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Real Exchange Rate Dynamics Revisited:
A Case with Financial Market Imperfections[*]

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Abstract

In this paper, we investigate the relationship between real exchange rate dynamics and financial market imperfections. For this purpose, we first construct a New Open Economy Macroeconomics (NOEM) model that incorporates staggered loan contracts as a simple form of the financial market imperfections. Our model with such a financial market friction replicates persistent, volatile, and realistic hump-shaped responses of real exchange rates, which have been thought very difficult to materialize in standard NOEM models. Remarkably, these realistic responses can materialize even with both supply and demand shocks, such as cost-push, loan rate, and monetary policy shocks. This implies that the financial market developments is a key element for understanding real exchange rate dynamics.

JEL codes: F31, F41

1 Introduction

Empirical studies conclude that real exchange rate dynamics are very volatile, persistent, and hump-shaped against shocks as shown in Eichenbaum and Evans (1995), Cheung and Lai (2000), Faust and Rogers (2003), and Steinsson (2007). Thus far in international finance, there has been intense debate as to whether theoretical dynamic general equilibrium models can reproduce such realistic exchange rate dynamics. Chari, Kehoe, and McGrattan (2002), focusing on the first two features, insist that New Open Economy Macroeconomics (NOEM) models may account for the volatility but not for the persistence to a monetary policy shock. In response to this critique, several studies have attempted to solve these three puzzles by introducing such features as strategic complementarity, nonoptimizing monetary policy, and optimal monetary policy into otherwise standard NOEM models as in Bergin and Feenstra (2000) and Benigno (2004a, b). These newly introduced mechanisms mitigate the persistence puzzle of the real exchange rate dynamics to some extent, but have not yet solved it completely. Actual persistence of real exchange rates is still higher than that simulated in those models. Furthermore, the mechanisms do not explain the significant hump-shaped responses of real exchange rates found in data. Steinsson (2007) stresses the importance of generating hump-shaped responses based on his autoregressive estimation of real exchange rates and shows that realistic levels of volatility, persistence, and hump-shaped responses of real exchange rates can be generated with the NOEM models when the cost-push shock is added to the economy, where the home bias is very strong.[1]

Analyses of the role of financial market imperfection for the real exchange rate dynamics, however, are very limited, even though Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2007) emphasize the role of a financial market imperfection to explain the business cycle tendencies found in data. In this paper, therefore we shed light on a sticky loan rate adjustment as a financial market imperfection for the real exchange rate dynamics. The loan rate stickiness is reported in Slovin and Sushka (1983) and Berger and Udell (1992) for the United States (US), Sorensen and Werner (2006) and

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[1] The importance of reproducing the hump-shaped responses are emphasized in, for example, Chrsitiano and Vigfusson (2003) and Vigfusson (2007).

Gambacorta (2008) for the euro area, and Bank of Japan (2007, 2009) for Japan.

We construct a NOEM model with an explicit role of banks in which we incorporate sticky loan interest rate contracts as in Teranishi (2007), which assumes it in the closed economy.[2] In our model, following Gadanecz (2004), McGuire and Tarashev (2006), and Lane and Milesi-Ferretti (2007, 2008), banks make loans to both domestic and foreign firms. The loan rate stickiness stems from imperfect (monopolistic) competition among banks for firms as shown in Sander and Kleimeier (2004), Gropp, Sorensen, and Lichtenberger (2007), and Gropp and Kashyap (2009).

We first estimate the loan interest rate stickiness for the United Kingdom (UK), the euro area, and Japan. Results show that banks, on average, take three quarters to adjust loan rates in these countries. Then we show that our model with such an estimated loan interest rate stickiness can replicate persistent, volatile, and realistic hump-shaped responses of real exchange rates even with both supply-side disturbances through cost-push and loan rate shocks and demand-side disturbances through a monetary policy shock. In particular, the fact that we can reproduce such a realistic response via a monetary policy shock merits attention. Previous studies, such as Chari, Kehoe, and McGrattan (2002) and Steinsson (2007), demonstrate that it is impossible to produce hump-shaped real exchange rate dynamics with such a shock. These results of hump-shaped real exchange rate dynamics are obtained in our model with sticky prices solely due to staggered loan contracts. Interestingly, by further incorporating the staggered price setting, which has been considered the important element for realistic real exchange rate dynamics in former studies, we cannot replicate the hump-shaped real exchange rate dynamics to the monetary policy shock.

This paper is structured as follows. In the next section, we show the outline of the model. In Section 3, after a brief survey of previous studies related to sticky loan rates, we estimate the degree of loan rate stickiness for the euro area, the UK, and Japan. Section

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[2]Graham and Wright (2007) also incorporate sticky loan interest rates into a closed-economy general equilibrium model. Contrary to Teranishi (2007), stickiness in interest rates is imposed for consumers rather than firms in Graham and Wright (2007). Thus, the loan interest rate in the IS curve is sticky in Graham and Wright (2007), but the loan interest rate in the Phillips curve is sticky in our model.

4 shows that realistic responses of real exchange rates are produced in our model with staggered loan contracts. In Section 5, we discuss the case with the staggered price setting. Finally, Section 6 summarizes the findings in this paper.

2 Model

The model consists of two symmetric countries. There are four types of agents in each country, consumers, firms, private banks, and the central bank as shown in Figure 1. In this section, we only show the outline of the model. A detailed derivation of the model is shown in Appendix A.

[Figure 1: Outline of the Model]

2.1 Consumers

A representative consumer chooses the amount of aggregate consumption, bank deposits, and investment in a set of risky assets given a deposit interest rate set by the central bank; consumes differentiated goods on both home- and foreign-produced consumer goods; provides differentiated labor services to both domestically financially supported (DFS) and internationally financially supported (IFS) projects, as well as offers wages to those differentiated types of labor with monopolistic power over labor supply; and owns banks and firms and receives dividends in each period.[3] In particular, the existence of the differentiated labor services is crucial for monopolistic competition in loan rate settings.[4] Thanks to this differentiated labor supply, the demand for loans is differentiated without assuming any restrictions on aggregate loans or loan interest rates.

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[3]Throughout our analyses, homotheticity of preferences is maintained.

[4]We assume that a representative household supplies differentiated labor to each differentiated project. On the other hand, each household supplies differentiated labor to one project (firm) in Erceg, Henderson, and Levin (2000), but risks from labor supply differentiation can be completely insured. Both specifications produce identical results.

A representative consumer in the home country H maximizes the following welfare W_t:

$$W_t = \mathrm{E}_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T) - \int_0^n V\left[l_T(h)\right] \mathrm{d}h - \int_n^1 V\left[l_T(\bar{h})\right] \mathrm{d}\bar{h} \right] \right\},$$

where E_t is the expectation operator conditional on the state of nature at date t. $U(\cdot)$ is an increasing and concave function in the consumption index C_t, and $V(\cdot)$ is an increasing and convex function in labor supply $l_t(\cdot)$. The budget constraint of the consumer is given by

$$
\begin{aligned}
P_t C_t + \mathrm{E}_t\left[X_{t,t+1} B_{t+1}\right] + D_t \quad \leq \quad & B_t + (1 + i_{t-1}) D_{t-1} + \int_0^n w_t(h) l_t(h) \mathrm{d}h \\
& + \int_n^1 w_t(\bar{h}) l_t(\bar{h}) \mathrm{d}\bar{h} + \Pi_t^B + \Pi_t^F,
\end{aligned}
$$

where P_t is a price index, B_t is a set of risky assets, D_t is the amount of deposits at private banks, i_t is the nominal deposit interest rate set by a central bank from $t-1$ to t, $w_t(h)$ is the nominal wage for labor supplied from the DFS group $l_t(h)$, $w_t(\bar{h})$ is the nominal wage for labor supplied from the IFS group $l_t(\bar{h})$, $\Pi_t^B = \int_0^1 \Pi_{t-1}^B(h) \mathrm{d}h$ is the nominal dividend stemming from the ownership of both local and international banks in the home country, $\Pi_t^F = \int_0^1 \Pi_{t-1}^F(f) \mathrm{d}f$ is the nominal dividend from the ownership of the firms in the domestic country, and $X_{t,t+1}$ is the stochastic discount factor. The labor type h and the firm's project type h match each other. Here, because we assume a complete financial market between the two countries, the consumer in each country can internationally buy and sell the state contingent securities to insure against country specific shocks. The consumption index that consists of bundles of differentiated goods produced by home and foreign firms is expressed as

$$C_t \equiv \frac{C_{H,t}^{\phi_H} C_{F,t}^{1-\phi_H}}{\phi_H^{\phi_H} \left(1 - \phi_H\right)^{1-\phi_H}},$$

where ϕ_H $(0 \leq \phi_H \leq 1)$ is a preference parameter that expresses the home bias. Here, $C_{H,t}$ and $C_{F,t}$ are consumption subindices of the continuum of differentiated goods produced by firms in the home country and the foreign country, respectively. They are defined as

$$C_{H,t} \equiv \left[\int_0^1 c_t\left(f\right)^{\frac{\sigma-1}{\sigma}} \mathrm{d}f \right]^{\frac{\sigma}{\sigma-1}},$$

and

$$C_{F,t} \equiv \left[\int_0^1 c_t\left(f^*\right)^{\frac{\sigma-1}{\sigma}} \mathrm{d}f^* \right]^{\frac{\sigma}{\sigma-1}},$$

where $c_t(f)$ is the demand for a good produced by firm f in the home country and $c_t(f^*)$ is the demand for a good produced by a firm f^* in the foreign country, where the asterisk denotes foreign variables. The consumer decides goods demand according to goods prices through a cost minimization under these indices. It is assumed that there is no trade friction and consumers in both countries have the same preferences over the differentiated goods.

The foreign consumers also solve a similar optimization problem.

2.2 Firms

There exists a continuum of firms in each country. Each firm decides the amount of differentiated labor to be employed from both the DFS and IFS groups through the cost minimization for the production cost. Part of the costs of labor must be financed by external loans from banks. For example, in country H, to finance the costs of hiring workers from the DFS group for the DFS project, the firm must borrow from local banks in the home country. However, to finance the costs of hiring workers from the IFS group, the firm must borrow from international banks in the foreign country. This reflects the fact that financial markets are globally integrated, namely, a significant increase has taken place in international borrowing and lending as shown in Gadanecz (2004), McGuire and Tarashev (2006), and Lane and Milesi-Ferretti (2007, 2008). We also know by looking into actual project finance that firms borrow funds with many different loan interest rates at the same time, depending on the nature of the projects. Firms tend to face different loan rates depending on when, why, for what, how long, or how much they require external funds. We interpret that these project differences are characterized by types of labor. For this point, we can assume a situation where each project factory in one region is forced to borrow from a neighbor bank by information or geographical segmentation in the bank evaluating the project, as in Mandelman (2006).[5] Labor is immobile between the two countries. Here, we

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[5]Kobayashi (2008) shows that a one-to-one relation between a firm and a bank due to monopoly induces the same loan rate curve as in our model. In this case, we assume that an intermediate good producer uses one type of loan and labor to produce an intermediate goods and a final goods producer uses all types of intermediate goods to produce a final good.

assume that firms must use all types of labors and therefore borrow from both local and international banks in the same proportion.

At the same time, each firm sets goods prices under monopolistically competitive markets under pricing-to-market. In the benchmark model explained here, we do not incorporate the staggered price setting, but prices are sticky due to staggered loan contracts that make marginal costs sticky, similar to the case with the sticky wages. We also examine the case with sticky prices later.

Let us first explain the determination of the marginal costs. Firms in both home and foreign countries optimally hire differentiated labor as price takers. This optimal labor allocation is carried out through two-step cost minimization problems. For example, domestic firm f hires all types of labor from both the DFS and IFS groups. When hiring from the DFS group, the γ portion of the labor cost associated with labor type h is financed by borrowing from the local bank h. Then, the first-step cost minimization problem on the allocation of differentiated labor from the DFS group is given by

$$\min_{l_t(h,f)} \int_0^n \left[1 + \gamma r_t\left(h\right)\right] w_t\left(h\right) l_t(h,f) \mathrm{d}h,$$

subject to the subindex regarding labor from the DFS group to firm f:

$$L_t\left(f\right) \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n l_t\left(h,f\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}h\right]^{\frac{\epsilon}{\epsilon-1}},$$

where $r_t(h)$ is the loan interest rate applied to employ a particular labor type h applied to differentiated labor supply. There $l_t(h,f)$ denotes the type of labor h employed by firm f. The local bank h has some monopoly power over setting loan interest rates. Thus, we assume monopolistic competition on the loan contracts between banks and firms. Based on the optimality conditions obtained from the above problem, the firms optimally choose the allocation of differentiated workers between the two groups. Because firms have some preference n to hire workers from the DFS group and $(1-n)$ to hire workers from the IFS group, the second-step cost minimization problem describing the allocation of differentiated labor between these two groups is given by

$$\min_{L_t, \bar{L}_t} \Omega_t L_t\left(f\right) + \bar{\Omega}_t \bar{L}_t\left(f\right),$$

where we define

$$\Omega_t \equiv \left\{ \frac{1}{n} \int_0^n \{[1 + \gamma r_t(h)] w_t(h)\}^{1-\epsilon} \, dh \right\}^{\frac{1}{1-\epsilon}},$$

and $\bar{\Omega}_t$ and $\bar{L}_t(f)$ are those for the IFS group, subject to the aggregate labor index:

$$\tilde{L}_t(f) \equiv \frac{[L_t(f)]^n \left[\bar{L}_t(f)\right]^{1-n}}{n^n (1-n)^{1-n}}.$$

Then, we have profit maximization in terms of price setting. Each domestic firm f sets its price $p_t(f)$ and $p_t^*(f)$ to maximize the profit, which is given by

$$p_t(f) c_t(f) + S_t p_t^*(f) c_t^*(f) - \tilde{\Omega}_t \tilde{L}_t(f),$$

where we define

$$\tilde{\Omega}_t \equiv \Omega_t^n \bar{\Omega}_t^{1-n},$$

S_t is the nominal exchange rate, and $c_t(f)$ and $c_t^*(f)$ are demand for the goods in domestic and foreign countries, respectively, that can be derived from the consumption bundles defined above. Although the firm sets $p_t(f)$ and $p_t^*(f)$ separately under the pricing-to-market assumption, without infrequent price changes we have

$$p_t(f) = S_t p_t^*(f).$$

We have a similar maximization problem for the foreign firm f^*.

