Contracting with a Time-Inconsistent Agent

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Abstract

I consider a repeated principal-agent model with moral hazard, in which the agent has $\beta\delta$-preferences, which are widely used to capture time-inconsistency. I first analyze the case where the agent is *sophisticated* in the sense that he is fully aware of his inconsistent discounting. I characterize the optimal wage scheme for such an agent and compare it to time-consistent benchmarks. The marginal cost of rewarding the agent for high output today exceeds the marginal benefit of delaying these rewards until tomorrow. In this sense, the principal does not smooth the agent's rewards over time. When facing a sophisticated agent, it is optimal for the principal to reward the good performance more and punish the bad performance more in the early period, relative to the optimal wage scheme for a time-consistent agent. I also consider the case where the agent is *naive* in the sense that he is not aware of his time-inconsistency. I show that the principal's maximum utility is the same from a sophisticated agent and a naive agent.

Keywords : repeated moral hazard, time-inconsistency, $\beta\delta$-preferences, sophisticated agent, naive agent.

JEL **Classification Numbers:** D03, D82, D86.

1 Introduction

There is a considerable amount of evidence showing that agents' behavior often exhibits time-inconsistency. When the benefits of an action are in the future and the costs are immediate, agents do not give the benefits much weight. That is, they tend to postpone costly actions and tough projects (e.g. finishing up writing a paper, filing taxes or going to the gym), but rarely tend to postpone gratification.[1] Although a substantial amount of work has been done regarding individual decision making under time-inconsistency, there is little work done on economic interactions that include time-inconsistent individuals. In particular, a natural question is how a dynamically consistent principal would contract with an agent who has time-inconsistent preferences.

The focus of the current paper is on the repeated principal-agent problem where there is moral hazard, with the standard trade-off between incentives and insurance.[2] I depart from the standard repeated moral hazard literature in allowing the agent's preferences to be time-inconsistent, specifically present-biased. The contribution of this paper is to characterize the effects of time-inconsistency on the optimal contract and on the principal's expected profits. I show that if the agent is aware of his time-inconsistency, the principal does not smooth the agent's rewards over time. It is optimal for the principal to provide the agent with more incentives in the earlier period and less in the later period, relative to the optimal wage scheme for a time-consistent agent. Also, the principal achieves the same expected profit from a time-inconsistent agent who is fully aware of his inconsistency and a time-inconsistent agent who is not aware of his inconsistency.

The agent's time-inconsistency is modeled by $\beta\delta$-preferences.[3] An agent with $\beta\delta$-preferences has a discount factor $\beta\delta$ between the current and the next period, and δ between any other pair of successive periods. In other words, the agent's preferences for a payoff in date t over a payoff in date $t + 1$ is stronger as date t gets closer. A time-inconsistent agent may or may not be aware of his inconsistency. A sophisticated agent is fully aware of his consistency in the sense that he correctly predicts how his future selves will behave. A naive agent is not aware of his inconsistency, in the sense that he mispredicts the behavior of his future selves through an overestimated β.

The principal is not able to observe the effort levels picked by the risk-averse agent; thus, she

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[1] See Frederick, Loewenstein, O'Donoghue (2002), for an exensive overview of the literature.

[2] See, for instance, Rogerson (1985) and Lambert (1983).

[3] $\beta\delta$-preferences are first developed by Phelps and Pollak (1968) and later used by Laibson (1997), O'Donoghue and Rabin (1999a,1999b,2001).

faces the natural trade-off between smoothing risk and providing incentives. The principal offers a wage scheme to the agent ensuring that he accepts it in the contracting stage, period 0, and exerts high effort in both of the two periods, 1 and 2, following the contracting stage. The wage scheme exhibits memory in the sense that the wages paid in period 2 depend on the performance in period 1.

When the agent is sophisticated, the principal does not intertemporally smooth the agent's rewards. More precisely, the marginal cost of rewarding the agent today exceeds the marginal benefit of delaying these rewards until tomorrow for a high level of output and vice versa for a low level of output. This is because the agent's discount factor is changing over time. In contrast the optimal contract for a time-consistent agent smoothes wages in this sense. Moreover, in the earlier period a sophisticated time-inconsistent agent receives bigger rewards for good performance and bigger punishments for bad performance under the optimal wage scheme, relative to a time-consistent agent. The intuition for this result is as follows. Suppose that the principal has two options for the sophisticated agent: She can either increase the incentives in period 1 or increase them in period 2, relative to the optimal contract for the time-consistent agent. To achieve the same overall incentives, the increase in period 2 must be bigger, because period 2 is discounted more by the agent. But the agent's individual rationality holds with slack under the former increase if it binds under the latter increase. This is because the agent discounts both changes to the contracting stage at the same rate since there is no payment made in the contracting stage. Thus, the principal can increase the wage differential in period 1, relative to the optimal scheme for the time-consistent agent, and still ensure that the agent accepts the contract.

When comparing the expected profit from a sophisticated agent to the one from a time-consistent agent, since the sophisticated agent discounts at either $\beta\delta$ or δ, there are two benchmark time-consistent agents, one with a discount factor δ and one with a discount factor $\beta\delta$. So I compare the expected profit from a sophisticated agent to that from these two time-consistent agents. The principal is better off with a time-consistent agent with a discount factor δ than with the sophisticated time-inconsistent agent, who in turn is better than a time-consistent agent with a discount factor $\beta\delta$, under a mild condition. This makes sense because the sophisticated agent with $\beta\delta$-preferences has an overall discount factor between δ and $\beta\delta$.

With a naive agent the principal's problem is more involved because she can deceive the agent

and potentially get information rents. First, I show that whenever the principal wants to implement high effort, the degree of the naivete of the agent, measured by the difference between the true β and his overestimated β, does not play a role. Moreover, the principal, implementing high effort in both periods, is indifferent between facing a naive agent and facing a sophisticated agent. This is particularly striking because with a naive time-inconsistent agent the principal can choose to deceive the agent. However, such an opportunity to manipulate does not provide the principal with higher profits and in the optimal contract the principal chooses not to deceive the agent at all.

This paper is related to a number of other papers in the literature. Rogerson (1985) considers a repeated moral hazard problem with a time-consistent agent, and shows that the optimal contract exhibits memory and that the marginal cost of rewarding the agent today equals the marginal benefit of delaying these rewards to tomorrow. Chade, Prokopovych and Smith (2008) study infinitely repeated games where players have $\beta\delta$-preferences. They characterize the equilibrium payoffs and show that the equilibrium payoff set is not monotonic in β or δ. O'Donoghue and Rabin (1999b) and Gilpatric (2003) both consider principal-agent relationship with time-inconsistent agents. The former introduces a moral hazard problem in the form of unobservable task-cost realizations and assumes that the agent is risk-neutral. The latter focuses on a contracting problem with time-inconsistent agents assuming that profit is fully determined by the effort, so effort is effectively observable. The current paper distinguishes itself from these papers by allowing the trade-off between risk and incentives. Eliaz and Spiegler (2006) characterize the optimal menu of contracts when a monopoly is contracting with time-inconsistent agents and show that it includes exploitative contracts for naive agents.

2   The Model

I consider a finitely repeated moral hazard problem. A principal is contracting with an agent to work on a two period project. Each period the agent can exert costly effort. The principal cannot observe the effort choices of the agent. The project, in each period, has an output which is publicly observed. The output in each period is stochastic and affected by the effort level picked by the agent in that period.

