Unemployment Fluctuations with Staggered Nash Wage Bargaining

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Abstract

Shimer (2005) and Hall (2005) have recently emphasized that the conventional model of unemployment dynamics due to Mortensen-Pissarides(1994) has difficulty acccounting for the relatively smooth behavior of wages and volatile behavior of employment over the business cycle. We address this issue by modifying the MP framework to allow for staggered multi-period wage contracting. What emerges is a tractable relation for wage dynamics that is a natural generalization of the period-by-period Nash bargaining outcome in the conventional formulation. An interesting side-product is the emergence of spillover effects of average wages on the bargaining process. We then show that reasonable calibration of the model can account for the reasonably well for the cyclical behavior of wages and labor market activity observed in the data. The spillover effects turn out to be important in this respect.

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1   Introduction

A long-standing challenge in macroeconomics is accounting for the relatively smooth behavior of real wages over the business cycle along with the relatively volatile behavior of employment. A recent body of research, beginning with Shimer (2005) and Hall (2005) has re-ignited interest in addressing this challenge. These authors show, among other things, that the conventional model of unemployment dynamics due to Mortensen and Pissarides (hereafter "MP") cannot account for the key cyclical movements in labor market activity. The basic problem is that the mechanism for wage determinism within this framework, period-by-period Nash bargaining between firms and workers, induces too much volatility in wages. This exaggerated procyclical movement in wages, in turn, dampens the cyclical movement in firms' incentives to hire. The authors proceed to show that with the introduction of ad hoc wage stickiness, the framework can account for employment volatility. Of course, this begs the question of what are the primitive forces that might underlies this wage rigidity.

A rapidly growing literature has emerged to take on this puzzle. Much of this work attempts to provide an axiomatic foundation for wage rigidity, explicitly building up from assumptions about the information structure, and so on. To date, due to complexity, this work has focused mainly on qualitative findings and has addressed quantitative issues only in a limited way.

In this paper we take a pragmatic approach to modelling wage rigidity, with the aim of developing a framework that is tractable for empirical analysis. In particular, we retain the empirically appealing feature of Nash bargaining, but modify the conventional MP model to allow for staggered multi-period wage contracting. Each period, only a subset of firms and workers negotiate a wage contract. Each wage bargain, further, is between a firm and it's existing workforce: Workers hired in-between contract settlements receive the existing wage. We restrict the form of the wage contract to call for a fixed wage per period over an exogenously given horizon. Though it would be undoubtedly preferable to completely endogenize the contract structure, these restrictions are reasonable from an empirical standpoint. The payoff is a simple empirically appealing wage equation that is an intuitive generalization of the standard Nash bargaining outcome. The gain over a simple ad hoc wage adjustment mechanism is that the key primitive parameter of the model is the average frequency of wage adjustment, as opposed to an arbitrary partial adjustment coefficient in a wage equation. In this way, the staggered contracting structure provides more discipline in evaluating the model than do simple ad hoc adjustment mechanisms.

The use of time dependent staggered price and wage setting, of course, is widespread in macroeconomic modelling, beginning with Taylor (1980) and Calvo (1983). More recently, Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2004) have found that staggered wage contracting is critical to the empirical performance of the recent vintage of dynamic general equilibrium (DGE) macro frameworks (i.e., sticky prices alone are not sufficient). There are, however, some important distinguishing features of our approach. First, macro models with staggered wage setting typically have employment adjusting along the intensive margin. That is, wage stickiness enhances

fluctuations in hours worked as opposed to total employment. As a consequence, these frameworks are susceptible to Barro's (1977) argument that wages may not be allocational in this kind of environment, given that firm's and workers have an on-going relationship. If wages are not allocational, of course, then wage rigidity does not influence model dynamics. By contrast, in the model we present, wages affect employment at the extensive margin: They influence the rate at which firms add new workers to their respective labor forces. As emphasized by Hall (1977), in this kind of setting the Barro's critique does not apply.

A second key difference involves the nature of the wage contracting process. In the conventional macro models, monopolistically competitive workers set wages. Here, firms and workers bargain over wages in a setting with search and matching frictions. As a consequence, some interesting "spillover" effects emerge of the average market wage on the contract wage. These spillover effects are a product of the staggered contract/bargaining environment. They introduce additional stickiness in the movement of real wages, much the same way that real rigidities introduce nominal price stickiness in models of price setting.

In section 2 we present the model. In section 3 we characterize the basic features of the model, including a set of simple dynamic equations for wages and the hiring rate, obtained by considering a local approximation of the model about the steady state. We also exposit the spillover effects that influence the wage bargaining process, contributing to overall wage stickiness. In section 4 we examine the empirical performance of the model and show that the framework does a good job of accounting for the basic features of the model, including wage dynamics. Concluding remarks are in section 5. Finally, the appendix provides an explicit derivation of all the key results.

2   The Model

The framework is a variation of the Mortensen-Pissarides search and matching model (Mortensen and Pissarides 1994, Pissarides 2000). The main difference is that we allow for staggered multiperiod wage contracting. Within the standard framework, workers and firms negotiate wages based on period-by-period Nash bargaining. We keep the Nash bargaining framework, but in the spirit of Taylor (1980), only a fraction of firms and workers re-set wages in any given period. As well, they strike a bargain that lasts for multiple periods. Workers hired in between contracting periods receive the existing contract wage

For technical reasons, there are two other differences from MP. First, because it will turn out to be important for us to distinguish between existing and newly hired workers at a firm, we drop the assumption of one worker per firm and instead allow firms to hire a continuum of workers. We assume constant returns to scale, however, which greatly simplifies the bargaining problem. Second, we drop the conventional assumption of a fixed cost per vacancy opened and instead assume that firms face quadratic adjustment costs of adjusting employment size. The reason is as follows: With staggered wage setting, there will arise a dispersion of wages across firms in equilibrium. Quadratic

costs of adjusting employment ensure a determinate equilibrium in the presence of wage dispersion.

Finally, we embed our search and matching framework within a simple intertemporal general equilibrium framework in order to study the dynamics of unemployment and wages. Following Merz (1995),we adopt the representative family construct, which effectively involves introducing complete consumption insurance.

2.1 Unemployment, Vacancies and Matching

Let us now be more precise about the details: We index firms by $i \in [0,1]$. Each firm i employs $n_t(i)$ workers at time t. It also posts $v_t(i)$ vacancies in order to attract new workers for the next period of operation. .The total number of vacancies and employed workers are $v_t = \int_0^1 v_t(i)di$ and $n_t = \int_0^1 n_t(i)di$. The total number of unemployed workers is u_t is given by

$$u_t = 1 - n_t \tag{1}$$

Following convention, we assume that the number of new hires or "matches", m_t, is a function of unemployed workers and vacancies, as follows:

$$m_t = \sigma_m u_t^\sigma v_t^{1-\sigma} \tag{2}$$

The probability a firm fills a vacancy in period t, q_t, is given by

$$q_t = m_t/v_t \tag{3}$$

Similarly, the probability an unemployed worker finds a job, s_t, is given by

$$s_t = m_t/u_t. \tag{4}$$

Both firms and workers take q_t and s_t as given. Finally, each firm exogenously separates from a fraction $(1 - \rho)$ of its workers each period where, ρ is the probability a worker "survives" with the firm until the next period.

2.2 Firms

Each period, firms produce output using capital and labor. To make the bargaining problem between firms and workers meaningful, we introduce costs of adjusting employment. As we noted earlier, because we will have wage dispersion across firms, we introduce quadratic labor adjustment costs, instead of the usual assumption of proportional hiring costs. For simplicity, we assume capital is perfectly mobile across firms and that there is a competitive rental market in capital.

