \quad m(\theta) \in [0,1] \text{ for all } \theta \in \Theta,$$

in the following propositions. We refer to such a contract as an efficient contract.

PROPOSITION 1. *There is an efficient contract with the following properties:* (i) $v_1(\hat{\theta},\theta) = \theta$ *and* $v_0(\hat{\theta}) = \hat{\theta}$, ($ii$) $x_1(\hat{\theta},\theta) = 0$ *and* $x_0(\hat{\theta},\theta) = \tau(\theta - \hat{\theta})$ *for* $\theta \neq \hat{\theta}$, *and* (iii) $x_0(\theta,\theta) = x_1(\theta,\theta)$.

Proof: To prove part (i), note that v_1 and v_0 only show up in the no-perks constraint (2), and that increasing their values to the maximum extent allowed by the limited liability constraint (1) relaxes the no-perks constraint. Hence, it is efficient to do so. Given (i) and the fact that it is efficient to punish misreporting to the maximum feasible extent, since it will not occur in equilibrium, (ii) follows. Given (i), it follows that under truthtelling the no-perks constraints cannot bind, and hence, fixing the expected utility provided to the manager in state θ at $y(\theta)$, where

$$y(\theta) = m(\theta)u(x_1(\theta,\theta)) + (1-m(\theta))u(x_0(\theta,\theta)),$$

it follows that any efficient contract must minimize the cost of delivering this utility. In other words, it must solve the following sub-problem

$$\min_{x_0,x_1} m(\theta)x_1 + (1-m(\theta))x_0 \text{ s.t. } m(\theta)u(x_1) + (1-m(\theta))u(x_0) \geq y(\theta).$$

The concavity of u implies that this minimization is achieved with $x_1 = x_0$. $Q.E.D.$

Given proposition 1, we can simplify the problem to be one of choosing $[m(\theta), w(\theta)]$ so as to

$$(7) \quad \max \int [\theta - w(\theta) - \gamma m(\theta)]\, p(\theta) d\theta$$

subject to

$$(8) \quad \int u(w(\theta)) p(\theta) d\theta = U$$

$$(9) \quad u(w(\theta)) \geq \sup_{\tilde{\theta} < \theta} (1 - m(\theta))\, u\left(\tau \left[\theta - \tilde{\theta}\right]\right),$$

where we now use $w(\theta)$ to denote $x(\theta, \theta)$.

PROPOSITION 2. *In any solution to the simplified problem:*

(i) $w(\theta)$ is such that there exists a $\bar{\theta}$ such that $w(\theta) = \bar{w}$ for all $\theta \leq \bar{\theta}$, and strictly increasing thereafter,

(ii) $m(\theta)$ is weakly decreasing.

If θ has finite upper support and if $\bar{w}$ is strictly greater than 0, then

(iii) $m(\theta) < 1$ and is strictly decreasing in the interior, and

(iv) there will exist a cutoff $\hat{\theta} < \sup(\Theta)$ such that $m(\theta) = 0$ for all $\theta > \hat{\theta}$.

Proof: We first form the Lagrangian, which we formulate in terms of choosing the monitoring probabilities $m(\theta)$ and the utility level of the manager $y(\theta)$ to yield a convex constraint set.

$$(10) \quad L = \int_{\theta} \left\{ \begin{array}{c} [\theta - C(y(\theta)) - \gamma m(\theta)] + \lambda [y(\theta) - U] \\ \int_0^{\theta} \delta(\theta, \tilde{\theta}) \left[y(\theta) - (1 - m(\tilde{\theta})) u(\tau \left[\theta - \tilde{\theta}\right]) \right] p(\tilde{\theta}) d\tilde{\theta} \\ + \chi^{+}(\theta)(1 - m(\theta)) + \chi^{-}(\theta) m(\theta)) \end{array} \right\} p(\theta) d\theta,$$

where $C(x) = u^{-1}(x)$, λ is the multiplier on the promise keeping constraint and $\delta(\theta, \tilde{\theta})$ is the multiplier on the incentive constraint (9) with respect to the deviation of reporting $\tilde{\theta}$ given a realization θ. The first-order conditions for this problem are

$$(11) \quad C'(y(\theta)) = \lambda + \int_0^{\theta} \delta(\theta, \tilde{\theta}) p(\tilde{\theta}) d\tilde{\theta}$$

(12) $\gamma = \int_{\theta}^{\infty} \delta(\theta', \theta) u(\tau(\theta' - \theta)) p(\theta') d\theta' - \chi^{+}(\theta) + \chi^{-}(\theta).$

Condition (11) implies that if constraint (9) doesn't bind, then $y(\theta) = \bar{u}$, and moreover that $y(\theta) \geq \bar{u}$ for all θ. In terms of the compensation of the manager, this implies that $w(\theta) = \bar{w}$ when the incentive constraint doesn't bind, where $\bar{w} = C(\bar{u})$. To see that if the constraint binds at θ_1 and if $\theta_2 > \theta_1$, then it binds at θ_2, and moreover that $w(\theta_1) < w(\theta_2)$, note that $(1 - m(\tilde{\theta}))u(\tau \left[\theta - \tilde{\theta}\right])$ is strictly increasing in θ, which implies that $y(\theta_2) > y(\theta_1)$, and hence

$$w(\theta_2) = C(u(\theta_2) > C(\theta_1) = w(\theta_1)$$

which this proves (i).

To prove (ii), note that condition (12) implies that if $m(\theta) > 0$, then it must be the case that $\delta(\tilde{\theta}, \theta) > 0$ for some $\tilde{\theta} > \theta$, or in other words that deviating and reporting θ binds on some type $\tilde{\theta}$. This result implies that if we take $y(\theta)$ as given, and define for each θ the probabilities of monitoring such that deviating is not weakly preferred, $\pi^{\theta}(\tilde{\theta})$, as follows

$$(13) \quad \pi^{\theta}(\tilde{\theta}) = \begin{cases} \max\left\{0, 1 - \frac{y(\theta)}{u(\tau\left[\theta - \tilde{\theta}\right])}\right\} & \text{if } \tilde{\theta} < \theta \\ 0 & \text{o.w.} \end{cases},$$

then, efficiency implies that $m(\theta) = \sup_{\tilde{\theta}}\{\pi^{\tilde{\theta}}(\theta)\}$. In other words, the probability of monitoring is positive only if it binds for some type. (Note that by filling in the required probabilities with zeros we removed the need to shrink the type space that could deviate to θ as θ increased.) The function $\pi^{\theta}(\tilde{\theta})$ is weakly decreasing in θ for fixed $\tilde{\theta}$, and the sup over a set of weakly decreasing functions is weakly decreasing.

