Amplification of Productivity Shocks:
Why Vacancies Don't Like to Hire the Unemployed?[1]

Éva Nagypál[2]

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PRELIMINARY — COMMENTS WELCOME

Abstract

In this paper I study a new amplification mechanism in search models that arises when workers can choose to search on the job and, despite the fact that all workers are ex-ante identical, employers prefer to hire already employed workers for endogenous reasons. The motivation for on-the-job search in the model is job-shopping, where workers look for jobs they find appealing, and the appeal of a job to the worker is not observed by the firm. In equilibrium, workers arriving from unemployment are more likely to leave a job for a more appealing job, and, knowing this, firms prefer to hire already employed, as opposed to unemployed, workers.

Employers' preference for hiring already employed workers introduces a new amplification mechanism into search models. This is because vacancies in the model with such preference respond more to aggregate shocks than in the standard search model due to the fact that employed workers reduce their search intensity in a recession, thereby making it less attractive for firms to post vacancies. Using simulations of the proposed model, I explore the extent that the presence of job-to-job transitions can help in explaining the volatility of unemployment and vacancies over the business cycle through this new amplification mechanism. The simulation results show that, for standard parameter values, this new mechanism can generate five times more amplification compared to the baseline model.

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1   Introduction

In this paper, I study a new amplification mechanism in search models that arises when workers can choose to search on the job and, despite the fact that all workers are ex-ante identical, employers prefer to hire already employed workers. Such a preference arises endogenously because the expected profits from hiring an unemployed worker are lower than those from hiring an employed worker. The reason for this is that workers hired from unemployment have higher expected turnover as they are willing to accept even low quality matches and then to continue to search on the job. Therefore, it is less profitable for firms to undertake the necessary investment needed to create employment relationships when the composition of searchers shifts towards unemployed workers during a recession, thereby stifling vacancy creation during these bad times.

The innovation of this research is to construct a search model that incorporates on-the-job search where, unlike in existing models of on-the-job search, employers prefer to hire already employed workers. There is both direct and indirect evidence that such a preference exists among employers, as argued in Eriksson and Lagerström (2004) and Nagypál (2004). Moreover, as I show, it has the theoretical appeal that it can give an explanation why vacancy creation is so low in recessions and why the resulting low job finding rate results in a burst of unemployment.

The puzzle that this paper addresses is fleshed out in detail in Shimer (2004), which is that textbook search models have essentially no internal amplification. The "standard" search model results in an elasticity of unemployment and of vacancies with respect to shocks to the productivity of employment relationships that is between .5 and 1.5, which is in sharp contrast with the elasticities observed in the data, which are on the order of 10 for both unemployment and vacancies.

Adding on-the-job search is a natural extension of the standard search model, especially in light of recent empirical findings using direct measures of job-to-job transition rates that argue that the extent of job-to-job movement has been underestimated using earlier indirect

methods. Existing search models with on-the-job search (such as those developed by Pissarides (1994), Mortensen (1994), Burdett and Mortensen (1998), and Barlevy (2002)) do not help in resolving the amplification puzzle, since they all involve a preference by firms for hiring unemployed workers, as opposed to employed workers.[3] This exacerbates the lack of amplification, as opposed to helping it, since it means that recessions are times when the pool of searchers changes *in favor* of vacancy creation, thereby further worsening the model's ability to explain the lack of vacancy creation during recessions in the data. In fact, for example, while Barlevy (2002) does not report the elasticity of unemployment to productivity shocks in his model, it can be calculated from the numbers that he reports to be below 0.10.

There are two channels through which preference for hiring unemployed workers arises in existing models of on-the-job search. First, since the alternatives of unemployed workers are worse on average than those of employed workers, they are more likely to accept a match of a given quality or productivity. Second, depending on the nature of wage setting, employed searchers have better outside options, thereby commanding higher wages than unemployed searchers. In a standard search model firms prefer higher acceptance rates and lower wages, thereby this means that they prefer unemployed searchers.

The key difference in the model developed in this paper is that firms do not always prefer to hire workers with higher acceptance rates. For this to happen, what is necessary is that some matches lead to a negative payoff to the firm, thereby making the acceptance of such matches undesirable. Such a negative payoff at the time of the creation of the employment relationship is not a feature of the standard search model, since, in that model, all costs of creating a vacancy are born prior to meeting a worker through vacancy creation costs. It is a natural extension to consider, however, that the firm has to expend some additional resources (on training, relocation, and other match-specific investments) at the time the match is formed. Of course, in order for firms to enter matches that lead to a negative

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[3]It should be noted, however, that the emphasis of these authors have not been on the role of on-the-job search in amplifying productivity shocks. A notable exception is Shimer (2003), who studies amplification with on-the-job search. The mechanism in his model is very different from the one studied here, since he departs in several ways from the standard search model, and argues that a preference for hiring *unemployed* workers can help in resolving the amplification puzzle.

payoff, it is necessary that the firm has less information about the match that is being formed than the worker. This is naturally the case when the quality of the match is only observed by the worker and enters directly only into the utility function of the worker.

