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The Disutility of International Debt: Analytical Results and Methodological Implications

by

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INTRODUCTION

Through most of history, even when interest payments were illegal, the costs of borrowing have been high (Anderson 2005). Among these costs have been servile relations to the lender, moral opprobrium, and, in the event of default, harsh punishment and destitution. In recent times, these costs have been reduced throughout the industrialized world. The benefits have arguably included more widespread homeownership, a reduction of the barriers of entry to the small business sector, and the intangible advantages of a more egalitarian society.

When developing nations borrow, they too face nonpecuniary costs. A number of such costs could be cited, but the strictures imposed by international financial institutions such as the International Monetary Fund would be high on any list. Demands by these organizations potentially count as a cost in two ways. First, they impose economic policies that are contrary to the interests of the borrowing nation; second, in doing so, they inflict the intangible costs of a loss of national autonomy. As an example, borrowing countries can be forced to adopt contractionary policies during recessions, often cutting needed social services in the process. The international financial institutions may compel countries to cut what they regard as distortionary taxes, exacerbating fiscal stress. Borrowers may be forced to adopt "reforms" that result in a more skewed distribution of wealth. One might argue that the intangible costs of a world divided into wealthy elite of nations and a subjugated borrower class are just as great as an individual nation so divided.

There exists a vast literature on the benefits of the opening of international debt markets. This literature focuses on certain benefits of capital market openness. These include a spreading of risk that purportedly reduces the volatility of consumption in individual nations. Also, financial openness enables countries to take advantage of differences in the rate of discount across economies, in the same way that trade in goods is mutually beneficial to countries with differing relative prices for goods. Accounts of these benefits of capital markets tend to overlook the effects of the exertion of power by lenders over borrowers, a relevant issue particularly in connection with small developing economies.

This paper attempts to incorporate the kind of nonpecuniary costs of borrowing discussed here into a model typical of the existing literature. Specifically, the paper models a small nation that has access to international financial markets. The nation is free to borrow money to smooth its consumption stream, and, in the second of the two models, to make an

agreement with lenders to repay a lessened amount during hard economic times. However, the representative nation suffers a loss of utility when it borrows or when it repays a relatively small amount of its debt. Several issues will be examined. First, are there any impacts of the revised utility function on the consumption stream of the borrowing nation? Second, how is local investment affected by the disutility of borrowing? Finally, what affect is there on moral hazard?

In tackling a largely nonpecuniary issue (national autonomy) using a standard rational choice model, this work follows up on and challenges the methodological approach pioneered by Lionel Robbins (1935). Robbins insisted that all conceivable values could be fit into the rational choice framework simply by adding them to the utility function. After developing the models, this paper uses them as test cases for this methodological claim. In particular, Robbins's claim that economists could and should maintain a rigid fact-value distinction, resisting the temptation to assess values, will be examined.

CONCEIVING INTERNATIONAL DEBT AS AN IMPERFECT SUBSTITUTE FOR LOCAL FUNDS

One tenet common to most neoclassical models is that a 10-percent return is the same as any other 10-percent return with the same risk, regardless of how one comes by it. In orthodox theory, countries invest until the return on investment is equal to the return on risk free bonds, which is in turn equal to the rate at which countries are willing to trade off current for future dollars. A key feature of the models of this paper is that countries have two different ways of transferring current wealth into future wealth (investing in capital equipment, and lending or borrowing on international debt securities markets), but these investments are not symmetric in all senses. This behavior can be quite rational, in the sense of arising from the successful maximization of an appropriate utility function. The result is that different social rates of discount apply to different types of assets. This is simply a recognition of the fact that conventional theory, in equating all rates of discount, inappropriately abstracts from normative distinctions between markets (bonds and capital stock) that arise owing to the differing international, political, and social consequences of operating in those markets. A return on capital of 10 percent may not compensate for an equal borrowing cost if the IMF appears to be lurking around the corner.

There are some apparent welfare implications of this recognition. Anytime all social and objective interest rates are not set equal, an economy suffers from what would be from a conventional standpoint a Pareto-inferior allocation of resources. In the first model, it will be shown that a country that suffers a loss of sovereignty in borrowing allocates insufficient resources to investment in capital equipment. On the other hand, it will be shown that in a model with investment, the issue is not that a utility cost of borrowing exists, but rather that a disutility exists that applies differentially to investment and to lending or borrowing. Once the stigma is also applied to domestic nonfinancial (dis)investment, the distortion of investment decisions disappears, and the only inefficiency is in consumption choices

In another model, we will show that a nation with a similar utility function will not take advantage of opportunities to use international borrowing to reduce economic risk, even if it has access to insurance provided by a perfectly competitive, risk neutral provider of contingent securities. It also may become more difficult for a borrower to establish creditworthiness on the basis of reputation.

The models raise some important methodological issues. One is their relationship to welfare. Here, since we are arguing that while the utility function is affected by the amount borrowed or by the amount a borrower fails to repay, utility would not be similarly affected if money could be borrowed without international consequences, such as IMF programs. This fact is relevant for policymaking, but it might be overlooked by an economist who simply observed an apparent aversion to borrowing. This reflects the fact that the source of utility in this case is relationships between nations, rather than the enjoyment they receive from consumption. The paper demonstrates that such concerns can be modeled in the manner of Robbins, but only at a cost of practicality and real understanding.

Such issues range from finding appropriate utility function for empirical prediction to characterizing the welfare of the agent. Agents motivated by a disutility of operating in international financial markets may defy the predictions of economists who incorrectly interpret a lack of borrowing as a preference for deferred utility, or an economically based lack of credibility. Such economists may find that their inferences cannot be generalized from one area of behavior (international borrowing) to a second, apparently identical field. They certainly cannot generalize an increasing distaste for foreign capital to a potentially more just system that garners the enthusiastic support of the developing world. With regard to welfare issues, the

economist can make many mistakes, including a failure to ascertain the appropriate conditions for efficient use of financial markets.