We turn now to the case in which θ has finite upper support. To prove (iii), note first that $y(\theta) \geq \bar{u} = u(\bar{w}) > 0$. Then note that $(1 - m(\theta))u\left(\tau\left[\theta - \tilde{\theta}\right]\right) \leq (1 - m(\theta))u(\tau(\sup(\theta))$, and that since $u(\tau(\sup(\theta))$ is finite, the rhs goes to zero as $m(\theta)$ goes to one. Hence, $m(\theta) < 1$ for all θ.

To prove that m is strictly decreasing in the interior when θ has finite upper support, assume that the reverse was true. That is, assume that for $\theta \in [\theta_1, \theta_2]$, $m(\theta) = a$, where

$0 < a < 1$ and $\theta_2 - \theta_1 > 0$. Then, note that since for all finite $\theta > \theta_1$,

$$(1-a)u(\tau\left[\theta - \theta_1\right)]) > (1-a)u(\tau\left[\theta - \min(\theta_2, \theta))\right]),$$

this contracts our earlier result that monitoring is only positive if it binds for some type.

To prove (iv), note that the incentive constraint (9) implies that if $\bar{w} > \tau(\theta - \hat{\theta})$, then deviating and making a report of $\hat{\theta}$ can only lower the payoff of the manager. Hence, for all θ such that $\tau(\sup(\Theta) - \theta) < \bar{w}$, monitoring is unnecessary and will optimally be set to zero. Note that since $w(\theta) \geq \bar{w}$, the interval over which $m(\theta) = 0$, may be substantially larger than this simple bound would imply. *Q.E.D.*

PROPOSITION 3. *Since monitoring goes to zero above some cutoff* $\hat{\theta}$; *i.e.* $m(\theta) = 0$ *for all* $\theta > \hat{\theta}$; *it is efficient for* $v_0(\theta) = \hat{\theta}$ *for all* $\theta > \hat{\theta}$.

Proof: The cessation in monitoring implies that

$$u\left(\tau\left[\theta - \hat{\theta}\right]\right) \geq \sup_{\tilde{\theta} \in [\hat{\theta}, \theta)} \left(1 - m(\tilde{\theta})\right) u\left(\tau\left[\theta - \tilde{\theta}\right]\right) = \sup_{\tilde{\theta} \in [\hat{\theta}, \theta)} u\left(\tau\left[\theta - \tilde{\theta}\right]\right),$$

and hence there is no further relaxation in the incentive constraint from higher payments in the second sub-period above the level of the monitoring threshold. *Q.E.D.*

As the gap between proposition 2 and proposition 3. makes clear, the exact form of $v_0(\theta)$ is only pinned down for θ such that $m(\theta) > 0$, and either of the two extremes indicated under proposition 2 or proposition 3. is efficient. This is because monitoring at a given θ is done to dissuade misreporting by managers with higher realizations of the shock. If monitoring ceases, above some threshold $\hat{\theta}$, then it is efficient to have the payments in the second sub-period take on a debt like structure in which up to the threshold the manager pays out everything, and above $\hat{\theta}$ he pays out the flat amount θ^*. However, for θ such that positive monitoring takes place, then setting $v_0(\theta) = \theta$ is necessary for efficiency for those realizations the no-perks constraint binds, and raising the second sub-period payment strictly relaxes this constraint. Hence, it must be set as high as possible for efficiency.

A. Contrasting Stochastic and Deterministic Monitoring

Here, we restrict ourselves to deterministic monitoring in order to exhibit the close connection between the two. Formally, we impose the restriction that $m(\theta) \in \{0, 1\}$. Now $v_0(\hat{\theta})$ is vacuous when $m(\hat{\theta}) = 1$ and $v_1(\hat{\theta}, \theta)$ is vacuous in the reverse case when $m(\hat{\theta}) = 0$. However, none of the results in proposition 1 depended on monitoring not being stochastic. So that characterization still holds. Moreover, the logic of propositions 2 and 3 carries over with this restriction on $m(\theta)$. We summarize these results in the following preposition. First through, a bit of useful notation. Let M denote the set of reports which induce monitoring; where $M = \{\theta : m(\theta) = 1\}$; and let θ^* denote the lower support of the set of reports that don't induce monitoring; where $\theta^* = \inf\{\theta : \theta \notin M\}$.

Proposition 4. *There is an efficient contract with the following properties: (i) $v_1(\hat{\theta}, \theta) = \theta$ for all $\hat{\theta}$ s.t. $m(\hat{\theta}) = 1$ and $v_0(\hat{\theta}) = \theta^*$ for all $\hat{\theta} \notin M$, where $\theta^* = \inf\left\{\hat{\theta} | \hat{\theta} \notin M\right\}$, (ii) M is an interval ranging from 0 to θ^*, and (iii) the payments to the manager $w(\theta)$ have the form $w(\theta) = \bar{w}$ for $\theta \leq \bar{\theta}$ and $w(\theta) = \tau(\theta - \theta^*)$ for $\theta > \bar{\theta}$, where $\bar{\theta}$ is the solution to $\bar{w} = \tau\left(\bar{\theta} - \theta^*\right)$.*

Proof: See Atkeson and Cole (1).

Thus the results with stochastic and deterministic monitoring are very similar. Moreover, the result that the stochastic monitoring schedule is strictly decreasing for all nonzero monitoring levels becomes, under deterministic monitoring the result that the monitoring is weakly decreasing, which implies that it is done on the an interval of shock reports starting from the lowest level. With respect to compensation, the results are somewhat more stark with deterministic monitoring. The efficient contract is completely specified by the base pay of the manager, $\bar{w}$ and the upper support of the monitoring set θ^*. Monitoring takes place for any report $\hat{\theta} < \theta^*$, and when monitoring takes place the outside investors take everything. In the case of a report that doesn't lead to monitoring, $\theta \geq \theta^*$, the second sub-period payment is θ^*. The compensation of the manager is the max $[\bar{w}, \tau(\theta - \theta^*)]$ so long as he tells the truth, and the minimum possible if he doesn't.