Given these elements of the model, the basic mechanism is simple. Workers can undertake job shopping at a cost both while unemployed and while employed, where job shopping simply means searching for a match with a higher idiosyncratic value to the worker. Unemployed workers are "desperate", as they are willing to accept any idiosyncratic value above some minimum threshold. Employed workers are more selective, and only accept matches that have a value above the value of their current match. Turnover, in turn, declines with the idiosyncratic value of the match for two reasons. First, the probability of finding a better match declines, and second, as a consequence, the incentives to search for a better job also decline, thereby leading to lower endogenous search effort. This then means that the expected turnover of previously unemployed workers is higher than that of previously employed workers, making them less attractive candidates for firms to hire. Of course, this higher turnover has to be weighed against the higher acceptance rate of unemployed workers. It is easy to argue, as I do below, that the turnover effect can outweigh the acceptance rate effect only if there is a possibility of the firm to make negative profits in a match, which naturally arises when the firm has no information about the idiosyncratic value of the match to the worker and has to bear some one-time match-specific costs to start the relationship. Amplification then is a direct results of this mechanism. Vacancies in the model with a preference for hiring employed workers respond more to aggregate shocks than in the standard search model due to the fact that employed workers reduce their search intensity in a recession, thereby making it less attractive for firms to post vacancies.

Using simulations of the proposed model, I explore the extent that the presence of job-to-job transitions can help in explaining the volatility of unemployment and vacancies over the business cycle through this new amplification mechanism. The simulation results show that this new mechanism can generate 5 times more amplification than the baseline model studied in Shimer (2004).

Finally, let us consider in some more detail how preference for employed workers can arise in an extension of the standard search model. When contemplating vacancy creation, if the firm believes that it will meet an unemployed worker, then the profits it can expect to make are

$$\int \Pi(\mu) A_u(\mu) dF(\mu), \tag{1}$$

where $\Pi(\mu)$ is the profit from creating a job of type μ, $A_u(\mu)$ is the probability that an unemployed worker will accept a type μ job, and $F(\mu)$ is the distribution of new matches. Correspondingly, the expected profits from meeting an employed worker are

$$\int \Pi(\mu) A_e(\mu) dF(\mu). \tag{2}$$

Since $A_u(\mu) \geq A_e(\mu)$ for all possible values of μ, if $\Pi(\mu) \geq 0$, then it is necessarily the case that the expected profits meeting an unemployed worker are at least as large as the expected profits from meeting an employed worker. Only if $\Pi(\mu)$ can become negative for some value of μ is there a possibility for the expected profits from meeting an unemployed worker to be lower than those from meeting an employed worker. The elements introduced in this model to allow for this possibility is a one-time start-up cost and asymmetric information.

2 Environment

Time is continuous and goes on forever. There is a unit measure of infinitely-lived workers, who are ex-ante identical. Workers can be either employed or unemployed, and the objective of workers is to maximize

$$\int_{t=0}^{\infty} e^{-rt} y_t, \tag{3}$$

where

$$y_t = \begin{cases} w_t + \mu_t - c(s_t) & \text{if employed,} \\ b - c(s_t) & \text{if unemployed.} \end{cases} \tag{4}$$

Here w_t is the wage received when employed at time t, μ_t denotes the attractiveness or appeal to employed workers of their current employment match, in other words, workers derive utility from being on a job they "like", and s_t denotes the search effort of the worker at time t. The appeal of a job to the worker, μ (which I will also call match quality below), is determined upon meeting a potential employer, and is drawn from the distribution $F(\cdot)$, where $F : [\underline{\mu}, \bar{\mu}] \to [0, 1]$ is a continuous, twice differentiable, strictly increasing distribution function, and $\underline{\mu} \in \mathbb{R} \cup \{-\infty\}$ and $\bar{\mu} \in \mathbb{R} \cup \{\infty\}$. In addition, workers can choose to engage in search at a flow cost of $c(s_t)$, where

$$c(s_t) = \begin{cases} c_0 + \hat{c}(s_t) & \text{if } s_t > 0 \\ 0 & \text{if } s_t = 0 \end{cases}, \tag{5}$$

where $\hat{c}(\cdot)$ is a strictly increasing, strictly convex, twice continuously differentiable function. This means that there is both a fixed cost and a variable cost of searching, so that when the incentives become sufficiently low, the worker stops searching all together. Finally, b denotes the constant utility flow (derived from leisure and/or from unemployment insurance benefits) that a worker receives while unemployed.