2.1 Timing

Unlike the standard two-period repeated moral hazard problem, there is need for at least three periods for the agent's time-inconsistent preferences to play a role. Consider the following timing of events:

$t = 0$	$t = 1$	$t = 2$
principal offers a contract	agent exerts effort, e_1	agent exerts effort, e_2
agent accepts or rejects	output is realized, q_1	output is realized, q_2
	wage is paid to agent, w_1	wage is paid to agent, w_2

More precisely:

- At $t = 0$, a contract, which is a wage scheme, is offered to the agent by the principal. Then the agent accepts or rejects. If she rejects, the game ends and both the principal and the agent get zero utility.[4] If she accepts, they move on to the next period.

- At $t = 1$, the agent chooses an effort level, e_1, which is not observed by the principal. The output, q_1, is realized which is observable by both the agent and the principal. The wage payment, $w_1(q_1)$, is made to the agent.

- At $t = 2$, the agent chooses an effort level, e_2, which is not observed by the principal. The output, q_2, is realized which is observable by both the agent and the principal. The wage payment, $w_2(q_1, q_2)$, is made to the agent.

In the contracting stage, $t = 0$, there is no effort decision. The only decision made by the agent is to accept or reject the contract offered by the principal. We can think of this as getting a job offer in March but starting in September, just as most economics Ph.D. candidates experience. Thus we have three periods with two effort decisions. I also assume that once the contract is accepted at $t = 0$, both agent and principal are committed to the contract until the end of period 2. That is, I focus on long-term contracts and abstract from renegotiation issues.

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[4]To justify this zero-outside option assumption, assume that the outside option has also a lag in payment. Then it would be natural to normalize the outside option utility to zero.

2.2 Agent

There are two effort levels, 0 and 1, and two outcomes, q_h and q_l with $q_h > q_l$. The agent receives utility $u(w)$ from the wage w, and disutility $\psi(e)$ from exerting effort e. He is risk-averse, that is, $u' > 0$, $u'' < 0$. The disutility from exerting effort is given by $\psi(1) = \psi$ and $\psi(0) = 0$, where $\psi > 0$. The agent has additively separable preferences, so, his net utility in period $t \in \{1, 2\}$ is given by $v_t = u(w_t) - \psi(e_t)$.

The agent's present value of a flow of future utilities as of period t will be

$$v_t + \beta \sum_{s=t+1} \delta^{s-t} v_s$$

The discount factor between the current period and the next period is $\beta\delta$, but the discount factor between two adjacent periods in the future is δ. The agent is *time-consistent* (exponential discounter) when $\beta = 1$, and *time-inconsistent* (quasi-hyperbolic discounter) when $\beta < 1$. A time-inconsistent agent can be fully aware, partially aware or fully unaware of his time inconsistency. Denote the agent's belief about his true β by $\widehat{\beta}$. As in the literature, we say that a time-inconsistent agent is *sophisticated* when he is fully aware of his inconsistency, that is, when $\widehat{\beta} = \beta < 1$. We say the agent is *partially naive* when $\beta < \widehat{\beta} < 1$ and *fully naive* agent when $\beta < \widehat{\beta} = 1$. When we say the agent is *naive,* we mean $\beta < \widehat{\beta} \leq 1$.

Denote the probability of getting *high output* under *high effort,* $e = 1$ by $\Pr(q = q_h | e = 1) = p_1$ and the probability of getting *high output* under *low effort,* $e = 0$ by $\Pr(q = q_h | e = 0) = p_0$, where I assume $p_1 > p_0$.

There is no lending or borrowing. So the agent spends whatever wage he earns in a period within that period. I also assume away limited liability.

2.3 Principal

The principal is risk-neutral, time-consistent and has a discount factor, δ_P. She knows what type of agent she is facing. That is, she knows whether the agent is time-consistent, sophisticated, fully naive or partially naive. However, she cannot observe the effort levels exerted by the agent as in the standard moral hazard problem. We assume that high effort level is highly valuable for the principal, so she wants to implement high effort in each period. The principal's problem is to find

an individually rational and incentive compatible contract which specifies wages in both periods for all possible output realizations. That is, she maximizes her expected profit subject to individual rationality and incentive compatibility, over all possible contracts, $\{w_i, w_{ij}\}_{i,j \in \{h,l\}}$, where w_i is the wage paid in the first period if the output is q_i in the first period, and w_{ij} is the wage paid in the second period if the first period output is q_i and the second period output is q_j. Her payoff from a contract, $\{(w_h, w_l), (w_{hh}, w_{hl}, w_{lh}, w_{ll})\}$, given that it is accepted and the agent exerts high effort in both periods is given by

$$p_1[q_h - w_h + \delta_P[p_1(q_h - w_{hh}) + (1 - p_1)(q_l - w_{hl})]]$$
$$+ (1 - p_1)[q_l - w_l + \delta_P[p_1(q_h - w_{lh}) + (1 - p_1)(q_l - w_{ll})]]$$

When the agent is sophisticated this problem is less complicated relative to the one where the agent is partially or fully naive. When the agent is naive, the principal has an opportunity to extract some information rents through deceiving the agent about what effort level his future self would pick. This option is not present when the agent is sophisticated.

3 Sophisticated Agent

The principal wants to implement high effort, $e = 1$, in both periods. The principal's problem when she is facing a sophisticated agent is

$$\max_{\{w_i, w_{ij}\}_{i,j \in \{h,l\}}} p_1[q_h - w_h + \delta_P[p_1(q_h - w_{hh}) + (1 - p_1)(q_l - w_{hl})]]$$
$$+ (1 - p_1)[q_l - w_l + \delta_P[p_1(q_h - w_{lh}) + (1 - p_1)(q_l - w_{ll})]]$$

subject to individual rationality, IR, and the incentive compatibility conditions, IC_1 and IC_2, for the two periods.

Start with the second period incentive compatibility constraint, IC_2. Given the output realization in the first period, IC_2 ensures that the agent exerts high effort in the second period. Denoting the utilities from wages with $u(w_{q_i}) = u_i$, and $u(w_{q_i q_j}) = u_{ij}$ where $i, j \in \{h, l\}$, IC_2 is given by

$$p_1 u_{ih} + (1 - p_1)u_{il} - \psi \geq p_0 u_{ih} + (1 - p_0)u_{il} \text{ for } i \in \{h, l\}$$

or

$$u_{ih} - u_{il} \geq \frac{\psi}{p_1 - p_0} \text{ for } i \in \{h, l\}$$

The first period incentive constraint, IC_1, that the agent will exert high effort in the second period, is given by

$$p_1[u_h + \beta\delta(p_1 u_{hh} + (1 - p_1)u_{hl} - \psi)] + (1 - p_1)[u_l + \beta\delta(p_1 u_{lh} + (1 - p_1)u_{ll} - \psi)] - \psi$$

$$\geq p_0[u_h + \beta\delta(p_1 u_{hh} + (1 - p_1)u_{hl} - \psi)] + (1 - p_0)[u_l + \beta\delta(p_1 u_{lh} + (1 - p_1)u_{ll} - \psi)]$$

which can be written as

$$u_h + \beta\delta[p_1 u_{hh} + (1 - p_1)u_{hl}] - u_l - \beta\delta[p_1 u_{lh} + (1 - p_1)u_{ll}] \geq \frac{\psi}{p_1 - p_0}$$

