Competition for Default*

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Abstract

This paper analyzes Markovian equilibria in a model of strategic lending where (i) agents cannot commit to long term contracts, (ii) contracts are incomplete, and (iii) incumbent lenders can coordinate their actions. The structure of the credit market changes endogenously over time along every equilibrium path. After a sequence of bad shocks, the borrower in a competitive market accumulates debt overhang and the incumbent lenders exercise monopoly power. Even though the incumbents could maintain this power forever, they find it profitable to let the borrower regain access to the competitive market after a sequence of good shocks.

Equilibria are computed numerically, and their attributes are qualitatively consistent with numerous known empirical facts on sovereign lending. In addition, the model predicts that a borrower who accumulates debt overhang will regain access to the competitive credit market only after good shocks. This prediction is shown to be consistent with data on emerging market economies.

Keywords: *sovereign debt, sovereign default, debt overhang, buybacks.*

1 Introduction

If a borrower accumulates more debt than the maximum surplus that a lender can extract from her, the borrower will be unable to roll over her debt in a competitive asset market. This is called

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debt overhang. However, the lenders to whom the debt is owed have incentives to continue to lend, because further lending may allow them to recover at least part of the debt. In fact, since the incumbent lenders will not face any competition in the credit market, they may exercise monopoly power over the borrower. When a lender offers a contract in a competitive market, he already anticipates recovering part of his investment from the ability to exercise monopoly power over the borrower rather than from direct repayment. The goal of this paper is to build a theory around this observation and explore its consequences.

One consequence is that the structure of credit markets changes endogenously over time: the borrower has access to competitive credit markets in some periods and is excluded in others. Furthermore, the time of exclusion from credit markets is potentially long. Indeed, in the context of sovereign lending, countries often accumulate debts so large that they cannot be rolled over. Such a country then temporarily loses access to credit markets. After some time, often more than a decade, lenders renegotiate the size of the debt with the borrower and let the borrower regain access to competitive asset markets. But later, the country accumulates large debts again, and the cycle is repeated. These cycles are illustrated in Table 1, which lists the years of defaults of several Latin American countries.

	1824-34	1867-82	1890-1900	1911-21	1931-40	1976-89	1998-2003
Argentina	1830		1890	1915	1930s	1982	2001
Bolivia		1874			1931	1980	
Brazil	1826		1898	1914	1931	1983	
Chile	1826	1880			1931	1983	
Colombia	1826	1879	1900		1932		
Costa Rica	1827	1874	1895		1937	1983	
Dominican Republic		1869	1899		1931	1982	
Ecuador	1832	1868		1911,1914	1931	1982	1999
Guatemala	1828	1876	1894		1933		
Honduras	1827	1873		1914		1981	
Mexico	1827	1867		1914		1982	
Nicaragua	1828		1894	1911	1932	1980	
Paraguay	1827	1874	1892	1920	1932	1986	
Peru	1826	1876			1931	1978, 1983	
Uruguay		1876	1891	1915	1933	1983	2003
Venezuela	1832	1878	1892, 1898			1982	

Table 1. Selected Government Defaults and Reschedulings of Privately Held Bonds and Loans, 1820-2003. Source: Sturzenegger and Zettelmeyer (2005).

There are numerous historical episodes of debt accumulation and renegotiation, many of them involving the same countries. One such episode is the 1930s interwar debt crisis. Bolivia, Chile, Columbia and Peru defaulted in 1931-1932 and received substantial debt relief through negotiated settlements as well as market-based buybacks during the 1940-50s.[1] Another example is the case of the Highly Indebted Countries (HICs). This group included several Latin American countries, among them Bolivia and Peru.[2] These countries defaulted at the beginning of the 1980s, and 18 of these countries received substantial debt relief by swapping their debts for Brady bonds in the 1990s.[3] Finally, the most recent example of debt reduction, and the largest in history, is the Argentine debt-swap deal that closed in February 2005, four years after Argentina defaulted. The face value of the country's debt was decreased by 70 to 75 percent. Argentina was also one of the HICs in the 1980s.

The specific model analyzed in this paper is the following. Two lenders compete for a contractual position with a single borrower. The borrower has a stochastic production technology that requires investment every period. Each period, each lender offers contracts to the borrower. A contract specifies a net transfer between the lender and the borrower, and the debt for the next period. Neither the lenders nor the borrower can commit to multi-period contracts.

After the lenders make their offers, the borrower chooses a contract or defaults. A borrower who defaults is excluded from the asset market forevermore. A lender whose contract is accepted must repay the debt that is owed this period. Notice that if the lender whose contract is accepted is the same as the one to whom this period's debt is owed, he does not actually have to repay this period's debt; from his point of view, the debt already accumulated is a sunk cost. This creates an asymmetry between the incentives of the two lenders when offering new debt contracts.

The single lender in our model who is in a contractual relationship with the borrower is interpreted as the entire group of lenders to whom an accumulated debt is owed in reality. The other lender is interpreted as the entire group of lenders who have no claims on the borrower. Modeling the incumbents as a single entity implies that the incumbents can coordinate their actions when dealing with the borrower. In general, our model should be applied to environments where such coordination is possible if the borrower accumulates debt overhang. In reality, coordination among lenders can be difficult to achieve because of a free-rider problem. A single lender might prefer not to participate in a debt reduction plan if he anticipates that the borrower will fully repay his claim after she reaches an agreement with the rest of the lenders. Although we recognize the importance of this free-rider problem, we abstract from it in the current paper.[4]

Some properties of all Markovian equilibria of this game are characterized. A subgame perfect

[1] For a discussion, see Jorgensen and Sachs (1989) and Eichengreen and Portes (1989, 1991).

[2] For a discussion, see Bulow and Rogoff (1990).

[3] Brady bonds were named after U.S. Treasury Secretary Nicholas Brady, who in 1989 announced a plan encouraging debt relief for the HICs.

[4] This abstraction is consistent with sovereign debt renegotiation made through the London Club or the Paris Club, as well as with the use of Common Action Clauses (CACs) in sovereign bond issues. The London and Paris Clubs organize coordination among lenders. The inclusion of CACs allows the terms of contract to change only if a predetermined supermajority of bondholders consent. For a detailed discussion of these issues see Chapter 1 in Sturzenegger and Zettelmeyer (2005).

equilibrium is Markovian if the actions of the agents depend only on the income and debt of the borrower. We show that the state of the economy can always be classified into one of two regimes: a monopoly regime and a competitive one. If the borrower faces a debt overhang, the incumbent lender gives a take-it-or-leave-it offer to the borrower. That is, if the debt is larger than the maximum surplus that a lender can extract from the borrower, the incumbent lender has full bargaining power and acts as a monopolist. On the other hand, if there is no debt overhang, the lenders are Bertrand competitors; the contract accepted by the borrower maximizes the borrower's surplus subject to the condition that the outsider lender makes zero profit.

Our main result is that the state of the economy keeps switching between the two regimes along every equilibrium path. More precisely: If the lenders are competing at some point of time, the borrower surely accumulates debt overhang and loses her access to the competitive market at a future date. Conversely, even if the incumbent has monopoly power over the borrower, the borrower eventually regains access to competition with probability one. The latter result might seem surprising at first. Indeed, if the borrower faces debt overhang, the incumbent lender could offer contracts that would guarantee him monopoly power forever. We show, however, that after a sequence of *good* shocks, the incumbent lender will find it profitable to let the borrower have access to the competitive credit market again. The intuition behind this result is the following. The contracts offered by a monopolist lender are inefficient from the point of view of the borrower. If the debt is lowered, the borrower has access to a larger set of contracts and can make more efficient investment decisions. This implies that the borrower's willingness to pay for a marginal debt reduction exceeds the lender's loss from the same debt reduction. Therefore, after a sequence of good shocks, when the borrower has enough liquidity to compensate the incumbent lender for a reduction in the next period's debt, the incumbent will offer a contract that specifies a large immediate payment in exchange for reducing the debt next period. This is a *buyback*.

This result may be important in the context of sovereign lending. Countries having debt overhang often lose access to credit markets for several years, sometimes for more than a decade. The current literature does not provide a satisfactory explanation for this observation.[5] The majority of the literature on debt renegotiation assumes that if there is a debt overhang, the game is changed, and the parties must engage in bilateral bargaining to reduce debt.[6] Since, in general, there is no delay in bargaining, the debt reduction is immediate.[7] In contrast, in our model, the

[5]For example, in their seminal paper, Eaton and Gersovitz (1981) assume that the penalty for default is a permanent exclusion from future credit. They say that an important elaboration would be to make the time of exclusion endogenous.

[6]See, e.g., Bulow and Rogoff (1989), Fernandez and Rosenthal (1990), Ozler (1989), and Yue (2005).

[7]Some papers make time spent outside the credit market exogenously long. In Arellano (2005), a country that has defaulted can reenter the asset market according to an exogenously given probability in each period. Yue (2005) assumes that after debt renegotiation (via Nash bargaining), the country cannot access credit markets immediately, but only after fully repaying the reduced debt. Cole, Dow, and English (1995) get exogeneously long exclusion by assuming that the borrower's type follows a Markov process. A borrower of a *bad* type defaults, while a borrower of a *good* type makes repayments in order to regain access to credit markets.

time elapsed from the onset of debt overhang to a buyback is endogenous and potentially long. The contracting protocol that describes the interaction between the borrower and the lenders in our game does not change through the play of the game. In particular, it does not depend on whether there is a debt overhang. The borrower has no access to competitive asset markets in some periods because new lenders have no incentive to offer contracts. The change in the regime, and hence the time spent outside the competitive credit market, is merely a consequence of equilibrium behavior.

We identify two sources of inefficiency. When the incumbent lender has monopoly power, he invests less than the socially optimal amount in general. This is because the lender takes into account only his own payoff and ignores the borrower's when making investment decisions. On the other hand, if the borrower has enough liquidity and small debt, she overinvests, because overinvesting can decrease the probability of liquidity problems and debt overhang in the future.

To investigate time series properties of the model, we numerically compute an equilibrium and simulate it. Our computational results are qualitatively consistent with many empirical regularities observed in emerging market economies. In particular, there is a negative relationship between the level of debt and investment, and investment is lower if there is a debt overhang. Arslanalp and Henry (2004), Borensztein (1990a), Deshpande (1997) and Fischer (1987) document that the 1980s debt crises in the HICs were associated with a fall in investment, and that investment rose after the debt reduction deals. In addition, empirical studies including Arrau (1990), Borensztein (1990b), Deshpande (1997), Fry (1989), and Greene and Villanueva (1991) find a negative effect of debt on investment. These studies are discussed in details in Section 8.

Our computations also suggest that periods of debt overhang are associated with lower income levels. This is consistent with finding that debt crises usually result in output losses, as suggested by, for example, Sturzenegger's (2004) growth regressions for developing countries during the 1980s. Our simulations predict a negative correlation between the borrower's income and the country interest rate. Studies that report countercyclicality of interest rates on developing countries' loans include Cline (1995), Cline and Barnes (1997), Edwards (1984), Neumeyer and Perri (2005), and Uribe and Yue (2003). We also find a negative relationship between investment and the country interest rate, as found, for example, by Edwards (1984).

Another prediction of our model is that debt reductions occur after a sequence of good productivity shocks. We provide empirical evidence that supports this finding. In particular, the average real GDP growth rate of the Brady countries was 4.1 percent in the last three years before the debt relief deal, compared with 1.8 percent during the six years preceding those three years. An even more striking example is the 2005 Argentine debt swap. Real GDP grew at 8 percent in the last two years before the deal, compared with an average of -4.9 percent during the four years preceding those two years.

The remainder of the paper is organized as follows. The rest of this section reviews related literature. Section 2 describes the model. Section 3 includes preliminary observations and defines the equilibrium concept and the state space. Section 4 characterizes the value functions. Section

5 contains the main analytical results. Section 6 establishes the existence of an equilibrium. The computational results are presented in Section 7. In Section 8, we present correlation coefficients obtained from the simulated time series and discuss empirical evidence consistent with them. Section 9 concludes. Most of the proofs are relegated to the Appendix.

Related Literature

Our model has many features similar to those analyzed in the corporate debt literature. In particular, agents are risk neutral, and the motive for borrowing is that the borrower needs to invest in a stochastic production technology. On the other hand, related papers on corporate lending usually analyze ex-ante optimal contracts. These papers implicitly assume that contracts cannot be renegotiated, that is, agents can commit to multi-period contracts.[8] Instead, we analyze Markovian equilibria in a game where new contracts are offered in every period and agents cannot commit to long-term contracts. To illustrate the differences between this literature and our paper in terms of conclusions, we review a few of these papers.

Thomas and Worrall (1994) and Albuquerque and Hopenhayn (2004) study an environment very similar to ours. The main differences are that there is only a single lender[9] and the parties can commit to long-term contracts subject to participation constraints in each period. The ex-ante optimal contract has the following features. The value to the borrower increases and the value to the lender decreases over time. The reason is that the borrower's participation constraint is easier to satisfy if payments are delayed. Furthermore, the amount of investment increases over time, often converging to the socially optimal level. Since we study Markovian equilibria, and contracts are renegotiated every period, neither the value of the agents nor the amount of investment depends on time in our model. All of these variables are the functions only of the state variables: the borrower's income and the debt.

Clementi and Hopenhayn (2002) also analyze lending contracts with limited commitment under the assumption that the lender does not observe either the use of borrowed funds or the output of the project. The authors show that the ex-ante optimal contracts have the following feature. There are two absorbing states: after some time, either the project is liquidated or only the first-best investment is made and the lender has no claim on the borrower.[10] Liquidation follows a sequence of bad shocks. A sequence of good shocks results in an increasing path for the borrower's value, eventually reaching the first-best social surplus. In sharp contrast with this result, in our equilibria there are no absorbing states; the economy keeps switching between the competitive and monopoly regimes. Furthermore, investment is always inefficient.

Another paper identifying ex-ante optimal debt contracts is Atkeson (1991). In his model, the

[8] Commitment in this literature is often limited, so that contracts must be subject to participation constraints in each period.

[9] In Albuquerque and Hopenhayn (2004), there are many competing lenders, but they only force the incumbent lender to earn zero profit ex-ante and play no strategic role otherwise.

[10] Quadrini (2004) shows that liquidation of the firm can be a feature of the optimal contract even if renegotiation is possible.

lenders do not observe whether the borrower invests or consumes borrowed funds. To provide proper ex-ante incentives to the borrower, the optimal contract specifies a fall in consumption and investment for the lowest realizations of output. As we mentioned earlier, our model produces a similar result. If the borrower's income is small and the debt is large, investment is inefficiently small. However, while in Atkeson (1991) incentives cause this problem, in our model the lender's monopoly power is to blame.[11]

Unlike in our model, the majority of the literature on sovereign lending uses consumption smoothing as a motive for borrowing. See, e.g., Atkeson (1991), Eaton and Gersovitz (1981), Calvo and Kaminsky (1991), Kletzer and Wright (2000), and Yue (2005). On the other hand, many papers, like ours, analyze Markovian equilibria in the presence of competitive credit markets, instead of characterizing optimal contracts.

The majority of the papers on sovereign debt focus mostly on the incentives and welfare of the borrower, while the creditors only play passive roles. This paper shifts the focus toward the lenders. A key feature of our model is the dynamics of competition among the lenders resulting from stochastic productivity shocks. This feature can be crucial in analyzing debt overhangs, debt roll-overs, and buybacks. A central question of the literature on sovereign lending is whether debt reductions are socially beneficial and whether the borrower or the lender collects the rents from them. Below, we briefly describe the debate on these issues and the contribution of this paper to the debate.

