WORKING PAPER SERIES

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Working Paper No. 17/2010

OTTO VON GUERICKE
UNIVERSITÄT
MAGDEBURG

FACULTY OF ECONOMICS
AND MANAGEMENT

Impressum (§ 5 TMG)

Herausgeber:
Otto-von-Guericke-Universität Magdeburg
Fakultät für Wirtschaftswissenschaft
Die Dekanin

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Bezug über den Herausgeber
ISSN 1615-4274

Portfolio Management under Asymmetric Dependence and Distribution*

Stefan Hlawatsch and Peter Reichling**

July 2010

Abstract

Aim of our paper is to analyze the enhancement of portfolio management by using more sophisticated assumptions about distributions and dependencies of stock returns. We assume a skewed t-distribution of the returns according to Azzalini and Capitanio (2003) and a dependency structure following a Clayton copula. The risk measure applied to our portfolio selection changed from traditional portfolio variance to downside-oriented conditional value-at-risk. The empirical results show a superior performance of our approach compared to the Markowitz approach and to the approach proposed by Hatherley and Alcock (2007) on a risk-adjusted basis. The approach is applied on daily stock returns of 16 stocks of the EURO STOXX 50.

Keywords Asymmetric Dependency · Copula · Skewed t-Distribution · Conditional Value-at-Risk · Portfolio Optimization

J.E.L. Classification C01 · C13 · C15 · C16 · C46 · G11

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*We are very grateful to Sebastian Ostrowski for his comments and suggestions.

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1 Introduction and Related Literature

The general idea of portfolio selection is based on the framework introduced by Markowitz (1952). This framework is known as mean-variance portfolio theory (MVPT) since risk is measured by the variances of the rates of return. In case that no assumptions on the investor's utility functions are made,[1] MVPT requires that rates of return are multivariate normally distributed.[2] Then, marginal distributions have to be univariate normally distributed and the dependency structure has to be constant for all observations and all marginal distributions for each point in time. Several empirical studies have shown that these assumptions are frequently violated.

Regarding normality of marginal distributions, Mandelbrot (1963) was one of the first who analyzed the distribution of price changes. He found that wool price changes were non-normally distributed but rather follow a stable Paretian distribution. He used monthly changes in wool prices over the time period from 1890 to 1937. Fama (1965) adopted the idea of Mandelbrot (1963) and showed that the return distributions of stock price changes are more peaked and have fatter tails than the normal distribution predicts. His data set contained daily prices of 30 stocks of the Dow Jones Industrial Average over the time period from 1957 to 1962.

Westerfield (1977) analyzed dividend-adjusted daily prices of 315 common stocks traded on the New York Stock Exchange over the time horizon from 1968 to 1969 and concluded that stock price returns were not driven by a stable symmetric distribution which implies that they were not normally distributed. Also Hagerman (1978) rejected the hypothesis of normally distributed stock returns and he suggested to use a mixture of a normal distribution and a t-distribution. This analysis was performed on a data set comprising 1,091 stocks traded on the New York Stock Exchange and the American Stock Exchange. A statistical test for stock returns to identify multivariate normality was introduced by Richardson and Smith (1993). They analyzed 30 stocks of the Dow Jones Industrial Average over the period from 1951 to 1968. They could reject both normality of the marginal distributions and normality of the joint distribution. A comprehensive analysis of different return distributions was carried out by Peiró (1994) who tested six equity markets for return distributions following a normal, logistic, power, Student's t or two mixture distributions. As a result, the normal distribution was always excluded and the Student's t-distribution fitted best. Boothe and Glassman (1987), Young and Graff

[1] Of course, risk aversion is required but we make no assumptions on the degree of risk aversion.

[2] Chamberlain (1983) showed that the so-called elliptical symmetric distributions imply a mean-variance criterium and that these distributions belong to the location-scale class. Meyer (1987) proved that the location and scale parameters can be expressed by mean and standard deviation if these variables exist.

(1995) and Bekaert et al. (1998) provide evidence of non-normally distributed returns for other asset classes, e.g. exchange rates, real estates or emerging markets.

The second crucial assumption concerning the constant dependency structure was reviewed and rejected by several empirical studies. Patton (2004) analyzed a portfolio consisting of two funds, namely a portfolio of small market capitalized stocks and a portfolio of large market capitalized stocks, respectively, and the risk-free asset. He assumed an investor exhibiting constant relative risk aversion. By applying different dependence measures using various copula types and assuming different degrees of relative risk aversion he could not find a clear preference of a certain model having short-sale constraints. Even without any constraints the preferred model heavily depended on the degree of relative risk aversion. Sun et al. (2008) compared simulated co-movements of six German equity indices, using different marginal distributions and different copulas, with the realized returns in the time period from January to September 2006 using high-frequency stock prices. As a result, the skewed Students t-copula was found to fit best to the data.

A similar approach analyzing monthly index return movements was carried out by Okimoto (2008). He mainly used US and UK stock index data over the time period of 1973 to 2003 where he combined a Markov switching model with copula theory. He found two types of asymmetric dependencies, namely an asymmetric dependence between bear markets and normal markets and an asymmetric lower tail dependence in bear markets. Hatherley and Alcock (2007) analyzed the effects of incorporating asymmetric dependence on portfolio optimization. Therefore, they constructed a portfolio consisting of three Australian branch indices assuming a Clayton copula and normal marginal distributions. They optimized the portfolio weights by using a downside-oriented risk measure and compared the performance to a Markowitz-optimized portfolio. By incorporating lower tail dependence the downside-oriented copula approach outperformed on both absolute and risk-adjusted return measures.

For a general overview of different goodness-of-fit tests for copulas we recommend Genest et al. (2009). To analyze asymmetric dependencies, Hong et al. (2007) developed a model-free test for asymmetric correlations. Since their test statistic only checks for the existence of asymmetric dependence but not for the direction, Alcock and Hatherley (2009) extended the test by incorporating also the direction of asymmetric dependence.

Due to the empirical findings, it seems obvious to include both asymmetries, asymmetric marginal distributions and an asymmetric dependency structure between the marginal distributions. We extend the approach presented in Hatherley and Alcock (2007) by fitting skewed t-distributions to the return distributions of single stocks instead of fitting normal distributions to index returns as in Hatherley and Alcock (2007). The reason for using normal marginal distributions was that they wanted to analyze the influence of asymmetric dependency on performance. Therefore, they compared their results to a Markowitz approach which also assumes normal marginal distributions. Since we want to examine both kinds of asymmetries, the assumption of skewed t-distributed marginal distributions seems more reasonable. Additionally, we choose stocks instead of indices to account for the higher diversification potential. Another crucial point in our methodology is the use of a time-varying copula instead of a fixed copula parameter proposed by Hatherley and Alcock (2007). For the portfolio construction we use a conditional value-at-risk (CVaR) approach according to Rockafellar and Uryasev (2000).

The remaining paper is organized as follows: Section 2 describes the skewed t-distribution, the copula approach, the optimization methodology, and the benchmark approaches. Subsequently, the data set is introduced in Section 3. The results of the portfolio optimizations are presented in Section 4. Section 5 concludes.

2 Basics and Methodology

The general optimization procedure is represented by the following steps:

- Step 1: Estimation of the distribution parameters of the marginal distributions,
- Step 2: Estimation of the copula parameters,
- Step 3: Simulation of a large realization sample obtained from the estimated copula,
- Step 4: Construction of the efficient frontier in terms of expected rate of return and CVaR,
- Step 5: Selection of the optimal portfolio for a given CVaR.

These steps are described in more detail below.

2.1 Skewed t-distributions

The first step of our optimization procedure involves the estimation of the marginal distributions of the stock returns. As mentioned in Section 1, we decide to use a skewed t-distribution. In general, there are several ways to construct skewed t-distributions. We want to present a short overview of different methodologies to incorporate the skewness within the t-distribution. One of the first to introduce a skewed t-distribution was Hansen (1994). His suggestion is described by the following density function:

$$f_{\text{Hansen}}(x) \quad = \quad \frac{2\beta\Gamma\left(\frac{\nu+1}{2}\right)}{(\beta^2+1)\,\Gamma\left(\frac{\nu}{2}\right)\sqrt{\pi}\sqrt{\nu}\delta}\left[1+\frac{\left(\frac{x-\mu}{\delta}\right)^2}{\nu}\left(\frac{1}{\beta^2}\mathbb{1}_{(x\geq\mu)}+\beta^2\mathbb{1}_{(x<\mu)}\right)\right]^{-\frac{\nu+1}{2}}, \quad (1)$$

where $\beta > 0$ is a kind of skewness parameter, $\Gamma(\cdot)$ denotes the Gamma function, ν denotes the degrees of freedom (df), δ is a scale parameter, μ is the mean of the non-central t-distribution and $\mathbb{1}_{(\cdot)}$ denotes the indicator function. In the case $\beta = 1$, $f_{\text{Hansen}}(x)$ becomes the density function of the non-central t-distribution with mean μ and variance $\delta^2 \frac{\nu}{\nu-2}$.