Let $F_t(i)$ denote the value of firm i, $y_t(i)$ output, $k_t(i)$ the capital stock, $w_t(i)$ the wage rate, z_t the rental rate of capital, a_t economy-wide productivity and $x_t(i)$ the hiring rate, specifically

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the percent change in the firm's workforce from t to $t + 1$. In addition, let $\beta E_t \Lambda_{t,t+1}$ be the firm's discount rate, where the parameter β is the household's subjective discount factor. Given quadratic costs of adjusting the workforce:

$$F_t(i) = y_t(i) - w_t(i) n_t(i) - \frac{\kappa}{2} x_t(i)^2 n_t(i) - z_t k_t(i) + \beta E_t \Lambda_{t,t+1} F_{t+1}(i) \tag{5}$$

with

$$y_t(i) = a_t k_t(i)^\alpha n_t(i)^{1-\alpha} \tag{6}$$

and

$$n_{t+1}(i) = \rho n_t(i) + q_t v_t(i) \tag{7}$$

where the product of the firm's probability of filling a vacancy, q_t and the number of vacancies it posts equals the number of new hires. The total workforce at $t + 1$ is then the sum of the number of surviving workers, $\rho n_t(i)$, and new hires $q_t v_t(i)$.

In turn, the hiring rate is simply given by the ratio of new hires to the existing work force:

$$x_t(i) = \frac{q_t v_t(i)}{n_t(i)} \tag{8}$$

Note that the firm knows the hiring rate with certainty at time t, since it knows that likelihood q_t that each vacancy it posts will be filled.

At any time, the firm chooses the hiring rate (by posting vacancies) and its capital stock, given its existing employment stock, $n_t(i)$, the probability of filling a vacancy, the rental rate on capital and the current and expected path of wages. If it is a firm that is able to renegotiate the wage, it bargains with its workforce over a new contract. If it is not renegotiating, it takes as given the wage at the previous period's level, as well the likelihood it will be renegotiating in the future.

We next consider the firm's hiring and capital rental decisions, and defer a bit the description of the wage bargain. Let $J_t(i)$ be the value to the firm of adding another worker at time t:

$$J_t(i) = (1 - \alpha)\frac{y_t(i)}{n_t(i)} - w_t(i) + \frac{\kappa}{2} x_t(i)^2 + \rho \beta E_t \Lambda_{t,t+1} J_{t+1}(i) \tag{9}$$

Then the first order condition for vacancy posting equates the marginal cost of adding a worker with the discounted marginal benefit:

$$\kappa x_t(i) = \beta E_t \Lambda_{t,t+1} J_{t+1}(i) \tag{10}$$

In turn, the first order condition for capital is simply:

$$z_t = \alpha\frac{y_t(i)}{k_t(i)} = \alpha\frac{y_t}{k_t} \tag{11}$$

With Cobb-Douglas production and perfectly mobile capital, output/capital ratios are equalized across firms. It follows that capital/labor ratios and output/labor ratios are also equalized.

Let f_{nt} denote the firm's marginal product of labor at t (i.e., $f_{nt} = (1-\alpha)y_t/n_t$). Then combining equations yields the following following forward looking difference equation for the hiring rate

$$x_t(i) = \frac{1}{\kappa}\beta E_t \Lambda_{t,t+1}\left[f_{nt+1} - w_{t+1}(i) + \frac{\kappa}{2}x_{t+1}(i)^2 + \rho\kappa x_{t+1}(i)\right] \tag{12}$$

The hiring rate thus depends on a discounted stream of the firm's expected future surplus from the marginal worker: the sum of net earnings at the margin $f_{nt+1} - w_{t+1}(i)$ and saving on adjustment costs $\frac{\kappa}{2}x_{t+1}(i)^2$.

2.3 Workers

Let $V_t(i)$ be the value to a worker of employment at firm i and let U_t be the value of unemployment. $V_t(i)$, is given by

$$V_t(i) = w_t(i) + \beta E_t \Lambda_{t,t+1}\left[\rho V_{t+1}(i) + (1-\rho)U_{t+1}\right] \tag{13}$$

Note that this value depends on the wage specific to firm i, $w_t(i)$, as well as the likelihood the worker will remain employed in the subsequent period. The average value of employment, V_t, which depends on the average wage w_t, is:

$$V_t = w_t + \beta E_t \Lambda_{t,t+1}\left[\rho V_{t+1} + (1-\rho)U_{t+1}\right] \tag{14}$$

In turn, the value of unemployment is given by

$$U_t = b + \beta E_t \Lambda_{t,t+1}\left[s_t V_{t+1} + (1-s_t)U_{t+1}\right] \tag{15}$$

where b is the worker's outside option, taken to be unemployment benefits, and s_t is the probability of finding a job for the subsequent period. Here we assume that the value of finding a job next period simply corresponds to the average value of working next period across firms. That is, unemployed workers do not have a priori knowledge of which firms might be paying higher wages. They instead just randomly flock to firms posting vacancies.

The worker's surplus at firm i, $H_t(i)$, and average worker surplus, H_t, are given by, respectively:

$$H_t(i) = V_t(i) - U_t \tag{16}$$

$$H_t = V_t - U_t \tag{17}$$

It follows that:

$$H_t(i) = w_t(i) - b + \beta E_t \Lambda_{t,t+1}\left(\rho H_{t+1}(i) - s_t H_{t+1}\right) \tag{18}$$

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2.4 Consumption and Saving

Following Merz and others, we use the representative family construct, which gives rise to perfect consumption insurance. In particular, the family has employed workers at all firms and unemployed workers pool their incomes before choosing per capita consumption and asset holdings. In addition to wage income and unemployment benefits, the family has a diversified ownership stake in firms, which pay out profits Π_t. Households also receive government transfers, TR_t, which may be negative. They may either consume, c_t, or save in the form of capital, which may rent to firms at the rate z_t. Let Ω_t be the value function for the representative household. Then the maximization problem may be expressed as

$$\Omega_t = \max_{\{c_t, d_{t+1}, k_{t+1}\}} \left[u\left(c_t\right) + \beta E_t \Omega_{t+1} \right] \tag{19}$$

subject to

$$c_t + k_{t+1} = w_t n_t + (1 - n_t) b + (z_t + 1 - \delta) k_t + \Pi_t + TR_t$$

Let $\lambda_t \equiv u'(c_t)$. Then the first necessary conditions for consumption/saving yields:

$$\lambda_t = \beta E_t \lambda_{t+1} \left(z_{t+1} + 1 - \delta\right) \tag{20}$$

2.5 Nash Bargaining and Wage Dynamics

We restrict the form of the wage contract to call for a fixed wage per period over an exogenously given length of time. Though it would be undoubtedly preferable to completely endogenize the contract structure, these restrictions are reasonable from an empirical standpoint. The payoff will be a simple empirically appealing wage equation that is an intuitive generalization of the standard Nash bargaining outcome. In particular, given these restrictions on the form of the contract, workers and firms determine the contract wage through Nash bargaining.

We introduce staggered multi-period wage contracting in a way that simplifies aggregation. In particular, each period a firm has a fixed probability $1 - \lambda$ that it may re-negotiate the wage. This adjustment probability is independent of its history. Thus while how long an individual wage contract lasts is uncertain, the average duration is fixed at $1/(1 - \lambda)$. The coefficient λ is thus a measure of the degree of wage stickiness that can be calibrated to match the data. The simple Poisson adjustment process, further, implies that it is not necessary to keep track of individual firms' wage histories, which makes aggregation simple. In the end, the model will deliver a simple relation for the evolution of wages that is the product of Nash bargaining in conjunction with staggered wage setting.

Firms that enter a new wage agreement at t negotiate with the existing workforce, including the recent new hires. Due to constant returns, all workers are the same at the margin. The wage is chosen so that the negotiating firm and the marginal worker share the surplus from the marginal

match. Given the symmetry to which we just alluded, all workers employed at the firm receive the same newly-negotiated wage. When firms are not allowed to renegotiate the wage, all existing and newly hired workers employed at the firm receive the wage paid the previous period. In the benchmark case where the contract length corresponds to just one period, wage dynamics are just as in the standard model (and behave counterfactually as recently argued.)

Let w_t^* denote the wage of a firm that renegotiates at t. Given constant returns, all sets of renegotiating firms and workers at time t face the same problem, and thus set the same wage. As we noted earlier, the firm negotiate with the marginal worker over the surplus from the marginal match. We assume Nash bargaining, which implies that the wage w_t^* is chosen to solve

$$\max H_t(r)^\eta J_t(r)^{1-\eta} \tag{21}$$

where $H_t(r)$ and $J_t(r)$ are the value of J and H for renegotiating firms.

