Complete Markets, Enforcement Constraints and Intermediation

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ABSTRACT.   As shown by Alvarez and Jermann (2003), the constrained efficient allocations of models with complete markets and limited commitment can be decentralized with trade in a complete set of contingent claims subject to trading constraints that are not too tight, in the sense that they allow for the maximum possible amount of borrowing such that there is no default in equilibrium. On the other hand, the previous decentralization is not possible in the presence of a capital accumulation. The reason is that a shift in resources from one period to the next leads to a change in the aggregate capital stock that affects the autarky utility and drives a wedge between the expected marginal rates of substitution and transformation in the standard capital Euler equation of the planner's problem. To take these effects into account, one needs to include a savings constraint on the capital holdings, which is, however, difficult to interpret. In the present paper, we study the competitive equilibrium (CE) with a competitive intermediation sector that operates the investment technology and with no savings constraints. We show that the CE allocations can be characterized with a similar system of equations to the one characterizing the optimal allocations. The only difference is that the effects of capital accumulation on the autarky utilities is not internalized. In addition, our numerical results show that these autarky effects are quantitatively negligible. Thus, the CE allocations in the absence of savings constraints are very similar to the constrained efficient solution, whereas our characterization considerably simplifies the computation of the equilibrium.

Keywords: Complete markets, Enforcement Constraints, Intermediation

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1. INTRODUCTION

During the recent years, models with limited commitment have been introduced to study several important economic issues. Among others, Thomas and Worrall (1988) study efficient wage contracts, Kocherlakota (1996) analyzes optimal risk sharing, Alvarez and Jermann (2001) investigate asset prices and Krueger and Perri (2003) study inequality. Mostly, these models have been studied in endowment economies, whereas limited commitment models with capital accumulation have received less attention. In the present paper, we introduce capital accumulation into a similar framework to the one studied by the previous authors and study its consequences. In particular, we focus on the relationship between the constrained efficient allocations and the competitive equilibrium allocations with (endogenous) borrowing limits and complete financial markets.

As shown by Alvarez and Jermann (2000), the constrained efficient allocations of models with complete markets and enforcement constraints can be decentralized with debt constraints that are not too tight, in the sense that they are the loosest possible limits that do not allow for default in equilibrium. We show that these decentralizations are not possible in a model with capital accumulation. The reason is that a shift in resources from one period to the next in the presence of enforcement constraints has two additional effects on the standard Euler equation determining aggregate capital accumulation. On the one hand, it increases the planner's marginal rate of substitution, raising the benefits of a higher aggregate capital stock. On the other hand, a higher capital stock tightens the enforcement constraints through an increase in the autarky value, reducing the benefits of more capital. Since the previous effects drive a wedge between the marginal rates of substitution and transformation, one needs to impose a savings constraint on the individual capital holdings to decentralize the model with debt constraints. Note that a similar result has been obtained by Kehoe and Perri (2002,2005) in a related multi-country model, where the different agents are interpreted as representative agents of different countries. Further, they also show that one can decentralize the constrained efficient allocations in their environment with partial governmental default on loans to foreign households and capital income taxes.

In the present paper, we focus instead on a decentralization with borrowing constraints for two reasons. First, since we assume that there is only one production sector, our agents cannot be interpreted as countries but rather as two different households that engage in a trading contract to smooth consumption over time. Consequently, governmental default would not make sense in this environment. Second, a decentralization with debt constraints leads to lower interest rates with respect to a decentralization with sovereign default, being therefore more promising to explain (closed economy) asset pricing moments.

First, a key extension of the present work to the existing decentralization literature is the introduction of financial intermediaries that operate the investment technology and set the trading limits. In contrast to the findings of Kehoe and Perri (2002,2005), we show that a decentralization of the constrained efficient allocations with borrowing constraints in the presence of financial intermediaries is not possible *only* due to the impact on capital on the autarky valuations. As shown by the previous authors, however, these effects can be taken into account if one imposes a savings constraint on the capital holdings of the intermediary.

Second, since there is little evidence of the presence of savings constraints in the data, while it is difficult to provide equilibrium micro-foundations for them, we also characterize the decentralized equilibrium allocations with no savings constraints and borrowing limits that are not too tight. In particular, we show that the equilibrium allocations solve almost

the same system of equations as the constrained efficient allocations, with the key difference that the aforementioned effects of aggregate capital accumulation on the autarky values are not internalized. In addition, we also show how the borrowing limits that are not too tight can arise as an equilibrium outcome when the intermediaries are free to choose them.

Note that this characterization results provides a relatively simple solution method, which is similar to the one used to solve constrained efficient allocations, for a potentially very complicated equilibrium problem. Further, our numerical results show that, in the long-run, the competitive equilibrium allocations with and without saving limits exhibit permanent risk sharing. The key difference between the two economies arises in the short run from the fact that the economy will over accumulate capital in the absence of the savings constraint, which in turns implies a lower range of possible wealth distributions in the short run. In addition, our numerical results suggest that the autarky effects are quantitatively unimportant, implying that the equilibrium allocations without saving constraints are very similar to the constrained efficient solution.

The paper is organized as follows. The next section introduces the model economy. Section three discusses the benchmark competitive equilibrium with borrowing constraints that are not too tight. Section 4 introduces the constrained efficient allocations and Section 5 shows how one can decentralize these with a competitive equilibrium with borrowing constraints and a capital accumulation constraints on the financial intermediaries. The competitive equilibrium in the absence of the capital accumulation constraint is characterized in Section 6, where we also provide micro foundations for the borrowing limits. Finally, Section 7 compares the two competitive equilibria quantitatively and Section 8 summarizes and concludes.

2. THE ECONOMY

We consider an infinite horizon economy with aggregate technology shocks and idiosyncratic labor income shocks. Time is discrete and indexed by $t = 0, 1, 2....$ Further, the resolution of uncertainty is represented by an information structure or event-tree N, where each node or date-state $s^t \in N$, summarizing the history of the environment through and including date t, has a finite number of immediate successors, denoted by $s^{t+1}|s^t$. We use the notation $s^r|s^t$ with $r \geq t$ to indicate that node s^r belongs to the sub-tree with root s^t. In addition, with the exception of the unique root node s^0 at date $t = 0$, each node has a unique predecessor, denoted by s^{t-1}. The probability as of period 0 of date-event s^t is denoted by $\pi(s^t)$, with $\pi(s^0) = 1$, since the initial realization s^0 is given. In addition, we denote by $\pi(s^r|s^t)$ the probability of s^r given s^t, and we let $x = \left\{ x \left(s^t \right) \right\}_{s^t \in N}$ throughout the section.

At each node s^t, there exists a spot market for a single consumption good $y(s^t) \in \mathbb{R}_+$, produced with the following aggregate technology:

$$y(s^t) = f(z(s^t), K(s^{t-1}), L(s^t)) \tag{1}$$

where $K(s^{t-1}) \in \mathbb{R}_+$ and $L(s^t) \in \mathbb{R}_+$ denote the aggregate capital and labor respectively, with $K\left(s^{-1}\right) \in \mathbb{R}_{++}$ given. Further, $z(s^t) \in \mathbb{R}_{++}$ is an aggregate productivity shock that follows a stationary Markov chain with N_z possible values.

We assume that the production function $f(z, \cdot, \cdot) : \mathbb{R}_+^2 \to \mathbb{R}_+$ is continuously differentiable on the interior of its domain, strictly increasing, strictly concave in K and L separately, and homogeneous of degree one in the two arguments. We also assume that $f_K(z, K, L) > 0$ and $f_L(z, K, L) > 0$ for all $K > 0$ and $L > 0$, and that $\lim_{K \to 0} f_K(z, K, 1) = \infty$ and

$\lim_{k\to\infty} f_K(z, K, 1) = 0$. To simplify notation, the total output including undepreciated capital is denoted by:

$$F(z(s^t), K(s^{t-1}), L(s^t)) = y\left(s^t\right) + (1-\delta)K(s^{t-1}) \tag{2}$$

where δ is the capital depreciation rate.

The economy is populated by two types of households indexed by $i = 1, 2$, each containing a continuum of identical infinitely lived consumers[1]. Households have additively separable preferences over sequences of consumption c_i of the form:

$$U(c_i) = \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t)\beta^t u\left(c_i(s^t)\right) = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_i(s^t)\right) \tag{3}$$

where $\beta \in (0, 1)$ is the subjective discount factor and E_0 denotes the expectation conditional on information at date $t = 0$. We assume that the period utility function u is strictly increasing, strictly concave and continuously differentiable, with $\lim_{c_i\to 0} u'(c_i) = \infty$, and $\lim_{c_i\to\infty} u'(c_i) = 0$.

At each date-state s^t, households receive a stochastic labour endowment $\epsilon_i(s^t)$, following a stationary Markov chain with N_ϵ possible values. We assume that households supply labor inelastically, and we therefore have that $L(s^t) = \sum_{i\in I} \epsilon_i(s^t)$. Further, they have a potentially history dependent outside option $V_i\left(s^t\right)$, implying that they are subject to a participation constraint of the form:

$$\sum_{r=t}^{\infty} \sum_{s^r|s^t} \beta^{r-t} \pi\left(s^r\right) u\left(c_i\left(s^r\right)\right) \geq V_i(s^t) \text{ for } i = 1, 2. \tag{4}$$

Finally, the resource constraint of the economy is given by:

$$\sum_{i\in I} c_i(s^t) + K(s^t) = y\left(s^t\right) + (1-\delta)K(s^{t-1}). \tag{5}$$

3. Competitive Equilibrium with Borrowing Constraints

This section defines a competitive equilibrium of the above economy where agents can trade in one period ahead Arrow securities subject to borrowing constraints. We assume that the economy is populated by a representative firm that operates the production technology and by a competitive financial intermediation sector operating the investment technology. The intermediaries are risk neutral, and we use the representative intermediary in what follows, since we want to focus on symmetric equilibria.