Many developing nations find it difficult to attract foreign capital. Since they often badly need funds, it may seem hard to believe that their behavior could be significantly affected by a disutility of borrowing or a disutility of incomplete repayment. But disenchantment with the conditions attached to borrowing may be growing. Argentina has recently challenged the financial establishment, expressing a refusal to deal with lenders on their own terms. The government has improved its fiscal position, but it has done so by raising taxes, a policy of which the IMF disapproves (Rohter 2004). Will a rejection of conditionality lead to a restriction in the availability of credit? It may be difficult for Argentina to reenter world capital markets without the support of the IMF. Will Argentina and other nations that have suffered ill consequences from foreign borrowing find some alternative way of obtaining capital? So far, the Argentine economy seems to be enjoying an increased flow of capital from its own citizens. This almost deliberate substitution of domestic for foreign capital may reflect the kind of disutility of international capital modeled in this paper. Perhaps a disutility that had previously been felt by citizens is finally finding political expression in number of nations.

Some nations have failed to attract much foreign capital in the first place. This may be due to the fact that the types of policies that attract foreign lenders are repugnant to the nations involved and are never adopted. Often, a failure to attract foreign capital is interpreted by economists as a symptom of a lack of credibility. But it is one lesson of this paper that an apparent lack of credibility may be due to the disutilities imposed by the international financial community.

If this fact is recognized, perhaps international institutions might eventually understand the situation and bow to the necessity of easing their demands. This seems unlikely. Alternatively, it may be that self-interest will bring international capital back to the table; the prospect of lending at a profit may overwhelm any desire to punish "bad behavior" on the part of recalcitrant nations such as Argentina. Neoclassical economists would interpret this as a lack of credibility on the part of lenders in a threat to cut a nation off from capital markets. The outcome may thus rest on a tougher test of all parties' credibility than they have encountered to date.

MODEL I: NON-PECUNIARY COSTS OF INTERNATIONAL DEBT IN A SMALL COUNTRY MODEL

We offer two models in this paper, both based on models in Obstfeld and Rogoff (1996), with slight modifications to functional form and the addition of unconventional arguments to the utility functions. We will sometimes refer to the latter additions as "sovereignty-loss disutility." Both involve infinitely lived representative agents. The purpose is not to suggest that such models are always insightful, but rather to measure various issues against a sort of neoclassical "baseline," in order to isolate certain concerns. For the most part, we will leave to one side other objections that have been raised against representative agent models. One claim sometimes made in defense of neoclassical models is that, in spite of various shortcomings, they offer tools to deal with welfare and policy questions that cannot be answered by nonneoclassical methods, such as Sraffian models. One could see this paper as a response to such an argument, that we must put up with various theoretical weaknesses to obtain the benefits of a useful theory of welfare and policy choice.

The agent, in this case a small state, chooses a consumption stream to maximize the discounted present value of its stream of utility, subject to its available resources. In the first model, agents have two means of transferring wealth over time. They can invest in domestic capital or in foreign bonds denominated in the consumption good. In the second model, which features an uncertain but given endowment, agents trade securities that pay different amounts to different parties depending upon the outcome of random events. This market is similar to an insurance market, but it shares many features of a market for securities such as equities that pay off different amounts depending upon the success of the borrower.

To start the first model, we assume the objective function:

$$\sum_{s=t}^{\infty} \beta^{s-t} (aC_s - (b/2)C_s^2 + cB_s)$$

where C_t is consumption in period t, B_s is the stock of outstanding bonds at time s, and β is a fixed discount factor between 0 and 1. B takes on a positive value if the country is a net lender, and a negative value if it is in debt. The utility function is conventional, with the exception of

the term involving B. This is a very crude representation of loss of sovereignty due to borrowing. Lenders have greater standing in the world the more they lend and borrowers lose sovereignty as their debt increases. It should be noted here that this term does not reflect a dislike of borrowing per se or a normal preference to defer consumption. It is a distinct motive. If a country could borrow without losing world standing then it would be more willing to borrow. The term is very similar to an addition to the interest rate, and it will have similar effects. But it is entered into the utility function to emphasize that it represents a nonmonetary cost of borrowing.

As mentioned before, the economy is constrained to consume no more than its current income plus its inherited assets. This leads to the budget constraints

$$C_t = B_t (1+r) - B_{t+1} + K_t + F(K_t) - K_{t+1}$$

for all t, where r is an exogenously given world rate of interest, K_t is the amount of the capital good, F() is the production function, and the t subscripts indicate a particular time period. (The notation for the models is for the most part borrowed from Obstfeld and Rogoff.) The representative agent's interest rate is constant because it is a small country and borrows too little to affect world financial markets. K is assumed to be the same as the consumption good, so that goods can be converted to capital and back again. Once capital is used it becomes available again for consumption, investment, or foreign lending. Capital is the only input into production. Note that investment (the last three terms of the equation) can be negative. Depreciation is neglected for simplicity. The constraint says that the economy inherits foreign debt or bonds, the capital stock, and newly produced goods. All of these resources can then be divided into consumption or next period's capital, bonds, or debts.

This sort of model is subject to a number of important critiques that are tangential to the ones emphasized here. First, it becomes analytically very difficult to conceptualize an input called "capital" when there are many goods that are produced by many capital goods. Normally, this problem renders the determination of the interest rate problematic (Garegnani 1990). Here, we take the interest rate as given by the world market. Also, the use of only one factor of production prevents some of the problems of this type, though it does not help us understand the real-world situation in which there are multiple produced inputs and outputs. Another problem with representative agent models is that many phenomena that occur in a world with a large

number of heterogeneous agents do not translate into a model with a single representative agent (Kirman 1992; Hartley 1997). Fortunately, here we are dealing with a nation as a whole; it is in this setting that concerns about sovereignty make the most sense. Still, none of this rescues neoclassical models from problems related to the capital controversy or critiques of the representative agent. We merely neglect such concerns here, so as to concentrate on another critique, which, we argue, carries a great deal of force in its own right.