The appendix shows that for renegotiating firms and workers we can write

$$J_t(r) = E_t \sum_{s=0}^{\infty} (\rho\beta) \Lambda_{t,t+s} \left[f_{nt+s} + \frac{\kappa}{2} x_{t+s}(r)^2 \right] - W_t(r) \tag{22}$$

$$H_t(r) = W_t(r) - E_t \sum_{s=0}^{\infty} (\rho\beta) \Lambda_{t,t+s} [b + s_{t+s}\beta\Lambda_{t+s,t+s+1}H_{t+s+1}] \tag{23}$$

where $W_t(r)$ denotes the sum of expected future wages, given by

$$W_t(r) = \Delta_t w_t^* + E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \left[(1-\lambda)(\rho\beta)\Lambda_{t+s,t+s+1}\Delta_{t+s+1}w_{t+s+1}^* \right], \tag{24}$$

with

$$\Delta_t = \sum_{s=0}^{\infty} (\rho\beta\lambda)^s \Lambda_{t,t+s}. \tag{25}$$

Intuitively, the firm's surplus from the marginal worker, $J_t(r)$, is discounted earnings, f_{nt+s}, plus savings on adjustment costs, $\frac{\kappa}{2}x_{t+s}(r)^2$, net expected wage payments, $W_t(r)$. Note that the latter takes into account the expected life of the current wage contract as well as expected renegotiations that will take place in the future. In turn, the marginal worker's surplus, $H_t(r)$, depends on the expected discounted value of wage payments over both the existing contract and subsequent contracts, net the discounted sum of flow value of unemployment, $b + s_{t+s}\beta\Lambda_{t+s,t+s+1}H_{t+s+1}$.

The solution to the Nash bargaining problem, then, is

$$\eta J_t(r) = (1-\eta) H_t(r) \tag{26}$$

As the appendix shows, combining equations yields the following first order forward looking difference equation for the contract wage:

$$\Delta_t w_t^* = w_t^{tar}(r) + \rho\lambda\beta E_t\Lambda_{t,t+1}\Delta_{t+1}w_{t+1}^* \tag{27}$$

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where the forcing variable $w_t^{tar}(r)$ can be thought of as the "target" wage and is given by

$$w_t^{tar}(r) = \eta\left(f_{nt} + \frac{\kappa}{2}x_t(r)^2\right) + (1-\eta)\left(b + s_t\beta\Lambda_{t,t+1}E_tH_{t+1}\right) \tag{28}$$

Observe that the target wage has the same form as the wage that would emerge under period-by-period Nash bargaining. In particular, it is a convex combination of the firm's surplus from the match and the value of the worker's outside option, where the weight depends on worker's relative bargaining power η. The firm's surplus is the sum of the worker's marginal product of labor and the saving on adjustment costs. With our quadratic cost formulation, this saving is measured by $\frac{\kappa}{2}x_t(r)^2$. The value of the worker's outside option in turn is the discounted sum of the flow value of unemployment, $b + s_{t+s}\beta\Lambda_{t+s,t+s+1}H_{t+s+1}$.

As in the conventional literature on time-dependent wage and price contracting (e.g. Taylor (1980), Calvo (1983)), the contract wage depends on an expected discounted sum of the target under perfectly flexible adjustment, in this case $w_t^{tar}(r)$. Iterating equation (27) yields

$$w_t^* = E_t\sum_{s=0}^{\infty}(\rho\lambda\beta)^s E_t\Lambda_{t+s,t+s+1}\frac{\Delta_{t+s+1}}{\Delta_t}w_{t+s}^{tar}(r) \tag{29}$$

Observe that in the limiting case of period by period wage negotiations, i.e., when $\lambda = 0$, w_t^* converges to $w_t^{tar}(r)$.

A significant difference from the traditional literature on wage contracting, however, is that spillover effects emerge directly from the bargaining problem that have the contract wage depend positively on the economy-wide average wage. We defer momentarily a characterization of these spillover effects. Finally, given that all firms that renegotiate at t choose the same contract wage w_t^* and given that the average wage of firms that do not renegotiate is simply last periods aggregate wage (since they are a random draw from the population), the aggregate wage is given by

$$w_t = (1-\lambda)w_t^* + \lambda w_{t-1} \tag{30}$$

2.6 Resource Constraints

We complete the model with the following resource constraint, which divides output between consumption and adjustment costs.

$$y_t = c_t + k_{t+1} - (1-\delta)k_t + \frac{\kappa}{2}x_t^2 n_t \tag{31}$$

This completes the description of the model.

3   Wage/Hiring Dynamics and Spillover Effects

To gain some intuition for the model, we next derive loglinear equations for wages and hiring. In doing so, we identify spillover effects that make the wage bargain sensitive to the average wage economy-wide in a way that works to enhance wage rigidity.

We begin by deriving an expression for the target wage, $w_t^{tar}(r)$, the forcing variable in the difference equation for wages. In particular, by making use of the definitions of H_t and $H_t(r)$, along with the Nash bargaining condition and the vacancy posting condition, the component of the flow value of unemployment that stems from the discounted surplus conditional on new employment next period, $\beta E_t \Lambda_{t,t+1} H_{t+1}$, may be expressed as (see the appendix)

$$\beta E_t \Lambda_{t,t+1} H_{t+1} = \frac{\eta}{1-\eta} \kappa x_t(r) + \lambda \beta E_t \Lambda_{t,t+1} \Delta_{t+1}(w_t - w_t^*) \tag{32}$$

Intuitively, the surplus value of moving from unemployment this period to employment next period is likely to be high next period if the hiring rate is high today (implying a high marginal product of labor tomorrow) and if economy-wide wages are high relative to the current contract wage (since the new job is likely to offer a more attractive wage relative to the existing wage).

The presence of the wage gap, $w_t - w_t^*$, in the expression for $\beta E_t \Lambda_{t,t+1} H_{t+1}$ introduces a direct spillover effect of economy-wide wages on $w_t^{tar}(r)$. This can be seen by combining equations (32) and (28):

$$w_t^{tar}(r) = w_t^{flex} + \eta[\frac{\kappa}{2}(x_t(r)^2 - x_t^2) + s_t \kappa(x_t(r) - x_t)] + (1-\eta)[s_t \lambda \beta E_t \Lambda_{t,t+1} \Delta_{t+1}(w_t - w_t^*)] \tag{33}$$

where w_t^{flex} is the wage that would arise under perfectly flexible wage adjustment economy wide, i.e., it is the wage that would arise from period-by-period Nash bargaining economy-wide and is given by

$$\begin{aligned} w_t^{flex} &= \eta(f_{nt} + \frac{\kappa}{2} x_t^2) + (1-\eta)(b + \frac{\eta}{1-\eta} \kappa s_t x_t) \\ &= \eta(f_{nt} + \frac{\kappa}{2} x_t^2 + \kappa s_t x_t) + (1-\eta)b \end{aligned} \tag{34}$$

Note that the expression for w_t^{flex} reflects that with period-by-period Nash bargaining economy-wide, wages and hiring rates will be identical across firms. Observe that the wage gap positively influences the target wage, reflecting that the worker's flow value of unemployment depends on the average wage to the contract wage.

The difference between $w_t^{tar}(r)$ and w_t^{flex} depends not only on the wage gap, but also on the difference between the hiring rate of re-negotiating firms, $x_t(r)$ and the economy-wide average x_t. To a first approximation, however, this latter difference also depends on the wage gap, introducing a second channel through which there is a spillover effect of the average wage on the contract wage.

In particular a loglinear expansion of the job creation condition yields the following relation, where $\widehat{z}_t(r)$ denotes the percent deviation of variable z from its steady state value:

$$\widehat{x}_t(r) - \widehat{x}_t = \lambda\beta w\,(\kappa x)^{-1}\,(1-(x+\rho)\,\lambda\beta)^{-1}\,(\widehat{w}_t - \widehat{w}_t^*) \tag{35}$$

Intuitively, if the contract wage is below the average wage, re-negotiating firms will be hiring at a faster rate than average. In contrast to the first spillover effect, which works by directly affecting the target wage, this second effect works indirectly by influence the hiring rate, which in turn affects the target wage.