Each period, after observing the realization of the productivity shock, the representative firm rents labor from the households and physical capital from the intermediary to maximize the period profits:

$$Max_{\{K,L\}} f(z(s^t), K(s^{t-1}), L(s^t)) - w\left(s^t\right) L(s^t) - r(s^t)K(s^{t-1}).$$

The profit maximization implies that the wages and capital rental rate are given by the following expressions at every node:

$$w(s^t) = f_L(s^t) \equiv f_L(z(s^t), K(s^{t-1}), L(s^t)) \tag{6}$$

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[1] All the results in the paper hold for a finite number I of agents, and the assumption that $I = 2$ is therefore without loss of generality. On the other hand, it simplifies both the notation and the computations.

$$r(s^t) = f_K(s^t) \equiv f_K(z(s^t), K(s^{t-1}), L(s^t)). \tag{7}$$

The representative intermediary lives for two periods. An intermediary born at node s^t first decides how much capital $k(s^t)$ to purchase at period t. The capital is rented to the firm, earning a rental revenue of $r(s^{t+1})k(s^t)$ and a liquidation value of $(1 - \delta)k(s^t)$ the following period. Further, to finance the capital purchases, the intermediary sells the future consumption goods in the spot market for one period ahead contingent claims traded at price $q(s^{t+1}|s^t)$, which represents the price of one unit of consumption good delivered at $t + 1$, contingent on the realization $s^{t+1}|s^t$. Thus, the intermediary solves the following problem:

$$Max_{\{k\}} \left\{ -k(s^t) + \sum_{s^{t+1}|s^t} q(s^{t+1}|s^t)[r(s^{t+1}) + (1 - \delta)]k(s^t) \right\}$$

In equilibrium, it must be the case that, in equilibrium,

$$1 = \sum_{s^{t+1}|s^t} q(s^{t+1}|s^t)[r(s^{t+1}) + (1 - \delta)]. \tag{8}$$

At each note s^t, households can trade through the financial intermediaries in a complete set of state contingent claims to one period ahead consumption. If we denote by $a_i(s^{t+1})$ the amount of contingent claims chosen by household $i \in I$ at the end of period t, the maximization problem is given by:

$$Max_{\{c_i, a_i\}} \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t)\beta^t u\left(c_i(s^t)\right) \text{ s.t.} \tag{Problem 1}$$

$$c_i(s^t) + \sum_{s^{t+1}|s^t} q(s^{t+1}|s^t)a_i(s^{t+1}) \leq w(s^t)\epsilon_i(s^t) + a_i(s^t) \tag{9}$$

$$a_i(s^{t+1}) \geq A_i(s^{t+1}) \text{ for } \forall s^{t+1}|s^t \tag{10}$$

where the initial holdings $\left\{a_i(s^0)\right\}_{i=1,2}$ of Arrow securities are given. Note that, for the state contingent debt issued by the intermediary to match the demand from the households, it must be the case that $\sum_i a_i(s^{t+1}) = [r(s^{t+1}) + (1 - \delta)]k(s^t)$. Further, the Arrow security holdings are subject to the borrowing constraints $A_i(s^{t+1})$, which are set by the intermediaries so that no intermediary has an incentive to deviate. The determination of these limits is discussed below.

If $\gamma_i^{ce}(s^{t+1}) \geq 0$ is the multiplier on the portfolio constraint (10) of security $a_i(s^{t+1})$, the necessary and sufficient first order conditions from the previous maximization problem imply that:

$$q(s^{t+1}|s^t) = \beta\pi(s^{t+1}|s^t)\left\{ \frac{u'\left(c_i(s^{t+1})\right)}{u'\left(c_i(s^t)\right)} \right\} + \frac{\gamma_i^{ce}(s^{t+1})}{u'\left(c_i(s^t)\right)} \tag{11}$$

whereas the transversality condition is given by:

$$\lim_{t \to \infty} \sum_{s^t} \beta^t \pi(s^t)u'\left(c_i(s^t)\right)[a_i(s^t) - A_i(s^t)] \leq 0. \tag{12}$$

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Clearly, the portfolio constraint can only be binding for one of the two households, implying that $\gamma_i^{ce}(s^{t+1}) = 0$ for at least one household. The first order condition in (11) can therefore be rewritten as follows:

$$q(s^{t+1}|s^t) = \beta\pi(s^{t+1}|s^t)\max_{i=1,2}\left\{\frac{u'\left(c_i(s^{t+1}|s^t)\right)}{u'\left(c_i(s^t)\right)}\right\}. \tag{13}$$

Definition 1 A *competitive equilibrium with borrowing constraints* $\{A_i\}$ and initial conditions $K(s^{-1})$ and $\{a_i(s^0)\}_{i=1,2}$ satisfying $\sum_i a_i(s^0) = [r(s^0) + (1-\delta)]K(s^{-1})$ is a vector of quantities $\{c_i, k, a_i, K\}$ and prices $\{w, r, q\}$ such that (i) given prices $\{c_i, a_i\}$ solves the problem for each household; (ii) the factor prices $\{w, r\}$ satisfy the optimality conditions of the firm; (iii) q and r satisfy the optimality condition of the intermediary; (iv) all markets clear, i.e. $k(s^t) = K(s^t)$, $\sum_i a_i(s^{t+1}) = [r(s^{t+1}) + 1 - \delta]K(s^t)$, $\sum_i \epsilon_i(s^t) = L(s^t)$ and $\sum_{i\in I} c_i(s^t) + K(s^t) = y\left(s^t\right) + (1-\delta)K(s^{t-1})$.

As stated earlier, we assume that each household has an outside option. In particular, we assume that households can leave the risk sharing arrangement at any date-state and go to financial autarky. In this case, they will only be able to consumer their labour income, and they are excluded from financial markets forever[2]. Similarly to Alvarez and Jermann (1997), we choose limits that are *not too tight*, in the sense that any further loosening will imply that agents with that level of debt prefer to go to financial autarky. In order to determine these limits, we can write (Problem 1) recursively as follows:

$$W^{ce}(a_i(s^t), S_i(s^t)) = u\left(c_i(s^t)\right) + \beta\sum_{s^{t+1}}\pi(s^{t+1}|s^t)W^{ce}(a_i(s^{t+1}), S_i(s^{t+1}))$$

$$\text{s.t. (9) and (10),}$$

$$K(s^t) = \Gamma(S_i(s^{t+1})) \tag{Problem 1'}$$

where $S_i(s^{t+1})$ is defined as before and Γ is the law of motion of the aggregate capital, which is taken as given by the households.

Definition 2 The borrowing constraints on Arrow securities are *not too tight* if they satisfy the following condition for all $i = 1, 2$ and all nodes $s^t \in N$:

$$W^{ce}(A_i(s^t), S_i(s^t)) = V^{ce}(S_i(s^t)) \tag{14}$$

where the value of the outside option is given by:

$$V^{ce}(S_i(s^t)) = \sum_{r=t}^{\infty}\sum_{s^r|s^t}\beta^{r-t}\pi\left(s^r|s^t\right)u\left(w\left(s^r\right)\epsilon_i(s^r)\right). \tag{15}$$

Note that, since the value of staying in the trading contract W^{ce} is increasing in wealth, whereas and V^{ce} is independent of wealth, the fact that $a_i\left(s^t\right) \geq A_i\left(s^t\right)$ implies that, for all $i = 1, 2$ and all $s^t \in N$, $\sum_{r=t}^{\infty}\sum_{s^r|s^t}\beta^{r-t}\pi\left(s^r|s^t\right)u\left(c_i\left(s^r\right)\right) \geq V^{ce}(S_i(s^t))$.

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[2]In section (7), we also consider a case where households are excluded from the complete market scenario described here, but where they can save by accumulating physical capital.

4. Constrained Efficient Allocations

This section characterizes the constrained efficient allocations of the previous economy. This allocation solves a central planning problem where the planner takes into account both the resource constraint and the participation constraints of the two households. If α_i is the initial Pareto weight assigned by the planner to each household, the problem of the planner can be written as follows:

$$Max_{\{c_i, K\}} \sum_{i \in I} \alpha_i \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t) \beta^t u \left(c_i(s^t) \right) \text{ s.t.} \qquad \text{(Problem 2)}$$

$$\sum_{i \in I} c_i(s^t) + K(s^t) = F(z(s^t), K(s^{t-1}), L(s^t)) \qquad (16)$$

$$\sum_{r=t}^{\infty} \sum_{s^r} \beta^{r-t} \pi(s^r) u(c_i(s^r)) \geq V(S_i \left(s^t \right)) \text{ for } i = 1, 2. \qquad (17)$$

As in the previous section, we have assumed that the value of the outside option for type $i \in I$ depends on $S_i \left(s^t \right) = \left(\epsilon_i(s^t); z \left(s^t \right), \epsilon_{-i}(s^t), K \left(s^{t-1} \right) \right)$. and this implies that we can write $V_i \left(s^t \right) = V(S_i \left(s^t \right))$.

The previous equations reflect that standard dynamic programming is inapplicable, since future decision variables appear in the participation constraints. However, following Marcet and Marimon (1999), we can write the Lagrangian of the above problem as follows:

$$\inf_{\{\gamma_i\}} \sup_{\{c_i, K\}} H \equiv \sum_{i \in I} \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t) \beta^t \left\{ u(c_i(s^t))(\mu_i(s^t) + \alpha_i) - \gamma_i(s^t) V(S_i \left(s^t \right)) \right\}.$$

In the previous equation, $\beta^t \gamma_i(s^t)$ denotes the Lagrange multiplier of the time t participation constraint for household $i \in I$, while $\mu_i(s^t) \equiv \sum_{r=0}^{t} \gamma_i(s^r)$, where the summation is over the particular history s^t. Further, this pseudo state variable can be defined recursively by:

$$\mu_i(s^t) = \mu_i(s^{t-1}) + \gamma_i(s^t), \ \mu_i(s^{-1}) = 0 \text{ for } i = 1, 2. \qquad (18)$$

It is easy to see that the solution to the previous problem can be characterized by the resource and participation constraints in (16)-(17) and by the following first order conditions[3]:

$$\frac{u' \left(c_1(s^t) \right)}{u' \left(c_2(s^t) \right)} = \lambda(s^t) = \frac{(1 + v_2(s^t))}{(1 + v_1(s^t))} \lambda(s^{t-1}) \qquad (19)$$

$$1 = \beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \left\{ \frac{u' \left(c_i(s^{t+1}) \right)}{u' \left(c_i(s^t) \right)} (1 + v_i(s^{t+1})) F_K \left(s^{t+1} \right) \right\} \qquad (20)$$

$$-\beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \left\{ \sum_{j=1,2} \frac{v_j(s^{t+1})}{u' \left(c_j(s^t) \right)} V_K(S_j \left(s^{t+1} \right)) \right\} \text{ for } i = 1, 2.$$

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[3] The first-order conditions for this problem are only necessary but not sufficient in general. The reason is that the constraint set defined by (16) and (17) is not necessarily convex. More precisely, convexity is guaranteed only if V is convex in capital. Unfortunately, V is a concave function in our application. So, we assume for now that the first-order conditions are sufficient and we will see later that this assumption is satisfied for the examples we consider.