The first-order condition for this problem is obtained by substituting for consumption in the utility function with the period budget constraint and differentiating by B_{t+1} . (We neglect the condition needed to prevent "Ponzi games.") For ease of analysis, we assume that $\beta(1+r)=1$. We obtain

$$C_{t+1} = C_t + \frac{\beta c}{b}$$

Clearly, the consumption path is "tilted" toward the future, in contrast to a similar model without bonds in the utility function, which would lead to equal consumption in each future period. Agents want to defer their consumption because of the sovereignty penalty of borrowing and the benefits of lending. The degree of the tilt depends upon the subjective rate of

¹ A word about the no-Ponzi constraint is in order. In this case, it is not restrictive. In models of this type, it is (Obstfeld and Rogoff 1996)

$$\lim_{t\to\infty} ((K_{t+1} + B_{t+1})/(1+r)^t) = 0$$

Now, at some point 0, as consumption grows, the total assets of the representative agent reach zero from above. Its debts will climb faster, the lower its income, Y. Suppose then (without loss of generality) that Y = 0 in all periods from time 0 onward. In time 0, K+B will become -C*, where C* is some constant level of consumption in that period. Subsequently, according to the Euler equation, consumption will rise by β c/b each period, and each period's consumption will be subtracted from K+B. Hence, in time 1, K+B will be -2C* - β c/b, then in time 2, -3C* - 3β c/b, etc. As Gauss realized as a schoolboy, the tth partial sum of this infinite arithmetic series in K+B is -.5t(C* + C* + t β c/b) (Weisstein 2005). Under our assumption that all consumption is financed by borrowing or capital decumulation, the condition above then becomes

$$\lim_{t\to\infty} (-.5(t+1)(2C*+(t+1)\beta c/b)/(1+r)^t) = 0$$

One can then apply l'Hopital's rule twice by differentiating both numerator and denominator of the expression after the limit sign twice with respect to t, then taking the limit as t goes to infinity. The limit is indeed zero, which can be verified numerically.

discount, β , and the parameters of the utility function. It is clear that consumption drifts upward indefinitely and in fact, $C_t \rightarrow \infty$.

The next first-order condition is obtained by differentiating with respect to K_{t+1}

$$\beta(a - bC_{t+1})(1 + F'(K_{t+1})) = a - bC_t$$

This equation determines the level of investment. Normally it would imply that the marginal product of capital should be set equal to the interest rate (1+r) = 1+F'(). Then, at the margin, the nation can transfer wealth from present to future at the same rate using international bond markets as it can via investment. Here, though, we combine with the first first-order condition to get

$$c = (a - bC_{t+1})(1 + F'(K_{t+1}) - (1 + r))$$

The first term in parentheses is the marginal utility of consumption and is assumed to be positive. Since c is a positive parameter, the marginal utility of capital, F'() exceeds the interest rate r. In other words, money could be borrowed at the world interest rate and be invested profitably at the margin. So there is an important welfare issue here: investment takes place at less than an efficient rate, because of the disamenities associated with the capital markets. If money could somehow be obtained without the disutility, a Pareto improvement could be made and a free lunch obtained. This contrasts with a situation in which investment is low because the populace heavily discounts its future well-being. It is pointed out below that there may be serious consequences of economists' failure to understand this distinction.

For the sake of comparison, consider a situation in which borrowing and disinvesting carry the same disutility, meaning that all means of moving consumption forward in time, regardless of their international ramifications, reduce utility. This would not apply in the previous model, because there we assumed that the disutility stemmed from a diminution of stature in the world due to borrower status, rather than immediate consumption as such. Here we drop the critical assumption that present-future commodity tradeoffs have different significance depending upon the international context of the tradeoff. To fix ideas, consider the following utility function

$$U = \sum_{s=t}^{\infty} \beta^{s-t} (aC_s - (b/2)C_s^2 + c'(K_s + B_s))$$

The first first-order condition is the same as before. Agents still tilt their consumption into the future owing to sovereignty concerns. The second first-order condition is obtained by differentiating with respect to K_{t+1} . The result is then substituted into the consumption first order condition, leading to

$$(1+r) = (1+F'())$$

In this case, there is no underinvestment. As in a conventional model without bonds in the utility function, investment is independent of the discount rate and equal to the world discount rate. There is no Pareto improvement to be made on the production side, even if money could somehow be sneaked into the country without compromising sovereignty. It is thus seen that the underinvestment result depends upon not only a disutility of borrowing but also *unequal* disutilities of borrowing and disinvesting. The conclusion rests on the theme of different utilities associated with the diverging international conditions under which future and current consumption are traded off. Concerns about such international circumstances of economic activity have consequences for the efficiency issues emphasized by economists.

MODEL II: AN ONUS ON INTERNATIONAL BORROWERS IN RECESSION

Having established some significant effects for borrowing disutility in a setting of certainty with investment, we now abstract from investment and add an element of what neoclassical economists usually call uncertainty. Here, this means that all random variables are drawn from a known distribution. In many settings, this model does not apply, because there are no known moments or distribution (Davidson 2002, Ch. 3). Still, we proceed, because we wish to investigate a different critique that deserves separate consideration.

Here we abstract from production and consider a situation in which each country receives a random endowment of the consumption good in each period. The country can also

buy securities on the world market that provide payments in certain states of the world (endowment levels), of which there are a discrete number. Formally, a country's endowment is

$$Y_t = Y * + \varepsilon_t$$

where epsilon is a mean-zero, independently and identically distributed discrete random variable with a symmetric distribution, and Y* is a known constant. Countries can enter into contracts on world markets in advance to receive particular amounts of the consumption good in particular states of the world (particular values of ϵ). The amount *paid* in period t is $P(\epsilon_i)$. So, the country presumably signs a contract to pay a positive P in good times in return for an insurance benefit – P in bad times, both of which will depend in general on the exact state of the world. Since nations are risk averse, they presumably wish to take advantage of this opportunity to at least some extent.