This indirect spillover reinforces the effect of the direct one. Loglinearizing the equation for the target wage (33) and combining with equation (35) yields:

$$\widehat{w}_t^{tar}(r) = \widehat{w}_t^{flex} + \frac{\tau_1 + \tau_2}{1 - \rho\lambda\beta}\,(\widehat{w}_t - \widehat{w}_t^*) \tag{36}$$

where τ_1 and τ_2 reflect the influence of the direct and indirect spillover effects on the target wage, respectively, and are given by

$$\begin{aligned}
\tau_1 &= (1-\eta)\,s\lambda\beta \\
\tau_2 &= \eta\,(x+s)\,\lambda\beta(1-\rho\lambda\beta)\,(1-(x+\rho)\,\lambda\beta)^{-1}
\end{aligned} \tag{37}$$

Next, loglinearizing the equation for the contract wage (27) and combining with equation (36) yields:

$$\widehat{w}_t^* = (1-\rho\lambda\beta)\,\widehat{w}_t^{flex} + \rho\lambda\beta E_t\widehat{w}_{t+1}^* + \tau\,(\widehat{w}_t - \widehat{w}_t^*) \tag{38}$$

where τ reflects the combined influence of the spillover effects:

$$\tau = \tau_1 + \tau_2 \tag{39}$$

The loglinearized wage index is in turn given by

$$\widehat{w}_t = (1-\lambda)\,\widehat{w}_t^* + \lambda\widehat{w}_{t-1} \tag{40}$$

Combining these equations then yields the following second order difference equation for the wage, with the frictionless wage, $\widehat{w}_t^{flex}$, as the forcing variable:

$$\widehat{w}_t = \gamma_b\widehat{w}_{t-1} + \gamma_{flex}\widehat{w}_t^{flex} + \gamma_f E_t\widehat{w}_{t+1} \tag{41}$$

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where

$$
\begin{aligned}
\gamma_b &= (1+\tau)\,\phi^{-1} \\
\gamma_{flex} &= \gamma\phi^{-1} \\
\gamma_f &= \rho\beta\phi^{-1} \\
\gamma &= \frac{(1-\lambda)(1-\rho\lambda\beta)}{\lambda} \\
\phi &= 1 + \tau + \gamma + \rho\beta
\end{aligned}
\tag{42}
$$

with $\gamma_b + \gamma + \gamma_f = 1$. Due to staggered contracting, $\widehat{w}_t$ depends on both the lagged wage $\widehat{w}_{t-1}$ as well as the expected future wage $E_t\widehat{w}_{t+1}$. Note that the spillover effects, measured by τ, reduce the sensitivity of the wage to movements in the both $\widehat{w}_t^{flex}$ and $E_t\widehat{w}_{t+1}$. In this respect, this spillover effects work much the same way as how real wage rigidities enhance price stickiness in monetary models with time-dependent pricing (see, e.g., Woodford, 2003).

Finally, the loglinearized frictionless wage is given by

$$
\widehat{w}_t^{flex} = \varphi_{f_n}\widehat{f}_{nt} + \varphi_x\widehat{x}_t + \varphi_s\widehat{s}_t
\tag{43}
$$

where $\varphi_{f_n} = \eta f_n w^{-1}$, $\varphi_x = \eta\kappa x\,(x+s)\,w^{-1}$ and $\varphi_s = \eta\kappa x s w^{-1}$. The key determinants of $\widehat{w}_t^{flex}$ are the labor productivity, $\widehat{f}_{nt}$, the hiring rate $\widehat{x}_t$, and the job finding rate, $\widehat{s}_t$. These are the cyclical factors that influence the value of the marginal worker to the firm and the worker's flow value of unemployment. With period-by-period Nash bargaining, the wage equals $\widehat{w}_t^{flex}$. (This can be seen by setting λ equal to zero in equations (38), (39), (37),(41), and (42).) With staggered contract, however, the wage depends on a weighted sum of the current and expected future values of $\widehat{w}_t^{flex}$, as well as the lagged wage.

Finally, loglinearizing the difference equation for the hiring rate (12) yields:

$$
\widehat{x}_t = E_t\widehat{\Lambda}_{t,t+1} + (\beta/\kappa x)\left(f_n\widehat{f}_{nt+1} - w E_t\widehat{w}_{t+1}\right) + \beta\,(x+\rho)\,E_t\widehat{x}_{t+1}
\tag{44}
$$

The hiring rate thus depends on current and expected movements of the marginal product of labor relative to the wage. The stickiness in the wage due to staggered contracting, everything else equal, that current and expected movement in the marginal product of labor will have a greater impact on the hiring rate, than would have been the case otherwise.

We defer to the appendix a complete presentation of the loglinear equations of the model.

4   Model Evaluation

4.1   Calibration

We choose a monthly calibration in order to properly capture the high rate of job finding in U.S. data. Our parametrization is summarized in Table 1. There are twelve parameters to which we

need to assign values. Five are conventional in the business cycle literature: the discount factor, β, the coefficient of relative risk aversion, γ, the depreciation rate, δ, the "share" parameter on capital in the Cobb-Douglas production function, α, and the autoregressive parameter for the technology shock, ρ_a. We use conventional values for all these parameters: $\beta = 0.99^{\frac{1}{3}}$, $\gamma = 1.0$, $\delta = 0.025/3$, $\alpha = 0.33$, and $\rho_a = 0.95^{\frac{1}{3}}$. Note that in contrast to the frictionless labor market model, the parameter α does not necessarily correspond to the labor share, since the latter will in general depend on the outcome of the bargaining process. However, here we simply follow convention by setting $\alpha = 0.33$ to facilitate comparison with the RBC literature. We also normalize the steady state value of output per person, a, to 1.

Table 1: Parameters and steady state values		
Output per person	a	1.0
Production parameter	α	0.33
Discount factor	β	$0.99^{\frac{1}{3}}$
CRRA parameter	γ^c	1.0
Capital depreciation rate	δ	0.025/3
Technology shock	ρ_a	$0.95^{\frac{1}{3}}$
Survival rate	ρ	0.035
Matching parameter	σ	0.5
Job finding probability	s	0.35
Replacement ratio	b/w	0.4
Bargaining power	η	0.5
Renegotiation frequency	$1-\lambda$	1/12

There are an additional five parameters that are related to the search and matching literature: the separation rate, $1 - \rho$, the job-finding probability, s, the matching function function parameter, σ, the bargaining power parameter, η, and the replacement ratio, b/w. First, we set $1 - \rho = 0.035$ and $s = 0.35$. The first is standard in the literature and is supported by strong evidence from alternative data sources. The second is higher then what has typically been used in the literature and relies on recent evidence about the high job finding rate in the U.S. (Hall, 2005 and Shimer, 2005). Second, we choose the match elasticity to unemployment, σ, to be equal to 0.5. Estimates of this parameter in the literature range from values of 0.4 (Blanchard and Diamond, 1989) to about 0.7 (Shimer, 2005), consistently with evidence summarized by Petrongolo and Pissarides (2001). Third, since there is no direct evidence on the value of the bargaining power, we initially assign equal bargaining power to workers and firms and set η to 0.5. Importantly, in our model with capital this value is consistent with a steady state labor share of about 2/3 as in the data. We then choose the replacement ratio b/w to be 0.4. Note that the value of the replacement ratio includes not only unemployment benefits but also the value of leisure associated with unemployment.

Finally, there are two parameters that are new and specific to this model. One is the cost of adjustment parameter, κ, and the other is the frequency of wage contract negotiations $1 - \lambda$. The adjustment cost parameter is pinned down by the steady state of the model: Given the steady state hiring rate and marginal product of labor, the steady state hiring rate and wage bargaining relations pin down the steady state value of κ and the wage rate (see the appendix). We pick the frequency of wage adjustment so that the average period wages are fixed, $1/(1 - \lambda)$, is within reason. We take $1 - \lambda$ to be 1/12, implying that wage contracts are renegotiated on average once a year. We also explore below shortening the contract length, along with varying the bargaining parameter.

