# Reputation and Price Dynamics: Cascades and Bubbles in Financial Markets

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#### Abstract

What are the equilibrium features of a dynamic financial market where traders care about their reputation for ability? We modify a standard sequential trading model to allow for career concerns. We show that the market must be informationally inefficient: there is no equilibrium in which prices converge to the true value, even in the long run. This finding, which stands in sharp contrast with the results for standard financial markets, is due to the fact that our traders face an endogenous incentive to behave in a conformist manner. We also show that each asset carries an endogenous reputational benefit or cost, which, if asset supply is sufficiently limited, translates into a price premium or discount and can generate bubbles.

### 1 Introduction

The substantial increase in the institutional ownership of corporate equity around the world in recent decades has underscored the importance of study-

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ing the effects of institutional trade on asset prices.<sup>1</sup> Institutions, and their employees, may be guided by incentives not fully captured by standard models in finance. For example, consider the case of US mutual funds which make up a significant proportion of institutional investors in US equity markets. An important body of empirical work highlights the fact that mutual funds (e.g. Chevalier and Ellison [12]) and their employees (Chevalier and Ellison [13]) both face *career concerns*: they are interested in enhancing their *reputation* with their respective principals and sometimes indulge in perverse actions (e.g. excessive risk taking) in order to achieve this. Given the importance of institutions in equity markets, it is plausible to expect that such behaviour may affect equilibrium quantities in financial markets. What are the equilibrium features of a market in which a large proportion of traders face reputational concerns?

While a growing body of literature examines the effects of agency conflicts on asset pricing, the explicit modeling of reputation in financial markets is in its infancy.<sup>2</sup> Dasgupta and Prat [15] present a two-period micro-founded model of careern concerns in financial markets to examine the effect of reputation in enhancing trading volume. However, that analysis is done for a static market: each asset is traded only once.

The present contribution is to study the equilibrium of a multi-period financial market in which some traders have reputational concerns. As we shall see, the dynamic properties of such market are very different from those of a standard market.

We present the most parsimonious model that captures the essence of our arguments. Much of our model is standard. We present a T-period sequential trade market for a single (Arrow) asset where all transactions occur via uninformed *market makers* who are risk neutral and competitive (following Glosten and Milgrom [18] and Kyle [19]) and quote bid and ask prices to reflect the informational content of order flow. In addition there is a large group of liquidity-driven *noise traders* who trade for exogenous

<sup>&</sup>lt;sup>1</sup>On the New York Stock Exchange the percentage of outstanding corporate equity held by institutional investors has increased from 7.2% in 1950 to 49.8% in 2002 (NYSE Factbook 2003). For OECD countries as a whole, institutional ownership now accounts for around 30% of corporate equity (Nielsen [23]). Allen [2] presents persuasive arguments for the importance of financial institutions to asset pricing.

<sup>&</sup>lt;sup>2</sup>Allen and Gorton [4] and Dow and Gorton [17] examine the asset pricing implications of non-reputational agency concern. Reputational concerns are implicit in the contractual forms assumed in the general equilibrium models Cuoco and Kaniel [14] and Vayanos [33].

reasons that are unrelated to the liquidation value of the asset. Our only innovation is that we introduce a large group of reputationally-concerned institutional traders (whom we call *fund managers*), who trade on behalf of other (inactive) investors. These traders receive a payoff that depends both on the direct profits they produce and on the reputation that they earn with their clients. These fund managers can be of two types (smart or dumb) and receive informative signals about the asset liquidation value, where the precision depends on their (unknown) type. In each trading round either a randomly selected fund manager or a noise trader interact with the market maker. The asset payoff is realized at time T and all payments are made. We present the following results.

- 1. We show that, in this market of reputationally-sensitive traders, prices never converge to true liquidation value, even in the long run. If fund managers trade according to their private signal, price evolves to incorporate such private information. Over time, price should converge to the true liquidation value. However, as the uncertainty over the liquidation value is resolved, two things happen. First, the fund managers have less opportunity to make trading profits because price is close to the liquidation value. The expected profit for a fund manager who trades according to his signal is always positive, but it tends to zero as the price becomes more precise. Second, taking a "contrarian" position (e.g. selling after price has been going up) starts to carry a reputational cost: with high probability, the trade will turn out to be incorrect and the fund manager will "look dumb" in the eyes of (rational) investors. Because of the combination of these two effects, when price becomes sufficiently precise fund managers begin to behave in a conformist way: their trade stops reflecting their private information. From then on, there is no information aggregation whatsoever and the price stays constant.
- 2. Having thus ruled out equilibria in which fund managers always trade sincerely, we proceed to construct examples of equilibria with conformist trading. In particular, we illustrate that there may exist an equilibrium in this model in which institutional traders trade sincerely until prices reach a high or low threshold. Once the price crosses such a threshold, they proceed to trade in a conformist manner.
- 3. We show that at any given price, there may be a difference between the

expected total value of the asset for a regular trader and for a trader with career concerns. We refer to this difference as the *reputational benefit or cost* carried by the asset at that price. We show that this reputational benefit is increasing in the asset price.

- 4. We demonstrate that career concerns can also lead to the formation of asset price bubbles. In the baseline model, fund managers have all the bargaining power in transactions because traders without career concerns are risk neutral and competitive. Hence, the asset price only reflects "fair" expectations of fundamentals. Once this assumption is relaxed, the reputational benefit or cost defined above affects the market price of the assets and generates a *reputational premium or* discount. Price bubbles may now arise. Our theory of bubbles yields testable predictions on the relation between the dynamics of the net trade by career traders and asset mispricing.
- 5. Finally, we subject our basic model to a number of robustness checks. The baseline model was presented under the assumption that institutional traders were unaware of their type. We illustrate that as long as self-knowledge is not extremely accurate, our core results remain unchanged. Also, while in the baseline model we assume that nonpecuniary rewards depend on the individual reputation, we extend the model to allow for rewards based on relative reputation and show that sincere trading cannot be sustained in equilibrium.

The present paper brings together two influential strands of the literature. The first strand concerns the theory of dynamic financial markets with asymmetrically informed traders (Glosten and Milgrom [18] and Kyle [19]). The second strand focusses on the analysis of career concerns in sequential investment decision-making (Scharfstein and Stein [25]). Models in the first strand consider a full-fledged financial market with endogenously determined prices but do not allow traders to have career concerns. Models in the second strand do the exact opposite: they analyze the role of reputational concerns in a partial equilibrium setting, where prices are exogenously fixed.

In the first strand, Glosten and Milgrom [18] have shown that in a dynamic financial markets the price must tend to the true liquidation value in the long term. More recently, Avery and Zemsky [8] have shown that statistical information cascades à là Banerjee [9] and Bikhchandani, Hirshleifer, and Welch [10] are impossible in such a market.<sup>3</sup> After every investment decision, the price adjusts to reflect the expected value of the asset based on information revealed by past trades. Thus, traders with private information stand to make a profit by trading according to their signals. But by doing so, they release additional private information into the public domain. In the long run, the market achieves informational efficiency.<sup>4</sup> Our work shows that the addition of career concerns changes things dramatically. The presence of a reputational motive can make the market informationally inefficient and can generate bubbles. Other authors (Lee [20] and Chari and Kehoe [11]) have argued that information cascades can occur when prices are endogenous.<sup>5</sup> However, their arguments hinge on a market breakdown: trade stops altogether. Instead, in our model trade continues even after the informational cascade has started.

In the second strand, Scharfstein and Stein [25] have shown that managers who care about their reputation for ability may choose to ignore relevant private information and instead mimic past investment decisions of other managers.<sup>6</sup> This is because a manager who possesses "contrarian" information (for instance he observes a negative signal for an asset that has experienced price growth) jeopardizes his reputation if he decides to trade according to his signal.<sup>7</sup> From this perspective, the contribution of the present paper

<sup>4</sup>Under additional assumptions, Avery and Zemsky [8] show that a form of herd behaviour may occur in the presence of prices, but in all versions of their model cascades are absent and prices always converge to true liquidation value.

<sup>5</sup>In Lee's [20] model the existence of a transaction cost to trading can prevent traders with relatively inaccurate signals from trading, thus trapping private information in an illiquid market. In Chari and Kehoe [11], traders have the option of exiting the market (by making an outside investment) and may in equilibrium find it optimal to exit before further information arrives, thus, again, trapping private information.

<sup>6</sup>Other more recent papers in this strand include, for example, Avery and Chevalier [7] and Trueman [32].

<sup>7</sup>The partial equilibrium reputational model that is closest to ours is Ottaviani and Sorensen [24]. The connection will be discussed in detail in Section 3.

 $<sup>^{3}</sup>$ A word of caution is in order here. There is almost universal agreement in the literature on the meaning of a *cascade*, which is the definition we have used above (an equilibrium event in which information gets trapped, and agents' actions no longer reveal any of their valuable private information). However, there is little agreement on the definition of the term *herds* (for example, substantively different definitions are used by Avery and Zemsky ([8]), Smith and Sorensen ([30]), and Chari and Kehoe ([11])). In the interest of clarity, throughout this paper we shall restrict attention to cascades *only*. On occasion, we shall make a distinction between efficient and inefficient cascades.

is to embed Scharfstein and Stein's model model into a standard dynamic market model such as Glosten and Milgrom [18]. This allows us to show that Scharfstein and Stein's insights on conformistic behavior are robust to the extension to market equilibrium. But more importantly, we are able to study the implications of micro-founded reputation-driven conformism over market behavior (prices, informational efficiency, trades, and profits), which opens the way to potentially interesting predictions on observable market variables.<sup>8</sup>

The rest of the paper is organized as follows. In the next section we present the model. Section 3 discusses the impossibility of full information aggregation, first through an example and then through the general result. Section 4 provides an example of an equilibrim price dynamics, while section 5 studies the dynamics of the reputational benefit or cost of the asset. Section 6 shows that, if asset supply is limited, the price can incorporate a reputational premium and bubbles can arise. Extensions are examined in section 7. Section 8 concludes.