An alternative skewed t-distribution, called epsilon-skew-t (EST) distribution was proposed by Gómez et al. (2007) following Arellano-Valle et al. (2005), where the density function is defined as:

$$f_{\text{EST}}(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\pi}\sqrt{\nu}\delta}\left[1 + \frac{\left(\frac{x-\mu}{\delta}\right)^2}{\nu}\left(\frac{1}{(1-\epsilon)^2}\mathbb{1}_{(x\geq\mu)} + \frac{1}{(1+\epsilon)^2}\mathbb{1}_{(x<\mu)}\right)\right]^{-\frac{\nu+1}{2}}. \quad (2)$$

If $\epsilon = 0$ the density function reduces to that of a non-central t-distribution with mean μ and variance $\delta^2 \frac{\nu}{\nu-2}$.

The skewed t-distribution introduced by Jones and Faddy (2003) shows the density function:

$$\begin{aligned} f_{\text{Jones}}(x) &= \frac{\Gamma(\nu+\beta)\, 2^{1-\nu-\beta}}{\Gamma\left(\frac{\nu}{2}\right)\Gamma\left(\frac{\nu}{2}+\beta\right)\sqrt{\nu+\beta}\delta}\left(1 + \frac{\left(\frac{x-\mu}{\delta}\right)}{\sqrt{\nu+\beta+\left(\frac{x-\mu}{\delta}\right)^2}}\right)^{\frac{\nu+1}{2}} \\ &\quad \left(1 - \frac{\left(\frac{x-\mu}{\delta}\right)}{\sqrt{\nu+\beta+\left(\frac{x-\mu}{\delta}\right)^2}}\right)^{\frac{\nu+2\beta+1}{2}}, \end{aligned} \quad (3)$$

where $\nu > 0$ and $\beta > -\nu/2$. If $\beta = 0$, $f_{\text{Jones}}(x)$ reduces to the density function of the non-central t-distribution with mean μ and variance $\delta^2 \frac{\nu}{\nu-2}$.

Another way to obtain a skewed t-distribution is the generalized hyperbolic skewed t-distribution proposed by Aas and Haff (2006) where the density function is defined as follows:

$$\begin{aligned} f_{\text{Aas/Haff}}(x) &= \frac{2^{\frac{1-\nu}{2}}\delta^{\nu}|\beta|^{\frac{\nu+1}{2}}K_{\frac{\nu+1}{2}}\left(\sqrt{\beta^2\left(\delta^2+(x-\mu)^2\right)}\right)\exp(\beta(x-\mu))}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\pi}\left(\sqrt{\delta^2+(x-\mu)^2}\right)^{\frac{\nu+1}{2}}}, \quad \beta \neq 0 \\ &\text{and} \\ f_{\text{Aas/Haff}}(x) &= \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi}\delta\Gamma\left(\frac{\nu}{2}\right)}\left[1 + \frac{(x-\mu)^2}{\delta^2}\right]^{-\frac{\nu+1}{2}}, \quad \beta = 0, \end{aligned} \quad (4)$$

where $K_j(\cdot)$ denotes the modified Bessel function of the third kind of order j. In the case of $\beta = 0$, $f_{\text{Aas/Haff}}(x)$ again reduces to the density function of the non-central t-distribution with mean μ and variance $\frac{\delta^2}{\nu-2}$.

Arslan and Genç (2009) and Theodossiou (1998) present a generalized skewed t (SGT) distribution which covers several well-known distributions, e.g. Hansen's skewed t-distribution, the generalized t-distribution, and the Gaussian distribution. The density function according to Arslan and Genç (2009) reads as:

$$f_{\mathrm{SGT}}(x) = \frac{p}{2B\left(1/p, q\right) q^{1/p}\sigma} \left[1 + \frac{|x-\mu|^p}{\left(1+\operatorname{sign}(x-\mu)\lambda\right)^p q\sigma^p}\right]^{-\frac{pq+1}{p}}, \tag{5}$$

where $B(\cdot)$ denotes the beta function, μ is a location parameter, $\sigma > 0$ is a scale parameter, $-1 < \lambda < 1$ is the skewness parameter, and $p, q > 0$ are the shape parameters. This density function can be transformed to the form given by Theodossiou (1998). If $\lambda = 0$ and $p = 2$, once again the density function becomes that of the non-central t-distribution with mean μ, degrees of freedom $2q$, and variance $\frac{\nu\sigma^2}{2\nu-4}$.

The skewed t-distribution applied in our paper corresponds to Azzalini and Capitanio (2003) where the density function is given by:

$$f_{\mathrm{A/C}}(x) = \frac{2\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi}\sqrt{\nu}\delta\Gamma\left(\frac{\nu}{2}\right)} \left[1 + \frac{(x-\mu)^2}{\delta^2\nu}\right]^{-\frac{\nu+1}{2}} T_{\nu+1}\left(\beta\frac{x-\mu}{\delta}\sqrt{\frac{\nu+1}{\left(\frac{x-\mu}{\delta}\right)^2+\nu}}\right), \tag{6}$$

where $T_n(\cdot)$ denotes the distribution function of the univariate t-distribution with n degrees of freedom. In the case $\beta = 0$, also $f_{\mathrm{A/C}}(x)$ reduces to the density function of the non-central t-distribution with mean μ and variance $\delta^2\frac{\nu}{\nu-2}$.

For obtaining the distribution parameters the maximum likelihood approach is applied.[3]. The log-likelihood function of Equation (6) for n observations equals:

$$\begin{aligned} l(\beta,\delta,\mu,\nu) = & \sum_{i=1}^{n}\left(\log(2) - \log(\delta) + \log\left\{g_\nu\left(\left(\frac{x_i-\mu}{\delta}\right)^2\right)\right\}\right. \\ & \left. + \log\left\{T_{\nu+1}\left(\beta\frac{x_i-\mu}{\delta}\sqrt{\frac{\nu+1}{\left(\frac{x_i-\mu}{\delta}\right)^2+\nu}}\right)\right\}\right), \end{aligned} \tag{7}$$

where g_n denotes the standard t-distribution with n degrees of freedom. The maximum is obtained via numerical optimization.[4]

2.2 Asymmetric Dependencies via Copulas

MVPT requires that the dependency structure between all assets is symmetric and linear. Under these assumptions empirical observations like tail dependencies and non-linear dependencies are not covered. To overcome these drawbacks economists often tend to

[3] For a theoretical background see Azzalini and Capitanio (2003)

[4] We use the R-package `sn` developed by Azzalini. We chose the skewed t-distribution according to Azzalini and Capitanio (2003) since for our data set the `sn`-package performs best.

use copulas to model the dependency structure between risky assets. The idea of copulas was introduced by Sklar (1959) who tried to find a function that combines univariate distributions to a multivariate distribution. In general, a copula is a joint distribution function with uniform marginal distributions. Thus, the joint distribution function of two random variables X and Y can be obtained by:

$$F(x, y) \quad = \quad C\left(F_1(x), F_2(y)\right), \tag{8}$$

where F denotes the distribution function and C denotes the copula. The general requirements for a function to be a copula function are as follows:

Definition 1. *A function $C : [0,1]^n \rightarrow [0,1]$ is a n-copula if it possesses the following properties:*

- $\forall\ u \in [0,1] : C(1,\dots,1,u,1,\dots,1) = u,$
- $\forall\ u_i \in [0,1] : C(u_1,\dots,u_n) = 0$ *if* $\exists\ j \in \{1,\dots,n\} : u_j = 0,$
- *C is grounded and n-increasing.*

There are three main categories of copulas which differ in the generation of the copula function. Elliptical copulas are copula functions derived from elliptical distribution functions like the Gaussian or the Student's t-distribution. Archimedean copula functions are constructed via generator functions. The third copula class is called extreme value copulas which are derived from multivariate extreme value distributions.[5]

Since asset returns frequently exhibit lower tail dependence as pointed out by Hartmann et al. (2004), Rodriguez (2007), Fortin and Kuzmics (2002) and Longin and Solnik (2001), we decide to model this dependency via a Clayton copula which belongs to the Archimedean copula class. Lower tail dependence in the two-dimensional case can be thought of high positive correlation between two random variables whenever small realizations are obtained. For instance, when analyzing rates of return, lower tail dependence refers to high positive correlation between extreme negative returns.

