(Relative Price) Lessons from Taking an AK Model to the Data*

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Abstract

This paper takes an AK model to the PWT data. Using the policy functions of the model, we recover time series for technology and investment shocks for a panel of countries and we isolate what we believe to be pervasive patterns in macroeconomics: i) the two shocks are negatively correlated, ii) consumption becomes ever cheaper relative to investment measured broadly, and iii) investment shocks matter less than technology shocks for postwar data. The widely researched relative price of equipment investment is an incomplete and potentially misleading part of the real relative intertemporal price.

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1 Introduction

This paper takes a simple two sector AK model to the data. Our model economy contains two stylized mechanisms that affect growth outcomes that are summarized by two different shocks: an intratemporal technology shock and an intertemporal technology shock. Our aim is to see what can be learned about the nature of these shocks and to study their joint characteristics.

Our prior is that if endogenous growth theory is correct, then a very stylized linear model should do well against the raw data, just as the early stylized concave models did against log detrended data. We assume that the countries in our sample are always sufficiently close to the balanced growth path to make transitional dynamics of second order in explaining movements in growth rates. This is a standard assumption in concave models as well.[1]

As argued by McGrattan (1998), the AK model is able to reproduce important features of the data, and as added by Fatás (2000), the empirical evidence is not compatible with a concave model with exogenous productivity shocks. For the purpose of this paper, one advantage of using the AK model is that its tractability allows us to be very agnostic when taking the model to the data; we assume the linearity of the production function and a logarithmic utility but we make no assumptions about stochastic processes. Instead, the structure we impose allows for an extremely simple and clear procedure to recover the shocks.

We are very explicit taking the model to the data. One necessary implication of the AK model is that capital must be viewed as a broad measure.[2] However, this is also true in concave models, as Mankiw, Romer and Weil (1992) show when studying human capital. This broad nature of capital has measurement consequences for both the stock and its price: the price of the broader stock is not the same as the price of physical capital. This is sometimes overlooked when looking at data. For instance, Chari, Kehoe and McGrattan (1996) mention that capital should be understood broadly but use data for investment on physical capital and for the price of investment goods; Felbermayr and Licandro (2002) use an AK model and compare its results to the fact established by Gordon (1990) and used by Greenwood, Hercovitz and Krussell (1997), that the relative price of equipment investment has been declining. In this paper we do not use data on physical capital or its price.

We use the policy functions implied by dynamic optimality to extract from the data the *exact* time series of both the technology shock and the in-

[1] Chari, Kehoe and McGrattan (1996) assume it for *all* countries in the PWT.

[2] See Rebelo (1991), Barro and Sala-i-Martin (1995), Parente and Prescott (1994), Mc Grattan (1998), and Klenow and Rodriguez-Clare (1997).

vestment shock. Our recovered technology shock grows exponentially, counteracted by an investment shock that declines, also exponentially. Over time consumption goods become cheaper relative to broad capital. This is a dramatic implication of understanding capital in a broad sense, and is the opposite pattern from the widely researched price of equipment investment: broad capital behaves very differently from equipment capital.

The two shocks are also negatively correlated over time and across countries, which suggests that interpretations of these technologies as reflecting institutional characteristics of countries are misleading, as one would expect good institutions to make both intratemporal and intertemporal technologies more efficient.

After uncovering the shocks and looking at their patterns, we evaluate their separate contributions to the variation of output growth. Here we come at the end of a long line of literature of which one-shock models are reviewed in King and Rebelo (1999). But one-shock models are singular models and so, cannot be used to obtain the shocks, as shown by Ingram, Kocherlakota and Savin (1994). These authors also note that if, in a multi-shock model, the different shocks are correlated, it is impossible to measure precisely the contribution of any individual shock to the variance of output. We show that their insights have important empirical implications, and provide a quantitative illustration of the bias when one tries to assign explanatory power to the different shocks. Nevertheless, our approach to this question still allows us to make a qualitative (yet precise) statement about the contribution of each shock to output variance: the investment shock is less important than the technology shock.

The paper proceeds with the description of the model and the data. Following that we generate the shocks and study them. Section 5 discusses extensions and part of the extensive literature in this area, and Section 6 concludes.

2 Model

We consider the simplest two sector AK model. Utility of the representative agent is maximized subject to a budget constraint where aggregate output is divided between consumption and savings, $y_t = A_t k_t = c_t + s_t$. Production is of the AK form, where A_t is the intratemporal technology shock. There is also an intertemporal technology that transforms current savings into investment, $I_t = \theta_t s_t$, and is summarized by the shock (θ_t).[3] An increase in

[3] If K is physical capital, the data counterpart of θ is p_c/p_I, as used in Chari, Kehoe and McGrattan (1996). For a broad measure of capital, we cannot obtain θ in the data.

θ constitutes an increase in the efficiency of the intertemporal technology or a decrease in investment "distortions". Finally, capital accumulation obeys, $k_{t+1} = (1-\delta) k_t + I_t$, and capital depreciates at rate δ.

The problem of the planner is to choose an investment path to maximize the sum of the present value of expected utility flows.[4] Given a discount factor β, and solving with respect to k_{t+1}, we obtain the Euler equation of this economy:

$$u'(c_t) = \theta_t \beta E_t \left\{ u'(c_{t+1}) \left[A_{t+1} + \frac{1-\delta}{\theta_{t+1}} \right] \right\}$$

The solution to this model presents a balanced growth path. Typically, the long-run growth rate is obtained and comparative statics are performed on this variable. For example, here, the more ineficient the intertemporal technology (lower θ), the lower the growth rate of consumption, g_c.

We now impose logarithmic utility in order to solve the dynamic programming problem analitically.[5] We can assume the pair (A, θ) follows a joint Markov process and solve the Bellman equation. Instead we solve the Euler equation forward as in this way we do not have to assume anything about the stochastic processes other than what is imposed by the Transversality condition. This allows the data all the freedom to determine what these processes are.

The policy function for this model is then:

$$k_{t+1} = \beta \theta_t \left[A_t + \frac{(1-\delta)}{\theta_t} \right] k_t$$

Optimal consumption $c_t(A_t, \theta_t, k_t)$ is then a function of the current values of the state variables. It is therefore not necessary to have information on c_t, A_t, θ_t, *and* k_t, to analyze this economy. One variable is redundant, and so we eliminate capital. Even if we want to look at the stock of capital as strictly physical capital, we know that measures of capital are the least reliable. As we take into account that in this model capital must be viewed broadly, we really need to eliminate these data. Consumption growth, output growth, and the consumption to output ratio are then given by

$$\begin{aligned} \frac{c_{t+1}}{c_t} &= \left[A_{t+1} + \frac{(1-\delta)}{\theta_{t+1}} \right] \theta_t \beta \\ \frac{y_{t+1}}{y_t} &= \frac{A_{t+1}}{A_t} \beta \left[\theta_t A_t + (1-\delta) \right] \\ \frac{c_t}{y_t} &= (1-\beta) + (1-\beta)(1-\delta) \left[\frac{1}{A_t \theta_t} \right] \end{aligned}$$

[4]For this economy, the planner solution is the same as the market solution.

