Climate change mitigation options and directed technical change: A decentralized equilibrium analysis

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Abstract

The paper considers a climate change growth model with three R&D sectors dedicated to energy, backstop and CCS (Carbon Capture and Storage) efficiency. First, we characterize the set of decentralized equilibria: A particular equilibrium is associated to each vector of public tools which includes a carbon tax and a subsidy to each R&D sector. Moreover, we show that it is possible to compute any equilibrium as the solution of a maximization program. Second, we solve the first-best optimum problem and we implement it by computing the vector of optimal tools. Finally, we illustrate the theoretical model using some calibrated functional specifications. In particular, we investigate the effects of various combinations of public policies (including the optimal ones) by determining the deviation of each corresponding equilibrium from the "laisser-faire" benchmark.

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1 Introduction

There exists a large strand of literature on economic growth, climate change and technological improvements (see for instance Bosetti et al., 2006 and 2009; Edenhofer et al., 2005 and 2006; Gerlagh 2006; Gerlagh and Van Der Zwaan 2006; Nordhaus, 2008; Popp, 2006a and 2006b). In these models, the analysis usually focuses on the optimal trajectories and their comparison with the business-as-usual scenario. For many reasons that will be discussed below, it may be relevant to examine some intermediate cases between these two polar ones. Nevertheless, a decentralized economy framework is required to perform such an analysis. The main objective of this paper is to complete the literature mentioned above by setting up a general equilibrium analysis that allows to compute any equilibrium in the decentralized economy.

A full description of the set of equilibria offers several advantages. Under a positive point of view, it allows to examine how the economy reacts to policy changes. We can thus look at the individual effects of a given policy instrument as well as a given subset of them, the other ones being kept unchanged. This will give some insights on the complementarity/substitutability of public tools. Under a normative point of view, as usual, this approach allows for the computation of the economic instruments that restore the first-best optimum. However, because of budgetary, socioeconomic or political constraints, the enforcement of first-best optimum can be difficult to achieve for the policy-maker that would rather implement second-best solutions. Finally, another advantage is the possibility to compare the outcome of a cost-benefit analysis in a partial equilibrium approach (e.g. Gerlagh et al., 2008) with the one obtained from a general equilibrium framework.

In line with the "top-down" approach and based on the DICE and ENTICE-BR models (Nordhaus, 2008, and Popp, 2006a, respectively), we develop an endogenous growth model in which energy services can be produced from a polluting non-renewable resource as well as a clean backstop. Moreover, we assume that carbon emissions can be partially released thanks to CCS (Carbon Capture and Storage) technology. We introduce three R&D sectors, the first one improving the efficiency of energy production, the second one, the efficiency of the backstop and the last one, the efficiency of the sequestration process. With this respect, we have to consider two types of market failures: the pollution associated with the atmospheric release of carbon and the research spillovers in each R&D sector. For this matter, in the decentralized equilibrium, we introduce two kinds of economic policy instruments in accordance: an environmental tax on the carbon emissions and a research subsidy for the energy, backstop and CCS R&D sectors. There is an equilibrium associated to each vector of instruments. Clearly, when public instruments are optimally set, the equilibrium of the decentralized economy coincides with the first best optimum. In particular, we provide a full expression of the optimal carbon tax and we analyze its dynamic properties. As in Goulder and Mathai (2000), we show that the tax can evolve non-monotonically over time and we characterize the driving forces that make it either growing or declining.

At this point, three remarks can be formulated. The first one is related to the way we deal with R&D sectors in the decentralized framework. In the standard endogenous growth theory (Aghion and Howitt, 1992; Romer, 1990...), the production of an innovation is associated with a particular intermediate good. Research is funded by the monopoly profits of intermediate producers who benefit from an exclusive right, like a patent, for the production and the sale of these goods. In this paper, to simplify the analysis, we do not explicitly introduce tangible intermediate goods in research sectors, as it is done for instance by Gerlagh and Lise (2005), Edenhofer et al. (2006) and Popp (2006a). Then, we adopt the shortcut proposed by Grimaud and Rougé (2008) in the case of growth models with polluting resources and environmental concerns. This approach is based on the comparison between the socially optimal value of innovations and the private one, which emerges at the decentralized equilibrium. Several empirical studies (Jones and Williams, 1998; Popp, 2006a) find that this last value is lower than the former one. This is justified in the standard literature by the presence of some failures that prevent the decentralized equilibrium to implement the first-best optimum. We use these studies to build the "laisser-faire" equilibrium. Finally, research subsidies can be enforced in order to reduce the gap between these social and private values[1].

The second remark is a technical one which concerns the computation of the economic variables, quantities and prices, in the decentralized economy. As usual, the first step consists in studying the behavior of agents and, under market clearing conditions, in characterizing the equilibrium trajectory. In a second step, we show that there exists an

[1]According to the OECD Science, Technology and R&D Statistics, publicly-funded energy R&D in 2004 among OECD countries amounted to 9.72 billion US$, which represented 4% of overall public R&D budgets. In the United States, energy investments from the private sector have shrunk during the last decade; governmental funding currently represents 76% of total US energy R&D expenditures (Nemet and Kammen, 2007).

optimization program whose the solution is the same as the equilibrium one. This allows the numerical computation of any equilibrium trajectories in a calibrated model.

The last remark is about the particular decarbonisation technology considered. As recommended by the IPCC, abatement technologies reveal crucial for the implementation of a cost-effective climate change mitigation policy. Such abatement technologies notably include renewable energy but also the possibility to reduce the carbon footprint of fossil fuel burning. According to the IPCC (2005), carbon capture and storage (CCS) offers promising prospects. This process consists in separating the carbon dioxide from other flux gases during the process of energy production. It is particularly adapted to large-scale centralized power stations but may also indirectly apply to non electric energy supply (cf. Hoffert et al., 2002). Once captured, the gases are then being disposed into various reservoirs, such as depleted oil and gas fields, depleted coal mines, deep saline aquifers, or oceans.

Next, we provide some numerical illustrations by calibrating the model to fit the world 2005 data. As suggested by the theory, the optimal carbon tax is generally non-monotonic over time. We find that the implementation of this tax alone leads to the expected effects on the fossil fuel use (and then on carbon emissions), but it does not provide incentive enough to hardly stimulate R&D activities. Similarly, research policies alone have high impacts on R&D activities, but their effect on the atmospheric carbon accumulation is very low. In other words, the crossed effects of each policy instrument are weak. Moreover, the simultaneous use of these two types of public tools reinforces the individual role of each one, thus revealing high complementarity between them. For instance, we observe numerically that the simultaneous implementation of a carbon tax and appropriate R&D subsidies can strengthen the role of the backstop and of the CCS. Finally, the recourse to these two abatement options is reinforced by a more ambitious carbon tax, in order to stabilize the atmospheric carbon concentration for instance.

The article is organized as follows. Section 2 presents the decentralized economy and studies the behavior of agents in each sector. In section 3, i) we characterize the decentralized equilibrium, ii) we identify the maximization program associated with this equilibrium, iii) we characterize the first-best optimum solutions, and iv) we compute the appropriate public tools that implement the optimum. In section 4, we present the calibration of the model and we derive a selection of numerical results. We conclude in section 5.

2 The decentralized economy

The model is mainly based on ENTICE-BR (Popp, 2006a) and on the last version of DICE (Nordhaus, 2008). We consider a worldwide economy containing four production sectors: final output, energy services, fossil fuel and carbon-free backstop. The fossil fuel combustion process releases CO_2 flows which accumulate into the atmosphere, inducing a rise of the average temperatures. Feedbacks on the economy are captured by a damage function measuring the continuous and gradual losses in terms of final output due to global warming (i.e. GWP losses). Moreover, an atmospheric carbon concentration cap can be eventually introduced to take into account the high levels of uncertainty and irreversibility that are generally avoided by the standard damage function. Industrial emissions can be partly sequestered and stored in carbon reservoirs owing to a CCS device. The production of final energy services, backstop and CCS require specific knowledge provided by three directed R&D sectors (in the sense of Acemoglu, 2002). We assume that all sectors, except the R&D's ones, are perfectly competitive. Finally, in order to correct the two types of distortions involved by the model – pollution and research spillovers – we introduce two types of policy tools: an environmental tax on the fossil fuel use and a subsidy for each R&D sector. Note that, because of CCS, the tax applies on the residual carbon emissions after sequestration and it is thus disconnected from the fossil resource use.

The model is sketched in Figure 1. Specific functional forms and calibration details are described in appendix A4. The following subsection derives the individual behaviors.

2.1 Behavior of agents

2.1.1 The final good sector

The production of a quantity Q_t of final good depends on three endogenous elements: capital K_t, energy services E_t, and a scaling factor Ω_t which accounts from climate-related damages, as discussed below. It also depends on exogenous inputs: the total factor productivity A_t and the population level L_t, growing at exogenous rates $g_{A,t}$ and $g_{L,t}$ respectively. We write $Q_t = Q(K_t, E_t, L_t, A_t, \Omega_t)$, where the production function $Q(.)$ is assumed to have the standard properties (increasing and concave in each argument).

Normalizing to one the price of the final output and denoting by $p_{E,t}$, w_t, r_t and δ, the price of energy services, the real wage, the interest rate[2] and the depreciation

[2] We assume here that the representative household holds the capital and rents it to firms at a rental price R_t. Standard arbitrage conditions imply $R_t = r_t + \delta$.

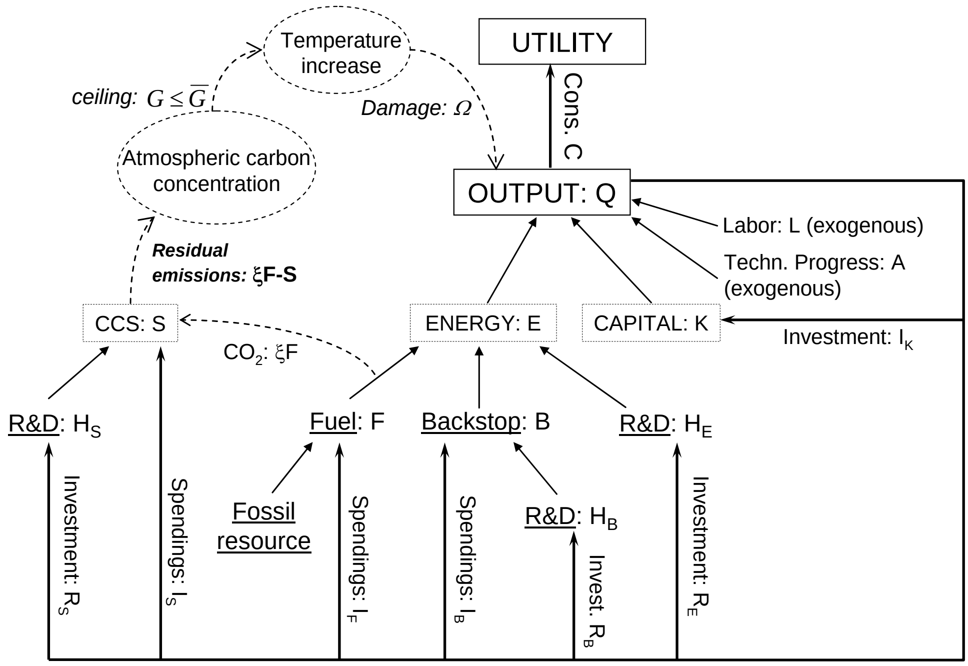

Figure 1: Description of the model

rate of capital, respectively, the instantaneous profit of producers is expressed as $\Pi_t^Q = Q_t - p_{E,t}E_t - w_tL_t - (r_t + \delta)K_t$. Maximization of this profit function with respect to K_t, L_t and E_t, leads to the following first-order conditions:

$$Q_K - (r_t + \delta) = 0 \quad (1)$$

$$Q_L - w_t = 0 \quad (2)$$

$$Q_E - p_{E,t} = 0 \quad (3)$$

where J_X stands for the partial derivative of function $J(.)$ with respect to X.