In general, Archimedean copulas are constructed by a generator function γ which is a strictly decreasing continuous function with the unit interval as its domain, range $[0,\infty]$ and $\gamma(1) = 0$. The resulting n-dimensional copula $C_n(u_1,\dots,u_n)$ reads as:

$$C_n(u_1,\dots,u_n) \quad = \quad \gamma^{[-1]}\left(\gamma(u_1) + \ldots + \gamma(u_n)\right), \tag{9}$$

where $\gamma^{[-1]}(\cdot)$ denotes the pseudo-inverse and $u_i = F_i(x)$ with $F_i(x)$ being the ith marginal

[5] For a general overview of copula concepts and different copula classes see Cherubini et al. (2004), Nelsen (2006), and Malervergne and Sornette (2006).

distribution function. The pseudo-inverse $\gamma^{[-1]}(x)$ is defined as:

$$\gamma^{[-1]}(x) \quad = \quad \begin{cases} \gamma^{-1}(x), & 0 \leq x \leq \gamma(0) \\ 0, & x \geq \gamma(0) \end{cases} . \tag{10}$$

The generator function for the Clayton copula reads as:

$$\gamma(u) = \frac{1}{\lambda}\left(u^{-\lambda} - 1\right). \tag{11}$$

So, the n-dimensional Clayton copula results:

$$C(u_1, \ldots, u_n) = \left(\sum_{i=1}^{n} u_i^{-\lambda} - (n-1)\right)^{-\frac{1}{\lambda}}. \tag{12}$$

Estimation of the copula parameter λ is done via a two-step maximum likelihood approach. The log-likelihood function of a n-dimensional copula with T observations is defined as:

$$\begin{aligned} l(\theta_1, \ldots, \theta_n; \lambda) \quad &= \quad \sum_{t=1}^{T} \log f\left[x(t); \theta_1, \ldots, \theta_n; \lambda\right] \\ &= \quad \sum_{t=1}^{T} \log c\left[F_1(x(t); \theta_1), \ldots, F_n(x(t); \theta_n); \lambda\right] \\ &\qquad + \sum_{t=1}^{T} \log f_1(x(t); \theta_1) \\ &\qquad \vdots \\ &\qquad + \sum_{t=1}^{T} \log f_n(x(t); \theta_n), \end{aligned} \tag{13}$$

where f denotes the density of the joint distribution, f_i denotes the density of the ith marginal distribution, θ_i denotes the vector of distribution parameters of the ith marginal distribution, and c denotes the density of the copula itself. The first step of the maximum likelihood approach involves the estimation of each vector θ_i of the n marginal distributions by maximizing $\sum_{t=1}^{T} \log f_i(x(t); \theta_i)$ with respect to θ_i. When using skewed t-distributions for the marginal distributions as in our case, maximization can be done according to our procedure from the end of Section 2.1.

The second step within the maximum likelihood approach involves the maximization of Equation (13) with respect to λ using the estimators of the marginal distribution parameters obtained in the first step. Since λ does only occur in the first summand of

Equation (13), maximization of the log-likelihood simplifies to:

$$l(\lambda) = \sum_{t=1}^{T} \log c\left[\widehat{F_1}(x(t);\widehat{\theta}_1),\ldots,\widehat{F_n}(x(t);\widehat{\theta}_n);\lambda\right]. \tag{14}$$

Applying the second step to our n-dimensional Clayton copula, the density function of the copula reads as:

$$\begin{aligned} c(\widehat{u_1},\ldots,\widehat{u_n});\lambda) &= \frac{\partial^n C}{\partial\widehat{u_1}\ldots\partial\widehat{u_n}} \\ &= \prod_{i=1}^{n-1}(1+i\lambda)\left(\prod_{j=1}^{n}\widehat{u_j}\right)^{-\lambda-1}\left(\sum_{k=1}^{n}\widehat{u_k}^{-\lambda}-n+1\right)^{-\frac{1}{\lambda}-n}, \end{aligned} \tag{15}$$

where $\widehat{u_i} = \widehat{F_i}(x;\widehat{\theta_i})$. Maximizing the logarithm of Equation (15) with respect to λ and assuming T observations finally results in:

$$\begin{aligned} \widehat{\lambda} &= \arg\max_{\lambda}\left[T\sum_{i=1}^{n-1}\ln(1+i\lambda)-(\lambda+1)\sum_{t=1}^{T}\sum_{j=1}^{n}\ln\widehat{u_{j,t}}\right. \\ &\quad \left.-\left(\frac{1}{\lambda}+n\right)\sum_{t=1}^{T}\ln\left(\sum_{k=1}^{n}\widehat{u_{k,t}}^{-\lambda}-n+1\right)\right]. \end{aligned} \tag{16}$$

2.3 Portfolio Optimization

After estimating the marginal distributions and the copula, we are now able to proceed with the portfolio optimization. First, we simulate a large sample consisting of Q observations of each asset's rate of return distribution in the copula. Therefore, we generate, for each of the n assets, Q realizations out of a $(0,1)$-uniform distribution resulting in n Q-dimensional vectors denoted as $X_k,\ k=1,\ldots,n$. Subsequently, we transform the independent uniformly distributed random variables X_k to dependent uniformly distributed random variables Y_k using Bayes' theorem. For an Archimedean copula the conditional probability of Y_k, under the assumption that $Y_1 = X_1$, can be obtained for each observation $q = 1,\ldots,Q$ by the following equation:

$$\begin{aligned} y_{k,q} &= C_k(y_{k,q}|y_{1,q},\ldots,y_{k-1,q}) \\ &= \frac{\partial^{k-1}\left[\gamma^{[-1]}\left(\sum_{j=1}^{k}\gamma(x_{j,q})\right)\right]/\partial u_1\ldots\partial u_{k-1}}{\partial^{k-1}\left[\gamma^{[-1]}\left(\sum_{j=1}^{k-1}\gamma(x_{j,q})\right)\right]/\partial u_1\ldots\partial u_{k-1}} \quad \forall k=2,\ldots,n;\ q=1,\ldots,Q. \end{aligned} \tag{17}$$

Applying Equation (17) to the n-dimensional Clayton copula results in:

$$
\begin{aligned}
y_{1,q} &= x_{1,q} \\
y_{2,q} &= \left(y_{1,q}^{-\lambda}\left(x_{2,q}^{-\frac{\lambda}{\lambda+1}}-1\right)+1\right)^{-\frac{1}{\lambda}} \\
&\vdots \\
y_{n,q} &= \left(\left(\sum_{k=1}^{n-1} y_{k,q}^{-\lambda}-n+2\right)\left(x_{n,q}^{-\frac{\lambda}{\lambda(1-n)-1}}-1\right)+1\right)^{-\frac{1}{\lambda}}.
\end{aligned} \tag{18}
$$

Following, the Q realizations of the n dependent uniformly distributed random variables are transformed into simulated rates of return by applying the inverses of the marginal distribution functions of the copula:

$$
r_{n,q} = F_{\mathrm{A/C},n}^{-1}\left(y_{n,q};\widehat{\theta}_n\right), \tag{19}
$$

where $F_{\mathrm{A/C},n}^{-1}(\cdot)$ denotes the inverse of the skewed t-distribution according to Azzalini and Capitanio (2003). This finishes the third step of our procedure and we now proceed with the construction of the efficient frontier.

Since we do not assume a multivariate normal distribution of portfolio returns, we cannot generate an efficient frontier with respect to expected returns and standard deviations of the portfolios.[6] Thus, we decide to use CVaR instead of standard deviation. As shown by Acerbi and Tasche (2002), CVaR is a coherent risk measure, i.e. it is sub-additive, monotonous, positively homogeneous and translation invariant. CVaR, also denoted as expected shortfall, is defined as the expected value of rates of return R lower or equal to the value-at-risk (VaR) for a given probability α:

$$
\mathrm{CVaR} = \mathrm{E}[R|R \leq \mathrm{VaR}_{\alpha}]. \tag{20}
$$

Optimization of the portfolio weights is done by minimizing CVaR for a given portfolio rate of return:

$$
\begin{aligned}
&\min_{\mathrm{VaR},w} \mathrm{CVaR} \\
&\text{s.t.} \\
&\mu w \leq -\bar{R} \\
&w^T \mathbf{1} = 1,
\end{aligned} \tag{21}
$$

where w denotes the vector of the asset's weights within the portfolio, μ denotes the vector

[6] We refer back to footnote 2 and omit the discussion about distributions of the location-scale class.

of expected rates of return of the single assets, $\bar{R}$ denotes the portfolio's target rate of return, and $\mathbf{1}$ denotes the vector of ones.