There is a large measure of ex-ante identical firms. Firms' objective is to maximize

$$\int_{t=0}^{\infty} e^{-rt}(\pi_t - K\xi_t),$$
(6)

where

$$\pi_t = \begin{cases} 0 & \text{if firm is inactive} \\ -c_v & \text{if firm is active with a vacant job} \\ p - w_t & \text{if firm is active with a filled job.} \end{cases}$$
(7)

$$\xi_t = \begin{cases} 1 & \text{if a new match is created at time } t \\ 0 & \text{otherwise} \end{cases}$$
(8)

This means that any firm can enter the market and become active by posting a vacancy at flow cost c_v. If a firm posts a vacancy, then it participates in the matching market for creating new matches. When a firm creates a match, it needs to pay a one-time match-specific start-up cost of K, and then it receives a flow profit of $p - w_t$ until the match dissolves. Here p is the output of a match, which is assumed to be the same for all matches. Matches dissolve for exogenous reasons at rate δ and endogenously when the worker decides to form a new employment relationship as a consequence of on-the-job search.

There is a single matching market with a meeting function determining the number of meetings (m_t) as a function of the total amount of search effort of workers (s_t) and the number of vacancies posted (v_t):

$$m_t = m(s_t, v_t),$$
(9)

such that $m_s(s, v) > 0$, $m(0, v) = 0$ for any v, $m_v(s, v) > 0$, $m(s, 0) = 0$ for any s. I assume that $m(s, v)$ has constant returns to scale, so that the meeting rate per unit of search effort for workers can be written as

$$\lambda_t = \lambda(\theta_t) = \frac{m(s_t, v_t)}{s_t} = m\left(1, \frac{v_t}{s_t}\right) = m(1, \theta_t),$$
(10)

where $\theta_t = \frac{v_t}{s_t}$ is market tightness at time t. Similarly, the meeting rate for firms can be written as

$$\eta_t = \eta(\theta_t) = \frac{m(s_t, v_t)}{v_t} = \frac{\frac{m(s_t, v_t)}{s_t}}{\frac{v_t}{s_t}} = \frac{\lambda(\theta_t)}{\theta_t}. \tag{11}$$

The timing of match formation is as follows. If a worker and a firm meet, the worker observes the appeal of the potential match and can decide whether or not to form the match. In order to form the match, an already employed worker needs to end his current relationship. The relevance of this assumption is that it implies that the outside option of all workers is unemployment.[4] The firm does not observe neither the appeal of the match to the worker nor whether the worker was previously unemployed or employed. If the worker agrees to forming a match, then wages are determined upon the formation of the match by splitting the expected surplus such that the worker receives β fraction of it. Wages are subsequently renegotiated only if otherwise the participation constraint of the parties would be violated conditional on the worker not being able to credibly communicate the existence of a possibility to form a new match. Firms rationally form and update their beliefs about the appeal of a job to the worker based on the information available to them.

It is worth commenting on the particular choice of the wage setting mechanism. While there is no micro-foundation or axiomatic basis for the chosen wage-determination mechanism, it is chosen to keep the model as close as possible to the standard search model that is studied in Shimer (2004). Without asymmetric information and on-the-job search, Nash bargaining implies the sharing of the surplus between the worker and the firm. Therefore, assuming surplus sharing in the environment studied provides the most direct comparison with the standard model. It would be worthwhile to study, and it is an issue I will consider in future work, how departing from surplus sharing affects the results reported below.

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[4]An alternative way to justify unemployment being the outside option of all workers is to assume that matches cannot be "recalled" and that there is scope for renegotiation between the worker and the firm immediately after the worker has moved from the old employer to their current employer.

3 Equilibrium

3.1 Definition of stationary equilibrium

For the sake of simplicity, and to keep the analysis tractable, I consider what happens in the above described economy in a stationary equilibrium. Clearly, given the assumptions on wage-setting above, in a stationary equilibrium, the only new information that arrives to a worker-firm pair while in a match is whether the match is still in existence or not, therefore the only state variable that enters the asset values of workers and firms, besides the wage and the quality of the match, is the length of the relationship. Let the value of unemployment be U, the value of a worker employed in a match of quality μ, tenure τ, and wage w be $W(w, \mu, \tau)$, the value of a vacancy be V, and the value of employment for the firm in a match of tenure τ and wage w be $J(w, \tau)$.

Definition 1. *A recursive stationary search equilibrium is unemployment rate u, vacancy rate v, asset values $\{U, V, W(w, \mu, \tau), J(w, \tau)\}$, wage function $w(\tau)$, workers' search decisions $s(w, \mu, \tau)$, and distribution of employed workers $G(w, \mu, \tau)$ such that*

- *U and $W(\cdot)$ are the value of unemployment and of working for workers making optimal matching, searching, and acceptance decisions given u, v, $w(\cdot)$, and $G(\cdot)$, and $s(\cdot)$ is the corresponding optimal search policy.*

- *V and $J(\cdot)$ are the value of a vacancy and of a filled job for firms making optimal vacancy creation and matching decisions given u, v, $w(\cdot)$, and $G(\cdot)$.*

- *Agents update their beliefs rationally.*

- *There is free entry of vacancies.*

- *Wages are determined by sharing of the expected surplus upon meeting and are subsequently renegotiated only if otherwise the participation constraint of the parties would be violated.*