#### 2 Model

The economy lasts T discrete periods: 1, 2, ...T. Trade can occur in periods 1, 2...T - 1. The market trades an Arrow security, which has liquidation value  $v \in \{0, 1\}$ , which is revealed at time T.

There are a large number of fund managers and noise traders. At each period  $t \in \{1, 2, ..., T - 1\}$  either a fund manager or a noise trade enters the market with probabilities  $1 - \delta$  and  $\delta \in (0, 1)$  respectively. The traders interact with a risk-neutral competitive market maker, and can issue market orders  $(a_t)$  to buy  $(a_t = 1)$  one unit or sell  $(a_t = 0)$  one unit of the asset. The market maker sets ask  $(p_t^a)$  and bid  $(p_t^b)$  prices equal to expected value of v conditional on order history. Denote the history of observed orders at the beginning of period t (not including the order at time t) by  $h_t$ . Let  $p_t = E(v|h_t), p_t^a = E(v|h_t, \text{buy}), p_t^b = E(v|h_t, \text{sell}).$ 

The fund manager can be of two types:  $\theta \in \{b, g\}$  with  $\Pr(\theta = g) = \gamma$ . The type is independent of v. If at time t a fund manager appears, he

<sup>&</sup>lt;sup>8</sup>There is a growing body of empirical literature on conformist behaviour by institutional traders. Sias ([29]) surveys this literature, reconciles differences in previous conclusions, and presents persuasive evidence for momentum trading by institutional traders.

receives a signal  $s_t \in \{0, 1\}$  with distribution

$$\Pr\left(s_t = v | v, \theta\right) = \sigma_{\theta},$$

where

$$\frac{1}{2} < \sigma_b < \sigma_g < 1.$$

Fund managers do not know their type. Noise trader also buy or sell a unit independent of v.

The returns obtained by the trader at time t is denoted  $\chi_t$  and is defined by:

$$\chi\left(a_{t}, p_{t}^{a}, p_{t}^{b}, v\right) = \begin{cases} v - p_{t}^{a} & \text{if } a_{t} = 1\\ p_{t}^{b} - v & \text{if } a_{t} = 0 \end{cases}$$

If a fund manager traded at time t, his actions are observed at time T. Investors form a posterior belief about the manager's type based upon the managers actions, which we define to be

$$\hat{\gamma}_t(a_t, h_t, v) = \Pr\left(\theta_t = g | a_t, h_t, v\right)$$

Suppose the fund manager at time t receives utility

$$u(a_t, p_t^a, p_t^b, v, h_t) = (1 - \beta) \pi \left( \chi_t \right) + \beta r \left( \hat{\gamma}_t \right),$$

where  $\beta \in (0,1)$  is a parameter, and the functions  $\pi$  and r measure the direct payoff and reputational payoff and are increasing and continuous in the relevant arguments.

We can show that in this setting, in contrast to well-known prior results, that the market cannot be fully informationally efficient *even in the long run*. We begin with an example to illustrate the intuition behind the main result.

# 3 The Impossibility of Full Revelation

We first present an example and then we discuss the general result.

#### 3.1 Example

Let  $\pi$  and r be linear. The manager maximizes  $\beta \chi^t + (1 - \beta) \hat{\gamma}^t$ .

A sincere equilibrium is one in which fund managers play according to their signals:  $a^t = s^t$  for all t at which a fund manager is active.

If  $\beta = 1$ , there is a sincere equilibrium (Avery-Zemsky 1998). Suppose instead that  $\beta < 1$ .

**Proposition 1** There is no sincere equilibrium.

**Proof.** Suppose there is a sincere equilibrium. Suppose that at t the price is  $p_t$  and the manager plays  $a_t = s^t$ . Let

$$\hat{v}_{1}^{t} = \Pr\left(v = 1 | s_{t} = 1, h_{t}\right) = \frac{\Pr\left(s_{t} = 1 | v = 1\right)}{\Pr\left(s_{t} = 1 | h_{t}\right)} p_{t} = \frac{\sigma}{p_{t}\sigma + (1 - p_{t})(1 - \sigma)} p_{t}$$
$$\hat{v}_{0}^{t} = \Pr\left(v = 1 | s_{t} = 0, h_{t}\right) = \frac{\Pr\left(s_{t} = 0 | v = 1\right)}{\Pr\left(s_{t} = 0 | h_{t}\right)} p_{t} = \frac{1 - \sigma}{(1 - p_{t})\sigma + p_{t}(1 - \sigma)} p_{t}$$

The bid-ask prices are

$$p_t^a = \delta_t^b p_t + \left(1 - \delta_t^b\right) \hat{v}_1^t$$
$$p_t^b = \delta_t^s p_t + \left(1 - \delta_t^s\right) \hat{v}_0^t$$

where  $\delta_t^b = \Pr(\text{noise trader}|\text{buy order}, h_t)$ , is the probability assigned by the market maker that he faces a noise trader upon receiving a buy order and observing the history of trades. Similarly,  $\delta_t^s = \Pr(\text{noise trader}|\text{sell order}, h_t)$ . Straightforward calculations show that

$$\delta_t^b = \frac{\frac{1}{2}\delta}{\frac{1}{2}\delta + (1-\delta)[p_t\sigma + (1-p_t)(1-\sigma)]}$$
  
$$\delta_t^s = \frac{\frac{1}{2}\delta}{\frac{1}{2}\delta + (1-\delta)[p_t(1-\sigma) + (1-p_t)\sigma]}$$

Suppose the current price is  $p_t$  and the manager in t observes  $s^t = 0$ . If he buys, his expected trading profit is  $\hat{v}_0^t - p_t^a$ , while if he sells it is  $p_t^b - \hat{v}_0^t$ . Thus the difference between the expected profit of buying and selling is

$$\Delta \pi = 2\hat{v}_0^t - p_t^a - p_t^b.$$

The reputational payoffs in a sincere equilibrium are:

$$\hat{\gamma} (a_t = 1, v = 1) = \frac{\sigma_g}{\sigma} \gamma$$
$$\hat{\gamma} (a_t = 0, v = 1) = \frac{1 - \sigma_g}{1 - \sigma} \gamma$$
$$\hat{\gamma} (a_t = 1, v = 0) = \frac{1 - \sigma_g}{1 - \sigma} \gamma$$
$$\hat{\gamma} (a_t = 0, v = 0) = \frac{\sigma_g}{\sigma} \gamma$$

Define

$$\Delta \hat{\gamma} = \left(\frac{\sigma_g}{\sigma} - \frac{1 - \sigma_g}{1 - \sigma}\right) \gamma.$$

The expected reputational benefit of choosing  $a^t = 0$  instead of  $a^t = 1$  when  $s^t = 0$  is

$$\begin{aligned} \Delta r &= \Pr\left(v = 1 | s^t = 0, h_t\right) \left(\hat{\gamma} \left(a_t = 1, v = 1\right) - \hat{\gamma} \left(a_t = 0, v = 1\right)\right) \\ &+ \Pr\left(v = 0 | s^t = 0, h_t\right) \left(\hat{\gamma} \left(a_t = 1, v = 0\right) - \hat{\gamma} \left(a_t = 0, v = 0\right)\right) \\ &= \hat{v}_0^t \left(\frac{\sigma_g}{\sigma} - \frac{1 - \sigma_g}{1 - \sigma}\right) \gamma + \left(1 - \hat{v}_0^t\right) \left(\frac{1 - \sigma_g}{1 - \sigma} - \frac{\sigma_g}{\sigma}\right) \gamma \\ &= \left(2\hat{v}_0^t - 1\right) \Delta \hat{\gamma} \end{aligned}$$

It is a best response to play  $a^t = 0$  instead of  $a^t = 1$  when  $s^t = 0$  when

$$\beta \Delta \pi + (1 - \beta) \,\Delta r < 0.$$

Let the price rise to one. Notice that as  $p_t \to 1$ ,  $\hat{v}_0^t \to 1$  and  $\hat{v}_1^t \to 1$ , that is, regardless of whether the manager has received the high or the low signal, the accumulated information in prices convinces him that the expected liquidation value is 1. Also notice that  $\delta_t^b$  and  $\delta_t^s$  are bounded, and thus as  $p_t \to 1$ , it must also be true that  $p_t^a \to 1$  and  $p_t^b \to 1$ . Now, taking limits we have

$$\lim_{p_t \to 1} \Delta \pi = \lim_{p_t \to 1} 2\hat{v}_0^t - p_t^a - p_t^b = 2 - 1 - 1 = 0$$
$$\lim_{p_t \to 1} \Delta r = \lim_{p_t \to 1} \left(2\hat{v}_0^t - 1\right)\Delta\hat{\gamma} = (2 - 1)\Delta\hat{\gamma} > 0$$

Hence, for  $p_t$  high enough it is a best response for a fund manager with  $s^t = 0$  to play  $a^t = 1$ . Thus the equilibrium cannot be sincere. A contradiction.