Sachs (1988, 1989, 1990) argues in favor of debt reductions. He claims that debt burden decreases domestic investment, impedes growth, and aggravates economical and political instability. Krugman (1988b) claims that debt reduction generally benefits both the debtor and the creditors if the debtor is on the wrong side of the "debt relief Laffer curve" – that is, if the nominal debt is so high that reducing it actually increases the expected payment. Krugman (1988a) finds that if the debtor can affect output through a costly effort, the creditor might find it profitable to forgive part of debt to increase the likelihood that the country will repay the rest. Froot (1989) argues that when a country is liquidity constrained, the creditors should combine debt reduction with new lending in order to stimulate investment, thereby increasing potential future repayments. Calvo and Kaminsky (1991) study a model in which debt reduction is a feature of the optimal contract as a means to improve consumption smoothing. Their calibration suggests that small risk premia could be consistent with relatively large debt reductions.

Bulow and Rogoff (1988), on the other hand, claim that open-market buybacks are likely to be a poor deal for a debtor country that faces debt overhang because the country simply uses its scarce resources to subsidize the creditors. This is because a buyback reduces the face value of the debt but the price of the remaining claims rises. Hence, the actual market value of outstanding debt changes little, reflecting low market expectations about the country's capacity to repay debt.

[11] A paper related to investment inefficiencies due to limited enforcement is Marcet and Marimon (1992). Their computational results suggest that the welfare loss associated to these inefficiencies can be severe.

The main criticism of Bulow and Rogoff (1988) is that they do not model inefficiencies associated with debt overhang, such as investment disincentives, limits on future borrowing and loss of access to credit markets.[12] However, Bulow and Rogoff (1991) argue that even if buybacks stimulate domestic investment, creditors will fully capture the efficiency gain from them.

Fernandez and Rosenthal (1990) present a model where only the lenders benefit from debt reductions. In their equilibria, if the borrower cannot service her debt, the lenders offer a deal that keeps the borrower indifferent between accepting the deal and defaulting. Since the borrower accepts the deal in equilibrium, all efficiency gains from debt reduction go to the lenders.

This paper's contribution to the debate is the following. Once debt overhang accumulates, it is socially efficient to reduce the debt instead of letting the borrower default. This is because, despite the debt overhang, it remains worthwhile to invest in the borrower's production technology. On the one hand, like Fernandez and Rosenthal (1990) and Bulow and Rogoff (1988, 1991), we find that only the lenders benefit from buybacks *ex post*.[13] The borrower is no better off accepting a debt reduction than defaulting. On the other hand, if debt buybacks are possible, the lenders rationally anticipate that the borrower will not default once debt overhang accumulates. This allows the lenders to recover at least part of their investments. Lenders offer contracts in the credit market that reflect these expectations. Since the credit market is competitive to start with, the borrower then extracts all of the *ex-ante* efficiency gain from buybacks. Our arguments emphasize that it is insufficient to analyze environments where debt overhang has already accumulated when one evaluates the welfare implications of debt reductions and buybacks.

2 The Model

There are three risk neutral agents: a borrower and two lenders. Time is discrete, and all agents discount the future according to the same discount factor β.

Production and Preferences

The borrower has a stochastic technology to transform capital goods into consumption goods. A production function f and a random variable S describe the technology. If the amount of capital investment is K, then the borrower's income in the next period, in terms of consumption goods, is

$$F(K) = sf(K),$$

where s is the realization of the shock S. The production function is strictly increasing and strictly concave and satisfies the Inada conditions. The random variable S is assumed to be distributed according to cdf G, with positive density g on $[0, 1]$. Capital completely depreciates every period.

[12] See the discussion in Bulow and Rogoff (1988), Sachs (1988), and Rotemberg (1991).

[13] It is important to distinguish the buyback mechanism used in this paper from other mechanisms. Bulow and Rogoff (1988, 1991) consider open-market buybacks, where the *face value* of debt is reduced by purchasing the debt at a discount. In our model, a buyback is a reduction in the *market value* of debt. This is the type of buyback implemented, for example, by letting the HICs swapping their debts for Brady bonds in the 1990s.

The lenders can instantaneously transform one unit of capital good into one unit of consumption good and vice versa. This means the lenders are indifferent between the two goods.[14] The lenders have enough capital to invest in production in every period. Each agent's goal is to maximize the discounted present value of expected consumption.

Timing and Contracts

Neither the borrower nor the lenders can commit to multi-period contracts. A typical contract consists of three numbers, (R, K, D'). R denotes the immediate payment from the borrower to the lender whose contract is accepted, K (≥ 0) is the lender's investment in the production technology, and D' (≥ 0) is the next period's debt. More generally, a contract can be a probability mixture of these triples.[15] The simple form of the contract implies that although output is observable, contracts cannot be conditioned on it.[16] In particular, the next period's debt, D', cannot be contingent on the realized output.

Suppose that at the beginning of a period, the realization of output is I and the debt is D. The borrower and both lenders observe I and D. Then the lenders simultaneously offer a set of contracts to the borrower. The borrower either chooses a contract from these sets or defaults. (If no contract is offered, the borrower must default.) If the borrower defaults, she consumes her current income, I, and the game ends. If she chooses a contract (R, K, D'), she gives R units of consumption good to the lender who offered this contract, and the lender invests K units of capital in the production technology. The lender whose contract is accepted also repays the debt D to last period's lender.[17] The borrower consumes $I - R$ and the new lender consumes R. Then the period ends.

Since the borrower is risk neutral, R can always be assumed to be deterministic even though contracts can be random. A contract (R, K, D') is *feasible* if $R \leq I$. We call a lender *the incumbent* at the beginning of a period if the borrower accepted his contract in the previous period. We call the other lender *the outsider*.

In reality, a borrower can service her debt if she can partially repay her outstanding debt and can contract with lenders such that her old creditors get repaid. Hence, the assumption that the new lender has to pay D to the old one corresponds to the borrower's ability to service her debt.[18]

[14] Having two different goods ensures that the borrower's continuation payoff upon default is simply her last-period output. This assumption is not uncommon in the literature. See, for example, Thomas and Worrall (1994). Although our main results would hold even if there were a single good and the borrower could invest in her own technology, we have not been able to prove that an equilibrium other than autarky exists in that case.

[15] Allowing randomization guarantees that the lender has a concave value function, which makes it possible to characterize some specific features of the policy functions.

[16] This assumption is often justified in the literature by pointing out that some shocks are observable but not verifiable.

[17] If the borrower accepts a random contract – that is, a lottery over (R, K, D') triples – then the outcome of the lottery is first observed.

[18] We do not allow the borrower to default on old lenders and contract with new ones. Old debts are assumed to be senior. We want to point out that this type of seniority can be a feature of equilibria without assuming it. That

One might ask: Why does the borrower not repay D, at least partially? Recall that the borrower gives the new lender R, which should be viewed as part of the repayment of D.

3 Preliminaries

The First-Best Investment

Since each agent is risk neutral, a social planner would invest such that the discounted present value of expected net outputs is maximized. The optimal (first-best) investment solves the following maximization problem:

$$\max_{K} -K + \beta \int_0^1 sf(K)\, dG(s).$$

The solution, K_{FB}, is defined by the following first-order condition:

$$\beta f'(K_{FB}) \int_0^1 s dG(s) = 1. \tag{1}$$

Equilibrium Concept

We restrict attention to Markovian equilibria. To be more specific, we are interested in subgame perfect equilibria in which the lenders offer sets of contracts that depend only on the borrower's income, I, and the current debt, D, and in which the borrower's choice of contract depends only on her income and the set of contracts offered. In addition, we require that the actual level of overhang debt has no influence on the incumbent lender's offers. That is, if the debt is so large that the lender cannot recover it (and hence the debt has no real meaning), the set of contracts offered by the incumbent lender does not depend on the actual level of the debt; all that matters is that there is a debt overhang. We lay this out formally below.

In the next sections, we operate under the assumptions that (i) Markovian equilibria exist, and (ii) autarky is not an equilibrium. We characterize some properties of all Markovian equilibria. These properties then play an important role in the equilibrium-existence proof in Section 6, where we also prove that autarky is indeed not an equilibrium. We will not show that there is a unique Markovian equilibrium in our game. However, all of our results hold for each Markovian equilibrium.

Fix a Markovian equilibrium and introduce the following notation. $W^L(I, D)$ denotes the value to the incumbent lender at the beginning of a period if the borrower's income is I and her debt is D. Similarly, $W^B(I, D)$ denotes the value to the borrower. Both of these functions depend on the equilibrium we have fixed. We refer to the pair (I, D) as a state.

is, the leders' equilibrium strategies deter new lenders from offering contracts to the borrower unless the old lenders are repaid, see Kletzer and Wright (2000).

Incentives

Suppose that the borrower's income is I. Then she prefers accepting a contract (R, K, D') to defaulting if

$$I - R + \beta E W^B \left(F(K), D'\right) \geq I.^{19} \tag{2}$$

The right-hand side of this inequality is the value of default, and the left-hand side is the expected payoff from accepting the contract. The contract must satisfy $R \leq I$ to be feasible. The incentive to accept such a contract is strict whenever (2) is a strict inequality.

The outsider lender is willing to offer only those contracts under which his expected payoff is large enough to cover the current debt, D, that he must pay to the incumbent lender. The following constraint describes these contracts:

$$R + E\left[-K + \beta W^L \left(F\left(K\right), D'\right)\right] \geq D, \tag{3}$$

where $R \leq I$. The left-hand side of the inequality is the outsider lender's expected payoff if the borrower accepts his contract. The right-hand side is the required payment to the incumbent.

If the outsider lender offers a contract satisfying (3), then it would also be profitable for the incumbent lender to offer the same contract. This is because if the incumbent offers this contract and it is accepted, his payoff is the left-hand side of (3), while his payoff otherwise is the right-hand side. Therefore, without loss of generality, we assume throughout the paper that the incumbent lender's contract is always the one accepted along the equilibrium path.[20] Most of our proofs use the following argument: A contract cannot be an equilibrium contract if the outsider lender can offer a contract that is strictly preferred and accepted by the borrower and hence the outsider becomes the incumbent.

D may be so high that no feasible contract can satisfy (2) and (3) simultaneously. In such a situation, the outsider lender cannot offer any contract that both guarantees him a non-negative payoff and is acceptable to the borrower. However, the incumbent lender may still be willing to offer a contract even if D cannot be fully recovered: The incumbent's payoff from an accepted contract is the left-hand side of (3), and whenever this quantity is positive, the incumbent has the incentive to offer a contract. This observation is central to our results. The current debt, D, is sunk cost to the incumbent but limits the outsider's ability to offer contracts.

Debt Overhang and the State Space

Let $V^L(I)$ and $V^B(I)$ denote the value to the incumbent lender and the borrower, respectively, conditional on the outsider lender offering no contract. Notice that $V^L(I)$ is the maximum payoff the incumbent can achieve if the income of the borrower is I, that is $V^L(I) = \sup_D W^L(I, D)$. The reason is the following. Suppose that the borrower's income is I. If the outsider offers no contract, the incumbent can offer the same set of contracts to the borrower as the one offered at state (I, D). The borrower accepts the same contract as the one when the debt is D because her

[20] Ponzi games are automatically excluded by this assumption.

decision depends only on the set of offered contracts. This shows that $V^L(I) \geq W^L(I, D)$. If the debt is very large, say larger than the first-best social surplus, then the outsider has no incentive to offer contracts. Hence, there is a D such that $V^L(I) = W^L(I, D)$. We refer to V^L and V^B as monopolistic value functions.

If the debt exceeds $V^L(I)$, it is so high that the lender can never extract full repayment from the borrower in the future. Therefore, the outsider lender cannot offer a contract that is acceptable to the borrower and that would make it profitable to repay the debt to the incumbent.

Definition 1 *A borrower with income I and debt D faces debt overhang if $D \geq V^L(I)$.*

Whether the borrower faces debt overhang depends not only on her income and debt but also on the equilibrium actually played, because $V^L(I)$ depends on actions to be taken in the future.

Whenever $D \geq V^L(I)$, the incumbent lender's payoff is $V^L(I)$. He might be able to achieve this payoff by offering different sets of contracts providing the borrower with different continuation values. The set of contracts offered by the lender potentially depends on the actual level of debt. But this would mean that although the lender faces the same environment from the point of view of payoff relevant variables, his actions depend on the history of debt accumulation. Hence, corresponding to the notion of Markovian equilibrium, we restrict attention to strategies such that whenever $D_1 > D_2 \geq V^L(I)$, the lenders offer the same contracts to the borrower when the debt is either D_1 or D_2. Therefore, to describe an equilibrium one must define the strategies of the players only on the set $\left\{(I, D) : D \leq V^L(I)\right\}$. We refer to this set as the *state space*.

Market Structure

In what follows, we show that the contracting protocol in our game implies perfect competition if $D < V^L(I)$ and monopoly power for the incumbent lender if $D \geq V^L(I)$.

If the borrower could offer a contract to the lenders subject to the constraint that the current debt D must be paid to the incumbent lender, her maximization problem would be:

$$\max_{R,K,D'} I - R + \beta E W^B(F(K), D') \tag{4}$$

subject to (3). We show below that if $D < V^L(I)$, the contract accepted by the borrower solves this constrained maximization problem. In addition, (3) holds with equality, which means the incumbent lender cannot have a larger payoff than D at state (I, D). This means that whenever there is no debt overhang, the lenders are Bertrand competitors.

On the other hand, if $D \geq V^L(I)$, we will show that the incumbent lender can give a take-it-or-leave-it offer to the borrower. Thus the contract accepted in equilibrium solves the following maximization problem:

$$\max_{R,K,D'} R + E\left[-K + \beta W^L(F(K), D')\right] \tag{5}$$

subject to (2). This implies that whenever the borrower faces debt overhang, the incumbent lender has monopoly power.

The following proposition lays this out formally.

Proposition 1 *Suppose that the income of the borrower is I, the debt is D, and the equilibrium contract is (R^*, K^*, D^*). Then*

(i) If $D < V^L(I)$, then (R^*, K^*, D^*) solves (4) subject to (3), and (3) holds with equality.

(ii) If $D \geq V^L(I)$, then (R^*, K^*, D^*) solves (5) subject to (2).

Proof. See the Appendix. ■

Proposition 1 suggests that the borrower may want to avoid debt overhang because the incumbent, not the borrower, will make decisions if $D \geq V^L(I)$. From the borrower's point of view, these decisions may well be inefficient. We will show that this in fact happens in every equilibrium.

According to Proposition 1, the economy is always in one of two regimes. The proposition does not imply, however, that the economy keeps switching between these two regimes. The main result of this paper is that, in fact, it does.

Also notice that although Proposition 1 makes a sharp distinction between the case when $D \geq V^L(I)$ and the case when $D < V^L(I)$, it turns out that the borrower becomes continuously worse off as the debt D increases. This is because the set of available contracts satisfying equation (3) shrinks continuously as D gets larger.

4 The Value Functions

This section characterizes the value functions. First, we describe some properties of the lender's monopolistic value function, V^L. Second, we prove that $W^L(I, D) = \min\{D, V^L(I)\}$. Then, as a function of V^L, we fully characterize the monopolistic value function of the borrower, V^B. Finally, we establish a dual relationship between V^B and W^B. Our strategy is to express all of the value functions in terms of V^L. This will play an important role in establishing the existence of an equilibrium. (See Section 6.)

4.1 The Value Functions of the Incumbent

The next lemma describes some attributes of the function V^L.

Lemma 1 *The function V^L has the following attributes:*

(i) V^L is increasing,

(ii) if $\delta > 0$, then $V^L(I + \delta) - \delta \leq V^L(I)$ for all I, and

(iii) V^L is concave.

