Endogenous TFP and Cross-Country Income Differences

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Abstract

Influential works by Klenow & Rodriguez-Clare (1997), Hall and Jones (1999), and Parente & Prescott (2000), among others, have argued that most of the cross country differences in output per worker is explained by differences in total factor productivity (TFP). This conclusion, however, is obtained in a framework that explicitly or implicitly assumes TFP to be exogenous. We study whether this conclusion holds when TFP is endogenous. We device a general model that can accomodate diverse endogeous and exogenous TFP models in the literature such as Romer (1990), Jones (1995), and Klenow & Rodriguez-Clare (2004). We show analitically that allowing for TFP endogeneity always increases the role of factors in explaining cross-country income differences. We also find that unless the benefits to backwardness are implausibly large, the main conclusion of the studies above is overturned. Most of the cross-country differences in output per worker are explained by differences in savings rates and human capital.

1 Introduction

Consider the almost 36-fold difference in output per worker between the United States and Niger in 1988. According to Hall and Jones (1999), physical capital per worker was around 80 times larger in the US than in Niger, and human capital was around 3 times larger in the US. For standard physical and human capital shares, of 1/3 and 2/3 respectively, differences in these two factors alone can explain a 9-fold gap in output per worker. The remaining 4-fold of the gap must be attributed to differences in the efficiency in the use of those factors, or total factor productivity

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(TFP). These simple calculations indicate that the main reason why the US produces much more output per worker than Niger is because of its larger endowments of physical and human capital.

Hall and Jones (1999), however, reached a completely different conclusion. They argued that part of 80-fold difference in capital per worker is actually due to the 4-fold difference in TFPs. This is because if saving rates are about the same, a country with lower TFP produces less output and can accumulate less capital. Consequently, Hall and Jones impute part of the capital differences to TFP differences, following a methodology proposed by Klenow and Rodriguez-Clare (1997). Taking into account the direct effect of TFP on output, and the indirect effect of TFP on capital, Hall and Jones find that the 4-fold TFP gap accounts for an almost 8-fold difference in output per worker between the US and Niger.[1] The remaining 4.5-fold gap is explained by factors intensity. As a result, they conclude that the main reason for Niger's relative poverty is its low TFP. More generally, they conclude that most of the differences in the productivity of labor across countries is explained by TFP differences. A similar conclusion is obtained by Klenow and Rodriguez-Clare (1997), and Parente & Prescott (2000), among others, using a similar methodology. These papers call for theories of TFP, and have been instrumental in changing the focus of researchers and policymakers toward TFP and away from factors.

A shortcoming of the studies mentioned above is that they explicitly or implicitly assume TFP to be exogenous: their models do not allow for a feedback from factor endowments, and their determinants, to explain TFP differences. This feedback clearly occurs in models of endogenous growth, such as Romer (1990), models in which the amount of factors affects the extent of research and development activities and, consequently, the TFP level of the economy.

The objective of this paper is to study and quantify the sources of cross-country income differences within a framework that allows TFP to be endogenous. Specifically, we reevaluate the role of differences in saving rates and human capital as a source of income dispersion when factors of production affect TFP levels. For this purpose, we generalize the textbook Solow growth model to allow for endogenous TFP. The model is sufficiently flexible so that for certain parameter values

[1] To be precise, Hall and Jones find a 7.7 difference.

TFP is actually exogenous. More importantly, the model can also accommodate the basic mechanisms of other endogenous growth models such as Romer (1990), Jones (1995), and Klenow and Rodriguez-Clare (2004).

In our model, there are two types of externalities to TFP accumulation: first, there is a "catch-up" externality that depends on the distance of TFP from the frontier, and that captures the idea that lagging behind the technology frontier facilitates technological progress via adoption. This effect has been called "benefits to backwardness:" a more backward country would have a higher catch-up term. The second type is a standard positive research externality along the lines of the endogenous growth literature, e.g., Jones (1995). The strength of the catching-up term turns out to be *the* critical parameter. We show that our model with endogenous TFP can replicate the results of Hall and Jones (1999), and Klenow and Rodriguez-Clare (1997) when the benefits to backwardness are extremely large.