3. Self-Enforcing Monitoring

Court systems require that one of the parties to an agreement bring a complaint before they will consider enforcing it. This fact has been seen as suggesting that a natural condition to require of our efficient contract is that the outside investors making the monitoring decision have a positive motivation to undertake it. For monitoring to be self-enforcing, the gap in the expected net payments for monitoring v.s. not monitoring must be weakly greater than the cost γ for any reports that lead to monitoring, and strictly equal to γ if monitoring is occurring with a probability between 0 and 1.

This requirement is in effect an incentive constraint on the principals to the contract (in this case, the outside investors). The addition of this incentive constraint and the resulting lack of complete enforcement means that we can no longer appeal to the revelation principal to determine the nature of the message space in the contract. In particular, the truth-telling equilibrium of a direct mechanism has the potential of giving the principals too much information which can make satisfying their incentive constraint with respect to their monitoring decision difficult.

Consider an arbitrary message space denoted by Ω. Given this message space, we can denote a contract by $v_0^\omega(\omega)$, $v_1^\omega(\omega,\theta)$, $m^\omega(\omega)$, $x_0^\omega(\omega,\theta)$ and $x_1^\omega(\omega,\theta)$, along with a reporting strategy $\sigma^\omega : \Theta \to \Omega$. The analog to our conditions under commitment are given by the following constraints:

The *feasibility conditions* for the ω-contract are

$$v_0^\omega(\omega) \leq \inf\left\{\theta : \sigma^\omega(\theta) = \omega\right\},$$

$$v_1^\omega(\omega,\theta) \leq \theta,$$

$$x_i^\omega(\omega,\theta) \geq 0$$

The *incentive condition* for the ω-contract is given by

$$m^\omega(\sigma^\omega(\theta))u\left(x_1^\omega(\sigma^\omega(\theta),\theta)\right) + (1 - m^\omega(\sigma^\omega(\theta)))u\left(x_0^\omega(\sigma^\omega(\theta))\right)$$

$$\geq m^\omega(\hat{\omega})u\left(x_1^\omega(\hat{\omega},\theta)\right) + (1 - m^\omega(\hat{\omega}))u\left(x_0^\omega(\hat{\omega})\right) \;\; \forall \hat{\omega} \in \Omega \;\&\; \theta \in \Theta.$$

The *no-perks constraint* for the ω-contract is given by

$$u\left(x_i^\omega(\omega,\theta)\right) \geq u\left(\tau(\theta - v_i(\omega,\theta))\right).$$

The *promise keeping constraint* for the ω-contract is given by

$$\int \left[m^{\omega}(\sigma(\theta))u(x_1^{\omega}(\sigma^{\omega}(\theta),\theta)) + (1 - m^{\omega}(\sigma(\theta)))u(x_0^{\omega}(\sigma^{\omega}(\theta),\theta)) \right] p(\theta)d\theta \geq U_0.$$

A. Claim Separation

We turn next to formalizing our the self-enforcing condition for the monitors. To make monitoring self-enforcing, it is necessary that the net expected payment be equal to zero if $m(\theta) \in (0,1)$, nonnegative if $m(\theta) = 1$ and nonpositive if $m(\theta) = 0$. However, the net expected gain depends upon how the claims to output are distributed among the outside investors and the mechanism that initiates monitoring.

With respect to the distribution of claims, it matters whether or not the second sub-period and the third sub-period claims can be thought of as being held by one joint investor, or whether or not there are two separate investors holding each of these claims (and of course all the convex combinations in between).

To make this point, it will be convenient to define the expected payment conditional on monitoring probability $m^{\omega}(\hat{\omega})$ as $\tilde{v}_1^{\omega}(\hat{\omega})$, where

$$\tilde{v}_1^{\omega}(\hat{\omega}) = \frac{\int_{\{\theta:\sigma^{\omega}(\theta)=\hat{\omega}\}} v_1^{\omega}(\hat{\omega},\theta)p(\theta)d\theta}{\int_{\{\theta:\sigma^{\omega}(\theta)=\hat{\omega}\}} p(\theta)d\theta}$$

Next consider the net payout in two extreme cases in which (i) an investor held all of the claims and (ii) one investor held the claims to second sub-period payouts and a second investor held the claims to third sub-period payouts:

1. **Unseparated Claims Condition:** The expected gain to the monitors is

   $$\left[\tilde{v}_1^{\omega}(\hat{\omega}) - v_0^{\omega}(\hat{\omega})\right]\tau - \gamma,$$

   since the agency cost will lose them at most only the fraction τ of what is not paid out in the second sub-period

2. **Separated Claims Condition:** The expected gain to the monitors is

   $$\left[\tilde{v}_1^{\omega}(\hat{\omega}) - v_0^{\omega}(\hat{\omega})\right] - \gamma,$$

since now the holders to the claim on the second sub-period payment will lose the full amount of anything them don't collect in this period. Hence, separated claims imply a larger gap in the expected net payments for a given gap between $v_1^\omega(\hat{\omega})$ and $v_0^\omega(\hat{\omega})$.

The actual extent of the separation of the claims need to induce the right incentives depends upon how monitoring decision is made. For example, if all of the holders of second sub-period claims had to agree to monitoring, then we would need complete separation. If only a majority of the holders had to want it, then we would need that a majority of the second sub-period claims were held by investors who didn't also hold claims to the third sub-period. If only a single debt holder could trigger monitoring, then we would need that only a small portion of the claims were held by an outside investor who didn't also hold claims to the third period payments.