#### 3.2 General Result

We now consider the general problem, and illustrate that prices can never converge to true liquidation value in the presence of reputational concerns. The expected utility of the t-period fund manager is

$$E(u(a_t, p_t^a, p_t^b, v, h_t)|s_t) = \sum_{v=0,1} \Pr(v|h_t, s_t) \left( (1-\beta) \pi \left( \chi \left( a_t, p_t^a, p_t^b, v \right) \right) + \beta r \left( \hat{\gamma} \left( a_t, v, h_t \right) \right) \right)$$

Define

$$E(\Delta u(p_t^a, p_t^b, v, h_t)|s_t) = E(u(1, p_t^a, p_t^b, v, h_t)|s_t) - E(u(0, p_t^a, p_t^b, v, h_t)|s_t)$$
  
= 
$$\sum_{v=0,1} \Pr(v|h_t, s_t) \left( (1 - \beta) \Delta \pi \left( \chi \left( p_t^a, p_t^b, v \right) \right) + \beta \Delta r \left( \hat{\gamma} \left( v, h_t \right) \right) \right)$$

where

$$\Delta \pi \left( \chi \left( p_t^a, p_t^b, v \right) \right) = \pi (v - p_t^a) - \pi (p_t^b - v)$$

and

$$\Delta r\left(\hat{\gamma}\left(v,h_{t}\right)\right) = r(\hat{\gamma}\left(1,v,h_{t}\right)) - r(\hat{\gamma}\left(0,v,h_{t}\right))$$

We restrict attention to non-perverse equilibria. We say that an equilibrium is perverse if for some priod t and some history  $h_t$  which occurs with positive probability on the equilibrium path, the fund manager at t is more likely to buy if he has a negative signal than is he has a positive signal  $(\alpha_t (h_t, s_t = 0) > \alpha_t (h_t, s_t = 1))$ . Thus, we are not excluding perverse behavior off the equilibrium path. Note that perverse equilibria are extremely unplausible in a financial context because perverse behavior given  $h_t$  implies that the bid price conditional on  $h_t$  is strictly lower than the ask price conditional on  $h_t$ .

**Proposition 2** In any non-perverse equilibrium there exists  $(\overline{p}, \underline{p}) \in (0, 1)^2$ such that if  $p_t > \overline{p}$  or  $p_t < \underline{p}$  then the actions of fund managers from period t onwards provide no information about their private signals.

The proposition is proven in several steps. We first show that as the price approaches either 0 or 1 the expected trading profit goes to zero. Second, we analyze the reputational incentives in an informative non-perverse equilibrium. As the price goes to one, the fund manager faces a positive and non-infinitesimal expected reputational benefit if he chooses to buy rather than to sell. Conversely, when the price goes to one, he faces a positive and non-infinitesimal benefit if he sells rather than buying. Putting together the profit motive and the reputational motive, we conclude that if the price is high enough or low enough there cannot be an informative non-perverse equilibrium. From then on, the market is stuck in an information cascade. No additional private information is revealed.

This result bears a connection to Ottaviani and Sorensen [24], who provide a general analysis of reputational cheap talk and show that full information transmission is generically impossible. Of particular interest to the present paper is their Proposition 9, where they consider a sequence of experts providing reports on a common state of the world and they show that informational herding must occur. As in our model, the experts receive signal that are uncorrelated with each other (given the state of the world). However, there are several differences. On the one hand, Ottaviani and Sorensen consider a space of signals and states that is more general than our binary space. On the other hand, we: (a) we show the necessity of informational cascades (while Ottaviani and Sorense prove that herding must occur but they cannot exclude that the true value is revealed in the limit); (b) our experts have a profit motive as well as a reputational motive; and (c) most important, our model is embedded in a financial market.

#### 3.3 Proof

We proceed by proving two preliminary results.

**Lemma 3** There exists a function  $f : [0,1] \times [0,1] \rightarrow [0,1]$  such that  $\Pr(v = 1|h_t, s_t) = f(p_t, s_t)$  and f satisfies the following properties: (a)  $f(p_t, s_t)$  is strictly increasing and continuous in  $p_t$ . (b)  $f(1, s_t) = 1 = 1 - f(0, s_t)$ (c)  $f(p_t, 1) > f(p_t, 0)$ 

Proof.

$$\Pr(v = 1|h_t, s_t) = \frac{\Pr(s_t, h_t|v = 1) \Pr(v = 1)}{\Pr(s_t, h_t)}$$

$$= \frac{\Pr(s_t|v = 1) \Pr(v = 1)}{\Pr(s_t, h_t)} \Pr(h_t|v = 1)$$

$$= \frac{\Pr(s_t|v = 1) \Pr(v = 1)}{\Pr(s_t, h_t)} \frac{\Pr(v = 1|h_t) \Pr(h_t)}{\Pr(v = 1)}$$

$$= \frac{\Pr(s_t|v = 1) \Pr(h_t)}{\Pr(s_t, h_t)} p_t$$

$$\Pr(s_t, h_t) = \Pr(h_t)[p_t \Pr(s_t | v = 1) + (1 - p_t) \Pr(s_t | v = 0)]$$

Thus

$$\Pr(v = 1|h_t, s_t) = \frac{p_t \Pr(s_t|v=1)}{p_t \Pr(s_t|v=1) + (1-p_t)\Pr(s_t|v=0)} = f(p_t, s_t)$$

Now parts (a) and (b) follow immediately. To see part (c) note that

$$f(p_t, 1) = \frac{p_t}{p_t + (1 - p_t) \frac{\Pr(s_t = 1|v=0)}{\Pr(s_t = 1|v=1)}}$$
$$= \frac{p_t}{p_t + (1 - p_t) \frac{1 - \sigma}{\sigma}}$$

where  $\sigma = \gamma \sigma_g + (1 - \gamma) \sigma_b$ . Similarly

$$f(p_t, 0) = \frac{p_t}{p_t + (1 - p_t)\frac{\sigma}{1 - \sigma}}$$

Since  $\sigma_g > \sigma_b > \frac{1}{2}$ ,  $\sigma > \frac{1}{2}$ . Thus  $\frac{1-\sigma}{\sigma} < 1 < \frac{\sigma}{1-\sigma}$ . This then implies  $f(p_t, 1) > f(p_t, 0)$  which completes the proof of the lemma.

The mixed strategy of manager t in this market will generally depend on both this history he observes and his signal. We denote this by  $\alpha_{s_t}^t(h_t)$ . A mixed strategy equilibrium is a sequence  $\{\alpha_{s_t}^t(h_t)\}_{t=1}^{T-1}$ . For notational convenience, we often omit the history argument and we denote the t fund manager's strategy as  $(\alpha_0^t, \alpha_1^t) \in [0, 1]^2$ .

Consider a set of equilibrium strategies  $(\alpha_0^t, \alpha_1^t) \in [0, 1]^2$ . We can now compute the posteriors regarding fund managers. The posterior belief is given by

$$\hat{\gamma}(a_t, v, h_t) = \frac{\Pr(a_t | \theta = g, v, h_t) \gamma}{\Pr(a_t | \theta = g, v, h_t) \gamma + \Pr(a_t | \theta = b, v, h_t) (1 - \gamma)}$$

Note that

$$\Pr(a_t = 1 | \theta = g, v = 1, h_t) = \alpha_1(h_t) \sigma_g + \alpha_0(h_t) (1 - \sigma_g)$$

and similarly for the other realizations of  $\hat{\gamma}$ . Therefore, we can write

$$\begin{split} \hat{\gamma} \left( a_t = 1, v = 1, h_t \right) &= \frac{\alpha_1 \left( h_t \right) \sigma_g + \alpha_0 \left( h_t \right) \left( 1 - \sigma_g \right)}{\alpha_1 \left( h_t \right) \sigma + \alpha_0 \left( h_t \right) \left( 1 - \sigma \right)} \gamma \\ \hat{\gamma} \left( a_t = 1, v = 0, h_t \right) &= \frac{\alpha_1 \left( h_t \right) \left( 1 - \sigma_g \right) + \alpha_0 \left( h_t \right) \sigma_g}{\alpha_1 \left( h_t \right) \left( 1 - \sigma \right) + \alpha_0 \left( h_t \right) \sigma} \gamma \\ \hat{\gamma} \left( a_t = 0, v = 1, h_t \right) &= \frac{\left( 1 - \alpha_1 \left( h_t \right) \right) \sigma_g + \left( 1 - \alpha_0 \left( h_t \right) \right) \left( 1 - \sigma_g \right)}{\left( 1 - \alpha_1 \left( h_t \right) \right) \sigma + \left( 1 - \alpha_0 \left( h_t \right) \right) \left( 1 - \sigma \right)} \gamma \\ \hat{\gamma} \left( a_t = 0, v = 0, h_t \right) &= \frac{\left( 1 - \alpha_1 \left( h_t \right) \right) \left( 1 - \sigma_g \right) + \left( 1 - \alpha_0 \left( h_t \right) \right) \sigma_g}{\left( 1 - \alpha_1 \left( h_t \right) \right) \left( 1 - \sigma \right) + \left( 1 - \alpha_0 \left( h_t \right) \right) \sigma_g} \gamma \end{split}$$

We now show that in all non-perverse equilibria either the manager with the high signal or the manager with the low signal play a pure strategy. **Lemma 4** There are no mixed strategy equilibria in which  $0 < \alpha_0^t < \alpha_1^t < 1$  for any t.

