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Implied Market Loss Given Default in the Czech Republic
Structural-Model Approach*

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Abstract
This paper focuses on the key credit risk parameter – Loss Given Default (LGD). We describe its general properties and determinants with respect to seniority of debt, characteristics of debtors and macroeconomic conditions. Furthermore, we illustrate how the LGD can be extracted from market observable information with help of the adjusted Mertonian structural approach. We present a derivation of the formula for the expected LGD and show its sensitivity with respect to other structural company parameters. Finally, we estimate the 5-year expected LGDs for companies listed on the Prague Stock Exchange and find that the average LGD for this analyzed sample is in the range of 20–45 %. To the authors' knowledge, these are the first implied market estimates of LGD in the Czech Republic.

1. Introduction

Awareness of credit risk has led to the development of procedures and mechanisms for determining the causality between the attributes and potential bankruptcy of a borrower. In the last decade, credit risk techniques have seen significant development concerning the estimation of risks and other parameters specifying possible losses. One of those parameters is Loss Given Default (LGD), expressing the percentage of an exposure which will be not recovered after a counterparty defaults. While the estimation of the probability of default (PD) has received considerable attention over the past 20 years, LGD has gained greater acceptance only in recent years, as the New Basel Accord identified it as one of the key risk parameters. Yet LGD modeling is still quite a new, open problem in credit risk management. LGD estimation is not straightforward because it depends on many driving factors, such as the seniority of the claim, the quality of collateral, and the state of the economy. Moreover, the insufficient database of realized LGDs makes it more difficult to develop accurate LGD estimates based on historical data. Hence, the extraction of LGDs for credit-sensitive securities based on market observable information is an important issue in the current credit risk area and may bring other improvements in credit risk management.

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The paper therefore discusses this key LGD parameter for single corporate exposures and deals with the possibility of extracting it from market information. This type of LGD is denoted as the *implied market LGD*. To estimate this parameter, we utilize the adjusted Merton framework, which has so far been used most often for PD evaluation, and empirically implement this contingent claim approach for a set of companies in the Czech Republic. As the result, we estimate 5-year expected LGDs for almost thirty companies listed on the Prague Stock Exchange in the period 2000–2008. To the authors' knowledge, these are the first estimates of LGD from market information in the Czech Republic.

2. Basic Characteristics of LGD

LGD is usually defined as the loss rate experienced by a lender on a credit exposure if the counterparty defaults.[1] Thus, despite the default the lender still recovers $1-\text{LGD}$ percent of the exposure. One minus LGD is therefore called the recovery rate (RR). In principle, LGD also comprises other costs related to default of the debtor, and the correct formula should rather be

$$LGD = 1 - RR + Costs \tag{1.1}$$

Nevertheless, costs are relevant only in a specific type of LGD and are not usually high enough to influence the losses markedly in comparison with the recovery rate. Therefore, we use the recovery rate as the complement of the LGD in the following text and take these two parameters as conceptually the same.

Usually three basic types of LGD for defaulted facilities are used. *Market LGD* employs the price of a bond after default as a proxy for the recovered amount. However, the post-default price is available only for the fraction of the debt that is traded and for which an after-default market exists.[2] Market LGD is therefore highly limited for defaulted bank loans, which are traditionally not traded. For them, one must turn to another approach.

Workout LGD considers all relevant facts that may influence the final economic value of the recovered part of the exposure arising in the long-running workout process. LGD is then determined by the loss of principal, the carrying costs of non-performing assets, and the workout expenses. However, the appropriate discount rate, which should reflect the risk of holding the defaulted asset, is not known. Therefore, we must speak only of an estimated LGD, even if we are trying to measure it from ex-post data.

The last method of measuring of LGD is the concept of *Implied Market LGD*, which is estimated ex ante from market prices of non-defaulted loans, bonds, or credit default instruments by structural or reduced-form models. The idea is that prices of risky instruments reflect the market's expectation of the loss and may be broken down into PD and LGD. Implied market LGD estimation does not rely on historical data and can be used especially for low default facilities.