2.1.2 The energy-CCS sector

At each time t, the amount E_t of energy services is produced from two primary energies – a fossil fuel F_t and a backstop energy source B_t – and from a stock $H_{E,t}$ of specific knowledge which can improve the energy efficiency. The energy supply is then $E_t = E(F_t, B_t, H_{E,t})$, where $E(.)$ is assumed to be increasing and concave in each argument.

The economic and climatic systems are linked in the model by anthropogenic CO_2 emissions, generated by fossil fuel burning. Without CCS, the carbon flow released into the atmosphere would be equal to ξF_t, where ξ is the unitary carbon content of fossil fuel. We postulate that, at each date t, the CCS device allows a reduction of these emissions by an amount S_t, $0 \leq S_t \leq \xi F_t$ and, for the sake of simplicity, that CCS activities are part of the energy sector. To change emissions into stored carbon, the sequestration device needs specific investment spendings, $I_{S,t}$, and knowledge, $H_{S,t}$. The CCS technology then writes $S(F_t, I_{S,t}, H_{S,t})$, with $S(.)$ increasing and concave in each argument[3]. Note that in our model, we consider neither limited capacity of carbon sinks nor leakage problems. These questions are addressed, for instance, by Lafforgue et al. (2008) and Keller et al. (2007) respectively.

Denoting by $p_{F,t}$ and $p_{B,t}$ the fossil fuel and backstop prices, and by τ_t the unitary carbon tax on the flow of carbon emissions $(\xi F_t - S_t)$, the energy producer chooses F_t, B_t and $I_{S,t}$ that maximizes its instantaneous profit $\Pi_t^E = p_{E,t}E_t - p_{F,t}F_t - p_{B,t}B_t - I_{S,t} - \tau_t(\xi F_t - S_t)$. The first order conditions write:

$$p_{E,t}E_F - p_{F,t} - \tau_t(\xi - S_F) = 0 \tag{4}$$

$$p_{E,t}E_B - p_{B,t} = 0 \tag{5}$$

$$-1 + \tau_t S_{I_S} = 0 \tag{6}$$

Condition (6) equalizes the private cost of one unit of stockpiled carbon, $1/S_{I_S}$, with the carbon tax. Moreover, the extended unit cost of fossil fuel use, including the fuel price, the environmental penalty and the sequestration cost, can be defined as:

$$c_{F,t} = p_{F,t} + \frac{\tau_t(\xi F_t - S_t)}{F_t} + \frac{I_{S,t}}{F_t} \tag{7}$$

2.1.3 The primary energy sectors

At each time t, the extraction flow F_t of fossil resource depends on specific productive investments, $I_{F,t}$, and on the cumulated past extraction, Z_t. As in Popp (2006a) or in Gerlagh and Lise (2005), we do not explicitly model an initial fossil resource stock that is exhausted, but we focus on the increase in the extraction cost as the resource is depleted. We denote by Z_t the amount of resource extracted from the initial date up to t:

$$Z_t = \int_0^t F_s ds \Rightarrow \dot{Z}_t = F_t \tag{8}$$

[3]In a Romer model with tangible intermediate goods, the energy and CCS production functions would write $E_t = E\left[F_t, B_t, \int_0^{H_{E,t}} f^E(x_{j,t}^E)dj\right]$ and $S_t = S\left[F_t, I_{S,t}, \int_0^{H_{S,t}} f^S(x_{j,t}^S)dj\right]$ respectively, where $x_{j,t}^n$ is the j^{th} intermediate good and $f^n(.)$ is an increasing and strictly concave function, for $n = \{E, S\}$.

The fossil fuel extraction function is denoted by $F(I_{F,t}, Z_t)$, where $F(.)$ is increasing and concave in I_F, decreasing and convex in Z. The fuel producer must choose $\{I_{F,t}\}_{t=0}^{\infty}$ that maximizes $\int_0^{\infty} \Pi_t^F e^{-\int_0^t r_s ds} dt$ subject to (8), where its instantaneous profit is $\Pi_t^F = p_{F,t}F_t - I_{F,t}$. Denoting by η_t the multiplier associated with (8), the static and dynamic first-order conditions are:

$$(p_{F,t}F_{I_F} - 1)e^{-\int_0^t r_s ds} + \eta_t F_{I_F} = 0 \tag{9}$$

$$p_{F,t}F_Z e^{-\int_0^t r_s ds} + \eta_t F_Z = -\dot{\eta}_t \tag{10}$$

Combining these two equations, and using the transversality condition $\lim_{t\to\infty} \eta_t Z_t = 0$, we get the following fossil fuel price expression:

$$p_{F,t} = \frac{1}{F_{I_F}} - \int_t^{\infty} \frac{F_Z}{F_{I_F}} e^{-\int_t^s r_x dx} ds \tag{11}$$

Differentiating (11) with respect to time, it comes:

$$\dot{p}_{F,t} = r_t \left(p_{F,t} - \frac{1}{F_{I_F}} \right) + \frac{1}{F_{I_F}} \left(F_Z - \frac{\dot{F_{I_F}}}{F_{I_F}} \right) \tag{12}$$

which reads as a generalized version of the Hotelling rule in the case of an extraction technology given by function $F(.)$. In particular, if the marginal productivity of investment spendings coincides with the average productivity, i.e. if $F_{I_F} = F(.)/I_F$, then it is easy to see that (12) reduces to $\dot{p}_{F,t} = r_t(p_{F,t} - 1/F_{I_F})$. In the limit case where the marginal productivity tends to infinity, i.e. the marginal extraction cost tends to zero, one gets the elementary Hotelling rule, $\dot{p}_{F,t}/p_{F,t} = r_t$.

The backstop production function $B(I_{B,t}, H_{B,t})$ is assumed to be increasing and concave in the investment spending $I_{B,t}$ and in the specific stock of knowledge $H_{B,t}$.[4] Maximization of the profit $\Pi_t^B = p_{B,t}B(I_{B,t}, H_{B,t}) - I_{B,t}$, yields the following first-order condition:

$$p_{B,t}B_{I_B} - 1 = 0 \tag{13}$$

2.1.4 The R&D sectors

As already mentioned in the introduction, R&D sectors generally face several distortions. Jones and Williams (2000) identify four of them: i) *the duplication effect*: the R&D sector does not account for the redundancy of some research projects; ii) *the intertemporal spillover effect*: inventors do not account for that ideas they produce are used to produce

[4] Again, in a model with tangible intermediate goods, the backstop technology would write $B_t = B\left[I_{B,t}, \int_0^{H_{B,t}} g(x_{i,t}^B) di\right]$.

new ideas; iii) *the appropriability effect*: inventors appropriate only a part of the social value they create; iv) *the creative-destruction effect.* The global effect resulting from these distortions explains why the social value of an innovation is generally different from the private one. On this point, there does not exist a clear theoretical consensus emerging from the standard literature on endogenous growth. For instance, in the Romer's model (1990) with horizontal innovations, the private value is lower than the social one. However, Benassy (1998) showed that a slight modification of the Romer's model can lead to the opposite result. In the Aghion and Howitt model with vertical innovations (1992), the private value can be either larger or smaller than the social one, depending upon the parameters of the model.

However, there is an empirical evidence for a smaller private value. Jones and Williams (1998) estimate that research investments are at least four times below what would be socially optimal (on this point, see also Popp, 2006a, or Hart, 2008). In the following, we base our analysis on this observation.

There are three stocks of knowledge, each associated with a specific R&D sector (i.e. the energy, the backstop and the CCS ones). We consider that each innovation is a non-rival, indivisible and infinitely durable piece of knowledge (for instance, a scientific report, a data base, a software algorithm...) which is simultaneously used by the sector which produces the good i and by the R&D sector i, $i = \{B, E, S\}$. Thus, an innovation is not directly embodied into tangible intermediate goods and it cannot be financed by the sale of these goods. To circumvent this obstacle, one solution would consist in assuming that firms simultaneously produce output and undertake research. In that case, under perfect competition and constant returns to rival inputs, once these inputs have been payed, residual profits are nil. An imperfect competition framework would thus be required to generate positive profits allowing the firms to buy innovations, as it is done in Grimaud and Rouge (2008). This type of development would lead to several difficulties which are out of the scope of the present study. Moreover, Grimaud and Rouge (2008) show that Cournot competition does note prevent optimality when the labor supply is exogenous[5], which is the case in our model.

In order to avoid any problem, we adopt a shortcut aiming at directly valuing innovations. Basically, we proceed in three steps: i) In each research sector, we determine the

[5]Under Cournot competition, the real wage is lower than the optimal one, which implies an income transfer from labor to capital activities. However, since we assume a single representative agent with exogenous labor supply, this transfer has no effect on the equilibrium quantities.

social value of an innovation. Since an innovation is a non-rival good, this social value is the sum of the marginal profitabilities of this innovation in each sector using it. ii) Because of the failures mentioned above, the private (or effective) value in the absence research policy is lower than the social one. iii) The research sectors are eventually subsidized in order to reduce the gap between these two values.

Let us apply this three-steps procedure to the backstop R&D sector for instance. Each innovation produced by this sector is used by this R&D sector itself as well as by the backstop production sector. Thus, at each date t, the instantaneous social value of this innovation is $\bar{v}_{B,t} = \bar{v}^{B}_{B,t} + \bar{v}^{H_B}_{B,t}$, where $\bar{v}^{B}_{B,t}$ and $\bar{v}^{H_B}_{B,t}$ are the marginal profitabilities of this innovation in the backstop production sector and in the backstop R&D sector, respectively. The social value of this innovation at t, or equivalently the optimal value at t of an infinitely lived patent, is $\bar{V}_{B,t} = \int_t^\infty \bar{v}_{B,s} e^{-\int_t^s r_x dx} ds$. The same procedure applies for any R&D sector i, $i = \{B, E, S\}$. We denote by γ_i, $0 < \gamma_i < 1$, the rate of appropriability of the innovation value by the market, i.e. the share of the social value which is effectively paid to the innovator, and by σ_i (assumed constant for the sake of simplicity) the subsidy rate that government can eventually apply. Note that if $\sigma_i = 1 - \gamma_i$, the effective value matches the social one. The instantaneous effective value (including subsidy) is:

$$v_{i,t} = (\gamma_i + \sigma_i)\bar{v}_{i,t} \tag{14}$$

and the intertemporal effective value at date t is:

$$V_{i,t} = \int_t^\infty v_{i,s} e^{-\int_t^s r_x dx} ds \tag{15}$$

Differentiating (15) with respect to time leads to the usual arbitrage relation:

$$r_t = \frac{\dot{V}_{i,t}}{V_{i,t}} + \frac{v_{i,t}}{V_{i,t}}, \quad \forall i = \{B, E, S\} \tag{16}$$

which equates the rate of return on the financial market to the rate of return on the R&D sector i.

We can now analyze the R&D sector behavior. We assume that the dynamics of the knowledge stock $H_{i,t}$ is governed by the following innovation function $H^i(.)$:

$$\dot{H}_{i,t} = H^i(R_{i,t}, H_{i,t}) \tag{17}$$

where $R_{i,t}$ is the R&D investment into sector i. Function $H^i(.)$ is assumed to be increasing and concave in each argument[6]. At each time t, sector i supplies the flow of innovations

[6] As previously, in a model with tangible intermediate goods, (17) would be replaced by $\dot{H}_{i,t} = H^i\left[R_{i,t}, \int_0^{H_{i,t}} h(x^H_{i,t}) di\right]$.