In addition, in our model technological progress is always costly: it occurs only if some resources are diverted to technological advancement. This view is supported by Keller (2004) and Lederman and Maloney (2003), who argue that there is no indication that technology diffusion is inevitable or automatic, but rather, domestic investments are needed. In this aspect we differ from Klenow and Rodriguez-Clare (2004), who allow for some free technology diffusion that does not require the investment of any resources.

We find that unless the benefits to backwardness are implausibly large, the main conclusion of the studies above is overturned: most of the cross country differences in output per worker is explained by differences in savings rates and human capital.

The remainder of the paper is organized as follows. Section 2 provides a review of the Solow model augmented with human capital and its implications for the decomposition of the sources of cross-country income differences. Section 3 is the core of the paper: it presents and characterizes a model of endogenous TFP. In addition, it explores how this model changes the variance decomposition of the world income distribution.

2 Models with Exogenous TFP

The neoclassical growth model has been the workhorse of most existing attempts to quantify the sources of cross-country levels of output per worker. Prominent examples of these attempts have found completely opposite conclusions: on the one hand Mankiw, Romer and Weil (1992) –MRW henceforth– found that 78% of the world income variance could be explained by differences in human capital and saving rates across countries. On the other hand, Klenow and Rodríguez-Clare (1997) –KR (1997) hereafter,– and Hall and Jones (1999) –HJ henceforth– found that productivity differences are the dominant source of the large world dispersion of output per worker, accounting for around 60% of the variance. The reason why conclusions differ in these studies can be traced back to the measurement of human capital: while MRW use only secondary schooling, KR (1997) use in addition primary and terciary schooling, as well as experience and schooling quality.

However, all studies cited above share the common feature of using a framework –namely the Solow model augmented with human capital– in which the growth rate of productivity is exogenous. More specifically, consider the following aggregate Cobb-Douglas production function

$$Y_t = K_t^{\alpha} \left(A_t H_t\right)^{1-\alpha} \equiv Z_t K_t^{\alpha} H_t^{1-\alpha}$$

where K is aggregate physical capital, H is aggregate human capital, A represents labor augmenting technological progress, Y aggregate output, and $Z \equiv A^{1-\alpha}$ is total factor productivity (TFP). In what follows, we use loosely the term TFP to refer both to A and Z. Aggregate human capital is defined as $H_t \equiv hL_t$, where L is the labor force, and h transforms L in efficiency units. Output per worker $y = Y/L$ is then given by

$$y_t = Z_t k_t^{\alpha} h^{1-\alpha}.$$

With TFP exogenous and k endogenous, differences in k across countries reflect differences in Z_t. Thus, to properly capture the contribution of TFP to world income variance, MRW, KR (1997) and HJ write output per worker as

$$y_t = Z_t^{1+\frac{\alpha}{1-\alpha}} \bullet \left(\kappa^{\frac{\alpha}{1-\alpha}} h\right) \equiv A_t \cdot X_t \tag{1}$$

where $\kappa \equiv K/Y$ is the capital-output ratio, and X represents factor intensity. KR (1997) and HJ argue that this transformation is appropriate because it attributes to Z variations in K generated by differences in Z, while it assigns to X variations in K not induced by Z. It is instructive to notice that the term $Z_t^{1+\frac{\alpha}{1-\alpha}}$ captures both the direct effect of TFP in output per worker Z_t, and the indirect effect through capital, $Z_t^{\alpha/(1-\alpha)}$. Since generally $\alpha = 1/3$, then $1 + \alpha/(1-\alpha) = 1.5$, which makes apparent that when TFP is assumed exogenous, its explanatory power of the magnitude of y increases. In other words, if TFP is actually endogenous, (1) assigns too much role to TFP.