Proof. See the Appendix. ■

The value to the monopolist incumbent is weakly increasing in the borrower's income, because the more income the borrower has, the more surplus can be extracted from her. This shows part (i). Part (ii) means that the slope of the function V^L is (weakly) less than one. If the borrower's income is I, the incumbent can give δ units of consumption good to the borrower, and then offer the same contract as the one at $I+\delta$. The incumbent's payoff from this contract is $V^L(I+\delta)-\delta$. Of course, the lender might be able to offer a contract that provides him with an even higher payoff, hence $V^L(I) \geq V^L(I+\delta)-\delta$. Part (iii) is merely a consequence of the possibility of random contracts.

Next, we characterize W^L in terms of V^L.

Proposition 2 $W^L(I,D) = \min\{D, V^L(I)\}$.

Proof. See the Appendix. ■

The argument of the proof is the following. If $D \geq V^L(I)$, the lender acts as a monopolist, in which case his payoff is $V^L(I)$. If $D < V^L(I)$, he faces competition from the outsider. In this case, the incumbent cannot offer a contract that the borrower accepts and that generates a payoff higher than D. Otherwise, the outsider could offer essentially the same contract with a slightly smaller immediate repayment R. The borrower would strictly prefer and accept the outsider's contract, showing that the incumbent's payoff cannot exceed D.

Next, we characterize the monopolist's capital investment K_0 when $I=0$ in terms of V^L. Computing K_0 is important for two reasons. First, we will show that the monopolist incumbent invests K_0 not only at $I=0$ but also at a large range of other income realizations. Second, K_0 plays an important role in the characterization of the borrower's monopolistic value function, V^B. (See the next subsection.) At the state $(0, V^L(0))$, the lender's maximization problem is

$$\max_K -K + \beta E W^L(F(K), D').$$

Since $I=0$, the borrower accepts any contract specifying $K>0$. This is because at $I=0$ her default option is zero, and if $K>0$, with probability one her income in the next period is positive. Hence, the incentive compatibility constraint, (2), is automatically satisfied. Since the contract will be accepted anyway, Proposition 2 implies that the incumbent is (weakly) better off specifying $D' = V^L(f(K))$. That is, the borrower has no access to the competitive market in the next period, no matter what the realization of her income is. Therefore, the maximization problem of the lender can be rewritten as

$$\max_K -K + \beta \int_0^1 V^L(sf(K))\, dG(s). \tag{6}$$

Next we show that $\int_0^1 V^L(sI)\, dG(s)$ is concave in I.

Claim 1 *Suppose v is a concave function defined on $[0,\infty)$. Then $\int_0^1 v(sI)\, dG(s)$ is also concave in I on $[0,\infty)$.*

Proof. See the Appendix. ■

The previous claim and the strict concavity of f imply that the maximization problem (6) has a unique solution, K_0, and can be characterized by the following first-order condition corresponding to (6):

$$1 = \beta f'(K_0) \int_0^1 V^{L\prime}(sf(K_0))\, s dG(s). \tag{7}$$

Also notice that K_0 is strictly positive if autarky is not an equilibrium, that is, when V^L is not constant zero.

From the concavity it also follows that

$$-K + \beta \int_0^1 V^L (sf(K))\, dG(s)$$

is strictly increasing on $[0, K_0]$ and strictly decreasing on $[K_0, \infty)$.

4.2 The Value Functions of the Borrower

First, we compute the value to the borrower at $\left(0, V^L(0)\right)$, $V^B(0)$. Then we characterize V^B.

Recall that at $\left(0, V^L(0)\right)$, the equilibrium contract is $\left(0, K_0, V^L(f(K_0))\right)$. Hence,

$$V^B(0) = \beta \int_0^1 V^B(sf(K_0))\, dG(s), \tag{8}$$

because the right-hand side is the expected discounted payoff of the borrower from the contract $\left(0, K_0, V^L(f(K_0))\right)$.

The next lemma characterizes the value function of the borrower under monopoly power.

Lemma 2

$$V^B(I) = \begin{cases} V^B(0) & \textit{if } I < V^B(0), \\ I & \textit{otherwise.} \end{cases}$$

Proof. See the Appendix. ■

As we have shown, if the borrower has no income ($I = 0$) the monopolist lender's investment, K_0, is positive, generating a positive value for the borrower at $\left(0, V^L(0)\right)$. That is, $V^B(0) > 0$. Therefore, whenever $I < V^B(0)$, the monopolist can offer the same contract as the one at $I = 0$, except that $R = I$. The borrower strictly prefers accepting this contract to defaulting since it guarantees a payoff of $V^B(0)\,(> I)$. This shows that $V^B(I) = V^B(0)$ whenever $I < V^B(0)$. If $I \geq V^B(0)$ the monopolist lender always offers a contract that makes the borrower exactly indifferent between accepting the contract and defaulting. Since the borrower's value of default is I, this implies that $V^B(I) = I$.

By (8) and Lemma 2, $V^B(0)$ is defined by the following equation:

$$V^B(0) = \beta G\left(\frac{V^B(0)}{f(K_0)}\right) V^B(0) + \beta \int_{V^B(0)/f(K_0)}^1 sf(K_0)\, dG(s). \tag{9}$$

The previous equation, (7) and Lemma 2 enable one to determine V^B as a function of V^L.

The following proposition establishes the relationship between the functions W^B, V^B and V^L.

Proposition 3 *Suppose $D < V^L(I)$. Then*

$$W^B(I, D) = V^B(I'),$$

where I' solves

$$V^L(I') - D = I' - I. \tag{10}$$

Proof. See the Appendix. ■

Figure 1 represents the statement of Proposition 3 graphically. The state space $\left\{\left(\widetilde{I},\widetilde{D}\right):\widetilde{D}\leq V^L\left(\widetilde{I}\right)\right\}$, and the current state of the economy (I,D), are depicted on the bottom panel. To find the value to the borrower at (I,D), we draw a 45 degree line through the point (I,D) until it intersects with the curve V^L. Proposition 3 states that the value to the borrower at (I,D) is the same as at $\left(I',V^L\left(I'\right)\right)$, that is $W^B\left(I,D\right)=V^B\left(I'\right)$.

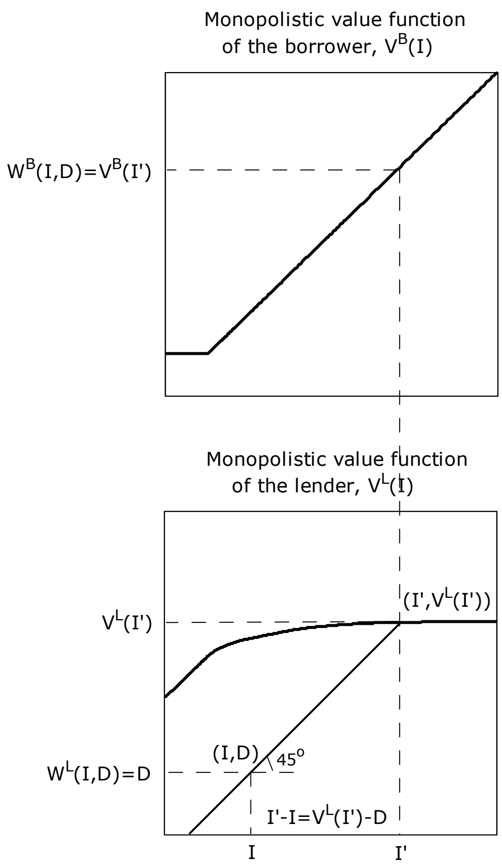

Figure 1. Competitive value functions.

The argument of the proof is based on establishing a dual relationship between the borrower's maximization problem at state (I,D) and the lender's maximization problem at $\left(I',V^L\left(I'\right)\right)$. Recall from Proposition 1 that the borrower's maximization problem at (I,D) is

$$\max_{(R,K,D')} I-R+\beta EW^B\left(F\left(K\right),D'\right)$$
$$\text{s.t. } D=R+E\left[-K+\beta W^L\left(F\left(K\right),D'\right)\right].$$

Suppose that (R^*,K^*,D^*) solves this problem. After substituting D from the previous constraint into (10), we get

$$V^L(I') = R^* + [I' - I] + E\left[-K^* + \beta W^L(F(K^*), D^*)\right].$$

This means that the contract $(R^* + [I' - I], K^*, D^*)$ maximizes the payoff of the incumbent lender at $\left(I', V^L(I')\right)$. (This contract satisfies the incentive compatibility constraint of the borrower, (2).) Essentially, the maximand in the borrower's optimization problem at (I, D) becomes the constraint in the lender's optimization problem at $\left(I', V^L(I')\right)$, and the constraint of the borrower becomes the maximand of the lender's problem. This shows the dual relationship between the constrained maximization problems of the borrower at (I, D) and the monopolist incumbent at $\left(I', V^L(I')\right)$.

An immediate consequence of the previous proposition is that the borrower's value is constant along the 45-degree line segment drawn in the bottom panel of Figure 1, which is defined as $\left\{\left(\widetilde{I}, \widetilde{D}\right) \in \mathbb{R}^2_+ : \ \widetilde{D} \leq V^L\left(\widetilde{I}\right), \ \widetilde{I} - I = \widetilde{D} - D\right\}$. In other words, as long as the borrower does not face a debt overhang, her value depends only on her net position, that is, on the difference between her income and debt. More precisely, if $D_1 \leq V^L(I_1)$, $D_2 \leq V^L(I_2)$ and $I_1 - D_1 = I_2 - D_2$, then $W^B(I_1, D_1) = W^B(I_2, D_2)$.

Let $H(I - D)$ denote the value to the borrower facing no debt overhang if her income is I and her debt is D. By Proposition 3,

$$H(x) = V^B\left(Q^{-1}(x)\right), \tag{11}$$

where $Q(I') = I' - V^L(I')$.

Indifference Curves

Figure 2 depicts indifference curves of the lender and the borrower on the (I, D) plane. The black lines plot the $V^L(I)$ curve, which divides the plane into two regions, $\left\{(I, D) : D \geq V^L(I)\right\}$ and $\left\{(I, D) : D < V^L(I)\right\}$. In the region where $D \geq V^L(I)$, the indifference curves of both agents are vertical, since the actual level of debt overhang is irrelevant. By Lemma 2, when $D \geq V^L(I)$ and $I \leq V^B(0)$, the borrower's payoff is constant at $V^B(0)$. This corresponds to the shaded area on the right panel. Similarly, to the right of the point where $V^L(I)$ becomes flat, the value to the lender is constant (see the left panel). In the next subsection, we show that such a point indeed exists. Since $W^L(I, D) = D$ whenever $D < V^L(I)$ (see Proposition 2), the indifference curves of the lender are horizontal below the $V^L(I)$ curve. By Proposition 3, in $D < V^L(I)$ region, the borrower's value depends only on her net position $I - D$, and hence her indifference curves are 45-degree lines.

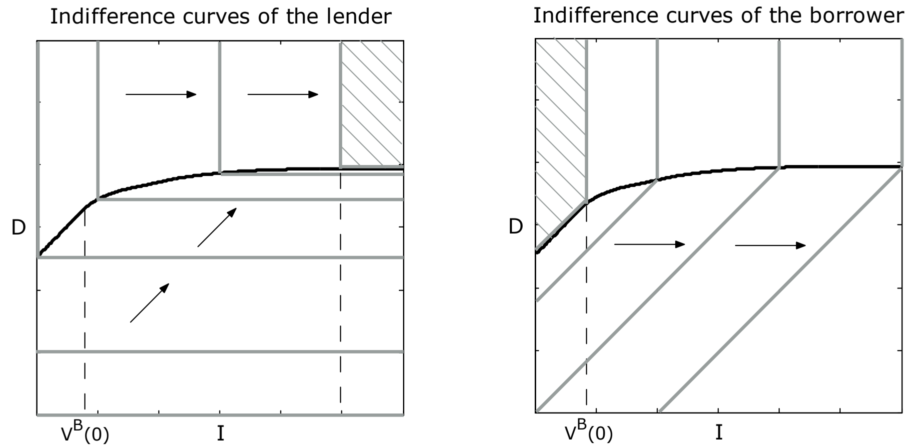

Figure 2. Indifference curves.

4.3 Marginal Values

In this subsection, we turn our attention to characterizing the marginal values of income to the borrower and to the lender. Identifying these marginal values is crucial because they determine investment decisions and hence social welfare. We show that the marginal value of income to the borrower is larger than one if there is no debt overhang. We also prove that the marginal value of income to the monopolist incumbent is strictly smaller than one if the income of the borrower is sufficiently large.

Lemma 3 *If $D < V^L(I)$, then*

$$H'(I-D) = \frac{1}{1 - V^{L\prime}(I')} \geq 1, \tag{12}$$

where I' is defined by (10). Furthermore, H is concave.

Proof. See the Appendix. ■

$H'(I-D)$ is the marginal value to the borrower of an additional unit of income in state (I, D). Recall from Lemma 1 that $V^{L\prime} \in [0,1]$, which implies that $H' \geq 1$. Since the borrower can always consume the additional unit of consumption good, the marginal value is obviously at least one. The more interesting observation is that whenever $V^{L\prime} > 0$, the marginal value is strictly bigger than one. The reason is that the borrower can use the additional income to reduce her debt. Reducing the debt gives the borrower access to a larger set of contracts, which allows her to make more efficient decisions. Hence, the borrower wants to reduce her debt as fast as possible.[21] The goal of the rest of this section is to characterize the set of income realizations for which $V^{L\prime} < 1$.

[21] If the borrower could save, she would do so to avoid debt accumulation. Given our game, $D' \geq 0$ since the lenders would default on any positive saving. Our results would hold even if we assumed that the lenders could not default on savings, as long as there were an upper bound on how much the borrower could save. (Without a bound equilibrium would not exist.) If the borrower were risk averse, our results would hold even without this bound.

Suppose that the borrower has no debt and that her income is so high that it does not constrain her when she decides how much capital to buy. Then her maximization problem becomes

$$\max_{(R,K,D')} -R + \beta E W^B \left(F\left(K\right), D'\right)$$
$$\text{s.t. } R = E\left(K - \beta W^L \left(F\left(K\right), D'\right)\right).$$

After plugging the constraint into the maximand, the problem can be rewritten as

$$\max_{(R,K,D')} E\left[\beta W^L \left(F\left(K\right), D'\right) + \beta E W^B \left(F\left(K\right), D'\right) - K\right].$$

From Lemma 3 and Proposition 2 it follows that $W_2^L + W_2^B \leq 0$, and hence the borrower always (weakly) prefers a contract with $D' = 0$ and $R = K$. Therefore the maximization problem can be further rewritten as

$$\max_K -K + \beta \int_0^1 H\left(sf\left(K\right)\right) dG\left(s\right).$$

The first-order condition is

$$-1 + \beta f'\left(K\right) \int_0^1 H'\left(sf\left(K\right)\right) s dG\left(s\right) = 0. \tag{13}$$

Since H is concave (see Lemma 3), by Claim 1 the second-order condition is automatically satisfied. Let K_M denote the solution of the previous equality, and I_M the continuation value to the borrower,

$$I_M = \beta \int_0^1 H\left(sf\left(K_M\right)\right) dG\left(s\right).$$

Lemma 4

$$V^{L\prime}\left(I\right) \begin{cases} = 1 & \textit{if } I \leq V^B\left(0\right), \\ \in \left(0,1\right) & \textit{if } I \in \left(V^B\left(0\right), I_M\right), \\ = 0 & \textit{if } I \geq I_M, \end{cases}$$

and $H'\left(I - D\right) > 1$ *whenever* $H\left(I - D\right)$ *is strictly smaller than* I_M*. Furthermore,* $H'\left(0\right) > 1$*.*

Proof. See the Appendix. ■

If $I \leq V^B\left(0\right)$, the lender offers the same contract as the one at state $\left(0, V^L\left(0\right)\right)$, except that it specifies $R = I$ instead of zero. That is, if the borrower's income increases by a dollar, the value to the lender also increases by a dollar because, by increasing the immediate repayment, he fully extract the additional income from the borrower. This shows that $V^{L\prime} = 1$ and $R\left(I\right) = I$ in this domain. Suppose now that $I \in \left(V^B\left(0\right), I_M\right)$. If the borrower's income increases by a dollar, the lender still wants to extract the increase from the borrower via an increase in the immediate repayment, R. However, since $I > V^B\left(0\right)$, unlike in the previous case, the increase in the repayment comes at a cost for the lender. In order to satisfy the incentive constraint of the borrower, the lender must compensate her for a larger repayment by either increasing the capital investment or decreasing the debt in the next period. Hence, the lender's continuation value increases by less than one, showing that $0 < V^{L\prime} < 1$ and $R(I) = I$ on $\left(V^B(0), I_M\right)$. If $I \geq I_M$, the lender no longer has an incentive to extract an increase in the borrower's income. This is because the equilibrium capital investment at $\left(I_M, V^L(I_M)\right)$ is already so high that its marginal product is very small. Furthermore, the next period's debt is already zero, so the lender cannot compensate the borrower by reducing D'. Therefore, whenever $I \geq I_M$, the lender offers the same contract as the one at state $\left(I_M, V^L(I_M)\right)$, which shows that $V^{L\prime} = 0$ and $R(I) = I_M$.