[1] In principle, we should refer to the loss rate given default as LGDR and use LGD for the absolute amount of the loss. However, LGD is used to indicate the loss rate by many practitioners, including Basel II, while the absolute loss is indicated as LGD.EAD, where EAD is the exposure at default (see BCBS, 2005).

[2] Moreover, outside the USA the market for defaulted bonds is either non-existent or does not have the required depth and liquidity.

Empirical evidence on recovery rates has confirmed that the RR increases with the seniority and security of the defaulted debts and decreases with the degree of subordination.[3] The results also tend to be rather similar in terms of average recovery rates – for bank loans (70–84 %) and for bonds: senior secured (53–66 %), senior unsecured (48–50 %), senior subordinated (34–38 %), and subordinated (26–33 %). All studies have also reported a high standard deviation characterizing the recovery rate across all bond debt classes, regularly exceeding 20 % (see Altman and Kishore, 1996, Castle and Keisman, 1999, or Keenan et al., 2000).

Recovery rates are ultimately determined by the value of the assets that can be seized in case of default. It is therefore intuitive to assume that the debtor's industry characteristic is a straightforward determinant of the LGD.[4] However, the literature does not give wholly unified answers (see Altman and Kishore, 1996, Grossman et al., 2001, or Acharya et al., 2003). Those studies have broken down the LGDs of corporate bonds by industry and have found evidence that some industries, such as *public utilities* and *chemicals*, do evidently better than others. Nonetheless, they have also shown that the standard deviation of the RR per industry and within a given industry is still very large.

An opposite view of the industry influence is presented by Gupton et al. (2000) and Araten et al. (2004), who, on the contrary, found no evidence of different LGDs across industries. They state that some sectors may enjoy periods of high recoveries, but can later fall below the average recoveries at other times. These unambiguous results of different studies might be due to LGD cyclicality in relation to the economic environment. Each industry can be at a different stage of the economic cycle, which can influence the LGD more than the industry-type itself. Acharya et al. (2003) showed that when the industry is in distress, the mean LGD is on average 10––20 % higher than otherwise.

Behind the cyclical variation is the fact that as the economy enters a recession, default rates increase. Recoveries from collateral will depend on the possibility of selling the relevant assets. We can generally suppose that a greater supply of collateral assets will lead to lower prices thereof, depending, of course, on the market size and structure observed for the particular asset. The result is that the macroeconomic situation can significantly influence the recovery rate, as has been demonstrated by several authors (see Araten et al., 2004, or Altman et al., 2005).

As has been shown, LGD is influenced by many factors, such as the facility's seniority and the presence of collateral, the borrower's industry characteristics, or more general factors such as the macroeconomic conditions. However, the previous research gives ambiguous results concerning some properties of LGD. It is clear that further research is needed. Hopefully, with the adoption of the Basel II accord, which sets rules for LGD data gathering and LGD estimation, this research will be based on

[3] The capital structure of the firm and the absolute priority rule (APR) are important determinants of the recovery rate. The APR states that the value of the bankrupted firm must be distributed to suppliers of capital so that "[...] *senior creditors are fully satisfied before any distributions are made to more junior creditors, and paid in full before common shareholders*" (Schuermann, 2004, p. 11). Eberhart and Weiss (1998) confirm that the APR is routinely violated in the interests of speed of resolution. Creditors agree to violate the APR to resolve bankruptcies faster.

[4] Firms in some sectors have a large amount of assets that can be easily sold on the market in case of default, while other sectors may be more labor-intensive, for example.

a better data sample offering more exact outcomes. Nevertheless, LGD predictions based on past data are not necessarily consistent with the evolution of fundamentals across time and can result in inaccurate estimates which do not capture the real trend in the economy.