$\dot{H}_{i,t}$ at price $V_{i,t}$, so that its profit function is $\Pi_t^{H_i} = V_{i,t}H^i(R_{i,t}, H_{i,t}) - R_{i,t}$. The first-order condition implies:

$$V_{i,t} = \frac{1}{H^i_{R_i}} \tag{18}$$

Using (18), we compute the marginal profitability of innovations in R&D sector i as:

$$\bar{v}_{i,t}^{H_i} = \frac{\partial \Pi_t^{H_i}}{\partial H_{i,t}} = V_{i,t}H^i_{H_i} = \frac{H^i_{H_i}}{H^i_{R_i}} \tag{19}$$

Finally, from the expressions of Π_t^B and Π_t^E, the marginal profitabilities of a backstop, energy and CCS innovation in the production sectors using them, are given respectively by $\bar{v}_{B,t}^B = \partial\Pi_t^B/\partial H_{B,t} = B_{H_B}/B_{I_B}$, $\bar{v}_{E,t}^E = \partial\Pi_t^E/\partial H_{E,t} = E_{H_E}/E_B B_{I_B}$ and $\bar{v}_{S,t}^E = \partial\Pi_t^E/\partial H_{S,t} = \tau_t S_{H_S}$. Therefore, the instantaneous effective values (including subsidies) of innovations are:

$$v_{B,t} = (\gamma_B + \sigma_B)\left(\frac{B_{H_B}}{B_{I_B}} + \frac{H^B_{H_B}}{H^B_{R_B}}\right) \tag{20}$$

$$v_{E,t} = (\gamma_E + \sigma_E)\left(\frac{E_{H_E}}{E_B B_{I_B}} + \frac{H^E_{H_E}}{H^E_{R_E}}\right) \tag{21}$$

$$v_{S,t} = (\gamma_S + \sigma_S)\left(\tau_t S_{H_S} + \frac{H^S_{H_S}}{H^S_{R_S}}\right) \tag{22}$$

2.1.5 The household and the government

Denoting by C_t the consumption at time t, by $U(.)$ the instantaneous utility function (assumed to have the standard properties) and by $\rho > 0$ the pure rate of time preferences, households maximize the welfare function $W = \int_0^\infty U(C_t)e^{-\rho}dt$ subject to its dynamic budget constraint:

$$\dot{K}_t = r_t K_t + w_t L_t + \Pi_t - C_t - T_t^a \tag{23}$$

where Π_t is the total profits gained in the economy and T_t^a is a lump-sum tax (subsidy-free) that allows to balance the budget constraint of the government. This maximization leads to the standard Keynes-Ramsey rule:

$$\rho - \frac{\dot{U}'(C_t)}{U'(C_t)} = \rho + \epsilon_t g_{C,t} = r_t \Rightarrow U'(C_t) = U'(C_0)e^{\rho t - \int_0^t r_s ds} \tag{24}$$

where ϵ_t is the inverse of the elasticity of intertemporal substitution of consumption, and $g_{C,t}$ is the instantaneous growth rate of consumption.

Assuming that the government's budget constraint is balanced at each time t (i.e. the sum of the various taxes equals R&D subsidies), then we have:

$$T_t^a + \tau_t(\xi F_t - S_t) = \sum_i Sub_{i,t}, \quad i = \{B, E, S\} \tag{25}$$

where $Sub_{i,t}$ denotes the amount of subsidy distributed to R&D sector i:

$$Sub_{i,t} = \left[\int_t^{\infty} \left(\frac{\sigma_i}{\gamma_i + \sigma_i} \right) v_{i,s} e^{-\int_t^s r_x dx} ds \right] H^i(R_{i,t}, H_{i,t}) \tag{26}$$

Finally, the balance equation of the final output writes:

$$Q_t = C_t + I_{F,t} + I_{B,t} + I_{S,t} + I_{K,t} + R_{E,t} + R_{B,t} + R_{S,t} \tag{27}$$

where $I_{K,t}$ is the instantaneous investment in capital, given by:

$$I_{K,t} = \dot{K}_t + \delta K_t \tag{28}$$

Hence, in our worldwide economy, the final output is devoted to aggregated consumption, fossil fuel production, backstop production, CCS, capital accumulation and R&D.

2.2 The environment and damages

Let G_t be the atmospheric carbon concentration at time t and ζ, $\zeta > 0$, the natural rate of decay. The increase in G_t drives the global mean temperature away from a given state, here the 1900 level. The difference between this state and the present global mean temperature, denoted by T_t, is taken here as the index of anthropogenic climate change. The climate dynamics can thus be captured by the following system:

$$\dot{G}_t = \xi F_t - S_t - \zeta G_t \tag{29}$$

$$\dot{T}_t = \Phi(G_t) - mT_t, \quad m > 0 \tag{30}$$

where $\Phi(.)$ is a simplified radiative forcing function, assumed to be increasing and concave in G, and m is a parameter of climatic inertia[7].

Global warming generates economic damages that are measured, by convention, in terms of final output losses through the scaling factor $\Omega(T_t)$, with $\Omega'(.) < 0$. In addition to the damage reflected by Ω_t, we will possibly be induced to impose a stabilization cap on the carbon pollution stock that society can not overshoot (see for instance Chakravorty et al., 2006):

$$G_t \leq \bar{G}, \quad \forall t \geq 0 \tag{31}$$

[7] In the analytical treatment of the model, we assume, for the sake of clarity, that the carbon cycle through atmosphere and oceans as well as the dynamic interactions between atmospheric and oceanic temperatures, are captured by the reduced form (29) and (30). However, in the numerical simulations, we adopt the full characterization of the climate module coming from the last version of DICE (Nordhaus, 2008).

This additional constraint can be justified by the fact that the social damage function is not able to reflect the entire environmental damages, but only part of it. In reality, uncertainty in the climatic consequences of global warming can imply some discontinuities in the damage, such as natural disasters or other strong irreversibilities, that are not taken into account by the standard functional representation of the damage.

3 Decentralized equilibrium and welfare analysis

3.1 Characterization of the decentralized equilibrium

From the previous analysis of individual behaviors, we can now study the set of equilibria. A particular equilibrium is associated with each quadruplet of policies $\{\sigma_B, \sigma_E, \sigma_S, \tau_t\}_{t=0}^{\infty}$. It is defined as a vector of quantity trajectories $\{Q_t, K_t, E_t, ...\}_{t=0}^{\infty}$ and a vector of price profiles $\{r_t, p_{E,t}, ...\}_{t=0}^{\infty}$ such that: i) firms maximize profits, ii) the representative household maximizes utility, iii) markets of private (i.e. rival) goods are perfectly competitive and cleared, iv) in each R&D sectors i, innovators receive a share $(\gamma_i + \sigma_i)$ of the social value of innovations. Such an equilibrium is characterized by the set of equations given by Proposition 1 below. Clearly, as analyzed in the following subsection, if the policy tools are set to their optimal levels, these equations also characterize the first-best optimum together with the system of prices that implements it.

Proposition 1 *At each time t, for a given quadruplet of policies $\{\sigma_B, \sigma_E, \sigma_S, \tau_t\}_{t=0}^{\infty}$, the equilibrium in the decentralized economy is characterized by the following seven-equations system:*

$$Q_E E_F - \tau_t(\xi - S_F) - \frac{1}{F_{I_F}} = \frac{-1}{U'(C_t)} \int_t^{\infty} \frac{F_Z}{F_{I_F}} U'(C_s) e^{-\rho(s-t)} ds \quad (32)$$

$$Q_E E_B = \frac{1}{B_{I_B}} \quad (33)$$

$$\frac{1}{S_{I_S}} = \tau_t \quad (34)$$

$$Q_K - \delta = \rho + \epsilon_t g_{C,t} \quad (35)$$

$$(\gamma_B + \sigma_B)\left(\frac{B_{H_B} H_{R_B}^B}{B_{I_B}} + H_{H_B}^B\right) - \frac{\dot{H}_{R_B}^B}{H_{R_B}^B} = \rho + \epsilon_t g_{C,t} \quad (36)$$

$$(\gamma_E + \sigma_E)\left(\frac{E_{H_E} H_{R_E}^E}{E_B B_{I_B}} + H_{H_E}^E\right) - \frac{\dot{H}_{R_E}^E}{H_{R_E}^E} = \rho + \epsilon_t g_{C,t} \quad (37)$$

$$(\gamma_S + \sigma_S)\left(\tau_t S_{H_S} H_{R_S}^S + H_{H_S}^S\right) - \frac{\dot{H}_{R_S}^S}{H_{R_S}^S} = \rho + \epsilon_t g_{C,t} \quad (38)$$

The associated system of prices $\left\{r_t^*, w_t^*, p_{E,t}^*, p_{F,t}^*, p_{B,t}^*, V_{i,t}^*\right\}_{t=0}^{\infty}$ *is obtained from the equations (1), (2), (3), (11), (13) and (18), respectively.*

Proof. See Appendix A1.

Equation (32) is an arbitrage condition that equalizes the marginal net profit from the increase by one unit of fossil fuel extraction (LHS) to the total marginal gain if there is no additional extraction (RHS)[8]. Equation (33) tells that the marginal productivity of the backstop (LHS) equals its marginal cost (RHS). As already mentioned, equation (34) formalizes the incentive effect of the carbon tax on the decision to invest in CCS. Equation (35) characterizes the standard trade-off between capital K_t and consumption C_t. Equation (36) (resp. (37) and (38)) characterizes the same kind of trade-off between specific investment into backstop R&D sector, $R_{B,t}$ (resp. energy R&D sector, $R_{E,t}$, and CCS R&D sector, $R_{S,t}$) and consumption. Obviously, the marginal return of each specific stock of knowledge H_i depends on the associated rate of subsidy σ_i.

3.2 The decentralized equilibrium under maximization form

In order to solve numerically the market outcome, we show that it is possible to transform the decentralized problem described above into a single maximization program. Proposition 2 explains how to proceed.

Proposition 2 *Solving the following program (we drop time subscripts for notational convenience):*

$$\max_{\{C,R_i,I_j,i=\{B,E,S\},j=\{F,E,S\}\}} \int_0^{\infty} U(C)e^{-\rho t}dt \quad \textit{subject to:}$$

$$\begin{aligned}
\dot{K} &= Q\left\{K, E[B(I_B,H_B)], F(I_F,Z), H_E], L, A, \Omega\right\} - C - \delta K - \sum_i R_i - \sum_j I_j \\
&\quad -\tau\left\{\xi F(I_F,Z) - S[F(I_F,Z), I_S, H_S]\right\}, \\
\dot{H}_i &= (\gamma_i + \sigma_i)H^i(R_i,H_i), \\
\textit{and } \dot{Z} &= F(I_F,Z)
\end{aligned}$$

leads to the same system of equations, (32)-(38), than in Proposition 1.

[8] If extraction increases by one unit, the associated revenue is $Q_E E_F$ and firms face two kinds of costs: the extraction cost, $1/F_{I_F}$, and the pollution cost, $\tau(\xi - S_F)$. Conversely, if no more fossil resource is extracted during the time interval dt, this generates an instantaneous gain due to the diminution in specific investment spending I_F corresponding to $(dI_F/dt)/F|_{dF=0} = -F_Z/F_{I_F}$. Multiplying this term by the marginal utility and integrating from t to ∞ with the discount rate ρ gives the total gain in terms of utility. Finally, dividing by $U'(C)$, this expression gives the gain in terms of output as specified in the RHS of (32).

Proof. See Appendix A2.

Proposition 2 can be read in fact as the welfare maximization program of a representative agent who would own all firms (final sector, energy-CCS, fossil fuel, backstop and R&D) and who would face the same incentive policies (carbon tax, research subsidies) than firms in the decentralized economy. This approach is the same than the one followed by Sinclair (1994) who also writes the market equilibrium under maximization form. The main difference with our model is that he assumes an exogenous rate of Hicks-neutral technical change.

3.3 First-best optimum and implementation

The social planner problem consists in choosing $\{C_t, R_{i,t}, I_{j,t}\}_{t=0}^{\infty}$ that maximizes the social welfare W, subject to the various technological constraints, the output allocation constraint (27), the state equations (8), (17), (28), (29), (30), and finally, the ceiling constraint (31). After eliminating the co-state variables, the first-order conditions leads to Proposition 3 below.