**Proof.** Consider a putative equilibrium in which  $1 > \alpha_0^t > 0$ , i.e. the agent at time t who receives signal zero is exactly indifferent between buying and selling. We will show that in this equilibrium, it must be the case that the agent who receives signal 1 at time t must strictly prefer to buy rather than sell. Consider the expected direct payoff difference between buying and selling:  $\sum_{v=0,1} \Pr(v|h_t, s_t) \Delta \pi (\chi(p_t^a, p_t^b, v))$ . This can be written as

$$f(p_t, s_t) \left( \pi \left( 1 - p_t^a \right) - \pi \left( p_t^b - 1 \right) \right) + \left( 1 - f(p_t, s_t) \right) \left( \pi \left( 0 - p_t^a \right) - \pi \left( p_t^b - 0 \right) \right)$$

Since  $\pi (1 - p_t^a) - \pi (p_t^b - 1) > 0 > \pi (0 - p_t^a) - \pi (p_t^b - 0)$ , and by Lemma 3  $f(p_t, 1) > f(p_t, 0)$ , it is clear that

$$\sum_{v=0,1} \Pr\left(v|h_t, s_t=1\right) \Delta \pi\left(\chi\left(p_t^a, p_t^b, v\right)\right) > \sum_{v=0,1} \Pr\left(v|h_t, s_t=0\right) \Delta \pi\left(\chi\left(p_t^a, p_t^b, v\right)\right)$$

Now consider the expected reputational payoff difference between buying and selling:  $\sum_{v=0,1} \Pr(v|h_t, s_t) \Delta r(\hat{\gamma}(v, h_t))$ . This can be expressed as:

$$f(p_t, s_t) \left[ r(\hat{\gamma}(1, h_t, 1)) - r(\hat{\gamma}(0, h_t, 1)) \right] + (1 - f(p_t, s_t)) \left[ r(\hat{\gamma}(1, h_t, 0)) - r(\hat{\gamma}(0, h_t, 0)) \right]$$

Notice that  $\hat{\gamma}(1, h_t, 1) > \hat{\gamma}(0, h_t, 1)$ ). To see why consider the expressions above. Suppose  $\frac{\alpha_1 \sigma_g + \alpha_0(1 - \sigma_g)}{\alpha_1 \sigma + \alpha_0(1 - \sigma)} < \frac{(1 - \alpha_1)\sigma_g + (1 - \alpha_0)(1 - \sigma_g)}{(1 - \alpha_1)\sigma + (1 - \alpha_0)(1 - \sigma)}$ . Algebraic manipulation shows that this implies that  $(\sigma_g - \sigma)(\alpha_1 - \alpha_0) < 0$  which is a contradiction since  $\sigma_g - \sigma > 0$  and  $\alpha_1 - \alpha_0 > 0$ . Similarly it is also true that  $\hat{\gamma}(1, h_t, 0) < \hat{\gamma}(0, h_t, 0)$ . To see why, consider again the expressions above. Suppose  $\frac{\alpha_1(1 - \sigma_g) + \alpha_0 \sigma_g}{\alpha_1(1 - \sigma) + \alpha_0 \sigma} > \frac{(1 - \alpha_1)(1 - \sigma_g) + (1 - \alpha_0)\sigma_g}{(1 - \alpha_1)(1 - \sigma) + (1 - \alpha_0)\sigma}$  This implies that  $-(\sigma_g - \sigma)(\alpha_1 - \alpha_0) > 0$  which is a contradiction. Thus,  $r(\hat{\gamma}(1, h_t, 1)) - r(\hat{\gamma}(0, h_t, 1)) > 0 > r(\hat{\gamma}(1, h_t, 0)) - r(\hat{\gamma}(0, h_t, 0))$ . Given Lemma 3, we know that  $f(p_t, 1) > f(p_t, 0)$ , and thus

$$\sum_{v=0,1} \Pr(v|h_t, s_t = 1) \Delta r(\hat{\gamma}(v, h_t)) > \sum_{v=0,1} \Pr(v|h_t, s_t = 0) \Delta r(\hat{\gamma}(v, h_t))$$

Finally, the arguments above imply that

$$\sum_{v=0,1} \Pr(v|h_t, s_t = 1) \left( (1-\beta) \,\Delta \pi + \beta \Delta r \right) > \sum_{v=0,1} \Pr(v|h_t, s_t = 0) \left( (1-\beta) \,\Delta \pi + \beta \Delta r \right) = 0$$

Thus, if  $0 < \alpha^0(h_t) < 1$ , then  $\alpha^1(h_t) = 1$ . An identical argument establishes that if  $0 < \alpha^1(h_t) < 1$ , then  $\alpha^0(h_t) = 0$ . This completes the proof of the lemma.

Let  $\{\alpha_{s_t}(h_t)\}_{t=1}^{T-1}$  be any non-perverse perfect Bayesian equilibrium of this game:  $\alpha_1(h_t) > \alpha_0(h_t)$  for all  $h_t$ .

**Lemma 5** For every  $\varepsilon > 0$  there exists  $\bar{p}_1(\varepsilon)$  and  $\bar{p}_2(\varepsilon)$  such that (for all  $h_t$ ):

$$\Delta \pi \geq -\varepsilon \text{ for all } p_t > \bar{p}_2(\varepsilon)$$

and

$$\Delta \pi \leq \varepsilon \text{ for all } p_t < \bar{p}_1(\varepsilon)$$

**Proof.** With terms defined as above, first consider the expected direct payoff difference between buying and selling. By a slight abuse of notation, we can write:

$$\Delta \pi = \sum_{v=0,1} \Pr(v|h_t, s_t) \left( \pi \left(v - p_t^a\right) - \pi \left(p_t^b - v\right) \right)$$
  
=  $f(p_t, s_t) \left( \pi \left(1 - p_t^a\right) - \pi \left(p_t^b - 1\right) \right) + (1 - f(p_t, s_t)) \left( \pi \left(0 - p_t^a\right) - \pi \left(p_t^b - 0\right) \right)$ 

Note that this function is *bounded above* by

$$U_{\pi} = \sum_{v=0,1} \Pr(v|h_t, s_t) (\pi(v) - \pi(-v))$$
  
=  $f(p_t, s_t) (\pi(1) - \pi(-1))$ 

And it is *bounded below* by

$$L_{\pi} = \sum_{v=0,1} \Pr(v|h_t, s_t) (\pi(v-1) - \pi(1-v))$$
  
=  $(1 - f(p_t, s_t)) (\pi(-1) - \pi(1))$ 

It is apparent that  $U_{\pi} > 0$ . From Lemma 3 we also conclude that  $U_{\pi}$  strictly increasing in  $p_t$  and  $\lim_{p_t \to 0} U_{\pi} = 0$ . Similarly,  $L_{\pi} < 0$ , strictly decreasing in  $p_t$  and  $\lim_{p_t \to 1} L_{\pi} = 0$ .

This means that for every  $\varepsilon > 0$  there exists  $\bar{p}_1(\varepsilon)$  and  $\bar{p}_2(\varepsilon)$  such that

$$\Delta \pi \geq -\varepsilon$$
 for all  $p_t > \bar{p}_2(\varepsilon)$ 

$$\Delta \pi \leq \varepsilon \text{ for all } p_t < \bar{p}_1(\varepsilon)$$

Second, consider the expected difference in reputational payoffs between buying and selling. Again, abusing notation, we write:

$$\Delta r = f(p_t, s_t) \left( v \left( \hat{\gamma} \left( a_t = 1, v = 1, h_t \right) \right) - v \left( \hat{\gamma} \left( a_t = 0, v = 1, h_t \right) \right) \right) \\ + \left( 1 - f(p_t, s_t) \right) \left( v \left( \hat{\gamma} \left( a_t = 1, v = 0, h_t \right) \right) - v \left( \hat{\gamma} \left( a_t = 0, v = 0, h_t \right) \right) \right)$$

**Lemma 6** For every number  $\epsilon > 0$  (but not too large), there exist  $\tilde{p}_1(\varepsilon)$  and  $\tilde{p}_2(\varepsilon)$  such that (for all  $h_t$ ):

$$\Delta r \geq \epsilon \text{ for all } p_t > \tilde{p}_2(\varepsilon)$$

and

$$\Delta r \leq -\epsilon \text{ for all } p_t < \tilde{p}_1(\varepsilon).$$

**Proof.** From Lemma 4 we know that there cannot be an equilibrium in which  $0 < \alpha_0(h_t) < \alpha_1(h_t) < 1$ . Thus, either  $0 = \alpha_0(h_t) < \alpha_1(h_t) < 1$  or  $0 < \alpha_0(h_t) < \alpha_1(h_t) = 1$ .

If  $0 = \alpha_0(h_t) < \alpha_1(h_t) < 1$ , the difference

$$r(\hat{\gamma}(a_t = 1, v = 1, h_t)) - r(\hat{\gamma}(a_t = 0, v = 1, h_t))$$

reduces to

$$r\left(\frac{\sigma_g}{\sigma}\gamma\right) - r\left(\frac{\left(1-\alpha_1\left(h_t\right)\right)\sigma_g + \left(1-\sigma_g\right)}{\left(1-\alpha_1\left(h_t\right)\right)\sigma + \left(1-\sigma\right)}\gamma\right),$$

which lies in the interval:

$$\left[r\left(\frac{\sigma_g}{\sigma}\gamma\right) - r\left(\gamma\right), r\left(\frac{\sigma_g}{\sigma}\gamma\right) - r\left(\frac{1 - \sigma_g}{1 - \sigma}\gamma\right)\right]$$