Notice that equation (1) is appropriate to study the sources of cross-country variation in y because it provides a meaningful decomposition in the sense that A and X are determined by different parameters in the model. To see, this assume that the following variables are exogenous: saving rate s, population growth γ_L, the growth rate of technological progress γ_A, and average human capital h. From the law of motion of physical capital

$$K_{t+1} = (1-\delta)K_t + sY_t$$

it follows that

$$\kappa_t = \frac{s}{\gamma_{Kt} + \delta - 1}.$$

Since along the balanced-growth path the net growth rate of physical capital is $\gamma_K = \gamma_A \gamma_L$, then the capital-output ratio is given by

$$\kappa = \frac{s}{\gamma_A \gamma_L + \delta - 1}.$$

The preceding equation shows that X is only a function of s, γ_A, γ_L, δ, and h, and more importantly, does not depend on the level of productivity A. Thus, using equation (1) provides a clear separation of productivity levels A on the one hand, and saving rates s and human capital h on the other hand.

KR (1997) use equation (1) to perform a variance decomposition exercise in order to measure the contributions of X and A to world income dispersion. Specifically, the world variance of output per worker is given by

$$var(\ln y) = var(\ln A) + var(\ln X) + 2cov(\ln A, \ln X),$$

where the covariance term $cov(\ln A, \ln X)$ should be zero under the model hypothesis that A is exogenous. However, it turns out that in the data $cov(\ln A, \ln X)$ is not zero: in fact, the two covariance terms in the equation above account for around 30% of the total variance of log output, a nonnegligible magnitude. As KR (1997) acknowledge, this suggests that the productivity level A is actually endogenous. In order to correct for this omission of the model, KR (1997) propose to assign half of the covariance term to the contribution of X and the other half to A, so that the contributions of factors Φ_X and productivity Φ_A are given by

$$\Phi_X = \frac{var(\ln X) + cov(\ln A, \ln X)}{var(\ln y)}$$

$$\Phi_A = \frac{var(\ln A) + cov(\ln A, \ln X)}{var(\ln y)}.$$

Using the database from HJ for 1988, and setting $\alpha = 1/3$, we obtain $\Phi_X = 40\%$ and $\Phi_A = 60\%$, which is the same variance decomposition reported by KC for 1985.[2] In addition, the covariance term $2 \times cov(\ln A, \ln X)$ accounts for around 30% of the total variance of log output, a nonnegligible magnitude. A theory of TFP should explain this large covariance between factor intensity X and productivity A, and should also reveal the appropriate distribution of the covariance term to the contributions of X and A. In the remainder of the paper, we formulate a model to relax the

[2]Hall and Jones (1999) data set is available at http://emlab.berkeley.edu/users/chad/HallJones400.asc. The data set contains information on output, physical capital, and human capital for 126 countries for the year 1988.

assumption that productivity A is exogenous, and asses how this variance decomposition changes. In particular, we explore the extent to which differences in s and h across countries are able to account for differences in output per worker in a model with endogenous A.

3 A Model of Endogenous TFP

3.1 The model

Consider the following Solow model extended to incorporate TFP accumulation along the lines of Jones (1995) and Klenow and Rodriguez-Clare (2004) –KR (2004) hereafter. Microfoundations for the reduced-form model presented below have been provided by Romer (1990) and Jones (1995). This is a two-sector economy: one of the sectors produces output Y that can be used for either consumption or investment, while the other sector produces knowledge A (expands productivity). Output production is given by

$$Y_t = K_{Yt}^{\alpha} \left(A_t H_{Yt}\right)^{1-\alpha}, \tag{2}$$

where the subscript Y indicates the sector to which factors are assigned. As in the Solow model, the law of motion of physical capital is

$$K_{t+1} = (1-\delta) K_t + sY_t, \tag{3}$$

and the one of human capital is

$$H_{t+1} = \gamma_L H_t, \tag{4}$$

where as before, $H_t = hL_t$ and γ_L is exogenous population growth.