Proposition 3 *At each time t, an optimal solution is characterized by the following seven-equations system:*

$$Q_E E_F - \frac{(\xi - S_F)}{S_{I_S}} - \frac{1}{F_{I_F}} = \frac{-1}{U'(C_t)} \int_t^{\infty} \frac{F_Z}{F_{I_F}} U'(C_s) e^{-\rho(s-t)} ds \tag{39}$$

$$Q_E E_B = \frac{1}{B_{I_B}} \tag{40}$$

$$\frac{1}{S_{I_S}} = \frac{-1}{U'(C_t)} \int_t^{\infty} \left[\Phi'(G_s) J_s - \varphi_{G,s} e^{\rho s}\right] e^{-(\zeta+\rho)(s-t)} ds \tag{41}$$

$$Q_K - \delta = \rho + \epsilon_t g_{C,t} \tag{42}$$

$$\frac{B_{H_B} H^B_{R_B}}{B_{I_B}} + H^B_{H_B} - \frac{\dot{H}^B_{R_B}}{H^B_{R_B}} = \rho + \epsilon_t g_{C,t} \tag{43}$$

$$\frac{E_{H_E} H^E_{R_E}}{E_B B_{I_B}} + H^E_{H_E} - \frac{\dot{H}^E_{R_E}}{H^E_{R_E}} = \rho + \epsilon_t g_{C,t} \tag{44}$$

$$\frac{S_{H_S} H^S_{R_S}}{S_{I_S}} + H^S_{H_S} - \frac{\dot{H}^S_{R_S}}{H^S_{R_S}} = \rho + \epsilon_t g_{C,t} \tag{45}$$

where $J_s = \int_s^{\infty} Q_\Omega \Omega'(T_x) U'(C_x) e^{-(m+\rho)(x-s)} dx \leq 0$ and where $\varphi_{G,s}$ is the Lagrange multiplier associated with constraint (31), thus satisfying $\varphi_{G,s} \geq 0$, with $\varphi_{G,s} = 0$ for any s such that $G_s < \bar{G}$.

Proof. See Appendix A3.

The interpretation of these conditions are almost the same than the ones formulated in Proposition 1, excepted that, now, all the trade-offs are optimally solved. In other respects, recall that, for a given set of public policies, a particular equilibrium is characterized by conditions (32)-(38) of Proposition 1. This equilibrium will be said to be optimal if it satisfies the optimum characterizing conditions (39)-(45) of Proposition 2. By analogy between these two sets of conditions, we can show that there exists a single quadruplet $\{\sigma_B, \sigma_E, \sigma_S, \tau_t\}_{t=0}^{\infty}$ that implements the first-best. These findings are summarized in Proposition 4 below.

Proposition 4 *The equilibrium characterized in Proposition 1 is optimal if and only if* $\{\sigma_B, \sigma_E, \sigma_S, \tau_t\}_{t=0}^{\infty} = \{\sigma_B^o, \sigma_E^o, \sigma_S^o, \tau_t^o\}_{t=0}^{\infty}$, *where* $\sigma_i^o = 1 - \gamma_i$ *for* $i = \{B, E, S\}$, *and where* τ_t^o *is given by:*

$$\tau_t^o = \frac{-1}{U'(C)} \int_t^{\infty} \left[\Phi'(G) \int_s^{\infty} Q_{\Omega} \Omega'(T) U'(C) e^{-(m+\rho)(x-s)} dx - \varphi_G e^{\rho s} \right] e^{-(\zeta+\rho)(s-t)} ds \quad (46)$$

Proof. First, if $\tau_t = \tau_t^o$, then conditions (39) and (41) are satisfied by using (32) and (34). Second, (40) and (42) are identical to (33) and (35), respectively. Third, if $\sigma_i = 1 - \gamma_i$, for $i = \{B, E, S\}$, then (43), (44) and (45) are identical to (36), (37) and (38), respectively.

Proposition 4 states first that, in any R&D sector, the optimal subsidy rate must be equal to the share of the social value of innovations which is not captured by the market, in order to entirely fill the gap between the private value and the social one. In section 4, according to several empirical studies, we will postulate that $\gamma_i = 0.3$, thus implying $\sigma_i^o = 0.7$ for $i = \{B, E, S\}$.

Second, the optimal trajectory of the carbon tax is given by (46). Since $\Omega'(T_t) < 0$, we have $\tau_t^o \geq 0$ for any $t \geq 0$. This expression reads as the ratio between the marginal social cost of climate change – the marginal damage in terms of utility coming from the emission of an additional unit of carbon – and the marginal utility of consumption. In other words, it is the environmental cost (in terms of final good) of one unit of carbon in the atmosphere. This carbon tax can be expressed as the sum of two components. The first one depends on the damage function and on the dynamics of the atmospheric carbon stock and temperatures. It gives the discounted sum of marginal damages from t to ∞ coming from the emission of an additional unit of carbon at date t. The second one is only related to the ceiling constraint through φ_G. It gives the social cost at t of one unit of carbon in the atmosphere due to a tightening in the ceiling constraint. Then, the sum of these two components is the instantaneous total social cost of one unit of carbon.

Log-differentiating (46) gives us the optimal growth rate of the tax:

$$\frac{\dot{\tau}_t}{\tau_t} = \zeta + \rho + \epsilon_t g_{C,t} + \frac{\left[\varphi_{G,t} e^{\rho t} - \Phi'(G_t) J_t\right]}{\int_t^\infty \left[\varphi_{G,s} e^{\rho s} - \Phi'(G_s) J_s\right] e^{-(\zeta+\rho)(s-t)} ds} \tag{47}$$

where $\rho + \epsilon_t g_{C,t}$ is equal to the interest rate r_t. As in Goulder and Mathai (2000), the dynamics of the optimal carbon tax results from the combination of three components. The sum of the two first ones, i.e. the optimal appropriate discount rate $\zeta + r_t$ in the terminology of Goulder and Mathai, is unambiguously positive. The last component in (47) reflects the full social cost of one unit of carbon, including both the direct marginal damage and the social cost of the carbon ceiling, and is unambiguously negative. It generalizes Goulder and Mathai's result to the case where a damage function and a carbon cap are simultaneously considered. To sum up, we have two opposite effects meaning that the carbon tax can either rise or fall over time[9,10]. In the following section, we illustrate this point by depicting some monotonous or non-monotonous trajectories depending on the relative weights of these effects. We will observe that, in the absence of carbon cap, the last component is relatively weak with respect to the discount term, and thus the tax is rising over time. Under ceiling constraint, this last term becomes stringent at the time the ceiling is reached and the tax exhibits an invert U-shape trajectory.

4 Numerical results

4.1 Calibration and scenarios

Functional forms and calibration of the associated parameters are mainly provided by the last version of DICE (Nordhaus, 2008) for the climate module, the final output, the social preferences, the feedbacks on economic productivity from climate change, the total factor productivity and demographic dynamics. The energy production and R&D characterizations come from ENTICE-BR (Popp, 2006a). For CCS technology, we use a specification derived from the sequestration cost function used in DEMETER (Gerlagh and van der Zwaan, 2006) and the calibration is updated from the IPCC special report on CCS (2005). Others calibrations are provided by IEA data. All these details are referred to appendix A4. The starting year is 2005.

To study the effects of policy instruments, we solve the equilibrium for various values

[9] In the case where the is only a ceiling and no damage, the tax is unambiguously rising over time as long as the ceiling is not reached since $\varphi_{G,t} = 0$ $\forall t$ such that $G_t < \bar{G}$.

[10] For discussions about the optimal time path of the carbon tax, see also for instance Sinclair, 1994, Ulph and Ulph, 1994, Farzin and Tahvonen, 1996, Hoel and Kverndokk, 1996, or Chakravorty et al., 2006.

of τ and σ, by using the method described in Proposition 2. Note that we restrict the scenarios to the case where, $\forall i$, $\gamma_i = 0.3$ and we will discuss later about the sensitivity of the model to this parameter. Moreover, we consider only symmetric R&D policies, i.e. the case where σ_i is independent of i.[11] The selected cases are listed in Table 1.

Scenario	τ_t	σ	Comment
A	0	0	Laisser-faire
B	τ_t^{sb}	0	Second-best tax, no R&D subsidy
C	0	0.7	R&D subsidies, no carbon tax
D	τ_t^{o}	0.7	First-best optimum (without ceiling)
E	τ_t^{550}	0.7	Optimum with a 550ppm cap
F	τ_t^{450}	0.7	Optimum with a 450ppm cap

Table 1: Summary of the various scenarios for $\gamma_i = 0.3$, $i = \{B, E, S\}$

The benchmark case A refers to the laisser-faire equilibrium (BAU), in which neither environmental tax nor R&D subsidies are set. In scenario B, we study the effect on the equilibrium of an environmental tax by assuming zero σ_i's and by setting τ_t equal to its second-best optimal level, τ^{sb}.[12] Similarly, in scenario C, we analyze the impact of R&D subsidy rates by assuming $\tau = 0$ and $\sigma = 1 - \gamma = 0.7$.[13] Scenario D refers to the first-best optimum without carbon cap. Finally, two stabilization caps of 450 and 550ppm, which are enforced owing to the specific tax trajectories τ_t^{550} and τ_t^{450} respectively, are also studied (cases E and F).

4.2 Summary of results

We adopt the following notations to summarize the effects of the various policy combinations. $\Delta X|_{A\to D}$ stands for the change in variable X due to a simultaneous increase of τ from 0 to τ^o and of the σ's from 0 to σ^o. These changes are illustrated in the following figures by a shift from the "scenario A" trajectories to the "scenario D" trajectories. $\Delta X|_{A\to B}$

[11] We do not discuss here about the differentiated effects of the R&D subsidies. In a model with two R&D sectors, Grimaud and Lafforgue (2008) show that cross effects are very weak, i.e. an R&D policy in a particular sector has no crowding out impact on the other sector. With more than two R&D sectors, a large number of scenarios can be considered, so that we let these developments for future research.

[12] Formally, it is the tax trajectory that maximizes social welfare given the constraint of zero research subsidy, in the set of decentralized equilibria. For more details on second-best policies, see Grimaud and Lafforgue (2008).

[13] Although the optimal subsidy rates are the same in scenarios C, D, E and F, the amount of subsidies that are distributed among R&D sectors may differ, cf. equation (26).

is the change of X due to an increase in τ from 0 to τ^{sb}, given $\sigma = 0$. Symmetrically, given $\tau = 0$, $\Delta X|_{A\rightarrow C}$ denotes the change in variable X due to a simultaneous increase of the σ's from 0 to σ^o. Finally, $\Delta X|_{D\rightarrow E/F}$ measures the change in X due to an increase in the tax level (i.e. the introduction of a ceiling constraint), given the optimal enforcement of the R&D policies. Table 3 provides the signs of the Δ's for the main variables of interest (where insignificant changes are depicted by $\sim$).

X	$\Delta X\|_{A\rightarrow D}$	$\Delta X\|_{A\rightarrow B}$	$\Delta X\|_{A\rightarrow C}$	$\Delta X\|_{D\rightarrow E}$
p_F	$-$	$-$	$\sim$	$-$
c_F	$+$	$+$	$\sim$	$+$
p_B	$-$	$\sim$	$-$	$-$
p_E	$+$	$+$	$-$	$+$
V_{H_B}	$+$	$\sim$	$+$	$+$
V_{H_E}	$-$	$\sim$	$-$	$\sim$
V_{H_S}	$+$	$+$	$\sim$	$+$
F	$-$	$-$	$-$(weak)	$-$
B	$+$	$+$(weak)	$+$	$+$
E	$-$	$-$	$+$	$-$
S	$+$	$+$(weak)	$\sim$	$+$
H_B	$+$	$\sim$	$+$	$+$
H_E	$+$	$\sim$	$+$	$\sim$
H_S	$+$	$+$(weak)	$\sim$	$+$
R_B	$+$	$\sim$	$+$	$+$
R_E	$+$	$\sim$	$+$	$\sim$
R_S	$+$	$+$(weak)	$\sim$	$+$
Q_B	$+$	$+$	$+$	$+$
Q_F	$-$	$-$	$-$(weak)	$-$
Q_S	$+$	$+$(weak)	$\sim$	$+$
G, T	$-$	$-$	$-$(weak)	$-$
Q	$-$ then $+$	$-$ then $+$	$+$	$-$ then $+$
C	$+$	$-$ then $+$	$+$	$-$ then $+$

Table 2: Summary of economic policy effects

4.3 Numerical simulations

4.3.1 Optimal carbon tax and energy prices

As depicted in Figure 2, the first-best tax level starts from 49\$/tC and follows a quite linear increase to reach 256\$/tC by 2105. The impossibility to enforce any research policy leads to a second-best tax which is slightly higher than the first-best one, starting from 49.6\$/tC and rising to 275\$/tC in 2105. The stabilization to 550 and 450 requires much higher tax levels. Starting from respectively 73 and 172\$/tC, they increase sharply, reach some high 550\$/tC and 735\$/tC in 2075 and 2055, before declining once the concentration ceiling has been reached. Naturally, the rate of increase of the carbon prices for the 450ppm target is more rapid than that of the 550ppm case. These carbon prices prove slightly higher than Nordhaus (2008) estimates for similar climate strategies.