3. LGD Modeling

In this section, we focus on analytical tools which enable us to obtain forward-looking estimates of LGD from market observable information. We employ asset-pricing models aimed at determining the equilibrium arbitrage-free price of risky assets. Each risky asset should offer an expected return corresponding to its degree of risk; therefore, all risky parameters must be evaluated by the market in order to get the equilibrium price. This assumption that prices include all information is then used in credit risk pricing models which utilize market information (e.g. share or bond prices) to measure credit risk and to extract the key risk parameters such as the PD or LGD from prices. These models are forward-looking, estimating the risk parameters expected by the market. Given the nature of this method, such an estimate of LGD is called the *implied market LGD*.

These credit risk pricing models can be further classified as *structural* and *reduced-form* models. The category of structural-form models is based on the framework developed by Merton in 1974 using the theory of option pricing presented by Black and Scholes (1973). The term "structural" comes from the fact that these models focus on structural characteristics of the company such as asset volatility or leverage, which determine the relevant credit risk elements. Default and the RR are therefore a function of those variables.

In contrast, *reduced-form* models generally assume that default is possible and is driven by some exogenous random variable. The result is that default and recovery are modeled independently of the firm's structural features, which lacks the clear economic intuition behind the default event. The basic input parameters for extracting LGD in the reduced-form approach are the prices of risky corporate bonds. However, companies in the Czech Republic still use traditional bank loans more than bonds as a source of finance. This results in a situation where the domestic corporate debt market is rather illiquid and incomplete and can thus barely reflect market expectations about the default and recovery risk of a particular company or its security (see Dvořáková, 2003). The result is that the reduced-form models are currently hardly applicable for LGD estimation in the Czech Republic.

The stock market provides an alternative source of information, assuming that share prices incorporate all the available information, including the future prospects and creditworthiness of a company.[5] Structural models for extracting a company's default risk typically utilize observed stock prices, stock volatility, and specifics about the company's capital structure. Even though the number of listed companies in the Czech Republic is also limited, some of them seem to be sufficiently liquid to apply structural models and estimate the required credit risk parameters. As a result,

[5] This is true only if the efficiency hypothesis holds, which has been questioned by some studies (see e.g. Sloan, 1996). There is also the question of whether stock price volatility is caused solely by the incorporation of new information about future stock returns, or if it is caused largely by trading itself (see French, 1980).

we will utilize the Merton structural approach to derive a formula for the implied market LGDs for particular companies.

The seminal structural Merton (1974) model relies on many assumptions, most of which derive from the Black-Scholes option pricing theory.[6] The total asset value of firm V is financed by equity E and one zero-coupon non-callable debt contract D, maturing at time T with face value F, i.e., $V_t = D_t + E_t$. With a no-taxes assumption this implies that the value of the firm's assets and the value of the firm are identical and do not depend on the capital structure itself. Some of those assumptions became a source of criticism and were later relaxed.[7] The dynamics of the firm's value through time can be described by a stochastic differential equation called geometric Brownian motion

$$dV_t = \mu_V V_t dt + \sigma_V V_t dW_t^V \tag{1.2}$$

where μ_V is the asset drift (i.e., the instantaneous expected rate of return on the firm's value V per unit time), σ_V is the standard deviation of its return, and dW_t^V is a standard Gauss-Wiener process.

In such a framework, credit risk concerns the possibility that the value of the company evolving stochastically will be less on the maturity day T than the repayment value of the loan F. The debt holders receive at T either the value F (if $V_T > F$) or the entire value of the firm and the owners of the firm receive nothing (if $V_T < F$). This means that the value of the equity is identical to the formula for pricing an European call option on the firm's value with exercise price F (Merton, 1974, p. 10). Indeed, at maturity time T, the equity holders will exercise the option and pay the debt holders the face value of the liabilities if $V_T \geq F$, otherwise they let this option expire. By applying the Black-Scholes option pricing formula, the expression for the value of the equity is

$$E(V,\tau) = V\Phi\left(d_1\right) - Fe^{-r\tau}\Phi\left(d_2\right) \tag{1.3}$$

where $d_1 = \dfrac{\ln(V/F) + \left(r + 0{,}5\sigma_V^2\right)\tau}{\sigma_V\sqrt{\tau}}$, $d_2 = d_1 - \sigma_V\sqrt{\tau}$, Φ (.) is a cumulative standard normal distribution, r is the instantaneous riskless interest rate, and $\tau = T - t$ is the length of time until maturity.