The second sector of the economy expands knowledge A according to

$$A_{t+1} - A_t = B_t K_{At}^{\theta} H_{At}^{1-\theta} L_{At}^{-\zeta}, \tag{5}$$

where

$$B_t = \lambda \left(\frac{A_t^*}{A_t} \right)^{\eta} A_t^{\sigma}. \tag{6}$$

Equations (5) and (6) describe how TFP is accumulated. Equation (5) states that TFP investments require capital K_{At}, and quality adjusted labor H_{At}. Romer (1990) and Jones (1995) do not include capital as an input in the production of knowledge. However, as Jones (1995) acknowledges, computers and other forms of capital play a complementary role in the discovery of knowledge, and so capital belongs in the R&D production function. Along the lines of the endogenous growth literature, (6) allows for the existence of externalities in the production of TFP which occurs when $\sigma \neq 0$. Moreover, the term $(A_t^*/A_t)^{\eta}$ captures international spillovers. Here, A_t^* is the technological frontier (which can be country specific, but it is assumed to be exogenous to the country). For $\eta > 0$, the the term $(A_t^*/A_t)^{\eta}$ captures the idea that lagging behind the technology frontier facilitates technological progress via adoption. $L_{At}^{-\zeta}$ is a term required to eliminate scale effects in levels if $\zeta = 1$. A natural benchmark assumption would be $\zeta = 0$.

Equation (5) also implies that technological progress is always costly: it only occurs if some resources are diverted to technological advancement (all inputs are necessary). This view is supported by Keller (2004) and Lederman and Maloney (2003), who argue that there is no indication that technology diffusion is inevitable or automatic, but rather, domestic investments are needed. In this aspect we differ from KR (2004), who allow for some free technology diffusion that does not require the investment of any resources.

Another aspect in which we differ from KR (2004) is the way in which the catch-up externality is specified. For our purpose, it is convenient to introduce parameter η in order to map our accounting results into the ones described in Section 2. In particular, as we discuss below, the standard accounting results obtained with the exogenous TFP model can be obtained in our framework by making $\eta \to \infty$. In contrast, KR (2004) follow Howitt's (2000) specification. In a way, our η plays a similar role to KR's (2004) ε, a parameter that determines the extent of free technological diffusion in their model.

Equation (6) can be written as

$$B_t = \lambda A_t^{*\eta} A_t^{\phi} \tag{7}$$

where $\phi \equiv \sigma - \eta$. As we show below, this parameter turns out to be critical: ϕ may be positive or negative, depending on whether the positive externalities associated to TFP accumulation outweight the negative externalities. The literature on endogenous growth, such as Romer (1990), stresses the positive externalities associated to having a large pool of ideas, or blueprints. On the other hand, the literature on technological diffusion stresses the negative externality associated with closing the technological gap, which makes technological progress more difficult. This negative effect has also been called the "benefits to backwardness."

Finally, the model is closed adding the following full-employment constraints:

$$L_{At} = \mu_L H_t; \quad L_{Yt} = (1 - \mu_L) H_t; \quad 0 < \mu_L < 1 \tag{8}$$

$$K_{At} = \mu_K K_t; \quad K_{Yt} = (1 - \mu_K) K_t; \quad 0 < \mu_K < 1. \tag{9}$$

which state that constant fractions of total labor and total capital are allocated to the production of goods and to the production of TFP.