Finally, the individual rationality constraint, IR, is given by

$$p_1[\beta\delta u_h + \beta\delta^2(p_1 u_{hh} + (1 - p_1)u_{hl} - \psi)] + (1 - p_1)[\beta\delta u_l + \beta\delta^2(p_1 u_{lh} + (1 - p_1)u_{ll} - \psi)] - \psi \geq 0$$

or

$$p_1[u_h + \delta(p_1 u_{hh} + (1 - p_1)u_{hl})] + (1 - p_1)[u_l + \delta(p_1 u_{lh} + (1 - p_1)u_{ll})] \geq (1 + \delta)\psi$$

Observe that β enters IC_1 but it does not enter IR or IC_2. There is no payment made at $t = 0$. So from the point of view of $t = 0$, periods 1 and 2 are effectively discounted to the contracting stage at 1 and δ, respectively. However, agent, being sophisticated, knows that at $t = 1$ he will discount period 2 at $\beta\delta$. Hence in IC_1, periods 1 and 2 are discounted at 1 and $\beta\delta$, respectively.

Denoting the inverse of the utility function by $h(u)$, the principal's problem becomes

$$\max_{\{u_i, u_{ij}\}_{i,j \in \{h,l\}}} p_1[q_h - h(u_h) + \delta_P\{p_1(q_h - h(u_{hh})) + (1 - p_1)(q_l - h(u_{hl}))\}]$$
$$+ (1 - p_1)[q_l - h(u_l) + \delta_P\{p_1(q_h - h(u_{lh})) + (1 - p_1)(q_l - h(u_{ll}))\}]$$

subject to

$$(IR) \quad p_1[u_h + \delta(p_1 u_{hh} + (1-p_1)u_{hl})] + (1-p_1)[u_l + \delta(p_1 u_{lh} + (1-p_1)u_{ll})] \geq (1+\delta)\psi$$

$$(IC_1) \quad u_h + \beta\delta[p_1 u_{hh} + (1-p_1)u_{hl}] - u_l - \beta\delta[p_1 u_{lh} + (1-p_1)u_{ll}] \geq \frac{\psi}{p_1 - p_0}$$

$$(IC_2) \quad u_{ih} - u_{il} \geq \frac{\psi}{p_1 - p_0} \text{ for } i \in \{h, l\}$$

For a given first period output level $q_i \in \{q_h, q_l\}$, the agent's continuation payoff will be $p_1 u_{ih} + (1-p_1)u_{il} - \psi$. If the agent has been promised Eu_i for the second period when the first period output realization is q_i, then u_{ih} and u_{il} are defined to be

$$p_1 u_{ih} + (1-p_1)u_{il} - \psi = Eu_i \text{ for } i \in \{h, l\}$$

Once the principal promises the agent a utility of Eu_i, the continuation of the optimal contract for the second period will be given by the solution to the following problem

$$\max_{u_{ih}, u_{il}} p_1(q_h - h(u_{ih})) + (1-p_1)(q_l - h(u_{il}))$$

subject to

$$u_{ih} - u_{il} \geq \frac{\psi}{p_1 - p_0}$$

$$p_1 u_{ih} + (1-p_1)u_{il} - \psi \geq Eu_i$$

This is a static problem and it is straightforward to show that both constraints bind. Hence, for a given first period output, q_i, the second period payoffs to the agent are

$$u_{ih} = \psi + Eu_i + (1-p_1)\frac{\psi}{p_1 - p_0}$$

$$u_{il} = \psi + Eu_i - p_1\frac{\psi}{p_1 - p_0}$$

Denote the cost of implementing the high effort level in the second period, given that the promised second period utility is Eu_i, by $C_2(Eu_i)$. Denote the continuation value of the contract

for the principal by $V_2(Eu_i)$. Then

$$C_2(Eu_i) = p_1 h(u_{ih}) + (1 - p_1)h(u_{il})$$

$$V_2(Eu_i) = p_1 q_h + (1 - p_1)q_l - C_2(Eu_i)$$

$$V_2'(Eu_i) = -C_2'(Eu_i)$$

Now the principal's problem can be reduced to

$$\max_{\{u_i, Eu_i\}_{i \in \{h,l\}}} p_1[q_h - h(u_h)] + (1 - p_1)[q_l - h(u_{hl})] + \delta_P[p_1 V_2(Eu_h) + (1 - p_1)V_2(Eu_l)]$$

subject to

$$(IR) \quad p_1[u_h + \delta Eu_h] + (1 - p_1)[u_l + \delta Eu_l] \geq \psi$$

$$(IC) \quad u_h - u_l + \beta\delta(Eu_h - Eu_l) \geq \frac{\psi}{p_1 - p_0}$$

Note that incentives in the first period depend on the second period only through Eu_i, not through u_{ih} or u_{il}. Attaching λ to IR and μ to IC_1 we have the following first order conditions with respect to u_h, u_l, Eu_h and Eu_l, respectively.

$$h'(u_h) = \frac{\mu}{p_1} + \lambda \tag{1}$$

$$h'(u_l) = -\frac{\mu}{1 - p_1} + \lambda \tag{2}$$

$$\frac{\delta_P}{\delta}C_2'(Eu_h) = \frac{\beta\mu}{p_1} + \lambda \tag{3}$$

$$\frac{\delta_P}{\delta}C_2'(Eu_l) = -\frac{\beta\mu}{1 - p_1} + \lambda \tag{4}$$

(1) and (2) imply that

$$\lambda = p_1 h'(u_h) + (1 - p_1)h'(u_l) \tag{5}$$

$$\mu = p_1(1 - p_1)(h'(u_h) - h'(u_l)) \tag{6}$$

(1) and (3) imply that

$$h'(u_h) = \frac{\delta_P}{\delta}C_2'(Eu_h) + \frac{\mu}{p_h}(1 - \beta) \tag{7}$$

(2) and (4) imply that

$$h'(u_l) = \frac{\delta_P}{\delta} C_2'(Eu_l) - \frac{\mu}{1 - p_1}(1 - \beta)$$ (8)

Lemma 1 $\lambda > 0$ *and* $\mu > 0$.

Proof. $\lambda > 0$ *follows immediately from (5). By construction* $\mu \geq 0$. *Note that* $\mu = 0$ *implies* $u_h = u_l$ *and* $Eu_h = Eu_l$. *But then IC of the reduced problem is violated.* ∎

Henceforth, we will denote a time-consistent agent who has discount factor δ with TC_δ and a sophisticated time-inconsistent agent who has discounting function represented by $(1, \beta\delta, \beta\delta^2)$ with SO. We will also use TC_δ and SO as superscripts whenever appropriate.