[5]For example, Jovanovic (2004) uses log utility in an AK model.

If we subtract consumption growth from output growth we can see that output grows faster than consumption if the growth rate of the technology shock is higher than the growth rate of the relative price of investment $(1/\theta_t)$, as in that case the relative price of consumption is rising so agents respond by consuming less and investing more. It is also immediate that the certainty model would have the same growth rates for consumption and output but in a stochastic environment this need not happen at any point. Using the data and the equations above we back out time series for the two shocks. If we use the p_c/p_I data to measure θ, we can then use data on $\frac{c_t}{y_t}$ and $\frac{c_{t+1}}{c_t}$ to recover two different but equally legitimate time series for (A_t). This is the singularity problem exposed by Ingram, Kocherlakota and Savin (1994). The linear structure simply illustrates clearly the powerful implication that the model cannot really be tested: a one-shock model always fails, while a two-shock model is never rejected.[6,7]

3 Data

The model we use can be summarized by a simple budget constraint where output equals consumption plus investment, with one relative price: $Y = C + PI$. Our data must be treated to fit this simple form, but that is not straightforward. We will use data from the PWT 6.1. in two different forms, and for further robustness checks we will use a variety of NIPA data for the United States only.[8]

The PWT 6.1. data are in real terms, in 1996 prices, and cover 24 countries (listed in the tables in the Appendix) for the years 1950 trough 2000. Since some countries in our restricted sample lack the observation for 1950, we actually used the sample only from 1951 to 2000, resulting in 50 observations for each of the 24 countries. In parentheses below are the labels in the PWT dataset. We extract the consumption, investment, and government expenditure shares of GDP (KC,KI,KG), and a GDP measure (RGDPL) to go with them.

We try two measures against our model. First, we use the data as they

[6] In an A-shock model we can have a second-order Markov process, which introduces (A_{t-1}) as an extra state variable. However, *we still get the same policy function:* A single shock, no matter how complex its process, will not make the model fit the data.

[7] Using the data on PC/PI from the PWT 6.1, as a measure of θ, we are left with a single shock (A) to recover. But then we have two versions of the same shock. We estimated (β, δ) pairs for each country by minimizing the distance between the two shock series. The estimates were unrealistic and the two resulting time series of A_t were very different.

[8] To tackle concerns raised by Whelan (2003) regarding price indices.

come, and these results are shown in the main text. We use the (KC) measure to denote C/Y in the data, which in the PWT is in fact $\frac{p_c C}{pY}$. pY matches Y in the model. Note that in the model the consumption price is normalized to equal the output price, and they are both one. So, taking this model construction literally, we can simply use the data as they come to build our measures of C_t/Y_t and Y_{t+1}/Y_t.

Our second measure tries to match the data against the model equation $Y = C + pI$. According to the PWT we can write $pY = p_c C + p_I I + p_G G$. Because the model does not have government, $p_G G$ is removed from our data. A common procedure is to impose balanced budget with income taxes and assume expenditure to be an exogenous additive shock, $p_G G = \tau pY$.[9] Also, in the data the three shares of consumption (KC), investment (KI) and government expenditure (KG) do not add up to 1. The missing element is the difference between exports and imports ($E = X - M$). We assume this object to be an independent component of aggregate expenditure proportional to output (at a random factor e) so that we can subtract it as another shock. We get

$$pY - p_G G - p_E E = (p - p\tau - p_E e)\, Ak = \tilde{A}k$$

In this case the consumption and investment shares must be recomputed. New output is now equal to $\tilde{Y} = pY \left[\frac{p_c C}{pY} + \frac{p_I I}{pY}\right]$, which removes the government component and the external balance. The corresponding consumption share of this measure of output is $\left[\frac{p_c C}{pY}\right] / \left[\frac{p_c C}{pY} + \frac{p_I I}{pY}\right]$. Ultimately, the consumption *level* measure used (C_t) is the same. It is output that is truncated to have only the consumption and investment components.

This construction has the obvious caveat that we are using the wrong measure of investment to make our case, but it has the virtue of being mechanically closer to the model. We show the numerical results of using this measure in the Appendix. The main point is that they are virtually the same as those from our first measure.

As a further robustness check, we look at NIPA data for the US only, which offers more detail and which is described below. We use data from Tables 1.1.3., 1.1.5, and also data from the National Center for Education Studies, Tables 29 and 35.

Prices and Data Facts

A partial measure of the intertemporal shock could be the price of investment goods (PI) relative to the price of consumption goods (PC). These

[9] However, this may not be the best reduced form as only 11 out of 24 countries have the same sign on the correlations $\rho(c, g)$ and $\rho(i, g)$, which is what we expect if government expenditure works as an intratemporal shock.

prices are PPP indexes divided by the exchange rate. Their ratio $\frac{p_c}{p_i}$ would be a proxy for the intertemporal shock. If we interpret the investment data as showing $k' - (1-\delta)k = I$, the shock θ described in the model would then be $\frac{p_c}{p_I}$. But, as we will see, this is a very incomplete measure of θ, no doubt because it considers only physical capital.

We performed a variety of unit root tests and their outcome points to stationarity in consumption (and output) growth (c_{t+1}/c_t), and to a unit root in the consumption share (c_t/y_t) and the relative price (p_c/p_I). Our unit root testing follows Baxter, Jermann and King (1998) who also investigate the stationarity of some NIPA ratios for eleven countries and find mixed evidence of non-stationarity. The tests were a variety of univariate Dickey-Fuller tests. We also computed 95% confidence intervals for the autoregressive root, following Stock (1991), which confirmed the results of the DF tests. This feature conditions the inference regarding the relative importance of the two shocks, and their relationship, but we again follow the reasoning of Baxter, Jermann and King, and proceed with our analysis assuming that the data are draws from stationary distributions.[10]

[10] We could use multivariate versions of these tests, which have higher power, but this is outside the range of this paper, because we do not need to filter the data in any way.