In the previous Euler condition, the terms $F_K\left(s^{t+1}\right)$ and $V_K(S_i\left(s^{t+1}\right))$ for $i = 1, 2$ on the right hand side represent the derivatives of total output F and of the outside option value V with respect to the aggregate capital stock K. Further, we have expressed the equations in terms of the normalized multipliers λ and v_i, which simplify the system of equilibrium equations and are given by:

$$v_i(s^t) = \frac{\gamma_i(s^t)}{\mu_i(s^{t-1}) + \alpha_i} \text{ for } i = 1, 2 \tag{21}$$

$$\lambda(s^t) = \frac{\mu_2(s^t) + \alpha_2}{\mu_1(s^t) + \alpha_1}, \text{ with } \lambda(s^{-1}) = \frac{\alpha_2}{\alpha_1}. \tag{22}$$

Here, it is important to note that $v_i(s^t) > 0$ only if $\gamma_i(s^t) > 0$ due to the fact that $\mu_i(s^{t-1}) + \alpha_i > 0$. This implies that $v_i(s^t)$ is positive only when the participation constraint of type $i \in I$ is binding. Note also that λ is the "temporary" relative Pareto weight of type 2 households relative to type 1 households. Thus, as usual in models with complete markets, condition (19) says that the relative consumptions of the two types are determined by their (temporary) relative Pareto weights. Further, as in other models with commitment (see for example Thomas and Worrall [1988] and Kocherlakota [1996]) whenever households belonging to type 1 have a binding participation constraint ($v_1(s^t) > 0$), λ is decreasing, and their relative Pareto weight is increasing. The opposite happens when the participation constraint of type 2 household is binding. Finally, notice that, whenever the aggregate technology and the idiosyncratic labor endowment shock are Markovian, the optimal allocation of this problem is recursive in $(\epsilon_1, \epsilon_2, K, \lambda)$.

As reflected by the Euler equation in (20), when the participation constraints are not binding for any household at any continuation history s^{t+1}, implying that $v_i(s^{t+1}) = 0$ for all $s^{t+1}|s^t$ and for $i = 1, 2$, the equation reduces to the standard capital Euler condition of the stochastic growth model. On the other hand, the presence of binding enforcement constraints at a any continuation state s^{t+1} introduces two additional effects on the intertemporal allocation of consumption and capital. First, it increases the planner's marginal rate of substitution between period t and $t+1$ goods, raising the benefits of a higher aggregate capital stock at $t+1$. This is due to the fact that a higher future capital increases output and consequently future consumption of each household type and it is reflected by *the presence of* $v_i\left(s^{t+1}\right)$ on the first part of the right hand side of the equation. Second, a higher capital stock at $t+1$ tightens the enforcement constraints through an increase in the autarky value, reducing the benefits of a higher capital at $t+1$. This is reflected by *the autarky effects* on the second part of the right hand side of the equation.

As shown by Alvarez and Jermann (1997) and Kehoe and Perri (2002,2004), who study the constrained efficient allocations of economies with complete markets and participation constraints, if $\{c_1, c_2\}$ is constrained efficient and $W(S_j\left(s^t\right)) > V(S_j\left(s^t\right))$, it has to be the case that:

$$\frac{u'\left(c_j(s^t)\right)}{u'\left(c_j(s^{t-1})\right)} = \max_{i=1,2} \frac{u'\left(c_i(s^t)\right)}{u'\left(c_i(s^{t-1})\right)}.$$

To see that the previous equality is true, note that equation (19) can be rewritten as:

$$\frac{u'\left(c_1(s^t)\right)}{u'\left(c_2(s^t)\right)} = \lambda(s^t) = \frac{(1 + v_2(s^t))}{(1 + v_1(s^t))} \frac{u'\left(c_1(s^{t-1})\right)}{u'\left(c_2(s^{t-1})\right)}.$$

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First, it is easy to see that $\frac{u'(c_1(s^t))}{u'(c_1(s^{t-1}))} = \frac{u'(c_2(s^t))}{u'(c_2(s^{t-1}))}$ if $v_i(s^t) = 0$ for $i = 1, 2$. Further, if $v_2(s^t) > 0$ and $v_1(s^t) = 0$, implying that the participation constraint is binding for agent two, we have that $\frac{u'(c_1(s^t))}{u'(c_1(s^{t-1}))} > \frac{u'(c_2(s^t))}{u'(c_2(s^{t-1}))}$, and the opposite holds if $v_2(s^t) = 0$ and $v_1(s^t) > 0$. This result also implies that

$$\frac{u'\left(c_1(s^{t+1})\right)(1 + v_1(s^{t+1}))}{u'\left(c_1(s^t)\right)} = \frac{u'\left(c_2(s^{t+1})\right)(1 + v_2(s^{t+1}))}{u'\left(c_2(s^t)\right)}$$

for all $s^{t+1}|s^t$. Thus, we can use the adjusted intertemporal marginal rate of substitution of either agent in the Euler condition (20), which can be rewritten as:

$$1 = \beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \left\{ \max_{i=1,2} \left\{ \frac{u'\left(c_i(s^{t+1})\right)}{u'\left(c_i(s^t)\right)} \right\} F_K\left(s^{t+1}\right) - \sum_{j=1,2} \frac{v_j(s^{t+1})}{u'\left(c_j(s^t)\right)} V_K(S_j\left(s^{t+1}\right)) \right\}. \tag{23}$$

In what follows, we focus on allocations that have high implied interest rates, in the sense that they have a finite present value when discounted with the appropriate present value prices. Following Alvarez and Jermann (1997), we say that an allocation x has high implied interest rates if:

$$\sum_{t=0}^{\infty} \sum_{s^t} Q_p(s^t|s^0)x(s^t) < \infty \text{ for } \forall s^t, t$$

where

$$q_p(s^{t+1}|s^t) = \max_{i=1,2} \beta \pi(s^{t+1}|s^t) \left\{ \frac{u'\left(c_i(s^{t+1})\right)}{u'\left(c_i(s^t)\right)} \right\} \tag{24}$$

$$Q_p(s^t|s^0) = q_p(s^t|s^{t-1})q_p(s^{t-1}|s^{t-2})...q_p(s^1|s^0). \tag{25}$$

In the next section, we show that set-up in Section 3 needs to be modified in order to decentralize the constrained efficient allocation as a competitive equilibrium with trading constraints that are not too tight. Further, the competitive equilibrium described in the previous section is further characterized in section 6.

5.   DECENTRALIZATION WITH SEQUENTIAL TRADE

In this section, we first show that constrained efficient allocations with an outside option of financial autarky cannot be decentralized as a competitive equilibrium with trade in one period ahead Arrow securities subject to borrowing constraints that are not too tight. We then argue that this decentralization is possible if one introduces a savings constraint on the capital holdings of the intermediary.

As to the first finding, a similar result has been shown by Kehoe and Perri (2002,2004), who study an economy with no financial intermediaries and with two production sectors that can be interpreted as countries. In their environment, the impossibility of a decentralization in the presence of participation constraints is due to the two effects on the standard Euler equation discussed above. In contrast to this, we show that this impossibility is solely due to the *autarky effects* in the presence of financial intermediaries.

In addition, the previous authors show that a decentralization is possible if one introduces either a savings constraints on the capital holdings of the households or government default on foreign debt and capital income taxes. In our framework, however, government default is not a viable option, since it requires that agents of a given type are able to coordinate. In addition, it is difficult to imagine that governments would default on behalf of some of the households against some other households in the same economy. Given this, we focus on a decentralization with borrowing constraints. Further, we show that this decentralization is also possible in our framework if one imposes a savings or accumulation constraint on the capital holdings of the intermediary.

Our first result is stated by the following proposition, which is analogous to proposition 5 in Kehoe and Perri (2002).

Proposition 1 *Let* $\{c_1, c_2, K\}$ *be a constrained efficient allocation where* $c\left(s^t\right) = \sum_i c_i(s^t)$ *has high implied interest rates. Then, it cannot be decentralized as a competitive equilibrium with trade in one period ahead Arrow securities subject to borrowing constraints on the Arrow security holdings that are not too tight.*

Proof of Proposition 1: To prove the proposition, recall that the capital Euler equation of the planner can be written as:

$$
\begin{aligned}
1 &= \beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \left\{ \max_{i=1,2} \left[\frac{u'\left(c_i(s^{t+1})\right)}{u'\left(c_i(s^t)\right)} \right] \left[f_K(s^{t+1}) + 1 - \delta \right] \right\} \\
&\quad - \beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \left\{ \sum_{j=1,2} \frac{v_j(s^{t+1})}{u'\left(c_j(s^t)\right)} V_K(S_j\left(s^{t+1}\right)) \right\}.
\end{aligned}
$$

On the other hand, the equilibrium condition of the intermediary in the competitive equilibrium (8) can be rewritten as:

$$
1 = \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t)\beta \left\{ \max_{i=1,2} \left[\frac{u'\left(c_i(s^{t+1})\right)}{u'\left(c_i(s^t)\right)} \right] \right\} \left[f_K\left(s^{t+1}\right) + 1 - \delta \right]
$$

where we have substituted for $q(s^{t+1}|s^t)$ and $r\left(s^{t+1}\right)$ from (13) and (7). Clearly, the previous two equations cannot be satisfied by the same allocation $\{c_1, c_2, K\}$ if the participation constraint is ever binding, that is, if $v_j(s^{t+1}) > 0$ for some s^{t+1} with $\pi(s^{t+1}) > 0$. It therefore follows that constrained efficient allocations cannot be decentralized as a competitive equilibrium with borrowing constraints on the Arrow security holdings that are not too tight.■

Several remarks are worth noting. First, the previous result is in contrast to the one obtained by Alvarez and Jermann (1997), who show that a decentralization of the constrained efficient allocations with borrowing constraints that are not too tight is possible in the absence of capital accumulation. Second, the equations reflect that the impossibility of a decentralization in the presence of financial intermediaries is solely due to the autarky effects on the planner's Euler equation, an observation that will prove to be useful in the next section. To see this, note that, if no intermediaries were present, the budget constraint of the households in the decentralized economy would be given by:

$$
c_i(s^t) + \sum_{s^{t+1}|s^t} q(s^{t+1}|s^t)a_i(s^{t+1}) + k_i\left(s^{t+1}\right) \le w(s^t)\epsilon_i(s^t) + a_i(s^t) + r\left(s^t\right) k_i\left(s^t\right).
$$

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Further, the first order conditions with respect to the individual capital holdings would imply that:

$$1 = \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t)\beta \left\{ \frac{u'\left(c_i(s^{t+1})\right)}{u'\left(c_i(s^t)\right)} \right\} [f_K\left(s^{t+1}\right) + 1 - \delta] \text{ for } i = 1,2.$$

Comparing the previous equation to the Euler equation of the planner, it is easy to see that both cannot be satisfied by the same allocation, even if the value of autarky is independent of capital and $V_K(S_j\left(s^t\right)) \equiv 0$ for all s^{t+1} and $j = 1,2$. Kehoe and Perri (2002,2004) suggests that imposing a savings constraint on the individual capital holdings takes care of both Euler equation effects. In what follows, we show that a similar result can also be obtained in our setup. In particular, we show that the constrained efficient allocations can be decentralized with borrowing constraints on the total asset wealth if one imposes a savings constraint on the capital holdings of the intermediary. This is stated by the following proposition.