In the case where a carbon target is introduced, the tax pace evolves non-monotonically over time. Indeed, as long as the ceiling is not reached, the Lagrange multiplier φ_G associated to the ceiling constraint is nil, and it becomes positive at the moment the constraint is binding. Since the last component in equation (47) is strengthened by this multiplier, the date at which the tax starts to decline and the date at which the carbon stabilization cap is reached are closed.

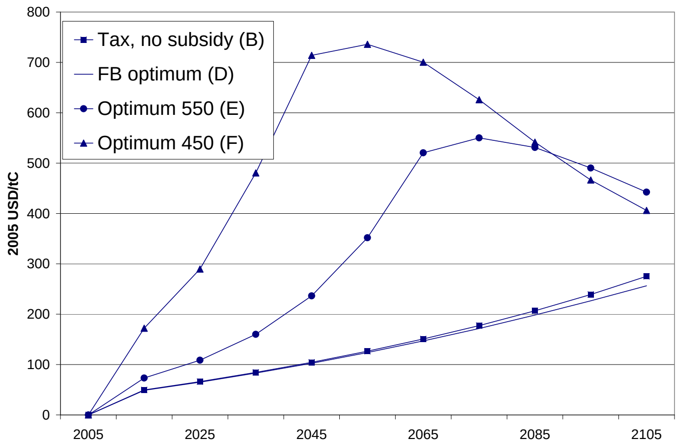

Figure 2: Optimal environmental taxes

Let us now analyze the effect of these tax trajectories on the prices of primary energies. First, the fossil fuel market price increases only slowly due to the relative flatness of our fossil fuel supply curve (see Figure 3-a). The implementation of a carbon tax reduces the producer price which induces substantial rent transfers from extractive industries to governments. In 2105, the revenues losses for the fossil energy producer amount to 55% and 52% when carbon caps are set at 550 and 450ppm, respectively. The concerns of oil-rich countries towards stringent climate mitigation commitments has already been commented and assessed in the literature (see for example Bergstrom, 1982, or Sinn, 2008). Moreover, an increment in the R&D subsidy rates has no effect on the fossil fuel price, thus illustrating the absence of crossed effects in this case.

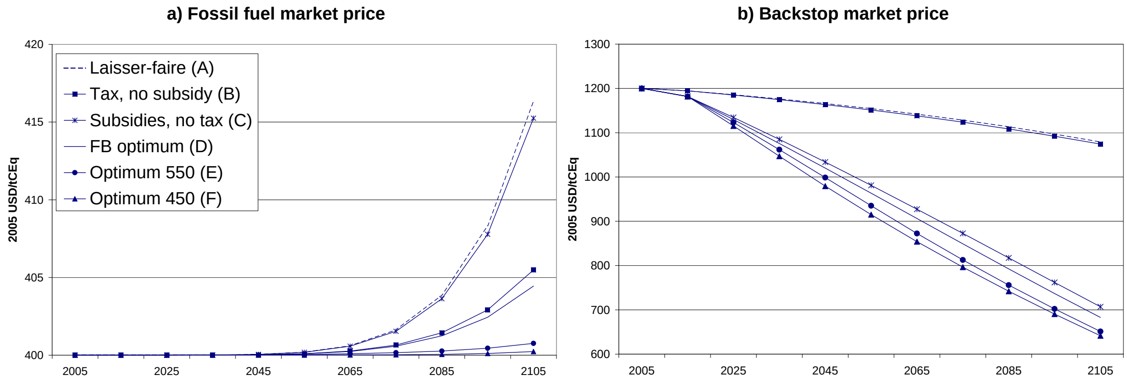

Figure 3: Fossil fuel and backstop prices

Simultaneously, introducing a carbon tax implies obviously a rise in the unit user cost of the fossil fuel (cf. $c_{F,t}$ as defined by (7)), as observed by comparing the upper trajectories of cases a to d in Figure 4. When carbon emissions are penalized, this creates an incentive for energy firms to store a part of these emissions so that their cost of using fossil fuel is obtained by adding two components to the fossil fuel market price: i) the tax on the emissions released in the atmosphere and ii) the unit cost of CCS. Such a decomposition is depicted in Figure 4. The incentives to use CCS devices, and thus the CCS unit cost, are contingent to an high level of tax, or equivalently to a constraining carbon target.

Second, the decreasing market price of the backstop energy reveals largely affected by the introduction of research subsidies, as can be seen from Figure 3-b. Such subsidies stimulate backstop research, thereby increasing its productivity and then, reducing production cost. They allow the backstop price to be cut by half by 2105. Moreover, two different streams of trajectories can be identified. The higher ones are drawn for cases A and B, i.e. when backstop R&D is not granted at all whereas the lower ones imply some positive σ_B. Then, R&D subsidies mainly matter to explain a decrease in the backstop price whereas the level of tax has only a weak depressive effect. Again, there is no crossed effect.

4.3.2 R&D

The effects of directed technical change can be portrayed by examining the effective value of an innovation in both CCS and backstop R&D, V_B and V_S, as depicted in Figure 5.[14]

[14] Results on energy R&D are less of interest and are not discussed here.

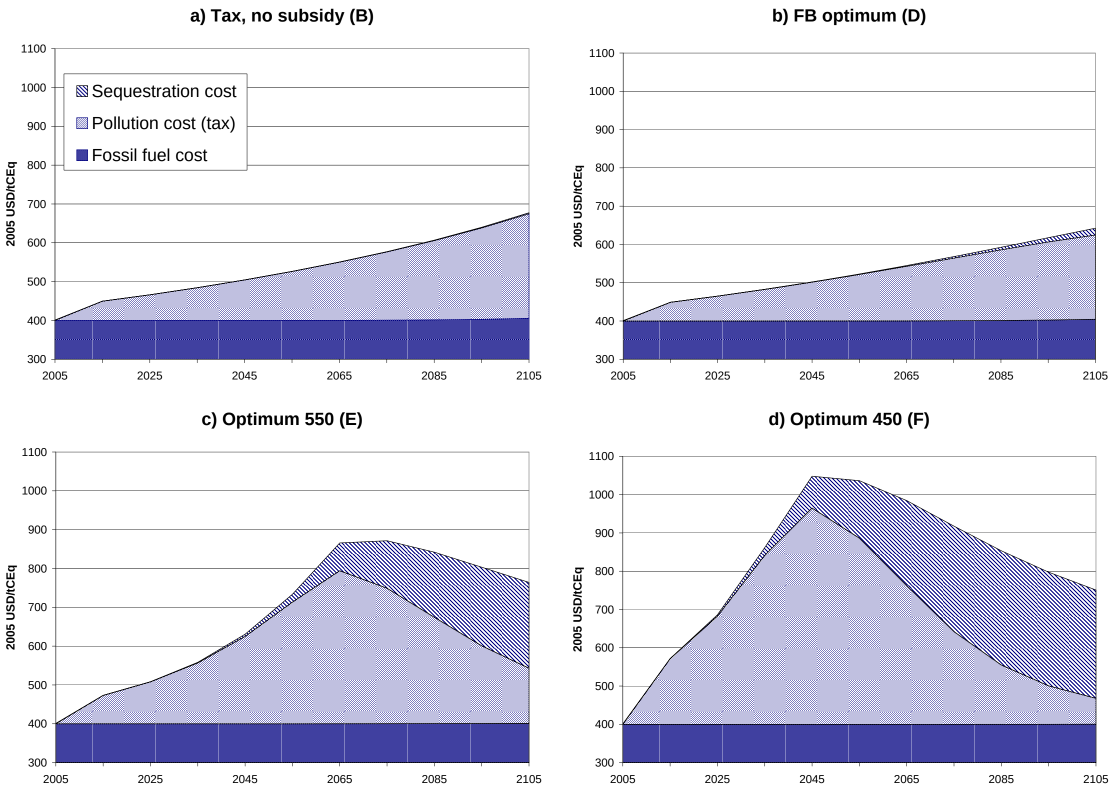

Figure 4: Decomposition of the unit cost of fossil fuel use

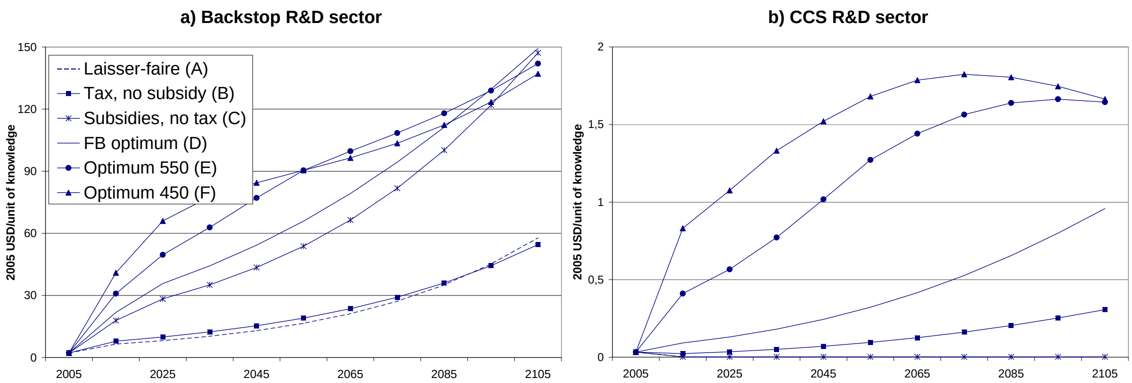

Figure 5: Effective innovation values in backstop and CCS R&D

The behavior of these innovation values provide insights on the allocation and the direction of R&D funding over time. First, the rising values demonstrate that the innovation activity grows strongly during the century, with the exception of the laisser-faire case which does not provide incentive for investing in CCS. Second, the increase in innovation values is strongly governed by the stringency of climate policy. Clearly, the introduction of a carbon ceiling induces the fastest increase in the effective value of innovations. Third, the role of each mitigation option can be inferred from the time-path of both CCS and backstop innovation values: CCS innovation value grows fast from the earliest periods, reaches a peak by around 2075 and starts declining thereafter. On the contrary, the backstop innovation value keeps on rising over time, though at a slow pace initially. A simple supply-demand argument is necessary to understand these behaviors. As the innovation activity is growing fast, due to the urgent need of developing carbon-free energy supply, and as the expected returns on CCS R&D are the highest initially because of relatively low cost of technology improvement relative to the backstop, a "technology push" in favor of CCS cause its innovation value to rise fast. In the longer run, backstop energy offers larger deployment potential and thus takes over CCS investments. Its value then develops at a faster pace while the CCS innovation is becoming less valued as its development shrinks.

These innovation values drive the R&D expenses flowing to each research sector. Figure 6 depicts such R&D investments for our major cases. In the polar laisser-faire case, hardly any R&D budget is dedicated to research and CCS R&D is not financed at all. A similar outcome occurs when an optimal tax is set while research subsidies are nil. When all research subsidies are optimally set without carbon tax, R&D allowances do not profit the CCS sector but mainly the backstop research sector that receives similar amounts to the first-best optimal case. The first-best optimum restoration calls for a continuous increase in R&D budgets that will mainly benefit the development of the backstop technology. By the end of the century, overall R&D budgets will then have been multiplied by a factor of roughly 10, amounting to slightly less than 1 billion USD. The energy efficiency sector and the CCS sector receive respectively 13 and 17% of total R&D budgets in 2100. Looking at the two stabilization cases, one notices drastic changes in R&D budgets allocation and volumes. By the end of century, the overall R&D budgets exceed the ones obtained when restoring the first-best solution. The necessity of curbing quickly the net polluting emissions flow leads to substantial investments in CCS R&D that constitutes the cheapest mid-term mitigation option. The more stringent the carbon target, the higher is the share of CCS R&D spending.