4 Recovering the shocks

Our capital stock must be understood as a broad measure. This implies that the true intertemporal shock in the model has many components, only one of which we observe in the data as the relative price of consumption to investment. Ideally, we would have a model that decomposed capital explicitly. But in order to apply our methodology, such a model would need other restrictive assumptions. One way of dealing with this problem is to do some robustness checks, which we discuss below.

The task now is to recover the shocks. Given our data construction, only two variables are independent, and in the model only two equations are independent. After some algebra we obtain:

$$\begin{aligned}\frac{\theta_t}{\theta_{t-1}} &= \frac{\frac{c_t}{y_t}}{\frac{c_t}{y_t}-(1-\beta)}\frac{\beta(1-\delta)}{\frac{c_t}{c_{t-1}}} \\ \theta_t A_t &= \frac{(1-\delta)}{\frac{c_t}{y_t}\frac{1}{1-\beta}-1}\end{aligned}$$

and to pin down the θ_t series we assume that the initial values equal the PWT 6.1 observed initial relative price between consumption and investment for each country for all countries. This is an innocuous assumption, and we also used an initial value of $\theta = 1$ with the same results. We impose a common β (0.94) and δ (0.1).

The shocks we back out are displayed in Figure 1. They show that A_t is growing exponentially, and that the broad intertemporal price (θ_t) is falling, also exponentially. Consumption goods become ever cheaper, and therefore the relative price of consumption is fast approaching zero.[11] This is triggered by fast growth in technology. In order to have stable growth it is necessary for the relative price of investment to rise quickly, for every country. This outcome is interesting because the literature has investigated what seems to be an opposite phenomenon, namely, the drop in the relative price of equipment investment.

It is possible that our θ is an index of two relative prices moving in opposite directions: the relative price of physical capital moving down and the relative price of human capital moving up. So, our results *are compatible* with Greenwood, Hercowitz and Krusell (1997), and others, who stress the relative decline in the price of equipment investment relative to the price of

[11] "The Economist", of December 13th-19th, 2003, has on pages 6 and 7 of its "Survey of Food", a very interesting article on "How technology pushes down the price", which is very much in the spirit of what we see in Figure 1.

consumption. However, the trends we obtain suggest that this emphasis on the relative price of equipment is a second-order phenomenon when compared to the accumulation of broad capital: perhaps an increasing price of human capital is the dominant factor.[12]

It is important to stress two points.

First*, we do not impose the behavior of the individual time series on our model*. We chose fairly standard parameter values. The data and the parameter values could have delivered shocks that looked very different. They could have been both stationary, or they could have been exponential with reverse slopes.

Second, *our results are robust*. The first test of robustness is to see if these results depend on the parameter values: low values of beta and delta may change the results. This is not the case for any δ between 10% and 1%, and any β between 0.99 and 0.94. This robustness is not as strong if we use our second measure of C/Y detailed in the Appendix. But it is still strong enough. With our $\beta = 0.94$ a value of $\delta = 0.02$ still delivers this clear pattern. If we have $\beta = 0.96$, a value of $\delta = 0.01$ still does the same. We must note that $\beta = 0.94$ implies an unrealistically high real interest rate, which biases the outcomes away from the pattern we find. In fact, Taylor (1993) suggests a real interest rate of about 2% for the US. Also, even though a broad measure of capital implies that we must redefine depreciation, only for unrealistically low depreciation rates is the pattern of behavior of the shocks inverted. For example, Whelan (2003) uses depreciaton of 3% for structures but of *13%* for equipment.

A second test of robustness is to use other data.

To show this we look at NIPA data for the USA: using the equations above, we can use nominal GDP data from column one in Table 1.1.5., and index data from Table 1.1.3. to obtain the same picture as for our basic data. For this, consumption can be defined either as only of nondurables and services (columns 4 and 5), or have durables added (column 3), with the same result. The c/y *real* ratio is simply the ratio of nominal values for GDP (column 1 in Table 1.1.5) and consumption (columns 4 plus 5, and perhaps also column 3 for durables). Real consumption growth is computed weighting the real indices from Table 1.1.3., by the nominal expenditure weights from Table 1.1.5. of each category of consumption, following the recomendations of Whelan (2003), but even changing this weighting construction does not change the picture we obtain or the correlations that follow. Furthermore, defining nominal output (all we need here since by assumption the price of

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[12]Note that we do not use data on physical capital or investment prices "which are well-known to be poorly measured" (Fisher, 2002, page 10).

consumption and the price of output are the same) not as the first column of the NIPA tables, but as the sum of the consumption and investment columns, also does not change the outcome.

Still with US NIPA data, and with respect to parameter values, if we define consumption to be just nondurables and services and include durables in investment, and set β at 0.93 and δ at 0.035, the two shocks come out close to flat. Somewhere below these values the trend pattern is reversed, *but these values are very low*: using consumption growth for the United States from NIPA Table 1.1.3, and matching it with our model expression for consumption growth, they imply an average real interest rate - given by $(c_{t+1}/c_t)(1/\beta) - 1$ - between 1950 and 2000 of 10.88%. Even at our benchmark value of beta (0.94) this real interest rate is still 9.7%. And, for example, using $\beta = 0.96$, even a 2% depreciation rate still delivers our pattern of increasing A and decreasing θ.

4.1 Characteristics of the shocks

We begin by obtaining the coefficient of correlation between the shocks. We remind the reader that the data are constructed using the raw series, that is, the consumption share is simply KC, and output is simply $Y = RGDPL$.

First, since the time series of both A and θ are exponential by construction, we take logs and remove a linear trend.[13] Then we compute the correlation coefficient of the log-detrended data for each country. The outcome is:

$Table1$	$\rho(A_t, \theta_t)$		$\rho(A_t, \theta_t)$		$\rho(A_t, \theta_t)$
AUS^*	-0.499	FRA	$\mathbf{-0.084}$	MEX	-0.917
AUT	-0.606	GBR	-0.683	NLD	-0.777
BEL^*	-0.845	GRC	-0.913	NOR	-0.950
CAN^*	-0.942	IRL	-0.798	NZL	-0.959
CHE^*	-0.873	ISL	-0.612	PRT	-0.833
DNK	-0.500	ITA	$\mathbf{-0.217}$	SWE	-0.449
ESP	-0.983	JPN	-0.970	TUR	-0.798
FIN	-0.951	LUX	-0.532	USA	-0.872

All but two countries have strongly significant negative correlations between the two shocks. The average correlation is -0.7320 and the standard deviation of these numbers across countries is 0.2465. Interestingly, this is also true by date. If we compute 50 cross sectional correlations, we get all

[13] The *individual* (θ_t, A_t) time series cannot be realizations of stationary processes, but *the product* $\theta_t A_t$ is stationary.

negative correlations, and only for 1993 (-0.231) is it below -0.28. The average *across dates* is -0.7087 with a standard deviation of 0.1477. So, countries with high values of A have low values of theta.