Equations (2)-(9) describe a general model of capital and TFP accumulation that could incorporate versions of other models in the literature. For example, the standard Solow model with exogenous technological change is obtained making $\mu_L = \mu_K = 0$, $\phi = 1$ and $\eta = 0$. A barebone version of Romer's (1990) model of endogenous growth is obtained when $\theta = \eta = \zeta = \mu_K = 0$, and $\phi = \gamma_L = 1$. Finally, Jones (1995) model of endogenous growth is obtained making $\theta = \eta = \zeta = \mu_K = 0$, and $\phi < 1$, $\gamma_L > 1$. Thus, our model allows us to reproduce KR (1997) and HJ results, but more importantly, allows us to check the robustness of those results.

3.2 Characterization of the Equilibrium

In this section we characterize the dynamics of this model economy and propose a way to use it in order to disentangle the sources of world income dispersion under endogenous TFP. Using (2) and

(8), output per worker y_t can be written as

$$y_t = (1-\mu_L)(1-\mu_K)^{\frac{\alpha}{1-\alpha}} A_t \left(\kappa_t^{\frac{\alpha}{1-\alpha}} h\right). \tag{10}$$

Equation (10) is a version of the equation employed by KR (1997) and HJ, as given by (1). The most important difference between these two equations is that in (10) A_t is endogenous. Thus, we cannot stop here and use (10) for variance decomposition, but we need to go further and rewrite (10) in a way appropriate for this purpose.

We now characterize the dynamics of the model economy. Let the gross growth rate of any variable V be $\gamma_{Vt} = V_{t+1}/V_t$, and $g_{vt} \equiv \gamma_{Vt} - 1$ its corresponding net growth rate. The following proposition states the main result of the paper, a generalized version of equation (1) that allows for endogenous TFP.

Proposition 1 Assume $1-\phi-\theta \neq 0$. Then y_t satisfies

$$y_t = \widehat{A}_t \cdot \widehat{X}_t \tag{11}$$

where

$$\widehat{A}_t \equiv (1-\mu_L)(1-\mu_K)^{\frac{\alpha}{1-\alpha}} \left(\frac{\lambda A_t^{*\eta}}{g_{At}} \mu_K^{\theta} (1-\mu_K)^{\frac{\alpha}{1-\alpha}} (1-\mu_L)^{\theta} \mu_L^{1-\theta-\zeta} L_t^{1-\zeta}\right)^{\frac{1}{1-\phi-\theta}}, \tag{12}$$

and

$$\widehat{X}_t \equiv \kappa_t^{\frac{\alpha}{1-\alpha}\left(1+\frac{\theta/\alpha}{1-\phi-\theta}\right)} h^{1+\frac{1}{1-\phi-\theta}}. \tag{13}$$

Proof: According to equations (5) and (6)

$$g_{At} = \lambda A_t^{*\eta} A_t^{\phi-1} (\mu_K K_t)^{\theta} (\mu_L H_t)^{(1-\theta)} L_{At}^{-\zeta}, \tag{14}$$

and solving for A_t,

$$A_t = \left(\frac{\lambda A_t^{*\eta} \mu_L^{-\zeta} L_t^{-\zeta}}{g_{At}} \left(\mu_K K_t\right)^{\theta} \left(\mu_L H_t\right)^{(1-\theta)} \right)^{\frac{1}{1-\phi}}. \tag{15}$$

On the other hand, (10) implies that

$$K_t = \kappa_t Y_t = \kappa_t \left(1-\mu_L\right)\left(1-\mu_K\right)^{\frac{\alpha}{1-\alpha}} \kappa_t^{\frac{\alpha}{1-\alpha}} A_t H_t,$$

or

$$K_t = \left(1-\mu_L\right)\left(1-\mu_K\right)^{\frac{\alpha}{1-\alpha}} \kappa_t^{\frac{1}{1-\alpha}} A_t H_t.$$

Substituting this result into (15), and solving for A_t we obtain

$$A_t = - {}_t \kappa_t^{\frac{1}{1-\alpha}\frac{\theta}{1-\phi-\theta}} h^{\frac{1}{1-\phi-\theta}}, \tag{16}$$

where

$$- {}_t = \left(\frac{\lambda A_t^{*\eta}}{g_{At}} \mu_K^{\theta} \left(1-\mu_K\right)^{\frac{\theta\alpha}{1-\alpha}} \left(1-\mu_L\right)^{\theta} \mu_L^{1-\theta-\zeta} L_t^{1-\zeta} \right)^{\frac{1}{1-\phi-\theta}}.$$

Equation (16) makes clear that TFP levels depend on the factor endowments of the economy. Substituting (16) into (10) one obtains the desired result.