Finally, given our parametrization, the steady values of unemployment, the hiring rate, the labor share, as well as the consumption/output and investment/output ratios are determined. Table 2 reports these values.

Table 2: Implied parameters and steady state values		
	Baseline calibration 4Q and $\eta = 0.5$	Alternative calibration 3Q and $\eta = 0.7$
Unemployment rate	0.09	0.09
Hiring rate	0.035	0.035
Labor share	0.646	0.658
Adjustment cost parameter	152	66
Investment/output ratio	0.24	0.24
Consumption/output ratio	0.74	0.75
Adjustment costs/output ratio	0.02	0.01

4.2   Results

We judge the model against quarterly U.S. data from 1951:1-2005:1. For series that are available monthly, we take quarterly averages. Since the artificial series that the model generates is based on a monthly calibration, we also take quarterly averages of this data.

Most of the data is from the BLS. All variables are measured in logs. Output y is production in the non-farm business sector. Employment n is all employees in the non-farm business sector. The labor share ls and output per worker a are similarly from the non-farm business sector. The wage w is the wage per person in non-farm business (i.e., $w = ls \cdot a$). Unemployed u is civilian unemployment 16 years old and over. Vacancies v are based on the help wanted advertising index from the Conference Board. Finally, the data are HP filtered with a conventional smoothing weight.

We examine the behavior of the model taking the technology shock as the exogenous driving force. To illustrate how the wage contracting process affects model dynamics, we first examine the impulse responses of the model economy to a unit increase in total factor productivity. The

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solid line in each panel of Figure 1 illustrates the response of the respective variable for our baseline model. For comparison, the dotted line reports the response of the conventional flexible wage model with period-by-period Nash bargaining (obtained by setting $\lambda = 0$).

Observe that in the conventional case with period-by-period wage adjustment, the response of employment is relatively modest, confirming the arguments of Hall and Shimer. There is also only a modest response of other indicators of labor market activity, such as vacancies, v, unemployment u, labor market tightness, $\theta = v/u$, and the hiring rate x. Wages, by contrast, adjust quickly. The resulting small adjustment of employment leads to output dynamics that closely mimic the technology shock.

By contrast, in the model with staggered multiperiod contracting, the hiring rate jumps sharply in the wake of the technology shock along with the measures of labor market activity. A substantial rise in employment follows, certainly as compared to the conventional flexible wage case. Associated with the rise in employment, is a smooth drawn out adjustment in wages, directly a product of the staggered multi-period contracting. The lagged rise in employment leads to a humped shaped response of output, i.e., output continues to rise for several periods before reverting to trend, in contrast to the technology shock which reverts immediately.

We next explore how well the model economy is able to account the overall volatility in the data. Table 3 reports the standard deviation, autocorrelation, and contemporaneous correlation with output for the nine key variables of the model. The standard deviations are normalized relative to output. Model Economy I is our baseline case, with wage contract lasting four quarters on average and bargaining weight of 0.5. Model Economy II is the alternative with three quarters average wage contract and a bargaining weight of 0.7.

Overall the model economies do well in capturing the basic features of the data. Both come reasonably close in capturing the standard deviations of the labor market variables relative to output. They also come close to capturing the autocorrelation of all these variables as well as the contemporaneous correlations with output. A distinguishing feature of our analysis is that we appear to capture wage dynamics. Note that both calibrations come very close the matching the relative volatility of wages (0.43 and 0.44 versus 0.46 in the data) and the contemporaneous correlation of wages with output (0.61 and 0.71 versus 0.65) in the data. The model autocorrelation of wages are a bit high (0.96 and 0.95 versus 0.84 in the data). This may be in part because the BLS measure of wages we used includes benefits. A pure measure of wages, average hourly earnings of production workers (often used in the literature but available only over a shorter sample) yields roughly the same standard deviation as the our measure, but a higher serial correlation, around 0.91, which is closer in line with the model-generated data.

Finally, we observe that Model Economy II performs about as well as I in explaining the data.[1]

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[1] In the new calibration, the weight on productivity in the loglinear flex wage equation is higher (φ_a goes from 0.52 to 0.71), so that the flex wage absorbs a larger share of changes in productivity. However, in the new calibration, the adjustment cost parameter κ (calculated from ss) and the marginal profits in ss (given by $f_n - w + \frac{\kappa}{2}x^2$) are lower. Marginal profits go from about 20% to 9%. If profits are smaller in equilibrium, a positive productivity shock induces

Thus one can rely on wage contracts of an average length of only three quarters, given a bargaining weight of 0.7, which lies within the range of model estimates.

Table 3: Aggregate Statistics								
	y	w	ls	n	u	v	θ	a
US Economy: US Data 1951:1-2005:01								
Relative Standard Deviation	1.00	0.46	0.51	0.62	5.81	6.59	12.10	0.62
Autocorrelation	0.84	0.84	0.73	0.92	0.86	0.90	0.89	0.77
Correlation with y	1.00	0.65	−0.29	0.77	−0.82	0.89	0.87	0.72
Model Economy I: 4Q and $\eta = 0.5$								
Relative Standard Deviation	1.00	0.43	0.55	0.47	4.70	6.55	10.76	0.73
Autocorrelation	0.86	0.96	0.67	0.92	0.92	0.85	0.90	0.79
Correlation with y	1.00	0.61	−0.58	0.94	−0.94	0.94	0.96	0.97
Model Economy II: 3Q and $\eta = 0.7$								
Relative Standard Deviation	1.00	0.44	0.43	0.52	5.15	7.49	11.94	0.68
Autocorrelation	0.86	0.95	0.59	0.89	0.89	0.78	0.86	0.79
Correlation with y	1.00	0.71	−0.48	0.95	−0.95	0.88	0.95	0.98

Finally, we consider the importance of the spillovers for model dynamics. To do so, we simulate the model, eliminating the spillover effects on wage dynamics. In particular, we set equal to zero the parameter τ in equations (41) and (42), which governs the magnitude of the spillover effect.

Table 4 presents the results. For comparison, in the top panel we show again the results for our baseline case, Model Economy I (with the spillover effects included.) The bottom panel shows the same economy, but with the spillovers gone. As the table makes clear, eliminating the spillovers significantly enhances wage flexibility and reduces employment volatility. The relative volatility of wages jumps nearly fifty percent, from 0.43 to 0.62. Conversely, the relative volatility of employment roughly in half, from 0.47 to 0.26. The other measures of labor activity, u, v and θ similarly fall by about half. Thus the wage inertia and resulting employment dynamics in our model are not only a product of staggered multi-period wage contracting, but also of the spillover effects from the Nash bargaining process.

---

a larger percentage increase in profits.

	y	w	ls	n	u	v	θ	a
			Table 4: The Spillover Effect					
			Model Economy I					
Relative Standard Deviation	1.00	0.43	0.55	0.47	4.70	6.55	10.76	0.73
Autocorrelation	0.86	0.96	0.67	0.92	0.92	0.85	0.90	0.79
Correlation with y	1.00	0.61	−0.58	0.94	−0.94	0.94	0.96	0.97
			Model Economy I - No Spillover					
Relative Standard Deviation	1.00	0.62	0.51	0.26	2.58	3.78	5.98	0.84
Autocorrelation	0.83	0.93	0.59	0.89	0.89	0.77	0.85	0.79
Correlation with y	1.00	0.81	−0.52	0.95	−0.95	0.93	0.97	0.99

5   Comparison with other literature (to be added)

6   Concluding Remarks

We have modified the Mortensen and Pissarides model of unemployment dynamics to allow for staggered multiperiod wage contracting. What emerges is a tractable relation for wage dynamics that is a natural generalization of the period-by-period Nash bargaining outcome in the conventional formulation. An interesting side-product is the emergence of spillover effects aggregate wages that influence the bargaining process. We then show that reasonable calibration of the model can account for the reasonably well for the cyclical behavior of wages and labor market activity observed in the data. The spillover effects turn out to be important in this respect.