Remark 1 *Let $R(I)$ denote the repayment of the borrower at state $\left(I, V^L(I)\right)$. Then*

$$R(I) = \begin{cases} I & \text{if } I < I_M, \\ I_M & \text{otherwise.} \end{cases}$$

Proof. See the Appendix. ■

5 Main Results

Inefficient Investments

Since each agent in our model is risk neutral, any welfare loss is due to inefficient investments. The next theorem identifies two sources of inefficiency.

Theorem 1 *The monopolist lender invests too little if the borrower has low income. The borrower invests too much if she has small debt and high income. Formally:*

$$K_0 < K_{FB} < K_M.$$

Proof. First, we show that $K_{FB} < K_M$. Notice that (12) implies

$$\int_0^1 H'(sf(K))\, sf'(K)\, dG(s) > \int_0^1 sf'(K)\, dG(s).$$

The inequality is strict because $H'(0) > 1$, by Lemma 4. Therefore

$$0 = -1 + \beta \int_0^1 H'(sf(K_M))\, sf'(K_M)\, dG(s) > -1 + \beta \int_0^1 sf'(K_M)\, dG(s).$$

The previous inequality together with (1) implies $K_M > K_{FB}$.

Second, we show that $K_0 < K_{FB}$. From Lemma 4,

$$\int_0^1 V^{L\prime}(sf(K))\, sf'(K)\, dG(s) < \int_0^1 sf'(K)\, dG(s).$$

The inequality is strict because $f(K_0) > V^B(0)$ (for otherwise the borrower would never consume). Therefore

$$0 = -1 + \beta \int_0^1 V^{L\prime}(sf(K_0))\, sf'(K_0)\, dG(s) < -1 + \beta \int_0^1 sf'(K_0)\, dG(s),$$

which together with (1) implies $K_0 < K_{FB}$. ■

The intuition behind these results is the following. If $I = 0$, the borrower cannot compensate the monopolist lender by up-front payments for his investments. The lender's investment decision is driven by his ability to extract the next period's income from the borrower. Since the borrower controls the income, the lender can extract it only at a cost. Hence, the marginal value of income for the monopolist is less than one, and thus he invests too little. On the other hand, if the net income of the borrower is high ($I - D > K_M$), the borrower overinvests. This is because, by overinvesting, the borrower can reduce the probability of debt overhang. The borrower wants to do so because she anticipates that if there is debt overhang, the incumbent lender will invest inefficiently little.

Investments K_0 and K_M are made on a large set of states. Investment K_0 is made if $I \in \left[0, V^B(0)\right]$ and the lender has monopoly power. Investment K_M is made if the net income of the borrower, $I - D$, lies in the interval $[K_M, \infty)$. We conjecture, but can show only by numerical computations, that in general the investment is positively correlated with income. Our computational results are reported in Section 7.

Atkeson (1991) also shows that investment is inefficiently small for the lowest realizations of output. However, in his model, a moral hazard problem generates this result. The lender does not observe whether the borrower invests or consumes the borrowed funds. The investment is small if the realized income is small, because this provides proper ex-ante incentives for the borrower.

Welfare Analysis

Recall that if $D \leq V^L(I)$, the value to the borrower is $H(I - D)$ and the value to the lender is exactly D. Thus the social surplus at state (I, D) is $H(I - D) + D$.

Theorem 2 *(i) Suppose $V^L(I) \geq D_1 > D_2$. Then*

$$H(I - D_1) + D_1 \leq H(I - D_2) + D_2.$$

Furthermore the inequality is strict whenever $H(I - D_2) < I_M$.

(ii) Suppose that $I_1 > I_2$ and $V^L(I_1) \geq D$. Then

$$H(I_1 - D) + D \geq H(I_2 - D) + D + [I_1 - I_2].$$

Furthermore, the inequality is strict whenever $H(I_1 - D) < I_M$.

Proof. It follows from (12) and Lemma 4 that

$$\frac{\partial H(I - D)}{\partial I} = -\frac{\partial H(I - D)}{\partial D} \geq 1,$$

with strict inequality whenever $H(I - D) < I_M$. Therefore,

$$\frac{\partial H(I - D) + D}{\partial D} = 1 - H'(I - D) \leq 0,$$

with strict inequality whenever $H(I-D) < I_M$. This proves part (i). Furthermore,

$$\frac{\partial H(I-D)+D}{\partial I} = H'(I-D) \geq 1,$$

with strict inequality whenever $H(I-D) < I_M$, showing part (ii). ■

The amount of debt D is not part of the physical environment. Hence, in a first-best world, where there are no commitment problems, the debt cannot have any effect on social surplus. Efficiency therefore requires that investment decisions do not depend on the debt D. According to part (i), an increase in debt decreases social welfare in our model. This is because a larger debt implies less efficient investment decisions.

The second part of the theorem says that, in general, an increase in the borrower's income I increases the social surplus by more than the increase in I. The intuition is the same as before: The borrower's value increases by more than the increase in her income because she can use the additional income to repay debt and avoid debt overhang.

Fluctuating Market Structures

In the next theorem, we show that the structure of the credit market keeps changing over time.

Theorem 3 *(i) Suppose that the borrower faces a debt overhang. Then there is surely a future date at which the borrower regains access to the competitive credit market. More precisely: Suppose that at time t the economy is at state* (I_t, D_t) *and* $D_t \geq V^L(I_t)$*. Then, with probability one, there is a future date,* τ*, at which the economy is at state* (I_τ, D_t) *such that* $D_\tau < V^L(I_\tau)$*.*

(ii) Suppose that the borrower has access to the competitive credit market. Then there is surely a future date at which the borrower faces a debt overhang. More precisely: Suppose that at time t the economy is at state (I_t, D_t) *and* $D_t < V^L(I_t)$*. Then, with probability one, there is a future date,* τ*, at which the economy is at state* (I_τ, D_t) *such that* $D_\tau \geq V^L(I_\tau)$*.*

Proof. See the Appendix. ■

When the borrower faces debt overhang, the incumbent lender acts as a monopolist. Clearly, the incumbent lender could offer a sequence of contracts that would guarantee him monopoly power forever no matter what shocks were realized.[22] The first part of the theorem says that the incumbent does not find such a sequence of contracts to be optimal; after a while, he offers a contract that gives the borrower access to the competitive market next period with positive probability. The incumbent lender sometimes specifies the next period's debt such that, at least in high-income states, the borrower will not face debt overhang.

The argument for this result is the following. Suppose that the lender never allowed the borrower to return to the competitive market. Then, at least in some states, the borrower's consumption would be positive, i.e., the optimal contract would specify $R < I$. Otherwise, the borrower would

[22] This is because the monopolist lender can always offer a contract $\left(R, K, V^L(f(K))\right)$ that the borrower prefers to default, that is, a contract such that $I - R + \beta E V^B(F(K)) \geq I$. An example of such a contract is $\left(\min\left\{I, V^B(0)\right\}, K_0, V^L(f(K_0))\right)$.

default whenever $I > 0$. On the other hand, the borrower's gain from a marginal debt reduction exceeds the lender's loss. (See Proposition 2 and Lemma 3.) Since $R < I$, the borrower has liquidity to compensate the lender for a debt reduction. Hence, the monopolist incumbent would be better off increasing R and decreasing next period's debt, D'. Hence, $R < I$ cannot be specified in the optimal contract, a contradiction.

Another way to understand why the monopolist incumbent lets the borrower have access to competitive markets is the following. The most efficient way to extract surplus from the borrower is to guarantee efficient investment decisions to her. If the lender could commit to multi-period contracts, he would promise to invest efficiently in the future in exchange for large immediate repayments. Unfortunately, such promises are not credible because of the lack of commitment. However, by lowering the next period's debt, the incumbent provides the borrower with access to the competitive market with positive probability. In the competitive market, the borrower can make more efficient investment decisions. Lowering the next period's debt serves as a commitment device for the incumbent. It can be viewed as a credible promise of the incumbent to invest efficiently in the future.

Part (ii) says that debt overhang occurs with probability one. This is because, with probability one, the borrower is hit by a sequence of bad shocks. Along that sequence, arbitrarily large debts can accumulate.

From the proof of the theorem, the following remark follows.

Remark 2 *If the borrower accumulates debt overhang, she regains access to the competitive credit market only after the realization of a sequence of good shocks.*

Buyback Boondoggle

As we mentioned in the introduction, several papers argue that borrowers do not benefit from organized buybacks. For example, Bulow and Rogoff (1988) report the following episode. In March 1988, Bolivia's foreign commercial bank debt was selling at a mega-discount, about 6 cents on the dollar. Benefactors donated several million dollars to the Bolivian government for purposes of repurchasing part of the debt at these secondary market prices, which would allow the government to retire nearly half of the nominal value of the debt. However, after the repurchase, the secondary market price on the remaining debt rose to 11 cents, and as a result, the market value of the total debt remained practically unchanged. That is, the burden of the Bolivians' external debt was no lower, and the buyback was just a transfer to the creditors. Bulow and Rogoff (1988) call this phenomenon a buyback boondoggle.

There are two ways to explain this phenomenon in our model, depending on whether the transfer from a third party is targeted at reducing debt directly or at increasing the borrower's income.[23] Suppose that the borrower accumulated debt overhang, that is, $D \geq V^L(I)$. First, assume that a third party *unexpectedly* reduces the debt by an amount less than $D - V^L(I)$. Since the new

[23] We are grateful to Casey Mulligan for drawing our attention to this observation.

debt still exceeds the maximum surplus that can be extracted from the borrower, $V^L(I)$, the debt reduction is simply a transfer to the incumbents, but has no impact on the borrower's welfare. The borrower's payoff is $V^B(I)$ before as well as after the debt reduction. Second, suppose that $I < V^B(0)$, and the donation increases the borrower's income by less than $V^B(0) - I$. Since the borrower's payoff is constant on $\left[0, V^B(0)\right]$ (see Lemma 2), the borrower does not benefit from the transfer. The monopolist lender extracts the transfer from the borrower via an increase in R without changing the continuation payoff of the borrower.

A shortcoming both of the analysis above and of Bulow and Rogoff (1988) is that they consider only what happens *after* debt overhang accumulates. Although the borrower might not benefit from buybacks ex-post, third-party transfers still have an impact ex-ante. The contracts offered in the competitive credit market will reflect the lenders' anticipation of such transfers. Since the credit market is competitive, the borrower extracts all of the surplus from buybacks ex-ante.

6 The Fixed-Point Theorem

We have not yet shown that a Markovian equilibrium exists in our game. Notice that by Propositions 2 and 3, it is enough to characterize the monopolistic value functions. These results immediately deliver the competitive value functions. Also notice that from (7), (8) and Lemma 2, the monopolistic value function of the borrower can be deduced from the monopolistic value function of the lender.

Recall that the lender's payoff when the borrower's income is I, $V^L(I)$, is the value of the following constrained maximization problem:

$$\max_{R,K,D'} R + E\left[-K + \beta E W^L\left(F(K), D'\right)\right]$$
$$\text{s.t. } I - R + \beta E W^B\left(F(K), D'\right) \geq I,\ I \geq R.$$

Since W^L is determined by V^L, this may seem to be a standard stochastic dynamic programming problem. However, the constraint, in particular W^B, is also influenced by V^L. As a result, Blackwell's conditions fail to hold for the corresponding fixed-point operator. The operator is non-monotonic. Hence, we cannot use the usual techniques to guarantee existence. This is also the reason why we were unable to establish uniqueness of the equilibrium.

The argument of the existence proof is the following. First, we show that the potential candidates for V^L are contained in a convex, compact set, called Γ. Then, we construct a fixed-point operator, T, as follows. For each element of Γ, say g, using Proposition 2, (7), (8), Lemma 2 and Proposition 3, we construct the rest of the value functions, V_g^B, W_g^L and W_g^B, as if g were the true V^L. Then, we define $Tg(I)$ as the value to the monopolistic lender solving a maximization problem if the borrower's income is I, assuming that the value functions are W_g^L and W_g^B. Finally, we show that this operator has a fixed point, and any fixed point determines an equilibrium.

First, all of the value functions can be bounded between zero and the discounted present value of the social surplus conditional on the first-best investment being made each period. Let us define M as

$$M = \sum_{i=0}^{\infty} \beta^i \left(\beta \int_0^1 sf\left(K_{FB}\right) dG\left(s\right) - K_{FB} \right).$$

Next, we define the set of possible candidates for $V^L - V^L(0)$. Consider the following set of functions:

$$\Gamma = \left\{ g \mid g \in C[0,\infty),\, g(0) = 0,\, g \text{ is concave},\, \frac{dg}{dx} \in [0,1]\,,\, \frac{dg}{dx} = 0 \text{ on } [M,\infty) \right\}.$$

Recall from Lemma 1 that we know the monopolistic lender's value function is increasing and concave, with slope less than one. If the income of the borrower exceeds M, then the monopolistic lender is clearly unable to offer a contract that specifies $R = I$, because the discounted present value from a contract cannot exceed M. Therefore, from Lemma 4 and Remark 1 it follows that $V^{L\prime} = 0$ on $[M,\infty)$. Hence, if there exists an equilibrium, $V^L - V^L(0) \in \Gamma$. Observe that Γ with the supremum norm, $\left(\Gamma, \|.\|_{\sup}\right)$, is a convex, compact set. We are going to define a fixed-point operator on Γ. The reader should think of the function $g \in \Gamma$ as a potential candidate for $V^L - V^L(0)$.

For all $g \in \Gamma$, let $K_0(g)$ be the solution to the maximization problem

$$\max_K -K + \beta \int_0^1 g\left(sf\left(K\right)\right) dG\left(s\right).$$

(From (7), $K_0(g)$ would be the investment made by the monopolist lender if $g = V^L - V^L(0)$ and $I = 0$.) Since g is concave, from Claim 1 it follows that $K_0(g)$ is well-defined. Next, define $V_g^B(0)$ by

$$V_g^B(0) = \beta G\left(\frac{V^B(0)}{f\left(K_0(g)\right)}\right) V^B(0) + \beta \int_{V^B(0)/f(K_0(g))}^1 sf\left(K_0(g)\right) dG\left(s\right).$$

(From (9), $V_g^B(0)$ would be $V^B(0)$ if $g = V^L - V^L(0)$.) Since $\beta < 1$, $V_g^B(0)$ is well-defined. Define $V_g^B(I)$, the monopolistic value function of the borrower if g were equal to $V^L - V^L(0)$, by

$$V_g^B(I) = \begin{cases} V_g^B(0) & \text{if } I \leq V_g^B(0), \\ I & \text{otherwise.} \end{cases}$$

Furthermore, define $V_g^L(0)$ by the following equation:

$$V_g^L(0) = -K_0(g) + \beta \int_0^1 \left[V_g^L(0) + g\left(sf\left(K_0(g)\right)\right)\right] dG\left(s\right).$$

(Clearly, $V_g^L(0)$ would be $V^L(0)$ if $g = V^L - V^L(0)$.) Define $V_g^L(I) = V_g^L(0) + g(I)$.