For USA NIPA data also, the correlation between the log-detrended shocks is always negative, large and significant, whichever data construction we use, and for *any* parameter values we use. The negative correlations make it difficult to accept an interpretation of these shocks based on the quality of institutions. Our prior is that a positive shock on institutions should simultaneously help the productivity of both the final output sector and the investment sector.

An alternative interpretation of these shocks may be to understand A as technological development, in the line of quality ladder models. Then the increasing complexity of capital goods (including physical and human capital) implies an increasing cost of investment and justifies its increasing price. In this interpretation the faster the technological development measured by A, the faster the decrease in θ, which is the inverse of the relative price of investment.

In addition, the fact that these correlations are strong and significant, regardless of their sign, implies that inference about their relative importance depends on the order of orthogonalization. But some lessons may nevertheless be extracted from such an exercise.

4.2 Which shock is more important?

Which shock affects output growth more? Casual examination of the expression for output suggests the answer: $Y_t = A_t K_t$. The intertemporal shock has only an indirect impact on output. But is this intuition correct? To find out we shut down one shock - by setting it at its country-specific exponential trend ($\bar{A}_t$, or $\bar{\theta}_t$) - and see how much variation of the actual output growth the active shock explains. If we do this and work some of the algebra we obtain

$$\begin{aligned}
\frac{y_{t+1}}{y_t}|_{\bar{\theta}} &= \frac{A_{t+1}}{A_t}\beta\left[\bar{\theta}_t A_t + (1-\delta)\right] = \frac{y_{t+1}}{y_t}\left[1 + \frac{(1-\delta)\left(\bar{\theta}_t/\theta_t - 1\right)}{c_t/y_t}\right] \\
\frac{y_{t+1}}{y_t}|_{\bar{A}} &= \frac{\bar{A}_{t+1}}{\bar{A}_t}\beta\left[\theta_t \bar{A}_t + (1-\delta)\right] = \frac{\bar{A}_{t+1}}{\bar{A}_t}\left[1 + \frac{y_{t-1}}{y_t}\frac{\frac{c_{t-1}}{y_{t-1}}\beta\bar{A}_t\theta_{t-1}}{c_t/y_t - (1-\beta)}\right]\beta(1-\delta)
\end{aligned}$$

and this exercise quickly shows that when we shut down θ_t by setting it to $\bar{\theta}_t$, we still preserve output growth, just adding some noise. That is not the case when we shut down A, in which case we effectively lose any time $t+1$ information.

With these constructed series we run an OLS regression of the true data, against the artificial series generated with only one shock and the other shock set to its trend. We then use the R squared as a measure of the ability of the shocks to explain the variance in the output rate of growth:[14]

$$\frac{y_{t+1}}{y_t} = \alpha_0 + \alpha_1 \left[\frac{y_{t+1}}{y_t} |_{\bar{\theta}} \right] + \epsilon_t \Rightarrow R^2 \left(A_t, \bar{\theta}_t \right)$$

$$\frac{y_{t+1}}{y_t} = \alpha_0 + \alpha_1 \left[\frac{y_{t+1}}{y_t} |_{\bar{A}} \right] + \epsilon_t \Rightarrow R^2 \left(\theta_t, \bar{A}_t \right)$$

In fact the noise we add in the first case is very small, and the R squared averaged over all countries is $R^2 \left(A_t, \bar{\theta}_t \right) = 0.9324$, whereas the R squared obtained from shutting down theta is on average across countries $R^2 \left(\theta_t, \bar{A}_t \right) = 0.0610$.

$Table2$	$R^2 \left(\theta_t, \bar{A}_t \right)$	$R^2 \left(A_t, \bar{\theta}_t \right)$
$mean$	0.0610	0.9324
std	0.0835	0.0451

We perform several experiments with these data and detail them in the Appendix, but they all point to one conclusion: *shutting down A affects the model considerably more than shutting down θ*.[15]

5 Discussion

Our point of departure was the realization that growth is always endogenous. This led us to the simplest endogenous growth model which, as argued by McGrattan (1998) and Fatás (2000), is a useful model. McGrattan used long time series to show that growth and savings rates are significantly correlated, and Fatás showed that a stochastic AK model is able to reproduce the positive correlation between long-term growth rates and the persistence of output fluctuations.[16]

[14] We could regress the actual data against θ (or A) directly. But that regression would be mispecified, as output growth is non-linear in the shocks. Note finally that if we put both shocks on the right hand side we get $R^2 = 1$.

[15] This is in line with the results of Chari, Kehoe and McGrattan (2002), but contrary to the conclusions of Chari, Kehoe and McGrattan (1996). It is also similar to Cooper and Ejarque (2000), and opposes Fisher (2002).

[16] Fatás (2000) actually reconciles Jones (1995) with McGrattan (1998): McGrattan smooths moments over several periods, which cannot distinguish the basic AK model from endogenous growth models with transitional dynamics. Jones rejects AK models by looking at short-run deviations from the balanced growth path.

We also follow a long literature in concave models that studies the impact of different shocks. Prescott (1986) finds that, for the USA, a technology shock explains a large fraction of output variation. This shock drives growth and cycles in Jones, Manuelli and Siu (2000). An early reference with the investment shock is Greenwood, Hercovitz and Huffman (1988), while Barlevy (2004) is a recent linear model with such shocks. Chari, Kehoe and McGrattan (1996) conclude that the investment shock is an important determinant of the variability of relative income levels across countries. Restuccia and Urrutia (2001) show that the investment shock is closely related to the investment to output ratio. More recently, Chari, Kehoe and McGrattan (2002) consider simultaneously three types of shocks, an efficiency shock, a labor shock and an investment shock in a concave model, and conclude that the first two types of shocks explain most of the output variation.

In the financial intermediation literature, Cooper and Ejarque (2000) explore the fact that US consumption and investment are positively correlated in postwar data to argue that investment shocks (since they induce negative correlations) must be of second order in explaining output variations. Fisher (2003), building on Greenwood, Hercovitz and Krussell (1997), takes into account information on the relative price of equipment investment to reach the opposite conclusion in a model where shocks have a stochastic trend. He performs a VAR exercise where he imposes two identification conditions on the data: that investment shocks are the only ones affecting the relative price between consumption and investment, and that technology shocks are the only shocks affecting productivity in the long run. Both these conditions are embodied in our model.