Definition: We therefore define an ω-contract to *self-enforcing* if there exists a $\phi \in [\gamma, \gamma/\tau]$ such that for all $\omega \in \Omega$, the expected differential between monitoring and not monitoring is equal to ϕ if the monitoring probability is interior, and greater than equal to ϕ if it is occurring with probability one. Formally, this is the requirement that for all $\hat{\omega} \in \Omega$,

$$[\tilde{v}_1^\omega(\hat{\omega}) - v_0^\omega(\hat{\omega})] = \phi \text{ if } 0 < m(\hat{\omega}) < 1,$$

$$[\tilde{v}_1^\omega(\hat{\omega}) - v_0^\omega(\hat{\omega})] \geq \phi \text{ if } m(\hat{\omega}) = 1,$$

$$(14) \quad [\tilde{v}_1^\omega(\hat{\omega}) - v_0^\omega(\hat{\omega})] \leq \phi \text{ if } m(\hat{\omega}) = 0$$

This definition imposes the weakest form of a self-enforcement constraint on the ω-contract. It merely requires that the contract be such that there exists degree of separation (and a voting rule for initiating monitoring) which will enable the contract to be self-enforcing. One of the impacts of this constraint will be to force similar ω reports for some θ values in order to satisfy this constraint. This will lead to the original direct optimal contract not being able to satisfy the monitoring self-enforcing condition. For example, if there was a distinct report $\hat{\omega}$ for any $\theta < \gamma$, then it would not be possible to satisfy this constraint and have a positive probability of monitoring.

B. Reconsidering our commitment solutions

Here we examine whether or not the analogs to the solutions with commitment with either stochastic or deterministic monitoring can satisfy the monitoring self-enforcement condition. We will show that trivially the stochastic monitoring solution cannot. However, we will show that for the case of deterministic monitoring, if the amount being paid out on the debt claim (which pays off in the second sub-period) is sufficient large, then the efficient contract with deterministic monitoring is self-enforcing. This result both provides a partial rationalization for deterministic monitoring and gives further insight into how the self-enforcing constraint is altering the efficient contract.

The commitment contract with stochastic monitoring violates the self-enforcement condition at every point at which monitoring is strictly interior. To see this note that if an ω-contract is to replicate the outcomes of the efficient contract with stochastic monitoring and commitment, it must be the case that (i) $m^{\omega}(\sigma^{\omega}(\theta)) = m(\theta)$, (ii) $v_1^{\omega}(\sigma^{\omega}(\theta), \theta) = v_1(\theta, \theta)$ and (iii) $v_0^{\omega}(\sigma^{\omega}(\theta)) = v_0(\theta)$. But, we have already shown that if $m(\theta)$ is strictly interior at θ, this implies that it is also is strictly decreasing at θ from proposition 2, which implies that $m(\theta)$ is invertible at this value of θ. Since the monitors must know the probability with which they are suppose to monitor, this implies that they will also be able to infer that the state is θ, since $m^{\omega}(\sigma^{\omega}(\theta))$ will also be invertible. Hence under the associated ω-contract, for $\omega = \sigma^{\omega}(\theta)$,

$$\tilde{v}_1^{\omega}(\omega') - v_0^{\omega}(\omega') = E\{v_1(\theta) - v_0(\theta)\} = 0.$$

Given that our earlier result that $m(\theta)$ being positive implies that (9) binds, this then leads to our result since there is no slack within which to adjust v_1 and v_0 to achieve self-enforcement. To see this note that since $v_1(\theta) = v_0(\theta) = \theta$, we cannot raise $v_1(\theta)$ without violating feasibility. But we cannot we lower $v_0(\theta)$ without violating (9), since this constraint binds.

The invertibility of the commitment stochastic monitoring schedule whenever it was interior was a key factor in precluding a self-enforcing ω-contract analog. However, the deterministic contract imposes a two part partition of the state space, which avoids the invertibility problem for the ω-contract analog. Hence, the only factor in whether or not it is self-enforcing is whether the expected payment under monitoring is sufficient to cover the costs of monitoring.

PROPOSITION 5. *If an efficient contract with deterministic monitoring satisfies*

$$E\{\theta|\theta \leq \theta^*\} > \gamma,$$

then monitoring in the associated ω-contract is self-enforcing.

Proof: We can construct the associated ω-contract as follows:

$$\begin{aligned}
\Omega &= \{0,1\} \\
\sigma^\omega(\theta) &= \begin{cases} 0 \text{ if } \theta \leq \theta^* \\ 1 \text{ o.w.} \end{cases} \\
m^\omega(\omega) &= \begin{cases} 1 \text{ if } \omega = 0 \\ 0 \text{ o.w.} \end{cases} \\
v_0^\omega(\omega) &= \begin{cases} 0 \text{ if } \omega = 0 \\ \theta^* - \gamma \text{ o.w.} \end{cases} \\
v_1^\omega(\omega,\theta) &= \theta \\
x_0^\omega(\omega,\theta) &= \begin{cases} x_0(\theta,\theta) \text{ if } \theta \in \sigma^{\omega^{-1}}(\omega) \\ \tau\theta \text{ o.w.} \end{cases} \\
x_1^\omega(\omega,\theta) &= \begin{cases} x_1(\theta,\theta) \text{ if } \theta \in \sigma^{\omega^{-1}}(\omega) \\ 0 \text{ o.w.} \end{cases}
\end{aligned}$$

By construction, the payoffs to the principals and the manager are the same since x_i^ω is the same as x_i for each θ if the manager reports according to σ^ω in the ω-contracting game and tells the truth under the direct mechanism in the commitment case. Similar logic also implies the no-perks constraint given that the strategies are followed. With respect to the case when an agent misreports, the only relevant misreport is to say $\omega = 0$ when $\theta > \theta^*$ in the ω-contracting game, and report some $\theta < \theta^*$ in the direct mechanism. In both cases the payment conditional on not monitoring is 0, and hence the ω-contracts no-perks constraint is implied by that under the direct mechanism. Similar logic show that the incentive constraint under the ω-contract is implied by that under the direct mechanism. Hence, all that remains to be verified is that the self-enforcing condition for the monitors is satisfied. Note that

$$E\{\tilde{v}_1^\omega(0) - v_0^\omega(0)|\omega = 0\} = E\{\theta|\theta \leq \theta^*\} - 0 > \gamma,$$

and that

$$E\{\tilde{v}_1^\omega(1) - v_0^\omega(1)|\omega = 1\} > E\{\theta|\theta > \theta^*\} - (\theta^* - \gamma) > \gamma$$

and, hence our self-enforcing condition is satisfied. $Q.E.D.$

C. An Efficient Message Space

Here we prove that a message space consisting of monitoring probabilities, which we denote by π, and no-monitoring payments, which we denote by v_0, is sufficient to replicate the outcomes of any other mechanism with an alternative message space. One thing to note here is that this message space the least revealing possible given the activities of the monitors that are being supported. The monitors must know with which probability they are suppose to monitor, and they must also know whether or not the appropraite payment in the case of nonmonitoring has been made, because otherwise they could not credibly threaten to monitor and take everything if this payment wasn't made. Hence, at a minimum, both π and v_0 must be conveyed to them. This is the opposite of the standard revelation principal in which the message space is the space of the agent's information, Θ, which is the most revealing possible.