Finally, we are ready to determine the competitive value functions as if g were $V^L - V^L(0)$. First, let $H_g(x)$ be described by the following equations, in the spirit of Proposition 3:

$$H_g(x) = V_g^B(I'),$$

where $V_g^L(I') - I' = -x$. Given V_g^L and V_g^B, one can define $W_g^L(I,D)$ and $W_g^B(I,D)$ by

$$\begin{aligned} W_g^L(I,D) &= \min\left\{D, V_g^L(I)\right\}, \\ W_g^B(I,D) &= \begin{cases} H_g(I-D) & \text{if } D < V_g^L(I) \\ V_g^B(I) & \text{if } D \geq V_g^L(I). \end{cases} \end{aligned}$$

Define the operator $T_0 : \Gamma \to C([0,M])$ as follows. Let $T_0 g(I)$ be the value of the maximization problem

$$\max_{(R,K,D')} -K + R + \beta \int_0^1 W_g^L(sf(K), D')\, dG(s) \tag{14}$$

subject to

$$R = \min\left\{I, \beta \int_0^1 W_g^B(sf(K), D')\, dG(s)\right\}. \tag{15}$$

Notice that $T_0 g(I)$ would be the value to the monopolistic lender if the value functions were W_g^L and W_g^B and he could use only deterministic contracts. For some values of income, the lender might be better off randomizing between optimal deterministic contracts at different income levels such that (15) is satisfied in expectation. Hence, to find the generated value function of the lender, one must consider the convex hull of $T_0 g$. Define the fixed-point operator $T : \Gamma \to \Gamma$ as follows:

$$Tg = conc(T_0 g) - (conc(T_0 g))(0),$$

where *conc* denotes the concavification of a function. To see that T indeed maps into Γ, one might show that Tg is continuous and concave with slope less than one, and that $(Tg)(0) = 0$. The concavity of Tg and $(Tg)(0) = 0$ follow immediately from the construction. The continuity of Tg and the conclusion that its slope is less than one follow from the proof of Lemma 1.

Proposition 4 *(i) The operator T has a fixed point.*
(ii) There is a bijection between fixed points and equilibria.
(iii) Autarky is not an equilibrium.

Proof. (i) We apply Schauder's Fixed-Point Theorem to the operator Γ. As we mentioned, the set Γ is a convex, compact set. It remains to show that the operator T is continuous with respect to the supremum norm. This would clearly follow from the continuity of T_0. Notice that the functionals $K_0(g)$, $V_g^B(0)$ and $V_g^L(0)$ are all continuous in g. The function H_g is also continuous. Hence the value functions W_g^L and W_g^B are continuous. Therefore the maximand in (14) and the constraint (15) are both continuous. Thus T_0 is a continuous operator.

(ii) If $g = V^L - V^L(0)$, where V^L is an equilibrium value function of the lender, then g is obviously a fixed point of T. If g is a fixed point of T, then it follows from the proof of Theorem 9.2 in Stokey and Lucas (1989) that V^L, where $V^L - V^L(0) = g$, corresponds to an equilibrium.

(iii) Suppose, to the contrary, that autarky is an equilibrium. This would mean that there is an equilibrium where $V^L \equiv 0$, and that $g \equiv 0$ is a fixed point of T, by part (ii) of this proposition.

To get a contradiction, we show that $(T_0 g)(I) > 0$ whenever I is large enough, in particular if $I \geq \beta \int_0^1 sf(K_{FB})\,dG(s)$. If $g \equiv 0$, then $K_0(g) = 0$, $W_g^L \equiv 0$, and $V_g^B(I) = W_g^B(I, D) = I$. Then (14) and (15) become:

$$\max_{(R,K,D')} -K + R$$
$$\text{s.t. } R = \min\left\{I, \beta \int_0^1 sf(K)\,dG(s)\right\}.$$

If I is large enough, $R = \beta \int_0^1 sf(K)\,dG(s)$, and the maximization problem can be rewritten as

$$\max_K -K + \beta \int_0^1 sf(K)\,dG(s).$$

The solution is clearly K_{FB}, and the feasibility constraint is indeed satisfied if $I \geq \beta \int_0^1 sf(K_{FB})\,dG(s)$. Therefore $(T_0 g)(I) = -K_{FB} + \beta \int_0^1 sf(K_{FB})\,dG(s) > 0$. ■

7 Numerical Computations

We have not been able to analytically characterize the time series properties of our equilibria. The goal of this section is to compute a numerical example and identify correlation coefficients of different variables. We provide intuitions behind these results and compare them with empirical facts on emerging market economies in the next section. Our purpose is merely to investigate whether the predictions of our model are qualitatively consistent with data, not to perform a serious calibration.

Below we present the computational results for a uniform productivity shock, the production function $f(K) = 2K^{.75}$ and the interest rate $r = 1/\beta - 1 = 0.1$. To find a fixed point of the operator T defined in the previous section, we use the value function iterations approach. Starting with an arbitrary function $g_0 \in \Gamma$, we compute $g_1 = Tg_0$, $g_2 = Tg_1$, etc., and iterate until $\|g_n - g_{n-1}\| \leq \varepsilon$ for ε sufficiently small. We have no analytical result guaranteeing that such an algorithm converges. However, we obtained convergence every time the program was run, with different initial guesses, and the fixed point seems to be unique.

Next we describe the value and policy functions. Then we analyze the generated time series. The reader who is only interested in the time series can skip the following subsection and go to the next subsection.

7.1 Solution to the Monopolist's Problem

In this subsection we describe numerically computed monopolistic value functions, $V^B(I)$ and $V^L(I)$, and the corresponding policy functions of investment and the next period's debt, $K(I)$ and $D'(I)$. From Proposition 3, the competitive contract at state (I, D) is $(R(I') - I' + I, K(I'), D'(I'))$, where $(R(I'), K(I'), D'(I'))$ is the equilibrium contract at $\left(I', V^L(I')\right)$ and $I' = H(I - D)$. This gives us the competitive policy functions. The competitive value functions are $W^B(I,D) = V^B(I')$ and $W^L(I,D) = D$.

Monopolistic Value Functions

Figure 3 plots the monopolistic value functions, $V^B(I)$ and $V^L(I)$. As Lemma 2 claims, from 0 to $V^B(0)$ the borrower's value function is constant (equal to $V^B(0)$), and from $V^B(0)$ on, the borrower's value function is just the borrower's income. The shape of the lender's value function is as described in Lemmas 1 and 4. In particular, $V^L(I)$ is increasing and concave. It has slope one on $\left[0, V^B(0)\right]$, then gradually becomes flat, and the slope becomes zero exactly at $I = I_M$.

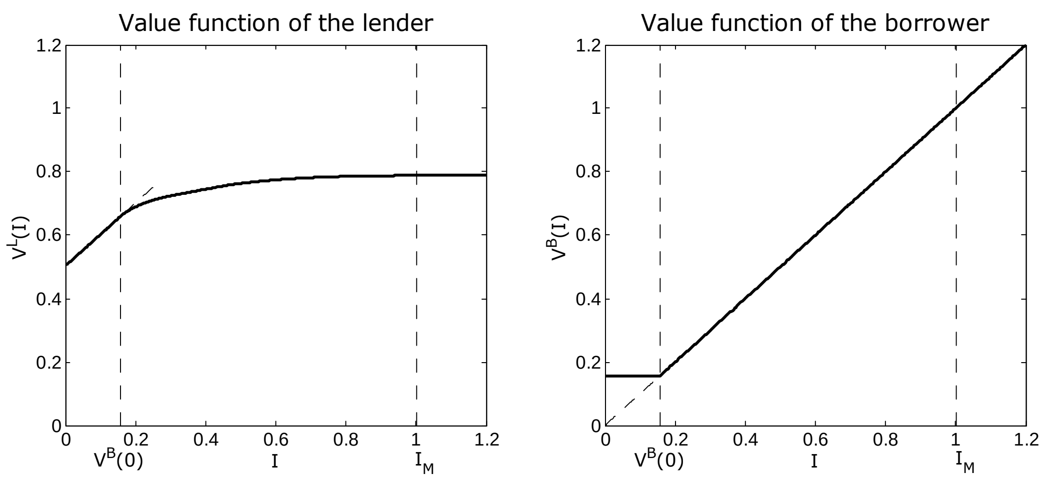

Figure 3. Monopolistic value functions.

Monopolistic Policy Functions

We now describe the equilibrium contracts offered by the monopolistic lender as a function of the borrower's income. Recall that the more income the borrower has, the more difficult it is for the lender to fully extract the income. The lender has two instruments to extract an increase in income through current repayment: He either increases the investment or decreases the next period's debt. In what follows, we analyze how the lender uses these instruments as a function of the borrower's income.

Figure 4 shows the monopolist's choice of investment and the next period's debt as functions of income. These functions are defined as if only deterministic contracts were allowed, but the value functions are generated by potentially random contracts. At the end of this subsection, we explain how the lender randomizes among these contracts.

On the investment graph, the upper horizontal line corresponds to the first-best investment level, and the lower horizontal line to K_0. The thick black line plots $K(I)$. On the debt graph, the thick black line corresponds $D'(I)$. The gray line depicts $V^L(f(K(I)))$. Notice that if the next period's debt is $V^L(f(K(I)))$ or higher, the lender will have monopoly power in the next period for sure. Whenever $D'(I) < V^L(f(K(I)))$, the borrower will have access to competitive markets in the next period with positive probability. The difference between $V^L(f(K(I)))$ and $D'(I)$ should be interpreted as the debt reduction. The thin solid line corresponds to $V^L(0)$. If the next period's debt lies below this level, the borrower surely avoids debt overhang in the next period.

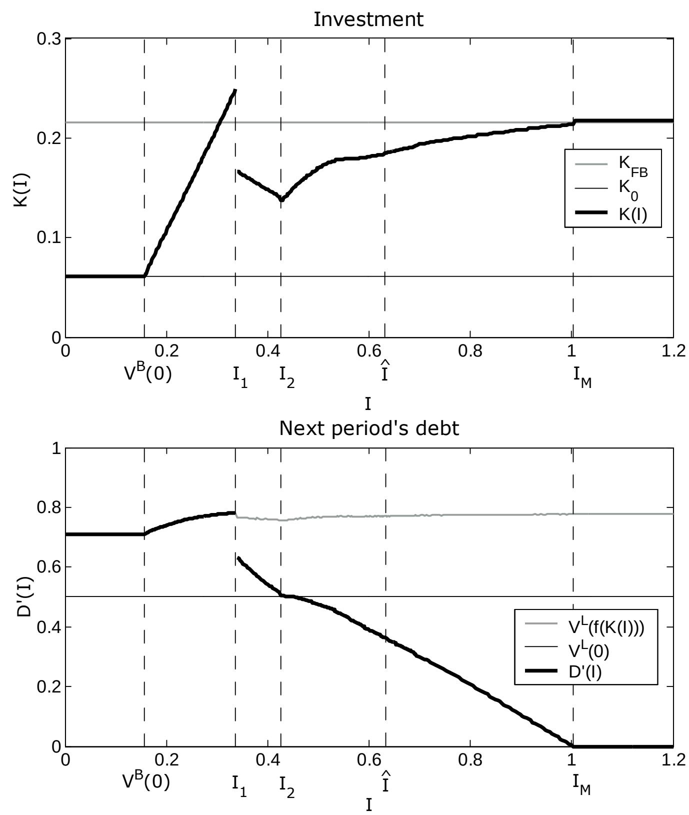

Figure 4. Monopolistic policy functions.

– *The interval* $\left[0, V^B(0)\right]$. As we have shown, when the borrower's income is zero, the lender receives no up-front payment but still invests a positive amount of capital, K_0. The investment is smaller than the socially efficient level, since the lender ignores the value of investment to the borrower. (See Theorem 1.) Notice that K_0 is the smallest investment the lender ever makes. Since the borrower cannot compensate the lender for a debt reduction, the next period's debt, D', is set to $V^L(f(K_0))$. This means the lender remains a monopolist next period no matter what the shock realization is. If $I \in \left[0, V^B(0)\right]$, the same decisions are made about K and D'.

– *The interval* $\left[V^B(0), I_1\right]$. It turns out that reducing debt only marginally is never profitable to the lender. This is because the borrower would regain access to the competitive market with only a very small probability, so providing more investment is a better way to increase the borrower's continuation value. As a result, if a debt reduction ever occurs, it has to be a discrete change.

This would correspond to a discontinuity of the curve $D'(I)$. On the interval $\left[V^B(0), I_1\right]$, the lender uses only investment to extract the borrower's income, and the next period's debt is always so high that the lender surely keeps his monopoly power. For income levels sufficiently close to I_1, the investment is actually higher than K_{FB}.

– *The interval* $[I_1, I_2]$. The lender uses only debt reduction to extract income from the borrower on this interval. As a matter of fact, the investment falls with income, because debt reduction is a more efficient tool to extract the borrower's income. Since there is a discontinuous drop in D' at I_1, there is also a discontinuous drop in investment in order to keep V^B continuous. The curve $D'(I)$ lies strictly between the curves $V^L(0)$ and $V^L(f(K(I)))$ on this interval. This means that if the next period's income is high, the borrower regains access to the competitive credit market, but if the next period's income is low, the incumbent keeps his monopoly power.

– *The interval* $[I_2, \infty)$. As income rises beyond I_2, $D'(I)$ falls below $V^L(0)$. That is, the borrower returns to competition for sure. On this interval, the lender uses both instruments to extract additional income from the borrower. The next period's debt decreases with income, and investment increases with income. Eventually the next period's debt reaches zero. Investment continues to rise shortly after that, until I_M, where investment settles at $K_M\,(> K_{FB})$. No additional surplus can be extracted from that point on, so the policy functions are constant to the right of I_M.

– *Randomization.* So far we have analyzed the monopolist's decisions as if only deterministic contracts were allowed. The discrete jump in the policy functions at I_1 translates into non-concavity of the lender's value function. Therefore, around I_1 the monopolist will use random contracts to achieve a higher expected value. It turns out that the left end of the interval of non-concavity, $\underline{I}$, is just to the left of the income level where investment intersects with the first-best level. The right end, $\overline{I}$, is just to the right of I_2. At income level $I \in \left(\underline{I}, \overline{I}\right)$, the lender offers a random contract that mixes $\left(R(I), K(\underline{I}), D'(\underline{I})\right)$ with probability π and $\left(R(I), K\left(\overline{I}\right), D'\left(\overline{I}\right)\right)$ with probability $1-\pi$, where $\pi = \left(I - \underline{I}\right) / \left(\overline{I} - \underline{I}\right)$. The lender uses only deterministic contracts everywhere else.

Along the equilibrium path, the maximum possible level of the next period's income realization under monopoly is $\hat{I} = f(K(\underline{I}))$. In other words, in the presence of debt overhang, tomorrow's income realizations to the right of $\hat{I}$ are off the equilibrium path. In particular, the monopolist never forgives the debt in full, and the repayment is always equal to the borrower's income (so that consumption is zero).