We choose Monte Carlo approximation to minimize CVaR according to Rockafellar and Uryasev (2000). CVaR of a continuous random variable can be described by the following integral:

$$\text{CVaR} = \frac{1}{1-\alpha} \int_{-\infty}^{f(R,w) \leq \text{VaR}_\alpha(R)} f(R,w)p(R,w)\ \mathrm{d}w, \tag{22}$$

where $f(R,w)$ denotes the portfolio's return function and $p(R,w)$ denotes the probability distribution function. Equation (22) can be linearly approximated by:

$$\widehat{\text{CVaR}} = \text{VaR} + \frac{1}{(1-\alpha)Q} \sum_{q=1}^{Q} \max\left[-w^T r_{n,q} - \text{VaR}\ , 0\right], \tag{23}$$

where $r_{n,q}$ denote our simulated rates of return described above. Given the simulated sample of $n \cdot Q$ rates of return, the optimization problem given in Equation (21) rearranges to:

$$\begin{aligned}
\min_{\text{VaR},w} \widehat{\text{CVaR}} &= \text{VaR} + \frac{1}{(1-\alpha)Q} \sum_{q=1}^{Q} z_q \\
\text{s.t.} & \\
z_q &\geq \sum_{i=1}^{n} -w_i r_{i,q} - \text{VaR},\ \ q = 1, \ldots, Q \\
z_q &\geq 0,\ \ q = 1, \ldots, Q \\
\sum_{i=1}^{n} \mu_i w_i &\geq \bar{R} \\
\sum_{i=1}^{n} w_i &= 1
\end{aligned} \tag{24}$$

To derive the efficient frontier we repeat the optimization with different target rates of return. To enhance computation time, the first optimized portfolio is the minimum CVaR-portfolio. Subsequently the target rate of return is increased by an incremental amount until the optimized CVaR equals the target CVaR.

2.4 Benchmark Models

Our approach described in the previous sections is repeated at each point in time where the portfolio is re-weighted. To analyze the influence of incorporating either asymmetries in the marginal distributions or asymmetries in the dependence structure we consider two benchmark models. The first benchmark model is portfolio optimization according to

Markowitz (1952). With this benchmark model we can analyze the additional value generated by incorporating both asymmetries. The second benchmark model is an adjusted approach according to Hatherley and Alcock (2007). Here, normally distributed rates of return and a dependence structure following a Clayton copula are assumed. Compared with our model, we can analyze the additional value generated by incorporating asymmetric marginal distributions instead of normal distributions.[7] To ensure comparability the target CVaR in our approach and in the second benchmark approach are identical. Additionally, portfolio optimization according to Markowitz (1952) is done for a variance equal to the variance of the optimal portfolio of the second benchmark model for each point in time. Thus, the risk level in every model should be equal.

3 Data Set and Preliminary Results

In our study we analyze portfolios consisting of 16 stocks from the EURO STOXX 50. The EURO STOXX 50 is the most commonly cited stock index for companies in the Eurozone and, therefore, "... provides a Blue-chip representation of supersector leaders in the Eurozone."[8] It is separated into 18 industry sectors and contains stocks from nine Eurozone countries. The time period of our data set ranges from March 2001 to December 2009, which equals 2,250 trading days (after excluding holidays). Due to diversification purposes we choose the largest company of each industry sector which was listed in the EURO STOXX 50 over the whole time period of the data set.[9] The remaining 16 stocks of our data set comprise approximately 44 percent of the EURO STOXX 50. Table 1 provides an overview of the chosen stocks, their industry sectors and their corresponding weights in the EURO STOXX 50.

The total time period of 2,250 trading days is subdivided into 351 subperiods each covering 500 trading days, starting with the first 500 trading days and than moving the time window five days on. Thus, we assume that the portfolio is re-weighted every five trading days and the previous 500 trading days are used to estimate the distribution parameter, the copula parameter and the optimal portfolio weights. The whole estimation and optimization procedure described in Section 2 is carried out for each subperiod. To analyze the impact of shorter time periods on the estimated distribution parameters, the copula parameter and the corresponding optimal portfolios, we additionally subdivide the total time period

[7] The benchmark model is called "adjusted" since Hatherley and Alcock (2007) use a constant copula parameter for their portfolio optimization instead of re-estimating the copula parameter for each point in time as we do.

[8] See www.stoxx.com.

[9] For two industry sectors there was only one company assigned to that industry sector which was not listed over the whole time period of the data set. Therefore, we excluded these two sectors of industry.

Table 1: Composition of the data set

The table shows the composition of the data set, the corresponding industry sector and the weight of each company in the EURO STOXX 50.

Number	Company	Sector	Weight in the EURO STOXX 50
(1)	Daimler AG	Automobiles & Parts	4.37%
(2)	Banco Santander SA	Banks	11.28%
(3)	BASF SE	Chemicals	5.92%
(4)	CRH PLC	Construction & Materials	1.82%
(5)	Deutsche Börse AG	Financial Services	1.50%
(6)	Unilever NV	Food & Beverage	4.90%
(7)	Sanofi-Aventis SA	Health Care	8.54%
(8)	Siemens AG	Industrial Goods & Services	9.00%
(9)	Allianz SE	Insurance	5.90%
(10)	Vivendi SA	Media	3.42%
(11)	Total SA	Oil & Gas	13.21%
(12)	Philips Electronics NV	Personal & Household Goods	3.23%
(13)	Carrefour SA	Retail	3.05%
(14)	Nokia Corp.	Technology	6.06%
(15)	Telefónica SA	Telecommunications	10.11%
(16)	E.ON AG	Utilities	7.68%

to 401 subperiods each covering 250 trading days where again each portfolio is re-weighted every five trading days.[10]

At first, we want to analyze the rates of return distributions. As Table 2 shows, all distributions exhibit large kurtosis. Additionally, 12 out of the 16 stocks exhibit a positive skewness indicating that these return distributions have more realizations on the left tail of the distribution. The total number of rejections of the Jarque-Bera test over all subperiods for the 250-days subperiods and the 500-days subperiods for each stock support the result of non-normally distributed rates of return. With more than 50 percent of the stocks the assumption of normally distributed rates of return is rejected for all subperiods for the 500-days subperiods.

We also analyze whether the skewed t-distribution is a good estimator for the rate of return distributions. Therefore, we apply both a two-sample Kolmogorov-Smirnov test and a two-sample Cramér-von Mises test to check whether the realized rates of return and a simulated sample of the corresponding skewed t-distribution belong to the same underlying distribution. We perform both tests for each stock and for each subperiod with

[10] We are aware of the fact, that this methodology leads to overlapping data samples. Nevertheless, this "rolling window" approach is frequently used for performance measurements.

Table 2: Descriptive statistics of the data set

The table shows a summary of statistics for the 16 stocks of the data set. For each stock's rates of return column 2 to 5 show the mean ($\hat{\mu}$), the standard deviation ($\hat{\sigma}$), the kurtosis (Kurt), and the skewness (Skew) for the total time period. Columns 6 and 7 show the total number of rejections of the Jarque-Bera test over all subperiods of the data set for each stock for the 500-days subperiods (out of 351 subperiods) and the 250-days subperiods (out of 401 subperiods), respectively.