Clearly, the stark difference between the recovered θ - a measure of the relative price of broad capital - and the observed relative price between consumption and investment in the PWT data, suggests that we must extend our model, either with human capital as Rebelo (1991) or incorporating optimal leisure/labor choice and taxes on labor income as in McGrattan (1998). Keeping the same structure, we added labor and a taste shock to the model in a three-shock and three-variable framework.[17] The results did not change.

A more substantial exercise is to decompose the capital stock. However, achieving this in a way that allows for an explicit solution implies additional restrictions on the problem. A good example of this issue is the work of Blackburn and Galindev (2003) or a generalization of the work of Felbermayr and Licandro (2002). These last authors build a different two sector

[17] In this case utility is $z_t log(c_t) - \gamma \log(L_t)$, where z_t is the taste shock (the third shock). Output is $y_t = A_t K_t L_t^{\alpha}$, and the planner's solution is not the competitive equilibrium. However, we feel that taste shocks are not the point here.

endogenous growth model that incorporates the idea that the relative price of investment is falling. Their model is built to deliver this feature of the data, while our model is built to infer such behavior from other data properties.[18]

Now, despite the fact that we do not tackle the disaggregation of broad capital here, we believe one reason to take seriously the picture we saw above is that *the model proposed is arguably the simplest and most standard, and provides the most agnostic way of incorporating the two types of shocks we use*. The key is to understand what our model may be telling us: broad capital is *not* just equipment, structures, and some types of durable consumption. One key question is whether the additional components of capital are becoming more expensive in terms of the number of apples and oranges we can eat. In support of this theory we have a direct measure of the price of human capital. The price index for higher education increased relative to the GDP deflator by about 65% between 1960 and 2000, while the fraction of GDP attributable to investment in human capital is substantial.[19] The fact that we obtain equal and fairly robust results for both data treatments - including or excluding government expenditure - suggests that education is not the only missing factor, as the bulk of society's investment on education is included in government expenditure. Finally, and looking again only at private consumption versus investment, our experiments with NIPA data for the US take durable consumption out of the consumption measure and include it in investment, and still the results do not change.

One interesting way to continue this research is to extend the Felbermayr and Licandro (2002) model. Their model, despite its analytical restrictions, does have the attractive property that it can be generalized to many capital goods, and allows treatment of the limited information we have on expenditures on education as a measure of investment in human capital, in a better disaggregation of the broad capital stock.

[18] Chari, Kehoe and McGrattan (1996), and Fisher (2003) use structural properties to recover characteristics of the shocks, but they impose more structure than we do.

[19] Data from the National Center for Education Statistics. Tables 29 and 35. http://nces.ed.gov. In nominal terms, at current prices, total education expenditures amount to 7.3% of GDP in 2000 while the aggregate of durable consumption (8.79%) and investment in equipment and software (9.36%) represents a fraction of 18.15% of GDP (NIPA data). Other data that we do not discuss here regard the relative price of structures, which is often implicitly taken to be the price of output. Finally, there is a literature social capital that may have a role in this work, through public investments in public goods, for example.

6 Conclusion

We consider two sources of fluctuations in an AK model, an intratemporal shock and an intertemporal shock. Once we take into account that capital should be understood as a broad measure, both shocks are unobserved. We solve explicitly for the optimal investment decision in the model and take the exact implications of this optimal decision to the data. This allows us to recover the *exact* time series for the technology shock and the investment shock. We are then able to investigate their properties and their impact on the growth rate.

The time series of the two shocks indicate that consumption has become relatively cheaper over time, suggesting that the widely researched drop in the relative price of equipment investment is not the main component of the trend of aggregate consumption and investment in a broad measure of capital. The fact that the relative prices of other components of broad capital may not behave like the equipment price is illustrated by the rising relative price of education.

The two shocks are also strongly negatively correlated. The strong correlation makes it impossible to make definitive statements about the contribution of each shock to output variance. Nevertheless, we are able to show that the technology shock contributes more to the variance of output growth than does the investment shock. Moreover, the negative sign of the correlation makes it difficult to accept an interpretation of these shocks based on the quality of institutions. In principle, a positive shock on institutions should simultaneously help the productivity of both the final output sector and the investment sector. An alternative interpretation links technological development to the increasing complexity of investment goods.

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7 Figures and Tables

7.1 Main text data

This section details the results shown in the main text, as well as showing several experiments we performed to evaluate the different contribution of each shock. The data are constructed using the raw series, that is, the consumption share is simply KC, and output is simply $Y = RGDPL$. These numbers are then used to construct consumption as $C = KC * RGDPL$, and this consumption series is then lagged to compute C_{t+1}/C_t.

We use the raw series of shocks that we extracted from the data and also transformed series that we obtain after an orthogonalization process, so that we get an independent impact of each shock. So, first we take logs of our raw series, and then the log series are decomposed into a linear trend $(\bar{A}_t^L, \bar{\theta}_t^L)$ and a deviation component (A_t^d, θ_t^d). After taking logs our shocks are written

$$\log(A_t) = \bar{A}_t^L + A_t^d$$

One interesting characteristic is the fact that these deviations from trend have roughly the same standard deviation. By construction their mean is zero, and their standard deviation has a mean across countries of 0.0771 for $\sigma\left(\theta_t^d\right)$ and 0.0847 for $\sigma\left(A_t^d\right)$. So, it is not because one shock has a much smaller volatility than the other that it matters more or less to the movement of output.

Part 1. Orthogonalizing

We conduct two different orthogonalizations because we have no prior on how the two shocks are related, and because we want to isolate the individual contribution of each shock. First we remove the exponential trend from both θ and A. Then, in one case we regress by OLS

$$\theta_t^d = a + bA_t^d + \epsilon_t$$

and then use the pair $(\hat{\theta}_t^d = \hat{a} + \hat{b}\bar{A}^d + \hat{\epsilon}_t, A_t^d)$ where $\bar{A}^d$ is the *mean* of A_t^d, (which is zero by construction), thereby removing from θ_t^d the component that can be explained by A_t^d. This is a "pure θ" shock, and is orthogonal to A.

In the other case we simply switch the shocks. We do this for every country. Again note that this filtering is applied to the log-detrended component only. It is this noise (orthogonalized or not) that is added to the exponential trend.