Proposition 2 Let $\{c_1, c_2, K\}$ be a constrained efficient allocation where $c\left(s^t\right) = \sum_i c_i(s^t)$ has high implied interest rates. Further, assume that the intermediary in the decentralized economy is subject to a savings constraint of the form $k(s^t) \leq B(s^t)$. Then, the constrained efficient allocations can be decentralized as a competitive equilibrium with borrowing constraints on the total asset wealth that are not too tight.

Before proving the proposition, we start by describing the economy in the presence of a savings constraint on the capital holdings of the intermediary. In particular, if we let $\psi(s^t)$ be the multiplier on the constraint, optimality requires that:

$$1 = \beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \left\{ \max_{i=1,2} \left[\frac{u'\left(c_i(s^{t+1})\right)}{u'\left(c_i(s^t)\right)} \right] \right\} [r(s^{t+1}) + (1-\delta)] - \psi(s^t.) \qquad (26)$$

Since $\psi(s^t) \geq 0$, the previous equation implies that $1 \leq \sum_{s^{t+1}|s^t} q(s^{t+1}|s^t)[r(s^{t+1}) + (1-\delta)]$, and the intermediary makes therefore nonnegative profits, given by:

$$d\left(s^t\right) = \sum_{s^{t+1}|s^t} q(s^{t+1}|s^t)[r(s^{t+1}) + (1-\delta)]k(s^t) - k(s^t) = \psi(s^t)k(s^t). \qquad (27)$$

As before, the intermediaries operate for two periods. Further, we assume that the profits they make are distributed to the households when they are realized, i.e. during the first period of the intermediary's life-cycle. The period before an intermediary starts its business, households own equal shares of it, which they can immediately trade at a price $p\left(s^t\right)$, representing the total value of an intermediary that will pay profits next period. Thus, if $\theta_i\left(s^t\right)$ denote the share representing a claim to the next period profits held by household $i \in I$ at the end of period t, the budget constraint can be written as:

$$c_i(s^t) + \sum_{s^{t+1}|s^t} q(s^{t+1}|s^t)a_i(s^{t+1}) + p\left(s^t\right)\left[\theta_i\left(s^t\right) - \frac{1}{2}\right] \leq d(s^t)\theta_i\left(s^{t-1}\right) + a_i(s^t) + w_i\left(s^t\right)$$

where $\theta_i\left(s^{-1}\right) = \frac{1}{2}$ for $i = 1,2$. The absence of arbitrage implies that the price of the shares has to satisfy:

$$p\left(s^t\right) = \sum_{s^{t+1}|s^t} q(s^{t+1}|s^t)d(s^{t+1}). \qquad (28)$$

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Further, if we define $\bar{c}_i(s^t) = c_i(s^t) - w_i\left(s^t\right) - p\left(s^t\right)\frac{1}{2}$ and we denote by $\omega_i\left(s^t\right) = d(s^t)\theta_i\left(s^{t-1}\right) + a_i(s^t)$ the total asset wealth of the household at the beginning of period t, we have that:

$$\bar{c}_i(s^t) + \sum_{s^{t+1}|s^t} q(s^{t+1}|s^t)\omega_i\left(s^{t+1}\right) \leq \omega_i\left(s^t\right) \tag{29}$$

Finally, we assume that the total asset wealth of the household at the end of period t is subject to borrowing constraints that are not too tight, i.e.,

$$\omega_i\left(s^{t+1}\right) \geq A_i\left(s^{t+1}\right) \tag{30}$$

where $W^{ce}(A_i(s^{t+1}), S_i(s^{t+1})) = V^{ce}(S_i(s^{t+1}))$, and the two value functions are defined as above. A competitive equilibrium can then be defined as in the previous section, noting that market clearing now implies that $\sum_i \omega_i(s^{t+1}) = \left[r\left(s^{t+1}\right) + (1-\delta)\right]K\left(s^t\right) + d\left(s^t\right)$ and $\sum_i c_i\left(s^t\right) + K\left(s^t\right) = y\left(s^t\right) + (1-\delta)K\left(s^{t-1}\right)$. Further, the transversality condition is now given by:

$$\lim_{t\to\infty}\sum_{s^t}\beta^t\pi(s^t)u'\left(c_i(s^t)\right)\left[\omega_i(s^t) - A_i(s^t)\right] \leq 0. \tag{31}$$

It is easy to see that constrained efficient allocations assuming that the outside option involves consuming the labor income can now be decentralized as a competitive equilibrium. The proof is presented in what follows, and it extends the ones in Alvarez and Jermann (1997) and Kehoe and Perri (2002) to the presence of financial intermediaries that operate the investment technology and are subject to a savings constraint.

Proof of Proposition 2: We first note that the savings constraint $B(s^t)$ can be set so that a constrained efficient allocation that satisfies the planner's capital Euler equation in (23) also satisfies the optimality condition of the intermediary in (26). In particular, when the enforcement constraint in the planner's problem does not bind for any household at period $t+1$, implying that $v_i\left(s^{t+1}\right) = 0$ for $i = 1, 2$ and all s^{t+1}, $B(s^t)$ is set to an arbitrary large number. Further, when the enforcement constraint in the planner's problem is binding for any of the two households, $B(s^t)$ is set to the level of capital that solves the optimal allocation. In particular, we can always choose $B(s^t)$ such that the accumulation constraint is given by[4]:

$$\psi(s^t) = \beta\sum_{s^{t+1}|s^t}\pi(s^{t+1}|s^t)\left\{\sum_{i=1,2}\frac{v_i(s^{t+1})}{u'\left(c_i(s^t)\right)}V_K(S_i\left(s^{t+1}\right))\right\}$$

where $\psi(s^t) \geq 0$ due to the fact that $v_i(s^{t+1}) \geq 0$, $u'\left(c_i(s^t)\right) \geq 0$ and $V_K(S_i\left(s^{t+1}\right)) \geq 0$ for $i = 1, 2$. Further, the last inequality follows from the fact that the marginal product of labor is increasing in capital[5].

Given the capital allocation from the planner's problem, we can construct the wage and capital rental rates $w\left(s^t\right) = f_L\left(s^t\right)$ and $r\left(s^t\right) = f_K\left(s^t\right)$ that satisfy the optimality conditions of the firm in the competitive equilibrium at each node. Further, given the

---

[4]This is possible due to the fact that $\lim_{B(s^t)\to 0}\psi(s^t) = +\infty$ and $\lim_{B(s^t)\to\infty}\psi(s^t) = 0$.

[5]It is important to note that, with all production functions in the CES family, wages are concave in capital and V is therefore a concave function of capital as well.

consumption allocations from the planner's problem, we can use (24) and (25) to define the prices $q(s^{t+1}|s^t) = q_p(s^{t+1}|s^t)$ and $Q(s^{t+1}|s^t) = Q_p(s^{t+1}|s^t)$. In addition, $q(s^{t+1}|s^t)$ can be used to define the multiplier $\gamma_i^{ce}(s^{t+1})$ so that the asset Euler condition of the agents is satisfied. Note that it will have the desired properties. In particular, if $v_i = 0$, $\gamma_i^{ce}(s^{t+1}) = 0$. Further, if $v_i(s^{t+1}) > 0$, it follows that $\gamma_i^{ce}(s^{t+1}) > 0$. To see this, suppose that $v_j(s^{t+1}) > 0$ for some $j = 1, 2$. Then,

$$\beta\pi(s^{t+1}|s^t)\frac{u'\left(c_j(s^{t+1})\right)}{u'\left(c_j(s^t)\right)} < \max_{i=1,2}\left\{\pi(s^{t+1}|s^t)\beta\frac{u'\left(c_i(s^{t+1})\right)}{u'\left(c_i(s^t)\right)}\right\}$$

and

$$q(s^{t+1}|s^t) = \beta\pi(s^{t+1}|s^t)\max_{i=1,2}\left\{\frac{u'\left(c_i(s^{t+1})\right)}{u'\left(c_i(s^t)\right)}\right\} = \beta\pi(s^{t+1}|s^t)\frac{u'\left(c_j(s^{t+1})\right)}{u'\left(c_j(s^t)\right)} + \frac{\gamma_i^{ce}(s^{t+1})}{u'\left(c_j(s^t)\right)}.$$

Since the high implied interest rate condition holds, we can then use the budget constraint of the households in the competitive equilibrium to construct the wealth levels $\omega_i(s^t)$ that support the constrained efficient consumption allocations at every node. To do this, we first construct the profits $d\left(s^t\right)$ from (27), the share price $p\left(s^t\right)$ from (28), and the individual labor incomes from $w_i\left(s^t\right) = w\left(s^t\right)\epsilon_i\left(s^t\right)$. Further, we iterate on the budget constraint of each household to obtain:

$$\omega_i(s^t) = \sum_{n=0}^{\infty}\sum_{s^{t+n}|s^t} Q(s^{t+n}|s^t)\bar{c}_i(s^{t+n}) \tag{32}$$

and we let:

$$\omega_i(s^0) = \sum_{t=0}^{\infty}\sum_{s^t|s^0} Q(s^t|s^0)\bar{c}_i(s^t). \tag{33}$$

Finally, note that we can also recover the individual asset levels $a_i\left(s^t\right)$ and $\theta_i\left(s^{t-1}\right)$ from the following set of equations:

$$\omega_i\left(s^t\right) = d\left(s^t\right)\theta_i\left(s^{t-1}\right) + a_i\left(s^t\right) \text{ for all } s^t|s^{t-1}$$

$$\bar{c}_i\left(s^{t-1}\right) + p\left(s^{t-1}\right)\theta_i\left(s^{t-1}\right) + \sum_{s^t|s^{t-1}} q\left(s^t|s^{t-1}\right)a_i\left(s^t\right) = \omega_i\left(s^{t-1}\right).$$

The initial asset holding $a_i\left(s^0\right)$ are given by $\omega_i\left(s^0\right) = \frac{1}{2}d\left(s^0\right) + a_i\left(s^0\right)$ (recall that $\theta_i\left(s^{-1}\right) = \frac{1}{2}$).