2.3 Private Banks

There exists a continuum of private banks. There are two types of banks in each country: local banks and international banks. Each private bank plays two roles. A bank collects the deposits from consumers in its country. It also sets differentiated nominal loan interest rates according to their individual loan demand curves under the monopolistically competitive loan market. This heterogeneity reflects the empirical evidence for dispersed loan rates through the imperfect competition between firms and private banks on loan contracts reported by Sander and Kleimeier (2004), Gropp, Sorensen, and Lichtenberger (2007), and Gropp and Kashyap (2009). Moreover, Sander and Kleimeier (2004) and Gropp, Sorensen, and Lichtenberger (2007) show that this imperfection induces staggered loan rate contracts, which gives a justification for staggered loan rates found in the empirical studies.

A local bank lends only to firms when they hire labor from the DFS group. However, an international bank only provides a loan to firms when they hire labor from the IFS group. We here describe the optimization problem of an international bank in the home country. The international bank takes on the exchange rate risk inherent in its loans. Each international bank can reset loan interest rates with probability $(1 - \bar{\varphi}^*)$ following the Calvo (1983)–Yun (1996) framework. Under the segmented environment, private banks can set different loan interest rates depending on the types of labor. As a consequence, the private bank holds some monopoly power over the loan interest rate to firms. Therefore, the international bank $\bar{h}^*$ chooses the loan interest rate $r_t\left(\bar{h}^*\right)$ to maximize the present discounted value of profit:

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\bar{\varphi}^*)^{T-t} X_{t,T} q_{t,T}\left(\bar{h}^*, f^*\right) \left\{ S_{T+1}\left[1 + r_t\left(\bar{h}^*\right)\right] - S_T\left(1 + i_T\right) \right\},$$

where $q_{t,T}\left(\bar{h}^*, f^*\right)$ is the amounts of loans borrowed by firm f^* for the labor types $\bar{h}^*$. The demand for loans is derived from the cost minimization problem by firms.

Local banks in the home country and international and local banks in the foreign country face a similar profit maximization problem.

2.4 Linearized Equilibrium Conditions

By log-linearly approximating the equilibrium conditions obtained from above problems around the steady states, we can derive linearized equilibrium conditions. Here, $\hat{A}_t \approx \log\left(\frac{A_t}{A}\right) \approx \frac{A_t - A}{A}$, where A is the steady state of A_t. They are as follows.

Aggregate consumption in each country is given by

$$\hat{C}_t = \mathrm{E}_t \hat{C}_{t+1} - \upsilon\left(\hat{\imath}_t - \mathrm{E}_t \pi_{t+1}\right), \tag{1}$$

and

$$\hat{C}_t^* = \mathrm{E}_t \hat{C}_{t+1}^* - \upsilon^*\left(\hat{\imath}_t^* - \mathrm{E}_t \pi_{t+1}^*\right). \tag{2}$$

$\pi_t\left(\pi_t^*\right)$ denotes home (foreign) consumer price index (CPI) inflation, u_t is a demand shock in the home country, and υ and υ^* are positive parameters regarding the elasticity of consumption with respect to the real interest rate.

The price setting equations in the home and foreign countries are given by

$$
0 = \phi_H \left(\Theta_1 \hat{R}_{H,t} + \Theta_2 \hat{R}^*_{H,t} \right) + (1 - \phi_H) \left(\Theta_1^* \hat{R}^*_{F,t} + \Theta_2^* \hat{R}_{F,t} \right)
$$
$$
+ \upsilon^{-1} \left[\phi_H \hat{C}_t^W + (1 - \phi_H) \hat{C}_t^{W*} \right] + 2\phi_H (1 - \phi_H) \hat{e}_t + m_t, \tag{3}
$$

and

$$
0 = \phi_F \left(\Theta_1 \hat{R}_{H,t} + \Theta_2 \hat{R}^*_{H,t} \right) + (1 - \phi_F) \left(\Theta_1^* \hat{R}^*_{F,t} + \Theta_2^* \hat{R}_{F,t} \right)
$$
$$
+ (\upsilon^*)^{-1} \left[\phi_F \hat{C}_t^W + (1 - \phi_F) \hat{C}_t^{W*} \right] - 2\phi_F (1 - \phi_F) \hat{e}_t, \tag{4}
$$

where we define

$$
\hat{C}_t^W \equiv \phi_H \hat{C}_t + (1 - \phi_H) \hat{C}_t^*, \tag{5}
$$

$$
\hat{C}_t^{W*} \equiv \phi_F \hat{C}_t + (1 - \phi_F) \hat{C}_t^*. \tag{6}
$$

Equations (3) to (4) simply show that marginal costs are always constant. An international perfect risk-sharing condition implies

$$
\pi_t^* - \pi_t + \triangle \hat{S}_t = \triangle \hat{e}_t, \tag{7}
$$

and the definitions of nominal and real exchange rates imply

$$
\upsilon \hat{e}_t = \hat{C}_t - \hat{C}_t^* + x_t. \tag{8}
$$

Here $\hat{R}_{H,t}$ ($\hat{R}_{F,t}$) denotes the loan rate offered by a home local (foreign international) bank to a home firm, $\hat{R}^*_{H,t}$ ($\hat{R}^*_{F,t}$) denotes the loan rate offered by a home international (foreign local) bank to a foreign firm, $\Delta \hat{S}_t$ denotes the growth rate of nominal exchange rates, $\hat{e}_t$ denotes the real exchange rate, m_t denotes the cost-push shock in the home country, and x_t denotes the UIP shock. For the reason and importance of the uncovered interest rate parity (UIP) shock in open economy models, we can see Lubik and Schorfheide (2005). Θ_1, Θ_2, Θ_1^*, and Θ_2^* are positive parameters regarding the cost channel of loan contract, and $\phi_F = 1 - \phi_H$.[6] As shown in Steinsson (2007), the cost-push shock plays an important role in economic dynamics.

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[6] When there is no loan contract with flexible price, the price setting equations given by equations (3) and (4) are reduced to

$$
0 = \upsilon^{-1} \left[\phi_H \hat{C}_t^W + (1 - \phi_H) \hat{C}_t^{W*} \right] + 2\phi_H (1 - \phi_H) \hat{e}_t + m_t,
$$

The (new Keynesian) loan supply curves in both countries are

$$\hat{R}_{F,t} = \bar{\lambda}_1^* \mathrm{E}_t \hat{R}_{F,t+1} + \bar{\lambda}_2^* \hat{R}_{F,t-1} + \bar{\lambda}_3^* \left(\hat{\imath}_t - \mathrm{E}_t \triangle \hat{S}_{t+1} \right), \tag{9}$$

$$\hat{R}_{H,t} = \lambda_1 \mathrm{E}_t \hat{R}_{H,t+1} + \lambda_2 \hat{R}_{H,t-1} + \lambda_3 \hat{\imath}_t + z_t, \tag{10}$$

$$\hat{R}_{H,t}^* = \bar{\lambda}_1 \mathrm{E}_t \hat{R}_{H,t+1}^* + \bar{\lambda}_2 \hat{R}_{H,t-1}^* + \bar{\lambda}_3 \left(\mathrm{E}_t \triangle \hat{S}_{t+1} + \hat{\imath}_t^* \right), \tag{11}$$

and

$$\hat{R}_{F,t}^* = \lambda_1^* \mathrm{E}_t \hat{R}_{F,t+1}^* + \lambda_2^* \hat{R}_{F,t-1}^* + \lambda_3^* \hat{\imath}_t^*. \tag{12}$$

Here z_t is the loan rate shock for the loan rate set by the home local banks for home firms.[7] $\bar{\lambda}_j^*$, λ_j, $\bar{\lambda}_j$, and λ_j^* for $j = 1$, 2, and 3 are positive parameters regarding loan rate curves. In Cúrdia and Woodford (2009) and Gerali *et al.* (2009), the loan rate shock comes from a default risk and a risk premium for the borrower.[8] It is worth mentioning that one reason why these loan rate curves include lags because the loan rate is like a price level in the new Keynesian Phillips curve. If the new Keynesian Phillips curve is expressed in terms of the price level rather than inflation, it must contain the lagged price level.

To close the model, the central banks in both countries set the deposit (policy) interest rates following the Taylor (1993)-type rules as

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + (1 - \rho_i) \Phi_1 \pi_t + (1 - \rho_i) \Phi_2 \hat{C}_t + v_t, \tag{13}$$

and

$$\hat{\imath}_t^* = \rho_i^* \hat{\imath}_{t-1}^* + (1 - \rho_i^*) \Phi_1^* \pi_t^* + (1 - \rho_i^*) \Phi_2^* \hat{C}_t^*. \tag{14}$$

Here v_t is a monetary policy shock in the home country. Φ_1, Φ_2, Φ_1^*, Φ_2^*, ρ_i, and ρ_i^* are positive policy parameters. Chari, Kehoe, and McGrattan (2002) show the reason and importance of the monetary policy shock.

$$0 = (v^*)^{-1} \left[\phi_F \hat{C}_t^W + (1 - \phi_F) \hat{C}_t^{W*} \right] - 2\phi_F (1 - \phi_F) \hat{e}_t.$$

Note that the policy rate cannot affect consumption since home and foreign consumption is determined by these two equations and equation (8) as implied by Woodford (2003).

[7] When we assume an international loan rate shock for the loan rate set by foreign international banks for home firms in equation (9), similar results are obtained in simulations.

[8] Recent studies such as Neumeyer and Perri (2005) and Uribe and Yue (2006) insist that international risk premium shocks can induce large business cycles. Moreover, Marston (1995) and Uribe and Yue (2006) state that interest rate spreads are quite persistent.

We have 14 equations, namely, equations (1) to (14), for 14 endogenous variables: $\hat{C}$, $\hat{C}^*$, π, π^*, $\hat{C}^W$, $\hat{C}^{W^*}$, $\hat{e}$, $\Delta \hat{S}$, $\hat{R}_F$, $\hat{R}_H$, $\hat{R}_H^*$, $\hat{R}_F^*$, $\hat{\imath}$, and $\hat{\imath}^*$.

3   Empirical Evidence for a Sticky Loan Rate

Stickiness in loan rates has been reported regardless of newly contracted or outstanding and short- or long-term contracts. For the US, using micro-data, Slovin and Sushka (1983) show that it takes two or more quarters for private banks to adjust loan rates even for newly contracted loans against changes in the market rate. This finding is consistent with the results in Berger and Udell (1992). For the euro area countries, Sorensen and Werner (2006) estimate the incompleteness in the pass-through from policy rates to loan rates via an error correction model using macro-data. They find the lags ranging between one and three quarters even for newly contracted long-term loans. Gambacorta (2008) conducts a similar analysis for Italy and shows the existence of sticky adjustment in loan rates for about two quarters for newly contracted short-term lending. For Japan, Bank of Japan (2007, Figure 50) reports that the major city banks need more than three quarters and local banks need more than five quarters to adjust their loan rates to a change in the policy rate. Similarly, Bank of Japan (2009, Figure 3–12) concludes that loan rates demonstrate considerable stickiness. Several reasons are also reported for the existence of the sticky loan interest rates. Sorensen and Werner (2006), Sander and Kleimeier (2004), and Gropp, Sorensen, and Lichtenberger (2007) claim that the lower degree of competition can induce a higher degree of loan rate stickiness. Less competition gives banks the ability to benefit through sluggish adjustment of the lending rate. On the other hand, Berger and Udell (1992) emphasize relationship banking, insisting that long-run (repeated) business relations between banks and firms can result in loan rate stickiness. The banks set lower interest rates to insure that risk-averse borrowers in a long-run relationship avoid bankruptcy when interest rates are high. In this paper, we take the former view when modeling the loan rate stickiness.

To understand the real exchange rate dynamics in our model, we need to obtain the parameters on the loan rate stickiness. We estimate the Calvo parameter implicit in the new

Keynesian loan rate curve in equation (10) by nonlinear maximum likelihood estimation for the UK, the euro area, and Japan due to the data availability. When estimating the Calvo parameters, we assume that the same loan rate stickiness should apply for both domestic and international lending. We use official policy rates provided by central banks. For the UK, we use the official bank rate and weighted average overdraft interest rate for non-financial corporations for the period from January 1999 to March 2009; for the euro area, we use the main refinancing operations fixed rate and average outstanding loan interest rate from banks for non-financial corporations from the monetary financial institution (MFI) interest rate statistics for the period from January 2003 to March 2009; for Japan, we use the overnight uncollateralized call rate and average stock loan interest rate for the period from January 1984 to December 1995.[9] All data are monthly average and are de-trended by the HP filter with the smoothing parameter 14,400, which is usually used for monthly data. For a forward (expected) variable, we use the one-period-ahead forecast obtained from an estimated AR(2) model.