Default occurs when the firm's value drops below some *default barrier* (DB), which in the seminal Merton model is represented by the face value of debt F at its maturity. The probability of default is therefore simply expressed as $PD = \Pr(V_T \leq F)$.

[6] There are no transaction costs, taxes, or short-selling restrictions. The term structure of the risk-free interest rate is flat and known with certainty. The price of a riskless bond paying 1 USD at time T is hence $B_0[T] = exp[-rT]$, where r is the instantaneous riskless interest rate.

[7] Black and Cox (1976) introduced the possibility of a more complex capital structure of the company's liabilities, Geske (1977) presented an interest-paying debt, and Vasicek (1984) established a distinction between short and long-term debt. All the previous authors also enhanced the model by treating default as an event that can occur any time before the debt's maturity. More recent improvements, such as the work by Longstaff and Schwartz (1995) and Hull and White (1995), reject the constant risk-free interest rate and consider the interest rate as a stochastic variable instead. For a detailed description of later structural models, see e.g. Altman et al. (2005) and the references therein.

Using the assumption that the value of the firm V is log-normally distributed,[8] we can get information about the probability distribution of $\ln V_T$,[9] which is

$$\ln V_T \sim \Phi\left[\ln V_0 + \left(\mu_V - 0,5\sigma_V^2\right)T, \sigma_V^2 T\right] \tag{1.4}$$

From the properties of the natural logarithm, the probability of default can be expressed as $PD = \Pr(\ln V_T \leq \ln F)$ and from that by using (1.4) we can get

$$PD = \Phi\left(-\frac{\ln(V_0/F) + \left(\mu_V - 0,5\sigma_V^2\right)T}{\sigma_V\sqrt{T}}\right) = \Phi(-d_2^*) \tag{1.5}$$

which is the PD of the company at the time of maturity T expected at time $t = 0$, $(\tau = T)$, when the value of the firm V_0 is known with certainty. While $\Phi(-d_2^*)$ in (1.5) gives the real-world (physical) probability of default, $\Phi(-d_2)$ presents the default probability in the risk-neutral world. This is caused by using the riskless interest rate r instead of the expected rate of return μ_V in the formula for d_2. In the real world, investors demand more than the risk-free rate of return and therefore $d_2^* > d_2$, which implies $\Phi(-d_2^*) < \Phi(-d_2)$ and the fact that the risk-neutral PD overstates its physical measure. Similarly, one has to distinguish between the physical and risk-neutral RR.[10]

The recovery rate, assuming no liquidation costs after default, will be given by the ratio of the firm's value at T to the debt F (V_T/F), more formally expressed as

$$RR = E\left(\frac{V_T}{F}\middle| V_T < F\right) = \frac{1}{F}E\left(V_T \middle| V_T < F\right) \tag{1.6}$$

As mentioned earlier, V is a log-normal variable, so to get an explicit formula for the RR we can use the method presented in Liu et al. (1997), which derives the conditional mean for a log-normal distributed variable, which is exactly the case of the previous equation (Resti and Sironi, 2007).