Company	$\hat{\mu}$	$\hat{\sigma}$	Kurt	Skew	No. of rejections (500)	No. of rejections (250)
(1)	0.00012	0.02438	9.29663	0.51536	337	283
(2)	0.00029	0.02261	8.36405	0.32338	351	278
(3)	0.00044	0.01997	10.26965	0.46718	277	359
(4)	0.00031	0.02211	7.77878	−0.01085	351	336
(5)	0.00077	0.02331	9.97090	0.53588	351	328
(6)	0.00017	0.01558	7.69843	−0.15476	351	380
(7)	0.00016	0.01876	7.71408	0.18904	351	377
(8)	0.00016	0.02475	8.13711	0.20673	351	211
(9)	−0.00022	0.02666	9.71189	0.53303	345	335
(10)	−0.00018	0.02679	21.41037	−0.56964	323	287
(11)	0.00023	0.01794	9.47238	0.34216	315	385
(12)	0.00008	0.02668	6.01912	0.18645	326	370
(13)	−0.00009	0.01912	6.78463	0.00764	351	303
(14)	−0.00007	0.02871	8.20392	−0.14698	336	367
(15)	0.00022	0.01791	8.51584	0.37633	351	401
(16)	0.00040	0.01960	10.88825	0.37169	351	292

the null hypothesis that realized and generated returns belong to the same underlying distribution. The results are presented in Table 3 for both the 500-days subperiods and the 250-days subperiods.

According to the two-sample Kolmogorov-Smirnov test the null hypothesis is only rejected in less than one percent of all subperiods over all stocks on a significance level of five percent for both the 500-days subperiods and the 250-days subperiods. The two-sample Cramér-von Mises test rejects the null hypothesis in 8.5% of all subperiods over all assets on a significance level of five percent for the 500-days subperiods and 10.8% for the 250-days subperiods. As a result, the skewed t-distribution is a much more accurate estimator for the rate of return distributions than the normal distribution. The behavior of the estimators on the left and right tail of the return distribution is analyzed via the presentation of the quantile-quantile (QQ) plots for each stock. Figure 4 in Appendix 1 shows the 16 QQ plots for the first subperiod assuming skewed t-distributed rates of return. Figure 5 in Appendix 1 shows the corresponding QQ plots assuming normally

Table 3: Nonparametric tests

The table shows a summary of statistics for the 16 stocks of the data set. For each stock column 2 and 3 show the total number of rejections of the two-sample Kolmogorov-Smirnov (KS) test and for the two-sample Cramér-von Mises (CM) test on a significance level of five percent over all 500-days subperiods of the data set (out of 351 subperiods), respectively. Column 4 and 5 show the total number of rejections of the two-sample KS test and for the two-sample CM test on a significance level of five percent over all 250-days subperiods of the data set (out of 401 subperiods), respectively.

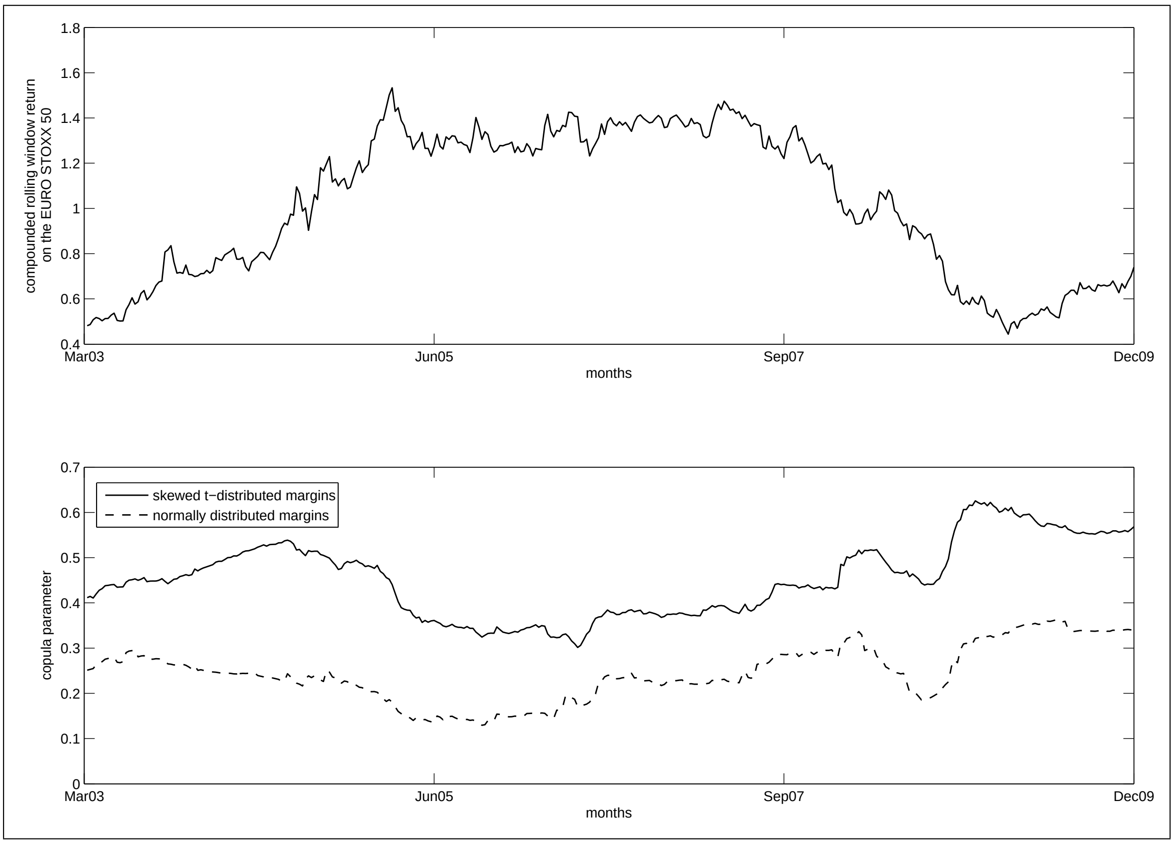
Company	351 subperiods		401 subperiods	
	KS test	CM test	KS test	CM test
(1)	5	24	2	44
(2)	1	56	1	54
(3)	1	47	2	56
(4)	4	25	8	28
(5)	2	22	0	56
(6)	1	27	2	44
(7)	3	24	3	39
(8)	2	45	2	42
(9)	1	26	8	34
(10)	4	15	3	39
(11)	2	27	7	36
(12)	10	13	5	37
(13)	6	22	1	58
(14)	3	25	2	33
(15)	3	44	2	46
(16)	4	26	3	47

distributed rates of return. Both figures are presented for the 500-days subperiods. Figures for the 250-days subperiods are omitted because they show similar findings. As the previous results indicate, again, the skewed t-distributions are much more accurate for the estimation of rates of return distributions. Especially the tail behavior of the skewed t-distribution fits better than the normal distribution resulting in less and lower outliers on the tails of the QQ plots.

After comparing the reliability of skewed t-distributions and normal distributions as estimators for the rate of return distributions we now analyze the dependence structure between the 16 stocks. Since we use a Clayton copula, we assume lower tail dependence between the marginal distributions. Additionally, we assume that the strength of the tail dependence changes with the business cycle of the economy as pointed out by, e.g., Ang and Cheng (2002), Hartmann et al. (2004), and Hu (2006). Thus, we expect the copula parameter to increase in bear markets and decrease in bull markets. To support our assumptions we present the development of our copula parameter in Figure 1. We present both, the copula parameter assuming skewed t-distributed rates of return and normally distributed rates of return. Furthermore, the compounded rates of return for the EURO STOXX 50 are also presented in Figure 1. The whole figure is derived by using the 500-days subperiods. Each compounded return equals the compounded return for the preceding 500 trading days.

Figure 1: Compounded rates of return of the EURO STOXX 50 and development of the copula parameter

The upper figure shows the compounded rates of return of the EURO STOXX 50 for each rolling time window. Here, the return is computed as the compounded return over the previous 500 trading days. The lower figure shows the development of the copula parameter assuming skewed t-distributed returns and normally distributed returns, respectively. Each point in time denotes the copula parameter estimated over the previous 500 trading days.



Both copula parameters tend to co-move inversely with the compounded rates of return of the EURO STOXX 50 indicating that both types of marginal distributions, namely the skewed t-distribution and the normal distribution, possess lower tail dependence where the strength of this interrelation depends on the business cycle of the economy. This result is verified by the correlation coefficients of the copula parameter with the compounded rate of return. For the Clayton copula assuming skewed t-distributed rates of return the correlation coefficient between the compounded rate of return of the EURO STOXX 50 and the copula parameter equals -0.7760 which is significant on the one percent significance level. The correlation between the copula parameter, assuming normally distributed rates of return, and the compounded return of the EURO STOXX 50 equals -0.6810 which is also significant on the one percent significance level.