The model once again features a disutility associated with certain sorts of financial dealings. In this case, we assume that the country loses standing in the world if it collects on its insurance contract. This insurance can be viewed as the payout of a variable-return bond. It may seem odd that a country could suffer a disutility for collecting money as specified in a previously signed contract. But often, contracts do not explicitly state what happens when a borrower experiences such hard times that it cannot repay for legitimate reasons. Sometimes both borrower and lender will deny that default is possible, when they know well that it is. Many more risks are knowingly incurred than are publicly acknowledged. When default happens, the borrower is often blamed, even if both parties were clearly gambling. Under current institutional arrangements, borrowers in this situation are often subjected to the kind of loss of autonomy modeled here. This is broadly the same sort of disutility that might arise owing to the social stigma of welfare receipt, when what is arguably a right is seen as a kind of largesse.

We now consider the utility function (for each individual period s)

$$U_s = aC_s - bC_s^2 + dP_s(\varepsilon_s)$$

if
$$P_s(\varepsilon_s) < 0$$

and

$$U_s = aC_s - bC_s^2$$

if
$$P_s(\varepsilon_s) \geq 0$$

The P term is added to the end of the objective function to indicate the sovereignty "cost" of recovering insurance benefits. Naturally, this disutility is suffered only if the country must receive a benefit, rather than paying a premium. The problem has two constraints: first, the insurance company or investor is risk neutral, and insurance is provided by a competitive world industry, so that profits are measured by their expected value, and the industry earns zero profit. The latter condition can be represented by

$$\sum_{i} \pi(\varepsilon_{i}) P_{s}(\varepsilon_{i}) = 0$$

The Lagrangean for the problem is the sum of the expected value of total discounted utility plus a Lagrange multiplier, u, times the constraint:

$$L = E_t \sum_{s=t}^{\infty} \beta^{s-t}(U_s) + \mu \sum_{i=1}^{N} \pi(\varepsilon_i) P_t(\varepsilon_i)$$

The second group of constraints for the problem shows that consumption in each period is equal to the resources households choose to spend.

$$C_s = B_s(1+r) - B_{s+1} - P_s(\varepsilon_s) + Y* + \varepsilon_s$$

Each period, the sovereign has the bonds or debt inherited from the previous period, with interest ($B_s(1+r)$). It also receives its income, which, as we saw earlier, is composed of a deterministic part, Y*, and a stochastic part, ϵ . Consumers also pay a premium, P, based upon the realization of ϵ . P may be negative or positive. A positive P can be interpreted as an

insurance premium and a negative P an insurance benefit. Also, P can also be thought of as the return to a security with a variable payout. The country has to pay back more of its borrowings when times are good. All available resources are split between consumption, C, insurance premiums, P, and borrowing or lending, B_{s+1}

The bond first-order condition is obtained by using the second constraint to substitute in the Lagrangean for C and differentiating with respect to B_{t+1} . We assume that $(1+r)\beta=1$. This gives

$$E_tC_{t+1} = C_t$$

which is the standard result that with a quadratic utility function, consumption does a random walk.

The other first-order condition is interesting. Suppose we differentiate by $P(\epsilon_s)$, where we assume $P(\epsilon_s)$ is negative. This means that the country is receiving insurance benefits, rather than paying a "premium." The first version of the condition

$$(a - bC_s(\varepsilon_s)) - d = \mu$$

The second version of this condition, for a country that is paying premiums, is

$$(a - bC_{s}(\varepsilon_{s})) = \mu$$

Equating the left-hand sides of the previous two equations shows a relationship between consumption in "good times" and consumption in "bad times."

$$C_{s}(\varepsilon) = d/b + C_{s}(\varepsilon')$$

Here ϵ represents any state in which P is positive and ϵ ' represents any state in which P is negative. From this we surmise that

$$P(\varepsilon) = P^* + \varepsilon$$

and

$$P(\varepsilon') = P^{**} + \varepsilon'$$

where the P's are constants. From the zero-profit constraint, we know that

$$\sum_{\varepsilon} \pi_i(P^* + \varepsilon_i) + \sum_{\varepsilon'} \pi_i(P^{**} + \varepsilon_i) = 0$$

where the summations are taken over positive or negative ε , respectively, or

$$-\sum \pi_i \varepsilon_i = \sum_{\varepsilon} \pi_i(P^*) + \sum_{\varepsilon'} \pi_i(P^{**})$$

Since the distribution has a mean and median of zero, this becomes

$$P* = -P**$$

So, the sovereign receives a certain average payment in bad times and pays an equivalent amount in good times. In addition to this payment, the insurer compensates the sovereign completely for the positive or negative ε , so that (after-insurance) income is Y*-P* or Y*-P**.

Now consider the effect of the disutility of low repayment on the ability of reputation to support the maintenance of capital markets. We suppose that the insurer can credibly commit to cutting off the sovereign from capital markets if the insured country fails to pay a premium. Default would occur, if at all, only if ε were positive. The ability of the sovereign to participate in world insurance markets depends in this case on its ability to credibly commit to pay its premiums. It can do so if the gain to be made by failing to pay a premium is less than the cost of the loss of access to world insurance markets infinitely far into the future. Note here that we are referring to an ability to commit to pay P*, which is different from the issue of the country's

loss of utility when it *receives* P**. The gain in the period in which the sovereign defaulted (did not pay P* when due) would be

immediategain =
$$a(Y^* + \varepsilon) - b(Y^* + \varepsilon)^2 - a(Y^* - P^*) + b(Y^* - P^*)^2$$

$$= a(\varepsilon + P^*) - b(2(Y^*\varepsilon + Y^*P^*) + \varepsilon^2 - P^{*2})$$