It is important to notice that according to equation (13), if $\phi + \theta < 1$ –as we show below must be the case along the balanced growth path– endogenizing TFP increases the role of factors (saving rates and human capital) in explaining world income dispersion. In particular, comparing the factor intensity terms $\widehat{X}$ in (13) and X in (1), it is easy to see that the exponents of both the capital-output ratio κ and average human capital h are larger in (13), the model with endogenous TFP. In other words, equation (13) implies that cross-country differences in human capital and capital-output ratios are now able to produce larger output differences. This result arises because the production function for output becomes in fact increasing returns to scale in factors when TFP is endogenous, even though it is constant returns to scale for a given TFP level.

A second important point to notice is that as the catching up parameter goes to infinity, i.e. as $\eta \to \infty$, then $\phi \to -\infty$ and $\widehat{X} \to X$. In other words, the variance decomposition of KR (1997) and HJ in equation (1) can be replicated in our model by making $\eta \to \infty$, which can be interpreted as extremely high (infinite) TFP investment costs. Thus, as $\eta \to \infty$ factors of production lose explanatory power. The reason is that when the negative externalities associated to TFP accumulation increase, then the technological progress brought about by additional factors is detrimental for additional technological progress. Closing the technology gap in this case is not beneficial because future losses outweight current gains.

A final point to notice is that while the factor intensity term X in equation (1) is analogous to $\widehat{X}$ in equation (11), this is not the case for the respective residuals A and $\widehat{A}$. These two terms are not analogous because A in (1) is exogenous and corresponds to $TFP^{\frac{1}{1-\alpha}}$, while there is no simple mapping between TFP and $\widehat{A}$ in (11) because our model endogenizes TFP. This implies that when performing the variance decomposition of output per worker using (1) and (11), we can only compare the contributions of X and $\widehat{X}$. In fact, (11) does not deliver a measure of the contribution of TFP to output dispersion; instead, it delivers a measure of the contribution of factors (κ and h) versus other determinants of output per worker (e.g., μ_L, μ_K, λ, A^*, and L).

At this point, we restrict the parameters of the model to preclude scale effects, in the sense of Jones (1995), from arising in our model. This is probably the most relevant case because scale effects are hard to defend empirically. KR (2004) also employ this assumption. Consider the case in which the scale economy, either in terms of population, capital, or output, does not affect the long term growth rate of the economy. It turns out that the following parameter restriction eliminates this type of scale effect from the model.

Assumption 1. $\phi + \theta < 1$.

Under Assumption 1, balanced growth is consistent with an increasing amount of K and H devoted to the production of knowledge. To see the origin of Assumption 1, consider the evolution of this economy along a balanced growth path. It is straightforward to check that along a balanced growth path $\gamma_K = \gamma_A \gamma_L$, i.e., K_t is a constant function of A_t and L_t. Also, since h is constant and exogenous, H_t will be a constant function of L_t. Using these results in equation (14) we can write g_{At} as some constant function s of the following level variables

$$g_{At} = s\left(A_t^{*\eta}, A_t^{\phi-1+\theta}, L_t^{1-\zeta}\right).$$

Since A_t^* and L_t grow at some exogenous rates, in order to avoid scale effects in the growth rate of the economy g_{At}, we need $\phi - 1 + \theta < 0$, which corresponds to Assumption 1.