Table 1 reports the values of the Calvo parameters on a quarterly basis. We can see that similar values are obtained for all three areas. On average, since the Calvo parameter on a quarterly basis is very close to 0.66, the loan interest rates are adjusted by about three quarters in each country. These findings are consistent with the former empirical studies. In the next section, we examine the responses of the real exchange rate to structural shocks under the estimated parameters of sticky loan rates in this section.

4   Simulation

The parameters are calibrated as in Table 2. β, υ, υ^*, α, α^*, σ, ϕ_H, Φ_1, Φ_1^*, Φ_2, Φ_2^*, ρ_i and ρ_i^* are from Steinsson (2007), ϵ and ϵ^* are from Rotemberg and Woodford (1997), and φ, φ^*, $\bar{\varphi}$, and $\bar{\varphi}^*$ are the estimated average value of loan rate stickiness across the countries in Table 1. We assume that the ratio of the dependency on external finance γ is unity and preferences regarding labor supply are the same between the DFS and IFS groups.

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[9]For the UK and the euro area, we use the whole sample available, but for Japan we only use the data until the end of 1995 to avoid a policy period of virtually zero interest rates.

We examine four types of positive 1 percent shocks. They are cost-push shock, loan rates shock, UIP shock, and monetary policy shock. Following Steinsson (2007), we set that each shock follows AR(1) process with a parameter of 0.9.

Table 3 shows the detailed simulation outcomes of the real exchange rate dynamics for the home shocks. We report five statistics defining the real exchange rate dynamics: (1) UL (up-life); (2) UL/HL (up-life over half-life); (3) QL–HL (quarter-life minus half-life); (4) AR(1) persistence; and (5) relative standard deviation of real exchange rates to consumption.[10] UL denotes the duration until the impulse response falls below the top (maximum) point, while HL shows the duration until the impulse response falls below half of the top point. QL denotes the duration until the impulse response falls below a quarter of the top point. These are the measures for how the impulse response functions are hump-shaped. For example, if the impulse responses are monotonically decreasing, UL and UL/HL should be zero. If the impulse responses demonstrate persistent hump-shaped dynamics, UL/HL and QL–HL become larger. For comparison, we also examine the case with flexible loan contracts ($\varphi = \varphi^* = \bar{\varphi} = \bar{\varphi}^* = 0$).

The results are shown in Table 3.[11] The first row in the table reports the average of key empirical features of real exchange rates for the UK, the euro area, and Japan shown in Steinsson (2007). The second to fifth rows illustrate outcomes from the model simulation when loan rates are flexible. These results demonstrate that the model can replicate neither hump-shaped dynamics of the real exchange rate nor AR(1) persistence when both the loan rate and price are flexible. On the other hand, the sixth to ninth rows show outcomes when banks cannot change loan interest rates every period. We can see that the hump-shaped dynamics of the real exchange rates can materialize for loan rate and monetary policy shocks. In particular, it is worth mentioning that we can reproduce hump-shaped and persistent responses to monetary policy shock, given the fact that it has been considered very difficult (see, for example, Chari, Kehoe, and McGrattan (2002) and Steinsson (2007)). Other statistics, namely, AR(1) persistence and the relative standard

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[10] As explained in Steinsson (2007), our model cannot replicate the low correlation of consumption between home and foreign countries, since we still maintain a complete financial market for consumers.

[11] We show impulse responses for loan rate shocks in Appendix B.

deviation of real exchange rates compared to that of consumption, are also in line with the empirical values with these two shocks.

The reason for the hump-shaped dynamics reflects the fact that the real exchange rate dynamics depend crucially on the marginal costs. To see this, we transform equations (3) and (4) as

$$
\begin{aligned}
0 = & \left(\phi_H - \phi_F\right)\left(\Theta_1 \hat{R}_{H,t} + \Theta_2 \hat{R}_{H,t}^*\right) + \left(\phi_H - \phi_F\right)\left(\Theta_1^* \hat{R}_{F,t}^* + \Theta_2^* \hat{R}_{F,t}\right) \\
& + \hat{e}_t + m_t - \upsilon^{-1}\left(\phi_H - \phi_F\right)^2 x_t,
\end{aligned} \tag{15}
$$

where we use equation (8) and $\upsilon = \upsilon^*$ from the assumption of symmetry. This equation simply implies that the real exchange rate dynamics depend on the loan rate dynamics. When loan rates are sticky, the loan rate dynamics are hump-shaped thanks to the lags of loan rates in equations (9)–(12) to loan rate shocks. This directly results in the hump-shaped real exchange rate dynamics. The monetary policy shock also produces such dynamics since the policy rate works as a shock in the loan rate curves. Moreover, the lag in the monetary policy rule further enhances the hump-shapedness of the loan rate dynamics.[12] Note that the parameters on the first and the second terms on the right-hand side of equation (15) imply the role of the home bias. When there is no home bias, namely, $\phi_H = \phi_F$, hump-shaped responses of real exchange rates cannot be created.

We observe that the hump-shaped responses do not appear for the cost-push shock. This will also become clear from equation (15). The cost-push shock directly affects the real exchange rate and does not work through the loan rate adjustments. For the UIP shock, loan rates applied to the home country move exactly in a manner opposite to those of the foreign country. Therefore, the UIP shock can be considered as the same scale but the different sign of monetary policy shocks to home and foreign countries, respectively. As a result, we have

$$
0 = \left(\phi_H - \phi_F\right)\left(\Theta_1 \hat{R}_{H,t} + \Theta_2 \hat{R}_{H,t}^*\right) + \left(\phi_H - \phi_F\right)\left(\Theta_1^* \hat{R}_{F,t}^* + \Theta_2^* \hat{R}_{F,t}\right),
$$

and

$$
0 = \hat{e}_t - \upsilon^{-1}\left(\phi_H - \phi_F\right)^2 x_t.
$$

---

[12]In detail, $\hat{R}_{H,t}$ and $\hat{R}_{H,t}^*$ with $\hat{\imath}_t$ reflecting the UIP condition, are the key to materializing the hump-shaped dynamics of real exchange rates. Here, UIP means $\mathrm{E}_t \triangle \hat{S}_{t+1} = i_t - i_t^*$.

Thus, the real exchange rate dynamics are directly determined by the UIP shock following the AR(1) process.

5   A Case with Infrequent Price Setting

How can the results obtained in the previous section be altered by the introduction of the staggered price setting in addition to the staggered loan contracts examined before? Following the Calvo (1983)–Yun (1996) framework, firms can reset a goods price with some positive probability every period. A detailed derivation is in Appendix A. Note that the model without the staggered price setting can produce inflation persistence thanks to persistent marginal cost. Thus, the model with staggered price setting holds the dual sticky mechanism for the price dynamics.

Two price setting equations given by equations (3) and (4) are now transformed into the augmented new Keynesian Phillips curves:

$$
\begin{aligned}
\pi_t &= \phi_H \kappa \left(\Theta_1 \hat{R}_{H,t} + \Theta_2 \hat{R}_{H,t}^* \right) + (1 - \phi_H) \kappa^* \left(\Theta_1^* \hat{R}_{F,t}^* + \Theta_2^* \hat{R}_{F,t} \right) \\
&\quad + \bar{\kappa} \left[\phi_H \hat{C}_t^W + (1 - \phi_H) \hat{C}_t^{W*} \right] + \varsigma \hat{e}_t + \beta \mathrm{E}_t \pi_{t+1} + m_t,
\end{aligned}
$$

and

$$
\begin{aligned}
\pi_t^* &= (1 - \phi_H) \kappa \left(\Theta_1 \hat{R}_{H,t} + \Theta_2 \hat{R}_{H,t}^* \right) + \phi_H \kappa^* \left(\Theta_1^* \hat{R}_{F,t}^* + \Theta_2^* \hat{R}_{F,t} \right) \\
&\quad + \bar{\kappa}^* \left[\phi_F \hat{C}_t^W + (1 - \phi_F) \hat{C}_t^{W*} \right] - \varsigma_t^* \hat{e}_t + \beta \mathrm{E}_t \pi_{t+1}^*.
\end{aligned}
$$

The parameters κ, $\bar{\kappa}$, ς, κ^*, $\bar{\kappa}^*$, and ς^* are given in Table 2. We set that firms can change the price once a year on average. When there is no loan contract, these augmented new Keynesian Phillips curves are reduced to standard international new Keynesian Phillips curves as in Steinsson (2007).

Table 4 shows the simulation outcomes from the model with both sticky price and loan.[13] The second to the fifth rows illustrate outcomes from the model when loan rates are flexible. On the other hand, the sixth to ninth rows show outcomes when both firms and banks cannot change goods prices and loan interest rates every period, respectively.

---

[13] We show impulse responses for loan rate shocks in Appendix B.

We can see that the real exchange rate dynamics are hump-shaped only with cost-push and loan rates shocks irrespective of whether the loan rates are sticky or not. The reason for this is exactly that given by Steinsson (2007). By setting the home bias parameter very high, the home cost-push shock has only negligible effects on foreign variables. Under such circumstances, the hump-shaped responses of real exchange rates come from the responses in home consumption as implied by equation (8). Therefore, eventually, it depends on the dynamics of real interest rates in the home country since the consumption can be represented as the discounted sum of future real interest rates according to the Euler conditions given by equation (1). Here, after a cost-push shock hits the economy, inflation and consumption can move in opposite directions while they co-move to the demand and monetary policy shocks. As a result, real interest rates affected naturally by short-term nominal interest rates set through the Taylor (1993)-type rule show non-monotonic responses.[14] Therefore, we can reproduce the hump-shaped responses in real interest rates and therefore in real exchange rates to cost-push shocks. A loan rate shock itself is not a cost-push shock, but is considered to be a more microfounded cost-push shock. Thus, a direct shock to loan interest rates works as if it were a cost-push shock and therefore results in hump-shaped responses in the real exchange rate.

At the same time, due to this very reason, a model with the infrequent price change cannot produce the hump-shaped real exchange rate dynamics for monetary policy shock. The effects of the monetary policy shock through real interest rates completely dominate those observed under the flexible price setting. By the same reason in the previous section, the real exchange rate dynamics are solely determined by the UIP shock that follows the AR(1) process.

6   Concluding Remarks

Empirical papers have shown the nontrivial roles of financial market imperfections, which can be represented as a staggered loan contract. We introduce this into an otherwise standard NOEM model in a tractable manner. Simulation results with such staggered loan

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[14]This logic is similar to Benigno (2004a).

contracts can generate persistent, volatile, and hump-shaped responses in real exchange rates for both supply and demand shocks, such as cost-push, loan rate, and monetary policy shocks. In particular, the fact that we can reproduce such a realistic response via a monetary policy shock merits attention, because reproduction has been considered very difficult by previous studies. According to the results in this paper, financial market imperfections are a very important element in understanding the realistic real exchange rate dynamics.

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Table 1: Estimation result (quarter basis)

Country	Mean	$\pm \sigma$ interval
UK	0.59	[0.45, 0.74]
Euro area	0.57	[0.55, 0.58]
Japan	0.57	[0.56, 0.59]
Average	0.58	-

Note: We estimate the Calvo parameter implicit in equation (10) using nonlinear maximum likelihood estimation. For the UK, we use the official bank rate and weighted average overdraft interest rate for non-financial corporations for the period from January 1999 to March 2009. For the euro area, we use the main refinancing operations fixed rate and average outstanding loan interest rate from banks for non-financial corporations from the MFI interest rate statistics for the period from January 2003 to March 2009. For Japan, we use the overnight uncollateralized call rate and average stock loan interest rate for the period from January 1984 to December 1995. All data are the monthly average and are de-trended by the HP filter with the conventional parameter 14,400, which is usually used for monthly data. For a forward (expected) variable, we use the one-period-ahead forecast obtained from an estimated AR(2) model.

Table 2: Parameter values

Parameters	Values	Explanation
β	0.99	Discount factor
υ, υ^*	0.2	Elasticity of output with respect to real interest rate
μ_1, μ_1^*	1	Marginal product of labor
η_1, η_1^*	0	Elasticity of marginal disutility for labor
η_2, η_2^*	0	Elasticity of marginal product of labor
α, α^*	0.75	Probability of price change
$\varphi, \varphi^*, \overline{\varphi}, \overline{\varphi}^*$	0.57	Probability of loan interest rate change
σ	7.66	Substitutability of differentiated consumption goods
ϵ, ϵ^*	7.66	Substitutability of differentiated laborers
$\gamma, \gamma^*, \overline{\gamma}, \overline{\gamma}^*$	1	Ratio of external finance to total finance
n	0.5	Preference for labors in the DFS group
ϕ_H	0.94	Preference for goods produced in the home country
Φ_1, Φ_1^*	2	Coefficient on inflation rate in the Taylor rule
Φ_2, Φ_2^*	0.5	Coefficient on the output gap in the Taylor rule
ρ_i, ρ_i^*	0.85	Lag parameter in the Taylor rule

Note: In particular, we define $\Theta_1 \equiv n \frac{\gamma(1+R_H)}{1+\gamma R_H}$, $\Theta_2 \equiv (1-n) \frac{\overline{\gamma}\left(1+R_H^*\right)}{1+\overline{\gamma} R_H^*}$, where R_H and R_H^* are the steady state values of loan rates. We have similar definitions for Θ_1^* and Θ_2^*. We have $\overline{\lambda}_1^* \equiv \frac{\overline{\varphi}^* \beta}{1+(\overline{\varphi}^*)^2 \beta}$, $\overline{\lambda}_2^* \equiv \frac{\overline{\varphi}^*}{1+(\overline{\varphi}^*)^2 \beta}$, and $\overline{\lambda}_3^* \equiv \frac{1-\overline{\varphi}^*}{1+(\overline{\varphi}^*)^2 \beta} \frac{\epsilon^*}{\epsilon^*-1} \frac{(1-\beta \overline{\varphi}^*)(1+i)}{1+R_F}$, where i is the steady state value of the policy rate as $\beta = (1+i)^{-1}$. We have $(1+\overline{\gamma}^* R_F) - \epsilon^* \overline{\gamma}^* \left\{(1+R_F) - (1+i)\right\} = 0$ in the steady state. We have similar definitions for λ_j, $\overline{\lambda}_j$, and λ_j^* for $j = 1, 2,$ and 3. We have $\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\eta_2\sigma)}$, $\overline{\kappa} \equiv \kappa \left(\eta_1 \mu_1^{-1} + \eta_2 + \upsilon^{-1}\right)$, and $\varsigma \equiv 2\kappa\phi_H (1 - \phi_H)$. For κ^*, $\overline{\kappa}^*$, and ς^*, we have similar definitions. Other details for parameters are shown in Appendix A.