The first two terms in the first equality show the utility that the sovereign enjoys by refusing to make its insurance payment. It simply consumes its total income, including the ϵ , which would have been partly surrendered if the country had paid its premium. The second two terms show the utility that the country would have received from paying its premium and consuming the remainder of its endowment. This immediate gain (the entire right-hand side of the equation) has to be weighed against the present discounted cost of being isolated from world capital markets in the future. This is a cost, because capital markets provide a utility-enhancing protection from the vicissitudes of income. The loss in the future from being cut off from world capital markets is

$$loss = E_t \sum_{s=t+1}^{\infty} \beta^{s-t} (a(Y^* + \varepsilon_s) - b(Y^* + \varepsilon_s)^2) - \sum_{s=t+1}^{\infty} \beta^{s-t} [a(-5P^* - 5P^{**} + Y^*) - b(5(Y^* - P^*)^2 + 5(Y^* - P^{**})^2) + 5dP^{**}]$$

Using the fact that $P^{**} = -P^*$ and that ε has a mean of zero,

$$= \beta (1 - \beta)^{-1} [aY^* - bE_t(Y^{*2} + \varepsilon^2) - aY^* + b(Y^{*2} + P^{*2}) - .5dP^{**}]$$

$$= -\beta(1-\beta)^{-1}[b(var(\varepsilon)-P^{*2})+.5dP^{**}]$$

Here, losses are negative numbers. The first sum on the right-hand side of the "loss" equation is the discounted value of the utilities obtained in the future when no insurance is available. The terms after the second summation indicate the discounted value of the future income that would have been obtained if the country had kept access to insurance markets. It would then have received Y* each period and paid P* half the time and P** (a negative "payment") the other half of the time.

We now wish to compare these results with the standard model in which there is no utility loss from the loss of autonomy. In the absence of the d term in the utility function, there is full insurance (Obstfeld and Rogoff 1996, p. 364). Specializing the model in Obstfeld and Rogoff to the quadratic functional form used here, the gain becomes

$$immediategain2 = a(Y^* + \varepsilon) - b(Y^* + \varepsilon)^2 - a(Y^*) + b(Y^*)^2 = a\varepsilon - 2bY^*\varepsilon - b\varepsilon^2$$

The difference between the gains in the two situations is

$$aP*-2bY*P*+bP*^2$$

The immediate payoff for nonpayment differs between the cases with and without sovereignty utility. If this difference is positive, the net gain from exiting insurance markets is greater with the d term than without. Since P* is positive, the sign of this expression will depend upon the parameter values. The loss in the model without disutility from sovereignty loss is

$$loss2 = -b\beta(1-\beta)^{-1} \operatorname{var}(\varepsilon)$$

which differs from the earlier loss by the amount

$$\beta(1-\beta)^{-1}(bP^{*2}-.5dP^{**})$$

Thus, since P** < 0, the loss of utility from default is unambiguously lower when the country has a sovereignty term in its utility function. The fact that the loss and gain change when sovereignty-loss disutility is considered indicates that it is more or less difficult to sustain lending on the basis of reputation. Recall that if the cost of defaulting is less than the benefit, then the borrower or insurance buyer will have an incentive to default, destroying its ability to borrow in the first place. First, consider the future loss from losing access to world capital markets. Without considering the unusual feature in the utility function, this loss (loss2) is proportional to the variance of the disturbance to income. This is logical, because the main sacrifice from world market withdrawal is in the steadiness of income in the face of the random

shock. Two terms reduce the loss, though. (1) The first term in the second pair of parentheses in the preceding equation shows that the loss is reduced owing to the variation in income due to the lack of full insurance. This term is larger, the larger the amount by which insurance payments fail to even out income across realizations of the disturbance, an amount reflecting the lack of full insurance. Recall that the lack of insurance is due to the unusual term in the utility function. (2) The second term shows that the loss is also reduced because of the disutility of participating in world markets, i.e., the loss of autonomy discussed above. It is interesting to note that the reduction in the comparative disutility of default involves not just the explicit reduction in the benefits of borrowing due to the modification of the utility function, but also a reduction due to imperfect insurance, a byproduct of the modification.

The gain also differs, by the amount shown in the equation below "immediategain2." Intuitively, this is a reduction in the premium one avoids paying by opting out of the capital market. The reward for breaking the contract to pay a premium in a period of high income is reduced because the premium is lower when insurance is incomplete.

The world capital market is inhibited by a lack of credibility on the part of the borrower, which arises from a disutility from *receiving* payments. The failure of credibility owes to the reduced value of insurance market participation to a potential purchaser of insurance. Sovereignty-loss disutility reduces the value of obtaining insurance by lowering the utility benefits of collecting a given insurance benefit and by reducing the amount of insurance a country will optimally choose. Since the benefits of insurance are low when sovereignty appears in the utility function, incentives may be higher for a nation to effectively opt out of the insurance market by refusing to pay a premium. Once again, the efficiency of the capital market is affected by the nonefficiency issues discussed here. Several policy issues are important here. For example, do the results above imply that recipients of insurance payments should not be forced to suffer a loss of sovereignty? In the next section, we argue that while conventional economics purports to subsume all types of utility functions, it does not do a good job in handling the policy issues raised by our model.

DOES INTERNATIONAL BORROWING DISUTILITY OR LOW-RETURN DISUTILITY HAVE METHODOLOGICAL IMPLICATIONS?