Concerning the trading limits, if $v_i(s^t) = 0$ for agent i, we first set:

$$A_i(s^{t+1}) = -\sum_{n=1}^{\infty}\sum_{s^{t+n}|s^t} Q(s^{t+n}|s^t)\left[w_i(s^{t+n}) + \frac{1}{2}p\left(s^{t+n}\right)\right]$$

and we will redefine the limit for these cases later. In addition, if $v_i(s^t) > 0$, we set $A_i(s^{t+1}) = \omega_i(s^{t+1})$, implying that it will be binding when the participation constraint in the planner's

problem is binding. The transversality condition is satisfied, since:

$$\lim_{t\to\infty} \sum_{s^t} \beta^t \pi(s^t) u'\left(c_i(s^t)\right) \left[\omega_i(s^t) - A_i(s^t)\right]$$

$$\leq \ \lim_{t\to\infty} \sum_{s^t} \beta^t \pi(s^t) u\left(c_i(s^t)\right) \left[\sum_{n=0}^{\infty} \sum_{s^{t+n}|s^t} Q(s^{t+n}|s^t) c_i(s^{t+n})\right]$$

$$\leq \ u'\left(c_i(s^0)\right) \lim_{t\to\infty} \sum_{s^t} \beta^t \pi(s^t) \frac{u'\left(c_i(s^t)\right)}{u'\left(c_i(s^0)\right)} \left[\sum_{n=0}^{\infty} \sum_{s^{t+n}|s^t} Q(s^{t+n}|s^t) \sum_i c_i(s^{t+n})\right]$$

$$\leq \ u'\left(c_i(s^0)\right) \lim_{t\to\infty} \sum_{s^t} Q(s^t|s^0) \left[\sum_{n=0}^{\infty} \sum_{s^{t+n}|s^t} Q(s^{t+n}|s^t) \sum_i c_i(s^{t+n})\right] = 0.$$

The first inequality follows from the fact that $[\omega_i(s^t) - A_i(s^t)]$ is equal to zero if the participation constraint is binding and it is equal to $\sum_{n=0}^{\infty} \sum_{s^{t+n}|s^t} Q(s^{t+n}|s^t) c_i(s^{t+n}) \geq 0$ otherwise, since $\omega_i(s^t) = \sum_{n=0}^{\infty} \sum_{s^{t+n}|s^t} Q(s^{t+n}|s^t) \bar{c}_i(s^{t+n})$. The second follows from the fact that $c_i(s^t) \leq \sum_i c_i(s^t)$. The third inequality follows from the the definition of $Q(s^t|s^0)$ and from the fact that $Q(s^t|s^0) \geq \beta^t \pi(s^t) \frac{u'\left(c_i(s^t)\right)}{u'(c_i(s^0))}$ by construction. Finally, the last equality follows form the high implied interest rate condition.

To show that markets clear, we can sum the total asset wealth in (32) and (33), obtaining that:

$$\sum_i \omega_i(s^t) = \sum_{n=0}^{\infty} \sum_{s^{t+n}|s^t} Q(s^{t+n}|s^t) \sum_i \bar{c}_i(s^{t+n}) = \left[f_K(s^t) + (1-\delta)\right] K(s^{t-1}) + d\left(s^t\right)$$

$$\sum_i \omega_i(s^0) = \sum_{t=0}^{\infty} \sum_{s^t} Q(s^t|s^0) \sum_i \bar{c}_i(s^t) = \left[f_K(s^0) + (1-\delta)\right] K(s^0) + d\left(s^0\right).$$

This also implies that $\sum_i a_i\left(s^t\right) = \left[r\left(s^t\right) + (1-\delta)\right] K\left(s^{t-1}\right)$ and $\sum_i \theta_i\left(s^t\right) = 1$. In addition, summing the two budget constraints, we have that:

$$\sum_i c_i(s^t) \ = \ \sum_i \omega_i\left(s^t\right) + p\left(s^t\right) + w\left(s^t\right) - \sum_{s^{t+1}|s^t} q\left(s^{t+1}|s^t\right) \sum_i \omega_i\left(s^{t+1}\right)$$

$$= \ y\left(s^t\right) + (1-\delta)K(s^{t-1}) - K\left(s^t\right).$$

It only remains to redefine the borrowing limits so that they are not too tight. To do this, we first construct the autarky values at each node using the allocations of the planner:

$$V^{ce}(S_i(s^t)) = \sum_{r=t}^{\infty} \sum_{s^r|s^t} \beta^{r-t} \pi\left(s^r\right) u\left(f_L\left(s^r\right) \epsilon_i\left(s^r\right)\right).$$

We can then use (Problem 1') to generate the value function $W^{ce}(\omega_i(s^t), S_i(s^t))$ and we use these two functions to redefine the borrowing constraints for the nodes where the limit is not binding. In particular, we can iterate on the constraint $A_i(s^t)$ until we find the one

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that satisfies $W^{ce}(A_i(s^t), S_i(s^t)) = V^{ce}(S_i(s^t))$. Since the new set of constraints constraint is (weakly) tighter than before, the new value of $\omega_i - A_i$ still satisfies the transversality condition. Further, since, these constraints do not bind for any household for whom the participation constraint is not binding in the planner's solution, the allocation derived above with the original constraints is still feasible and optimal.∎

6.   Characterization of the CE without Savings Constraints

The previous section shows that a decentralization of the constrained efficient allocations with sequential trade and borrowing constraints is possible in the presence of financial intermediaries that are subject to a savings constraint on the capital holdings. These type of constraints, however, are difficult to interpret and are not typically found in the data. In particular, it is difficult to imagine how these upper bounds would arise as an equilibrium outcome. On the other hand, Proposition 5 at the end of this section Proposition shows that, if the intermediaries can set the borrowing limits on households, they will choose the ones which are not to tight. Given this, the present section characterizes the competitive equilibrium allocations with no savings constraints. In particular, it is shown that it satisfies the same system of equations as the constrained efficient problem except the Euler condition in (20), which can be replaced by:

$$1 = \beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \left\{ \frac{u'\left(c_i(s^{t+1})\right)}{u'\left(c_i(s^t)\right)} \left(1 + v_i\left(s^{t+1}\right)\right) [f_K(s^{t+1}) + 1 - \delta] \right\}. \qquad (34)$$

This result is stated by the following proposition.

Proposition 3 *Let $\{c_1, c_2, K\}$ be a solution to equations (16), (17), (19), (21), (22) and (34) where $c = \sum_i c_i$ has high implied interest rates. Then, this allocation can be decentralized as a competitive equilibrium with trade in one period ahead Arrow securities subject to borrowing constraints on the Arrow security holdings that are not too tight.*

Proof of Proposition 3: The proof follows the same arguments as the proof of proposition 2, and we therefore only sketch it in what follows. First, given the consumption allocations $\{c_i\}_{i=1,2}$ from the planner's problem, we can use (24) and (25) to define the prices $q(s^{t+1}|s^t) = q_p(s^{t+1}|s^t)$ and $Q(s^{t+1}|s^t) = Q_p(s^{t+1}|s^t)$ for all nodes. Further, since the high implied interest rate condition holds, we can then use the prices and the budget constraint of the households to construct the holdings $\{a_i\}_{i=1,2}$ so that the constrained efficient consumption allocations $\{c_i\}_{i=1,2}$ are feasible at every node. Note that, in the absence of a savings constraint, the profits of the intermediary are always equal to zero. Concerning the trading limits, if $v_i(s^t) = 0$ for agent i, we first set $A_i(s^{t+1}) = -\sum_{n=1}^{\infty} \sum_{s^{t+n}|s^t} Q(s^{t+n}|s^t) w_i(s^{t+n})$ and we will redefine this limit later. Further, if $v_i(s^t) > 0$, we set $A_i(s^{t+1}) = a_i(s^{t+1})$, implying that it will be binding when the participation constraint in the planner's problem is binding. To make sure that the sufficient Euler equations are satisfied, we can first use $q(s^{t+1}|s^t)$ to define the multiplier $\gamma_i^{ce}(s^{t+1})$ so that the Euler condition of the agents in (11) is satisfied. It is easy to see that an allocation that satisfies (34) also satisfies the equilibrium condition of the intermediary in (8). Further, using the same arguments as in the proof of proposition 2, we can check that the transversality condition in (12) is satisfied. Finally, we can construct the value functions $W(a_i(s^t); S_i(s^t))$ and $V(S_i(s^t))$ from the value functions of the planner's problem and redefine the borrowing constraints on Arrow security holdings

so that they satisfy $W(A_i(s^{t+1}); S_i(s^{t+1})) = V(S_i(s^{t+1}))$ at every node. Since these limits do not bind for the originally unconstrained consumers, the allocations obtained under the natural borrowing limits are still feasible and optimal.■

The following proposition shows that the reverse is also true.

Proposition 4 *Let $\{c_1, c_2, K, q, r, w\}$ be a competitive equilibrium with borrowing constraints $\{A_i\}$ that are not too tight. Then $\{c_1, c_2, K\}$ is a solution to equations (16), (17), (19), (21)-(22) and (34). Further, $c\left(s^t\right) = \sum_i c_i(s^t)$ satisfies the high implied interest rates condition with respect to the price $Q(s^t|s^0)$ defined by:*

$$Q(s^t|s^0) = q(s^t|s^{t-1})q(s^{t-1}|s^{t-2})...q(s^1|s^0).$$

Proof of Proposition 4: To prove the proposition, we first note that the resource constraint in (16) is satisfied by the equilibrium allocations. Since the asset holdings are subject to portfolio restrictions $\{A_i\}$ that are not too tight, the value functions in the competitive equilibrium satisfy:

$$W^{ce}(a_i(s^t), S_i(s^t)) \geq V^{ce}(S_i(s^t))$$

for all $i = 1, 2$ and all $s^t \in N$, where:

$$W^{ce}(a_i(s^t), S_i(s^t)) = \sum_{r=t}^{\infty} \sum_{s^r} \beta^{r-t} \pi(s^r|s^t) u(c_i(s^r))$$

$$V^{ce}(S_i(s^t)) = \sum_{r=t}^{\infty} \sum_{s^r} \beta^{r-t} \pi(s^r|s^t) u(w(s^r)\epsilon_i(s^r)).$$

It therefore follows that the functions defined by $W(S_i(s^t)) = W^{ce}(a_i(s^t), S_i(s^t))$ and $V(S_i(s^t)) = V^{ce}(S_i(s^t))$ satisfy the participation constraints in (17). We also note that the competitive equilibrium allocations still solve the same problem if the borrowing constraints on the Arrow securities of the unconstrained households are substituted for the natural borrowing limits, defined by:

$$A_i(s^{t+1}) = -\sum_{n=1}^{\infty} \sum_{s^{t+n}|s^t} Q(s^{t+n}|s^t) w_i\left(s^{t+n}\right).$$

Optimality implies that the previous limit is finite[6]. In addition, since the shocks z and ϵ_i lie in a compact set, the present values of K and $f_L\left(s^t\right)$ are finite. Using the resource constraint, it is then easy to see that the competitive equilibrium allocation satisfies the high implied interest rate condition.