However the level of lower tail dependence is quite different. The amount of the copula parameter, assuming skewed t-distributed rates of return, is almost twice as large as the copula parameter assuming normally distributed rates of return. As already mentioned, the empirical return distributions of the 16 stocks are mostly positively skewed indicating more observations at the lower tail. This tail behavior is incorporated when using skewed t-distributed return distributions. Thus, we suggest that the assumption of normally distributed rates of return underestimates co-movements of stock returns especially in bear markets.[11]

4 Portfolio Results

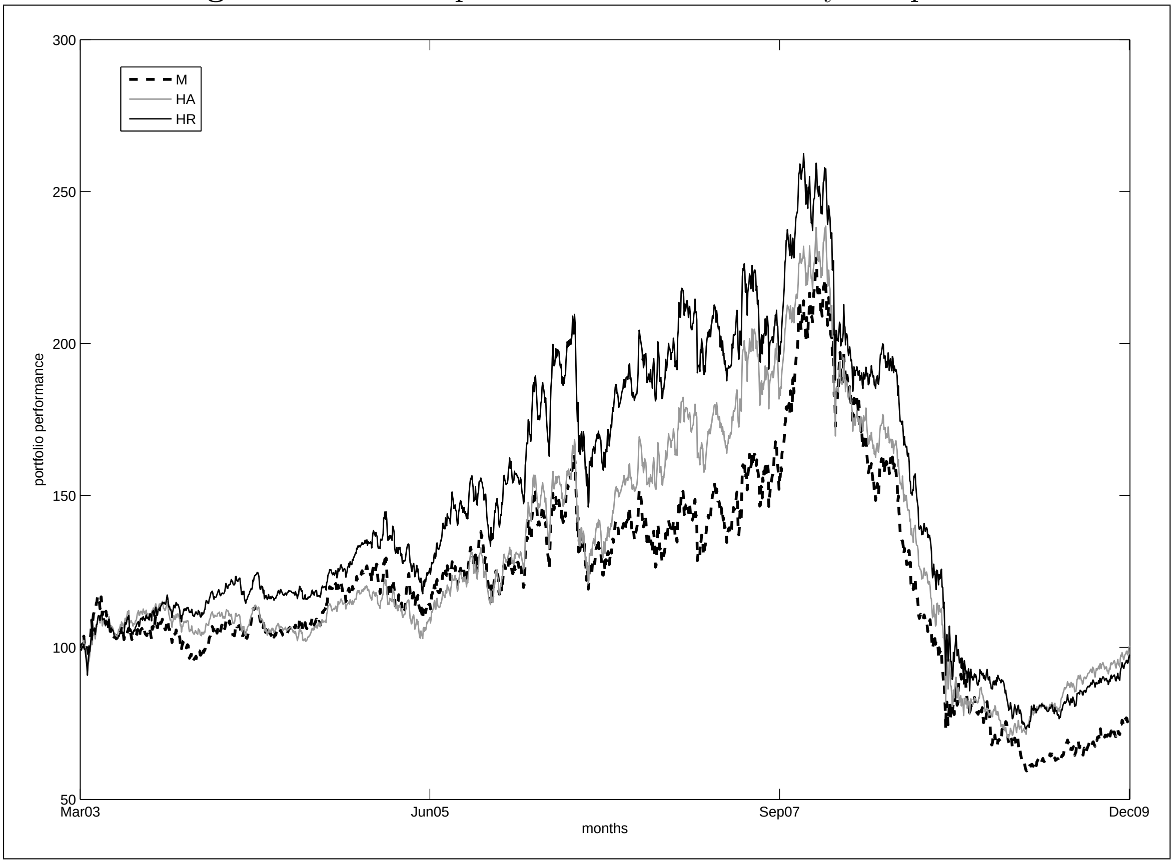
We are now in the position to discuss the results of our portfolio optimization (HR) approach and those of the benchmark approaches. The benchmark approaches are the Markowitz (M) approach and the adjusted approach according to Hatherley and Alcock (2007) denoted with HA. All approaches are derived for the 250-days subperiods and the 500-days subperiods, respectively. For the two downside-oriented optimization approaches, a CVaR of 0.0225 is arbitrarily chosen. As described in Section 3 the optimal portfolio weights are computed for a five days interval with either the previous 250 or 500 trading days. So, the portfolio is assumed to be re-weighted every five days with an initial amount of 100€. All gains in each subperiod are reinvested.

At first, we present the overall portfolio performance of the three approaches for the 500-days subperiods in Figure 2. Our approach and the HA approach dominate the Markowitz approach over nearly the total time period. Additionally, our approach also dominates the adjusted approach according to Hatherley and Alcock (2007) except for the last quarter year. These results suggest that accounting for asymmetric dependence leads to improved portfolio performance. The performance can be further improved by accounting for asymmetric rates of return distributions.[12]

[11] The figure for the 250-days subperiods is omitted since it leads to similar results. Again, the compounded rates of return of the EURO STOXX 50 co-move inversely with both copula parameters. The correlation coefficients between the compounded rate of return of the EURO STOXX 50 and the copula parameter assuming skewed t-distributed rates of return and normally distributed rates of return equal -0.5636 and -0.5543, respectively and are also significant on the one percent significance level.

[12] The result for the 250-days subperiods is presented in Appendix 2.

Figure 2: Portfolio performance for the 500-days subperiods



To emphasize our results we compute risk-adjusted mean returns for the three approaches. As mentioned in Section 2.4, the three approaches are assumed to have similar risk but only for the period used for optimization. Since we use optimized portfolio weights for the next five days, the approaches need not to be at the same level of risk. Therefore, we compute the average return per unit of downside risk, measured by lower semivariance, for different time periods. We choose semivariance because asymmetric rates of return distributions are applied and so the portfolio's rate of return distribution may not be symmetric. Table 4 shows the monthly, semiannually and annually average risk-adjusted return and their corresponding standard deviations.

The presented results are slightly ambivalent. Regarding the 500-days subperiods, the average downside-risk-adjusted returns are the highest for our approach, no matter which time period is considered. However, standard deviations are the highest of all approaches for the monthly and semiannually time frame and the second highest for the annually time period. The Markowitz approach is dominated by both downside-oriented approaches regarding the average risk-adjusted return and only possess the lowest standard deviation for the semiannually time period.

Regarding the 250-days subperiods again our approach outperformed the benchmark strategies on a monthly basis regarding the average risk-adjusted return. For the other two

Table 4: Risk-adjusted returns

The table shows for both the 500-days and the 250-days subperiods the average downside-risk-adjusted return and the standard deviation for monthly, semiannually and annually time spans, respectively.

	Approach	500-days subperiods		250-days subperiods	
		$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$
Monthly	M	0.0187	0.2410	0.0499	0.2148
	HA	0.0257	0.2348	0.0495	0.2338
	HR	0.0351	0.2508	0.0510	0.2414
Semiannually	M	0.0140	0.0894	0.0366	0.0880
	HA	0.0277	0.0944	0.0375	0.0868
	HR	0.0319	0.0968	0.0323	0.0772
Annually	M	0.0112	0.0739	0.0363	0.0651
	HA	0.0237	0.0694	0.0345	0.0662
	HR	0.0264	0.0721	0.0308	0.0614

time periods our approach shows the lowest standard deviation where the HA approach possesses the highest average return for the semiannually time period and the Markowitz approach the highest average return for the annually time period. The Markowitz approach is dominated for nearly every time span for both lengths of subperiods. One possible reason for the lower performance of our approach for the 250-days subperiods might be that, against the usual empirical findings, the rates of return distributions tend to be more Gaussian since the average of the estimated degrees of freedom for the skewed t-distributions increases significantly for nearly all stocks.

In the next, step we separate the portfolio return changes in positive and negative changes and analyze their means and standard deviations. This is done to check whether the downside-oriented optimization approaches lead to better results for negative portfolio returns. The findings are presented in Table 5. Regarding the downside changes, the downside-oriented optimization approaches dominate the Markowitz approach with respect to the mean downside change. Additionally, for the 500-days subperiods the standard deviations of both approaches are also smaller than the one of the Markowitz approach. When analyzing the upside changes our optimization approach and the Markowitz approach perform similar in case of the 500-days subperiods but the downside-oriented approaches fail to outperform the Markowitz approach for the 250-days subperiods.

Finally, Figure 3 presents the portion of long positions within the portfolio for the 500-days subperiods.[13] Here, the portion of long positions for the Markowitz approach is always smaller than the portions of long positions for the two downside-oriented approaches.

[13] The figure for the 250-days subperiods is omitted since the results are nearly identical.

Table 5: Upside and downside changes of the portfolio returns

The table shows the means and standard deviations for upside and downside changes of the portfolio returns for both the 250-days subperiods and the 500-days subperiods, respectively.