As suggested by Robbins, we have incorporated a nonmaterial value into our analysis by putting it into the objective function, as we would with any other good. Does a slightly unusual utility

function have any wider implications? Does Robbins's technique adequately deal with the issue at hand? The model leads to nonstandard conclusions largely because sovereignty disutility drives a wedge between what might be called experienced utility and action utility (Sen 1973; Kahneman 1994). Action utility is the utility function that drives behavior. Actors behave "as if" they maximize this utility function; hence it is the correct function for modeling market behavior. In this case, countries act as if they maximize a function with a penalty for borrowing (model I) or with a penalty for making a negative payment on a security or collecting on insurance (model II). This is the function that Robbins says we ought to be concerned with. On the other hand, this utility function does not reflect actual welfare (experienced utility) in a certain sense: countries do not actually lose welfare from borrowing per se, as an observation of their action utility function might suggest. They judge in fact that they would be better off if they were able to borrow; but their desire to borrow is offset by an overriding dislike of submitting to the wishes of the lender. This fact is not apparent to an economist who judges welfare strictly on the basis of observed market behavior (the "revealed" action utility function) and does not take into account that agents are concerned with their relations with other countries, as opposed to simply smoothing their consumption streams. Robbins would not be particularly concerned with such welfare issues, because he rejected Pigouvian welfare economics (Robbins 1927). But we will argue that it is difficult to avoid such questions, if economics is to provide any meaningful answers to policy problems.

The misunderstanding of observed capital market behavior induced by these methodological mistakes leads the economist to make several mistakes about appropriate policies, a logical outcome of a failure to consider the appropriate measure of utility.

(1) When the utility function should properly be perceived as containing a penalty for international capital-market participation, the incorrect conclusion is drawn that this function is arbitrary or irrational.

The neoclassical (Robbinsite) economist does not attempt to investigate the reasonable basis for a particular utility function (in this case the concern about sovereignty). We use the term "reasonable," as against rational, to refer to the motivation or justification for a particular preference. In some cases, a preference may have no more justification than the tastes of the individual, as in the preference for chocolate over vanilla. Neoclassical economics does not distinguish this case from others, where certain reasons lie behind choices. This fact is reflected

in the use of the word "taste" for matters of morality such as discrimination. Hence, in this case, the disutility of capital market participation appears to be equivalent to an arbitrary or irrational whim. If it is seen only as such, the appropriate policy response would be to educate sovereign governments that there is no reason to have an irrational desire not to borrow. Indeed, nations might come to that conclusion on their own. On the other hand, once the correct theory (in the sense of a justification for the utility function) is obtained, it would be recognized that the correct policy response would be to eliminate the imposition of sovereignty loss when it is clearly counterproductive. What was initially perceived as an irrational superstition on the part of potential borrowers is revealed to be the logical response to the onerous way repayment is enforced, and the burden of change is shifted from borrower to lender. This policy conclusion hinges on appropriately judging the agent's rationale for a choice, not merely observing the choice and its implied utility function. The methodological implications of this are discussed later.

(2) The presumption is made that when capital markets are competitive, a Pareto optimum is attained. In fact, resources are misallocated.

In the neoclassical model, free capital markets are good from a policy perspective because they allow capital to be used where it is most productive and because they allow a diversification of risk. No possible Pareto improvement is possible in this model once capital markets are freed. But this policy conclusion is incorrect. This is due to the fact that countries are participating less in financial markets for a particular reason. They are not discounting future utility to a small degree. If they were, the allocation in the first model would be optimal. Free capital markets respond well to differences in rates of time discount. Instead, there is a disutility to borrowing under current international arrangements that results in several failures of efficiency: a) underinsurance; b) underborrowing; and c) underinvestment. The implication is that the world should not be complacent about observed problems in capital markets.

(3) "Deep Parameters" are miscalibrated

Many neoclassical macroeconomists insist that empirical models should be functions of deep parameters such as utility function parameters, rather than behavioral relationships such as consumption functions or Phillips curves. This tack is thought to be an answer to the so-called

"Lucas critique," which argues that Keynesian behavioral relationships can vary as the policy regime varies (Lucas 1991 [1976]). For example, if the monetary authority starts an aggressive program of seeking high employment, workers will respond only temporarily, as long as it takes for them to grasp the changes in the statistical process generating policy. Thus, an empirically estimated Phillips curve will break down once a new policy has been in effect for some time. It is far better to estimate the preference and technology parameters that stay constant over time and across policy regimes. A seminal paper, by Hansen and Sargent (1980, p. 7, quoted in Hartley 1997), "describes research which aims to provide tractable procedures for combining econometric models with dynamic economic theory for the purpose of modeling and interpreting economic time series. That we are short of such methods was a message of Lucas's (1976) criticism of procedures for econometric policy evaluation... The implication of Lucas's observation is that instead of estimating the parameters of decision rules, what should be estimated are the parameters of agents' objective functions and of the random processes they faced historically." Observations of lending behavior are often used as a means of obtaining measures of such "deep" parameters. For example, in one of the seminal papers on real business cycle theory, Kydland and Prescott (1982) estimate the risk aversion parameter of their utility function using the equation

$$r = \rho + (1 - \gamma)(\dot{c}/c)$$

where r is the interest rate, ρ is subjective rate of time discount, γ is the rate of relative risk aversion, and c is consumption, with the overdot representing differentiation by time. This equation is based upon the Euler equation for a consumer's dynamic optimization problem with a constant-relative-risk-aversion utility function. This estimation method will not work if borrowing appears in the utility function. If the equation above is estimated in that case, the upward "tilt" in consumption induced by the form of the utility function in the model above will be incorrectly attributed to a low rate of time discount or a high degree of risk aversion. If the estimate is then used to simulate the behavior of the economy, as called for in certain types of papers, the results could be misleading.

Moreover, estimates such as this one are often used for welfare analysis. For example, cost-benefit analysis often uses estimates of behavioral parameters to judge the merits of public

investments. If a rate of subjective discount or degree of relative risk aversion is incorrectly estimated, the cost-benefit analysis would be incorrect.

One way of looking at this point is simply as a factor that renders parameter estimates inconsistent across the contexts in which they are apparently observed. A discount factor or risk-aversion parameter estimated using data on borrowing will not be valid in predicting other types of behavior, such as investment. Nor will these parameters provide correct policy prescriptions when applied to still other intertemporal problems, such as cost-benefit evaluations.