To recover the multipliers in the planner's problem, we can first use the equilibrium consumption allocations to define $\lambda(s^t) = \frac{u'\left(c_1(s^t)\right)}{u'(c_2(s^t))}$. Further, $\{v_i\}_{i=1,2}$ can be recovered as

---

[6]In an exchange economy context with sequential trade and potentially incomplete financial markets, Santos and Woodford (1997) show that the natural borrowing limit implied by the optimal allocations has to be finite. Otherwise, one can construct a portfolio that yields more utility than the optimal allocation. The same proof can be used in the present setup.

follows. If the portfolio constraint is not binding for household i at node s^t in the decentralized problem, we set $v_i(s^t) = 0$. Otherwise, if it is binding for agent two, we set $v_1(s^t) = 0$ and $v_2(s^t)$ is recovered from:

$$\frac{u'\left(c_1(s^t)\right)}{u'\left(c_2(s^t)\right)} = (1 + v_2(s^t))\frac{u'\left(c_1(s^{t-1})\right)}{u'\left(c_2(s^{t-1})\right)}$$

Similarly, if it is binding for agent one, we set $v_2(s^t) = 0$ and $v_1(s^t)$ is recovered from:

$$\frac{u'\left(c_1(s^t)\right)}{u'\left(c_2(s^t)\right)} = \frac{1}{(1 + v_1(s^t))}\frac{u'\left(c_1(s^{t-1})\right)}{u'\left(c_2(s^{t-1})\right)}.$$

Clearly, this implies that equations (19) and (21)-(22) are satisfied. In addition, it also follows that:

$$\max_{i=1,2}\frac{u'\left(c_i(s^t)\right)}{u'\left(c_i(s^{t-1})\right)} = (1 + v_2(s^t))\frac{u'\left(c_2(s^t)\right)}{u'\left(c_2(s^{t-1})\right)} = \frac{u'\left(c_1(s^t)\right)}{u'\left(c_1(s^{t-1})\right)}(1 + v_1(s^t)).$$

Finally, since equation (8) in the decentralized solution implies that:

$$
\begin{aligned}
1 &= \beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \left\{ \max_{i=1,2} \left[\frac{u'\left(c_i(s^{t+1})\right)}{u'\left(c_i(s^t)\right)}\right] \left(f_K\left(s^{t+1}\right) + 1 - \delta\right) \right\} \\
&= \beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \left\{ \left[\frac{u'\left(c_i(s^{t+1})\right)}{u'\left(c_i(s^t)\right)}(1 + v_i(s^{t+1}))\right] \left(f_K(s^{t+1}) + 1 - \delta\right) \right\}
\end{aligned}
$$

the modified Euler equations (34) is also satisfied. ∎

Several remarks are worth noting. First, whereas the competitive equilibrium without savings constraints solves a system of equations that is very similar to the optimal planner's problem, considerably simplifying the equilibrium computations, the solution is still suboptimal due to the fact that it ignores the autarky effects. The key here that financial intermediaries do not internalize the effect of capital accumulation on default incentives, whereas the planner internalizes this effect in the (constrained) optimal allocation. Further, the next section shows that these autarky effects are quantitatively unimportant. Thus, there is little difference between the optimal allocations of the CE model without savings constraints.

Second, $\lambda(s^t)$ measures the relative wealth of the two types of households. To see this, we can define the Lagrange multipliers of (9) by $\beta^t \xi_i(s^t)$. In the competitive equilibrium, we then have that:

$$\frac{\xi_1(s^t)}{\xi_2(s^t)} = \frac{u'\left(c_1(s^t)\right)}{u'\left(c_2(s^t)\right)} = \lambda(s^t),$$

where the second inequality is a consequence of Proposition 4. The above identity implies that $\lambda(s^t)$ measures the relative wealth of type 2 versus type 1, since the bigger is $a_i(s^t)$ the smaller is $\xi_i(s^t)$, which measures the marginal utility of wealth. Therefore a higher $\lambda(s^t)$ implies that agent 1 has a smaller initial wealth compared to type 2 households.

Third, our equilibrium concept and the characterization provided above does not necessarily require that the limits are not too tight. In particular, the participation constraints

will be satisfied for any limits that are equal or tighter to the limits defined by (14). In what follows we provide some micro foundations for these limits by letting the intermediaries set them. We first show that no intermediary has incentives to loosen or tighten the limits individually when they are set to be not too tight. This implies that this choice of the constraints is indeed an equilibrium decision of the intermediaries. We also show that no symmetric equilibrium exists where some or all of the limits are looser than the ones dictated by (14).

Proposition 5 (i) *The CE with borrowing constraints that are not too tight remains to be a competitive equilibrium if intermediaries can set the borrowing limits. **(ii)** No symmetric competitive equilibrium exists for limits that are looser than the ones that are no too tight.*

Proof of Proposition 5: (i) We now show that there are no profitable deviations from the equilibrium allocation with limits that are looser then the ones defined by (14). To see this, first notice that tightening the limits will not increase the profits of any intermediary. In what follows we show that no intermediary can make positive profits by making loosening the limits, that is, by setting $\overline{A}_i(s^t) \leq A_i(s^t) < 0$ for all s^t and assume (without a loss of generality) that $\overline{A}_1(\widehat{s}) < A_1(\widehat{s})$ for some $\widehat{s}|\widetilde{s}$ where the participation constraint was binding for agent at wealth level $A_1(\widehat{s})$. This implies that under the original prices $q(s^{t+1}|\widetilde{s})$, type 1 agents would default if next period node $\widehat{s}|\widetilde{s}$ occurs. Since type 1 households would choose $a_1(\widehat{s}) < A_i(\widehat{s}) < 0$ and default if $\widehat{s}$ occurs, it is easy to see that the intermediary would make negative profits. First define $\overline{a}_1(s^{t+1}|\widetilde{s})$ as the asset decision of type 1 households under the new limits and observe that $\overline{a}_1(\widehat{s}) < A_i(\widehat{s}) < 0$ under $q(\widehat{s}|\widetilde{s})$. Then default of type 1 households imply that the profits of the intermediary is given by:

$$
\begin{aligned}
\overline{d}(\widetilde{s}) &= -k(\widetilde{s}) + \sum_{s^{t+1}|\widetilde{s}} q(s^{t+1}|\widetilde{s})[r(s^{t+1}) + (1-\delta)]k(\widetilde{s}) + q(\widehat{s}|\widetilde{s})\overline{a}_1(\widehat{s}) \\
&< -k(\widetilde{s}) + \sum_{s^{t+1}|\widetilde{s}} q(s^{t+1}|\widetilde{s})[r(s^{t+1}) + (1-\delta)]k(\widetilde{s}) = 0.
\end{aligned}
$$

For the second equality we used the intermediaries equilibrium condition (8).

(ii) Now we show that there does not exist any symmetric equilibrium with limits that are looser than the limits that are not too tight. To do this, we assume that the limits are such that some agent would default in equilibrium. In particular, we assume that there exists an equilibrium with prices q and limits $\{A_i\}_{i=1,2}$ such that agents of type 1 would default under some continuation history $s^{t+1}|s^t = \widehat{s}|s^t$ if the current history is $s^t = \widetilde{s}$. First, notice that perfect competition would still require that intermediaries will make zero profits. Theirs profits are given by:

$$
d(\widetilde{s}) = -k(\widetilde{s}) + \sum_{s^{t+1}|\widetilde{s}} q(s^{t+1}|\widetilde{s})[r(s^{t+1}) + (1-\delta)]k(\widetilde{s}) + q(\widehat{s}|\widetilde{s})a_1(\widehat{s}) = 0.
$$

Since a household would only default at node $\widehat{s}$ if $a_1(\widehat{s}) < 0$, the previous equation implies that:

$$
-k(\widetilde{s}) + \sum_{s^{t+1}|\widetilde{s}} q(s^{t+1}|\widetilde{s})[r(s^{t+1}) + (1-\delta)]k(\widetilde{s}) > 0.
$$

Thus, in any symmetric equilibrium with default, it must be the case that:

$$
\sum_{s^{t+1}|\widetilde{s}} q(s^{t+1}|\widetilde{s})[r(s^{t+1}) + (1-\delta)] - 1 > 0.
$$

The previous condition implies that any intermediary could make arbitrarily positive profits by trading only with agents of type 2 and by demanding arbitrary large amounts of total deposits $(\sum_{s^{t+1}|\tilde{s}} q(s^{t+1}|\tilde{s})a_2(\tilde{s}))$ from them. However, this contradicts the fact that the original portfolio was optimal for the intermediaries under $q(s^{t+1}|s^t)$. ∎

7. Quantitative Comparison of the Competitive Equilibria

In this section we solve numerically for both competitive equilibrium allocations (with and without savings constraints). The parameters of the economy are calibrated following the asset pricing and real business cycle literature. The time period is assumed to be one quarter, and the discount factor and depreciation rate are therefore set to $\beta = 0.99$ and $\delta = 0.025$. Concerning the functional forms, we assume that the production function is Cobb-Douglas, with a constant capital share of $\alpha = 0.36$. Further, the utility function of the households is assumed to be $u(c) = \log(c)$. Finally, the exogenous shock processes are assumed to be independent. In particular, the aggregate technology shock follows a two state Markov chain with $z \in \{z_l, z_h\} = \{0.99, 1.01\}$, and its transition matrix is given by:

$$\Pi_z = \begin{bmatrix} \pi_{ll} & \pi_{lh} \\ \pi_{hl} & \pi_{hh} \end{bmatrix} = \begin{bmatrix} 0.875 & 0.125 \\ 0.125 & 0.875 \end{bmatrix}.$$

We assume that aggregate labor supply is constant and we normalize it to 1. As to the idiosyncratic income process, it is assumed to follow a seven state Markov chain. Further, the values and transition matrix are obtained by using the Hussey and Tauchen (1991) procedure to discretize the following process:

$$\epsilon^{i\prime} = (1 - \psi_\epsilon)\mu_\epsilon + \psi_\epsilon \epsilon^i + u, \ u \sim N(0, \sigma_u^2).$$

where the shock parameters are set to $\psi_\epsilon = 0.956$ and $\sigma_u^2 = 0.082$, corresponding to quarterly adjusted estimates from annual data used by Aiyagari (1994). Constant aggregate labor supply implies that $\epsilon^{-i} = 1 - \epsilon^i$, and the values for ϵ^1 were chosen to be symmetric around $\mu_\epsilon = 0.5$. Consequently, the idiosyncratic productivity of the two types follows the same process and the shocks are perfectly negatively correlated across the two types.

Note that Proposition 3 and 4 provide us with a relatively easy and analogous solution method for both models. In the competitive equilibrium with saving constraint (*autarky effects*) we use equations (16), (17), (19), (21), (22) and (20). Further, to solve for the competitive equilibrium with no savings constraints (*no autarky effects*), we use the same system of equations but (20) is replaced by (34).