- *The distribution $G(\cdot)$ is consistent with the decisions of the agents in the economy.*

3.2 Characterization of equilibrium

3.2.1 Wage determination

Let the expected value of a worker upon forming a match be

$$W_0(w) = \int_0^{\bar{\mu}} W(w, \mu, 0) dH_0(\mu), \qquad (12)$$

where $H_0(\mu)$ is the distribution of match quality at the formation of a match conditional on the worker accepting the match, to be derived below. Similarly,

$$J_0(w) = \int_0^{\bar{\mu}} J(w, 0 \mid \mu) dH_0(\mu), \qquad (13)$$

is the expected value of a firm upon forming a match, where $J(w, 0 \mid \mu)$ is the value to the firm from matching with a worker with match quality μ.

The wage is set such that

$$(1 - \beta)(W_0(w) - U) = \beta(J_0(w) - V). \qquad (14)$$

Since the information that the wage is conditioned on is the same in all initial matches, and all workers have the same outside option, there is a single initial wage in equilibrium. Clearly, conditional on the worker having accepted the match, this initial wage satisfies the participation constraint of both parties. Moreover, it is straightforward to show that wages will never be renegotiated, since the initial wage will satisfy the participation constraint of the agents at all future tenure conditional on the match having continued. This follows from the fact that match continuation in the model is always favorable information to a firm, meaning that the firm's posterior belief improves as the match lasts longer and longer. Formally, the lack of renegotiation follows from the lack of firm-initiated separations, as stated in the proposition below, which is proved in the Appendix.

Proposition 2. *If workers' search decision has the reservation property, there are no firm-initiated separations in a stationary equilibrium.*

3.2.2 Worker side

Given that there is a unique wage in the stationary equilibrium as argued above, the wage w and tenure τ can be dropped as state variables from the value of an employed worker. The Bellman equation characterizing the value of being a worker with quality μ is then

$$rW(\mu) = \max_{s \geq 0} \left\{ \mu + w - c(s) + \lambda(\theta)s \int_{\underline{\mu}}^{\bar{\mu}} \max[W(\mu') - W(\mu), 0]dF(\mu') + \delta(U - W(\mu)) \right\}.$$
(15)

The flow payoff from working is the utility derived from being in a match of quality μ and from the wage w. An employed worker needs to choose her search effort, and if a new firm is encountered, she needs to decide whether to form the new match given its quality μ' that is drawn from the distribution F, or to stay with her current employer. Moreover, at rate δ the worker suffers a loss of asset value due to exogenous separation.

The Bellman equation characterizing the value of being an unemployed worker is

$$rU = \max_{s \geq 0} \left\{ b - c(s) + \lambda(\theta)s \int_{\underline{\mu}}^{\bar{\mu}} \max[W(\mu') - U, 0]dF(\mu') \right\}.$$
(16)

An unemployed worker needs to also choose her search effort, and if a firm is encountered, she needs to decide whether to form the new match given its quality μ' that is drawn from the distribution F, or to remain unemployed.

Equation (15) defines a contraction, and therefore the Contraction Mapping Theorem implies that $W(\mu)$ is increasing in μ and, given the assumptions on $F(\cdot)$ and $c(\cdot)$, differentiable except at the points where the search decision changes discontinuously. This in turn implies that acceptance decisions have the reservation property with the quality of the current match being the reservation match quality.

Let us next turn to studying the worker's search decision. Given the structure of the search cost and using the reservation property of acceptance decisions, the worker's decision problem

can be rewritten as follows:

$$rW(\mu) = \max\left\{\max_{s>0}\left\{\mu + w - c(s) + \lambda(\theta)s\int_{\mu}^{\bar{\mu}}W(\mu') - W(\mu)dF(\mu') + \delta(U - W(\mu))\right\};\right.$$
$$\left.\mu + w + \delta(U - W(\mu))\right\}, \tag{17}$$

where the search decision has been broken down into two steps: a decisions whether to search at all, and a decision of how much to search if searching. I assume that the worker chooses to search if she is indifferent between searching and not searching. The first-order condition characterizing the second of these maximization problems is given by

$$c'(s(\mu)) = \lambda(\theta)\int_{\mu}^{\bar{\mu}}W(\mu') - W(\mu)dF(\mu') = \lambda(\theta)\int_{\mu}^{\bar{\mu}}W'(\mu')\bar{F}(\mu')d\mu', \tag{18}$$

where the second equality follows from integration by parts and $\bar{F} = 1 - F$ is the survival function of the distribution F. With regards to the first maximization problem, clearly, the payoff from search is declining with μ, hence the optimal policy with respect to whether to search at all has the reservation property. This means that that there exists a μ_s above which the worker will choose not to search at all and below which she will choose to search. At μ_s, the condition of optimality states that

$$c(s(\mu_s)) = \lambda(\theta)s(\mu_s)\int_{\mu_s}^{\bar{\mu}}W'(\mu')\bar{F}(\mu')d\mu'. \tag{19}$$

From these two optimality conditions, and given the properties of $c(\cdot)$, the following Lemma follows.