Table 3: Properties of real exchange rate under basic model

Setting	HL	UL/HL	QL–HL	AR(1)	Std(e)/Std(C)
Empirical values	2.8	0.42	1.51	0.79	4.97
FL and CPS	1.5	0	1.75	0.67	4.94
FL and LRS	1.5	0	1.75	0.67	4.62
FL and UIPS	1.5	0	1.75	0.67	51.5
FL and MPS	1.5	0	1.75	0.67	4.62
SL and CPS	1.75	0	1.75	0.7	4.97
SL and LRS	2.75	0.27	1.75	0.89	4.64
SL and UIPS	1.75	0	1.5	0.69	41.8
SL and MPS	2.75	0.27	1.75	0.89	4.64

Note: HL expresses half-life (measured in years), UL/HL expresses up-life over half-life, QL-HL expresses quarter-life minus half-life, AR(1) expresses the first-order autocorrelation of the HP-filtered series, and Std(e)/Std(C) expresses the standard deviation of HP-filtered e (real exchange rate) divided by the standard deviation of HP-filtered C (consumption). The point estimates for AR(1) and Std(e)/Std(C) are calculated by simulating 1,000 data series from each model, in which each data length is 127, and the point estimate is the median value of the resulting distribution exactly following Steinsson (2007). FL and SL mean flexible and staggered loan contracts, respectively. CPS, LRS, UIPS, and MPS denote cost-push shock, loan rate shock, UIP shock, and monetary policy shock, respectively.

Table 4: Properties of real exchange rate under infrequent price change model

Setting	HL	UL/HL	QL–HL	AR(1)	Std(e)/Std(C)
Empirical values	2.8	0.42	1.51	0.79	4.97
FL and CPS	2.75	0.18	1.75	0.84	4.96
FL and LRS	2.75	0.18	1.75	0.84	4.64
FL and UIPS	1.25	0	1.75	0.66	99.9
FL and MPS	0.25	0	0.75	0.52	5.02
SL and CPS	2.75	0.25	1.75	0.84	4.97
SL and LRS	3.25	0.31	1.75	0.86	4.64
SL and UIPS	1.25	0	1.75	0.66	91.2
SL and MPS	0.25	0	0.75	0.52	5.02

Note: HL expresses half-life (measured in years), UL/HL expresses up-life over half-life, QL-HL expresses quarter-life minus half-life, AR(1) expresses the first-order autocorrelation of the HP-filtered series, and Std(e)/Std(C) expresses the standard deviation of HP-filtered e (real exchange rate) divided by the standard deviation of HP-filtered C (consumption). The point estimates for AR(1) and Std(e)/Std(C) are calculated by simulating 1,000 data series from each model, in which each data length is 127, and the point estimate is the median value of the resulting distribution exactly following Steinsson (2007). FL and SL mean flexible and staggered loan contracts, respectively. CPS, LRS, UIPS, and MPS denote cost-push shock, loan rate shock, UIP shock, and monetary policy shock, respectively.

Appendix

A   Derivation of the model

The model consists of two symmetric countries.

A.1   Consumer

A.1.1   Cost minimization

The utility of the representative consumer[15] in the home country H is increasing and concave in the aggregate consumption index C_t. The consumption index that consists of bundles of differentiated goods produced by home and foreign firms is expressed as:

$$C_t \equiv \frac{C_{H,t}^{\phi_H} C_{F,t}^{1-\phi_H}}{\phi_H^{\phi_H} \left(1 - \phi_H\right)^{1-\phi_H}},$$

where ϕ_H $(0 \leq \phi_H \leq 1)$ is a preference parameter that expresses the home bias. Here, $C_{H,t}$ and $C_{F,t}$ are consumption subindices of the continuum of differentiated goods produced by firms in the home country and the foreign country, respectively. They are defined as:

$$C_{H,t} \equiv \left[\int_0^1 c_t\left(f\right)^{\frac{\sigma-1}{\sigma}} \mathrm{d}f\right]^{\frac{\sigma}{\sigma-1}},$$

and

$$C_{F,t} \equiv \left[\int_0^1 c_t\left(f^*\right)^{\frac{\sigma-1}{\sigma}} \mathrm{d}f^*\right]^{\frac{\sigma}{\sigma-1}},$$

where $c_t\left(f\right)$ is the demand for a good produced by firm f in the home country and $c_t\left(f^*\right)$ is the demand for a good produced by a firm f^* in the foreign country, where the asterisk denotes foreign variables. It is assumed that there are no trade frictions and consumers in both countries have the same preferences over the differentiated goods. Following the standard cost minimization problem on the aggregate consumption index of home and foreign goods as well as the consumption subindices of the continuum of differentiated goods, we can derive the consumption-based price indices:

$$P_t \equiv P_{H,t}^{\phi_H} P_{F,t}^{1-\phi_H}, \tag{16}$$

---

[15]The same optimal allocations are obtained even by assuming that each homogenous consumer provides differentiated labor supply to each firm.

with

$$P_{H,t} \equiv \left[\int_0^1 p_t\,(f)^{1-\sigma}\,\mathrm{d}f \right]^{\frac{1}{1-\sigma}},$$

and

$$P_{F,t} \equiv \left[\int_0^1 p_t\,(f^*)^{1-\sigma}\,\mathrm{d}f^* \right]^{\frac{1}{1-\sigma}},$$

where $p_t\,(f)$ is the price on $c_t\,(f)$, and $p_t\,(f^*)$ is the price on $c_t\,(f^*)$. Then, we can obtain the following Hicksian demand functions for each differentiated good given the aggregate consumption:

$$c_t(f) = \phi_H \left[\frac{p_t\,(f)}{P_{H,t}} \right]^{-\sigma} \left(\frac{P_{H,t}}{P_t} \right)^{-1} C_t, \tag{17}$$

and

$$c_t(f^*) = (1 - \phi_H) \left[\frac{p_t\,(f^*)}{P_{F,t}} \right]^{-\sigma} \left(\frac{P_{F,t}}{P_t} \right)^{-1} C_t.$$

Here, as in other applications of the Dixit and Stiglitz (1977)[16] aggregator, consumers' allocations across differentiated goods at each time are optimal in terms of cost minimization.

We can derive similar optimality conditions for the foreign counterpart. For example, the consumption-based price index is now expressed as:

$$P_t^* \equiv \left(P_{H,t}^* \right)^{\phi_F} \left(P_{F,t}^* \right)^{1-\phi_F}, \tag{18}$$

and the demand functions for each differentiated good given the aggregate consumption are:

$$c_t^*\,(f) = \phi_F \left[\frac{p_t^*\,(f)}{P_{H,t}^*} \right]^{-\sigma} \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-1} C_t^*, \tag{19}$$

and

$$c_t^*\,(f^*) = (1 - \phi_F) \left[\frac{p_t^*\,(f^*)}{P_{F,t}^*} \right]^{-\sigma} \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-1} C_t^*.$$

A.1.2 Utility maximization

A representative consumer in the home country maximizes the following welfare W_t:

$$W_t = \mathrm{E}_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T) - \int_0^n V(l_T(h))\mathrm{d}h - \int_n^1 V(l_T(\bar{h}))\mathrm{d}\bar{h} \right] \right\},$$

---

[16] Dixit, A., Stiglitz, J., 1977. Monopolistic competition and optimum product diversity. American Economic Review 67, 297-308.

where E_t is the expectation operator conditional on the state of nature at date t. The budget constraint of the consumer is given by:[17]

$$
\begin{aligned}
P_t C_t + \mathrm{E}_t\left[X_{t,t+1} B_{t+1}\right] + D_t \quad &\leq \quad B_t + (1 + i_{t-1}) D_{t-1} + \int_0^n w_t(h) l_t(h) \mathrm{d}h \\
&\quad + \int_n^1 w_t(\bar{h}) l_t(\bar{h}) \mathrm{d}\bar{h} + \Pi_t^B + \Pi_t^F,
\end{aligned} \tag{20}
$$

where B_t is a set of risky assets, D_t is the deposit to private banks, i_t is the nominal deposit interest rate set by a central bank from $t-1$ to t, $w_t(h)$ is the nominal wage for labor supplied from the DFS $l_t(h)$, $w_t(\bar{h})$ is the nominal wage for labor supplied from the IFS $l_t(\bar{h})$, $\Pi_t^B = \int_0^1 \Pi_{t-1}^B(h) \mathrm{d}h$ is the nominal dividend stemming from the ownership of both local and international banks in the home country, $\Pi_t^F = \int_0^1 \Pi_{t-1}^F(f) \mathrm{d}f$ is the nominal dividend from the ownership of the firms in the domestic country, and $X_{t,t+1}$ is the stochastic discount factor. Here, because we assume a complete financial market between the two countries, the consumer in each country can internationally buy and sell the state contingent securities to insure against country specific shocks. Consequently, there only exists a unique discount factor. The relationship between the deposit interest rate and the stochastic discount factor is now expressed as:

$$
\frac{1}{1 + i_t} = \mathrm{E}_t\left[X_{t,t+1}\right]. \tag{21}
$$

Given the optimal allocation of differentiated consumption expenditures, the consumer now optimally chooses the total amount of consumption, risky assets, and deposits in each period. Necessary and sufficient conditions, when the transversality condition is satisfied, for those optimizations are given by:

$$
U_C(C_t) = \beta(1 + i_t)\mathrm{E}_t\left[U_C(C_{t+1})\frac{P_t}{P_{t+1}}\right], \tag{22}
$$

$$
\frac{U_C(C_t)}{U_C(C_{t+1})} = \frac{\beta}{X_{t,t+1}}\frac{P_t}{P_{t+1}}. \tag{23}
$$

Together with equation (21), we see that the condition given by equation (22) defines the intertemporally optimal allocation on aggregate consumption. Then, the standard New

---

[17]For simplicity, we do not explicitly include the amount of contingency claims under complete financial markets.

Keynesian IS curve for the home country, by log-linearizing equation (22) around steady states, is obtained as follows:

$$\hat{C}_t = \mathrm{E}_t \hat{C}_{t+1} - \upsilon\left(\hat{\imath}_t - \mathrm{E}_t \pi_{t+1}\right),$$

where each variable is defined as the log deviation from its steady state where price is flexible and loan rate is constant. We define $\hat{\imath}_t \equiv \ln(1 + i_t)/(1 + i)$ where i is a steady-state value of i_t, $\pi_t \equiv \ln \frac{P_t}{P_{t-1}}$ which is an aggregate inflation in the home country, and $\upsilon \equiv -\frac{U_C}{U_{CC}C} > 0$.

In this model, a representative consumer provides all types of differentiated labor to each firm, and therefore maintains some monopoly power over the determination of his own wage, as in Erceg, Henderson, and Levin (2000). There are two types of labor group: the DFS and the IFS. The workers populated on $[0, n)$ belong to the DFS and other labor populated on $[n, 1]$ belong to the IFS.[18] We assume that each firm hires all types of labor in the same proportion from the two groups. The consumer sets each wage $w_t(h)$ for any h and $w_t(\bar{h})$ for any $\bar{h}$ to maximize its utility subject to the budget constraint given by equation (20) and the labor demand functions given by equations (31) and (32) in the next section. Here, although differentiated labor supply is assumed in this paper, consumers change wages in a flexible manner. Then we have the optimality conditions for labor supply as follows:

$$\frac{w_t(h)}{P_t} = \frac{\epsilon}{\epsilon - 1}\frac{V_l\left[l_t(h)\right]}{U_C(C_t)}, \tag{24}$$

and

$$\frac{w_t(\bar{h})}{P_t} = \frac{\epsilon}{\epsilon - 1}\frac{V_l\left[l_t(\bar{h})\right]}{U_C(C_t)}. \tag{25}$$

As written above, thanks to this heterogeneity in labor supply, we can model the differentiated demand for loans without assuming any restrictions on aggregate loans and loan interest rates. In this paper, consumers supply their labor only for firms, not for banks.

---

[18] The difference of these two groups is characterized by somewhat wider properties of workers, like English speaking or Japanese speaking, though the differences of workers within each group are characterized by narrower properties of workers, like person who has knowledge of accounting in bank or person who has skill of making automobile in plant.