In what follows, we let $s_1 = [\epsilon, \lambda; z, K]$ and $s_2 = [1 - \epsilon, 1/\lambda; z, K]$. Under our Markovian assumption on the shocks, the previous set of equations implies that we can describe the optimal allocations in both models by the consumption functions $\{c_i(s_i)\}_{i=1,2}$, the normalized multipliers on the participations constraints $\{\nu_i(s_i)\}_{i=1,2}$ and the laws of motion for the relative wealth $\lambda'(s_1)$ and aggregate capital $K'(s_1)$. To solve for these functions, we have used policy functions iterations in both models

Our numerical results are presented on Figures 1 to 4 in the Appendix. All the optimal policies are conditioned on the low aggregate technology shock $z = 0.99$ and on $K = 38.6$, which is in the stationary distribution of capital. For expositional convenience, we have plotted the results for only three levels of the labour endowment, where ϵ_1 is the lowest and ϵ_7 is the highest labor endowment. Recall that type 2 households have the highest labor

endowment when type 1 households have the lowest. Note also that both types have equal endowments when $\epsilon_4 = 1 - \epsilon_4 = 0.5$.

Figure 1 displays $\lambda' \equiv \lambda(s^{t+1})$ as a function of $\lambda(s^t)$ for the three different levels of the idiosyncratic income shocks. The first thing that is reflected by the figure is that agents enjoy permanent perfect risk sharing in the long run in both models. To see this, assume first that our initial λ is inside its ergodic set, which is equal to $\lambda \in [0.8368, 1.195]$ and $\lambda \in [0.8366, 1.1953]$ for the models without and with the savings constraint respectively. As we see on the graph, $\lambda' = \lambda$ inside this region, independently of the labor income shocks. Condition (19) implies then that this can only happen if neither agent's participation constraint is binding. Second, the same condition implies that the ratio of marginal utilities remains constant over time if this is the case. These facts, however, are the defining feature of a perfect risk sharing allocation. Assume now that we start with $\lambda > 2.5$, implying that type 1 households hold significantly lower initial assets, and they are therefore entitled to less consumption than agent 2. In this case, Figure 1 implies that λ' depends on the idiosyncratic income of the agent, and that it will drop to a new level depending on the shock realization. In particular, the higher the idiosyncratic income, the lower will be the new level of the relative wealth, since type 1 agents require a higher compensation for staying in the risk sharing arrangement. Note that, whenever λ jumps, type 1 agents' participation constraint is binding, and these new level of λ' pins down the borrowing constraint faced by type 1 households in the previous period. This process will go on until the highest income (ϵ^7) is experienced by the type 1 agents. In this case, λ will enter the stationary distribution[7] ($\lambda = 1.195$) and remain constant forever, implying that agents enjoy permanent perfect risk sharing from that period on. In addition, a symmetric argument implies that whenever $\lambda < 0.83$, λ will become 0.83 and remain constant forever after finite number of periods. Given this, the present framework implies that agents will obtain full insurance in the long-run, independently of the initial wealth distribution. On the other hand, the economy may experience movements in consumption and the relative wealth in the short run.

The second important observation is that, comparing the two economies, we observe minor differences only. First of all, the long-run behavior is practically identical. Both economies enjoy perfect risk sharing in the long run. In addition, if $\lambda(s^0) \in [0.8368, 1.195]$, then the long-run allocations are identical. This is due to the fact that the borrowing constraints (and therefore the savings constraint of the intermediary in the constrained efficient economy) will never bind in this case, implying that individual consumptions will be determined by $\lambda(s^0)$, and capital accumulation will be (unconstrained) efficient. On the other hand, if $\lambda(s^0)$ is outside the above interval, the long-run allocations are only slightly different, since the bounds of the stationary distribution are slightly different in the two models. As we see, the model with savings constraint allows for a slightly wider range of the wealth distribution. Also, the model with autarky effects allows for a wider range of λ' outside the stationary distribution. As we will see below, this is the consequence of the different capital accumulation pattern in the two economies.

Figure 2, shows the optimal consumption of type 1 households in the two economies as a function of λ for different levels of the labor endowment. Obviously, as the relative wealth

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[7]We use the terms ergodic set and the stationary distribution loosely in this paper. Notice, however that we defined these sets as the possible values of λ in the long run. In fact, the initial condition λ_0 will pin down a unique long-run value for the relative wealth, that is, for any given initial value, the long run distribution is degenerate.

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of type 1 households decreases (λ increases) their consumption decreases. Also, since we have perfect risk sharing in the stationary distribution, consumption does not depend on the idiosyncratic labour endowment there. For the same reason, the optimal consumption allocations are identical across the two models in this range. Outside the stationary distribution, as expected, consumption is increasing in the labour endowment. Also, we observe that the model with autarky effects allows for a higher consumption for every λ and ϵ outside the stationary distribution. On the one hand, this is not surprising, since these allocations correspond to the constrained efficient allocations, where we should expect a higher consumption. On the other hand, the resource constraint then implies that, in the model without saving constraints, aggregate capital accumulation will be higher.

Figure 3 displays the next period's aggregate capital K' as a function of λ and ϵ and it documents the previously mentioned pattern. Not surprisingly, aggregate capital is again independent of both the wealth distribution and labour endowments in the stationary distribution, where it is set to its efficient level. On the other hand, markets are effectively incomplete outside the stationary distribution, where we see a higher capital accumulation. This result is well-documented in models with exogenously incomplete markets, see Aiyagari (1994) for a model without aggregate uncertainty and Ábrahám and Cárceles-Poveda (2005) for a model with a similar set-up but trade in physical capital only. Note also that these effects are the biggest when low idiosyncratic labour endowment coincides with low wealth (this is the case for type 1 households on the upper right corner of the figure and for type 2 households in the upper left corner).

To see why this happens, we can look at Figure 1 and at the Euler equation of the constrained efficient problem in (20). It is clear for Figure 1 that, when type one households have labour endowment ϵ_7 and a high wealth ($\lambda < 0.5$), the participation constraint of type 2 households is going to be binding in many continuation states $((v_i(s^{t+1}) > 0)$. In turn, this implies that the return of investment is higher and more capital will be accumulated. In the decentralized problem this is equivalent to an increase of most of the Arrow security prices $q(s^{t+1}|s^t)$, implying that intermediaries have to pay lower return to the agents and therefore invest more. In the model without autarky effects, this is the only effect. On the other hand, this over accumulation is mitigated by the autarky effects in the constrained efficient allocation. In that case, the planner internalizes that increasing capital will increase the value of autarky for the agent. Since $\frac{v_j(s^{t+1})}{u'(c_j(s^t))} V_K(S_j(s^{t+1})) > 0$ for some s^{t+1}, the presence of this effect will then imply a lower capital accumulation. In the decentralized solution, this is internalized with a binding upper limit on capital accumulation, which deters intermediaries from excessively overinvesting. This is the indirect reason of why a higher range of the wealth distribution (a higher range of λ) results in the model with saving constraints. As an example, if type 1 households have a low labour endowment (labor income) and a low wealth in the model with no autarky effects, they will have less incentives to default because capital accumulation is lower and the value of their outside option is therefore lower.

Finally, Figure 4 shows the life-time utilities of the agents with different labour endowments and initial wealths (measured by λ). Obviously, welfare is identical across the two economies in the stationary distribution, since the allocations are identical. Outside the stationary distribution, however, agents gain some utility in the suboptimal allocation compared to the constrained efficient allocation (autarky effects) if they are relatively wealthy ($\lambda < 1$), and they loose some utility when they are less wealthy ($\lambda > 1$). The reason for the

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utility loss is that, although agents can enjoy a higher current consumption in the economy with saving constraints, there is also less capital accumulation, affecting their life-time utility negatively. When $\lambda > 1.195$, this second effect dominates. Notice that these utility gains and losses are quantitatively very small, even outside the stationary distribution. In other words, the competitive equilibrium allocation without the savings constraint is close to be optimal.

Overall, we conclude that both economies have practically identical allocations in the long run (stationary distribution), and they have some (small) differences in the short run. The model without saving constraints leads to a higher short run capital accumulation and consequently to a lower consumption. We checked the robustness of these findings by allowing agents to accumulate physical capital through the intermediaries in autarky, increasing the value of the outside option and limiting the scope of risk sharing in both economies. In this case, we obtain a somewhat narrower range of λ in the stationary distribution. However, none of the other key findings are influenced by this change. We still find a perfect risk sharing in the long-run in both models, and we find the same type of qualitative changes and the same relatively small differences between the two models.

8. Conclusions

In this paper, we show two key theoretical results. First, in the presence of capital accumulation, the constrained efficient allocation of a model with limited commitment cannot be decentralized by a competitive equilibrium with borrowing constraints that are not tight, in contrast to the finding in endowment economies. On the other hand, this decentralization is possible with the introduction of financial intermediaries and an upper limit on their capital holdings. Second, we characterize the competitive equilibrium with only borrowing constraints that are not too tight. We show that these limits are micro founded, since the intermediaries have no incentives to loosen or tighten them. Furthermore, we show that the key inefficiency in this economy is coming from the fact that intermediaries do not internalize the effect of aggregate capital on the autarky value of the agents.

We think that this second result is particularly important, since it provides an empirically plausible decentralization which can be used to analyze several applied questions where financial intermediation is important. In addition, in spite of the fact that this economy is suboptimal, the solution for the equilibrium allocation does not require any extra computational burden as compared to the optimal solution due to our characterization result.

Finally, we show that there are no significant differences between the equilibrium allocations with and without the saving constraint in our framework, especially in the long run. This is mostly due to the fact, the in our production economy, autarky is not an attractive enough outside option, even if agents can save after default. One key direction of future research should be to identify other applications where the differences are more significant. In particular, models where the long run optimal allocation does not display permanent risk sharing, such as Kehoe and Perri (2002). They can be studied using this methodology to further understand the important differences between the two equilibria.