Lemma 3. *There exists a μ_s, such that for all $\mu > \mu_s$, $s(\mu) = 0$. Moreover, the optimal search effort of the worker, $s(\mu)$, is continuous and strictly declining in μ for all $\mu < \mu_s$.*

In what follows, I assume that the variable part of the search cost function takes on the form $\hat{c}(s) = c_1 s^{1+\rho}$, for some $\rho > 0$. Substituting in this functional form of c, Equations (18)

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evaluated at $\mu = \mu_s$ and (19) imply that

$$s(\mu_s) = \left[\frac{c_0}{c_1 \rho} \right]^{\frac{1}{1+\rho}}, \tag{20}$$

which in turn together with Equation (18) evaluated at $\mu = \mu_s$ implies that

$$c_1(1+\rho) \left[\frac{c_0}{c_1 \rho} \right]^{\frac{\rho}{1+\rho}} = \frac{\lambda(\theta)}{r+\delta} \int_{\mu_s}^{\bar{\mu}} \bar{F}(\mu') d\mu', \tag{21}$$

an equilibrium condition that determines μ_s as a function of $\lambda(\theta)$ and of exogenous parameters.

Taking derivatives with respect to μ on both sides of the worker's asset equation and rearranging gives

$$\frac{dW(\mu)}{d\mu} = \begin{cases} \frac{1}{r+\delta+\lambda(\theta)s(\mu)(1-F(\mu))} & \text{if} \quad \mu < \mu^s \\ \frac{1}{r+\delta} & \text{if} \quad \mu > \mu^s \end{cases} \tag{22}$$

Substituting into the optimality condition for search for $\mu < \mu_s$, taking derivatives on both sides with respect to μ, and using the functional form for $\hat{c}(s)$ gives the differential equation

$$s'(\mu) = -\frac{\lambda(\theta) \bar{F}(\mu) s(\mu)^{1-\rho}}{(1+\rho)\rho \left(r+\delta+\lambda(\theta)s(\mu)\bar{F}(\mu) \right)}. \tag{23}$$

This differential equation together with the boundary condition in (20) fully characterizes the search decision of workers as a function of the quality of their match, and can be solved numerically for any value of $\rho > 0$.

Notice that while $W(\mu)$ is continuous everywhere, there is a kink in the function $W(\mu)$ at μ^s, since there is a positive difference between its derivative from the left (the first expression evaluated at μ^s) and the derivative from the right (the second expression evaluated at μ^s).

Finally, given that $W(\mu)$ is increasing as argued above, an unemployed worker will clearly

adopt a reservation match quality policy when searching for a job, hence

$$rU = \max_{s \geq 0} \left\{ b - c(s) + \lambda(\theta)s \int_{\mu_m}^{\bar{\mu}} [W(\mu') - U]dF(\mu') \right\}, \tag{24}$$

where μ_m is an unemployed worker's reservation match quality implicitly defined by

$$W(\mu_m) = U. \tag{25}$$

Comparing the asset equation of a worker at match quality μ_m and that of an unemployed worker, it is clear that

$$s_u = s(\mu_m) \tag{26}$$

$$\mu_m = b - w. \tag{27}$$

3.2.3 Firm side

Again, given that there is a unique wage in the stationary equilibrium as argued above, the wage w can be dropped as a state variable from the value of employment for the firm. The value of being a firm with a match of tenure τ is then

$$J(\tau) = \int_{\underline{\mu}}^{\bar{\mu}} J(\mu)dH_\tau(\mu) \tag{28}$$

where $H_\tau(\mu)$ is the distribution of match quality for a match of tenure τ. $J(\mu)$ in turn satisfies the Bellman equation

$$rJ(\mu) = \begin{cases} p - w + \lambda(\theta)s(\mu)\bar{F}(\mu)(V - J(\mu)) + \delta(V - J(\mu)) & \text{if} \quad \mu \leq \mu_s \\ p - w + \delta(V - J(\mu)) & \text{if} \quad \mu > \mu_s \end{cases}. \tag{29}$$

The flow payoff of a match to the firm is $p - w$. In addition, the firm needs to take into account that the match might end for exogenous reasons at rate δ and endogenously if the worker decides to move to another job, where the latter happens at rate $\lambda(\theta)s(\mu)\bar{F}(\mu)$. Since endogenous turnover is decreasing with μ (and becomes zero once $\mu > \mu_s$), the value of a match to the firm increases in μ.