7.2 Time Series

This subsection presents the results of time series simulations of the model. Starting from the initial condition $I = D = 0$ and generating a random sequence of productivity shocks, we calculate the resulting paths of the main variables using the policy functions described above. The numerical results are based on simulating our model for 100,000 periods. The graphs in this section show only the first 80 periods of this simulation.

Figure 5 plots the time series of productivity shocks, output, consumption of the borrower, investment and debt. The black dots appear at the top of each panel when there is monopoly, and at the bottom when there is competition.

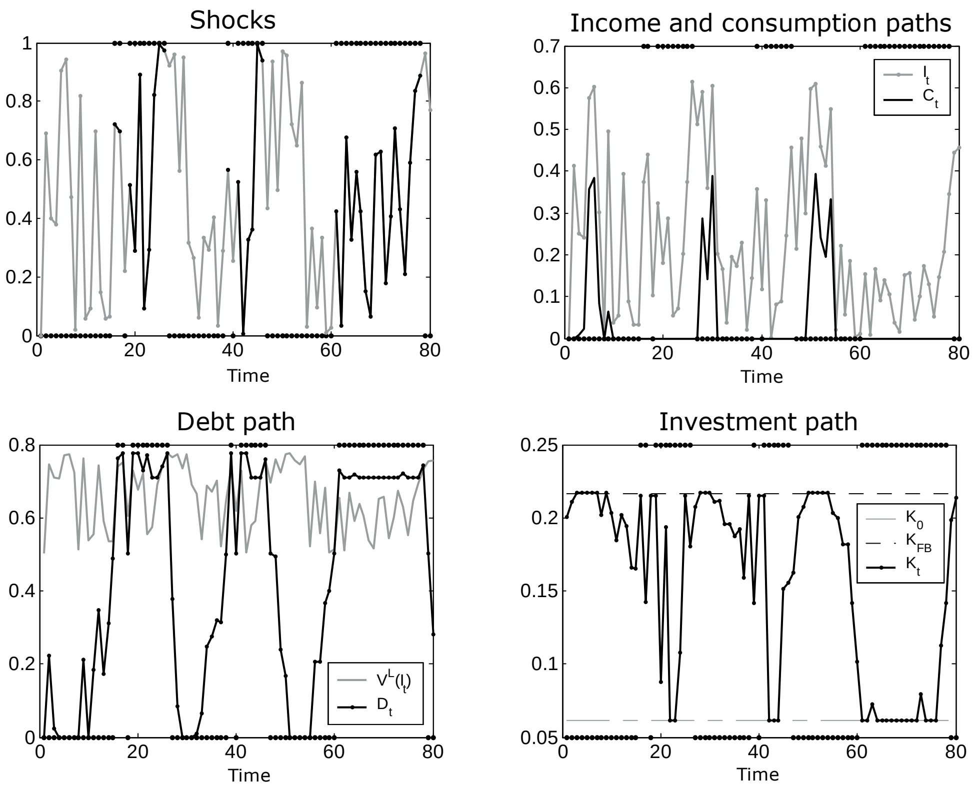

Figure 5. Time series.

– *Market Structure.* Figure 5 shows how the structure of the credit market fluctuates between the two regimes, as Theorem 3 predicts. The time series of shocks is highlighted in black during periods of monopoly. Notice that debt overhang occurs after a sequence of bad shocks, and competition is restored after good shocks. The proportion of time with monopolistic market structure is 11.6 percent, and the average length of the monopolistic regime is 10.3 periods. A switch in regime (from monopoly to competition or vice versa) occurs in 2.3 percent of the periods. There are fewer monopoly periods than competitive periods because the borrower repays her debt as fast as possible to avoid debt overhang.

– *Consumption.* The upper right panel shows the paths of the borrower's income (in gray) and consumption (in black). Observe that consumption is often zero. This is because whenever the borrower has positive debt, she prefers repaying her debt to consuming. This feature of the equilibrium is merely the consequence of the assumption that the borrower is risk neutral.

– *Debt.* The black line on the lower left panel shows the time path of the borrower's debt, D_t. The gray line depicts $V^L(I_t)$. The debt level is below $V^L(I_t)$ when the asset market is competitive. If there is monopoly, $D_t \geq V^L(I_t)$, the black line is above the gray line. Notice that even after the borrower returns to competition, she continues to reduce her debt.

– *Investment.* The lower right panel of Figure 5 plots investment over time. The two horizontal lines again correspond to K_{FB} and K_0. On average, monopoly involves lower investment than competition. The average investment during competition is 96.4 percent of the first-best level of investment, K_{FB}, and the average investment during monopoly is 45 percent of K_{FB}. Notice that periods of zero consumption during competition also involve lower investment. On the one hand, the borrower has little income to pay for capital up front, and on the other hand, she does not want to borrow too much because she does not want to accumulate debt.

– *Welfare.* Figure 6 plots the time series of the welfare loss in percentage terms, computed as

$$(SS_{FB} - SS)/SS_{FB} \cdot 100,$$

where SS_{FB} denotes the first-best social surplus, achieved if the first-best investment is made every period,

$$SS_{FB}(I_t, D_t) = I_t + \frac{-K_{FB} + \beta \int_0^1 sf(K_{FB})\,dG(s)}{1-\beta},$$

and SS is the social surplus in equilibrium,

$$SS(I_t, D_t) = W^B(I_t, D_t) + W^L(I_t, D_t).$$

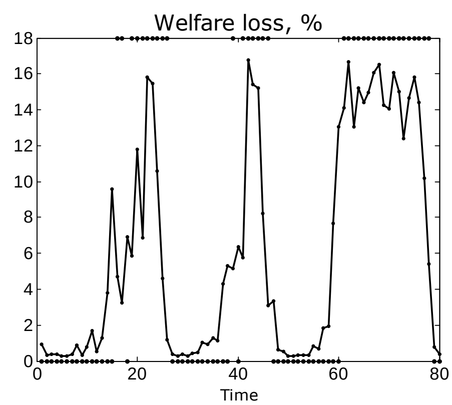

Figure 6. Welfare loss relative to the first best.

The average welfare loss is about 2 percent. In other words, on average about 2 percent of the present discounted value of the output stream is lost due to the inefficiencies of this economy. However, conditional on the monopoly regime, this loss is 12.1 percent, while conditional on competition the loss is only 0.8 percent. Moreover, the investment graph of Figure 5 shows that the welfare loss is almost solely due to underinvestment, not to overinvestment.

8 Empirical Evidence

8.1 Correlations

In this subsection, we report correlation coefficients between certain variables for the time series simulations described above and compare the results with stylized facts described in the literature. Again, since the model parameters were chosen ad hoc rather than calibrated, the correlation results should be viewed purely qualitatively. Table 2 presents the correlation coefficients.

	I_t	D_t	K_t	D_{t+1}	C_t
Market structure indicator,					
= 1/0 if monopoly/competition	$-.25$	.83	$-.86$	.79	$-.24$
Investment, K_t	.46	$-.83$			
Next period's debt, D_{t+1}	$-.55$	.90	$-.85$		
Probability of debt overhang	$-.32$	.80	$-.83$	.84	$-.25$
Country interest rate,					
$r_t = D_{t+1}/\left(D_t + K_t - R_t\right) - 1$	$-.30$	.73	$-.77$	.78	$-.23$
Consumption, $C_t = I_t - R_t$	.78	$-.42$	.36	$-.49$	
Net borrowing, $K_t - R_t$	$-.62$	$-.40$			

Table 2. Correlation coefficients.

The first row of Table 2 shows the correlation coefficient between the indicator of the market structure (= 1 if monopoly and = 0 if competition) and other variables. In particular, the correlation between the monopoly indicator and investment is -0.86. This is because monopoly is associated with lower investment on average compared with competition. (See Theorem 1.) This is consistent with empirical observations. Deshpande (1997) documents that the 1980s debt crises were associated with a drastic fall in investment. In particular, for the HICs, investment declined at an average annual rate of 5.3 percent from 1980 to 1987. Per capita investment fell by about 40 percent. Fischer (1987) reports that investment fell by 5 to 7 percent of GNP between 1981 and 1985 for some indebted countries. According to Borensztein (1990a), the average investment to GDP ratio for 15 HICs was 18 percent in 1982-87, compared to a 24 percent average ratio in 1971-81. Furthermore, Arslanalp and Henry (2004) show that the countries that received debt relief under the Brady deal between 1989 and 1995 experienced an investment boom soon after the deals were made. The average growth rate of capital in these countries increased from 1.6 percent per year five years before the debt relief, to 3.5 percent per year during the five years after the debt relief. This evidence is consistent with our result that investment rises after a debt buyback.

The correlation coefficient between investment, K_t, and the amount of debt, D_t, is -0.83. This can partly be explained by the association of periods of monopoly power with higher debt levels. (The correlation coefficient between the indicator function for a monopoly period and current debt is 0.83.) Also, as we have shown, investment is small during the monopoly regime. However, even during the competitive regime, investment is smaller when debt is larger, because the borrower uses her money to repay debt instead of buying capital. Empirical studies have confirmed this negative effect of debt on investment. For example, Arrau (1990) finds this effect for the 1980s debt crisis in Mexico. Borensztein (1990b) estimates a positive impact of debt reduction on investment in the Philippines. Deshpande (1997) finds a negative relationship between debt and investment for 13 heavily indebted countries between 1971 and 1991. Fry (1989) confirms this finding for 28 developing countries that were highly indebted to the World Bank in 1986. Greene and Villanueva (1991) show that the rate of private investment is negatively related to the debt-service ratio and the ratio of debt to GDP in a sample of 23 developing countries over the period 1975 to 1987.[24]

Due to low investment, income is also low under the monopoly regime. The correlation coefficient between the income of the borrower, I_t, and the monopoly indicator is -0.25. Arslanalp and Henry (2004) report that after debt relief, the Brady countries experienced a sharp rise in per capita GDP growth rates, in line with this prediction. In addition, our result is consistent with the observation that sovereign debt crises result in output losses. Sturzenegger (2004) provides evidence from growth regressions for a large sample of developing countries and finds that countries that defaulted in the 1980s experienced lower GDP growth compared with those that did not default. The author estimates an accumulated 4 percent drop in output over the immediately following four years.

There is a positive relationship between investment, K_t, and the income of the borrower, I_t; the corresponding correlation coefficient is .46. When the borrower has more income, she can afford more investment. Similarly, when income is higher, the borrower is more likely to be able to afford the next period's debt reduction if there is monopoly, and to repay debt if there is competition. The correlation coefficient between I_t and D_{t+1} is negative: -0.55.

Many authors have argued that a large amount of debt increases the probability of default and hence must be associated with higher risk premia. Recent papers that discuss this issue include Arellano (2005) and Yue (2005). Empirical support for this argument includes, for example, Edwards (1984), who estimates a positive effect of the debt-output ratio on the country interest rate spread over LIBOR for 19 countries between 1976 and 1980. Edwards (1986) confirms this finding using data on Eurocurrency loans granted to 26 developing countries and on bonds issued by 13 developing countries during the same time period. Our model also generates a negative relationship between debt and country risk. Higher debt today makes debt overhang tomorrow more likely. (The correlation between D_t and probability of debt overhang next period is 0.80.) This in turn raises the risk premium on the country's loans. (The correlation between D_t and the country interest rate is 0.73.) In addition, Edwards (1984) finds that country bond spreads are negatively related to gross domestic investment over GDP. The author's explanation is that

[24] Cohen (1993) disagrees that large debt is a predictor of low investment. However, Deshpande (1997) argues that a negative effect of debt burden on investment is expected to operate only for countries experiencing debt overhang (the severely indebted countries). Cohen's sample includes 81 developing countries, many of which, presumably, do not experience debt overhang.

higher investment indicates that the country has good prospects for future growth, which should decrease the probability of default and hence the risk premium. Consistent with this finding, we find a strong negative correlation between investment and the country interest rate: -0.77.

A number of studies document that default risk and default premia are highly countercyclical in emerging economies. Neumeyer and Perri (2005) report countercyclicality of real interest rates for Argentina (1983-2001) and Brazil, Mexico, Korea and the Philippines (1994-2001). The authors find that eliminating country risk would lower Argentine output volatility by around 27 percent. Uribe and Yue (2003) also document a negative relationship between country spreads and output for Argentina, Brazil, Ecuador, Mexico, Peru, the Philippines and South Africa over the period 1994 to 2001. Estimating a VAR model, they find that country spread shocks explain 12 percent of movements in output, and output explains 12 percent of the movements of country interest rates.[25] Cline (1995), Cline and Barnes (1997), and Edwards (1984) find that GDP growth is a significant determinant of country spreads in developing countries. In our simulations, we obtain a correlation coefficient of -0.30 between the borrower's income and the country interest rate, and a correlation coefficient of -0.32 between income and the probability of debt overhang.

Our model also generates a negative correlation between consumption and the country interest rate (-0.23), consistent with empirical observations. For example, Arellano (2005) reports it for Argentina for the last decade. She also documents that the dynamics of interest rates, consumption, output and current account around the 1999 default episodes in Russia and Ecuador are similar to those in Argentina.

Aguiar and Gopinath (2004a,b), Arellano (2005) and Yue (2005) suggest that an empirical regularity of emerging market economies, in particular Argentina, is a countercyclical current account. In other words, countries tend to borrow more (and at lower interest rates) in booms than in recessions. The models of Aguiar and Gopinath (2004a) and Yue (2005) introduce persistent shocks. Hence, when the borrower receives a good endowment shock, her permanent income rises by more than her current income, which induces her to borrow in order to smooth consumption. In our model, good shocks are associated with higher investment. However, net borrowing, $K_t - R_t$, is negatively correlated with output. (The correlation coefficient is -0.62.) This is because the borrower always wants to repay her debt, and in good states the borrower can afford to repay more. However, the shocks in our setup are i.i.d. Introducing persistence in the shock process would have two effects. First, a high shock today would predict a high shock tomorrow, which would make investment in the current period more productive. This would induce higher investment and higher borrowing. Second, higher income today would translate into higher future expected income, which would decrease the threat of debt overhang. The borrower could then choose to delay part of the debt repayment and use her income to buy more capital. This again would increase net borrowing. Our conjecture is that persistence in productivity shocks can result in more borrowing in good

[25] Aguiar and Gopinath (2004a), Arellano (2005) and Yue (2005) build models that match the countercyclicality of country interest rates in the Argentine economy.

states, but for reasons other than consumption smoothing, in contrast to the existing literature.

8.2 Debt Reductions After a Sequence of Good Shocks

One of the predictions of our model is that the debt reductions occur after a sequence of good shocks, see Remark 2. In this subsection, we show that empirical evidence supports this prediction.

The Brady Plan

One major example of debt relief is the Brady Plan announced in 1989 in response to the 1980s sovereign debt crisis. This plan resulted in debt-reduction deals for 18 countries during the following decade. The 1980s debt crisis was caused mainly by worldwide events in the 1970s and 1980s, including oil price shocks, high interest rates, recession in industrial countries and low commodity prices. The HICs accumulated debt overhang, and the crisis became apparent when in 1982 Mexico announced that it could not honor its debt obligations. From 1982 to 1988, debtor countries and commercial bank creditors engaged in repeated rounds of debt rescheduling that proved useless in solving the sovereign debt crisis.