On the other hand if  $0 < \alpha_0(h_t) < \alpha_1(h_t) = 1$ , then the difference reduces to

$$r\left(\frac{\sigma_g + \alpha_0(h_t)(1 - \sigma_g)}{\sigma + \alpha_0(h_t)(1 - \sigma)}\gamma\right) - r\left(\frac{1 - \sigma_g}{1 - \sigma}\gamma\right)$$

which lies in the interval:

$$\left[r\left(\gamma\right) - r\left(\frac{1-\sigma_g}{1-\sigma}\gamma\right), r\left(\frac{\sigma_g}{\sigma}\gamma\right) - r\left(\frac{1-\sigma_g}{1-\sigma}\gamma\right)\right]$$

and

Now, defining

$$U_r^1 = r\left(\frac{\sigma_g}{\sigma}\gamma\right) - r\left(\frac{1-\sigma_g}{1-\sigma}\gamma\right) > 0$$

and

$$L_{r}^{1} = \min\left[r\left(\gamma\right) - r\left(\frac{1 - \sigma_{g}}{1 - \sigma}\gamma\right), r\left(\frac{\sigma_{g}}{\sigma}\gamma\right) - r\left(\gamma\right)\right]$$

we note that in all non-perverse equilibria, the difference

$$r(\hat{\gamma}(a_t = 1, v = 1, h_t)) - r(\hat{\gamma}(a_t = 0, v = 1, h_t))$$

is bounded below by  $L_r^1$  and bounded above by  $U_r^1$ . Now consider the case where v = 0. If  $0 = \alpha_0 (h_t) < \alpha_1 (h_t) < 1$ , the difference

$$r(\hat{\gamma}(a_t = 1, v = 0, h_t)) - r(\hat{\gamma}(a_t = 0, v = 0, h_t))$$

reduces to

$$r\left(\frac{1-\sigma_g}{1-\sigma}\gamma\right) - r\left(\frac{(1-\alpha_1(h_t))\sigma_g + \sigma_g}{(1-\alpha_1(h_t))\sigma + \sigma}\gamma\right)$$

which lies in the interval

$$\left[r\left(\frac{1-\sigma_g}{1-\sigma}\gamma\right)-r\left(\frac{\sigma_g}{\sigma}\gamma\right), r\left(\frac{1-\sigma_g}{1-\sigma}\gamma\right)-r\left(\gamma\right)\right]$$

On the other hand if  $0 < \alpha_0(h_t) < \alpha_1(h_t) = 1$ , then the difference reduces to

$$r\left(\frac{(1-\sigma_g)+\alpha_0(h_t)\sigma_g}{(1-\sigma)+\alpha_0(h_t)\sigma}\gamma\right)-r\left(\frac{\sigma_g}{\sigma}\gamma\right)$$

which lies in the interval:

$$\left[r\left(\frac{1-\sigma_g}{1-\sigma}\gamma\right)-r\left(\frac{\sigma_g}{\sigma}\gamma\right), r\left(\gamma\right)-r\left(\frac{\sigma_g}{\sigma}\gamma\right)\right]$$

Now, defining

$$L_r^0 = r\left(\frac{1-\sigma_g}{1-\sigma}\gamma\right) - r\left(\frac{\sigma_g}{\sigma}\gamma\right) < 0$$

and

$$U_{r}^{0} = \max\left[r\left(\gamma\right) - r\left(\frac{\sigma_{g}}{\sigma}\gamma\right), r\left(\frac{1 - \sigma_{g}}{1 - \sigma}\gamma\right) - r\left(\gamma\right)\right] < 0$$

we note that in all non-perverse equilibria, the difference

$$r(\hat{\gamma}(a_t = 1, v = 0, h_t)) - r(\hat{\gamma}(a_t = 0, v = 0, h_t))$$

is bounded below by  $L_r^0$  and bounded above by  $U_r^0$ .

Thus  $\Delta r$  is bounded above by

$$f(p_t, s_t)U_r^1 + (1 - f(p_t, s_t))U_r^0$$

and bounded below by

$$f(p_t, s_t)L_r^1 + (1 - f(p_t, s_t))L_r^0$$

Finally, we note that  $U_r^1 = -L_r^0 > 0$  and  $L_r^1 = -U_r^0 > 0$ . Thus,  $\Delta r$  is bounded above by

$$U_r = f(p_t, s_t)U_r^1 - (1 - f(p_t, s_t))L_r^1$$

and bounded below by

$$L_r = f(p_t, s_t) L_r^1 - (1 - f(p_t, s_t)) U_r^1$$

Now, utilizing Lemma 1 above, we know that  $U_r$  strictly increasing in  $p_t$  and  $\lim_{p_t\to 0} U_r = -L_r^1 < 0$ . Similarly,  $L_r < 0$ , strictly decreasing in  $p_t$  and  $\lim_{p_t\to 1} L_r = L_r^1 > 0$ .

Thus, for every number  $0 < \epsilon < L_r^1$  there exists a price  $\tilde{p}_2(\epsilon) < 1$  such that for  $p_t > \tilde{p}_2(\epsilon), \Delta r \ge \epsilon$ .

The part of the proof concerning  $\tilde{p}_1(\epsilon)$  is analogous and it is omitted. But by appeal to the arguments above, for a given positive number  $\frac{\beta}{1-\beta}(\epsilon-\delta)$  (where  $\delta \in (0,\epsilon)$ ) there exists  $\overline{p}_2(\frac{\beta}{1-\beta}(\epsilon-\delta)) < 1$  such that if  $p_t > \overline{p}_2(\frac{\beta}{1-\beta}(\epsilon-\delta))$ , then  $\Delta \pi \ge -\frac{\beta}{1-\beta}(\epsilon-\delta) > -\frac{\beta}{1-\beta}\epsilon$ . Thus for  $p_t > \max[\widetilde{p}_2(\epsilon), \overline{p}_2(\frac{\beta}{1-\beta}(\epsilon-\delta))]$ ,

$$\Delta u = (1 - \beta)\Delta \pi + \beta \Delta r > (1 - \beta)\left(-\frac{\beta}{1 - \beta}\epsilon\right) + \beta \epsilon = 0$$

Thus, for any price greater than  $\tilde{p}_2(\epsilon)$  the fund manager would always choose to buy. The case for selling is symmetric.

### 4 Price Dynamics: An Example

We have demonstrated a general impossibility result – there exists no equilibrium in which prices converge to fundamentals. A natural question is: What equilibria exist, and what are the dynamics of prices in such equilibria. We provide an example of such an equilibrium. Assume that  $\gamma = \frac{1}{2}$ ,  $\sigma_b = \frac{1}{2}$ ,  $\sigma_g = 1$ ,  $\beta = \frac{1}{2}$ , and  $\delta = 0$ .

**Proposition 7** There is an equilibrium in which fund managers play sincerely as long as  $p_t \in [\bar{p}, 1-\bar{p}]$ . If at some t trade occurs at  $p_t < \bar{p}$  ( $p_t > 1-\bar{p}$ ), all fund managers after t sell (buy) and the price stays at  $p_t$ . The threshold  $\bar{p}$  is  $\frac{13}{20} - \frac{1}{20}\sqrt{109} = 0.128$ . Within the bands, the price evolves according to

$$p_{t+1} = \frac{p_t + 2a_t p_t}{3 - 2p_t - 2(1 - 2p_t)a_t}$$

**Proof.** We then have that the conditional expectation of v becomes

$$\hat{v}_1^t = \Pr(v = 1 | s_t = 1, h_t) = \frac{3p_t}{1 + 2p_t};$$
  
 $\hat{v}_0^t = \Pr(v = 1 | s_t = 0, h_t) = \frac{p_t}{3 - 2p_t};$ 

and the reputational benefit of making the right trade rather than the wrong trade in a sincere equilibrium becomes:

$$\Delta \hat{\gamma} = \left(\frac{\sigma_g}{\sigma} - \frac{1 - \sigma_g}{1 - \sigma}\right) \gamma = \frac{2}{3}$$

From the analysis above, we know that a fund manager who observes  $s_t = 1$  prefers to buy rather than sell (in a sincere equilibrium) if

$$\beta \underbrace{\left(2\delta\left(\hat{v}_{1}^{t}-p_{t}\right)+\left(1-\delta\right)\left(\hat{v}_{1}^{t}-\hat{v}_{0}^{t}\right)\right)}^{\Delta \pi}+\left(1-\beta\right)\underbrace{\left(2\hat{v}_{1}^{t}-1\right)\Delta\hat{\gamma}}^{\Delta r}\geq0$$

In the present numerical example, the condition simplifies to

$$\left(\frac{3p_t}{1+2p_t} - \frac{p_t}{3-2p_t}\right) \ge \left(1 - 2\frac{3p_t}{1+2p_t}\right)\frac{2}{3},$$

which holds if and only if

$$p_t \ge \bar{p} = \frac{13}{20} - \frac{1}{20}\sqrt{109} = 0.127.$$





It remains for us to check that for  $p_t < \bar{p}$  it is actually optimal for the fund manager who observes  $s_t = 1$  to sell instead of buy. This needs to be checked separately, since for  $p_t < \bar{p}$  equilibrium behaviour is not contingent on signals, and thus reputational payoffs (as well as direct profits) are different than in the sincere part of the equilibrium. Note that if the manager who has observed the high signal chooses to sell, so will the manager who observes the low signal.