Similar to the above case with cost minimization, we can derive the optimality conditions for the foreign counterpart. For example, the standard New Keynesian IS curve for the foreign country is:

$$\hat{C}_t^* = \mathrm{E}_t \hat{C}_{t+1}^* - \upsilon^*(\hat{i}_t^* - \mathrm{E}_t \pi_{t+1}^*),$$

which is derived from the optimality condition on the foreign asset holdings:

$$U_C^*(C_t^*) = \beta(1 + i_t^*)\mathrm{E}_t \left[U_C^*(C_{t+1}^*) \frac{P_t^*}{P_{t+1}^*} \right],$$

$$\frac{U_C^*(C_t^*)}{U_C^*(C_{t+1}^*)} = \frac{\beta}{X_{t,t+1}^*} \frac{P_t^*}{P_{t+1}^*}. \tag{26}$$

Furthermore, the optimality conditions for labor supply are expressed now as:

$$\frac{w_t^*(h^*)}{P_t^*} = \frac{\epsilon^*}{\epsilon^* - 1} \frac{V_l^* \left[l_t^*(h^*)\right]}{U_C^*(C_t^*)},$$

and

$$\frac{w_t^*(\bar{h}^*)}{P_t^*} = \frac{\epsilon^*}{\epsilon^* - 1} \frac{V_l^* \left[l_t^*(\bar{h}^*)\right]}{U_C^*(C_t^*)}.$$

A.2   Firms

A.2.1   Cost minimization

Firms in both home and foreign countries optimally hire differentiated labor as price takers. This optimal labor allocation is carried out through two-step cost minimization problems. Domestic firm f hires all types of labor from both the DFS and IFS groups. When hiring from the DFS group, γ portion of the labor cost associated with labor type h is financed by borrowing from the local bank h. Then, the first-step cost minimization problem on the allocation of differentiated labor from the DFS is given by

$$\min_{l_t(h,f)} \int_0^n \left[1 + \gamma r_t(h) \right] w_t(h) \, l_t(h, f) \mathrm{d}h,$$

subject to the sub-index regarding labor from DFS to firm f:

$$L_t(f) \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n l_t(h, f)^{\frac{\epsilon-1}{\epsilon}} \, \mathrm{d}h \right]^{\frac{\epsilon}{\epsilon-1}},$$

where $r_t(h)$ is the loan interest rate applied to employ a particular labor type h applied to differentiated labor supply. There $l_t(h, f)$ denotes type of labor h employed by firm f. The

local bank h has some monopoly power over setting loan interest rates. Thus, we assume the monopolistic competition on the loan contracts between banks and firms. The relative demand on differentiated labor is given as follows:

$$l_t(h, f) = \frac{1}{n} L_t \left\{ \frac{[1 + \gamma r_t(h)] w_t(h)}{\Omega_t} \right\}^{-\epsilon}, \tag{27}$$

where

$$\Omega_t \equiv \left\{ \frac{1}{n} \int_0^n \{[1 + \gamma r_t(h)] w_t(h)\}^{1-\epsilon} \, dh \right\}^{\frac{1}{1-\epsilon}}.$$

As a result, we can derive

$$\int_0^n [1 + \gamma r_t(h)] w_t(h) l_t(h, f) \, dh = \Omega_t L_t(f).$$

Through a similar cost minimization problem, we can derive the relative demand for each type of differentiated labor from the IFS as

$$l_t(\bar{h}, f) = \frac{1}{1-n} \bar{L}_t \left\{ \frac{[1 + \bar{\gamma} r_t^*(\bar{h})] w_t(\bar{h})}{\overline{\Omega}_t} \right\}^{-\epsilon}, \tag{28}$$

where

$$\overline{\Omega}_t \equiv \left\{ \frac{1}{1-n} \int_n^1 \{[1 + \bar{\gamma} r_t^*(\bar{h})] w_t(\bar{h})\}^{1-\epsilon} \, d\bar{h} \right\}^{\frac{1}{1-\epsilon}}.$$

Then

$$\int_n^1 [1 + \bar{\gamma} r_t^*(\bar{h})] w_t(\bar{h}) l_t(\bar{h}, f) \, d\bar{h} = \bar{\Omega}_t \bar{L}_t(f).$$

According to the above two optimality conditions, the firms optimally choose the allocation of differentiated workers between the two groups. Because firms have some preference n to hire workers from the DFS and $(1 - n)$ to hire workers from the IFS, the second-step cost minimization problem describing the allocation of differentiated labor between these two groups is given by

$$\min_{L_t, \bar{L}_t} \Omega_t L_t(f) + \bar{\Omega}_t \bar{L}_t(f),$$

subject to the aggregate labor index:

$$\tilde{L}_t(f) \equiv \frac{[L_t(f)]^n [\bar{L}_t(f)]^{1-n}}{n^n (1-n)^{1-n}}.$$

Then, the relative demand functions for each differentiated type of labor are derived as follows:

$$L_t(f) = n \tilde{L}_t(f) \left(\frac{\Omega_t}{\tilde{\Omega}_t} \right)^{-1}, \tag{29}$$

6

$$\bar{L}_t(f) = (1-n)\tilde{L}_t(f)\left(\frac{\bar{\Omega}_t}{\tilde{\Omega}_t}\right)^{-1}, \tag{30}$$

and

$$\tilde{\Omega}_t \equiv \Omega_t^n \bar{\Omega}_t^{1-n}.$$

Therefore, we can obtain the following equations:

$$\Omega_t L_t(f) + \bar{\Omega}_t \bar{L}_t(f) = \tilde{\Omega}_t \tilde{L}_t(f),$$

$$l_t(h,f) = \left\{\frac{[1+\gamma r_t(h)]\, w_t(h)}{\Omega_t}\right\}^{-\epsilon}\left(\frac{\Omega_t}{\tilde{\Omega}_t}\right)^{-1}\tilde{L}_t(f), \tag{31}$$

and

$$l_t(\bar{h},f) = \left\{\frac{[1+\gamma r_t(\bar{h})]\, w_t(\bar{h})}{\bar{\Omega}_t}\right\}^{-\epsilon}\left(\frac{\bar{\Omega}_t}{\tilde{\Omega}_t}\right)^{-1}\tilde{L}_t(f), \tag{32}$$

from equations (27), (28), (29), and (30). We can now clearly see that the demand for each differentiated worker depends on wages and loan interest rates, given the total demand for labor.

Finally, by the assumption that the firms finance part of the labor costs by loans, we can derive

$$\begin{aligned} q_t(h,f) &= \gamma w_t(h)\, l_t(h,f) \\ &= \gamma w_t(h)\left\{\frac{[1+\gamma r_t(h)]\, w_t(h)}{\Omega_t}\right\}^{-\epsilon}\left(\frac{\Omega_t}{\tilde{\Omega}_t}\right)^{-1}\tilde{L}_t(f), \end{aligned}$$

and

$$\begin{aligned} q_t(\bar{h},f) &= \bar{\gamma} w_t(\bar{h})\, l_t(\bar{h},f) \\ &= \bar{\gamma} w_t(\bar{h})\left\{\frac{[1+\bar{\gamma} r_t^*(\bar{h})]\, w_t(\bar{h})}{\bar{\Omega}_t}\right\}^{-\epsilon}\left(\frac{\bar{\Omega}_t}{\tilde{\Omega}_t}\right)^{-1}\tilde{L}_t(f). \end{aligned}$$

$q_t(h,f)$ and $q_t(\bar{h},f)$ denote amounts of loan borrowed by firm f to the labor types h and $\bar{h}$, respectively. These conditions demonstrate that the demands for each differentiated loan also depend on the wages and loan interest rates, given the total labor demand.

We can obtain similar conditions for the foreign country. For example

$$l_t(h^*,f^*) = \left\{\frac{[1+\gamma^* r_t^*(h^*)]\, w_t^*(h^*)}{\Omega_t^*}\right\}^{-\epsilon^*}\left(\frac{\Omega_t^*}{\tilde{\Omega}_t^*}\right)^{-1}\tilde{L}_t^*(f^*),$$

and

$$l_t\left(\bar{h}^*, f^*\right) = \left\{\frac{\left[1 + \bar{\gamma}^* r_t\left(\bar{h}^*\right)\right] w_t^*\left(\bar{h}^*\right)}{\bar{\Omega}_t^*}\right\}^{-\epsilon^*} \left(\frac{\bar{\Omega}_t^*}{\tilde{\Omega}_t^*}\right)^{-1} \tilde{L}_t^*\left(f^*\right),$$

where

$$L_t^*\left(f^*\right) \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n l_t^*(h^*, f)^{\frac{\epsilon^*-1}{\epsilon^*}} dh^*\right]^{\frac{\epsilon^*}{\epsilon^*-1}},$$

$$\bar{L}_t^*\left(f^*\right) \equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\epsilon}} \int_n^1 l_t^*\left(\bar{h}^*, f\right)^{\frac{\epsilon^*-1}{\epsilon^*}} d\bar{h}^*\right]^{\frac{\epsilon^*}{\epsilon^*-1}},$$

$$\tilde{L}_t^*\left(f^*\right) \equiv \frac{\left[L_t^*\left(f^*\right)\right]^n \left[\bar{L}_t^*\left(f^*\right)\right]^{1-n}}{n^n(1-n)^{1-n}},$$

$$\Omega_t^* \equiv \left\{\frac{1}{n} \int_0^n \left\{\left[(1 + \gamma^* r_t^*\left(h^*\right)\right] w_t^*\left(h^*\right)\right\}^{1-\epsilon^*} dh^*\right\}^{\frac{1}{1-\epsilon^*}},$$

$$\bar{\Omega}_t^* \equiv \left\{\frac{1}{1-n} \int_n^1 \left\{\left[1 + \bar{\gamma}^* r_t\left(\bar{h}^*\right)\right] w_t\left(\bar{h}^*\right)\right\}^{1-\epsilon^*} d\bar{h}^*\right\}^{\frac{1}{1-\epsilon^*}},$$

and

$$\tilde{\Omega}_t^* \equiv \left(\Omega_t^*\right)^n \left(\bar{\Omega}_t^*\right)^{1-n}.$$

Furthermore, loan demand conditions are

$$q_t^*\left(h^*, f^*\right) = \gamma^* w_t^*\left(h^*\right) l_t^*\left(h^*\right) = \gamma^* w_t^*\left(h^*\right) \left\{\frac{\left[1 + \gamma^* r_t^*\left(h^*\right)\right] w_t^*\left(h^*\right)}{\Omega_t^*}\right\}^{-\epsilon} \left(\frac{\Omega_t^*}{\tilde{\Omega}_t^*}\right)^{-1} \tilde{L}_t^*\left(f^*\right),$$

and

$$
\begin{aligned}
q_t\left(\bar{h}^*, f^*\right) &= \bar{\gamma}^* w_t^*\left(\bar{h}^*\right) l_t^*\left(\bar{h}^*\right) \\
&= \bar{\gamma}^* w_t^*\left(\bar{h}^*\right) \left\{\frac{\left[1 + \bar{\gamma}^* r_t\left(\bar{h}^*\right)\right] w_t^*\left(\bar{h}^*\right)}{\bar{\Omega}_t^*}\right\}^{-\epsilon} \left(\frac{\bar{\Omega}_t^*}{\tilde{\Omega}_t^*}\right)^{-1} \tilde{L}_t^*\left(f^*\right).
\end{aligned}
$$

As aggregate labor demand conditions, we can obtain

$$\tilde{L}_t = \int_0^1 \tilde{L}_t\left(f\right) df,$$

and

$$\tilde{L}_t^* = \int_0^1 \tilde{L}_t^*\left(f^*\right) df^*.$$

A.2.2 Price setting (profit maximization)

As is standard in the New Keynesian model following the Calvo (1983) - Yun (1996) framework, each firm f resets its price with probability $(1 - \alpha)$ and maximizes the present discounted value of profit, which is given by

$$\mathrm{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} X_{t,T} \left[p_t(f) c_{t,T}(f) + S_T p_t^*(f) c_{t,T}^*(f) - \tilde{\Omega}_T \tilde{L}_T(f) \right], \qquad (33)$$

where we use equations (17) and (19) for any time t. Here, the firm sets $p_t(f)$ and $p_t^*(f)$ separately under the pricing-to-market assumption. There S_t is the nominal exchange rate. The present discounted value of the profit given by equation (33) is further transformed into

$$\mathrm{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} X_{t,T} \left\{ \begin{array}{c} \phi_H p_t(f) \left[\frac{p_t(f)}{P_{H,T}} \right]^{-\sigma} \left[\frac{P_{H,T}}{P_T} \right]^{-1} C_T \\ + \phi_F S_T p_t^*(f) \left[\frac{p_t^*(f)}{P_{H,T}^*} \right]^{-\sigma} \left[\frac{P_{H,T}^*}{P_T^*} \right]^{-1} C_T^* - \tilde{\Omega}_T \tilde{L}_T(f) \end{array} \right\}.$$

It should be noted that price setting is independent of the loan interest rate setting of private banks.