The Bellman equation characterizing the value of a vacancy can be expressed as

$$rV = -c_f + \eta(\theta)P_a \int_{\underline{\mu}}^{\bar{\mu}} \left(J(\mu) - K - V \right) dH_0(\mu), \tag{30}$$

where P_a is the probability that a match is accepted by the worker. Given the free-entry condition, $V = 0$, and the acceptance policy of the worker, we can write the above as

$$J(\mu) = \begin{cases} \frac{p-w}{r+\delta+\lambda(\theta)s(\mu)\bar{F}(\mu)} & \text{if} \quad \mu \leq \mu_s \\ \frac{p-w}{r+\delta} & \text{if} \quad \mu > \mu_s \end{cases}, \tag{31}$$

and

$$\frac{c_f}{\eta(\theta)} = P_a \int_{\mu_m}^{\bar{\mu}} \left(J(\mu) - K \right) dH_0(\mu). \tag{32}$$

3.2.4 Equilibrium distribution of workers

Next, I turn to the derivation of $G(\mu)$, which denotes the stationary measure of employed workers below match quality μ, and u, which is the stationary unemployment rate. Clearly, the support of G is $[\mu_m, \bar{\mu}]$ and $G(\bar{\mu}) = 1 - u$.

In the model, the stationary measure of unemployment can be derived from equating the flow into and out of unemployment

$$u\lambda(\theta)s(\mu_m)\bar{F}(\mu_m) = \delta\left(1 - u\right), \tag{33}$$

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so that

$$u = \frac{\delta}{\delta + \lambda(\theta)s(\mu_m)\bar{F}(\mu_m)}. \tag{34}$$

To determine the distribution $G(\mu)$, one can equate the flow into and out of $G(\mu)$ (just as in the Burdett and Mortensen (1998) model). The flow into the pool of employed workers with match quality μ or lower is

$$u\lambda(\theta)s(\mu_m)(\bar{F}(\mu_m) - \bar{F}(\mu)), \tag{35}$$

while flow out of the pool of employed workers with match quality μ or lower is

$$\delta G(\mu) + \int_{\mu_m}^{\min(\mu,\mu^s)} \lambda(\theta)s(\mu')\bar{F}(\mu)dG(\mu'). \tag{36}$$

The inflow clearly consists only of unemployed workers, while the outflow consists of workers that separate exogenously, and workers that find a match that is better than μ, where one has to take into account that only workers below match quality μ_s are searching.

Equating these two flows when $\mu \leq \mu_s$ means

$$\frac{u\lambda(\theta)s(\mu_m)(\bar{F}(\mu_m) - \bar{F}(\mu))}{\bar{F}(\mu)} = \frac{\delta G(\mu)}{\bar{F}(\mu)} + \lambda(\theta)\int_{\mu_m}^{\mu} s(\mu')dG(\mu'). \tag{37}$$

Differentiating both sides with respect to μ and rearranging gives

$$G'(\mu) = \frac{u\lambda(\theta)s(\mu_m)\bar{F}(\mu_m) - \delta G(\mu)}{\delta + \lambda(\theta)s(\mu)\bar{F}(\mu)}\frac{f(\mu)}{\bar{F}(\mu)}. \tag{38}$$

For $\mu > \mu_s$ the same steps give

$$G'(\mu) = \frac{u\lambda(\theta)s(\mu_m)\bar{F}(\mu_m) - \delta G(\mu)}{\delta}\frac{f(\mu)}{\bar{F}(\mu)} \tag{39}$$

These differential equations together with the boundary condition in $G(\mu_m) = 0$ fully characterize the distribution of workers.

Given the distribution G, the firm's initial belief that a match of quality $\mu \geq \mu_m$ is accepted can be expressed as

$$A(\mu) = \frac{u + G(\min(\mu, \mu^s))}{u + G(\mu_s)}. \tag{40}$$

Using Bayes' rule, the probability density function corresponding to the distribution H_0 can be written as

$$h_0(\mu) = \frac{A(\mu)f(\mu)}{P_a}, \tag{41}$$

where P_a is the probability that a worker accepts a match, which can be written as

$$P_a = \int_{\underline{\mu}}^{\bar{\mu}} A(\mu)f(\mu)d\mu, \tag{42}$$

3.3 Summary of equilibrium conditions

For the numerical exercise below, it is useful to note that two parameters, c_1 and m_0 can be eliminated from the equilibrium conditions. In other words, two more normalizations are possible. Let then $\hat{s}(\mu) = s(\mu)c_1^{\frac{1}{1+\rho}}$, $\hat{\lambda} = \lambda c_1^{-\frac{1}{1+\rho}}$, and $\hat{c}_f = c_f \left(\frac{c_1^{\frac{\alpha}{1+\rho}}}{m_0} \right)^{\frac{1}{1-\alpha}}$. Then the complete set of equilibrium conditions can be rewritten as

$$0 = (1+\rho)\left[\frac{c_0}{\rho}\right]^{\frac{\rho}{1+\rho}} - \frac{\hat{\lambda}}{r+\delta}\int_{\mu_s}^{\bar{\mu}} \bar{F}(\mu')d\mu', \tag{43}$$