Let $\Lambda = m(\Omega) \times v_0^\omega(\Omega)$. Define the new λ-contract by setting

$$v_1^\lambda(\pi, v_0, \theta) = v_1^\omega(\hat{\omega}, \theta),$$
$$x_i^\lambda(\pi, v_0, \theta) = x_i^\omega(\hat{\omega}, \theta),$$

and reporting strategy

$$[\pi^\lambda(\theta), v_0^\lambda(\theta)] = [m^\omega(\theta), v_0^\omega(\theta)],$$

where

$$\hat{\omega} : m^{\omega}(\hat{\omega}) = \pi \text{ and } v_0^{\omega}(\hat{\omega}) = v_0.$$

PROPOSITION 6. *The λ-contract deliverers the same payoffs as the ω-contract, and if the ω-contract satisfies our constraints, then so too does the λ-contract.*

Proof: The payoff part follows trivially. With respect to the constraints, the feasibility, no-perks, and promise-keeping constraints also follows trivially. Hence, all that needs to be checked is the incentive constraints on the manager and the monitors.

To see that the analog incentive constraint on the manager holds for the λ-contract, assume otherwise. That is, for some θ there exists an $(\pi', v_0') \in \Lambda$ report, where $\pi' \neq \pi^{\lambda}(\theta)$ or $v_0' \neq v_0^{\lambda}(\theta)$, for which

$$\begin{aligned}&\pi^{\lambda}(\theta) u\left(x_1^{\lambda}(\pi^{\lambda}(\theta), v_0^{\lambda}(\theta), \theta)\right) + (1 - \pi^{\lambda}(\theta)) u\left(x_0^{\lambda}(\pi^{\lambda}(\theta), v_0^{\lambda}(\theta), \theta)\right) \\ &< \pi' u\left(x_1^{\lambda}(\pi', v_0', \theta)\right) + (1 - \pi') u\left(x_0^{\lambda}(\pi', v_0', \theta)\right).\end{aligned}$$

But then this implies that the incentive constraint cannot hold for θ when the manager considers the report

$$\omega' : m^{\omega}(\omega') = \pi' \text{ and } v_0^{\omega}(\omega') = v_0'.$$

Hence, this cannot be the case.

Next consider the incentive constraint for the monitors and assume that it does not hold with respect to the λ-contract, despite holding for the original ω-contract. We start first with the case in which the monitoring probability is interior, and assume that there exists a $(\pi, v_0) \in \Lambda$ report for which

$$\left[E\left\{v_1^{\lambda}(\pi, v_0, \theta) | \theta : \pi^{\lambda}(\theta) = \pi \,\&\, v_0^{\lambda}(\theta) = v_0\right\} - v_0\right] \neq \phi \text{ for } \pi \in (0, 1).$$

But, note that

$$\begin{aligned} & E\left\{v_1^\lambda(\pi, v_0, \theta)|\theta : \pi^\lambda(\theta) = \pi \ \& \ v_0^\lambda(\theta) = v_0\right\} \\ = \ & \frac{\sum_{\omega \in \hat{\Omega}} E\left\{v_1^\omega(\omega, \theta)|\omega\right\} \Pr\{\theta : \sigma^\omega(\theta) = \omega\}}{\sum_{\omega \in \hat{\Omega}} \Pr\{\theta : \sigma^\omega(\theta) = \omega\}}, \end{aligned}$$

where

$$\hat{\Omega} = \{\omega \in \Omega : m^\omega(\omega) = \pi \ \& \ v_0^\omega(\omega) = v_0\}.$$

But, $E\left\{v_1^\omega(\omega, \theta)|\omega\right\} - v_0^\omega(\omega) = \phi$ from the monitoring incentive constraint on the ω-contract, which generates a contradiction. *Q.E.D.*

D. Efficiency and Partitions

The message space along with the reporting strategy partition the type space. Here we start with a given partition of the type space and, taking the separation parameter ϕ in the monitoring self-enforcement condition as given, characterize the efficient contract.

Let $\{\Theta_1, \Theta_{2,}...\}$ denote such a partition. For it to be physically feasible to satisfy the monitoring condition, it must be the case that

$$E\{\theta|\Theta_i\} \geq \gamma \text{ for all } i.$$

Given a physically feasible partition, and our results with respect to the efficient message space, we can formulate the contracting problem as choosing $[\pi_i, v_{0i}]_{i=1,\ldots,n}$ and functions $[v_1(\theta), x_0(\theta), x_1(\theta)]$ so as to

$$\max \sum_i \int_{\Theta_i} \left[\theta - \gamma\pi_i - \pi_i x_1(\theta) - (1 - \pi_i)x_0(\theta)\right] p(\theta)d\theta$$

subject to

$$\sum_i \int_{\Theta_i} \left[\pi_i u(x_1(\theta)) + (1 - \pi_i)u(x_0(\theta))\right] p(\theta)d\theta \geq U$$

$$\pi_i u(x_1(\theta)) + (1-\pi_i)u(x_0(\theta)) \geq (1-\pi_j)u(\tau(\theta - v_{0j}))$$
$$\text{for each } \theta, \text{ and each } j \neq i$$

$$x_0(\theta) \geq \tau(\theta - v_{0i}) \text{ for each } \theta$$
$$x_1(\theta) \geq \tau(\theta - v_1(\theta)) \text{ for each } \theta$$

$$v_{0i} \leq \inf\{\theta \in \Theta_i\} \text{ for } i = 1, ..., n$$

$$v_1(\theta) \leq \theta \text{ for each } \theta$$

$$\int [v_1(\theta) - v_{0i} - \phi] \frac{p(\theta)}{\Pr\{\Theta_i\}} d\theta \begin{array}{l} \geq 0 \text{ if } \pi_i = 1 \\ = 0 \text{ if } \pi_i \in (0,1) \\ \leq 0 \text{ if } \pi_i = 0 \end{array} \text{ for } i = 1, ..., n.$$

PROPOSITION 7. *It is efficient to set* $v_{0i} = \min[E\{\theta|\theta \in \Theta_i\} - \phi, \min(\theta \in \Theta_i)]$. *Additionally, if* $v_{0i} < \min(\theta \in \Theta_i)$, *then it is efficient to set* $v_1(\theta) = \theta$ *for all* $\theta \in \Theta_i$.