All of these problems have a bearing on several general methodological strategies employed by neoclassical economists.

(1) "Preferences can be deduced from behavior and are therefore not unobservable."

In modern economics, this view is best represented by Samuelson and his "revealed preference" theory. While Robbins disapproved of this theory (1953, 101–102), it is consistent with Robbins's notion that preferences should be linked to actual desires, rather than to hedonic utility (the pleasure of the agent). In this paper, preferences can be observed, but the observations may lead to incorrect conclusions about the underlying reasons for behavior. This is due to the fact that welfare preferences are different from action preferences. Because observations of market behavior cannot discriminate between a disutility of sovereignty loss and a low rate of time preference (two observationally equivalent sets of welfare preferences), they do not provide enough information to make policy judgments through means such as costbenefit analyses.

The real problem is not that certain aspects of preferences are unobservable, but rather that the types of observation used must be expanded to gather information that would allow economists to distinguish between otherwise observationally equivalent cases. For example, it may help to study how agents justify their own behavior publicly and to other agents. An interview with a policymaker might reveal that he or she is averse to being subject to an IMF structural adjustment program. Another appropriate methodological strategy would be to study the history of relations between countries such as the United States, on the one hand, and small, developing economies on the other. Putting the current situation in its proper historical context would illuminate the cultural sensitivities and existing imbalances of power that are exacerbated by lending relationships.

(2) "Economists should not concern themselves with justifications for various preferences. Preferences should be used as data, not questioned. There is no arguing about the rationales for preferences. (De gustibus non est disputandum.)"

This precept was stated well by Lionel Robbins: "The general absurdity of the belief that the world contemplated by the economist is peopled only by egoists or 'pleasure machines' should be sufficiently clear from what has been said already... We do not regard it as part of our problem to explain why... particular valuations exist. We take them as data. So far as we are concerned, our economic subjects can be pure egoists, pure altruists, pure sensualists or—what is much more likely—mixed bundles of all these impulses" (Robbins 1935, p. 95). Robbins goes on to give an example of a consumer who cares about the well-being of the baker from whom he buys bread. As long as we strive to achieve our ends in a consistent manner, according to Robbins, it does not matter from the standpoint of economic theory what those ends are. Economic theory applies regardless of the utility functions, ends, or "valuations" of the economic agent. Here we have used this conception of value to adapt a conventional utility function to an unconventional "taste" for sovereignty.

But the analysis above shows that rationales for preferences must be discerned and tested. Different rationales lead to different conclusions about the values of various parameters. One rationale would interpret a low level of borrowing as the result of skewed period utility functions (precautionary savings). Another possible rationale is the one in the model above. As we have seen the different rationales have different policy implications. Further, to deduce proper policy, it is necessary to judge the validity of rationales for preferences. The policy implications of the above model will differ depending upon whether one believes that the aversion of nations to borrowing is a legitimate one. If the strictures imposed by an international financial institution are in fact justifiable or insignificant, then one would *not* draw the policy conclusion from the model above that such strictures should be eliminated.

The preferences-as-data approach also runs the risk of begging the question. Certainly, economics is consistent with many unusual sorts of preferences. One strategy of Robbins's is to argue that knowledge of ends and their validity should be drawn from outside economics: "[Economics] does not itself deal with the value of ends...But I have not urged you to *ignore* such considerations. On the contrary, I would suggest that the whole drift of my discussion goes to show that the economist who is to make most use of his materials must be prepared to be more than an economist, to transcend his subject. Unless he is prepared to go beyond the

technique of his subject, to live widely and intensely, to steep himself in the intellectual atmosphere of his time he will not be in a position to apply the machinery at his disposal—he will not be alive to the ends which he will be asked to take account of' (lecture notes on "Economics and Political Theory" 1932, quoted in Howson 2004). An economist must take account of the proper ends, but such ends must be drawn from outside economics, which consists of "technique" or "machinery".

Those who might accept this methodology must realize that a theory that simply adopts whatever preferences are found empirically or discovered by other disciplines runs the risk of being overly dependent on reasoning from outside the theory. If behavior can be motivated by any variables suggested by extra-economic work, one might justifiably adopt the explanation offered by that work, rather than simply "cooking" preferences to mimic or obtain extraeconomic results. Where is the value-added in a theory consistent with any desires at all? Should we not be uneasy with using information that is not somehow theoretically justified? Is it rational to have any arbitrary preferences, which may not make sense from the perspective of economic theory alone? If we have an economic theory, why not examine preferences to determine if they are consistent with that theory? What if the preference in question was an aversion to maximizing? Have we truly explained an economic outcome if we simply attribute it to desires that make no sense? To truly explain behavior, is it sufficient to say that the agent prefers to behave that way? It seems that economics is flexible enough to accommodate many types of preferences but leaves us without a compass to discern their deeper implications. This is reflected in the results above. Certain analytical results have been derived, but few would regard them as a full analysis of what some regard as imperialism.

As one last question, suppose we observe a preference for sovereignty. We can then rationalize the nation's behavior in rejecting loans. But is this really rational? In the model above, the preference to avoid loans is "rational" when there is a sovereignty disincentive. But that disincentive would not be present in the absence of an *irrational* economic doctrine, subscribed to by international financial institutions, that says countries benefit by submitting to the demands of lenders. Thus, the sort of behavior described here might be, quite literally, the result not of (rational) microeconomic optimization, but of macroeconomics (an illogical theory of what is the best policy toward debtor nations). (See Anderson 2002 for another example.) The ultimate source of behavior is irrational, despite the fact that nations are doing the best they can with the resources they have.

(3) "Institutional choice is a matter of efficiency. Since institutions are instruments to achieve individual ends, they can be judged solely on the basis of the goods they 'deliver.'"