If the agent is time-consistent, then the conditions (7) and (8) correspond to the relationship between contingent wages across periods presented in Rogerson (1985). If $\beta = 1$, then conditions (7) and (8) imply $h'(u_i^{TC_\delta}) = \frac{\delta_P}{\delta} C_2'(Eu_i^{TC_\delta})$ for $i \in \{h, l\}$. The definition of C_2 implies that

$$C_2'(Eu_i^{TC_\delta}) = p_1 h'(u_{ih}^{TC_\delta}) + (1 - p_1)h'(u_{il}^{TC_\delta}) = E_{q_j}(h'(u_{ij}^{TC_\delta}))$$

Therefore, for a time-consistent agent with discount factor δ, we have

$$h'(u_i^{TC_\delta}) = \frac{\delta_P}{\delta} E_j(h'(u_{ij}^{TC_\delta})) \text{ for } i \in \{h, l\}$$ (9)

This relationship is often referred to as the *martingale property*. This property says that the principal intertemporally smooths the agent's rewards over time in a way that the cost of promising one more unit of utility today is exactly equal to the gain, discounted appropriately, from having one less unit of utility to promise tomorrow, following any realization of the output. That is, the marginal cost of rewarding the agent in the first period for an output realization of q_i must be equal to marginal benefit of delaying these awards, discounted appropriately, in the continuation of the contract given that the first period output is q_i. The principal intertemporally spreads the rewards to the agent to minimize the cost of implementing high effort in the first period.

Now, consider a time-consistent agent with a discount factor $\beta\delta$, with $\beta < 1$. Then (9) implies

$$h'(u_h^{TC_{\beta\delta}}) = \frac{\delta_P}{\beta\delta} E_{q_j}(h'(u_{hq_j}^{TC_{\beta\delta}}))$$

$$h'(u_l^{TC_{\beta\delta}}) = \frac{\delta_P}{\beta\delta} E_{q_j}(h'(u_{lq_j}^{TC_{\beta\delta}}))$$

Put differently,

$$h'(u_h^{TC_{\beta\delta}}) > \frac{\delta_P}{\delta} E_{q_j}(h'(u_{hq_j}^{TC_{\beta\delta}}))$$

$$h'(u_l^{TC_{\beta\delta}}) > \frac{\delta_P}{\delta} E_{q_j}(h'(u_{lq_j}^{TC_{\beta\delta}}))$$

So for a time-consistent agent, the principal needs to shift the payments to the first period for both output realizations, as the agent gets more impatient.[5] However shifting payments to the first period for SO is not as easy as it is for $TC_{\beta\delta}$. Both $TC_{\beta\delta}$ and SO have the same IC_1. But for $TC_{\beta\delta}$, IR uses $(1, \beta\delta)$ discounting, whereas IR for SO uses $(1, \delta)$ discounting. That is, for SO the second period is more important than it is for $TC_{\beta\delta}$ which makes it harder for the principal to shift payments to the first period for *both* realizations. This intuition suggests that if the principal shifts the payments to the first period for one output realization, then for the other output realization, she must shift the payments to the second period. Now, we can show that the martingale property does not hold for a sophisticated agent.

Proposition 1 *The martingale property fails when the agent is sophisticated. More precisely,*

$$h'(u_h^{SO}) > \frac{\delta_P}{\delta} E_j(h'(u_{hj}^{SO}))$$

$$h'(u_l^{SO}) < \frac{\delta_P}{\delta} E_j(h'(u_{lj}^{SO}))$$

Proof. Since $\mu > 0$, (7) and (8) imply $h'(u_h) > \frac{\delta_P}{\delta} C_2'(Eu_h)$ and $h'(u_l) < \frac{\delta_P}{\delta} C_2'(Eu_l)$. By definition $C_2'(Eu_i) = E_j(h'(u_{ij}))$. Hence $\delta h'(u_h^{SO}) > \delta_P E_j(h'(u_{hj}^{SO}))$ and $C_2'(Eu_l) = E_{q_j}(h'(u_{lq_j}))$. ∎

Suppose $\delta_P = \delta$. Then this proposition says that the marginal cost of rewarding the sophisticated agent for a high level of output in the first period is higher than the expected marginal cost of

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[5]This is because $h'' > 0$.

promising these awards in the second period given that the first period output level is high and symmetrically when the first period output level is low.

Using the above result, I compare the first period utility for a time-consistent agent with discount factor δ to that for a sophisticated time-inconsistent agent with $(1, \beta\delta, \beta\delta^2)$ in the optimal contract that implements high effort in both periods.

Proposition 2 *Assume $\delta_P = \delta$. Then $u_h^{SO} > u_h^{TC_\delta}$ and $u_l^{SO} < u_l^{TC_\delta}$.*

Proof. See the Appendix. ∎

For a time-inconsistent sophisticated agent with discounting $(1, \beta\delta, \beta\delta^2)$, the principal should increase the wage for good performance and decrease it for bad performance, relative to the wage scheme for a time-consistent agent with a discount factor δ. That is, it's optimal for the principal to increase the risk the sophisticated time-inconsistent agent faces in the first period relative to the time-consistent agent. The intuition for this result is the following. First set $\delta = 1$ for simplicity. Denote the wage differential and expected wage differential with $\triangle u = u_h - u_l$ and $\triangle EU = EU_h - EU_l$, respectively. Now suppose that $\{u_i, EU_j\}_{i,j \in \{h,l\}}$ is the optimal contract for a time-consistent agent with discount factor $\delta = 1$. Then consider the following two options for the sophisticated agent. In the first option increase $\triangle u$ by $\triangle_1$ and keep $\triangle EU$ the same. In the second option, increase $\triangle EU$ by $\triangle_2$ and keep $\triangle u$ the same. If both options provide the agent with the same incentives, it should be the case that $\triangle_1 = \beta \triangle_2$. This is because the sophisticated agent discounts the second period to the first period by a factor $\beta\delta$ which is β, since $\delta = 1$. Hence, $\triangle_1 < \triangle_2$. Sophisticated agent's individual rationality constraint will hold with slack under the first option if it binds under the second option. This is because $\beta\triangle_1 < \beta \triangle_2$. Then, the principal can increase $\triangle u$ by $\triangle_2$, and hence provide more incentives and still not violate the individual rationality.

Now I compare the expected profits that the principal achieves in the optimal contracts with a time-consistent agent and with a sophisticated time-inconsistent agent. However, there are two sensible comparisons. I will compare the expected profit from SO to the expected profit from TC_δ, and to that from $TC_{\beta\delta}$. The proposition below compares the expected profit from SO to the expected profit from TC_δ.

Proposition 3 *The expected profit from TC_δ is higher than that from SO.*

Proof. Recall the optimization problem when the agent is time-inconsistent and sophisticated.

$$\max_{u_i, Eu_i} p_1[q_h - h(u_h)] + (1 - p_1)[q_l - h(u_{hl})] + \delta_P[p_1 V_2(Eu_h) + (1 - p_1)V_2(Eu_l)]$$

subject to

$$(IR) \quad p_1[u_h + \delta Eu_h] + (1 - p_1)[u_l + \delta Eu_l] \geq \psi$$

$$(IC) \qquad u_h - u_l + \beta\delta(Eu_h - Eu_l) \geq \frac{\psi}{p_1 - p_0}$$

The solution $\{u_h^*, u_l^*, EU_h^*, EU_l^*\}$ and the Lagrange multipliers are continuously differentiable functions of β.[6] Also the non-degenerate constraint qualification holds.[7] Hence by the envelope theorem, we get $\frac{dE\pi(\beta)}{d\beta} = \mu\delta(Eu_h - Eu_l)$ where μ is the Lagrange multiplier attached to the IC. Both μ and δ are positive. We have already shown that $Eu_h > Eu_l$, so $\frac{dE\pi(\beta)}{d\beta} > 0$. Hence, $E\pi(\beta = 1) > E\pi(\beta < 1)$. ∎

This result is driven by the fact that the sophisticated time-inconsistent agent discounts the second period more than the consistent agent does when incentives are considered. Hence the effect of the second period on the first period incentives is lower for SO compared to TC_δ. But both agents discount the second period the same when the contract is evaluated in the contracting stage. Therefore it's harder to implement high effort with SO, which gives rise to lower profits.