The optimal price setting of $\bar{p}_t(f)$ under the situation in which managers can reset their prices with probability $(1 - \alpha)$ is given by

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{U_C(C_T)}{P_T} y_{t,T}(f) \left[\frac{\sigma - 1}{\sigma} \bar{p}_t(f) - \tilde{\Omega}_T \frac{\partial \tilde{L}_T(f)}{\partial y_{t,T}(f)} \right] = 0, \qquad (34)$$

where we substitute equation (23). By further substituting equations (24) and (25) into equation (34), equation (34) can be now rewritten as

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} U_C(C_T) y_{t,T}(f) \frac{P_{H,T}}{P_T} \left[\frac{\sigma - 1}{\sigma} \frac{\bar{p}_t(f)}{P_{H,t}} \frac{P_{H,t}}{P_{H,T}} - \frac{\epsilon}{\epsilon - 1} Z_{t,T}(f) \right] = 0, \qquad (35)$$

where

$$
\begin{aligned}
Z_{t,T}(f) &= \left\{ \left(\frac{1}{n} \right) \int_0^n [1 + \gamma r_t(h)]^{1-\epsilon} \left\{ \frac{P_T}{P_{H,T}} \frac{V_l[l_T(h)]}{U_C(C_t)} \frac{\partial \tilde{L}_T(f)}{\partial y_{t,T}(f)} \right\}^{1-\epsilon} \mathrm{d}h \right\}^{\frac{n}{1-\epsilon}} \\
&\quad \times \left\{ \left(\frac{1}{1-n} \right) \int_n^1 [1 + \bar{\gamma} r_t^*(\bar{h})]^{1-\epsilon} \left\{ \frac{P_T}{P_{H,T}} \frac{V_l[l_T(\bar{h})]}{U_C(C_t)} \frac{\partial \tilde{L}_T(f)}{\partial y_{t,T}(f)} \right\}^{1-\epsilon} \mathrm{d}\bar{h} \right\}^{\frac{1-n}{1-\epsilon}}.
\end{aligned}
$$

By log-linearizing equation (35), we derive

$$\frac{1}{1-\alpha\beta} \widehat{\bar{p}}_t(f) = \mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\Theta_1 \hat{R}_{H,T} + \Theta_2 \hat{R}_{H,T}^* + \widehat{mc}_{H,t,T}(f) + \sum_{\tau=t+1}^{T} \pi_{H,\tau} \right], \qquad (36)$$

where $\Theta_1 \equiv n\frac{\gamma(1+R_H)}{1+\gamma R_H}$ and $\Theta_2 \equiv (1-n)\frac{\bar{\gamma}(1+R_H^*)}{1+\bar{\gamma}R_H^*}$ are positive parameters, and we define the real marginal cost as

$$\widehat{mc}_{H,t,T}(f) \equiv \int_0^n \widehat{mc}_{H,t,T}(h,f)\,dh + \int_n^1 \widehat{mc}_{H,t,T}(\bar{h},f)\,d\bar{h},$$

where

$$mc_{H,t,T}(h,f) \equiv \frac{P_T}{P_{H,T}}\frac{V_l[l_T(h)]}{U_Y(C_T)}\frac{\partial \tilde{L}_{t,T}(f)}{\partial y_{t,T}(f)},$$

and

$$mc_{H,t,T}(\bar{h},f) \equiv \frac{P_T}{P_{H,T}}\frac{V_l[l_T(\bar{h})]}{U_Y(C_T)}\frac{\partial \tilde{L}_{t,T}(f)}{\partial y_{t,T}(f)}.$$

We also define

$$R_{H,t} \equiv \frac{1}{n}\int_0^n r_t(h)\,dh,$$

$$R_{H,t}^* \equiv \frac{1}{1-n}\int_n^1 r_t^*(\bar{h})\,d\bar{h},$$

$$\tilde{p}_t(f) \equiv \frac{\bar{p}_t(f)}{P_{H,t}},$$

and

$$\pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}},$$

where $R_{H,t}$ is the aggregate loan interest rate by local banks in the home country, $R_{H,t}^*$ is the aggregate loan interest rate by international banks in the home country, and $\pi_{H,t}$ is inflation of goods produced and consumed in the home country. Then, equation (36) can be transformed into

$$\frac{1}{1-\alpha\beta}\widehat{\tilde{p}}_t(f) = \mathrm{E}_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left[(1+\eta_2\sigma)^{-1}\left(\widehat{mc}_{H,T} + \Theta_1\hat{R}_{H,T} + \Theta_2\hat{R}_{H,T}^*\right) + \sum_{\tau=t+1}^{T}\pi_{H,\tau}\right], \tag{37}$$

where we make use of the relationship:

$$\widehat{mc}_{H,t,T}(f) = \widehat{mc}_{H,T} - \eta_2\sigma\left[\widehat{\tilde{p}}_t(f) - \sum_{\tau=t+1}^{T}\pi_{H,\tau}\right],$$

where $\eta_2 \equiv -\frac{f_{YY}^{-1}(Y_H)Y_H}{f_Y^{-1}(Y_H)}$. We define $\hat{R}_{H,t} \equiv \ln(1+R_{H,t})/(1+R_H)$. We further denote the average real marginal cost as:

$$\widehat{mc}_{H,T} \equiv \int_0^n \widehat{mc}_{H,T}(h)\,dh + \int_n^1 \widehat{mc}_{H,T}(\bar{h})\,d\bar{h},$$

where

$$mc_{H,T}(h) \equiv \frac{P_T}{P_{H,T}} \frac{V_l[l_T(h)]}{U_Y(C_T)} \frac{\partial \tilde{L}_T}{\partial Y_{H,T}},$$

and

$$mc_{H,T}(\bar{h}) \equiv \frac{P_T}{P_{H,T}} \frac{V_l[l_T(\bar{h})]}{U_Y(C_T)} \frac{\partial \tilde{L}_T}{\partial Y_{H,T}}.$$

The point is that unit marginal cost is the same for all firms in the situation where each firm uses all types of labor and loans with the same proportion. Thus, all firms set the same price if they have a chance to reset their prices at time t.

In the Calvo (1983) - Yun (1996) setting, the evolution of the aggregate price index P is described by the following law of motion:

$$\int_0^1 p_t(f)^{1-\sigma} df = \alpha \int_0^1 p_{t-1}(f)^{1-\sigma} \, \mathrm{d}f + (1-\alpha) \int_0^1 \bar{p}_t(f)^{1-\sigma} \, \mathrm{d}f,$$

$$\implies P_{H,t}^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha)(\bar{p}_t)^{1-\sigma}, \tag{38}$$

where

$$P_{H,t}^{1-\sigma} \equiv \int_0^1 p_t(f)^{1-\sigma} \, \mathrm{d}f,$$

and

$$\bar{p}_t^{1-\sigma} \equiv \int_0^1 \bar{p}_t(f)^{1-\sigma} \, \mathrm{d}f.$$

The current aggregate price is given by the weighted average of changed and unchanged prices. Because the chances of resetting prices are randomly assigned to each firm with equal probability, an aggregate price change at time t should be evaluated by an average of price changes by all firms. By log-linearizing equation (38), together with equation (37), we can derive the following New Keynesian Phillips curve:

$$\pi_{H,t} = \kappa \left(\widehat{mc}_{H,t} + \Theta_1 \hat{R}_{H,t} + \Theta_2 \hat{R}_{H,t}^* \right) + \beta \mathrm{E}_t \pi_{H,t+1}, \tag{39}$$

where the slope coefficient $\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\eta_2\sigma)}$ is a positive parameter. This is quite similar to the standard New Keynesian Phillips curve, but contains loan interest rates as cost components.

Similarly, regarding the optimal price setting of $\bar{p}_t^*(f)$, we can derive

$$\pi_{H,t}^* = \kappa \left(\widehat{mc}_{H,t}^* + \Theta_1 \widehat{R}_{H,t} + \Theta_2 \widehat{R}_{H,t}^* - \hat{S}_t \right) + \beta \mathrm{E}_t \pi_{H,t+1}^*, \tag{40}$$

where $\widehat{mc}_{H,t}^*$ is given by replacing P by P^* in $\widehat{mc}_{H,t}$. $\pi_{H,t}^*$ is inflation of goods produced in the home country and consumed in the foreign country.

Furthermore, from optimal price settings by firm f^* in the foreign country, the New Keynesian Phillips curves are derived as

$$\pi_{F,t}^* = \kappa^* \left(\widehat{mc}_{F,t}^* + \Theta_1^* \hat{R}_{F,t}^* + \Theta_2^* \hat{R}_{F,t} \right) + \beta \mathrm{E}_t \pi_{F,t+1}^*, \tag{41}$$

$$\pi_{F,t} = \kappa^* \left(\widehat{mc}_{F,t} + \Theta_1^* \hat{R}_{F,t}^* + \Theta_2^* \hat{R}_{F,t} + \hat{S}_t \right) + \beta \mathrm{E}_t \pi_{F,t+1}, \tag{42}$$

where $\kappa^* \equiv \frac{(1-\alpha^*)(1-\alpha^*\beta)}{\alpha^*(1+\eta_2^*\sigma)}$, $\Theta_1^* \equiv n\frac{\gamma^*(1+R_F^*)}{1+\gamma^* R_F^*}$, $\Theta_2^* \equiv (1-n)\frac{\bar{\gamma}^*(1+R_F)}{1+\bar{\gamma}^* R_F}$, and $\eta_2^* \equiv -\frac{f_{YY}^{-1}(Y_F)Y_F}{f_Y^{-1}(Y_F)}$. $R_{F,t}$ is the aggregate loan interest rate by international banks in the foreign country, $R_{F,t}^*$ is the aggregate loan interest rate by local banks in the foreign country, $\pi_{F,t}^*$ is inflation of goods produced and consumed in the foreign country, and $\pi_{F,t}$ is inflation of goods produced in the foreign country and consumed in the home country.

As for CPI inflation rates, from equations (16) and (18), we can derive the following log-linearized relations as

$$\pi_t = \phi_H \pi_{H,t} + (1 - \phi_H) \pi_{F,t},$$

and

$$\pi_t^* = \phi_F \pi_{H,t}^* + (1 - \phi_F) \pi_{F,t}^*,$$

under the assumption of $\phi_H = 1 - \phi_F$; namely, symmetric home bias. Then, by considering the weighted average of equations (39) and (42) and the weighted average of equations (40) and (41), respectively, we can finally obtain the following two New Keynesian Phillips curves for consumer prices:

$$
\begin{aligned}
\pi_t = {} & \phi_H \kappa \left(\Theta_1 \hat{R}_{H,t} + \Theta_2 \hat{R}_{H,t}^* \right) + (1 - \phi_H) \kappa^* \left(\Theta_1^* \hat{R}_{F,t}^* + \Theta_2^* \hat{R}_{F,t} \right) \\
& + \bar{\kappa} \left[\phi_H \hat{C}_t^W + (1 - \phi_H) \hat{C}_t^{W*} \right] + \varsigma \hat{e}_t + \beta \mathrm{E}_t \pi_{t+1} + m_t,
\end{aligned} \tag{43}
$$

$$
\begin{aligned}
\pi_t^* = {} & (1 - \phi_H) \kappa \left(\Theta_1 \hat{R}_{H,t} + \Theta_2 \hat{R}_{H,t}^* \right) + \phi_H \kappa^* \left(\Theta_1^* \hat{R}_{F,t}^* + \Theta_2^* \hat{R}_{F,t} \right) \\
& + \bar{\kappa}^* \left[\phi_F \hat{C}_t^W + (1 - \phi_F) \hat{C}_t^{W*} \right] - \varsigma^* \hat{e}_t + \beta \mathrm{E}_t \pi_{t+1}^* + m_t^*,
\end{aligned} \tag{44}
$$

12

where $\bar{\kappa} \equiv \kappa \left(\eta_1 \mu_1^{-1} + \eta_2 + \upsilon^{-1} \right) > 0$, $\bar{\kappa}^* \equiv \kappa^* \left[\eta_1^* (\mu_1^*)^{-1} + \eta_2^* + (\upsilon^*)^{-1} \right] > 0$, $\varsigma \equiv 2\kappa\phi_H (1 - \phi_H)$, and $\varsigma^* \equiv 2\kappa^*\phi_F (1 - \phi_F)$, where $\mu_1 \equiv \frac{f_L(\tilde{L})\tilde{L}}{f(\tilde{L})}$, $\eta_1 \equiv \frac{V_{ll}l}{V_l}$, $\mu_1^* \equiv \frac{f_L(\tilde{L}^*)\tilde{L}^*}{f(\tilde{L}^*)}$, $\eta_1^* \equiv \frac{V_{ll}^* l^*}{V_l^*}$, and

$$\hat{C}_t^W = \phi_H \hat{C}_t + (1 - \phi_H) \hat{C}_t^*,$$

and

$$\hat{C}_t^{W*} = \phi_F \hat{C}_t + (1 - \phi_F) \hat{C}_t^*.$$

m_t and m_t^* are cost-push shocks in home and foreign countries from productivity shocks, respectively. Here, we assume a linear production function as $y_t(f) = A_t \tilde{L}_t(f)$, where A_t is a common productivity shock among domestic firms. We have similar production functions for firms in the foreign country. As shown in Appendix of Steinsson (2007), m_t and m_t^* are given by weighted average according to home bias of the home and foreign productivity shocks. In the simulation, we assume productivity shocks in two countries, satisfying $m_t > 0$ and $m_t^* = 0$ as in Steinsson (2007).

The definition of real exchange rate is given by

$$e_t \equiv S_t \frac{P_t^*}{P_t}.$$

When goods price is flexible, equations (43) and (44) are reduced to

$$
\begin{aligned}
0 = {} & \phi_H \left(\Theta_1 \hat{R}_{H,t} + \Theta_2 \hat{R}_{H,t}^* \right) + (1 - \phi_H) \left(\Theta_1^* \hat{R}_{F,t}^* + \Theta_2^* \hat{R}_{F,t} \right) \\
& + \upsilon^{-1} \left[\phi_H \hat{C}_t^W + (1 - \phi_H) \hat{C}_t^{W*} \right] + 2\phi_H (1 - \phi_H) \hat{e}_t + m_t,
\end{aligned}
$$

$$
\begin{aligned}
0 = {} & (1 - \phi_H) \left(\Theta_1 \hat{R}_{H,t} + \Theta_2 \hat{R}_{H,t}^* \right) + \phi_H \left(\Theta_1^* \hat{R}_{F,t}^* + \Theta_2^* \hat{R}_{F,t} \right) \\
& + (\upsilon^*)^{-1} \left[\phi_F \hat{C}_t^W + (1 - \phi_F) \hat{C}_t^{W*} \right] - 2\phi_F (1 - \phi_F) \hat{e}_t + m_t^*,
\end{aligned}
$$

since firms change goods prices in each period to maximize a following period profit with respect to $p_t(f)$ and $p_t^*(f)$ as

$$
\left\{
\begin{aligned}
& \phi_H p_t(f) \left[\frac{p_t(f)}{P_{H,t}} \right]^{-\sigma} \left[\frac{P_{H,t}}{P_t} \right]^{-1} C_t \\
& + \phi_F S_t p_t^*(f) \left[\frac{p_t^*(f)}{P_{H,t}^*} \right]^{-\sigma} \left[\frac{P_{H,t}^*}{P_t^*} \right]^{-1} C_t^* - \tilde{\Omega}_t \tilde{L}_t(f)
\end{aligned}
\right\},
$$

where we finally have $\widehat{\tilde{p}}_t(f) = 0$.