In this model, we deal with an institution called the world capital market. The usual analysis of institutions (at least in the "new institutional economics") sees configurations of institutions as a reflection of their efficiency properties (their ability to reduce transactions costs). By contrast, the subject of this paper is not the best instrument for delivering capital to its most efficient use. Rather, this issue is disutilities imposed by the capital market itself. An analogy would be a country that disliked central planning on the grounds of its violation of individual autonomy. In such a country, even though a central planner may be able to effect a Pareto-improving reallocation, citizens would reject the planner on the grounds of the undesirability of planning itself. In the same way, according to this paper and previous work by Elizabeth Anderson (2004), borrowers lose when they are entangled in certain demeaning social relations or institutions. In this sense, the models above invoke extra-economic considerations, even though they are obtained simply by a modification of the utility function. In turn, these extra-economic factors influence the economic properties of the system. Because of a concern about international institutions, certain standard efficiency results do not hold, as we have shown here. The analysis of institutions as strictly neutral means of attaining independent ends fails.

CONCLUSION

Two models were developed above. In both, the concerns of the representative agent, a nation in this case, went beyond consumption streams to the types of external relations involved in international financial market participation. In both models, the implications of the objective function were shown to go beyond intangibles such as standing in the world. In the first model, the nation did not sufficiently take advantage, from an efficiency point of view, of the opportunity to finance investment with foreign capital. It also tilted its consumption stream toward the future specifically because of the presence of a borrowing variable in the utility function. In the second model, the representative nation either bought insufficient insurance or obtained too little financial investment from a welfare point of view. One result was that it was

more difficult to maintain international lending markets strictly on the basis of reputation for repayment.

What are the implications of these findings for the line of argument that supporters of open capital markets develop using similar models? With conventional models, the models studied here agree that capital markets can benefit both borrowers and lenders. On the other hand, this paper shows that to reap these benefits, capital markets must somehow be adapted to eliminate the disutilities of borrowing or paying a small return. To some extent, the disutilities may be the result of unenlightened policy driven by the Washington consensus. But the disamenities for which the international financial institutions are responsible are to some extent the result of an imbalance of power that enables lenders to dominate borrowers, a problem that has vexed debt markets for some time and is largely invisible to the developers of standard models of international finance or the new open economy macroeconomics. A sure sign that existing disutilities of debt are the symptom of the power that the developed economies exert over the less developed is that the developed countries, including the United States and the members of the European Monetary and Economic Union, have chosen not to enforce fiscal strictures on themselves.

In spite of whatever insights the models above may offer, it was argued that the developed model revealed some potential limitations of an often-adopted strategy for dealing with noneconomic issues. The model thus had broader implications. A summing up of the methodological points demonstrated by the mathematical analysis is in order. All of these points addressed neoclassical stratagems that have been deployed to handle arguments that economics cannot handle certain types of preferences. Mostly such arguments suggest that economics is neutral among various values, preferences, or ends, and can be adapted to suit any of them. The first point made above was that when the justifications for the preferences in the model are not investigated, they are treated as equivalent to mere whims or "likings"; this unfairly denigrates the moral legitimacy of the concerns of potential borrowers. If developing nations lack the proper "taste" for foreign capital, they cannot reasonably lay the blame for a lack of capital on the lenders. Second, the optimality properties of a competitive allocation are different when an unusual form of preference is involved; an economist interested in welfare must dirty his or her hands and dig into the matter of motivations. Only once preferences are fully understood, will it be realized that the attenuation of capital markets is not merely an optimal response to a nation's desire to defer consumption or its lack of credibility. Third, the failure to understand the

reasoning behind behavior leads the economist to believe he is estimating one parameter (perhaps the subjective discount rate) when he or she is in fact measuring a tendency that does not apply outside of a particular international context.

A second list of methodological implications followed. First, the paper noted that economists often defend the empirical legitimacy of preference analysis by arguing that utility functions can be discerned by observing a consumer's choice behavior; it was shown that in order to draw interesting conclusions from the choices of the nations modeled here, one would have to distinguish between apparently indistinguishable motivations for preferences, a requirement that would lead one beyond the narrow sorts of fact-gathering associated in economics with empiricism. Second, the paper pointed out the limitations of the view that values, while important, were outside the purview of economics. In particular, we argued that the policy implications of the models would depend upon whether the fear of loss of sovereignty was a legitimate concern; if not, there would be no reason to deviate from the laissez-faire allocation derived in the model. Another challenge to the view that values were distinct from economics and should be drawn from other fields was that this sort of question-avoidance amounted to question-begging: the adoption of noneconomic value judgments causes one to question the value-added offered by a theory that is silent on such crucial matters. If justice and power are important in the relations between international lenders and borrowers, neoclassical economics may not be the best vehicle for understanding the issues at stake. Finally, the sorts of preferences modeled here are simply irreconcilable with the standard view of institutions in neoclassical analysis as neutral instruments for achieving individual ends; the institutions in this case are themselves bearers of value.

All of these observations suggest that in dealing with such a fundamental issue as the autonomy of a nation, economics cannot pretend to be neutral about the values at stake. An approach that purports to investigate the implications of a certain set of preferences without judging their validity or investigating their rationale fails to answer even the most basic policy questions at issue, is unable to draw empirical lessons that apply to other spheres of action, and is overly optimistic about the optimality of the status quo political economy.

A modeler who simply observes that foreign governments have an increasing distaste for borrowing, as depicted in this paper, has only gone a small part of the way toward understanding the moral, political, and economic issues at stake. Foreign voters, to the economist, may seem to be rationally maximizing an unreasonable value in rejecting

governments that accept the demands of foreign and international institutions. A just and efficient solution to frictions between small developing nations and international financial institutions may rest on the development of an understanding on all sides that, to the contrary, a "taste" for autonomy is both rational and reasonable. That achievement might allow small developing countries to garner the rewards of open financial markets touted by open-economy macroeconomists and finance specialists.

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