Now, along the balance growth path g_{At} is constant so that, according to equation (14)

$$\frac{g_{At+1}}{g_{At}} = 1 = \gamma_{A^*}^{\eta}\gamma_A^{\phi-1}\left(\gamma_K\right)^{\theta}\left(\gamma_L\right)^{(1-\theta)}\gamma_L^{-\zeta}.$$

Using the fact that $\gamma_K = \gamma_A\gamma_L$, it follows that the long term growth rate of the economy is given by

$$\gamma_A = \left(\gamma_{A^*}^{\eta}\gamma_L^{1-\zeta}\right)^{\frac{1}{1-\theta-\phi}}. \tag{17}$$

This is analogous to the result in Jones (1995), but for a model with capital in the R&D sector (i.e., $\theta \neq 0$) and technological diffusion. The long term growth rate of the economy is thus scale free. Moreover, one would like to restrict $\gamma_{A^*} = \gamma_A$ to preclude that along the balance growth path, the growth rate of the economy is larger (or smaller) than the growth rate of the technology frontier. Since we have assumed that the frontier A^* and its growth rate γ_{A^*} are exogenous to the country, the only way to guarantee $\gamma_{A^*} = \gamma_A$ is to impose further restrictions in the parameters, as summarized by Assumption 2.

Assumption 2. $\zeta = 1$ and $\sigma = 1 - \theta$.

Finally, equation (17) together with (3) imply that along the balanced growth path

$$\kappa = \frac{s}{\gamma_{A^*}^{\frac{\eta}{1-\theta-\phi}}\gamma_L^{\frac{2-\theta-\phi-\varsigma}{1-\theta-\phi}} + \delta - 1}.$$

3.3 Variance Decomposition

Our model has the advantage of being parsimonious. It turns out that in order to use equation (11) to perform a variance decomposition of output per worker only a few parameters are needed. Specifically, using (11) and (13) we obtain

$$y_t = \widehat{A}_t \widehat{X}_t = \widehat{A}_t \kappa_t^{\frac{\alpha}{1-\alpha}\left(1+\frac{\theta/\alpha}{1-\phi-\theta}\right)} h^{1+\frac{1}{1-\phi-\theta}} \tag{18}$$

where y, κ and h are observable (and taken from HJ data set for 1988), and by choosing α, θ and ϕ, we can construct $\widehat{A}$ as a residual.

A natural choice for the capital share in the production of knowledge θ is to set $\theta = \alpha = 1/3$, so that the production of knowledge and output are equally capital intensive. We show below how results change for different values of θ. The only parameter left for calibration is ϕ. Equation (18) makes clear that the critical parameter for the variance decomposition exercise is $\phi \equiv \sigma - \eta$, and in particular η. Recall from Assumption 2 that $\sigma = 1 - \theta = 2/3$. As discussed above, if $\eta \to \infty$ then $\phi \to -\infty$ and equation (18) collapses to $y_t = \widehat{A}_t \kappa_t^{\frac{\alpha}{1-\alpha}} h$, which would deliver the same variance decomposition as in the model with exogenous TFP. In contrast, as ϕ converges from below to $1 - \theta$, which is the bound consistent with Assumption 1, then the explanatory power of the saving rate (as captured in κ) and human capital increases.

We now provide some criteria to pin down sensible values of η. Assumptions 1 and 2 together imply that $\eta \in (0, \infty)$. We show that for all sensible values of η, the variance decomposition of KR (1997) and HJ is reversed. Specifically, we obtain results more in line with MRW: the main sources of cross-country income differences are differences in saving rates and human capital.