$$\hat{s}(\mu_s) = \left[\frac{c_0}{\rho}\right]^{\frac{1}{1+\rho}} \tag{44}$$

$$\hat{s}'(\mu) = -\frac{\hat{\lambda}\bar{F}(\mu)\hat{s}(\mu)^{1-\rho}}{(1+\rho)\rho\left[r+\delta+\hat{\lambda}\hat{s}(\mu)\bar{F}(\mu)\right]} \tag{45}$$

$$\mu_m = b - w. \tag{46}$$

$$0 = \hat{c}_f\hat{\lambda}^{\frac{\alpha}{1-\alpha}} - \int_{\mu_m}^{\bar{\mu}} (J(\mu)-K)A(\mu)f(\mu)d\mu \tag{47}$$

$$0 = \beta\int_{\mu_m}^{\bar{\mu}} J(\mu)A(\mu)f(\mu)d\mu - (1-\beta)\int_{\mu_m}^{\bar{\mu}} (W(\mu)-U)A(\mu)f(\mu)d\mu \tag{48}$$

$$A(\mu) = \frac{u+G(\mu)}{u+G(\mu_s)} \quad \text{if } \mu \le \mu_s. \tag{49}$$

$$G'(\mu) = \begin{cases} \frac{u\hat{\lambda}\hat{s}(\mu_m)\bar{F}(\mu_m)-\delta G(\mu)}{\delta+\hat{\lambda}\hat{s}(\mu)\bar{F}(\mu)}\frac{f(\mu)}{\bar{F}(\mu)} & \text{if } \mu \le \mu^s \\ \frac{u\hat{\lambda}\hat{s}(\mu_m)\bar{F}(\mu_m)-\delta G(\mu)}{\delta}\frac{f(\mu)}{\bar{F}(\mu)} & \text{if } \mu > \mu^s \end{cases} \tag{50}$$

$$u = \frac{\delta}{\delta+\hat{\lambda}\hat{s}(\mu_m)\bar{F}(\mu_m)}. \tag{51}$$

Using the asset equations, the value of employment to a worker and a firm can be written as

$$J(\mu) = \begin{cases} \frac{p-w}{r+\delta+\hat{\lambda}\hat{s}(\mu)\bar{F}(\mu)} & \text{if } \mu \le \mu^s \\ \frac{p-w}{r+\delta} & \text{if } \mu > \mu^s \end{cases} \qquad W(\mu) = \begin{cases} U + \int_{\mu_m}^{\mu} \frac{1}{r+\delta+\hat{\lambda}\hat{s}(\mu')\bar{F}(\mu')}d\mu' & \text{if } \mu \le \mu^s \\ W(\mu_s) + \frac{\mu-\mu_s}{r+\delta} & \text{if } \mu > \mu^s \end{cases} \tag{52}$$

4   Representative simulations

In this section, I simulate the above economy, and look at what happens when I change the aggregate productivity parameter p and the exogenous job destruction rate δ.

For the distribution of match qualities, I use a normal distribution with mean zero and variance 1. For the choice of the other parameters, unless otherwise mentioned, I follow Shimer (2004) as closely as possible, to facilitate direct comparison of the results. The model is set to generate quarterly series, so r is chosen to be 1.2%, giving an annual discount rate of 4.8%. The aggregate productivity is chosen to be $p = 5$, implying that in terms of the total payoff from production, the probability of drawing a match quality that is a half as important as output (i.e. equal to 2.5) is 0.62%. Of course, the endogenous distribution of match qualities first-order stochastically dominates the distribution of the draw of match qualities, which is standard normal, so the question of how important match quality is compared to output is determined endogenously. The value of leisure or of unemployment insurance, b, is set to 2.5 at 50% of the match output. This is higher than the number used by Shimer (2004) (he uses 40%), but recall that in this model the flow payoff from unemployment is b less the cost of search, where this cost is strictly positive in this model, while it is 0 in Shimer's work. In fact, taking into account the search cost, the flow payoff from unemployment is less than 40% of output. The exogenous job destruction rate is set to 6%.

The cost of posting a vacancy is set to 2.5% of output, though recall that this number is affected by the normalization of c_1 and m_0. The fixed cost of creating a match is set to 10, or 2 periods of output. The fixed cost of search for a worker is set close to 0 at 0.001, though even such a small fixed cost leads to no more search once the 70th percentile in the quality distribution is reached. The parameter ρ is set equal to 0.2.

The matching function is chosen to be Cobb-Douglas

$$m(s,v) = m_0 s^\alpha v^{1-\alpha}, \tag{53}$$

with elasticity with respect to unemployment α of 0.62. The workers' share of the expected surplus is also set to equal 0.62.

For these parameter values, the endogenous values of interest are reported in Table 1.