In 1989, U.S. Treasury Secretary Nicholas Brady announced a plan that encouraged the creditors to engage in voluntary debt-reduction schemes. Existing loans were swapped for either discount bonds (lesser face value but with a market-based floating rate of interest) or par bonds (equal face value but with a fixed, below-market interest rate). Both types of bonds had a 30-year maturity. The principal and interest of the new bonds were securitized by U.S. Treasury bonds. In some cases, commercial banks and multilateral agencies provided new loans. Brady bonds were issued by Argentina, Bolivia, Brazil, Bulgaria, Costa Rica, the Dominican Republic, Ecuador, Ivory Coast (Côte d'Ivoire), Jordan, Mexico, Nigeria, Panama, Peru, the Philippines, Poland, Uruguay, Venezuela and Vietnam. The total face value of the Brady bonds was more than $160 billion. A typical deal resulted in forgiving 30 to 35 percent of a country's debt.[26]

In accordance with our model, we view the time when the counties were unable to service their debt as the period of debt overhang and monopoly power. To see whether our theory's prediction that a debt reduction occurs after a sequence of good shocks holds in this case, we look at the real GDP growth rate of the 18 countries that issued Brady bonds. Different countries received debt reductions in different years, between 1990 and 1998. We average the growth rates of these countries such that the last year in each country's time series is the year when the Brady deal was made for that country. We plot the obtained average growth rate in Figure 7. Notice how growth is higher a few years before the Brady deal, just as the model predicts. The average real GDP growth rate of the Brady countries three years before the debt relief deal was 4.1 percent, compared with 1.8 percent during the six years preceding those three years, a difference of 2.3 percent. Case by case, the experience in more than two-thirds of the countries is consistent with the model's prediction. In addition, for 8 out of the 18 countries, the difference in average growth

[26] For a discussion, see Chuhan and Sturzenegger (2003), and Vasquez (1996).

rates between the last three years before the deal and the preceding six years exceeds 3.3 percent.

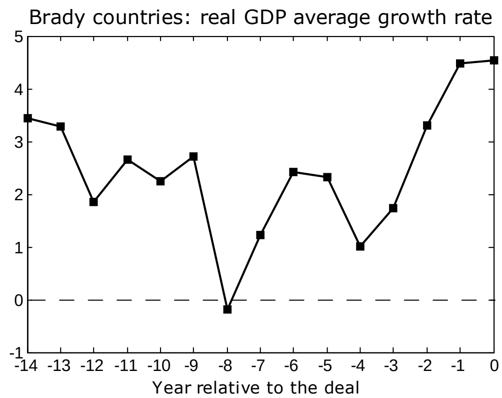

Figure 7. Real GDP average growth rate of the 18 Brady countries. Data source: UN.

2005 Argentine Debt Swap

The most recent sovereign debt reduction, and the largest in history, is the 2005 Argentine debt swap deal. In December 2001, after four years of deepening recession, Argentina's government ceased all debt payments. After three years of default, Argentina made an offer that would involve a 70 to 75 percent reduction in the net present value of its debt, provided at least 70 percent of the bondholders agreed with the arrangement. The \$102.6 billion debt swap closed on February 25, 2005, and was the largest write-down in the history of sovereign restructurings.

Looking at Argentina's real GDP growth rate (Figure 8), one can see that 2003 and 2004 are the years of good shocks, following our model's language. During these two years, real GDP in Argentina grew at 8 percent, compared with an average of -4.9 percent during the years 1999 - 2002, a difference of almost 13 percent. Again, just as our theory predicts, the debt relief follows a sequence of good shocks.

The Brady deal and the recent debt swap by Argentina empirically support our model's predictions that the borrower achieves a debt reduction after favorable shocks.[27] This observation is in sharp contrast to the predictions of the rest of the literature on sovereign debt renegotiation. Most of the existing models assume that once the borrower decides not to serve debt in full, the parties engage in bilateral renegotiation. Since there is no delay in bargaining, the debt reduction agreement is achieved immediately. Hence these models predict that the debt reduction occurs when the borrower cannot repay the debt, that is, after a sequence of bad shocks.

[27] Russia provides another example of a substantial increase in GDP growth a few years before debt relief. In 2000, after two years of rapid GDP growth, the London Club forgave about 50 percent of the present value of Russia's debt.

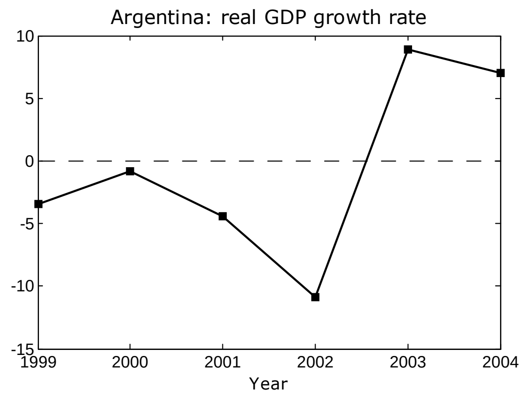

Figure 8. Argentina's real GDP growth rate. Data source: IMF.

9 Discussion

The main idea of this paper was based on the observation that accumulated debt is sunk cost for the incumbent lenders but not for new potential lenders. This asymmetry gives ex-post market power to the incumbents. Our goal was to explore the consequences of this phenomenon for financial contracting. Our main result is that the structure of the asset market keeps changing over time. In particular, even if the incumbents have monopoly power over the borrower, they find it optimal to let the borrower have access to the competitive credit market in a future date. This seems to be particularly relevant in explaining defaults and debt reductions in the context of sovereign lending.

Our model captures a fundamental conflict between the borrower and the lenders. On the one hand, the borrower needs the capital of the lenders; on the other hand, the borrower receives the realized income. The marginal value to the monopolist lender of additional income for the borrower is less than one, because it is costly for the lender to extract income from the borrower. Therefore, in general, the monopolist lender invests less than the socially efficient amount. Because of this inefficiency, the borrower wants to avoid debt overhang by repaying her debt as fast as possible. The borrower's marginal value of income is bigger than one, because she can use extra income to lower her debt. By lowering debt, the borrower may avoid debt overhang and can make more efficient investment decisions than the lenders would.

The borrower's large marginal value of income, which is also her marginal value of debt reduction, explains our main result. If the borrower has enough liquidity, the monopolist lender can extract income from her by reducing debt. Since the lender's marginal value of debt is one, which is less than the marginal value to the borrower, the monopolist lender can extract more current repayment from the borrower than the lender's loss due to a debt reduction.

Two assumptions were essential in obtaining our results. First, the contracts in our model are incomplete. Although we assumed that debt cannot be contingent on the realized income at all, what we really need is that debt cannot be perfectly indexed by the shocks. This would already imply that debt overhang can be accumulated, in turn giving market power to the incumbents.

Second, we assume that the incumbent lenders can coordinate their actions. Indeed, we model the incumbents as a single agent. We believe that neither of these assumptions is far-fetched.

Our model can be interesting in the contexts of both sovereign and corporate lending. Nonetheless, we have applied it only to sovereign lending. We have shown that our computational results are consistent with many empirical observations on emerging market economies. In addition, unlike other papers, our model predicts that debt reductions should happen only after a sequence of good shocks. We show this to be consistent with data.

One of our major goals in terms of future research is to analyze the role of international lending institutions in the environments described above. As we have shown, the investment decisions in our model are socially inefficient. Therefore, the question naturally arises: Can an organization such as the IMF intervene to reduce or eliminate the welfare loss?

10 Appendix

Proof of Lemma 1. (i) Suppose that $I' > I$, and the contract accepted at I is (R, K, D'). Then, if the income of the borrower is I' the lender can offer $(R - \varepsilon, K, D')$ to the borrower, where $\varepsilon > 0$. Since (R, K, D') was accepted at I, it must satisfy (2) and be feasible at I. Hence, $(R - \varepsilon, K, D')$ is feasible at I'. Furthermore,

$$\begin{aligned} I' - [R - \varepsilon] + \beta E W^B (F(K), D') &= I - R + \beta E W^B (F(K), D') + [I' - I] + \varepsilon \\ &\geq I + [I' - I] + \varepsilon > I', \end{aligned}$$

where the weak inequality follows from (R, K, D') satisfying (2) at I. But the above inequality chain shows that the borrower is strictly prefers accepting the contract $(R - \varepsilon, K, D')$ to defaulting at I'. The lender's payoff from this contract would be $V^L(I) - \varepsilon$, showing that $V^L(I') \geq V^L(I) - \varepsilon$. Since this is true for all $\varepsilon > 0$, $V^L(I') \geq V^L(I)$ follows.

(ii) Suppose that $\delta > 0$, and the contract accepted at $I + \delta$ is (R, K, D'). Then, the lender can offer the contract $(R - \delta, K, D')$ at I. Since (R, K, D') is feasible at $I + \delta$, $(R - \delta, K, D')$ is feasible at I. Notice that

$$\begin{aligned} I - (R - \delta) + \beta E W^B (F(K), D') &= I - R + \beta E W^B (F(K), D') + \delta \\ &\geq I + \delta > I, \end{aligned}$$

where the weak inequality holds because (R, K, D') satisfies (2) at $I + \delta$. The above inequality chain shows that the borrower strictly prefers accepting $(R - \delta, K, D')$ to defaulting at I. The lender's payoff from this contract is

$$\begin{aligned} & R - \delta + E\left[-K + \beta W^L (F(K), D')\right] \\ = \; & V^L (I + \delta) - \delta. \end{aligned}$$

This shows that $V^L(I) \geq V^L(I + \delta) - \delta$.

(iii) The concavity of V^L follows from the assumption that random contracts are allowed. ■

Proof of Proposition 2. First, suppose that the incumbent offers the same contract at (I, D), as the monopolist would at I. If the borrower accepted this contract, the incumbent's payoff would be $V^L(I)$. If the outsider offers a more attractive contract to the borrower, the incumbent would receive D. This proves $W^L(I, D) \geq \min\{D, V^L(I)\}$.

It remains to show that $W^L(I, D) \leq \min\{D, V^L(I)\}$. If the outsider lender does not offer a contract and the borrower's income is I, the incumbent lender can offer the same set of contracts as offered when the income is I and the debt is D. By the definition of Markovian equilibrium, the borrower accepts the same contract providing the lender with the same payoff. Hence, $V^L(I) \geq W^L(I, D)$. In order to establish $W^L(I, D) \leq \min\{D, V^L(I)\}$, we have to show that $W^L(I, D) \leq D$. Suppose by contradiction that $W^L(I, D) < D$, and the contract accepted at (I, D) is (R, K, D'). Then the outsider could offer a contract $(R - \varepsilon, K, D')$ where $\varepsilon \in (0, W^L(I, D) - D)$. This contract is feasible at (I, D) and the borrower strictly prefers it to (R, K, D') because $\varepsilon > 0$. The outsider's payoff from this contract is clearly $W^L(I, D) - D - \varepsilon$, which is strictly positive because $\varepsilon < W^L(I, D) - D$. This contradicts to the hypothesis that the equilibrium contract was (R, K, D'). ■

Proof of Proposition 1. (i) Suppose that $D < V^L(I)$. First, notice that

$$D = W^L(I, D) = R^* + E\left[-K^* + \beta W^L(F(K^*), D^*)\right],$$

where the first equality follows from Proposition 2, and the second one from the definition of W^L. This shows that (R^*, K^*, D^*) satisfies (3) with equality. It remains to show that (R^*, K^*, D^*) maximizes (4) subject to (3).

Suppose by contradiction that there is a feasible contract (R_2, K_2, D_2'), satisfies (3), and is strictly preferred by the borrower to (R^*, K^*, D^*). The problem can be that this contract is not offered in equilibrium, in particular, the outsider cannot make a strictly positive profit by offering this contract. This implies that (3) must be satisfied with equality. If $R_2 < I$, the outsider could offer $(R_2 + \varepsilon, K_2, D_2')$ and if $\varepsilon\,(> 0)$ is small enough it would still be accepted, and would provide the outsider with a strictly positive payoff. Hence, $R_2 = I$. Similarly, $D_2' = V^L(f(K_2))$ for otherwise, by Proposition 2, the outsider could achieve a strictly positive profit by offering a contract with a slightly higher next period's debt[28]. Hence, if the outsider offers the contract (R_2, K_2, D_2'), his payoff would be

$$-D + I + E\left[-K_2 + \beta W^L\left(F(K_2), V^L(f(K_2))\right)\right] = -D + I + E\left[-K_2 + \beta V^L(F(K_2))\right].$$

Recall, that $-K + \beta E V^L(F(K))$ is strictly increasing on $[0, K_0]$ and strictly decreasing on $[K_0, \infty)$. Therefore, if $K_2 \neq K_0$, the outsider can change K_2 marginally, such that the borrower's value is

[28] More precisely, the lender can offer (R_2, K_2, D_2'') where D_2'' is D_2' with probability $1 - \varepsilon$, and $D_2'' > D_2'$ with probability ε. If ε is small, the borrower still strictly prefers this contract to (R, K, D').

still larger than I but the lender's value from the contract increases. Hence, $K_2 = K_0$. But this implies $(R_2, K_2, D_2') = \left(I, K_0, V^L(f(K_0))\right)$, and this contract would provide the outsider with a payoff of $I + V^L(0) - D$. By Lemma 1, $I + V^L(0)$ must be equal to $V^L(I)$, and hence the outsider payoff would be $V^L(I) - D$. Since (R_2, K_2, D_2') satisfies (3) it follows that $D = V^L(I)$, a contradiction.

(ii) Suppose now that $D \geq V^L(I)$ and the equilibrium contract is (R^*, K^*, D^*). Then it must satisfy (2). Suppose by contradiction that (R^*, K^*, D^*) does not maximize (5) subject to (2). Then, there exists a contract (R_2, K_2, D_2') satisfying (2) and

$$R_2 + E\left[-K_2 + \beta W^L(F(K_2), D_2')\right] > R^* + E\left[-K^* + \beta W^L(F(K^*), D^*)\right].$$

The reason why this contract might not be offered in equilibrium is that the borrower is indifferent between accepting it and defaulting, and would choose to default. Let δ denote the difference between the two sides of the previous inequality. Consider a contract $(R_2 - \varepsilon, K_2, D_2')$, where $\varepsilon \in (0, \delta)$. Since (R_2, K_2, D_2') satisfies (2) the borrower strictly prefers accepting the contract $(R_2 - \varepsilon, K_2, D_2')$ to defaulting. Furthermore, this contract generates strictly higher payoff to the lender than (R^*, K^*, D^*) because $\varepsilon < \delta$. This shows that (R^*, K^*, D^*) cannot be an equilibrium contract. ■

Proof of Claim 1. It is obviously enough to show that $d\int_0^1 v(sI)\, dG(s)/dI$ is decreasing in I. Notice that

$$\frac{d\int_0^1 v(sI)\, dG(s)}{dI} = \int_0^1 v'(sI)\, s dG(s).$$

Since v is concave, $v'(sI)$ is decreasing in I for all $s \in [0,1]$. Hence the integral is also decreasing in I. ■

Proof of Lemma 2. Suppose first, that $I < V^B(0)$. Then the contract $\left(I, K_0, V^L(f(K_0))\right)$ is feasible and would be accepted by the borrower. This is because the expected payoff of the borrower is exactly $V^B(0)$ which is larger than I. Can the incumbent achieve a higher payoff? Suppose there is a feasible contract, (R, K, D'), generating a higher payoff for the lender, that is,

$$R + E\left[-K + \beta W^L(F(K), D')\right] \geq I - K_0 + \beta E V^L(F(K_0)).$$

Rearranging the terms,

$$\begin{aligned} E\left[-K + \beta W^L(F(K), D')\right] &\geq -K_0 + \beta E V^L(F(K_0)) + (I - R) \\ &\geq -K_0 + \beta E V^L(F(K_0)) = V^L(0), \end{aligned}$$

where the second inequality follows from the feasibility of (R, K, D'). The left-hand side would be the payoff of the lender if he offers the contract $(0, K, D')$ to the borrower at $\left(0, V^L(0)\right)$. The inequality chain implies that at $\left(0, V^L(0)\right)$ the lender could have achieved a (weakly) higher payoff by offering $(0, K, D')$ than with the contract $\left(0, K_0, V^L(f(K_0))\right)$. This contradicts to the uniqueness of the equilibrium contract $\left(0, K_0, V^L\left(f\left(K_0\right)\right)\right)$ at $\left(0, V^L\left(0\right)\right)$ established in the body text of the paper.