Proof: The logic here is largely in the case with commitment. The only caveat now is the incentive constraint for monitoring. However, if both v_{0i} and $v_1(\theta)$ can be raised without violating the upper bound constraints on v_{0i} and v_1 or the incentive constraint for the monitors, then it is always efficient to do so. If $E\{\theta|\theta \in \Theta_i\} - \phi < \min(\theta \in \Theta_i)$, then it is possible to raise v_{0i} to $E\{\theta|\theta \in \Theta_i\} - \phi$ and set $v_1(\theta) = \theta$. Moreover, it is not possible to set these variables any higher. If the reverse is true, then is always possible to raise v_{0i} to $\min(\theta \in \Theta_i)$ without violating the incentive constraint for monitoring by also raising some of those $v_1(\theta)$ who were below their upper bound of θ. *Q.E.D.*

Taking v_{0i} as set according to the above proposition, we can form the Lagrangian for this problem as

$$
\begin{aligned}
L = \min_{\lambda,\delta,\gamma_0,\gamma_1} & \max_{\{\pi_i, v_{0i}\}\{x_0(\theta), x_1(\theta)\}} \\
& \sum_i \int_{\Theta_i} \left[\theta - \gamma\pi_i - \pi_i x_1(\theta) - (1-\pi_i)x_0(\theta)\right] p(\theta)d\theta \\
& + \lambda \left\{ \sum_i \int_{\Theta_i} \left[\pi_i u(x_1(\theta)) + (1-\pi_i)u(x_0(\theta))\right] p(\theta)d\theta \geq U \right\} \\
& + \sum_i \sum_{j \neq i} \int_{\Theta_i} \left[\begin{array}{c} \pi_i u(x_1(\theta)) + (1-\pi_i)u(x_0(\theta)) \\ -(1-\pi_j)u(\tau(\theta - v_{0j})) \end{array} \right] \delta(\theta, j)p(\theta)d\theta \\
& + \sum_i \int_{\Theta_i} \left[x_0(\theta) - \tau(\theta - v_{0i})\right] \gamma_0(\theta)p(\theta)d\theta \\
& + \sum_i \int_{\Theta_i} \left[x_1(\theta) - \tau(\theta - v_1(\theta))\right] \gamma_1(\theta)p(\theta)d\theta \\
& + \sum_i \int_{\Theta_i} \left[\theta - v_1(\theta)\right] \phi_1(\theta)p(\theta)d\theta \\
& + \sum_i \left\{ \int_{\Theta_i} (v_1(\theta) - v_{0i} - \phi) \frac{p(\theta)}{\Pr(\Theta_i)} d\theta \right\} \psi_i \Pr\{\Theta_i\}
\end{aligned}
$$

where λ is the multiplier on the promise-keeping condition, $\delta(\theta, j)p(\theta)$ is the multiplier on the incentive constraint for the manager who has income θ and is considering report j, $\gamma_0(\theta)p(\theta)$ and $\gamma_1(\theta)p(\theta)$ are the multipliers on the no-perks constraints on $x_0(\theta)$ and $x_1(\theta)$ respectively, $\phi_1(\theta)p(\theta)$ are the multipliers on the physical upper bounds on $v_1(\theta)$, and $\psi_i \Pr\{\Theta_i\}$ is the multiplier on the incentive constraint with respect to monitoring.

The associated first-order conditions are

$$
\begin{aligned}
\pi_i : 0 = & \int_{\Theta_i} \left\{ \begin{array}{c} -\gamma - (x_1(\theta) - x_0(\theta)) + \\ \left(\lambda + \sum_{j \neq i} \delta(\theta, j)\right) \left[u(x_1(\theta)) - u(x_0(\theta)\right] \end{array} \right\} p(\theta)d\theta \\
& + \sum_{j \neq i} \left\{ \int_{\Theta_j} \delta(\theta, i) \left[u(\tau(\theta - v_{0i}))\right] \right\}
\end{aligned}
$$

$$
x_1(\theta) : 0 = \left\{ -\pi_i + \pi_i \left(\lambda + \sum_{j \neq i} \delta(\theta, j) \right) u'(x_1(\theta)) + \gamma_1(\theta) \right\}
$$

$$x_0(\theta) : 0 = \left\{ -(1-\pi_i) + (1-\pi_i)\left(\lambda + \sum_{j \neq i} \delta(\theta, j)\right) u'(x_0(\theta)) + \gamma_0(\theta) \right\}$$

$$v_1 : 0 = [\gamma_1(\theta) - \phi_1(\theta) + \psi_i]\, p(\theta) d\theta$$

These conditions are the analogs of what we saw with respect to the commitment case. If none of the incentive, and no-perks constraints bind, then once again it is easy to see that there is an efficient level of consumption for the manager which is constant.

Denote this level of consumption by $\bar{x}$, where $\lambda u'(\bar{x}) = 1$. Even when the incentive constraints on the manager, bind, thereby inducing a higher level of consumption for that θ, so long as the $v_1(\theta)$ and v_{0i} can be set high enough so that the no-perks constraints don't bind, then $\gamma_1(\theta) = \gamma_0(\theta) = 0$, and it is straightforward from the first-order conditions to see that $x_0(\theta) = x_1(\theta)$. However, to the extent that the introduction of the incentive constraint on monitoring and the resultant use of partitions precludes setting $v_1(\theta)$ and v_{0i} high enough to prevent the no-perks constraints from binding, there is an additional factor that can raise consumption above the efficient level, $\bar{x}$.