Baseline case	
unemployment rate	9.73%
vacancy rate	0.328%
fraction of unemployed searchers	18.13%
fraction of unemployed new hires	30.61%
job-to-job transition rate	2.057 %
wage rate	4.185
lowest accepted match quality	-1.685 ($\bar{F} = 95.40\%$)
search threshold match quality	0.417 ($\bar{F} = 33.83\%$)
average match quality	0.504

4.1 Comparative statics results — aggregate productivity

Next, I allow aggregate productivity to vary. As stated earlier, I rely on comparisons of stationary equilibria to assess the response of the model to aggregate shocks. In the standard search model such a comparative static exercise invariably gives results that are very close to the dynamic response of the full stochastic model. This is due to the fact that transition dynamics are very swift in the standard model due to the forward-looking and instantaneous adjustment of the vacancy-unemployment ratio and the resulting high job-finding rate. Due to the presence of on-the-job search, the full stochastic version of the model of this paper is much more complex. In particular, the complete distribution of match qualities across employed workers enters the state space, and hence the dynamics become more gradual. This is due to the fact that the vacancy-unemployment ratio does not adjust instantaneously to its new long-run value, and while the job-finding rate of unemployed workers is equally high as in the standard model, that is not true of the job-to-job transition rate of employed workers, and hence the adjustment towards the new steady state could be much more prolonged.

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	1% increase productivity	
	stationary equil. value	elasticity
unemployment rate	9.20%	-5.44
vacancy rate	0.351	3.63
fraction of unemployed searchers	17.18%	-5.23
fraction of unemployed new hires	29.71%	-2.92
job-to-job transition rate	2.12 %	3.19
wage rate	4.234	1.18
lowest accepted match quality	-1.734 ($\bar{F} = 95.86\%$)	-2.93
search threshold match quality	0.426 ($\bar{F} = 33.50\%$)	2.24
average match quality	.5102	1.32

These simulations show that on-the-job search and job-to-job transitions vary positively with aggregate productivity shocks when comparing stationary equilibria for different aggregate productivity levels.

The amplification mechanism embedded in the model shows up clearly when considering the response of the unemployment and the vacancy rate to changes in aggregate productivity. In the standard model, as Shimer (2004) has shown, the elasticity of the vacancy-unemployment ratio with respect to labor productivity (which in the standard model is equal to market tightness) is below 2 for reasonable parameter values. Here, the elasticity of the vacancy-unemployment ratio is 9.07, which is both due to the decline in unemployment (elasticity of -5.44) and to an increase in the vacancy rate (elasticity of 3.63).[5]

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[5]With regards to the level of the vacancy rate, recall again that these numbers correspond to the normalization where $c_1 = 1$ and $m_0 = 1$. Varying these parameters — which do not have clear equivalents in the data — would result in varying the level of the vacancy rate, but not its elasticity.

4.2 Comparative statics results — destruction shocks

	1% decline in destruction	
	stationary equil. value	elasticity
unemployment rate	9.35%	-3.92
vacancy rate	0.348	2.85
fraction of unemployed searchers	17.42%	-3.91
fraction of unemployed new hires	29.89%	-2.37
job-to-job transition rate	2.10 %	2.29
wage rate	4.19	0.13
lowest accepted match quality	-1.6904 ($\bar{F} = 95.45\%$)	-0.31
search threshold match quality	0.432 ($\bar{F} = 33.50\%$)	3.50
average match quality	.5141	2.11

In these simulations the vacancy-unemployment ratio varies negatively with job destruction shocks and with the unemployment rate. Shimer (2004) argues that destruction shocks induce a positive correlation between vacancies and unemployment rate in the standard model, thereby causing the vacancy-unemployment ratio to have a positive elasticity with respect to adverse destruction shocks. That conclusion no longer holds up in this model in the presence of job-to-job movements. This brings back destruction shocks into the picture as a potential source of business cycle variation in the vacancy-unemployment ratio.

5 Conclusions

INCOMPLETE.

6 Appendix

6.1 Proof of Proposition 1

Define conditional distributions

$$H^m(\mu) = \frac{H_0(\mu)}{H_0(\mu_s)} \qquad \mu \leq \mu_s \tag{54}$$

$$H^n(\mu) = \frac{H_0(\mu) - H_0(\mu_s)}{1 - H_0(\mu_s)} \qquad \mu \geq \mu_s \tag{55}$$

Then the belief of firm in match of tenure T

$$H_T(\mu) = \begin{cases} H^m(\mu)\omega_T & \text{if} \quad \mu \leq \mu_s \\ H^n(\mu)(1 - \omega_T) & \text{if} \quad \mu > \mu_s \end{cases} \tag{56}$$

where $\omega_0 = H_0(\mu_s)$. Applying Bayes's rule, the differential equation for the evolution of ω_T is

$$\dot{\omega}_T = -\eta\omega_T(1 - \omega_T), \tag{57}$$

where η is the rate at which workers below match quality μ_s find better matches. Solving the differential equation in Equation (57) gives

$$\omega_T = \frac{\omega_0}{\omega_0 + e^{\eta T}(1 - \omega_0)}. \tag{58}$$

Clearly, ω_T is decreasing in T, hence H_T first-order stochastically dominates H_0. This in turn means that a firm is always willing to continue a relationship at the initial wage.

Corollary 4. *There is a single wage in equilibrium.*