A comparison between the expected profit from SO to that from $TC_{\beta\delta}$ is given in the following proposition.

Proposition 4 *The expected profit from $TC_{\beta\delta}$ is lower(higher) than that from SO, whenever the expected second period promised utility, $p_1 Eu_h + (1 - p_1)Eu_l$, is positive for $TC_{\beta\delta}$(negative for SO).*

Proof. The individual rationality and incentive constraints are as follows

$$(IR) \quad p_1 u_h + (1 - p_1)u_l + \alpha\delta[p_1 Eu_h + (1 - p_1)Eu_l] \geq \psi$$

$$(IC) \qquad u_h - u_l + \beta\delta(Eu_h - Eu_l) \geq \frac{\psi}{p_1 - p_0}$$

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[6] If the agent is naive, then there is a jump at β.

[7] The rank of the augmented matrix of the constraint system is 2.

where $\alpha = 1$ represents SO and $\alpha = \beta$ represents $TC_{\beta\delta}$. We get $\frac{dE\pi(\alpha)}{d\alpha} = \lambda(p_1 Eu_h + (1 - p_1)Eu_l)$ where λ is the Lagrange multiplier attached to the individual rationality constraint. Note that α does not enter the objective function. If $p_1 Eu_h + (1 - p_1)Eu_l > 0$ at $\alpha = \beta$, then $E\pi(\alpha = \beta) < E\pi(\alpha = 1)$. That is, the principal prefers SO to $TC_{\beta\delta}$. Likewise, if $p_1 Eu_h + (1 - p_1)Eu_l < 0$ at $\alpha = 1$, then $E\pi(\alpha = \beta) > E\pi(\alpha = 1)$, so the principal prefers $TC_{\beta\delta}$ to SO. ∎

Whenever the optimal contract promises the time-consistent agent with a discount factor $\beta\delta$ a positive second period expected utility, then it is less costly to implement high effort with a sophisticated agent who has a discounting function given by $(1, \beta\delta, \beta\delta^2)$. This is because the contract for the consistent agent will be accepted by the inconsistent agent too. That is, the individual rationality constraint will hold for an inconsistent agent with slack. Then the principal can alter the wage scheme by appropriately reducing payments for high and low outputs without violating the incentive constraint. Note that both agents have the same incentive constraint. Hence, the principal can do better with SO than with $TC_{\beta\delta}$, when optimal contract for $TC_{\beta\delta}$ promises him a positive second period expected utility. A positive second period expected utility is an indication of the fact that it is relatively harder to implement high effort in both periods. When it is negative for the sophisticated agent, then the principal can do better with $TC_{\beta\delta}$, because $TC_{\beta\delta}$ will value the second period less than SO from the contracting stage point of view. Hence the contract for SO will make the $TC_{\beta\delta}$ to exert high effort, but with slack in the individually rationality constraint. Therefore, the principal facing $TC_{\beta\delta}$ can implement high effort with lower wages when the second period expected utility is negative for SO.

Example 1 *Suppose that the inverse utility function is $h(u) = u^2/2$ and for simplicity $\delta_P = \delta$. For parameters, $\delta = 0.8$, $\beta = 0.5$, $p_h = 0.8$, $p_l = 0.2$ and $\psi = 1$ we get the following approximate expected costs to implement high effort in both periods:*

$$TC_\delta \quad SO \quad TC_{\beta\delta}$$

$$\text{Expected cost} \quad 1.20 \quad 1.26 \quad 1.32$$

Hence the principal prefers a TC_δ over SO, and prefers SO over $TC_{\beta\delta}$. For $TC_{\beta\delta}$, expected second period promised utility, that is, $p_1 Eu_h + (1 - p_1)Eu_l$, is approximately 0.14 which is positive.

4 Naive Agent

When the agent is naive, his anticipation of his future self's behavior is not correct.[8] At $t = 0$ he believes that he has a discounting function given by $(1, \widehat{\beta}\delta, \widehat{\beta}\delta^2)$ where $\beta < \widehat{\beta} \leq 1$, so he underestimates his inconsistency. In the contracting stage, his decision to accept or reject the contract depends on the set of wages offered and on the actions he thinks his future selves will take. Hence IR should be based on his (incorrect) anticipation of his future actions. However IC should consider only the agent's actual behavior at the effort stage. This is because the principal knows agent's exact discounting function and, from $t = 1$ on, the agent behaves according to his true β. More precisely, the principal will pick a set of wages which makes the agent believe that his future selves will pick a certain effort scheme, the *artificial* one, denoted a. The same wage scheme will actually implement a potentially different effort scheme, the *effective* one, denoted e. The principal's expected profit will be based on the set of wages and on the effective effort schedule, not on the artificial one. The agent will use the discounting $(1, \widehat{\beta}\delta, \widehat{\beta}\delta^2)$ at the contracting stage expecting that his discounting from $t = 1$ on will be $(1, \widehat{\beta}\delta)$; however, he will end up behaving according to $(1, \beta\delta)$ from $t = 1$ on.

There are two sets of artificial effort levels. One is the effort schedule that the agent believes at $t = 0$, his future selves (the self at $t = 1$ and the self at $t = 2$) will pick at $t = 1$ and $t = 2$, denoted $\{a_1, (a_2^i)_{i\in\{h,l\}}\}$. The other is the effort schedule that the agent thinks at $t = 1$, his future self will pick at $t = 2$, denoted $\{(\widehat{a}_2^i)_{i\in\{h.l\}}\}$. Since there is no further time-inconsistency between $t = 1$ and $t = 2$, we have $\widehat{a}_2^i = e_2^i$ for $i \in \{h, l\}$. By the same argument, the effective second period effort will be the same as the artificial second period effort, from $t = 0$ point of view. That is, $a_2^i = e_2^i$ for $i \in \{h, l\}$. Therefore, the only artificial effort level that matters is a_1.

The principal has an opportunity to manipulate the agent's naivete by making him believe that his future self at $t = 1$ exerts no effort, but when he actually arrives at $t = 1$ he exerts high effort as the principal desires. That is, the principal may set a contract such that $a_1 = 0$ but $e_1 = 0$. In this case, the principal's problem is

$$\min_{\{u_i, Eu_i\}_{i\in\{h,l\}}} p_1[h(u_h) + \delta_P C_2(Eu_h)] + (1 - p_1)[h(u_l) + \delta_P C_2(Eu_l)]$$

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[8]Here, we consider a naive agent who, looking at the contract, does not infer anything about his future self's type. Instead he sticks to his prior belief about the future self.

subject to

$$(IR^0) \quad p_0[u_h + \delta E u_h] + (1 - p_0)[u_l + \delta E u_l] \geq 0$$

$$(IC_a^0) \quad u_h - u_l + \delta\widehat{\beta}(E u_h - E u_l) \leq \frac{\psi}{p_1 - p_0}$$

$$(IC_e^0) \quad u_h - u_l + \delta\beta(E u_h - E u_l) \geq \frac{\psi}{p_1 - p_0}$$

IR^0 ensures that the agent accepts the contract given that he believes that his self at $t = 1$ will pick low effort. IC_a^0 convinces the agent, from $t = 0$ perspective, that he will choose low effort at $t = 1$. Finally, IC_e^0 ensures that when the agent actually arrives at $t = 1$, he changes his mind and picks high effort.