The incentive constraint on monitoring can bind from either of two directions, and there are three cases correspondingly. Case 1: If $\psi_i > 0$, then $\phi_1(\theta) > 0$ and $v_1(\theta)$ must be at the upper bound for all $\theta \in \Theta_i$. In this case the no-perks constraint on $x_1(\theta)$ cannot bind and $\gamma_1(\theta) = 0$. Case 2: If $\psi_i < 0$, then $\gamma_1(\theta)$ must be positive and hence the no-perks constraint constraint on $x_1(\theta)$ must bind, which implies that $v_1(\theta) < \theta$. Case 3: $\psi_i = 0$, and the incentive constraint on monitoring doesn't bind. However, as the following proposition makes clear, Case 2 in which $\psi_i < 0$ is caused by having too coarse a partition, and will not arise with an efficient partition.

PROPOSITION 8. *If in the solution for a given partition, there exists an* i *such that,* $E\{\theta|\theta \in \Theta_i\} - \phi > \min(\theta \in \Theta_i)$ *and* $\pi_i > 0$, *then there is an efficiency improving sub-division of this element of the partition. Additionally, it is efficient to make* ϕ *as small as possible, which is* γ.

Proof: To prove the first statement, assume that this was the case, and consider the following two cases. *Case 1:* There is a subset of relatively high θ points $\tilde{\Theta} \subset \Theta_i$ such that $E\{\tilde{\Theta}\} > E\{\theta|\theta \in \Theta_i\} - \phi$, and $E\{\Theta_i/\tilde{\Theta}\} > \phi$. Then we can remove the remove the points $\tilde{\Theta}$ from Θ_i

and put them in their own partition. This will not lower v_{0i}, and we can construct $v_1(\theta)$ so that it is weakly higher on Θ_i, while we can set the v_0 and v_1 strictly higher for $\theta \in \tilde{\Theta}$. This means that the same monitoring and managerial consumption choices are still feasible, while it relaxes the our incentive and no-perks constraints. Hence, we can lower π for $\theta \in \tilde{\Theta}$ which raises the value of the objective. Note that Case 1 applies to the case in which $\Pr\{\Theta_i\} > 0$.

Case 2: The set Θ_i is not of positive measure and there does not exist an appropriate subset $\tilde{\Theta}$ to break off. In this case, one can achieve the same result by simply having the manager randomize in his reports. For example assume that $\Theta_i = \{\theta_1, \theta_2\}$, where $\theta_2 > \theta_1$. Then by choosing a probability κ of reporting partition i, where

$$\frac{\kappa p(\theta_2)\theta_2 + p(\theta_1)\theta_1}{\kappa p(\theta_2) + p(\theta_1)} - \phi > \theta_1,$$

the same choice of v_{0i} is still feasible, and $v_1(\pi_i, \theta)$ can be raised. Furthermore, for the report that is made with probability $1 - \kappa$, $v_1 = \theta_2$ and $v_0 = \theta_2 - \phi$. This relaxes that no-perks and incentive constraints for this alternative report, allowing for a lower monitoring probability.

To prove the second statement, assume that $\phi > \gamma$ and consider lowering it γ. So long as $E\{\theta|\theta \in \Theta_i\} - \gamma \le \min(\theta \in \Theta_i)$, this will lead to a raising of v_{0i}, which relaxes the incentive and no-perks constraints. If this the reverse inequality is true, then we can construct efficiency improving sub-divisions of our current partition along the lines discussed above. *Q.E.D.*

E. Efficient Partition Size

Assume that the partitions are intervals of the form

$$\Theta_i = [a_{-i}, a_i],$$

where $a_0 = 0$ and $a_{I+1} = 1$, and the vector of $a = \{a_1, ..., a_I\}$ (where $a_i > a_{i-1}$) defines the partitions. Then assume that the partitions are sufficiently small that $E\{\theta|\Theta_i\} - \gamma < a_{-i}$, so we can set $v_1(\theta, i) = \theta$ and $v_{0i} = E\{\theta|\Theta_i\} - \gamma$. Also, conjecture that U is sufficiently high that the no-perks constraints do not bind (something which we can verify once we have the solution). As was noted above, this implies that $x_0(\theta, i) = x_1(\theta, i)$. Given this, we can write the Lagrangean in terms of the utility of the agent, $y(\theta, i)$, as we did in the commitment case. In this case, the Lagrangean can be written as

$$
\begin{aligned}
L = & \sum_i \int_{a_{i-1}}^{a_i} [\theta - \gamma\pi_i - C(y(\theta, i))] \, p(\theta) d\theta \\
& + \lambda \left\{ \sum_i \int_{a_{i-1}}^{a_i} y(\theta, i) p(\theta) d\theta - U \right\} \\
& + \sum_i \sum_j \int_{a_{i-1}}^{a_i} [y(\theta, i) - (1 - \pi_j) u(\tau(\theta - E\{\theta | \theta \in [a_{j-1}, a_j]\} + \gamma))] \, \delta(\theta, j, i) p(\theta) d\theta.
\end{aligned}
$$

In this case, the f.o.c. for π_i can be written as

$$
\int_{a_{-i}}^{a_i} -\gamma p(\theta) d\theta + \sum_j \int_{a_{j-1}}^{a_j} u(\tau(\theta - E\{\theta | \theta \in [a_{i-1}, a_i]\} + \gamma)) \delta(\theta, i, j) p(\theta) d\theta = 0
$$

The f.o.c. for $y(\theta, i)$ can be written as

$$
-C'(y(\theta, i)) + \lambda + \sum_j \delta(\theta, j, i) = 0.
$$

And, the f.o.c. for a_i can be written as

$$
\begin{aligned}
& \{[-\gamma\pi_i - C(y(a_i, i))] - [-\gamma\pi_{i+1} - C(y(a_i, i+1))]\} \, p(a_i) \\
& + \lambda \, [y(a_i, i) - y(a_i, i+1)] \, p(a_i) \\
& + \sum_j \left\{ \begin{array}{c} [y(a_i, i) - (1 - \pi_j) u(\tau(a_i - E\{\theta | \theta \in [a_{j-1}, a_j]\} + \gamma))] \, \delta(\theta, j, i) \\ - [y(a_i, i+1) - (1 - \pi_j) u(\tau(a_i - E\{\theta | \theta \in [a_{j-1}, a_j]\} + \gamma))] \, \delta(\theta, j, i+1) \end{array} \right\} p(a_i) \\
& + \sum_j \int_{a_{j-1}}^{a_j} \left[-(1 - \pi_i) u'(\tau(\theta - E\{\theta | \theta \in [a_{i-1}, a_i]\} + \gamma)) \frac{dE\{\theta | \theta \in [a_{i-1}, a_i]\}}{da_i} \right] \delta(\theta, i, j) \\
& + \sum_j \int_{a_{j-1}}^{a_j} \left[-(1 - \pi_{i+1}) u'(\tau(\theta - E\{\theta | \theta \in [a_i, a_{i+1}]\} + \gamma)) \frac{dE\{\theta | \theta \in [a_i, a_{i+1}]\}}{da_i} \right] \delta(\theta, i+1, j) \\
= & \; 0.
\end{aligned}
$$