In the region where equilibrum strategies require all managers to sell, the reputational payoff to selling is simply  $\gamma$ . Buying, on the other hand, is an off-equilibrium event. Let us assume that such an event generates the following off-equilibrium belief: the manager who chooses to buy is assumed to do so sincerely, i.e., is assumed to have observed signal  $s_t = 1$ . Thus, the reputational payoff from buying is  $v_1^t \frac{\sigma_g}{\sigma} \gamma - (1 - v_1^t) \frac{1 - \sigma_g}{1 - \sigma} \gamma$ . Thus

$$\Delta r = v_1^t \Delta \gamma - (1 - \frac{1 - \sigma_g}{1 - \sigma})\gamma$$

Now consider direct profits. Since all managers sell in equilibrium, the market maker offers a bid price of simply  $p_t$ . Thus profits from selling are  $p_t - v_1^t$ . However, upon seeing a buy order, in the absence of noise traders, the market maker assumes (according to the off-equilibrium beliefs assumed above) that the order is from a fund-manager with signal  $s_t = 1$ . and thus the ask price is set to  $v_1^t$ . Thus, profits from buying are  $v_1^t - v_1^t = 0$ . We conclude that

$$\Delta \pi = 0 - (p_t - v_1^t) = v_1^t - p_t$$

Thus, the fund manager will prefer to sell rather than buy if and only if

$$v_1^t - p_t < -v_1^t \Delta \gamma + (1 - \frac{1 - \sigma_g}{1 - \sigma})\gamma$$

which in this example reduces to

$$\frac{3p_t}{1+2p_t} - p_t < -\frac{3p_t}{1+2p_t}\frac{2}{3} + (1-0)\frac{1}{2}$$

which holds if and only if  $p_t < 0.191$ . Since  $\overline{p} < 0.191$ , whenever  $p_t < \overline{p}$ , it must be true that  $p_t < 0.191$ . It is thus optimal for the manager to behave sincerely for  $p \ge \overline{p}$  and sell if  $p < \overline{p}$ .

A similar argument can be developed for a fund manager who observes  $s_t$ . She is willing to play sincerely only as long as  $p_t \leq 1 - \bar{p}$ .

In a sincere equilibrium, the evolution of  $p_t$  is as follows:

$$p_{t+1} = \begin{cases} p_t^a = \frac{3p_t}{1+2p_t} & \text{if } a_t = 1\\ p_t^b = \frac{p_t}{3-2p_t} & \text{if } a_t = 0 \end{cases}$$

Hence, we can write

$$p_{t+1} = \frac{p_t + 2a_t p_t}{3 - 2p_t - 2(1 - 2p_t)a_t}$$

When  $p_t > 1 - \bar{p}$ , the fund manager always buys, irrespective of his private signal. When  $p_t < \bar{p}$ , the fund manager always sells.

### 5 Prices and Reputational Benefits

From the viewpoint of fund managers, the expected total value of the asset can differ from expected liquidation value. The difference captures the expected *reputational benefit or cost* that the fund manager incurs if he buys or sells the asset.

Let  $w_{s_t}^t$  be the price at which a fund manager in t who observes signal  $s_t$  is exactly indifferent between buying and selling. We refer to  $w_{s_t}^t$  as the fund manager's *expected total value of the asset*. It is the solution of

$$\beta \left( \hat{v}_{s_t}^t - w_{s_t}^t \right) + (1 - \beta) E \left( \hat{\gamma}^t (a_t = 1, v, p_t) \right) = \beta \left( w_{s_t}^t - \hat{v}_{s_t}^t \right) + (1 - \beta) E \left( \hat{\gamma}^t (a_t = 0, v, p_t) \right).$$

If the fund manager has no career concerns ( $\beta = 1$ ), we simply have that  $w_{s_t}^t = \hat{v}_{s_t}^t$ : the expected total value is just the expected liquidation value. However, if  $\beta < 1$  there may be a wedge between the two values, which we indicate with

$$\rho_{s_t}^t = w_{s_t}^t - \hat{v}_{s_t}^t = \frac{1-\beta}{2\beta} \left( E\left(\hat{\gamma}^t \left(a_t = 1, v, p_t\right)\right) - E\left(\hat{\gamma}^t \left(a_t = 0, v, p_t\right)\right) \right).$$

We refer to  $\rho_{s_t}^t$  as the reputational benefit or cost of the asset for the manager in t if he observes  $s_t$ . We now characterize the dynamics of the reputational benefit in a large class of reasonable equilibria. Define *piecewise-stationary non-perverse equilibria* as follows:

**Definition 8** A piecewise-stationary non-perverse equilibrium is a non-perverse equilibrium in which  $\frac{\partial \alpha_{s_t}(p_t)}{\partial p_t} = 0$  except for a finite number of prices  $p_t \in \Gamma$ .

Note that all pure strategy equilibria and the truncated sincere equilibrium identified above fall within this class of equilibria.

**Proposition 9** In any piecewise-stationary non-perverse equilibrium, the reputational benefit  $\rho_{s_t}^t$  is strictly increasing in the asset price  $p_t$ , except possibly at a finite set of points.

**Proof.** Consider any piecewise-stationary non-perverse equilibrium, and suppose that  $p_t \notin \Gamma$ . For such  $p_t$ , there exists a small interval  $(p_t - \epsilon, p_t + \epsilon)$ such that for all  $\hat{p}_t \in (p_t - \epsilon, p_t + \epsilon)$ ,  $\alpha_{s_t}(\hat{p}_t) = \alpha_{s_t}(p_t)$ , by definition of piecewise-stationarity. Thus, when considering derivatives evaluated at  $p_t$ , we can suppress the history dependence of the strategy, and write  $\alpha_{s_t}(p_t) = \alpha_{s_t}$ . In addition, note that we have already shown that there are no totally mixed non-perverse equilibria. Thus, we can restrict attention to either  $\alpha_1 = 1$  and  $\alpha_0 \in (0, 1)$ , or  $\alpha_1 \in (0, 1)$  and  $\alpha_0 = 0$ . We demonstrate the result for the first case. The second is symmetric.

Using an earlier characterization of posteriors in all non perverse equilibria, it is easy to see that when  $\alpha_1 = 1$  and  $\alpha_0 \in (0, 1)$ ,  $E(\hat{\gamma}^t (a_t = 1, v, p_t)) - E\hat{\gamma}^t (a_t = 0, v, p_t)$  can be rewritten as

$$v_{s_t}^t \left[ \frac{\sigma_g + \alpha_0 (1 - \sigma_g)}{\sigma + \alpha_0 (1 - \sigma)} - \frac{(1 - \sigma_g) + \alpha_0 \sigma_g}{(1 - \sigma) + \alpha_0 \sigma} + \frac{\sigma_g}{\sigma} - \frac{1 - \sigma_g}{1 - \sigma} \right] + K$$

where K proxies for terms not involving  $p_t$ . We have shown earlier that  $v_{s_t}^t$  is an increasing function of  $p_t$  for all  $p_t$  and  $s_t$ . Thus, all that remains to be

shown is that the multiplier of  $v_{s_t}^t$  is strictly positive. Since  $\frac{\sigma_g}{\sigma} > \frac{1-\sigma_g}{1-\sigma}$  it is sufficient to show that  $\frac{\sigma_g + \alpha_0(1-\sigma_g)}{\sigma + \alpha_0(1-\sigma)} \ge \frac{(1-\sigma_g) + \alpha_0\sigma_g}{(1-\sigma) + \alpha_0\sigma}$ , which is only true if and only if

$$[\sigma_g + \alpha_0(1 - \sigma_g)][(1 - \sigma) + \alpha_0\sigma] \ge [(1 - \sigma_g) + \alpha_0\sigma_g][\sigma + \alpha_0(1 - \sigma)]$$

but

$$(\sigma_g + \alpha_0(1 - \sigma_g))((1 - \sigma) + \alpha_0\sigma) - ((1 - \sigma_g) + \alpha_0\sigma_g)(\sigma + \alpha_0(1 - \sigma))$$
  
=  $(1 - \alpha_0^2)(\sigma_g - \sigma) \ge 0$ 

Thus

$$\rho_{s_t}^t = w_{s_t}^t - \hat{v}_{s_t}^t = \frac{1 - \beta}{2\beta} \left( E\left(\hat{\gamma}^t \left(a_t = 1, v, p_t\right)\right) - E\left(\hat{\gamma}^t \left(a_t = 0, v, p_t\right)\right) \right)$$

is always increasing in  $p_t$ , except possibly for  $p_t \in \Gamma$ .

There is a systematice difference between the valuation of the asset for traders with career concerns and regular traders. The difference is increasing in the price, except possibly at a countable number of points. In general, therefore, the higher the price of an asset, the greater is the reputational surplus from purchasing it. We shall see below that if the market making sector has market power, and thus can extract at least part of this surplus from career concerned managers, trade can sometimes occur at seemingly irrational prices.

# 6 Reputational Benefits and Asset Price Bubbles

So far, we have made an extreme assumption: during any transaction careerdriven traders have all the bargaining power. This is a consequence of including a competitive market making sector. While this is a standard modeling choice, it has extreme consequences in the present setting.

In this model, there are two types of traders, the ones with career concerns and the ones without. As we have argued already, there exists a wedge between the valuations of these two classes of investors. However, because of the "fair pricing" consequence of competitive market making, the price only reflects the valuation of traders without career concerns. This is clearly an extreme position. In reality, we should expect that the bargaining power is shared and that therefore the price reflects the valuations of both classes of investors.

While it would be desirable to model bargaining power as shared, such setting would involve a high degree of analytical complexity. In this section we limit ourselves to study the other extreme case, when all the bargaining power is on the side of traders without career concerns. This is sufficient to have an idea of what happens to equilibrium dynamics when the reputational benefit is factored into asset price.

We employ a simple way to generate limited supply of the asset. We assume that in every period t the fund manager faces one short-lived monopolistic trader who has an asset to sell and no buyers (perhaps because no one is willing to let her go short). The monopolistic seller operates only in period t. If he does not sell the asset at t, he will keep it until T when he will receive the liquidation value v. The monopolistic seller does not know the liquidation value, and infers it from the market by observing past order flow and also from the current period order flow.

We face a further modeling choice. If there are noise traders who submit market orders, a monopolistic seller can set an inifinitely high price and make infinite profits. We could assume that noise traders submit limit orders (with stochastic limits), but we choose to simplify the analysis by assuming that there are no noise traders. The results can be taken as the limit of a model in which there is a vanishing probability of a noise trader who submits a limit order.