When deriving equations (43) and (44), we assume that marginal cost elasticities of productions are zero, namely that the production function is linear, and $\kappa = \kappa^*$ and $\upsilon = \upsilon^*$, as is demonstrated in Table 2.[19] Furthermore, in this transformation, we use the optimality conditions on bond holdings:

$$\hat{C}_t - \hat{C}_t^* = \upsilon \hat{e}_t. \tag{45}$$

Equation (45) is obtained from two Euler equations in the domestic and foreign countries, namely, equations (23) and (26), under the internationally complete financial market.[20] We also use the log deviation of the real exchange rate from its steady state value as

$$\pi_t^* - \pi_t + \triangle \hat{S}_t = \triangle \hat{e}_t.$$

A.3 Private banks

A local bank lends only to firms when they hire labor from the DFS. However, an international bank only provides a loan to firms when they hire labor from the IFS.

First, we describe the optimization problem of an international bank in the home country. Here, the international bank takes on the exchange rate risk inherent in its loans. Each international bank can reset loan interest rates with probability $(1 - \bar{\varphi}^*)$ following the Calvo (1983) - Yun (1996) framework. Under the segmented environment stemming from differences in labor supply, private banks can set different loan interest rates depending on the types of labor. As a consequence, the private bank holds some monopoly power over the loan interest rate to firms. Therefore, the international bank $\bar{h}^*$ chooses the loan interest rate $r_t\left(\bar{h}^*\right)$ to maximize the present discounted value of profit:

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\bar{\varphi}^*)^{T-t} X_{t,T} q_{t,T}\left(\bar{h}^*, f^*\right) \left\{ S_{T+1}\left[1 + r_t\left(\bar{h}^*\right)\right] - S_T\left(1 + i_T\right) \right\}.$$

The optimal loan condition is now given by

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\varphi\beta)^{T-t} \frac{P_t}{P_T} \frac{U_C\left(C_T\right)}{U_C\left(C_t\right)} q_{t,T}^*\left(\bar{h}^*\right) \left\{ \begin{array}{c} S_{T+1}\left[1 + \bar{\gamma}^* r_t\left(\bar{h}^*\right)\right] \\ -\epsilon^* \bar{\gamma}^* \left\{ S_{T+1}\left[1 + r_t\left(\bar{h}^*\right)\right] - S_T\left(1 + i_T\right) \right\} \end{array} \right\} = 0. \tag{46}$$

---

[19] $\eta_2 = 0$, $\eta_1 = 0$, $\eta_2^* = 0$, and $\eta_1^* = 0$. If the marginal cost elasticity of production is nonzero, the Phillips curves become more complicated. However, the qualitative outcomes of simulations do not change even under a positive marginal cost elasticity of production.

[20] Following the convention, we assume $e_0 = 1$.

Because the international private banks that have the opportunity to reset their loan interest rates will set the same loan interest rate, the solution of $r_t \left(\bar{h}^* \right)$ in equation (46) is expressed only with $\bar{r}_t$. In this case, we have the following evolution of the aggregate loan interest rate index by international banks in the home country:

$$1 + R_{F,t} = \bar{\varphi}^* \left(1 + R_{F,t-1} \right) + \left(1 - \bar{\varphi}^* \right) \left(1 + \bar{r}_t \right). \tag{47}$$

By log-linearizing equations (46) and (47), we can determine the relationship between the loan and deposit interest rate as follows:

$$\hat{R}_{F,t} = \bar{\lambda}_1^* \mathrm{E}_t \hat{R}_{F,t+1} + \bar{\lambda}_2^* \hat{R}_{F,t-1} + \bar{\lambda}_3^* \left(\hat{\imath}_t - \mathrm{E}_t \triangle \hat{S}_{t+1} \right),$$

where $\bar{\lambda}_1^* \equiv \frac{\bar{\varphi}^* \beta}{1+(\bar{\varphi}^*)^2 \beta}$, $\bar{\lambda}_2^* \equiv \frac{\bar{\varphi}^*}{1+(\bar{\varphi}^*)^2 \beta}$, and $\bar{\lambda}_3^* \equiv \frac{1-\bar{\varphi}^*}{1+(\bar{\varphi}^*)^2 \beta} \frac{\epsilon^*}{\epsilon^*-1} \frac{(1-\beta \bar{\varphi}^*)(1+i)}{1+R_F}$ are positive parameters. This equation describes the foreign country's loan interest rate (supply) curve by the international bank in the home country.

Similarly, the optimization problem for a local bank h in the home country is given by

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\varphi)^{T-t} X_{t,T} q_{t,T} \left(h, f \right) \left\{ r_t \left(h \right) - M_T i_T \right\}.$$

M_t is an exogenous disturbance for net interest rate in collecting deposits, where M_t is one in the steady state. Cúrdia and Woodford (2009) and Gerali, Neri, Sessa, and Signoretti (2009) emphasize importance of loan rates shock from a default risk and a risk premium for borrower. Through the optimization problem of a local bank h in the home country, we can obtain the relationship between loan and deposit interest rates as follows:

$$\hat{R}_{H,t} = \lambda_1 \mathrm{E}_t \hat{R}_{H,t+1} + \lambda_2 \hat{R}_{H,t-1} + \lambda_3 \hat{\imath}_t + z_t,$$

where $\lambda_1 \equiv \frac{\varphi \beta}{1+\varphi^2 \beta}$, $\lambda_2 \equiv \frac{\varphi}{1+\varphi^2 \beta}$, and $\lambda_3 \equiv \frac{1-\varphi}{1+\varphi^2 \beta} \frac{\epsilon}{\epsilon-1} \frac{(1-\beta \varphi)(1+i)}{1+R_H}$ are positive parameters. z_t is loan rates shock for the loan rate set by the home local bank to a home firm, which comes from M_t. This equation describes the home country's loan interest rate (supply) curve by the local bank in the home country.

The domestic market loan clearing conditions are expressed as

$$q_{t,T}(h) = \int_0^1 q_{t,T}(h, f) \mathrm{d}f,$$

$$q_{t,T}^* \left(\bar{h}^* \right) = \int_0^1 q_{t,T}^* \left(\bar{h}^*, f^* \right) \mathrm{d}f^*,$$

$$\int_0^n q_{t,T}(h) \mathrm{d}h = n D_T,$$

and

$$S_T \int_n^1 q_{t,T}^* \left(\bar{h}^* \right) \mathrm{d}\bar{h}^* = (1 - n) D_T.$$

For international banks in the foreign country, we can derive the following loan interest rate curve:

$$\hat{R}_{H,t}^* = \bar{\lambda}_1 \mathrm{E}_t \hat{R}_{H,t+1}^* + \bar{\lambda}_2 \hat{R}_{H,t-1}^* + \bar{\lambda}_3 \left(\mathrm{E}_t \triangle \hat{S}_{t+1} + \hat{\imath}_t^* \right),$$

where $\bar{\lambda}_1 \equiv \frac{\bar{\varphi}\beta}{1+(\bar{\varphi})^2\beta}$, $\bar{\lambda}_2 \equiv \frac{\bar{\varphi}}{1+(\bar{\varphi})^2\beta}$, and $\bar{\lambda}_3 \equiv \frac{1-\bar{\varphi}}{1+(\bar{\varphi})^2\beta} \frac{\epsilon}{\epsilon-1} \frac{(1-\beta\bar{\varphi})(1+i^*)}{1+R_H^*}$ are positive parameters. This equation describes the home country's loan interest rate (supply) curve by the international bank in the foreign country. Similarly, for local banks in the foreign country, we can obtain

$$\hat{R}_{F,t}^* = \lambda_1^* \mathrm{E}_t \hat{R}_{F,t+1}^* + \lambda_2^* \hat{R}_{F,t-1}^* + \lambda_3^* \hat{\imath}_t^*,$$

where $\lambda_1^* \equiv \frac{\varphi^*\beta}{1+(\varphi^*)^2\beta}$, $\lambda_2^* \equiv \frac{\varphi^*}{1+(\varphi^*)^2\beta}$, and $\lambda_3^* \equiv \frac{1-\varphi^*}{1+(\varphi^*)^2\beta} \frac{\epsilon^*}{\epsilon^*-1} \frac{(1-\beta\varphi^*)(1+i^*)}{1+R_F^*}$ are positive parameters. This equation describes the foreign country's loan interest rate (supply) curve by the local bank in the foreign country. It should be noted that the four types of private bank in both the home and foreign countries can have different probabilities for resetting their loan interest rates. The foreign market loan clearing conditions are expressed as

$$q_{t,T}^* \left(h^* \right) = \int_0^1 q_{t,T}^*(h^*, f^*) \mathrm{d}f^*,$$

$$q_{t,T} \left(\bar{h} \right) = \int_0^1 q_{t,T}^* \left(\bar{h}, f \right) \mathrm{d}f,$$

$$\int_0^n q_{t,T}^* \left(h^* \right) \mathrm{d}h^* = n D_T^*,$$

and

$$\frac{1}{S_T} \int_n^1 q_{t,T} \left(\bar{h} \right) \mathrm{d}\bar{h} = (1 - n) D_T^*.$$

B   Impulse response

We show the impulse response to a loan rate shock for the loan rate set by the home local bank to a home firm. Figure A1 shows the case for the model with a flexible loan rate

and flexible goods price settings, and corresponds to the third row of Table 3. Figure A2 shows the case for the model with a sticky loan rate and flexible goods price settings, and corresponds to the seventh row of Table 3. Figure A3 shows the case for the model with a flexible loan rate and sticky goods price settings, and corresponds to the third row of Table 4. Figure A4 shows the case for a model with a sticky loan rate and sticky goods price settings, and corresponds to the seventh row of Table 4.

[Figure A1: Impulse response to a loan rate shock for the loan rate set by the home local bank for a home firm in the model with a flexible loan rate and flexible goods price settings]

[Figure A2: Impulse response to a loan rate shock for the loan rate set by the home local bank for a home firm in the model with a sticky loan rate and flexible goods price settings]

[Figure A3: Impulse response to a loan rate shock for the loan rate set by the home local bank for a home firm in the model with a flexible loan rate and sticky goods price settings]

[Figure A4: Impulse response to a loan rate shock for the loan rate set by the home local bank for a home firm in the model with a sticky loan rate and sticky goods price settings]
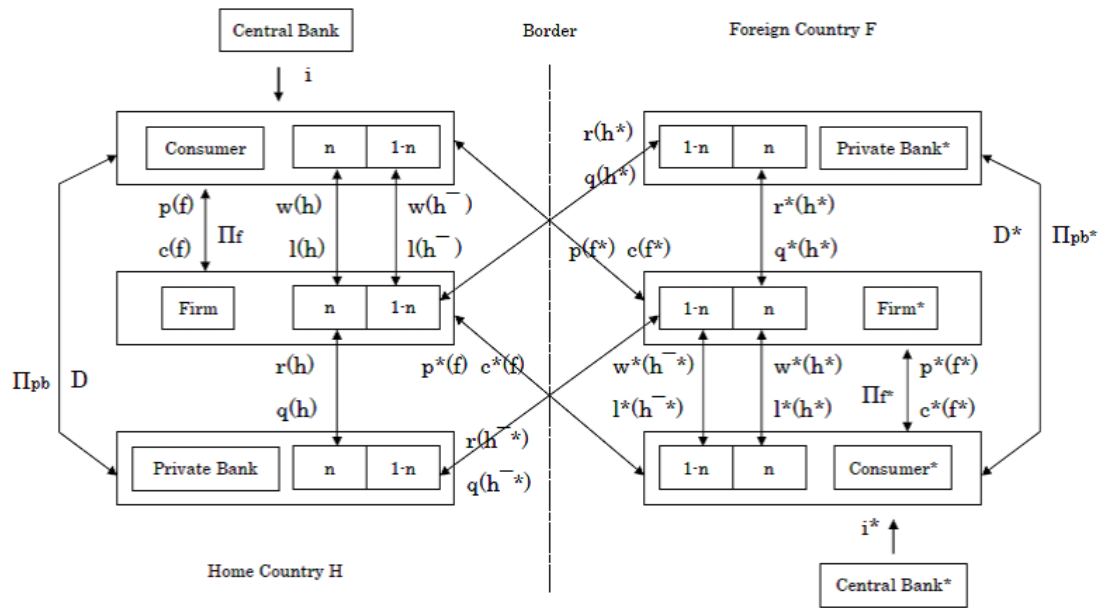
Figure 1: Outline of the model