	Approach	500-days subperiods		250-days subperiods	
		$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$
Positive	M	0.0281	0.0241	0.0332	0.0356
	HA	0.0268	0.0219	0.0288	0.0354
	HR	0.0277	0.0249	0.0292	0.0324
Negative	M	−0.0278	0.0269	−0.0321	0.0366
	HA	−0.0250	0.0234	−0.0309	0.0380
	HR	−0.0259	0.0240	−0.0320	0.0385

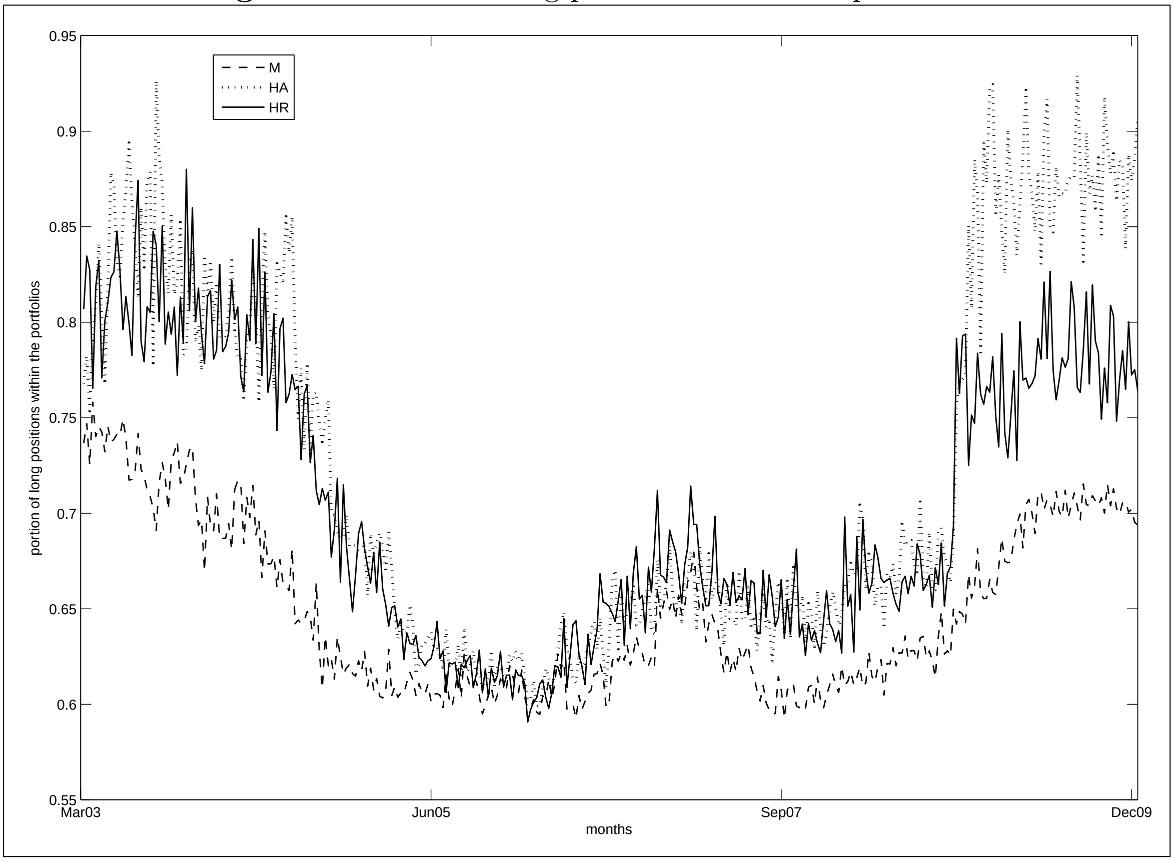
Short positions are taken to profit from negative stock returns. However, the potential of diversification is reduced with increasing correlation of the assets. The figures of Table 3 lead to the suggestion that diversification benefits for downside movements of the stocks are overestimated by the Markowitz approach. Furthermore, there may occur some regulatory difficulties regarding the high portion of short positions within a portfolio since some funds are subject to short-sale constraints or limits.

5 Conclusion

The idea of the paper was to analyze whether incorporating asymmetric dependencies and asymmetric distributions will lead to an improved portfolio selection. As a benchmark served the well-known portfolio optimization by Markowitz (1952) where multivariate normally distributed portfolio returns are assumed. We applied an adjusted version of the approach proposed by Hatherley and Alcock (2007) where asymmetric dependencies are modelled via a Clayton copula and stock returns are assumed to be normally distributed. In our approach, we additionally assumed skewed t-distributed rates of return for each single asset. We chose the skewed t-distribution according to Azzalini and Capitanio (2003) since computational stability was best for this version of the distribution.

We found that the incorporation of asymmetric dependencies between single assets improved the portfolio performance for both different lengths of the subperiods and different time horizons. The enhancement of the portfolio performance depended on the chosen length of the subperiod when incorporating asymmetric distributions. In general, the skewed t-distribution proved to be a more accurate estimator for the rate of return distributions than the Gaussian distribution especially with respect to the tail behavior. Therefore, we observed higher risk-adjusted average returns compared to the Markowitz

Figure 3: Portion of long positions within the portfolios



approach at least for the 500-days subperiods. Regarding the 250-days subperiods the rate of return distributions tend to be more Gaussian-style and so the adjusted approach according to Hatherley and Alcock (2007) was superior. All results were obtained by analyzing a portfolio consisting of stocks from the EURO STOXX 50 where we diversified the portfolio by including only one stock of each industry sector.

All portfolio weights were optimized by using estimators of the preceding period. This was done to ensure that all approaches exhibit a similar amount of model biases. The performance of our approach may be improved when using more sophisticated estimation models for future stock returns. However, the incorporation of asymmetry in return distributions as well as in their dependence structure leaded to better overall portfolio performance compared to the standard symmetric Markowitz approach.

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Appendix 1

Figure 4: QQ plots using skewed t-distributions

The figure shows the QQ plots for each stock assuming skewed t-distributed rates of return for the first subperiod. Realized returns are plotted along the x-axis and simulated returns are plotted on the y-axis.