As we noted earlier, in addition to the presence of the spillover effects, another important difference from existing macroeconomic models that rely on staggered multi-period wage setting is that within in our framework wages affect the adjustment of employment on the extensive margin as opposed to the intensive margin. As Hall has recently emphasized, for adjustment on the intensive margin, wages may not be allocational, as originally argued by Barro (1997). The same criticism, however, does not apply to adjustment on the extensive margin. For this reason it may be interesting to consider our approach and employment adjustment along the extensive margin as a way to shore up a potential weakness of these conventional macroeconomic models.

[Incomplete]

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APPENDIX I

A. Sum of expected future wages for a renegotiating firm, $W_t(r)$

- Let $W_t(i)$ denote the discounted sum of expected future wages at firm i:

$$W_t(i) = E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} w_{t+s}(i) = w_t(i) + (\rho\beta) E_t\Lambda_{t,t+1} w_{t+1}(i) + (\rho\beta)^2 E_t\Lambda_{t,t+2} w_{t+2}(i) + ...$$

- For a firm renegotiating at time t, the current and future expected wages are given by:

$$w_t(r) = w_t^*$$

$$\begin{aligned}
E_t w_{t+1}(r) &= \lambda w_t(r) + (1-\lambda) E_t w_{t+1}^* \\
&= \lambda w_t^* + (1-\lambda) E_t w_{t+1}^*
\end{aligned}$$

$$\begin{aligned}
E_t w_{t+2}(r) &= \lambda E_t w_{t+1}(r) + (1-\lambda) E_t w_{t+2}^* \\
&= \lambda \left[\lambda w_t^* + (1-\lambda) E_t w_{t+1}^* \right] + (1-\lambda) E_t w_{t+2}^* \\
&= \lambda^2 w_t^* + \lambda(1-\lambda) E_t w_{t+1}^* + (1-\lambda) E_t w_{t+2}^*
\end{aligned}$$

$$\begin{aligned}
E_t w_{t+3}(r) &= \lambda E_t w_{t+2}(r) + (1-\lambda) E_t w_{t+3}^* \\
&= \lambda \left[\lambda^2 w_t^* + \lambda(1-\lambda) E_t w_{t+1}^* + (1-\lambda) E_t w_{t+2}^* \right] + (1-\lambda) E_t w_{t+3}^* \\
&= \lambda^3 w_t^* + \lambda^2(1-\lambda) E_t w_{t+1}^* + \lambda(1-\lambda) E_t w_{t+2}^* + (1-\lambda) E_t w_{t+3}^*
\end{aligned}$$

and so on....

- Using these expressions, we can write:

$$\begin{aligned}
W_t(r) = \ & w_t^* \\
& + (\rho\beta) \Lambda_{t,t+1} \left[\lambda w_t^* + (1-\lambda) E_t w_{t+1}^* \right] \\
& + (\rho\beta)^2 \Lambda_{t,t+2} \left[\lambda^2 w_t^* + \lambda(1-\lambda) E_t w_{t+1}^* + (1-\lambda) E_t w_{t+2}^* \right] \\
& + (\rho\beta)^3 \Lambda_{t,t+3} \left[\lambda^3 w_t^* + \lambda^2(1-\lambda) E_t w_{t+1}^* + \lambda(1-\lambda) E_t w_{t+2}^* + (1-\lambda) E_t w_{t+3}^* \right] \\
& + ...
\end{aligned}$$

- Collecting terms:

$$\begin{aligned}
W_t(r) = \ & \left[1 + (\rho\beta\lambda) \Lambda_{t,t+1} + (\rho\beta\lambda)^2 \Lambda_{t,t+2} + ... \right] w_t^* \\
& + (1-\lambda)(\rho\beta) \Lambda_{t,t+1} \left[1 + (\rho\beta\lambda) \Lambda_{t+1,t+2} + (\rho\beta\lambda)^2 \Lambda_{t+1,t+3} + ... \right] w_{t+1}^* \\
& + (1-\lambda)(\rho\beta)^2 \Lambda_{t,t+2} \left[1 + (\rho\beta\lambda) \Lambda_{t+2,t+3} + (\rho\beta\lambda)^2 \Lambda_{t+2,t+4} + ... \right] w_{t+2}^* \\
& + (1-\lambda)(\rho\beta)^3 \Lambda_{t,t+3} \left[1 + (\rho\beta\lambda) \Lambda_{t+3,t+4} + (\rho\beta\lambda)^2 \Lambda_{t+3,t+5} + ... \right] w_{t+3}^*
\end{aligned}$$

- Letting

$$\Delta_t = \sum_{s=0}^{\infty} (\rho\lambda\beta)^s \Lambda_{t,t+s}$$

  we have

$$W_t(r) = \Delta_t w_t^* + (1-\lambda)(\rho\beta) E_t \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}^* + (1-\lambda)(\rho\beta)^2 E_t \Lambda_{t,t+2} \Delta_{t+2} w_{t+2}^* + \dots$$

- Finally, rearranging:

$$W_t(r) = \Delta_t w_t^* + E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \left[(1-\lambda)(\rho\beta) \Lambda_{t+s,t+s+1} \Delta_{t+s+1} w_{t+s+1}^* \right]$$

B. Value of a marginal worker for a renegotiating firm, $J_t(r)$

- Value of an additional worker for a firm

$$
\begin{aligned}
J_t(r) &= f_{nt} - w_t(r) + \frac{\kappa}{2} x_t(r)^2 + \rho\beta E_t \Lambda_{t,t+1} J_{t+1}(r) \\
&= E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \left[f_{nt+s} + \frac{\kappa}{2} x_{t+s}(r)^2 \right] - W_t(r)
\end{aligned}
$$

- Substituting the expression for $W_t(r)$, we get

$$J_t(r) = E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \left[f_{nt+s} + \frac{\kappa}{2} x_{t+s}(r)^2 - (1-\lambda)(\rho\beta) \Lambda_{t+s,t+s+1} \Delta_{t+s+1} w_{t+s+1}^* \right] - \Delta_t w_t^*$$

C. Worker surplus at a renegotiating firm, $H_t(r)$

- Worker surplus

$$
\begin{aligned}
H_t(r) &= w_t(r) - b - s_t \beta E_t \Lambda_{t,t+1} H_{t+1} + \rho\beta E_t \Lambda_{t,t+1} H_{t+1}(r) \\
&= W_t(r) - E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \left[b + s_{t+s} \beta \Lambda_{t+s,t+s+1} H_{t+s+1} \right]
\end{aligned}
$$

- Substituting the expression for $W_t(r)$, we get

$$H_t(r) = \Delta_t w_t^* - E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \left[b + s_{t+s} \beta \Lambda_{t+s,t+s+1} H_{t+s+1} - (1-\lambda)(\rho\beta) \Lambda_{t+s,t+s+1} \Delta_{t+s+1} w_{t+s+1}^* \right]$$

D. The contract wage

- The Nash first-order condition is

$$\eta J_t (r) = (1 - \eta) H_t (r)$$

- Substituting $J_t (r)$ and $H_t (r)$ and rearranging, we obtain:

$$
\begin{aligned}
\Delta_t w_t^* &= E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \left[\eta \left(f_{nt+s} + \frac{\kappa}{2} x_{t+s} (r)^2 \right) + (1 - \eta) (b + s_{t+s}\beta\Lambda_{t+s,t+s+1}H_{t+s+1}) \right. \\
&\quad \left. - (1 - \lambda) (\rho\beta) \Lambda_{t+s,t+s+1}\Delta_{t+s+1}w_{t+s+1}^* \right]
\end{aligned}
$$

- The above equation can be written in a recursive form in the following way:

$$
\begin{aligned}
\Delta_t w_t^* &= \eta \left(f_{nt} + \frac{\kappa}{2} x_t (r)^2 \right) + (1 - \eta) (b + s_t\beta E_t\Lambda_{t,t+1}H_{t+1}) \\
&\quad - (1 - \lambda) (\rho\beta) E_t\Lambda_{t,t+1}\Delta_{t+1}w_{t+1}^* + (\rho\beta) E_t\Lambda_{t,t+1}\Delta_{t+1}w_{t+1}^*
\end{aligned}
$$

- Simplifying, we obtain

$$\Delta_t w_t^* = w_t^{tar} (r) + \rho\lambda\beta E_t\Lambda_{t,t+1}\Delta_{t+1}w_{t+1}^*$$

with

$$w_t^{tar} (r) = \eta \left(f_{nt} + \frac{\kappa}{2} x_t (r)^2 \right) + (1 - \eta) (b + s_t\beta E_t\Lambda_{t,t+1}H_{t+1})$$

E. The expected average worker surplus

- Consider now the relation between $E_t H_{t+1}$ and $E_t H_{t+1} (r)$.