Attaching λ to the IR^0, μ_a to the IC_a^0 and μ_e to the IC_e^0, we get the following

$$h'(u_h) = \lambda\frac{p_0}{p_1} + \frac{\mu_e - \mu_a}{p_1}$$

$$h'(u_l) = \lambda\frac{1 - p_0}{1 - p_1} - \frac{\mu_e - \mu_a}{1 - p_1}$$

$$\delta_P C_2'(E u_h) = \lambda\delta\frac{p_0}{p_1} + \delta\frac{\beta\mu_e - \widehat{\beta}\mu_a}{p_1}$$

$$\delta_P C_2'(E u_l) = \lambda\delta\frac{1 - p_0}{1 - p_1} - \delta\frac{\beta\mu_e - \widehat{\beta}\mu_a}{1 - p_1}$$

Alternatively, the principal can simply choose to implement high effort in both periods, without trying to make the agent believe that his future self will pick low effort at $t = 1$. That is, $a_1 = 1$. In this case, the principal's problem is

$$\min_{\{u_i, E u_i\}_{i \in \{h,l\}}} p_1[h(u_h) + \delta_P C_2(E u_h)] + (1 - p_1)[h(u_l) + \delta_P C_2(E u_l)]$$

subject to

$$(IR^1) \quad p_1[u_h + \delta E u_h] + (1 - p_1)[u_l + \delta E u_l] \geq 0$$

$$(IC_a^1) \quad u_h - u_l + \delta\widehat{\beta}(E u_h - E u_l) \geq \frac{\psi}{p_1 - p_0}$$

$$(IC_e^1) \quad u_h - u_l + \delta\beta(E u_h - E u_l) \geq \frac{\psi}{p_1 - p_0}$$

Having constructed the principal's problem for both cases, namely for $a_1 = 0$ and $a_1 = 1$, I now show that in the case where high effort is highly valuable to the principal in both periods, the degree of the agent's misperception does not matter.

Lemma 2 *Suppose that the principal faces a naive agent with $0 < \beta < \widehat{\beta} \leq 1$, and that she wants to implement high effort in both periods. Then, the optimal contract does not depend on the degree of agent's misperception, $\widehat{\beta} - \beta$.*

Proof. See the Appendix. ■

When the principal is trying to convince the agent that his future self will pick low effort, she has to write a contract which has a higher value when low effort is exerted, from the perspective of the naive agent who expects to discount the last period less than he will. So the contract must have a relatively lower second period promised utility when the first period output is high compared to the second period promised utility when the first period output is low. This is captured by the fact that $Eu_h \leq Eu_l$. Since the agent at $t = 0$ highly values the second period, such a scheme makes him think that he should pick low effort in the first period. But, the same contract must also make sure that the self at $t = 1$ actually picks high effort. So the principal must use the first period utilities to ensure that. Therefore, the first period utility when the output is high is higher than the first period utility when the output is low. That is, $u_h > u_l$.

When writing a contract with $a_1 = 0$, the principal will ignore $\widehat{\beta}$, because a wage scheme that makes an agent with $\widehat{\beta}$ believe that his future self will pick low effort is also going to make any other agent with $\widetilde{\beta} > \beta$ believe that his future self will pick low effort. Since the effective effort level will be picked through the incentives from the $t = 1$ point of view, in which $\widehat{\beta}$ does not show up, the optimal wage scheme is the solution to the problem given for $a_1 = 0$ with the two binding constraints, IC_e^0 and IR^0, with $Eu_h \leq Eu_l$ and $u_h > u_l$. This solution satisfies IC_a^0, with slack, for any $\widehat{\beta}$ with $\widehat{\beta} > \beta$. That is, in the case where the principal tries to deceive the agent, she writes a contract based on the true incentives and the individual rationality for $a_1 = 0$. But, this contract makes any naive agent believe that his future self will pick low effort.

For the contract with $a_1 = 1$, the analysis is relatively straightforward. Any wage scheme that implements high effort and ensures that the agent believes his future self will pick high effort will definitely make any other naive agent with different degree of misperception believe that his future self will pick high effort. This is because for $a_1 = 1$, we have $Eu_h \geq Eu_l$. The principal, offering the same contract, will be able to make all naive agents with different degrees of naivete believe that they will choose a_1 in the first period.

When $Eu_h \neq Eu_l$, we can see that IC_e^0 binds but IC_a^0 does not, in Figure 1 below. The graph is drawn on a u_h, u_l space given the optimal Eu_h and Eu_l. Note that $Eu_h < Eu_l$. Both IC_e^0 and IC_a^0 have slope equal to 1. IR has slope equal to $-\frac{p_0}{1-p_0}$. However the isocost curve has slope $-\frac{p_1}{1-p_1}\frac{h'(u_h)}{h'(u_l)}$ where $u_h > u_l$ at the optimum. Since $\frac{p_1}{1-p_1}\frac{h'(u_h)}{h'(u_l)} > \frac{p_0}{1-p_0}$, the isocost curve should be steeper than IR at the optimum. That is possible only when IC_e^0 binds and IC_a^0 does not.

Figure 1

Having established the result that the optimal contract that implements high effort in both periods does not depend on the degree of the agent's misperception, I turn to the question of whether the principal is better off by choosing $a_1 = 0$ rather than $a_1 = 1$. When $a_1 = 1$, the contract will be the same as the optimal contract in the sophisticated agent case. If principal, facing a naive agent, can do better with $a_1 = 0$ rather than $a_1 = 1$, then she will be better off with a naive agent rather than a sophisticated agent. Otherwise, naivete and sophistication will make no difference at all. Therefore, the question whether the principal can do better convincing the agent that his future self will pick low effort, but eventually implementing high effort, is equivalent to the question of whether the principal is better off with a naive agent or a sophisticated agent.

Proposition 5 *If the principal implements high effort in both periods, she is indifferent between facing a sophisticated time-inconsistent agent and facing a naive time-inconsistent agent.*

Proof. Showing that $a_1 = 1$ is optimal will prove this result because in this case, the contract for sophisticated agent and the contract for naive agent with $a_1 = 1$ are going to be exactly the same. So I only need to show that

The cost of implementing high effort in both periods with $a_1 = 1$ is smaller than the cost with $a_1 = 0$. To see this, first note that IC_a does not bind for either problem with $a_1 = 1$ and $a_0 = 0$ as established in Lemma 2, and that both IR and IC_e do bind regardless of a_1. Now look at the following problem

$$\min_{\{u_i, Eu_i\}_{i \in \{h,l\}}} p_1[h(u_h) + \delta_P C_2(Eu_h)] + (1 - p_1)[h(u_l) + \delta_P C_2(Eu_l)]$$

19

subject to

$$(IR) \quad (p_1 - \eta)[u_h + \delta E u_h] + (1 - p_1 + \eta)[u_l + \delta E u_l] = 0$$

$$(IC_e) \quad u_h - u_l + \delta\beta(E u_h - E u_l) - \frac{\psi}{p_1 - p_0} = 0$$