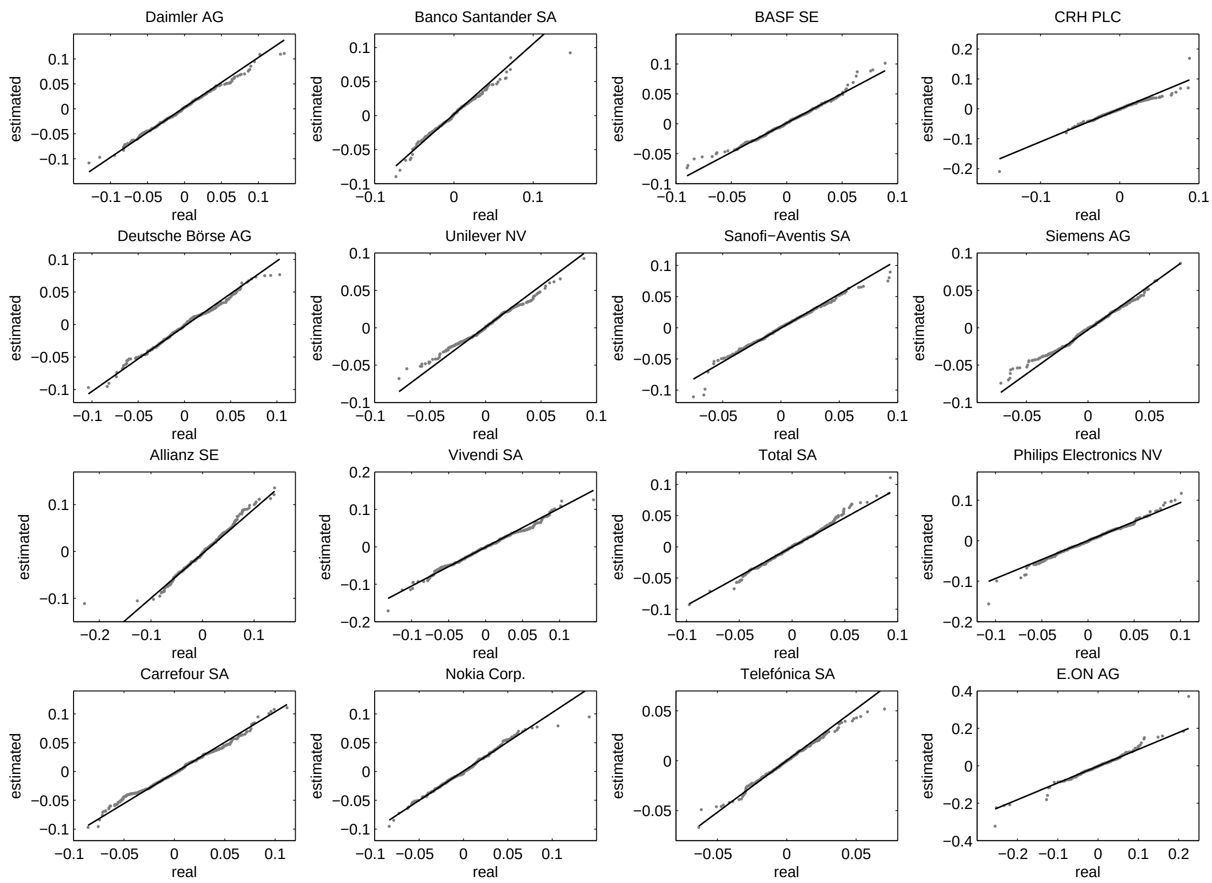
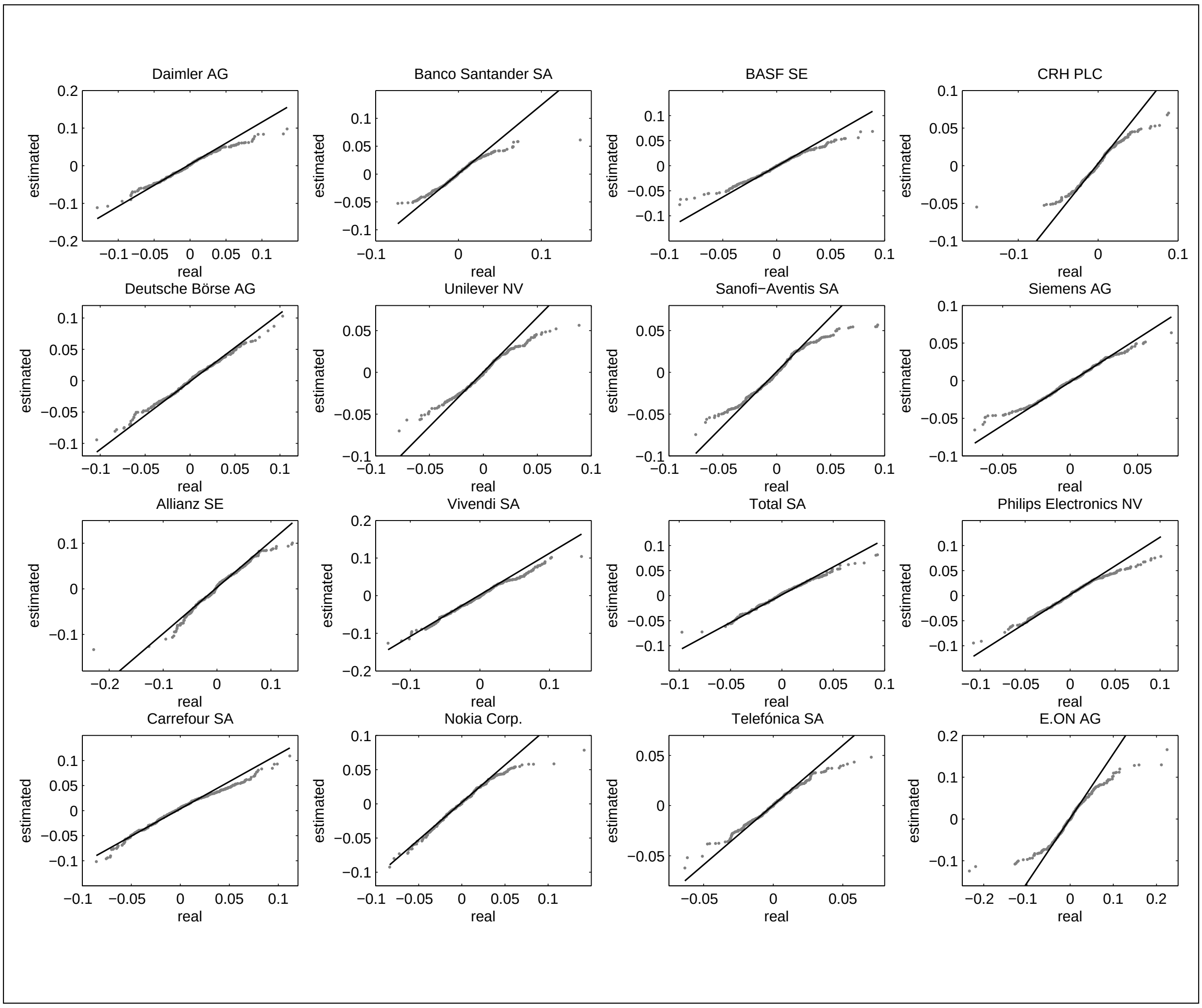


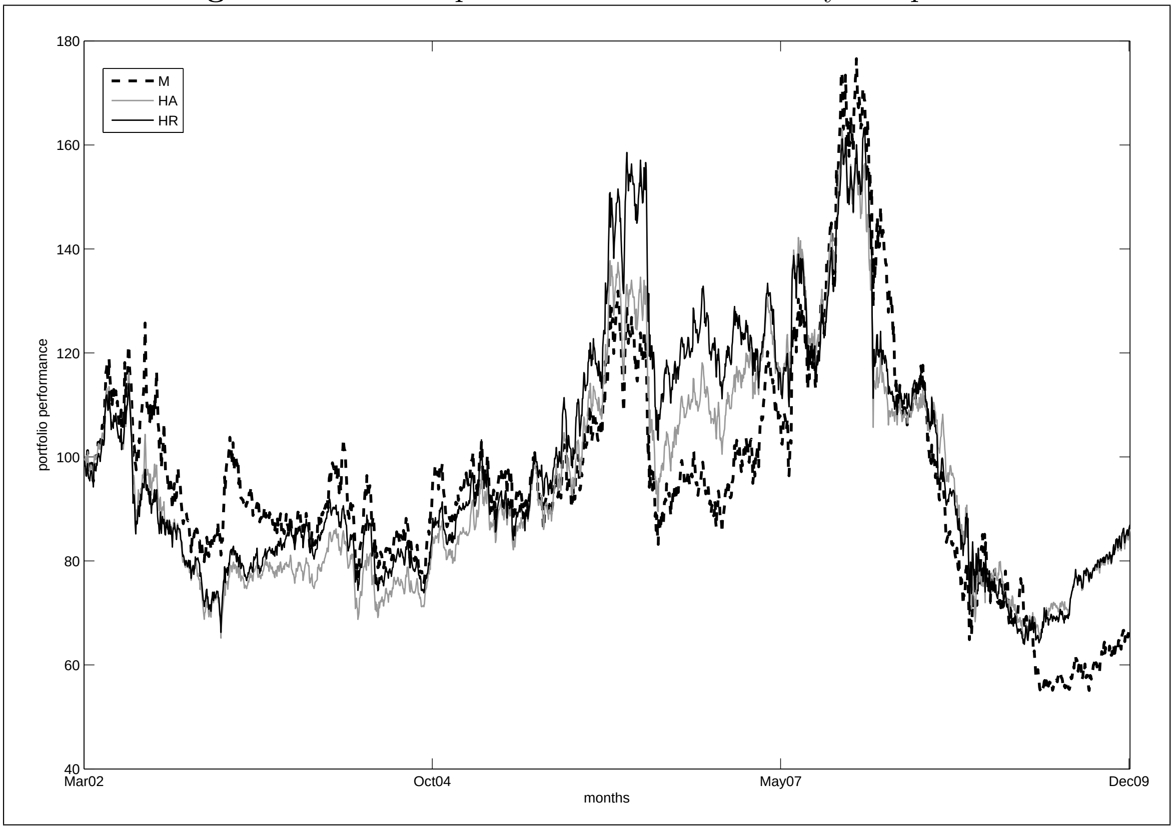
Figure 5: QQ plots using normal distributions

The figure shows the QQ plots for each stock assuming normally distributed rates of return for the first subperiod. Realized returns are plotted along the x-axis and simulated returns are plotted on the y-axis.



Appendix 2

Figure 6: Portfolio performance for the 250-days subperiods



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ISSN 1615-4274