Part 2. Evaluating the impact of each shock

Below we denote by $\left(\bar{A}_t, \bar{\theta}_t\right)$ the exponential trend $\exp(\bar{A}_t^L, \bar{\theta}_t^L)$. We are interested in the impact of each shock on the movement of the different data series. We evaluate it by comparing the true data with an adequately generated artificial series. These artificial data are produced by shutting down one of the shocks at its country trend $(\bar{A}_t, \bar{\theta}_t)$. We do this for the output growth equation: we compare the true $\frac{yt+1}{yt}$ data to the following two alternatives:[20]

$$\begin{aligned} \frac{y_{t+1}}{y_t}|_{\bar{A}} &= \frac{\bar{A}_{t+1}}{\bar{A}_t}\beta\left[\theta_t\bar{A}_t + (1-\delta)\right] \\ \frac{y_{t+1}}{y_t}|_{\bar{\theta}} &= \frac{A_{t+1}}{A_t}\beta\left[\bar{\theta}_t A_t + (1-\delta)\right] \end{aligned}$$

where here $\overline{x}$ stands for the exponential trend of x with no noise, and variables without upper bars denote either the raw series (θ_t, A_t) or the orthogonalized $(\hat{\theta}_t, \hat{A}_t)$ series which are constructed by adding to the exponential trend the exponentiated orthogonalized noise, $\exp(\hat{\theta}_t^d, \hat{A}_t^d)$.

We run an OLS regression of the true data, against the artificial series generated with only one shock and the other shock set to its trend. We run

[20] Chari, Kehoe and McGrattan (1996) and Restuccia and Urrutia (2001) shut down the technology shock.

the following estimations and use the R squared as a measure of the ability of the shocks to explain the variance in the output rate of growth:[21]

$$\frac{y_{t+1}}{y_t} = \alpha_0 + \alpha_1 \left[\frac{y_{t+1}}{y_t} |_{\bar{\theta}} \right] + \epsilon_t \Rightarrow R^2 \left(A_t \right)$$

$$\frac{y_{t+1}}{y_t} = \alpha_0 + \alpha_1 \left[\frac{y_{t+1}}{y_t} |_{\bar{A}} \right] + \epsilon_t \Rightarrow R^2 \left(\theta_t \right)$$

Part 3. Results

Table 3 shows the R squared of a series of regressions for every country. Column 1 shows the regression of the true $\frac{y_{t+1}}{y_t}$ data against $\frac{y_{t+1}}{y_t}|_{\bar{A}}$ as defined above, where the technology shock (A) is set at the country-specific trend with no noise. We effectively shut down A. This produces $R_1^2 \left(\theta_t, \bar{A}_t \right)$, which is a measure of the explanatory power of θ_t where θ_t is the series we get directly from using the data on our equations.

Column 2 regresses the true $\frac{y_{t+1}}{y_t}$ data against data constructed using the projection of the residual of $\hat{\theta}_t$, that gives $\hat{\theta}$ the least explanatory power (removing from θ_t the component that can be explained by A_t, which biases the explanatory away from θ). We get $R_2^2 \left(\hat{\theta}_t, \bar{A}_t \right)$. Note that in columns 1 and 2, the artificial $\left[\frac{y_{t+1}}{y_t} |_{\bar{A}} \right]$ is constructed using the trend of A, plus the respective θ series for each country.

Column 3 fixes θ to its country-specific trend and uses the raw A series, while column 4 uses the orthogonalized series for the residual of A, which gives A the least explanatory power by removing the orthogonal θ component inside A.

[21] We could regress the actual data against θ (or A) directly, but that regression would be mispecified, as output growth is non-linear in the shocks. Therefore, the R squared of our regression is a better measure of what is missing when we shut down one shock.

$Table3$	$R_1^2\left(\theta_t, \bar{A}_t\right)$	$R_2^2\left(\hat{\theta}_t, \bar{A}_t\right)$	$R_3^2\left(A_t, \bar{\theta}_t\right)$	$R_4^2\left(\hat{A}_t, \bar{\theta}_t\right)$
AUS	0.046	0.017	0.964	0.706
AUT	0.101	0.083	0.943	0.373
BEL	0.013	0.026	0.912	0.222
CAN	0.011	0.001	0.941	0.387
CHE	0.010	0.009	0.969	0.655
DNK	0.000	0.001	0.856	0.368
ESP	0.036	0.019	0.914	0.038
FIN	0.058	0.048	0.950	0.054
FRA	0.212	0.207	0.933	0.921
GBR	0.026	0.001	0.978	0.747
GRC	0.010	0.039	0.963	0.024
IRL	0.144	0.206	0.953	0.301
ISL	0.158	0.096	0.974	0.210
ITA	0.351	0.334	0.933	0.865
JPN	0.001	0.037	0.857	0.046
LUX	0.033	0.030	0.975	0.284
MEX	0.001	0.045	0.948	0.047
NLD	0.027	0.013	0.920	0.307
NOR	0.008	0.024	0.845	0.148
NZL	0.060	0.005	0.956	0.044
PRT	0.002	0.000	0.858	0.074
SWE	0.097	0.141	0.863	0.280
TUR	0.035	0.000	0.984	0.202
USA	0.020	0.062	0.987	0.347
$mean$	**0.0610**	0.0603	**0.9324**	0.3189
std	0.0835	0.0834	0.0451	0.2721

Although we cannot say how much each shock explains of total output variance, *one clear pattern emerges*: shutting down A affects the model considerably more than shutting down θ. Note that we could pick column four and argue that theta is responsible for $1 - R_4^2\left(\hat{A}_t\right) = 0.6811$ of the variation in output growth, but that, despite being acceptable, would be a biased inference, as casual observation of the output growth expression naturally suggests.

7.2 Correlations using Y=C+pI.

We again take logs and remove a linear trend. Then we compute the correlation coefficient of the log-detrended data for each country. The outcome is:

$Table4$	$\rho(A_t, \theta_t)$		$\rho(A_t, \theta_t)$		$\rho(A_t, \theta_t)$
AUS^*	-0.744	FRA	-0.865	MEX	-0.959
AUT	-0.925	GBR	-0.754	NLD	-0.821
BEL^*	-0.682	GRC	-0.929	NOR	-0.889
CAN^*	-0.938	IRL	-0.943	NZL	-0.929
CHE^*	-0.798	ISL	-0.758	PRT	-0.941
DNK	-0.796	ITA	-0.736	SWE	-0.755
ESP	-0.986	JPN	-0.986	TUR	-0.748
FIN	-0.961	LUX	-0.647	USA	-0.913