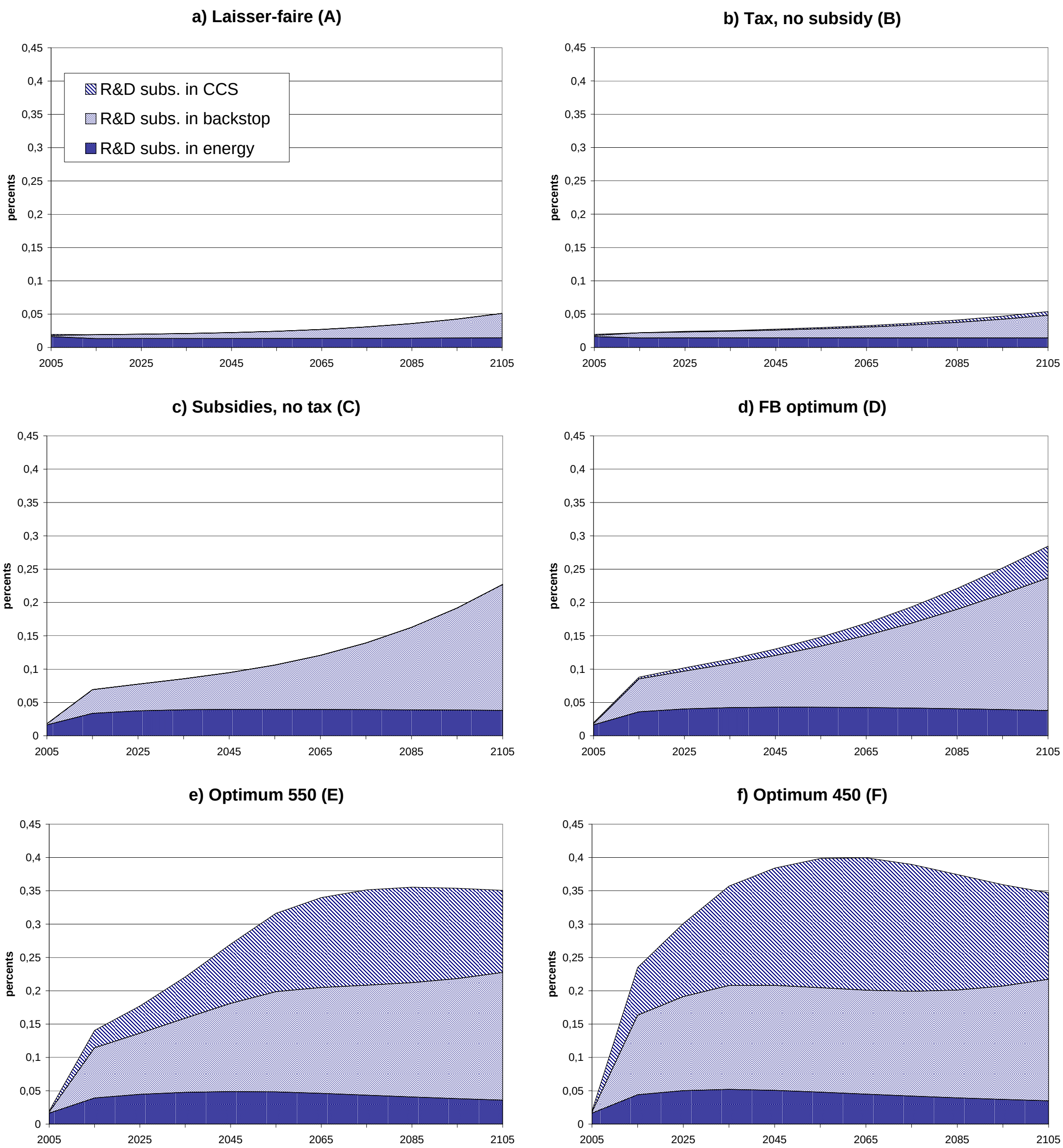

Figure 6: Intensity of dedicated R&D investments (i.e. R_i/Q)

Two conclusions can be drawn so far. The implementation a carbon tax alone hardly provides any incentive to proceed with R&D activities. In order to provide enough R&D incentives, one needs first to correct for the externality by imposing a carbon tax and second by subsidizing the research sectors. Moreover, short term investment in carbon-free R&D, namely in CCS activities, can become relevant when imposing a stringent cap on carbon accumulation, or equivalently, an higher level of tax.

4.3.3 Impacts on the energy mix

Let us now turn to the development of primary energy use throughout the century. As seen from Figure 7, the laisser-faire case induces a five-fold increase in energy use over the century, driven by strong economic growth and the absence of policy restrictions. Because of the lack of incentive (no carbon tax), the CCS technology is not utilized at all. In addition, despite the fossil fuel price growth over time, the backstop technology remains marginal because it is not competitive enough.

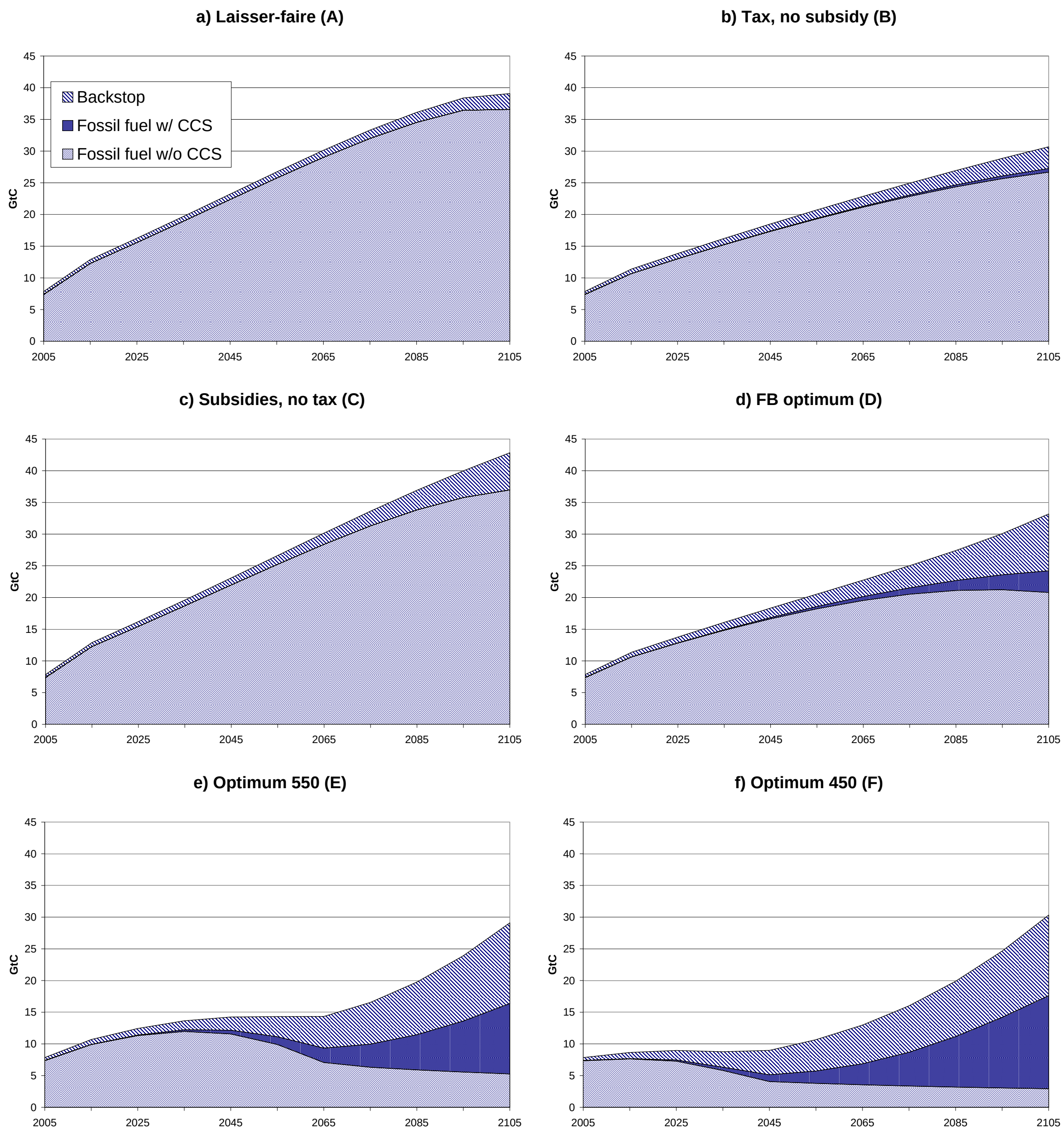

Figure 7: Primary energy use

When moving from case A to case B, the implementation of the optimal carbon tax alone does not result in substantial carbon sequestration, and/or backstop penetration. However, the fossil fuel share, and then the total primary energy use, are strongly reduced. Symmetrically, the implementation of research policies alone (i.e. moving from case A to case C) does not affect the fossil fuel use, but it slightly stimulates the backstop.

The simultaneous implementation of all optimal instruments (i.e. from case A to case D) reveals a complementarity effect between research grants and carbon taxation. Indeed, this scenario reinforces the effect of the tax on the fossil fuel use as observed in case B, and it increases the fraction of carbon emissions that are effectively sequestered (up to 4% of total carbon emissions in 2100). In addition, such a policy mix strengthens the role of backstop.

Finally, the two stabilization cases induce radical changes in world energy supply because of the sharp increase of carbon prices. This results in strong reductions of fossil fuel use, and thus of energy use, especially in the short-term where substitution possibilities with carbon-free energy are not yet available. By 2050, energy demand will have been reduced by 47% in the 550 ppm case, and by 60% in the 450 ppm case, as compared with the unconstrained optimum. In addition, the large amounts of R&D budgets allocated to CCS and backstop research sectors produce the expected benefits and allow for a deep mitigation of climate change owing to the decarbonisation of the economy both via the massive introduction of sequestration and via the backstop. When these carbon-free alternatives become economical, energy use rises again to reach similar levels to the laisser-faire ones in 2100. By that time, the backstop energy supplies 46% and 42% of total energy consumption. In the 550 and 450ppm cases, the CCS-based fossil fuel use accounts for 40% and 49% of total energy use in the 550 and 450ppm cases respectively. Therefore the lower the carbon target, the higher is the share of emission-free fossil fuel use.

4.3.4 Climate feedbacks on output

The environmental consequences of alternative scenarios are represented in Figure 8-a. The decentralized market outcome without any policy intervention involves a more intensive energy use without CO_2 removal and thus a faster carbon accumulation above to some dangerous 1000ppm level (IPCC, 2007). The implementation of optimal instruments limits the increase of atmospheric carbon accumulation to 800ppm by 2100. The implementation of the sole optimal tax without further R&D subsidies leads a slightly higher level of 850ppm. Notice that the sole optimal subsidies without CO_2 pricing just prove as inefficient from the environmental point of view.

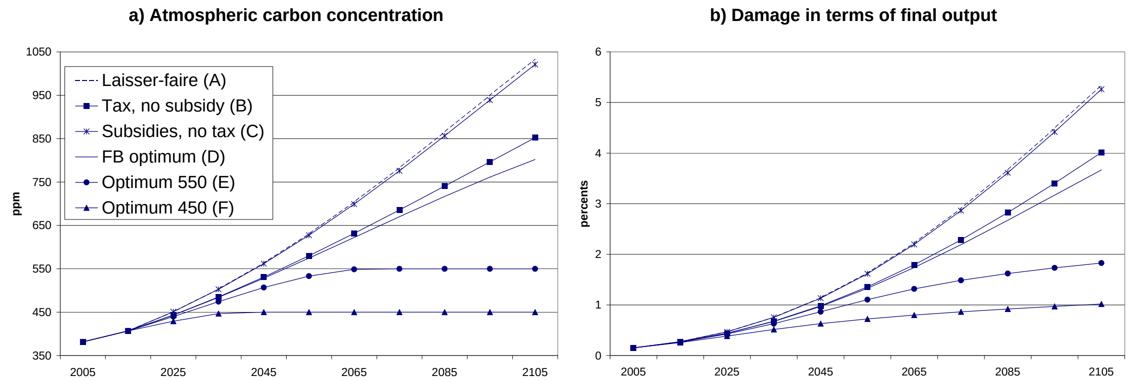

Figure 8: Atmospheric carbon concentration and damages

Figure 8-b shows the feedbacks of these atmospheric carbon concentrations on the economic damages, as measured in terms of final output. Policy inaction would lead to 5% of gross world product (GWP) losses per year by 2100, which is slightly lower then the forecasts established by Stern (2006). At the opposite, the implementation of the more stringent carbon cap, i.e. 450 ppm, limits these damages to 1% of GWP by 2100. Between these two extreme cases, the ranking of the trajectories among the various scenarios is the same than the one depicted in Figure 8-a.