If $\eta = 0$, then the problem above is exactly the one for $a_1 = 1$. If $\eta = p_1 - p_0$, then it is the problem that corresponds to the case with $a_1 = 0$. Then we have, $\frac{dEC(\eta)}{d\eta} = \lambda(u_h - u_l + \delta(E u_h - E u_l))$, where λ is the Lagrange multiplier attached to the IR, and $EC(\eta)$ is the expected cost in the relevant problem. We have already shown that $u_h - u_l + \delta(E u_h - E u_l) > 0$ for the sophisticated agent. So, this derivative evaluated at $\eta = 0$ gives us a positive value. Therefore the cost will be higher for the case with $\eta = p_1 - p_0$; that is, for the case with $a_1 = 0$. Thus, $a_1 = 1$ is optimal. The contract for the sophisticated agent and the contract for naive agent with $a_1 = 1$ are exactly the same. Thus, the principal is indifferent between facing a sophisticated time-inconsistent agent and facing a naive time-inconsistent agent. ∎

This is particularly striking because with a naive time-inconsistent agent the principal has the power to manipulate the agent through his misperception. However, such an opportunity to manipulate does not provide the principal with higher profits. In fact, the principal chooses not to deceive the agent at all.

5 Conclusion

In this paper, I analyzed a repeated moral hazard problem with a time-inconsistent agent. To capture time-inconsistency, I assumed $\beta\delta$-*preferences* which is simple enough and widely used in the literature. I posed the following question: Would a risk neutral principal prefer to face a time-consistent agent or a time-inconsistent agent? To answer this question, first I looked at the case where the time-inconsistent agent is fully aware of his inconsistency. I showed that the optimal contract for the time-inconsistent agent, relative to the optimal contract for the time-consistent agent, has a higher reward for high performance and a lower reward when the output is low, in the first period. The principal is better off facing a time-consistent agent with a discount factor δ, than facing a sophisticated time-inconsistent agent with discounting $(1, \beta\delta, \beta\delta^2)$. She is worse off facing a time-consistent agent with a discount factor $\beta\delta$, than facing a sophisticated time-inconsistent agent with discounting $(1, \beta\delta, \beta\delta^2)$ if the promised second period expected utility is positive for the

consistent agent.

I also looked at the case where the inconsistent agent is partially or fully naive. That is, he misperceives his inconsistency. The principal can consider two possible contracts: One contract tries to convince the agent that he is going to choose low effort in the future but when the time arrives actually induces high effort, while the other contract does not try to deceive the agent. I show that the latter contract is optimal, so the principal prefers not to try to deceive the agent. This implies that the principal is indifferent between facing a naive agent and facing a sophisticated agent.

I have assumed that the naive agent does not have the ability to infer something about his inconsistency from the contract. A more concrete way of modelling the problem at hand would be by considering a naive agent who can update his beliefs about his future behavior based on the contract. This obviously is a more involved problem and is an open question for now.

6   Appendix

Proof of Proposition 2. Recall IR and IC of the reduced problem for sophisticated agent.

$$(IR) \quad p_1[u_h^{SO} + \delta(Eu_h^{SO})] + (1 - p_1)[u_l^{SO} + \delta(Eu_l^{SO})] \geq \psi$$

$$(IC) \quad u_h^{SO} + \delta\beta[Eu_h^{SO}] - u_l - \delta\beta[Eu_l^{SO}] \geq \frac{\psi}{p_1 - p_0}$$

Multiplying IC with $1 - p_1$, and adding it up with IR we get

$$u_h^{SO} = \psi(1 + \frac{1 - p_1}{p_1 - p_0}) - \delta[(\beta + (1 - \beta)p_1)Eu_h^{SO} + (1 - (\beta + (1 - \beta)p_1))Eu_l^{SO}]$$

Multiplying IC with p_1, and subtracting it from IR we get

$$u_l^{SO} = \psi(1 - \frac{p_1}{p_1 - p_0}) - \delta[(1 - \beta)p_1 Eu_h^{SO} + (1 - (1 - \beta)p_1)Eu_l^{SO}]$$

And corresponding utilities for the time-consistent agent are

$$u_h^{TC_\delta} = \psi(1 + \frac{1 - p_1}{p_1 - p_0}) - \delta E u_h^{TC_\delta}$$

$$u_l^{TC_\delta} = \psi(1 - \frac{p_1}{p_1 - p_0}) - \delta E u_l^{TC_\delta}$$

Hence

$$u_h^{SO} + \delta[(\beta + (1 - \beta)p_1)E u_h^{SO} + (1 - (\beta + (1 - \beta)p_1))E u_l^{SO}] = u_h^{TC_\delta} + \delta E u_h^{TC_\delta}$$

$$u_l^{SO} + \delta[(1 - \beta)p_1 E u_h^{SO} + (1 - (1 - \beta)p_1)E u_l^{SO}] = u_l^{TC_\delta} + \delta E u_l^{TC_\delta}$$

Note that $1 > \beta + (1 - \beta)p_1 > 0$ and $1 > (1 - \beta)p_1 > 0$. Since $E u_h^{SO} > E u_l^{SO}$, [9] we get

$$u_h^{SO} + \delta E u_h^{SO} > u_h^{TC_\delta} + \delta E u_h^{TC_\delta} \qquad (10)$$

$$u_l^{SO} + \delta E u_l^{SO} < u_l^{TC_\delta} + \delta E u_l^{TC_\delta} \qquad (11)$$

Now using proposition 1 above we have

$$h'(u_h^{SO}) > C_2'(E u_h^{SO}) \qquad\qquad h'(u_h^{TC_\delta}) = C_2'(E u_h^{TC_\delta})$$

$$h'(u_l^{SO}) < C_2'(E u_l^{SO}) \qquad\qquad h'(u_l^{TC_\delta}) = C_2'(E u_l^{TC_\delta})$$

Define $g = (h')^{-1}$. Note that g and $C_2'(.)$ are increasing functions.[10] Therefore

$$u_h^{SO} > (g \circ C_2')(E u_h^{SO}) \qquad\qquad u_h^{TC_\delta} = (g \circ C_2')(E u_h^{TC_\delta}) \qquad (12)$$

$$u_l^{SO} < (g \circ C_2')(E u_l^{SO}) \qquad\qquad u_l^{TC_\delta} = (g \circ C_2')(E u_l^{TC_\delta}) \qquad (13)$$

---

[9] One can easily see this from equations (3) and (4) by using the fact that $\mu > 0$, $\lambda > 0$ and C_2' is an increasing function.