Suppose now that $I \geq V^B(0)$. Notice that $V^B(I) \geq I$ for otherwise the borrower defaults. Suppose by contradiction that $V^B(I) > I$ and the contract accepted at $\left(I, V^L(I)\right)$ is (R, K, D'). Hence,

$$V^B(I) = I - R + \beta E W^B\left(F(K), D'\right) > I.$$

If $I > R$, the monopolist incumbent could raise R and the contract would still be accepted. This shows $R = I$. Similarly, $D' = V^L(f(K))$ for otherwise, by Proposition 2, the lender could increase his continuation value by increasing the next period's debt. The value to the lender is

$$I + E\left[-K + \beta W^L\left(F(K), V^L(f(K))\right)\right] = I + E\left[-K + \beta V^L(F(K))\right].$$

Recall, that $-K + \beta E V^L(F(K))$ is strictly increasing on $[0, K_0]$ and strictly decreasing on $[K_0, \infty)$. Therefore, if $K \neq K_0$, the lender can change K marginally, such that the borrower's continuation value is still larger than I but the lender's value from the contract increases. Hence, $K = K_0$ and $V^B(I) = V^B(0) \leq I$, a contradiction. ■

Proof of Proposition 3. Notice that whenever $D < V^L(I)$, I' in (10) is well-defined (see Figure 1).

First, we show that $W^B(I, D) \geq V^B(I')$. Suppose that the equilibrium contract at $\left(I', V^L(I')\right)$ is (R, K, D'), where $R \leq I'$. From the definition of V^L,

$$V^L(I') = R + E\left[-K + \beta W^L\left(F(K), D'\right)\right].$$

Then the contract (R', K, D'), where $R' = R - [I' - I]$, satisfies

$$\begin{aligned} R' + E\left[-K + \beta W^L\left(F(K), D'\right)\right] &= R + E\left[-K + \beta W^L\left(F(K), D'\right)\right] - (I' - I) \\ &= V^L(I') - (I' - I) = D, \end{aligned}$$

where the last equality follows from (10). This means that the contract (R', K, D') satisfies the the zero-profit condition for the outsider, (3), which is the constraint of the maximization problem of the borrower, (4). Furthermore, since (R, K, D') was feasible at $\left(I', V^L(I')\right)$, (R', K, D') is feasible at (I, D). Therefore,

$$\begin{aligned} W^B(I, D) &\geq I - R' + \beta E W^B\left(F(K), D'\right) \\ &= I' - R + \beta E W^B\left(F(K), D'\right) = V^B(I'), \end{aligned}$$

where the last equality follows from (R, K, D') being the equilibrium contract at $\left(I', V^L(I')\right)$.

It remains to show that $W^B(I, D) \leq V^B(I')$. Suppose by contradiction that $W^B(I, D) > V^B(I')$ and the contract accepted at (I, D) is (R, K, D'). Since (R, K, D') is feasible at (I, D), $(R + [I' - I], K, D')$ is feasible at $\left(I', V^L(I')\right)$. Furthermore, $(R + [I' - I], K, D')$ would provide the borrower with a payoff of exactly $W^B(I,D) > V^B(I')$ at $\left(I', V^L(I')\right)$. The incumbent's payoff would be

$$W^L(I,D) + (I'-I) = D + (I'-I) = V^L(I'),$$

where the first equality follows Proposition 2 and the second one from (10). Using the same argument as the one in the proof of part (ii) of Proposition 1, $R + [I'-I] = I'$, $D' = V^L(f(K))$, and $K = K_0$. But then $W^B(I,D) = V^B(0) \leq V^B(I')$, a contradiction. ■

Proof of Lemma 3. From (11) and the derivative rule for inverse functions it follows that

$$H'(I-D) = V^{B\prime}\left(Q^{-1}(I-D)\right)\frac{1}{Q'\left(Q^{-1}(I-D)\right)}.$$

Notice that

$$\begin{aligned} Q\left(V^B(0)\right) &= V^B(0) - V^L\left(V^B(0)\right) = V^B(0) - \left[V^L(0) + V^B(0)\right] \\ &= -V^L(0) \leq -\left[V^L(I) - I\right] < I - D. \end{aligned}$$

The second equality follows from $V^L(I) = V^L(I) + I$ on $\left[0, V^B(0)\right]$ established in the proof of Lemma 2. The weak inequality follows from parts (i) and (ii) of Lemma 1, and the last inequality follows from $D < V^L(I)$. Since Q is increasing, $Q^{-1}(I-D)$ is strictly larger than $V^B(0)$. Hence, from Lemma 2 we know $V^{B\prime}\left(Q^{-1}(I-D)\right) = 1$. From the definition of Q and (10), $Q^{-1}(I-D) = I'$. Therefore $H'(I-D) = 1/\left(1 - V^{L\prime}(I')\right)$. From part (i) and (ii) of Lemma 1, $V^{L\prime} \in [0,1]$, yielding $H'(I-D) \geq 1$. The concavity of V^L (established in Lemma 1) and (12) imply the concavity of H. ■

Proof of Lemma 4. **Step 1:** $I \leq V^B(0)$. If $I < V^B(0)$, then obviously $V^{L\prime}(I) = 1$ and $R(I) = I$. This is because the monopolist lender can offer the contract $\left(I, K_0, V^L(f(K_0))\right)$. Since $I < V^B(0)$ the borrower strictly prefers accepting it to defaulting. That is, the monopolist can offer the same contract as at $I = 0$, except that $R = I$ instead of zero. Also notice that this contract is optimal, since this contract provides the incumbent with a payoff of $V^L(0) + I \geq V^L(I)$ by part (ii) of Lemma 1. Notice that the contract $\left(I, K_0, V^L(f(K_0))\right)$ is a unique maximizer of (5) subject to (2) for the following reason. Suppose the contract (R, K, D') is also a solution to (5) subject to (2) at $\left(I, V^L(I)\right)$. But then $(0, K, D')$ is optimal at $\left(0, V^L(0)\right)$. Hence, $K = K_0$ and $D' = f(K_0)$ because the equilibrium contract at $\left(0, V^L(0)\right)$ was shown to be unique in the text. $R = I$ immediately follows. By continuity, the equilibrium contract at $\left(V^B(0), V^L\left(V^B(0)\right)\right)$ is $\left(V^B(0), K_0, V^L(f(K_0))\right)$.

Step 2: $I \in \left(V^B(0), \infty\right)$. Next, we show that if $I > V^B(0)$, then $V^L(I) < V^L\left(V^B(0)\right) + I - V^B(0)$. Suppose by contradiction that $V^L(I) = V^L\left(V^B(0)\right) + I - V^B(0)$. Let (R, K, D') be the equilibrium contract at $\left(I, V^L(I)\right)$. Then, $\left(R - \left(I - V^B(0)\right), K, D'\right)$ would solve (5) subject to (2) at $\left(V^B(0), V^L\left(V^B(0)\right)\right)$. This contract is also feasible at $\left(V^B(0), V^L\left(V^B(0)\right)\right)$. In Step 1, we established that the contract solving (5) subject to (2) is unique at $\left(V^B(0), V^L\left(V^B(0)\right)\right)$, and is $\left(V^B(0), K_0, f(K_0)\right)$. Therefore, $R = I$, $K = K_0$, and $D' = V^L(f(K_0))$. But

$$I > V^B(0) = \beta E W^B\left(F(K_0), V^L(f(K_0))\right),$$

and hence the contract would be rejected by the borrower, a contradiction. Therefore, $V^L(I) < V^L\left(V^B(0)\right) + I - V^B(0)$. Since V^L is concave, $V^{L\prime}(I) \leq \left[V^L(I) - V^L\left(V^B(0)\right)\right] / \left[I - V^B(0)\right] < 1$ on $\left(V^B(0), \infty\right)$.

Step 3: $I \in [I_M, \infty)$. Suppose now, that the borrower has so large income, that $R < I$. Such an I exists, since $\lim_{K\to\infty} f'(K) = 0$. Notice that $D' = 0$ can be assumed since $H' \geq 1$ (as explained in the body text before the statement of this lemma.) Then, the monopolist's problem is to maximize

$$\begin{aligned} &\max_{R,K} R - EK \\ &\text{s.t. } R = \beta \int_0^1 H(sf(K))\, dG(s). \end{aligned}$$

Plugging the constraint into the maximand, the problem becomes

$$\max_K -EK + \beta \int_0^1 H(sf(K))\, dG(s).$$

As we have shown, the unique solution to this problem is K_M defined by (13). Therefore, the unique equilibrium contract is $(I_M, K_M, 0)$ whenever $I \geq I_M$. This shows that $V^L(I) = V^L(I_M)$ whenever $I \geq I_M$, and hence $V^{L\prime} = 0$ on $[0, \infty)$. Furthermore, $V^L(I_M) = I_M - K_M$.

Step 4: $I \in [0, I_M)$. Next, we show that if $I < I_M$, then $V^L(I_M) > V^L(I)$. Suppose by contradiction that $V^L(I) = V^L(I_M)$ and $I < I_M$. Then, the contract accepted at I, must also solve (5) subject to (2) at $\left(I_M, V^L(I_M)\right)$. Also notice that this contract must specify $R \leq I < I_M$. But the solution to (5) subject to (2) at $\left(I_M, V^L(I_M)\right)$ was shown to be unique, with $R = I_M$, a contradiction. Hence $V^L(I_M) > V^L(I)$. Therefore, the concavity of V^L implies $V^{L\prime} \geq \left[V^L(I_M) - V^L(I)\right] / [I_M - I] > 0$ on $[0, I_M)$.

Since $V^{L\prime}(I) < 1$ if $I > V^B(0)$ by step 2, and $V^{L\prime} > 0$ on $[0, I_M)$ by step 4, it follows that $V^{L\prime}(I) \in (0,1)$ whenever $I \in \left(V^B(0), I_M\right)$.

Recall that $H(I - D) = V^B(I') = I'$, where $I' - V^L(I') = I - D$. From steps 1 and 2, and (12) it follows that $H'(I - D) > 1$ if $I' = H(I - D) < I_M$.

It remains to show that $H'(0) > 1$. Notice that $H(0) = I^*$, where I^* is defined by $V^L(I^*) = I^*$. Since V^L is concave, $V^L(0) > 0$, and $V^{L\prime}(I) = 0$ if $I > I_M$, I^* is well-defined. From the previous paragraph we know that $H'(x) > 1$ if $H(x) < I_M$. Therefore, it is enough to show that $I^* < I_M$. In step 3, we showed that $V^L(I_M) - I_M = -K_M$. Since $V^L(I) - I$ is decreasing (by $V^{L\prime} \leq 1$), it follows that $I^* < I_M$. ■

Proof of Remark 1. In step 1 of the proof of Lemma 4 we have shown that $R(I) = I$ whenever $I \leq V^B(0)$. Suppose that $I \in \left(V^B(0), I_M\right)$ and the equilibrium contract is (R, K, D'), where

$R < I$. Then the same contract must also solve (5) subject to (2) at $\left(R, V^L(R)\right)$. This is because (R, K, D') is obviously feasible and satisfies (2) at $\left(R, V^L(R)\right)$. Hence, $V^L(I) = V^L(R)$. Since V^L is increasing by part (i) of Lemma 1, this implies that $V^{L\prime} = 0$ on (R, I). This contradicts to $V^{L\prime}(I) > 0$ on $\left[V^B(0), I_M\right]$ established in Lemma 4. In step 3 of the proof of Lemma 4 we have also shown that $R(I) = I_M$ if $I \geq I_M$. ■

Proof of Theorem 3. (i) We prove this statement by contradiction. Suppose that a monopolist lender offers a sequence of contract which provides him with monopoly power forever. The borrower's consumption must sometimes be positive, for otherwise she would choose to default. Therefore, with probability one there is a date, in which a contract of the following form is accepted: (R, K, D'), such that $R < I$ and $D' = V^L(f(K))$. If $I \leq I_M$, then $R(I) = I$ by Remark 1, a contradiction. If $I \geq I_M$, then the unique equilibrium contract is $(I_M, K_M, 0)$ (established in the proof of Lemma 4). This contradicts to $D' = V^L(f(K))$.

(ii) First, we show that there exists an $\varepsilon > 0$, such that whenever $D < V^L(I)$, in the equilibrium contract $EK > \varepsilon$. Suppose by contradiction that there exists a sequence (I_n, D_n) $\left(D_n < V^L(I_n)\right)$, such that in the corresponding contracts (R_n, K_n, D'_n), $\lim_{n\to\infty} EK_n = 0$. Notice that $R_n, I_n \leq I_M$ can be assumed and ED_n, $ED'_n \leq V^L(I_M)$, therefore there is a subsequence of (n), (n_k), such that $\lim_{n_k\to\infty} I_{n_k} = I^*$, $\lim_{n_k\to\infty} ED_{n_k} = ED^*$, $\lim_{n_k\to\infty} R_{n_k} = R^*$ and $\lim_{n_k\to\infty} ED'_{n_k} = ED'^*$. (Recall that $\lim_{n\to\infty} EK_n = 0$.) Then the contract $(R^*, 0, D'^*)$ is optimal at (I^*, D^*) by continuity. (We do not claim that it is actually the equilibrium contract, but it must be payoff equivalent.) If there is no investment ever after accepting the contract $(R^*, 0, D'^*)$, then autarky would be an equilibrium, a contradiction. Let t be the first date at which there is a contract $(0, K_2, D'_2)$ accepted such that $EK_2 > 0$. We show that the contract (R^*, K_2, D'_2) is strictly preferred by the borrower to $(R^*, 0, D'^*)$, and satisfies (3) at (I^*, D^*). Hence, by Proposition 1, it cannot be payoff equivalent to the equilibrium contract. The borrower's payoff is

$$I^* - R^* + \beta^{t+1} EW^B(F(K_2), D'_2) < I^* - R^* + \beta EW^B(F(K_2), D'_2),$$

because $\beta < 1$ and $EW^B(F(K_2), D'_2) > 0$. Notice that $E\left[-K_2 + \beta W^L(F(K_2), D')\right] \geq 0$, otherwise the lender would not offer $(0, K_2, D'_2)$ at time t. The lender's payoff is

$$R^* + \beta^t E\left[-K_2 + \beta W^L(F(K_2), D')\right] \geq R^* + E\left[-K_2 + \beta W^L(F(K_2), D')\right].$$

Choose $\delta = \min\left\{\varepsilon / \left(2f\left(\overline{K}\right)\right), 1\right\}$, where $\overline{K}$ denotes the largest investment made. Hence the borrower's income is less than $\varepsilon/2$ at least with probability δ in every period. But since the investment is larger than ε, the debt increases by at least $\varepsilon/2$ in such periods if there was no debt overhang. This is because if the debt is D in the current period and D' in the next one,

$$D = R - K + \beta EW^L(F(K), D') < \varepsilon/2 - \varepsilon + D'$$

and hence $D' - D > \varepsilon/2$. Therefore with the probability of at least δ^n, the debt becomes larger than $n\varepsilon/2$ after n periods. (This is because the shocks are independently distributed across periods.) Fix

n such that $n\varepsilon/2 > V^L(I_M)$. Then, no matter what the current state (I, D) is, with probability of at least δ^n the borrower faces debt overhang n periods later. Since there are infinitely many periods, one can conclude that the borrower faces debt overhang with probability one. ■

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