Figure 9-a gives the GWP time-development as a percentage of the one from the laisser-faire case. The sole implementation of optimal subsidies improves the GWP at any date. The implementation of the optimal tax alone reveal costly until the end of the century. More importantly, setting economic instruments to their optimal values leads to further GWP losses in the short and mid term compared to the market outcome without intervention. In the longer run though, GWP increases significantly again and catches up the laisser-faire trajectory by 2095, to reach even higher gains eventually, up to 8% in 2145. To sum up: i) The presence of a carbon tax implies some GWP losses for the earlier generations, and some gains for the future ones. In other words, The long run economic growth is always enhanced when climate change issue is addressed with a carbon tax. ii) The larger the tax is, i.e. the lower the carbon ceiling is, the stronger the initial losses but also the higher the long run gains.

Figure 9-b depicts the same kind of variations, but now applied to consumption, and thus to welfare. Except for the optimal case D, this figure drives to the same conclusions

than the previous one. However, we observe now that the simultaneous implementation of the optimal public instruments allows to avoid the losses in consumption for the first generations.

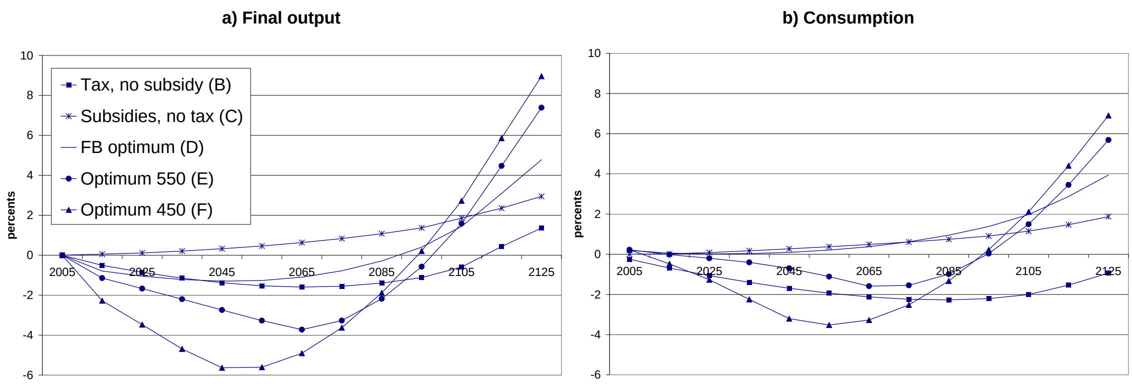

Figure 9: Final output and consumption variations as compared with the laisser-faire

4.4 Sensitivity analysis

As the choice of the parameter γ, i.e. the rate of appropriability of the innovation value by the market, is crucial, it is worth examining how a change in this parameter affects other key variables. This last section is thus devoted to such a sensitivity analysis. Until now this rate of appropriability was set at 0.3. We explore the implications of two alternative values: 0.2 and 0.4. Table 3 summarizes the percentage deviation of some selected key variables. Since the penetration of CCS technology is only modest in scenario A, we here focus on scenario B. Given the model structure, an increase or a decrease by 10 percentage points do not have symmetric effects on other variables but still produce some changes in the same order of magnitude. More importantly, the percentage deviations increase substantially over time. As seen from equations (14) and (15), the innovations values (variables V_{H_B} and V_{H_S}) are directly and largely affected by such parameter changes. And therefore knowledge in backstop and CCS technologies (variables H_B and H_S) accumulates much faster (for $\gamma = 0.4$). This is particularly true for CCS which plays a key role by the middle of the century and requires fast improvement prior to its wide-scale deployment. As a consequence, when parameter γ is set at 0.2, backstop use decreases by 10% in 2105 while CCS use decreases by 36%. Alternatively, when parameter γ is set at 0.4, backstop and CCS use increase by 12% and up to 47% respectively within the same time horizon.

	$\gamma = 0.2$			$\gamma = 0.4$		
X	2015	2055	2105	2015	2055	2105
τ^{sb}	0,2	0,4	1,1	-0,2	-0,3	-1
p_B	0,2	1,8	4,9	-0,2	-2	-5,3
V_{H_B}	-34,8	-33,6	-31,9	34,5	34	30,4
V_{H_S}	-37,8	-36,3	-35,6	42,2	38,7	38,2
F	0,1	0,4	0,8	-0,1	-0,5	-1
B	-0,3	-3,8	-9,7	0,3	4,6	12,3
S	-4,4	-21,3	-35,9	4,4	26,1	46,9
H_B	-0,2	-1,8	-4,7	0,2	2	5,6
H_S	-4,4	-21,4	-36,1	4,5	26,6	47,5
$\sum_i R_i$	-24,2	-40	-41,5	41,4	43,7	48,9

Table 3: Deviation (in %) of variable X when γ moves from 0.3 to 0.2 and 0.4, respectively, for scenario B.

5 Conclusion

Our analysis primarily consisted in decentralizing the "top-down" ENTICE-BR model (Popp, 2006a) in order to characterize the full set of equilibria. In addition to the backstop, we also considered a second abatement possibility by adding to the original model a CCS sector, together with an associated dedicated R&D activity. Simultaneously, in order to account for further climate change damages that are not integrated in the damage function, we imposed a cap on the atmospheric carbon accumulation. Since the economy faces two types of market failures, global warming and R&D spillovers, the regulator uses two types of public tools to correct them, a carbon tax and a subsidy for each R&D sector. A particular equilibrium is associated with each vector of instruments. First, we provided a characterization of this set of equilibria (Proposition 1). Second, we showed that we can obtain any decentralized equilibrium as the solution of a maximization program (Proposition 2). Third, we characterized the first-best optimum (Proposition 3) and we showed that there exists a unique vector of policy tools that implements it (Proposition 4). We calculated the optimal tax and subsidies analytically and we investigated their dynamic properties. In the line with Goulder and Mathai (2000) and Ulph and Ulph (1994), we verified that the optimal carbon tax is generally non-monotonic over time and follows an inverted U-shaped time-path. It falls once the ceiling is reached.

In a second step, we have used a calibrated version of the theoretical model based on DICE 2007 (Nordhaus, 2008), ENTICE-BR (Popp, 2006a) and DEMETER (Gerlagh et al., 2006), to assess the environmental and economic impacts of various climate change policies.

In addition to the standard comparison of the first-best outcome with the laisser-faire, we also provide some intermediate scenarios in which we analyze the separated impacts of the policy tools. Our main findings are the following.

i) Our results do not exhibit significant crossed effects in the sense that the implementation of a carbon tax alone hardly provides any incentive to conduct R&D activities and backstop production, when R&D policies used alone have only weak effects on the fossil fuel and CCS sectors.

ii) The simultaneous use of the two types of tools reinforces the individual effects of each one used alone, thus revealing complementarity between research grants and carbon taxation.

iii) The first-best case (without ceiling) does not result in substantial carbon sequestration.

iv) A carbon cap reinforces the role of CCS as a mid-term option for mitigating the climate change. In the longer term, if the policy-maker aims at stabilizing the climate, the massive introduction of backstop energy is necessary.

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Appendix

A1. Proof of Proposition 1

Integrating (10) and using (9) and the transversality condition on Z_t, we find:

$$\eta_t = \int_t^{\infty} \frac{F_Z}{F_{I_F}} e^{-\int_0^s r_x dx} ds.$$

The first characterizing condition (32) is obtained by replacing η into (9) by the expression above, and by noting that $p_F = Q_E E_F - (\xi - S_F)/S_{I_S}$ from (3), (4) and (6), and that $\exp(-\int_0^t r ds) = U'(C)\exp(-\rho t)/U'(C_0)$ from (24). Combining (3), (5) and (13) leads to condition (33). Condition (34) directly comes from (6). Next, using (1) and (24), we directly get condition (35). Finally, the differentiation of (18) with respect to time leads to:

$$\frac{\dot{V}_i}{V_i} = -\frac{\dot{H}^i_{R_i}}{H^i_{R_i}}, \quad i = \{B, E, S\}. \tag{48}$$

Substituting this expression into (16) and using (18) again, it comes:

$$r = -\frac{\dot{H}^i_{R_i}}{H^i_{R_i}} + v_i H^i_{R_i}, \quad \forall i = \{B, E, S\}. \tag{49}$$

We thus obtain conditions (36), (37) and (38) by replacing into (49) v_B, v_E and v_S by their expressions (20), (21) and (22), respectively.

A2. Proof of Proposition 2

Let $\mathcal{J}$ be the discounted value of the Hamiltonian of the maximization program of Proposition 2:

$$\mathcal{J} = U(C)e^{-\rho t} + \tilde{\lambda}\Big\{Q[K, E[B(I_B, H_B), F(I_F, Z), H_E], L, A, \Omega] - C - \delta K - \sum_i R_i - \sum_j I_j - \tau[\xi F(I_F, Z) - S[F(I_F, Z), I_S, H_S]]\Big\} + \sum_i \tilde{\nu}_i(\gamma_i + \sigma_i)H^i(R_i, H_i) + \tilde{\eta}F(I_F, Z)$$

The associated first order conditions are:

$$\frac{\partial \mathcal{J}}{\partial C} = U'(C)e^{-\rho t} - \tilde{\lambda} = 0 \quad (50)$$

$$\frac{\partial \mathcal{J}}{\partial I_F} = \tilde{\lambda}[Q_E E_F F_{I_F} - 1 - \tau F_{I_F}(\xi - S_F)] + \tilde{\eta}F_{I_F} = 0 \quad (51)$$

$$\frac{\partial \mathcal{J}}{\partial I_B} = \tilde{\lambda}(Q_E E_B B_{I_B} - 1) = 0 \quad (52)$$

$$\frac{\partial \mathcal{J}}{\partial I_S} = -\tilde{\lambda}(1 - \tau S_{I_S}) = 0 \quad (53)$$

$$\frac{\partial \mathcal{J}}{\partial R_i} = -\tilde{\lambda} + \tilde{\nu}_i(\gamma_i + \sigma_i)H^i_{R_i} = 0, \quad i = \{B, E, S\} \quad (54)$$

$$\frac{\partial \mathcal{J}}{\partial K} = \tilde{\lambda}(Q_K - \delta) = -\dot{\tilde{\lambda}} \quad (55)$$

$$\frac{\partial \mathcal{J}}{\partial H_i} = \tilde{\lambda}Q_E E_{H_i} + \tilde{\nu}_i(\gamma_i + \sigma_i)H^i_{H_i} = -\dot{\tilde{\nu}}_i, \quad i = \{B, E\} \quad (56)$$

$$\frac{\partial \mathcal{J}}{\partial H_S} = \tilde{\lambda}\tau S_{H_S} + \tilde{\nu}_S(\gamma_S + \sigma_S)H^S_{H_S} = -\dot{\tilde{\nu}}_S \quad (57)$$

$$\frac{\partial \mathcal{J}}{\partial Z} = \tilde{\lambda}[Q_E E_F F_Z - \tau F_Z(\xi - S_F)] + \tilde{\eta}F_Z = -\dot{\tilde{\eta}} \quad (58)$$

and the transversality conditions are:

$$\lim_{t\to\infty} \tilde{\lambda}K = \lim_{t\to\infty} \tilde{\nu}_i H_i = \lim_{t\to\infty} \tilde{\eta}Z = 0 \quad (59)$$

Replacing into (58) $\tilde{\eta}$ by its expression coming from (51), we find $\dot{\tilde{\eta}} = -\tilde{\lambda}F_Z/F_{I_F}$. Integrating this expression and using (50) and (59) implies:

$$\tilde{\eta} = \int_t^\infty \frac{F_Z}{F_{I_F}} U'(C)e^{-\rho s}ds. \quad (60)$$

Plugging this expression in (51) and using (50) again, one gets condition (32). Equations (52) and (53) directly imply (33) and (34). Using (50) and (55), one gets (35). The log-differentiation of (54) gives:

$$-\rho + \frac{\dot{U}'(C)}{U'(C)} = \frac{\dot{\tilde{\nu}}_i}{\tilde{\nu}_i} + \frac{\dot{H}^i_{R_i}}{H^i_{R_i}} \quad (61)$$

Replacing into (56), $\tilde{\lambda}/\tilde{\nu}_i$ and $\dot{\tilde{\nu}}_i/\tilde{\nu}_i$ by their expressions coming from (54) and (61), we obtain conditions (36) and (37). The same calculation applied to (57) finally leads to (38).