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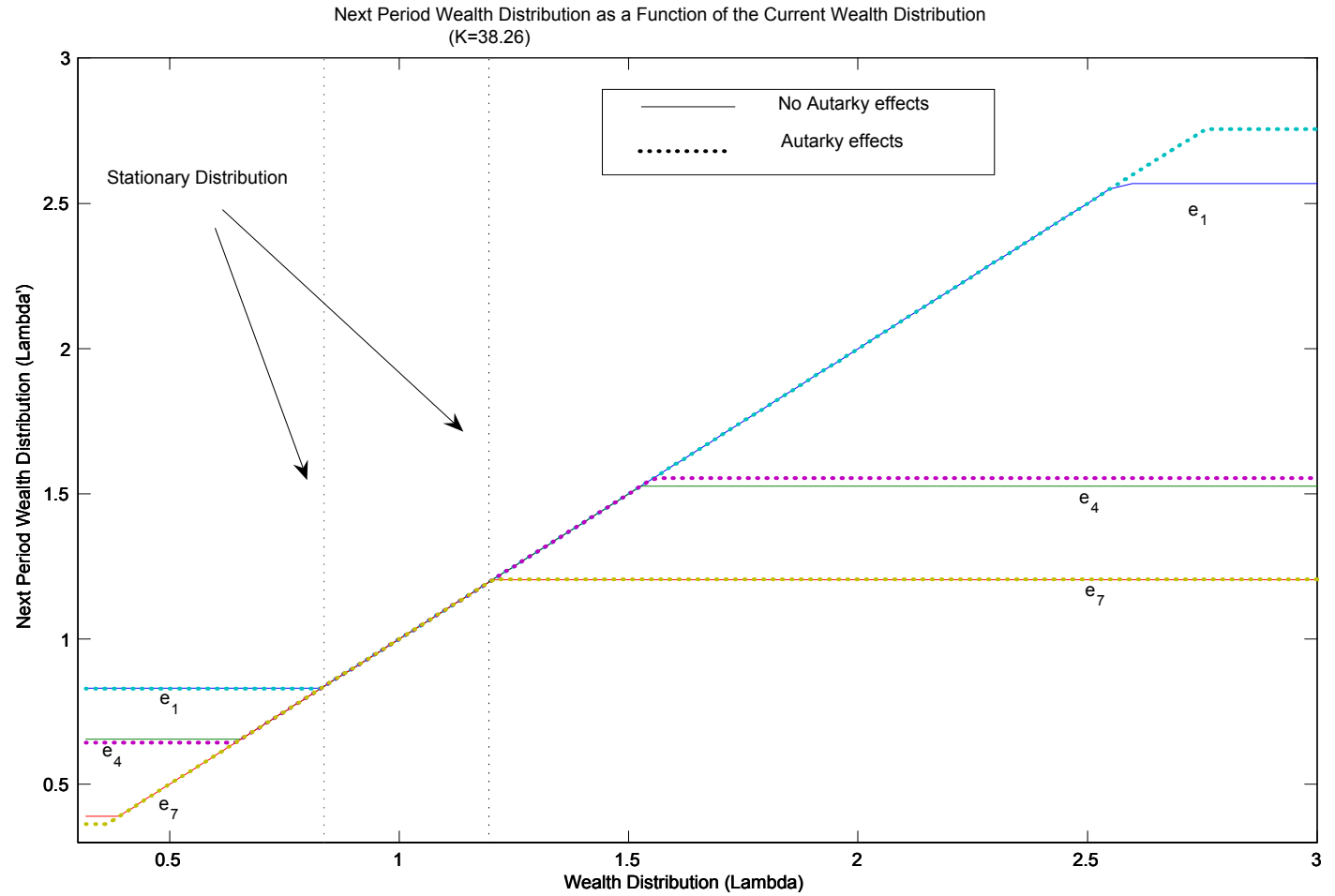Figure 1: Next Period Wealth Distribution λ' as a Function of λ and

Next Period Wealth Distribution as a Function of the Current Wealth Distribution
(K=38.26)

Figure 2: Optimal Consumption (c_1) as a Function of λ and ϵ

Consumption as a Function of the Current Wealth Distribution
(K=38.26)
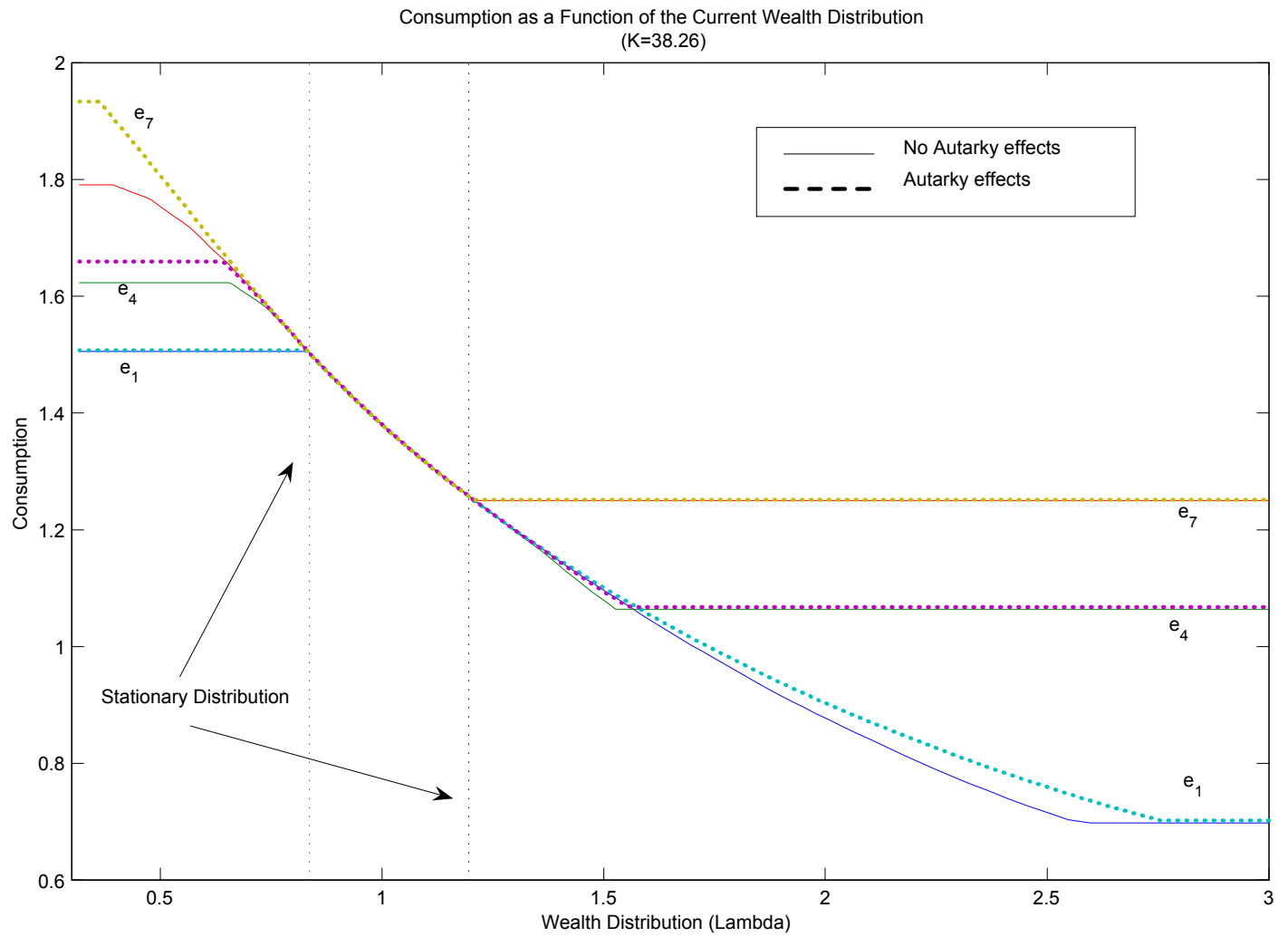
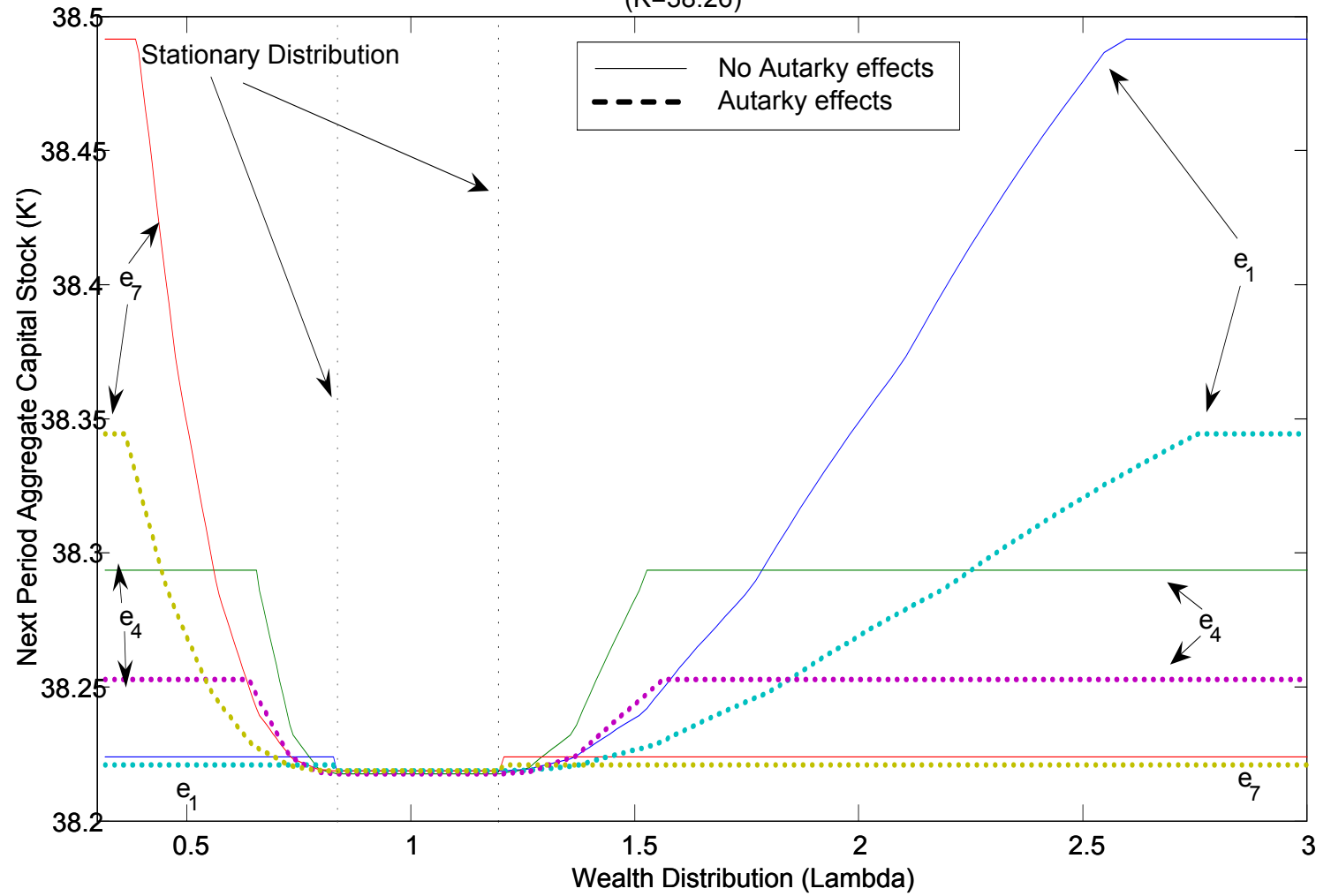Figure 3: Aggregate Capital Accumulation (K') as a Function of λ and ϵ

Figure 4: Life-Time Utilities (W) as a Function of λ and ϵ

Life-Time Utilities as a Function of the Current Wealth Distribution
(K=38.26)