- First note that, for a firm renegotiating at time t, we have:

$$
\begin{aligned}
E_t (w_{t+1} - w_{t+1} (r)) &= E_t \left[\lambda w_t + (1 - \lambda) w_{t+1}^* \right] - E_t \left[\lambda w_t^* + (1 - \lambda) w_{t+1}^* \right] \\
&= \lambda (w_t - w_t^*)
\end{aligned}
$$

$$
\begin{aligned}
E_t (w_{t+2} - w_{t+2} (r)) &= E_t \left[\lambda w_{t+1} + (1 - \lambda) w_{t+2}^* \right] - E_t \left[\lambda w_{t+1} (r) + (1 - \lambda) w_{t+2}^* \right] \\
&= E_t \lambda (w_{t+1} - w_{t+1} (r)) \\
&= \lambda^2 (w_t - w_t^*)
\end{aligned}
$$

and so on....

- Then, we can write:

$$
\begin{aligned}
E_t\left(H_{t+1} - H_{t+1}\left(r\right)\right) &= E_t\left(W_{t+1} - W_{t+1}\left(r\right)\right) \\
&= E_t \sum_{s=0}^{\infty} \left(\rho\beta\right)^s \Lambda_{t+1,t+s+1}\left(w_{t+s+1} - w_{t+s+1}\left(r\right)\right) \\
&= \lambda\left[1 + \left(\rho\lambda\beta\right) E_t\Lambda_{t+1,t+2} + \left(\rho\lambda\beta\right)^2 E_t\Lambda_{t+1,t+3} + ...\right]\left(w_t - w_t^*\right) \\
&= \lambda\Delta_{t+1}\left(w_t - w_t^*\right)
\end{aligned}
$$

- Using this equation in the Nash condition, we get:

$$
\begin{aligned}
\eta E_t J_{t+1}\left(r\right) &= \left(1 - \eta\right) E_t H_{t+1}\left(r\right) \\
&= \left(1 - \eta\right)\left(E_t H_{t+1} - \lambda\Delta_{t+1}\left(w_t - w_t^*\right)\right)
\end{aligned}
$$

which can be rearranged to:

$$
E_t H_{t+1} = \frac{\eta}{1 - \eta} E_t J_{t+1}\left(r\right) + E_t\lambda\Delta_{t+1}\left(w_t - w_t^*\right)
$$

- Finally, using the vacancy posting condition yields the following expression:

$$
\beta E_t\Lambda_{t,t+1} H_{t+1} = \frac{\eta}{1 - \eta}\kappa x_t\left(r\right) + \beta E_t\Lambda_{t,t+1}\lambda\Delta_{t+1}\left(w_t - w_t^*\right)
$$

F. The hiring rate at a renegotiating firm, $x_t\left(r\right)$

- Consider now the relation between $\widehat{x}_t$ and $\widehat{x}_t\left(r\right)$.

- First note that loglinearizing the job creation condition yields:

$$
\widehat{x}_t\left(i\right) = E_t\widehat{\Lambda}_{t,t+1} + \beta\left(\kappa x\right)^{-1}\left(f_n E_t\widehat{f}_{nt+1} - w E_t\widehat{w}_{t+1}\left(i\right)\right) + \beta\left(x + \rho\right) E_t\widehat{x}_{t+1}\left(i\right)
$$

We can then write:

$$
\widehat{x}_t\left(r\right) - \widehat{x}_t = \beta w\left(\kappa x\right)^{-1} E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}\left(r\right)\right) + \beta\left(x + \rho\right) E_t\left(\widehat{x}_{t+1}\left(r\right) - \widehat{x}_{t+1}\right)
$$

which can be iterated forward to give:

$$
\begin{aligned}
\widehat{x}_t\left(r\right) - \widehat{x}_t &= \beta w\left(\kappa x\right)^{-1} E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}\left(r\right)\right) \\
&\quad + \beta\left(x + \rho\right)\beta w\left(\kappa x\right)^{-1} E_t\left(\widehat{w}_{t+2} - \widehat{w}_{t+2}\left(r\right)\right) \\
&\quad + \beta^2\left(x + \rho\right)^2\beta w\left(\kappa x\right)^{-1} E_t\left(\widehat{w}_{t+2} - \widehat{w}_{t+3}\left(r\right)\right) \\
&\quad + ...
\end{aligned}
$$

- Substituting the expressions for the expected future wages at a firm renegotiating at time t and rearranging, we obtain:

$$
\begin{aligned}
\widehat{x}_t\left(r\right) - \widehat{x}_t &= \beta w\left(\kappa x\right)^{-1}\left[\lambda\left(\widehat{w}_t - \widehat{w}_t^*\right) + \lambda\beta\lambda\left(x + \rho\right)\left(\widehat{w}_t - \widehat{w}_t^*\right) + \lambda\left(\beta\lambda\right)^2\left(x + \rho\right)^2\left(\widehat{w}_t - \widehat{w}_t^*\right) + \dots\right] \\
&= \lambda\beta w\left(\kappa x\right)^{-1}\left[1 + \beta\lambda\left(x + \rho\right) + \left(\beta\lambda\right)^2\left(x + \rho\right)^2 + \dots\right]\left(\widehat{w}_t - \widehat{w}_t^*\right) \\
&= \lambda\beta w\left(\kappa x\right)^{-1}\left(1 - \left(x + \rho\right)\lambda\beta\right)^{-1}\left(\widehat{w}_t - \widehat{w}_t^*\right)
\end{aligned}
$$

APPENDIX II

Steady state calculation

- Given the calibrated parameters and steady state values in Table 1, we obtain implied values of $n, u, x, r, z, ls, k/y, c/y, I/y, \left(\kappa/2\right)\left(x^2 n/y\right), f_n, \kappa$ and w from steady state calculations.

- First obtain

$$
n = \frac{s}{1 - \rho + s}
$$

$$
u = 1 - n
$$

$$
x = \frac{su}{n}
$$

- Then get

$$
r = \frac{1}{\beta}
$$

$$
z = r - 1 + \delta
$$

$$
\frac{k}{y} = \frac{\alpha}{z}
$$

$$
\frac{I}{y} = \delta\frac{k}{y}
$$

$$
\frac{k}{n} = \left(a\frac{k}{y}\right)^{\frac{1}{1-\alpha}}
$$

$$
f_n = \left(1 - \alpha\right)a\left(\frac{k}{n}\right)^{\alpha}
$$

- Then κ and w solve the following system (equations (12) and (34))

$$
\begin{cases}
\kappa x = \beta\left(f_n - w + \frac{\kappa}{2}x^2 + \rho\kappa x\right) \\
w = \eta\left(f_n + \frac{\kappa}{2}x^2 + s\kappa x\right) + \left(1 - \eta\right)\frac{b}{w}w
\end{cases}
$$

- The staedy state labor share is calculated from

$$ls = \frac{wn}{y} = w\frac{n}{k}\frac{k}{y}$$

- Finally

$$\frac{c}{y} = 1 - \frac{I}{y} - \frac{\kappa}{2}\frac{x^2 n}{y}$$

APPENDIX III

The complete loglinear model

Variables $\left\{ \widehat{m}_t, \widehat{n}_t, \widehat{u}_t, \widehat{v}_t, \widehat{q}_t, \widehat{s}_t, \widehat{x}_t, \widehat{\lambda}_t, \widehat{c}_t, \widehat{r}_t, \widehat{z}_t, \widehat{w}_t, \widehat{w}_t^{flex}, \widehat{y}_t, \widehat{f}_{nt}, \widehat{k}_t, \widehat{I}_t, \widehat{a}_t \right\}$