A3. Proof of Proposition 3

Let $\mathcal{H}$ be the discounted value of the Hamiltonian of the optimal program:

$$\begin{aligned}\mathcal{H} \;=\;& U(C)e^{-\rho t} + \lambda \left\{ Q[K, E[B(.), F(.), H_E], L, A, \Omega(T)] - C - \delta K - \sum_i R_i - \sum_j I_j \right\} \\ & + \sum_i \nu_i H^i(R_i, H_i) + \eta F(.) + \mu_G \{\xi F(.) - S[F(.), I_S, H_S] - \zeta G\} \\ & + \mu_T[\Phi(G) - mT] + \varphi_G(\bar{G} - G)\end{aligned}$$

The associated first order conditions are:

$$\frac{\partial \mathcal{H}}{\partial C} = U'(C)e^{-\rho t} - \lambda = 0 \tag{62}$$

$$\frac{\partial \mathcal{H}}{\partial I_F} = \lambda(Q_E E_F F_{I_F} - 1) + \eta F_{I_F} + \mu_G F_{I_F}(\xi - S_F) = 0 \tag{63}$$

$$\frac{\partial \mathcal{H}}{\partial I_B} = \lambda(Q_E E_B B_{I_B} - 1) = 0 \tag{64}$$

$$\frac{\partial \mathcal{H}}{\partial I_S} = -\lambda - \mu_G S_{I_S} = 0 \tag{65}$$

$$\frac{\partial \mathcal{H}}{\partial R_i} = -\lambda + \nu_i H^i_{R_i} = 0, \quad i = \{B, E, S\} \tag{66}$$

$$\frac{\partial \mathcal{H}}{\partial K} = \lambda(Q_K - \delta) = -\dot{\lambda} \tag{67}$$

$$\frac{\partial \mathcal{H}}{\partial H_i} = \lambda Q_E E_{H_i} + \nu_i H^i_{H_i} = -\dot{\nu}_i, \quad i = \{B, E\} \tag{68}$$

$$\frac{\partial \mathcal{H}}{\partial H_S} = \nu_S H^S_{H_S} - \mu_G S_{H_S} = -\dot{\nu}_S \tag{69}$$

$$\frac{\partial \mathcal{H}}{\partial Z} = \lambda Q_E E_F F_Z + \eta F_Z + \mu_G F_Z(\xi - S_F) = -\dot{\eta} \tag{70}$$

$$\frac{\partial \mathcal{H}}{\partial G} = -\zeta \mu_G + \mu_T \Phi'(G) - \varphi_G = -\dot{\mu}_G \tag{71}$$

$$\frac{\partial \mathcal{H}}{\partial T} = \lambda Q_\Omega \Omega'(T) - m\mu_T = -\dot{\mu}_T \tag{72}$$

The complementary slackness condition and the transversality conditions are:

$$\varphi_G(\bar{G} - G) = 0, \quad \text{with } \varphi_G \geq 0, \forall t \geq 0 \tag{73}$$

$$\lim_{t\to\infty} \lambda K = \lim_{t\to\infty} \nu_i H_i = \lim_{t\to\infty} \eta Z = \lim_{t\to\infty} \mu_G G = \lim_{t\to\infty} \mu_T T = 0 \tag{74}$$

From (63), we find that $\eta = -\mu_G(\xi - S_F) - \lambda(Q_E E_F - 1/F_{I_F})$. Replacing this expression into (70) and using (62) leads to the following differential equation: $\dot{\eta} =$

$-(F_Z/F_{I_F})U'(C)\exp(-\rho t)$. Integrating this expression and using the transversality condition (74), we obtain:

$$\eta = \int_t^{\infty} \frac{F_Z}{F_{I_F}} U'(C) e^{-\rho s} ds. \tag{75}$$

Replacing into (63) λ, μ_G and η by their expressions coming from (62), (65) and (75), respectively, gives us the equation (39) of Proposition 2. Equation (40) directly comes from condition (64). From (62) and (72), we have: $\dot{\mu}_T = m\mu_T - Q_\Omega \Omega'(T)U'(C)\exp(-\rho t)$. Using (74), the solution of such a differential equation is given by:

$$\mu_T = \int_t^{\infty} Q_\Omega \Omega'(T) U'(C) e^{-[m(s-t)+\rho s]} ds. \tag{76}$$

Next, using the transversality condition (74), we determine the solution of the differential equation (71) as:

$$\mu_G = \int_t^{\infty} \left[\mu_T \Phi'(G) - \varphi_G\right] e^{-\zeta(s-t)} ds \tag{77}$$

where μ_T is defined by (76) and φ_G must be determined by looking at the behavior of the economy once the ceiling have been reached. Condition (41) is then obtained by replacing into (65) λ and μ_G by their expressions coming from (62) and (77), respectively. Log-differentiating (62) with respect to time implies:

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{U}'(C)}{U'(C)} - \rho = \epsilon g_C - \rho \tag{78}$$

Condition (42) is a direct implication of equations (67) and (78). Finally, the log-differentiation of (66) with respect to time yields:

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\nu}_i}{\nu_i} + \frac{\dot{H}^i_{R_i}}{H^i_{R_i}}. \tag{79}$$

Conditions (43) and (44) come from (66), (68), (78), (79) and from (64) by using $Q_E E_B = 1/B_{I_B}$. Similarly, condition (45) is obtained from (65), (66), (69), (78) and (79).

A4. Calibration of the model

Based on DICE (Nordhaus, 2008), ENTICE-BR (Popp, 2006a) and DEMETER (Gerlagh, 2006), we use the following specified forms[15]:

$$
\begin{aligned}
Q(K,E,L,A,\Omega) &= \Omega A K^{\gamma} E^{\beta} L^{1-\gamma-\beta}, \quad \gamma,\beta \in (0,1) \\
L &= L_0 e^{\int_0^t g^L ds} \quad A = A_0 e^{\int_0^t g^A ds}, \quad g_j = g_{j0} e^{-d_j t}, \quad d_j > 0, \forall j = \{A, L\} \\
E(F,B,H_E) &= \left[(F^{\rho_B} + B^{\rho_B})^{\frac{\rho_H}{\rho_B}} + \alpha_H H_E^{\rho_H} \right]^{\frac{1}{\rho_H}}, \quad \alpha_H, \rho_H, \rho_B \in (0,1) \\
F(I_F, Z) &= \frac{I_F}{c_F + \alpha_F (Z/\bar{Z})^{\eta_F}}, \quad c_F, \alpha_F, \eta_F > 0 \\
B(I_B, H_B) &= \alpha_B I_B H_B^{\eta_B}, \quad \alpha_B, \eta_B > 0 \\
S(F, I_S, H_S) &= \kappa(\xi F) \left[\left(1 + \frac{2 I_S H_S}{\kappa(\xi F)} \right)^{1/2} - 1 \right], \quad \kappa > 0 \\
H^i(R_i, H_i) &= a_i R_i^{b_i} H_i^{\phi_i}, \quad a_i > 0, \ b_i, \phi_i \in [0,1], \forall i = \{B, E, S\} \\
W &= v_1 \int_0^{\infty} L \frac{(C/L)^{1-\epsilon}}{(1-\epsilon)} e^{-\rho} dt + v_2, \quad v_1, v_2 > 0 \\
\Omega(T) &= \left[1 + \alpha_T T^2 \right]^{-1}, \quad \alpha_T > 0
\end{aligned}
$$

Next, let us provide some calibration details here. According to IEA (2007), world carbon emissions in 2005 amounted to 17.136 GtCO2. We retain 7.401 GtCeq as the initial fossil fuel consumption, given in gigatons of carbon equivalent. In addition, carbon-free energy produced out of renewable energy, excluding biomass and nuclear, represented 6% of total primary energy supply. We thus retain another 0.45 GtCeq as the initial amount of backstop energy use. We retain the Gerlagh's assumption for the cost of CCS that is worth 150US$/tC. According to IEA (2006), the cumulative CO_2 storage capacity is in the order of 184 million tons per year. This value serves as a seed value for sequestration level, S_0, in the initial year, which is then fixed at 0.05 GtC. The cost of CCS sequestration and the initial storage level allow for the calibration of the initial sequestration effort using the following relation: $I_{S,0}/S_0$ =CCS cost, which implies $I_{S,0}$ = 0.05GtC×150$/tC=7.5G$. The total factor productivity has been adjusted so as to produce a similar pattern of GWP development until 2100 to the one from DICE-08. The rates of return on both R&D spending and knowledge accumulation have been set to 0.3 and 0.2 respectively so as provide long term sequestration in line with IPCC (2007) projections. Without loss of generality, the initial stock of knowledge dedicated to CCS is set equal to 1. Calibration

[15]We replace the cost function of fossil fuel and backstop from Popp (2006a) and the cost function of sequestration from Gerlagh (2006) by their corresponding production functions in order to derive an utility/technology canonical model. With our notations, these unit cost functions are, respectively: $I_F/F = c_F + \alpha_F \left(Z/\bar{Z}\right)^{\eta_F}$, $I_B/B = 1/(\alpha_B H_B^{\eta_B})$, and $I_S/S = \left[1 + S/(2\kappa\xi F)\right]/H_S$.

of the other parameters come from DICE or ENTICE-BR. Table 4 below provides some more details.

Param.	Value	Description	Source
γ	0.3	Capital elasticity in output prod.	DICE
β	0.07029	Energy elasticity in output prod.	DICE
α_T	0.0028388	Scaling param. on damage	DICE
ρ_B		Elasticity of subs. for backstop	Calibrated
ρ_E	0.38	Elasticity of subs. for energy	ENTICE-BR
α_H	0.336	Scaling param. of H_E on energy	ENTICE-BR
F_0	7.401	2005 fossil fuel use in GtC	IEA
c_F	400	2005 fossil fuel price in USD	IEA
α_F	700	Scaling param. on fossil fuel cost	ENTICE-BR
η_F	4	Exponent in fossil fuel prod.	ENTICE-BR
B_0	0.45	2005 backstop use in GtC	IEA
α_B	1200	2005 backstop price in USD	DICE
η_B		Exponent in backstop prod.	Calibrated
a_B	0.0122	Scaling param. in backstop innovation	ENTICE-BR
a_E	0.0264	Scaling param. in energy innovation	ENTICE-BR
b_B	0.3	Rate of return of backstop R&D	ENTICE-BR
b_E	0.2	Rate of return of energy R&D	ENTICE-BR
c_S	150	Sequestration cost in 2005 USD/tC	DEMETER
S_0	0.05	2005 sequestration in GtC	IPCC
$Q_{S,0}$	7.5	2005 sequestration effort in bill. USD	IPCC
$H_{S,0}$	1	2005 level of knowledge in CCS	Calibrated
$R_{S,0}$	0.5	2005 R&D investment in CCS in bill. USD	Calibrated
a_S	0.5	Scaling param. in CCS innovation	Calibrated
b_S	0.3	Rate of return of CCS R&D	Calibrated
ϕ_S	0.2	Elasticity of knowledge in CCS innovation	Calibrated
Φ_i	0.54	Elasticity of knowledge in innovation	ENTICE-BR
ϵ	2	Elasticity of intertemporal subst.	DICE
ρ	0.015	Time preference rate	DICE
A_t		Total factor productivity trend	DICE
L_t		World population trend	DICE

Table 4: Calibration of the main parameters