In order to pin down η, consider first the case in which $\phi > 0$, so that the positive externalities from the production of knowledge σ exceed the negative (catching up) externalities η. This case of overall positive externalities is the one suggested by Romer (1990) and other follower papers. Under the restriction imposed by Assumption 1, we can consider values of ϕ in the range $(0, 1 - \theta)$. Since by Assumption 2 $\sigma = 1 - \theta$, this corresponds to $\eta \in (0, 1 - \theta)$. Table 1 presents the variance decomposition of our model under this case. As the table shows, for these values of η factor intensities (s and h) can explain all of the world income differences, and even more. For instance, when $\eta = 0.62$, then $\widehat{X}$ accounts for 105% of the total variance of (log) output per worker. Thus, if any component of $\widehat{A}$ is to play a role in explaining income differences, we would need to have $\eta > \sigma$ and $\phi \leq 0$.

As a second criteria to pin down η, consider next $\phi = 0$ (i.e., $\eta = \sigma = 1 - \theta = 2/3$). This a case in which the positive and negative externalities derived from the production of knowledge cancel out. Jones (1995) suggest this is a natural benchmark to consider. Table 1 shows that in this case factor intensities explain 100% of world income variance.

A third calibration criteria we use is to estimate the value of η by requiring the model to replicate the observed world distribution of A. Suppose that the technology frontier for all countries is the same, and that the US is at its technology frontier, i.e., suppose $A^*_{it} = A^*_{USAt}$ for any country i, and $A_{USAt} = A^*_{USAt}$. Then, under Assumption 2 and denoting $k_{iA} = K_{iA}/H_{iA}$, equation (14) implies that for any country i,

$$g_{i,At} = \lambda A^{*\eta}_{USAt} A^{\sigma-\eta-1}_{it} k^{\theta}_{iA} h_i.$$

Now, for $\alpha = \theta$, we have $k_{iA} = k_i = K_i/H_i$, so that the equation above can be written

$$g_{i,At} = \lambda \left(\frac{A^*_{USAt}}{A_{it}} \right)^{\eta} A^{-\theta}_{it} k^{\theta}_i h_i,$$

and thus,

$$\frac{g_{i,At}}{g_{USA,At}} = \frac{\left(A^*_{USAt}/A_{it}\right)^{\eta} A^{-\theta}_{it} k^{\theta}_i h_i}{A^{-\theta}_{USAt} k^{\theta}_{USA} h_{USA}}.$$

Since along the balanced growth path $g_{iAt} = g_{At}$ for all i, then

$$\frac{A_{it}}{A_{USAt}} = \left(\left(\frac{k_i}{k_{USA}} \right)^{\theta} \frac{h_i}{h_{USA}} \right)^{\frac{1}{\eta+\theta}},$$

where the coefficient $1/(\eta + \theta)$ can be backed-out from the data by computing a simple correlation. Such computation for our sample of countries in 1988 delivers $1/(\eta + \theta) = 0.73$, and with $\theta = 0.33$

we have an implied $\eta = 1.04$. As shown in Table 1, this value of η, which approximately implies $\phi = -\alpha$, assigns 78% of the variance of world output per worker to differences in saving rates and human capital. Interestingly, this is the same contribution to variance found by MRW, although under a different model and with other assumptions.

A fourth criteria to pin down the value of η is by matching the correlation between A and X in equation (1), i.e., the correlation between TFP (or more precisely $Z^{1/(1-\alpha)}$) and the factor intensity component as defined by KR (1997). This yields $\eta = 2$, and $\phi = -1.33$, i.e., $\phi = -(1+\alpha)$. Table 1 reports the variance decomposition for this case. Again, the main source of cross-country income differences are differences in saving rates and human capital, which account for 60% of the variance.

To summarize, all sensible values obtained for η deliver the same robust message: most of the world income differences are explained by differences in saving rates and human capital.

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Development Accounting

	X	A	2cov(lnX,LnA)/Var(lny)
Exogenous TFP	40%	60%	30%
ϕ=.05	105%	-5%	-72%
ϕ=.0	100%	0%	-61%
ϕ=-.33	80%	20%	-23%
ϕ=-.1.33	60%	40%	2%

Data: Hall and Jones (1999)