- Technology
$$\widehat{y}_t = \widehat{a}_t + \alpha\widehat{k}_t + (1 - \alpha)\widehat{n}_t \qquad\qquad (E1)$$

- Resource constraint
$$\widehat{y}_t = cy\widehat{c}_t + iy\widehat{I}_t + (1 - cy - iy)(2\widehat{x}_t + \widehat{n}_t) \qquad (E2)$$

where $cy = \frac{c}{y}$, $iy = \frac{I}{y}$ and $1 - cy - iy = \frac{\kappa}{2}\frac{x^2 n}{y}$

- Matching
$$\widehat{m}_t = \sigma\widehat{u}_t + (1 - \sigma)\widehat{v}_t \qquad\qquad (E3)$$

- Employment dynamics
$$\widehat{n}_{t+1} = \rho\widehat{n}_t + (1 - \rho)\widehat{m}_t \qquad\qquad (E4)$$

- Transition probabilities
$$\widehat{q}_t = \widehat{m}_t - \widehat{v}_t \qquad\qquad (E5)$$

$$\widehat{s}_t = \widehat{m}_t - \widehat{u}_t \qquad\qquad (E6)$$

- Unemployment
$$\widehat{u}_t = -\frac{n}{u}\widehat{n}_t \qquad\qquad (E7)$$

- Capital dynamics
$$\widehat{k}_{t+1} = (1 - \delta)\widehat{k}_t + \delta\widehat{I}_t \qquad\qquad (E8)$$

- Aggregate vacancies

$$\widehat{x}_t = \widehat{q}_t + \widehat{v}_t - \widehat{n}_t \tag{E9}$$

- Consumption-saving

$$\widehat{\lambda}_t = E_t\widehat{\lambda}_{t+1} + \widehat{r}_{t+1} \tag{E10}$$

$$E_t\widehat{r}_{t+1} = \frac{r-1+\delta}{r}E_t\widehat{z}_{t+1} \tag{E11}$$

- Marginal utility

$$\widehat{\lambda}_t = \widehat{u}_{c_t} \tag{E12}$$

- Aggregate hiring rate

$$\widehat{x}_t = E_t\widehat{\Lambda}_{t,t+1} + (\beta/\kappa x)\left(f_n\widehat{f}_{nt+1} - wE_t\widehat{w}_{t+1}\right) + \beta\left(x+\rho\right)E_t\widehat{x}_{t+1} \tag{E13}$$

- Marginal product of labor is

$$\widehat{f}_{nt} = \widehat{y}_t - \widehat{n}_t \tag{E14}$$

- Capital renting

$$\widehat{y}_t - \widehat{k}_t = \widehat{z}_t \tag{E15}$$

- Aggregate wage

$$\widehat{w}_t = \gamma_b\widehat{w}_{t-1} + \gamma_{flex}\widehat{w}_t^{flex} + \gamma_f E_t\widehat{w}_{t+1} \tag{E16}$$

where

$$\gamma = \frac{(1-\lambda)(1-\rho\lambda\beta)}{\lambda}$$

$$\tau = (1-\eta)\,s\lambda\beta + \eta\left(x+s\right)\lambda\beta\left(1-\rho\lambda\beta\right)\left(1-\left(x+\rho\right)\lambda\beta\right)^{-1}$$

$$\phi = 1 + \tau + \gamma + \rho\beta$$

$$\gamma_{flex} = \gamma\phi^{-1} \quad \gamma_f = \rho\beta\phi^{-1} \quad \gamma_b = (1+\tau)\,\phi^{-1}$$

- Flexible wage

$$\widehat{w}_t^{flex} = \varphi_{f_n}\widehat{f}_{nt} + \varphi_x\widehat{x}_t + \varphi_s\widehat{s}_t \tag{E17}$$

where $\varphi_{f_n} = \eta f_n w^{-1}$, $\varphi_x = \eta\kappa x\left(x+s\right)w^{-1}$ and $\varphi_s = \eta\kappa xsw^{-1}$

- Technology process

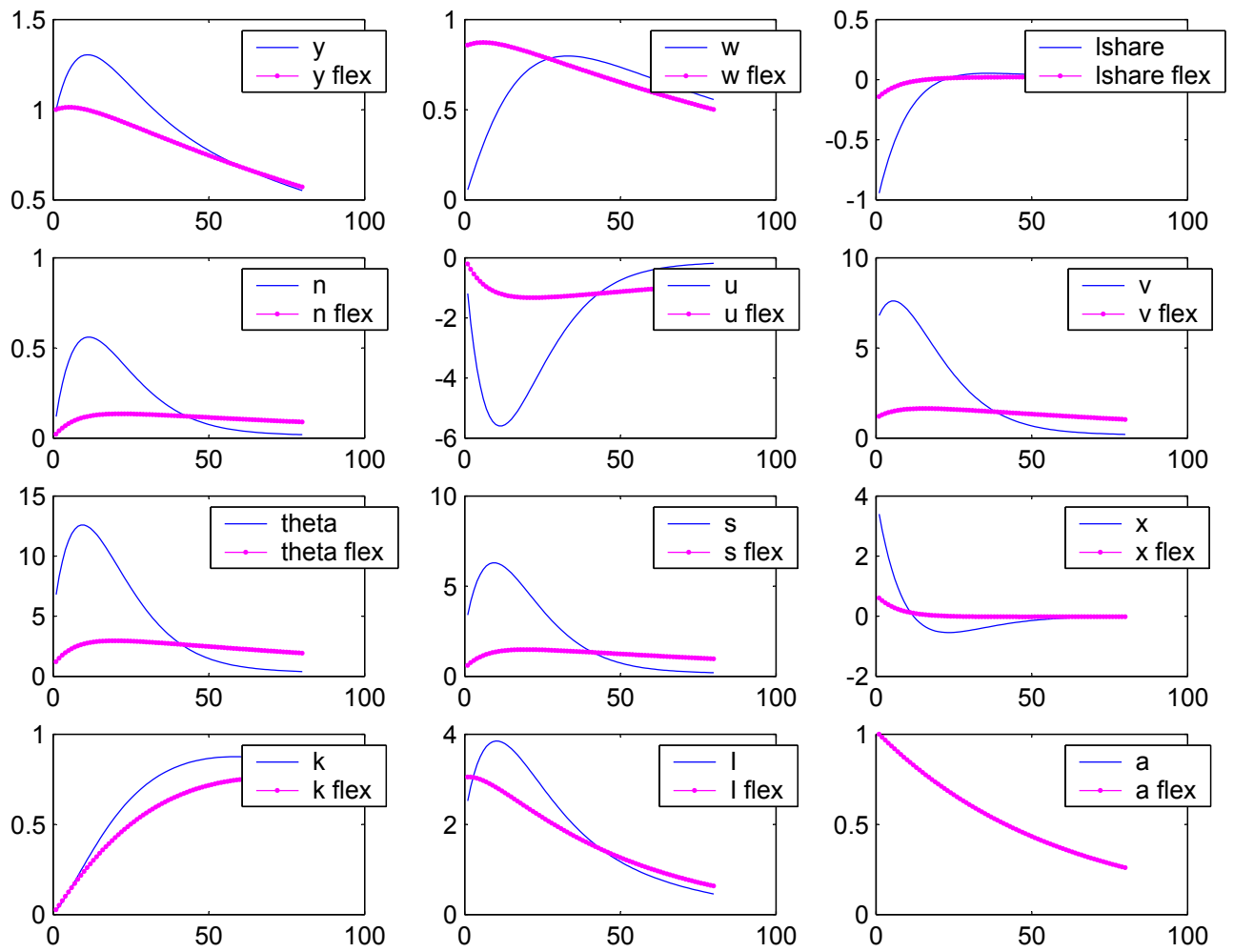$$\widehat{a}_t = \rho_a\widehat{a}_{t-1} + \varepsilon_t^a \tag{E18}$$

Figure 1: Impulse responses to a technology shock

Figure 2: The spillover effect