The analysis is carried out within the example used previously:  $\gamma = \frac{1}{2}$ ,  $\sigma_b = \frac{1}{2}$ ,  $\sigma_g = 1$ ,  $\beta = \frac{1}{2}$ , and  $\delta = 0$ .

#### **Proposition 10** There exists an equilibrium in which:

(i) A fund manager with  $s^t = 0$  never buys;

(ii) A fund manager with  $s^t = 1$  buys if and only if the expected liquidation value is above a certain threshold  $(\hat{v}^t > \frac{1}{4});$ 

(iii) Trades occur at a price higher than the expected liquidation value;

(iv) Trades can occur at price higher than 1 (for instance, when the market opens at t = 1 the seller quotes a price of  $\frac{13}{12}$  and makes a sale with probability  $\frac{1}{2}$ );

(v) The price can go up to  $\frac{3}{2}$ ;

(vi) An information cascade occurs only if the expected liquidation value goes below the threshold  $(\hat{v}^t < \frac{1}{4})$ .

**Proof.** Suppose we are in the equilibrium outlined above. A fund manager with  $s^t = 1$  is then willing to buy as long as:

$$\beta \left( v_1^t - p_t^a \right) + (1 - \beta) \left( v_1^t \hat{\gamma} \left( v = 1, a^t = 1 \right) + \left( 1 - v_1^t \right) \hat{\gamma} \left( v = 0, a^t = 1 \right) \right)$$
  
 
$$\geq 0 + (1 - \beta) \left( v_1^t \hat{\gamma} \left( v = 1, a^t = 0 \right) + \left( 1 - v_1^t \right) \hat{\gamma} \left( v = 0, a^t = 0 \right) \right)$$

which can be rewritten as

$$\beta \left( v_1^t - p_t^a \right) + (1 - \beta) \left( 2v_1^t - 1 \right) \Delta \hat{\gamma} \ge 0.$$

Thus, the maximum ask price for a fund manager with  $s^t = 1$ 

$$w_1^t = v_1^t + \frac{1-\beta}{\beta} \left(2v_1^t - 1\right) \Delta \hat{\gamma}.$$

Similarly, the maximum ask price for a fund manager with  $s^t = 0$  is

$$w_0^t = v_0^t + \frac{1-\beta}{\beta} \left(2v_0^t - 1\right) \Delta \hat{\gamma}.$$

Recall that

$$\hat{v}_1^t = \frac{3\hat{v}^t}{1+2\hat{v}^t}$$

$$\hat{v}_0^t = \frac{\hat{v}^t}{3-2\hat{v}^t}$$

$$\Delta\hat{\gamma} = \frac{2}{3}.$$

Hence

$$w_1^t = \frac{3\widehat{v}^t}{1+2\widehat{v}^t} + \left(2\frac{3\widehat{v}^t}{1+2\widehat{v}^t} - 1\right)\frac{2}{3} = \frac{17\widehat{v}^t - 2}{3(2\widehat{v}^t + 1)}.$$
$$w_0^t = \frac{\widehat{v}^t}{3-2\widehat{v}^t} + \left(2\frac{\widehat{v}^t}{3-2\widehat{v}^t} - 1\right)\frac{2}{3} = \frac{7\widehat{v}^t}{9-6\widehat{v}^t} - \frac{2}{3} = \frac{11\widehat{v}^t - 6}{3(3-2\widehat{v}^t)}.$$

Plot the two ask prices



Expected liquidation value of **v** 

Value of asset to fund manager

We need to check that the market maker cannot make more money by decreasing the sale price  $\hat{v}^t$  to a level at which the fund manager would buy even if  $s^t = 0$ . Suppose that the fund manager buys even if  $s^t = 0$ . The reputational payoffs are

$$\hat{\gamma} (a^t = 1, v = 1) = \hat{\gamma} (a^t = 1, v = 0) = \gamma$$

The action  $a^t = 0$  is out of equilibrium and we assume as before that the market believes that it is associated with a fund manager who has observed  $s^t = 0$ . As before,

$$\hat{\gamma} \left( a^t = 0, v = 1 \right) = \frac{1 - \sigma_g}{1 - \sigma} \gamma$$
$$\hat{\gamma} \left( a^t = 1, v = 1 \right) = \frac{\sigma_g}{\sigma} \gamma.$$

The reputational benefit for a fund manager with  $s^t = 1$  who chooses  $a^t = 1$  instead of  $a^t = 0$  is

$$\begin{aligned} \Delta \tilde{v} &= \left( v_1^t \left( 1 - \frac{1 - \sigma_g}{1 - \sigma} \right) + \left( 1 - v_1^t \right) \left( 1 - \frac{\sigma_g}{\sigma} \right) \right) \gamma \\ &= \left( v_1^t \left( \frac{\sigma_g}{\sigma} - \frac{1 - \sigma_g}{1 - \sigma} \right) + \left( 1 - \frac{\sigma_g}{\sigma} \right) \right) \gamma \\ &= v_1^t \Delta \hat{\gamma} - \left( \frac{\sigma_g}{\sigma} - 1 \right) \gamma \end{aligned}$$

Thus, the maximum ask price for a fund manager with  $s^t = 1$ 

$$\begin{split} \tilde{w}_1^t &= v_1^t + \frac{1-\beta}{\beta} \left( v_1^t \Delta \hat{\gamma} - \left( \frac{\sigma_g}{\sigma} - 1 \right) \gamma \right) \\ &= \frac{3 \hat{v}^t}{1+2 \hat{v}^t} + \frac{3 \hat{v}^t}{1+2 \hat{v}^t} \frac{2}{3} - (2-1) \frac{1}{2} \\ &= 5 \frac{\hat{v}^t}{2 \hat{v}^t + 1} - \frac{1}{2} = \frac{8 \hat{v}^t - 1}{2 \left( 2 \hat{v}^t + 1 \right)}. \end{split}$$

Similarly, the maximum ask price for a fund manager with  $s^t = 0$  is

$$\begin{split} \tilde{w}_0^t &= v_0^t + \frac{1-\beta}{\beta} \left( v_0^t \Delta \hat{\gamma} - \left( \frac{\sigma_g}{\sigma} - 1 \right) \gamma \right) \\ &= \frac{\hat{v}^t}{3 - 2\hat{v}^t} + \frac{\hat{v}^t}{3 - 2\hat{v}^t} \frac{2}{3} - (2-1)\frac{1}{2} \\ &= -5\frac{\hat{v}^t}{6\hat{v}^t - 9} - \frac{1}{2} = \frac{16\hat{v}^t - 9}{6\left(3 - 2\hat{v}^t\right)} \end{split}$$



Value of asset to fund manager

We now check which sales strategy is optimal for the seller. In the sincere equilibrium the seller will wish to sell if and only if

$$\frac{17\hat{v}^t - 2}{3\left(2\hat{v}^t + 1\right)} > \frac{3\hat{v}^t}{1 + 2\hat{v}^t}$$

because if he is able to make a sale this reveals that he is facing a fund manager with signal  $s_t = 1$ , and he is able to charge him a price of  $\frac{17\hat{v}^t - 2}{3(2\hat{v}^t + 1)}$ . This holds when  $\hat{v}^t > 0.25$ .

Instead if the seller sold at a price low enough to induce managers with either signal to buy from him, then he would be able to charge only  $\frac{16\hat{v}^t-9}{6(3-2\hat{v}^t)}$ and upon finding a buyer (with probability 1) would value the asset at  $\hat{v}^t$ (since the current period order flow conveys no information to him). Thus he would wish to sell if and only if

$$\frac{16\widehat{v}^t - 9}{6\left(3 - 2\widehat{v}^t\right)} > \widehat{v}^t$$

which holds when  $\hat{v}^t > 0.95336$ .

The strategies thus produce the same payoffs for  $\hat{v}^t \leq 0.25$  but potentially different payoffs everywhere else. Consider the region  $\hat{v}^t \in [0.25, 0.95336]$ . In this region, the strategy of excluding the low signal fund managers leads to a payoff of

$$\frac{1}{2} \frac{17\hat{v}^t - 2}{3\left(2\hat{v}^t + 1\right)} + \frac{1}{2} \frac{\hat{v}^t}{3 - 2\hat{v}^t}$$

since a sale is made with probability  $\frac{1}{2}$  and if a sale is not made then the seller knows the manager facing him has received signal  $s_t = 0$  and thus the asset he is forced to keep is worth only  $\hat{v}_0^t = \frac{\hat{v}^t}{3-2\hat{v}^t}$  to him. On the other hand, the strategy of not selling leads to a payoff of  $\hat{v}^t$ . For  $\hat{v}^t \in [0.25, 0.95336]$ , it is easy to see that

$$\frac{1}{2} \frac{17\hat{v}^t - 2}{3\left(2\hat{v}^t + 1\right)} + \frac{1}{2} \frac{\hat{v}^t}{3 - 2\hat{v}^t} > \hat{v}^t$$

Finally, then, it remains to check that in the region  $\hat{v}^t \in (0.95336, 1]$  it is better to exclude the fund managers with  $s_t = 0$ . The payoff from the strategy of excluding the low signal fund managers leads to an expected payoff of

$$\frac{1}{2} \frac{17 \hat{v}^t - 2}{3 \left( 2 \hat{v}^t + 1 \right)} + \frac{1}{2} \frac{\hat{v}^t}{3 - 2 \hat{v}^t}$$

as before. On the other hand, the payoff from the strategy of including all fund managers is

$$\frac{16\widehat{v}^t - 9}{6\left(3 - 2\widehat{v}^t\right)}$$

since he makes a sale with probability one. Now we note that

$$\frac{1}{2} \frac{17\hat{v}^t - 2}{3\left(2\hat{v}^t + 1\right)} + \frac{1}{2} \frac{\hat{v}^t}{3 - 2\hat{v}^t} > \frac{16\hat{v}^t - 9}{6\left(3 - 2\hat{v}^t\right)}$$

in the region  $\hat{v}^t \in (0.95336, 1]$ . Thus, the monopolistic seller retains the asset for  $\hat{v}^t \leq 0.25$  and the market ceases to function, and sells at the proposed equilibrium prices only to managers with high signals otherwise. The starting price is

$$p^{1} = \frac{17\left(\frac{1}{2}\right) - 2}{3\left(2\left(\frac{1}{2}\right) + 1\right)} = \frac{13}{12}$$

# 7 Extensions

#### 7.1 Self Knowledge

What happens if the fund managers know, at least partially, their type?