[10] $g'(.) = \frac{1}{h''(g(.))} > 0$ since $h'' > 0$. To see that $C_2'(.)$ is increasing:

$$C_2(E u_i) = p_h h(u_{q_i h}) + (1 - p_h)h(u_{q_i l})$$

$$= p_h h(\psi + E u_i + \frac{(1 - p_h)\psi}{p_h - p_l}) + (1 - p_h)h(\psi + E u_i - \frac{p_h \psi}{p_h - p_l})$$

Hence $C_2'(.) = p_h h'(.) + (1 - p_h)h'(.)$. Since $h'' > 0$, we also have $C_2'' > 0$.

where $g \circ C_2'$ is an increasing function. Now suppose $u_h^{SO} \leq u_h^{TC\delta}$. Then (10) implies $Eu_h^{SO} > Eu_h^{TC\delta}$, whereas (12) implies $(g \circ C_2')(Eu_h^{SO}) < (g \circ C_2')(Eu_h^{TC\delta})$ which in turn implies $Eu_h^{SO} < Eu_h^{TC\delta}$. Hence $u_h^{SO} > u_h^{TC\delta}$. And $u_l^{SO} < u_l^{TC\delta}$ follows similarly using (11) and (13).[11]

Proof of Lemma 2. Case 1: Suppose $a_1 = 0$. IC_a^0 and IC_e^0 together imply $\delta \widehat{\beta}(Eu_h - Eu_l) \leq \delta \beta(Eu_h - Eu_l)$. Hence $Eu_h \leq Eu_l$. If $Eu_h = Eu_l = Eu$, then the optimal $\{u_h, u_l, Eu\}$ will solve the following problem

$$\min_{u_h, u_l, Eu} p_1 h(u_h) + (1 - p_1) h(u_l) + \delta_P C_2(Eu)$$

subject to

$$p_0 u_h + (1 - p_0) u_l + \delta Eu \geq 0$$
$$u_h - u_l = \frac{\psi}{p_1 - p_0}$$

The second condition is directly implied by plugging $Eu_h = Eu_l = Eu$ into IC_a^0 and IC_e^0. $\widehat{\beta}$ does not enter the calculation. If $Eu_h < Eu_l$, then both IC^0's cannot be binding. Both cannot be non-binding as well. If they are both non-binding then $\mu_a = \mu_e = 0$. But then $h'(u_h) - h'(u_l) = \lambda(\frac{p_0}{p_1} - \frac{1-p_0}{1-p_1}) = \lambda(\frac{p_0 - p_1}{p_1(1-p_1)}) < 0$ since $p_0 < p_1$. Hence $u_h - u_l < 0$, which violates IC_e^0 since $Eu_h < Eu_l$ as well. Therefore, one of the IC_a^0 and IC_e^0 should be binding and the other should be non-binding. Suppose IC_a^0 binds and IC_e^0 does not. Then $\mu_a > 0$ and $\mu_e = 0$. But then we get $h'(u_h) - h'(u_l) = \lambda(\frac{p_0 - p_1}{p_1(1-p_1)}) - \mu_a(\frac{1}{p_1(1-p_1)}) < 0$ since $p_0 < p_1$ and $\mu_a > 0$. Again $u_h - u_l < 0$, which again violates IC_e^0 since $Eu_h < Eu_l$. Hence, if there is a solution to the problem with $a_1 = 0$, it should be the case that IC_e^0 binds and IC_a^0 does not. Therefore $\widehat{\beta}$ does not enter into the calculation of the solution.

Case2: Suppose $a_1 = 1$. In this case $Eu_h \geq Eu_l$.[12] If $Eu_h = Eu_l$, then similarly $\widehat{\beta}$ drops from the calculation. If $Eu_h > Eu_l$, then trivially IC_e^1 binds and IC_a^1 does not, hence $\widehat{\beta}$ drops again.

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[11] Suppose $u_l^{SO} \geq u_l^{TC\delta}$. Then (11) implies $Eu_l^{SO} < Eu_l^{TC\delta}$, whereas (13) implies $(g \circ C_2')(Eu_l^{SO}) > (g \circ C_2')(Eu_l^{TC\delta})$ which in turn implies $Eu_l^{SO} > Eu_l^{TC\delta}$. Therefore $u_l^{SO} < u_l^{TC\delta}$.

[12] Attaching appropriate multipliers to the constraints, we get $C'(Eu_h) = \lambda + \frac{\widehat{\beta}\mu_a + \beta\mu_e}{p_1}$ and $C'(Eu_l) = \lambda - \frac{\widehat{\beta}\mu_a + \beta\mu_e}{p_1}$. Since λ, μ_a and μ_e are both positive, $C'(Eu_h) \geq C'(Eu_l)$. That is, $Eu_h \geq Eu_l$.

REFERENCES

1. Chade, H., Prokopovych, P., and Smith, L., "Repeated Games with Present-Biased Preferences", *Journal of Economic Theory*, Vol. 139, (2008), pp 157-175.

2. Eliaz, K. and Spiegler, R., "Contracting with Diversely Naive Agents", *The Review of Economic Studies*, Vol.73, (2006), pp. 689-714.

3. Frederick, S., Loewenstein, G. and O'Donoghue, T., "Time Discounting and Time Preference: A Critical View", *Journal of Economic Literature*, Vol. 40, (2002), pp. 351-401

4. Fudenberg, D., Holmstrom, B. and Milgrom, P., "Short-Term Contracts and Long-Term Agency Relationships", *Journal of Economic Theory*, Vol. 51, No.1. (1990).

5. Gilpatric, S., "Present-Biased Preferences, Self-Awareness and Shirking in a Principal-Agent Setting", Mimeo, University of Tennessee, (2003).

6. Harris, M. and Holmstrom B., "A Theory of Wage Dynamics", *The Review of Economic Studies*, Vol. 49, No.3. (1982), pp. 315-333.

7. Holmstrom, B. and Milgrom P., "Aggregation and Linearity In The Provision of Intertemporal Incentives", *Econometrica*, Vol.55, No.2. (1987), pp. 303-328.

8. Laffont, J-J. and Tirole, J., "The Dynamics of Incentive Contracts", *Econometrica*, Vol.56, No.5. (1988), pp. 1153-1175.

9. Laibson, D., "Golden Eggs and Hyperbolic Discounting", *The Quarterly Journal of Economics*, Vol. 112, No.2. (1997), pp. 443–477.

10. Lambert, R. A., "Long-Term Contracts and Moral Hazard", *The Bell Journal of Economics*, Vol. 14, No.2. (1983), pp. 441-452.

11. Loewenstein, G. and Prelec, D., "Anomalies in Intertemporal Choice: Evidence and an Interpretation", *The Quarterly Journal of Economics*, Vol. 107, No.2. (1992), pp. 573-597.

12. Malcomson, J. M. and Spinnewyn, F., "The Multiperiod Principal-Agent Problem", *The Review of Economic Studies*, Vol.55, No.3. (1988), pp. 391-407.

13. O'Donoghue, T. and Rabin, M., "Choice and Procrastination", *The Quarterly Journal of Economics*, Vol. 116, No.1. (2001), pp. 121-160.

14. O'Donoghue, T. and Rabin, M., "Doing It Now or Later", *American Economic Review*, Vol. 89, No.1. (1999a), pp. 103-124.

15. O'Donoghue, T. and Rabin, M., "Incentives for Procrastinators", *The Quarterly Journal of Economics*, Vol. 114, No.3. (1999b), pp. 769-816.

16. Phelps, E. S. and Pollak, R. A., "On Second-Best National Saving and Game-Equilibrium Growth", *The Review of Economic Studies*, Vol.35, No.2. (1968), pp. 185-199.

17. Rogerson, W. P., "Repeated Moral Hazard", *Econometrica,* Vol.53,No.1. (1985), pp.69-76.

18. Sarafidis, Y., "Games with Time-Inconsistent Players", SSRN WP (2006).

19. Spear, S. E. and Srivastava, S., "On Repeated Moral Hazard with Discounting", *The Review of Economic Studies*, Vol. 54, No.4. (1987), pp. 599-617.

20. Strotz, R. H., "Myopia and Inconsistency in Dynamic Utility Maximization", *The Review of Economic Studies*, Vol.23, No.3. (1956), pp. 165-180.