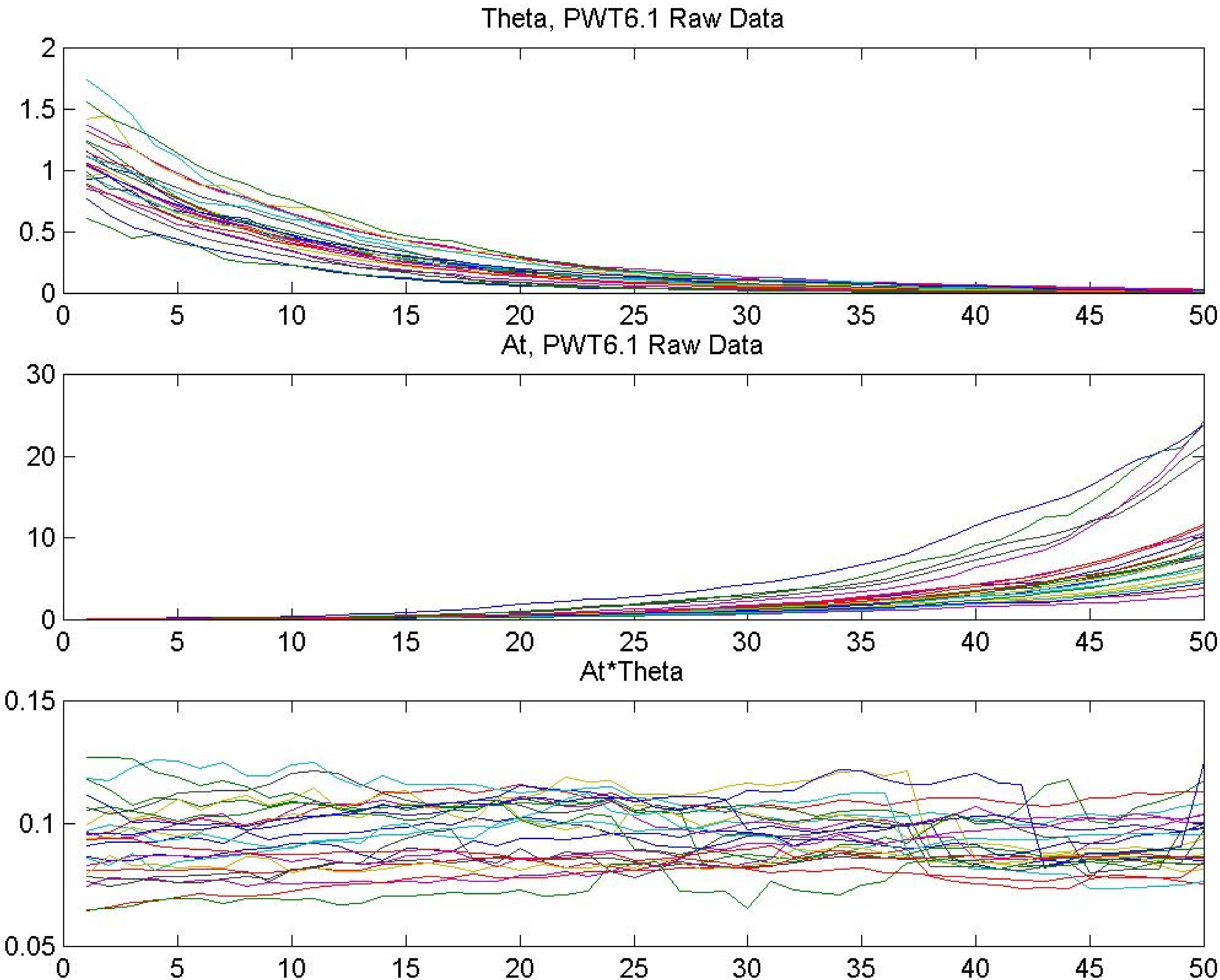
All countries have strongly significant negative correlations between the two shocks. The mean correlation is -0.8503 with a standard deviation of 0.1035. Across countries, the mean of the 50 - time constant - cross section correlations between the shocks is -0.8528 with a standard deviation of 0.0696. The picture we recover of the (A, θ) shocks is exactly the same in every qualitative sense, and so is its robustness to parameter values.

Which shock is more important?

R squared of regressions of true output growth against constructed output growth:

$Table5$	$R_1^2\left(\theta_t, \bar{A}_t\right)$	$R_2^2\left(\hat{\theta}_t, \bar{A}_t\right)$	$R_3^2\left(A_t, \bar{\theta}_t\right)$	$R_4^2\left(\hat{A}_t, \bar{\theta}_t\right)$
AUS	0.13	0.00	0.99	0.66
AUT	0.01	0.02	0.98	0.09
BEL	0.09	0.06	0.98	0.75
CAN	0.00	0.06	0.98	0.76
CHE	0.01	0.00	0.99	0.79
DNK	0.03	0.00	0.98	0.18
ESP	0.05	0.03	0.95	0.01
FIN	0.04	0.00	0.98	0.10
FRA	0.02	0.03	0.96	0.03
GBR	0.03	0.00	0.99	0.52
GRC	0.01	0.02	0.98	0.04
IRL	0.06	0.02	0.99	0.33
ISL	0.11	0.00	0.99	0.22
ITA	0.16	0.10	0.99	0.45
JPN	0.06	0.38	0.89	0.01
LUX	0.00	0.00	0.99	0.27
MEX	0.07	0.00	0.99	0.34
NLD	0.09	0.03	0.99	0.08
NOR	0.00	0.01	0.98	0.18
NZL	0.06	0.04	0.99	0.43
PRT	0.03	0.00	0.97	0.10
SWE	0.09	0.01	0.99	0.06
TUR	0.11	0.00	0.99	0.47
USA	0.04	0.00	0.99	0.66
$mean$	0.0546	0.0346	0.9802	0.3155
std	0.0438	0.0781	0.0220	0.2634