Central Bank

Border

Foreign Country F

i

Consumer | n | $1-n$

$r(h^*)$
$q(h^*)$

$1-n$ | n | Private Bank*

$p(f)$ $w(h)$ $w(\bar{h})$

Π_f

$c(f)$ $l(h)$ $l(\bar{h})$

$r^*(h^*)$

D^* Π_{pb^*}

$p(f^*)$ $c(f^*)$

$q^*(h^*)$

Firm | n | $1-n$

$1-n$ | n | Firm*

$r(h)$ $p^*(f)$ $c^*(f)$

$w^*(\bar{h}^*)$ $w^*(h^*)$ $p^*(f^*)$

Π_{pb} D

$q(h)$

$l^*(\bar{h}^*)$ $l^*(h^*)$

Π_{f^*} $c^*(f^*)$

Private Bank | n | $1-n$

$r(\bar{h}^*)$
$q(\bar{h}^*)$

$1-n$ | n | Consumer*

Home Country H

i^*

Central Bank*

Figure A1: Impulse response to a loan rate shock for the loan rate set by the home local bank for a home firm in the model with a flexible loan rate and flexible goods price settings
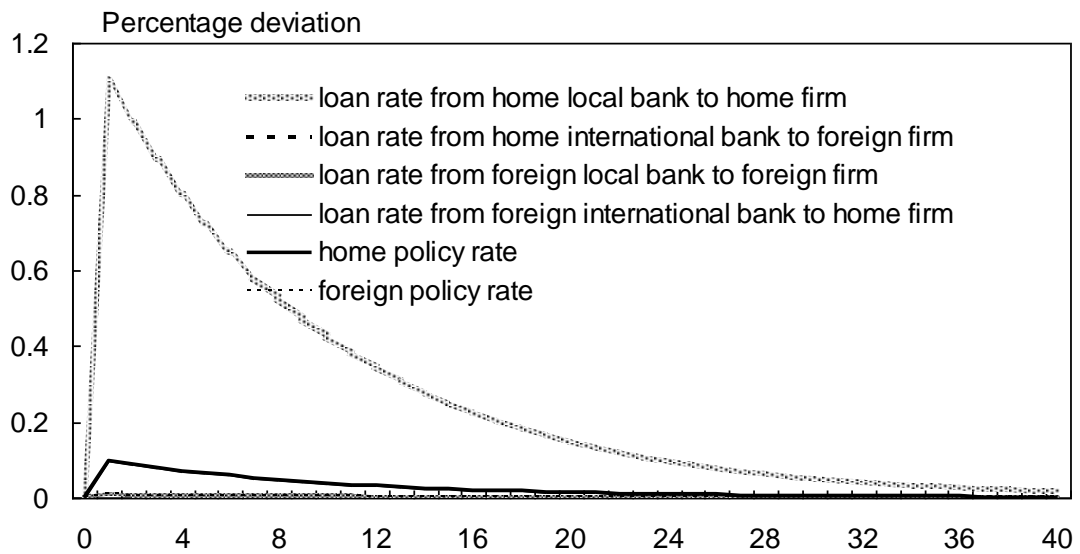
Percentage deviation

- real exchange rate
- home consumption
- foreign consumption
- home inflation
- foreign inflation

Percentage deviation

- loan rate from home local bank to home firm
- loan rate from home international bank to foreign firm
- loan rate from foreign local bank to foreign firm
- loan rate from foreign international bank to home firm
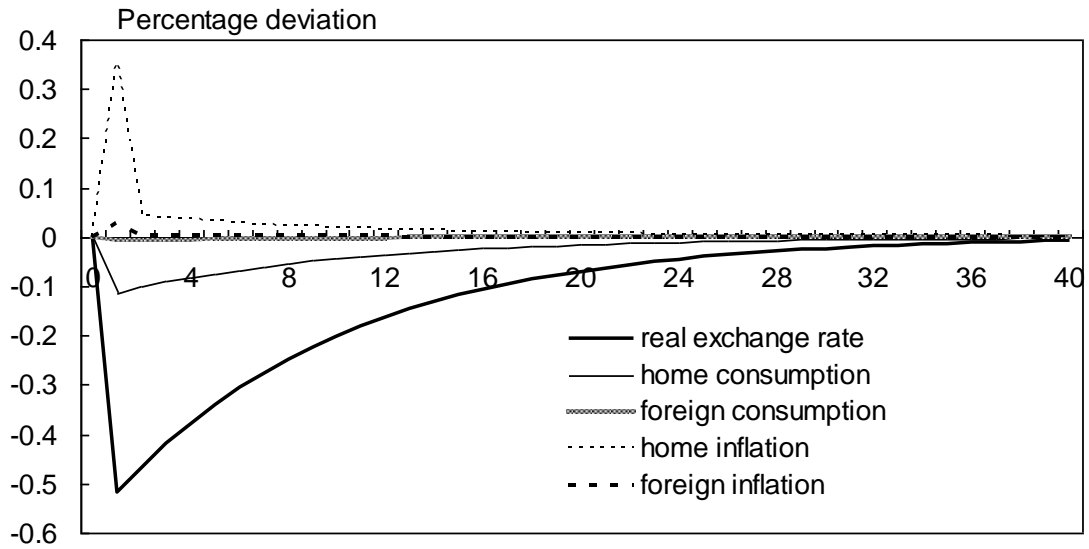- home policy rate
- foreign policy rate

Figure A2: Impulse response to a loan rate shock for the loan rate set by the home local bank for a
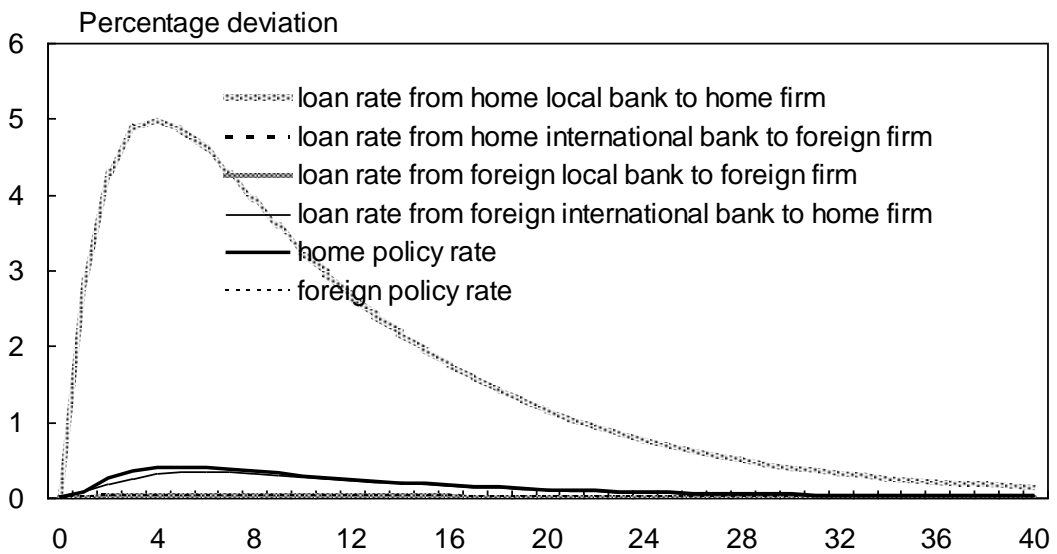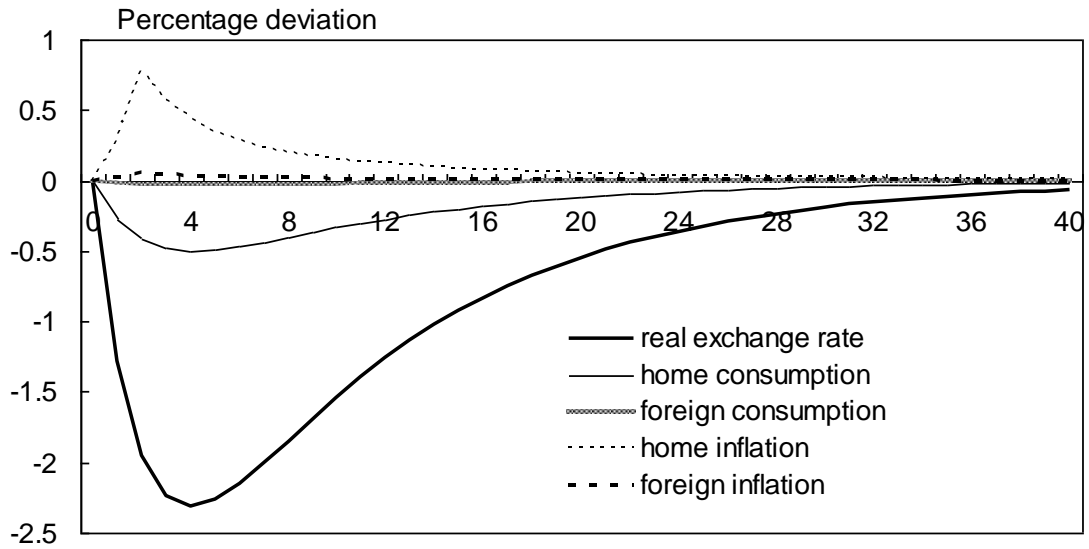home firm in the model with a sticky loan rate and flexible goods price settings

Figure A3: Impulse response to a loan rate shock for the loan rate set by the home local bank for a
home firm in the model with a flexible loan rate and sticky goods price settings

Percentage deviation

real exchange rate
home consumption
foreign consumption
home inflation
foreign inflation

Percentage deviation

loan rate from home local bank to home firm
loan rate from home international bank to foreign firm
loan rate from foreign local bank to foreign firm
loan rate from foreign international bank to home firm
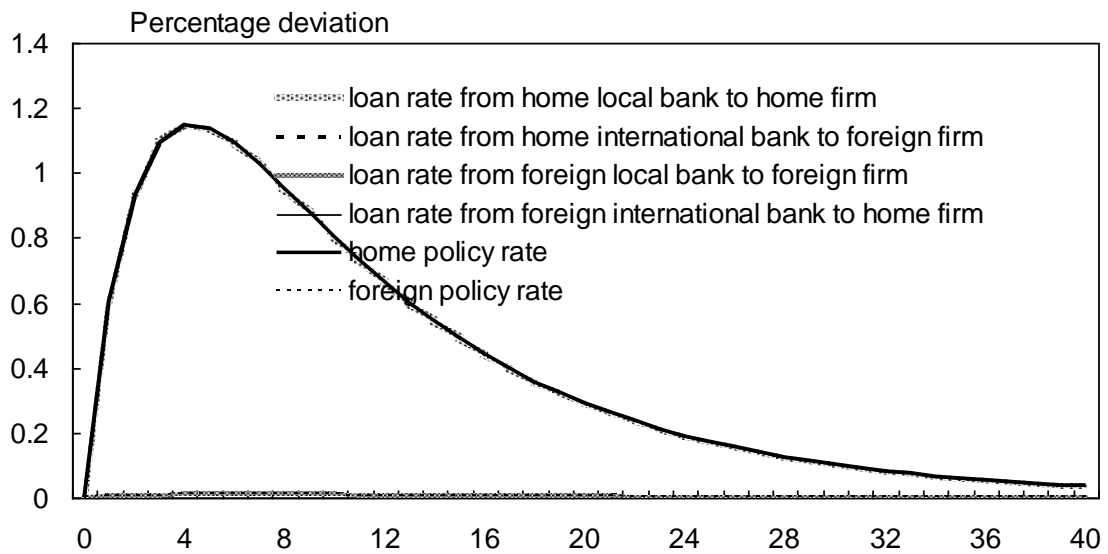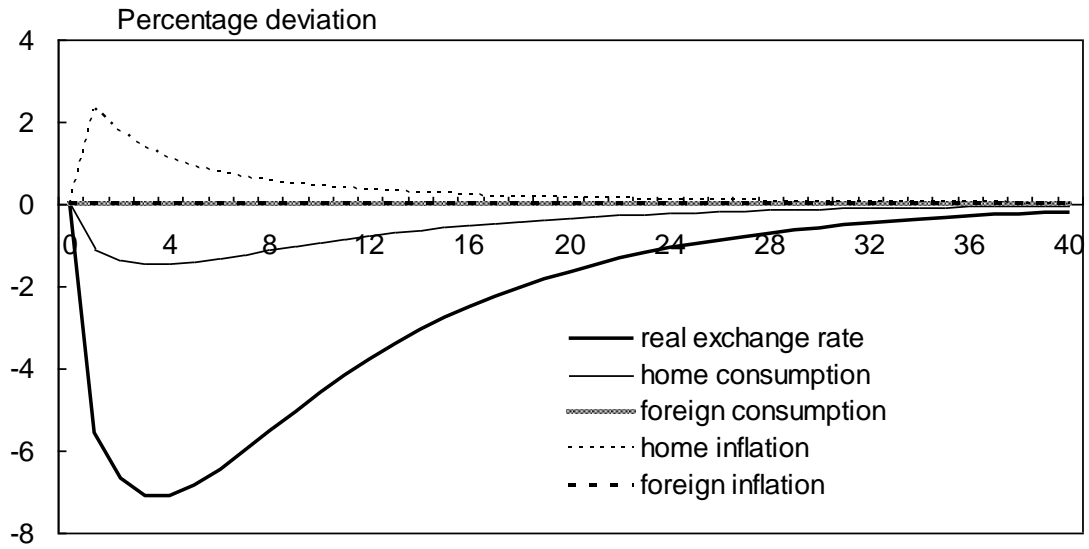home policy rate
foreign policy rate

Figure A4: Impulse response to a loan rate shock for the loan rate set by the home local bank for a home firm in the model with a sticky loan rate and sticky goods price settings