If the partitions are small, then the incentive constraints cannot bind locally: for $\theta \in [a_i, a_{i+1}]$ if a_{i+1} is close enough to a_{-i}, then since $\tau(\theta - a_{-i}) < C'^{-1}(\lambda)$, and the constraint doesn't bind with respect to the next lower partition. But, this implies that changing

$\delta(\theta, i, j) = \delta(\theta, i+1, j)$, and hence $y(a_i, i) = y(a_i, i+1)$. In this case, the above expression simplifies to

$$\begin{aligned}
& -\gamma(\pi_i - \pi_{i+1}) \\
& +\sum_j \int_{a_{j-1}}^{a_j} \left[(1-\pi_i) u'(\tau(\theta - E\{\theta | \theta \in [a_{i-1}, a_i]\} + \gamma)) \frac{dE\{\theta | \theta \in [a_{i-1}, a_i]\}}{da_i} \right] \delta(\theta, i, j) \\
& +\sum_j \int_{a_{j-1}}^{a_j} \left[(1-\pi_{i+1}) u'(\tau(\theta - E\{\theta | \theta \in [a_i, a_{i+1}]\} + \gamma)) \frac{dE\{\theta | \theta \in [a_i, a_{i+1}]\}}{da_i} \right] \delta(\theta, i+1, j) \\
= \; & 0
\end{aligned}$$

The monitoring probabilities will be declining in i, hence the first term is negative. But,

$$\frac{dE\{\theta | \theta \in [a_{i-1}, a_i]\}}{da_i} > 0$$

and

$$\frac{dE\{\theta | \theta \in [a_i, a_{i+1}]\}}{da_i} > 0.$$

What is happening at the margin is that the first-order condition is trading off the increases probability of monitoring from expanding partition Θ_i at the expense of Θ_{i+1} against the increase in v_{0i} and v_{0i+1} that this leads too.

4. Numerical Example

Consider the following example: The set of sets $\Theta = [0, 0.1, 0.2, ..., 2]$ and each state is equally likely. $\tau = .5$, $\gamma = 0.1$, and preferences are $u(c) = \sqrt{c}$, which implies that $C(u) = u^2$. We set the reservation utility $U = \sqrt{.2}$, which was high enough so that the no-perks condition did not bind in either commitment and self-enforcing stochastic monitoring problems. We find that stochastic monitoring with commitment yielded a 5.87% higher solution value relative to deterministic monitoring. We also find that at the optimal partition, self-enforcing monitoring yielded 5.57% higher solution value than deterministic monitoring, and for this solution the associated partition was nonminimal. We report the monitoring probabilities and the wage payments to the manager below for each of our solutions. What one see's is that base wages are lower with deterministic monitoring than with self-enforcing monitoring, which in turn has lower base wages that stochastic monitoring with commitment. Correspondingly, the solutions with lower base wages had higher wages for high levels of θ, when the incentive conditions bound. In the stochastic monitoring cases - both with commitment and self-enforcement - monitoring probabilities are always below 1/2, and monitoring is used over a wider range of states than in the deterministic case.

Deterministic Monitoring: The solution in terms of monitoring probabilities (m) and the manager's payment (w) is given by

θ	m	w	θ	m	w
0	1	.160	1.1	1	.160
.1	1	.160	1.2	0	.160
..2	1	.160	1.3	0	.160
.3	1	.160	1.4	0	.160
.4	1	.160	1.5	0	.200
.5	1	.160	1.6	0	.250
.6	1	.160	1.7	0	.300
.7	1	.160	1.8	0	.350
.8	1	.160	1.9	0	.400
.9	1	.160	2.0	0	.450
1.0	1	.160			

Stochastic Monitoring with Commitment: The value of the solution probabilities of monitoring (m) and the payment (w) are given in the following table.

θ	m	w	θ	m	w
0	.450	.165	1.1	.112	.168
.1	.429	.165	1.2	.058	.181
..2	.408	.165	1.3	0	.197
.3	.385	.165	1.4	0	.212
.4	.360	.165	1.5	0	.228
.5	.334	.165	1.6	0	.246
.6	.305	.165	1.7	0	.266
.7	.274	.165	1.8	0	.290
.8	.238	.165	1.9	0	.319
.9	.201	.165	2.0	0	.355
1.0	.158	.165			

Self-Enforcing Stochastic Monitoring: The value of the solution is 0.7766178 at the optimal partition

$$\begin{aligned}
\Theta_1 &= \{0, .1, .2\} \\
\Theta_2 &= \{.3, .4, .5\} \\
\Theta_3 &= \{.6, .7\} \\
\Theta_4 &= \{.8, .9\} \\
\Theta_5 &= \{1.0\} \\
\Theta_6 &= \{1.1\} \\
&\vdots
\end{aligned}$$

and the monitoring probabilities, wages, and the nonmonitoring payment v_0 were given by

θ	m	w	v_0	θ	m	w	v_0
0	.445	.162	0	1.1	.155	.170	1.0
.1	.445	.162	0	1.2	.110	.185	1.1
..2	.445	.162	0	1.3	.056	.200	1.2
.3	.379	.162	0.3	1.4	0	.216	1.3
.4	.379	.162	0.3	1.5	0	.231	1.4
.5	.379	.162	0.3	1.6	0	.250	1.5
.6	.315	.162	0.55	1.7	0	.270	1.6
.7	.315	.162	0.55	1.8	0	.293	1.7
.8	.253	.163	0.75	1.9	0	.321	1.8
.9	.253	.163	0.75	2.0	0	.357	1.9
1.0	.199	.164	0.9				