Again we see that shutting down A reduces the ability of the model to explain the data far more than shutting down theta.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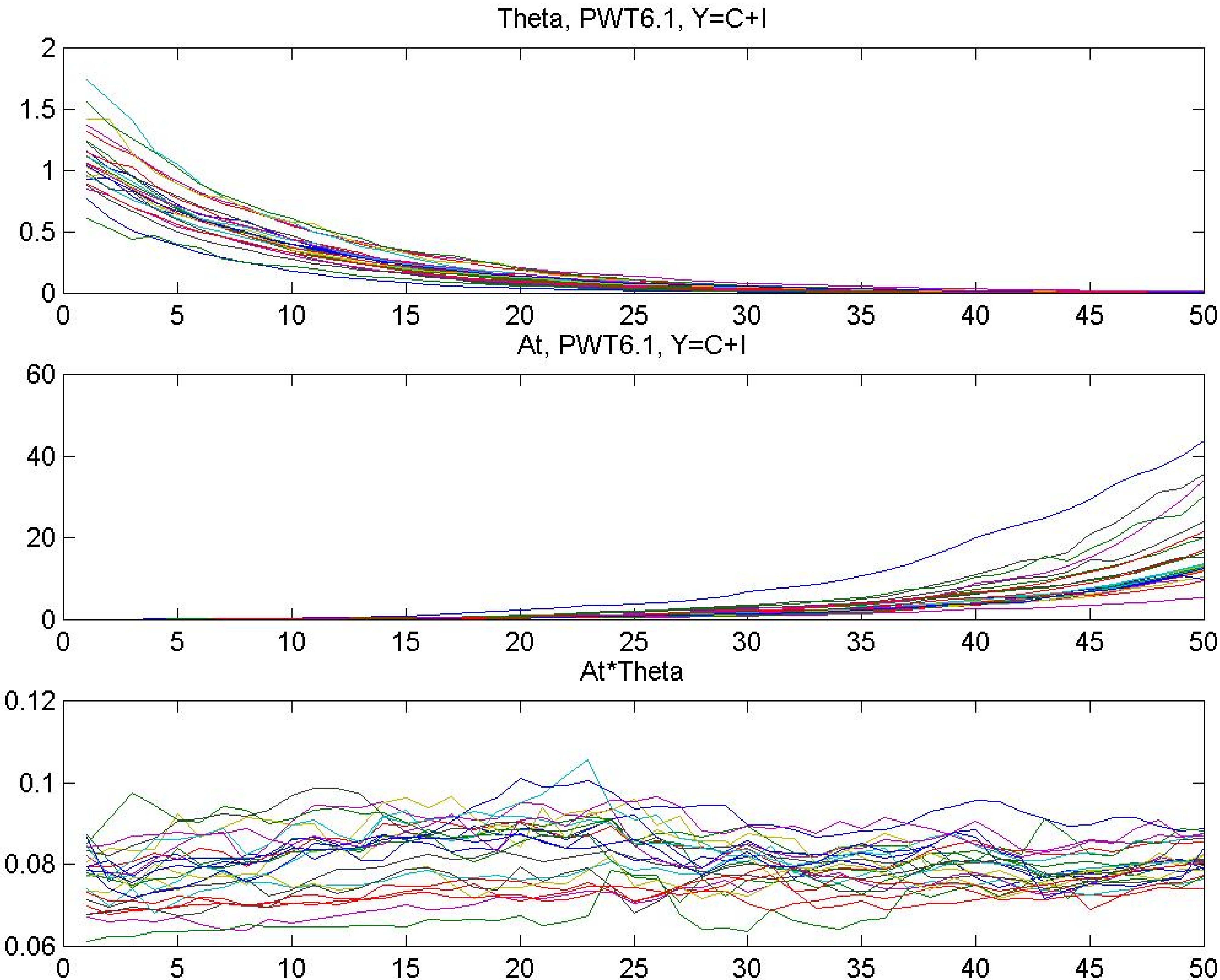
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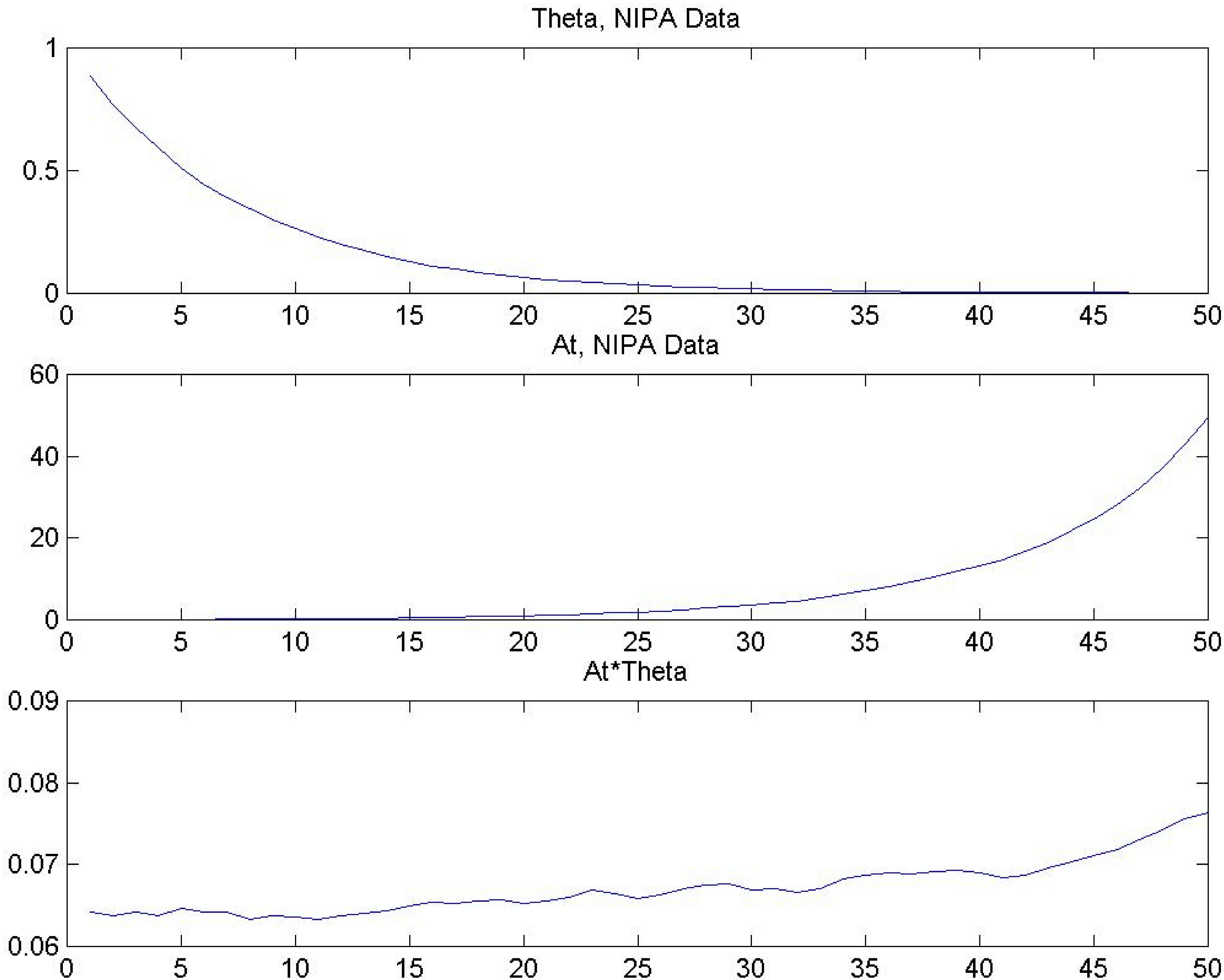

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