Suppose that the fund manager receives two signals: the now familiar  $s^t$  and a new signal  $z^t$ , with  $\Pr(z^t = \theta | \theta) = \rho$ .

**Proposition 11** If self-knowledge is not too accurate, there exists no equilibrium in which a fund manager with  $z_t = 1$  plays sincerely. **Proof.** Suppose there exists a non-perverse equilibrium in which a fund manager who observes  $z_t = g$  always plays  $a_t = s_t$ .

Consider a fund manager with  $z_t = 1$  and  $s_t = 0$  and suppose that the current price is  $p_t$ . Let

$$\hat{v}_t^{s_t, z_t} = E\left[v|s_t, z_t, h_t\right].$$

It is easy to see that

$$\lim_{p_t \to 1} \hat{v}_t^{s_t, z_t} = 1 \qquad \qquad \text{for all } s_t, z_t.$$

Hence, the expected benefit in terms of trading profit of playing  $a_t = 0$  instead of  $a_t = 1$  for a fund manager with  $z_t = g$  and  $s_t = 0$  goes to zero as price approaches 1:

$$\lim_{p_t \to 0} \Delta \pi = 0.$$

The expected reputation benefit/cost of playing  $a_t = 0$  instead  $a_t = 1$  for a fund manager with  $z_t = g$  and  $s_t = 0$  is

$$\Delta r = \hat{v}_t^{0,g} \left( \left( \hat{\gamma}_t \left( a_t = 0, v = 1 \right) - \hat{\gamma}_t \left( a_t = 1, v = 1 \right) \right) \right) \\ + \left( 1 - \hat{v}_t^{0,g} \right) \left( \hat{\gamma}_t \left( a_t = 0, v = 0 \right) - \hat{\gamma}_t \left( a_t = 1, v = 0 \right) \right).$$

Thus,

$$\lim_{p_t \to 1} \Delta r = \hat{\gamma}_t \left( a^t = 0, v = 1 \right) - \hat{\gamma}_t \left( a^t = 1, v = 1 \right).$$

As  $p_t \to 1$ , a fund manager with  $z_t = g$  and  $s_t = 0$  plays  $a_t = 0$  only if

$$\hat{\gamma}_t \left( a^t = 0, v = 1 \right) \ge \hat{\gamma}_t \left( a^t = 1, v = 1 \right).$$
 (1)

In a non-perverse equilibrium in which a fund manager with  $z_t = g$  plays  $a_t = s_t$ , as  $p_t \to 1$ , beliefs have the following bounds (based on the assumption that all agents with  $z_t = b$  play  $a_t = 1$ ):

$$\hat{\gamma}^t (a^t = 1, v = 1) \ge \Pr(\theta = g | \text{not} (z^t = 1 \text{ and } s^t = 0), v = 1) \hat{\gamma}^t (a^t = 0, v = 1) \le \Pr(\theta = g | z^t = 1, s^t = 0, v = 1).$$

It is easy to see that

$$\Pr\left(\theta = g | \text{not} \left(z^t = 1 \text{ and } s^t = 0\right), v = 1\right)$$
  
> 
$$\Pr\left(\theta = g | z^t = 0\right) = \frac{(1-\rho)\gamma}{(1-\rho)\gamma + \rho(1-\gamma)},$$

and

$$\Pr\left(\theta = g | z^{t} = 1, s^{t} = 0, v = 1\right) = \frac{(1 - \sigma_{g}) \rho \gamma}{(1 - \sigma_{g}) \rho \gamma + (1 - \sigma_{b}) (1 - \rho) (1 - \gamma)}$$

Inequality (1) is satisfied only if

$$\frac{(1-\sigma_g)\,\rho\gamma}{(1-\sigma_g)\,\rho\gamma+(1-\sigma_b)\,(1-\rho)\,(1-\gamma)} \ge \frac{(1-\rho)\gamma}{(1-\rho)\gamma+\rho(1-\gamma)}.$$

If  $\rho \rightarrow \frac{1}{2}$ , the inequality reduces to

$$\frac{(1-\sigma_g)\gamma}{(1-\sigma_g)\gamma+(1-\sigma_b)(1-\gamma)} \ge \gamma,$$

which is false.  $\blacksquare$ 

#### 7.2 Relative Reputation

Suppose that the reputational component of the fund manager's payoff depends on her relative reputation. The payoff is now

$$(1-\beta)\pi(\chi_t) + \beta r_t(\hat{\gamma}_1, ..., \hat{\gamma}_T).$$

We assume that r is still continuous and differentiable in its components and that, for fund manager t,

$$\begin{array}{lll} \displaystyle \frac{\partial r_t}{\partial \hat{\gamma}_t} &> & 0\\ \displaystyle \frac{\partial r_t}{\partial \hat{\gamma}_\tau} &\leq & 0 & \qquad {\rm for} \ \tau \neq t. \end{array}$$

This formulation encompasses a situation in which the reputational payoff is an increasing (and perhaps convex) function of the difference between the reputation of a particular manager and the average reputation of all managers:

$$r_t\left(\hat{\gamma}_1,...,\hat{\gamma}_T\right) = R\left(\hat{\gamma}_\tau - \frac{\sum_{\tau=1}^T \hat{\gamma}_\tau}{T}\right).$$

**Proposition 12** There is no sincere equilibrium.

**Proof.** Suppose a sincere equilibrium exists and consider the fund manager in the last period, T. As the agent's action does not affect the reputations of all othere agents, the analysis is identical to the case without relative performance. The proof of nonexistence for  $p_T \rightarrow \{0, 1\}$  follows familiar lines.

#### 7.3 Informed Individual Traders

To date, we have restricted attention to a market in which uninformed market makers are faced with either noise traders or reputationally sensitive traders. A natural added member of the marketplace would be informed individual investors, who do not face career concerns. How would the results change if informed individual investors operated in our baseline model?

It is clear that informed inviduals devoid of career concerns would trade sincerely, and thus, in the presence of such traders prices would eventually converge to true value. However, the basic intuition of the main result in unchanged in this case: once prices were close enough to true value, career concerned institutional traders would begin to ignore their own information. Thus convergence to true value would take place much more slowly than in the case without fund managers, and the extent of slowdown in convergence would depend on the proportion of career concerned traders in the market. In addition, conformist trading by institutional traders would still occur in the presence of informed individual traders. Thus, our model would still predict information cascades by institutional traders, in keeping with the empirical literature on herding and momentum trading by institutions (for example, Sias [28] and [29]).

## 8 Conclusion

The central message of this paper is that the presence of (even small amounts of) reputational concerns will prevent institutional traders from trading sincerely when prices become close enough to liquidation value. Such a tendency to neglect valuable private information is an endogenous (and pervasive) obstacle to the convergence of prices to liquidation values in the long run, and can be the basis of herd-like behaviour by institutional traders, along the lines already documented in the empirical literature.

Further, we have argued that the presence of reputational incentives can

drive a wedge between the expected liquidation value of an asset and its total value to fund managers. We have presented an example of how such reputational premia can be incorporated into prices, thus leading to asset price bubbles. While we have carefully related our central results on conformist trading to existing theoretical explanations in the introduction, it remains for us to do the same for our explorations on asset price bubbles. In conclusion, therefore, we briefly relate our example of asset price bubbles to the existing theoretical literature on bubbles in financial markets.

While the classical no-trade arguments of Milgrom and Stokey [21] and Tirole [31] preclude bubbles in markets with asymmetric information and rational agents in general, a number of papers construct examples of bubbles while examining the role of higher order beliefs in asset pricing. Allen, Morris, and Postlewaite [5] build on the no-trade theorems to develop necessary conditions for the existence of bubbles, and provide examples of economies Morris, Postlewaite, and Shin [22] illustrate in which bubbles can exist. the connection between bubbles and higher order uncertainty. Prices are biased statistics of true value in the recent work of Allen, Morris, and Shin [6].There are also a large number of papers in which bubbles arise and persist because some traders are irrational (e.g. DeLong, Shleifer, Summers, and Waldmann [16], Shleifer and Vishny [27], Abreu and Brunnermeier [1], and Scheinkman and Xiong [26], amongst others).

More closely related to our work, a few papers construct examples of bubbles based on agency conflicts. Most notably, Allen and Gorton [4] develop a model in which prices can diverge from fundamentals due to churning by portfolio managers. In their model, bad fund managers buy bubble stocks at prices above their known liquidation value in the hope of reselling them before they die — at even higher prices — to other bad fund managers. Their behavior is the result of an option-like payoff structure under which profits are shared with managers but losses are not. Churning thus creates the possibility of short-term speculative profits. A related principal-agent conflict leads to bubbles in Allen and Gale [3]. In contrast to both of these papers, in our example, bubbles arise without option-like payoffs purely due to reputational concerns of financial traders.

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