Considering the normal distribution of $\ln V$ stated in (1.4), we can write the conditional mean of V_T, given $V_T < F$, as

$$E\left(V_T \middle| V_T < F\right) = V_0 \exp[\mu_V T]\frac{\Phi(-d_1^*)}{\Phi(-d_2^*)} \tag{1.7}$$

The derivation of (1.7) is presented in the *Appendix*. Using the term in equation (1.6) we get the final expression for the expected recovery rate at time $t = 0$ in the form

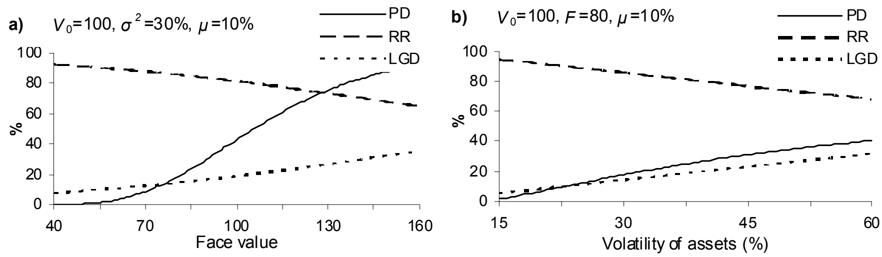
$$RR = \frac{1}{F}E\left(V_T \middle| V_T < F\right) = \frac{V_0}{F}\exp[\mu_V T]\frac{\Phi(-d_1^*)}{\Phi(-d_2^*)} \tag{1.8}$$

[8] According to Crouhy et al. (2000) this is quite a robust hypothesis confirmed by actual data.

[9] Itô's Lemma can again be used to get the dynamics for $d\ln V_t$ and from that the parameters of the normal distribution for $\ln V_t$ can be determined.

[10] As, for example, Delianedis and Geske (2003) state, risk-neutral default probabilities can serve as an upper bound to physical default probabilities. For recoveries the reverse relation holds – the risk-neutral expected recovery rate is less than its physical (real-world) counterpart (see Madan et al., 2006, p. 5).

Figure 1 Sensitivity Analysis for PD and RR (LGD)



Source: computed from eq. (1.5) and (1.8)

which is the physical recovery rate. The risk-neutral RR would be obtained by replacing μ_V with r. The RR function is homogeneous of degree zero in V_0 and F, which means that a proportional change in those variables does not influence its value (*ceteris paribus*). Moreover, the RR, like the PD, is dependent on the uncertain development of the firm's value and therefore is not constant through time but stochastic.

Using the presented expression for the PD and RR, sensitivity analyses can be made with respect to the company's other structural parameters. Consider a firm with $F=80$, $V_0=100$, $\sigma^2=30$ %, $\mu=10$ %, and $T=1$. The variables will be shocked to see how the PD and RR change.

The *Figure 1* presents the results for the RR and PD in physical measures. It can be supposed that the higher is the nominal value of the debt which the firm has to pay back at maturity T, the higher is the expected LGD and PD – part a). Thus, an increase in the firm's leverage (leaving asset volatility unchanged) brings about a higher PD and LGD. An increase in asset volatility has a similar impact, causing higher uncertainty of the firm's future value at maturity T and therefore a fall in the RR – part b).

To sum up, Merton's approach evidently generates a negative correlation between the PD and the RR, because both variables depend on the same structural characteristics of the firm. The RR is significantly determined by the value of the firm's assets at the maturity time T.

However, the original Merton model does not include any payouts to security holders. Since the interest payouts occur over the life of the debt and they are considerably lower than the principal amount, they represent a lower default risk. Neglecting them should therefore not introduce an important bias into our analysis. However, disregarding the dividend stream, as Hillegeist et al. (2004) state, could introduce significant errors into the estimation of the current market value of the firm and its volatility and influence the resulting LGD estimate. Therefore, it is necessary to modify the seminal Merton approach and incorporate dividend payouts into the model.

If we denote the dividend rate δ as the ratio between the sum of the previous year's common and preferred dividends and the market value of the firm's assets, then the equation for the equity value reflecting the dividend stream paid by the firm to equity holders would change as proposed by Hillegeist et al. (2004) to

$$E(V,T) = V\exp[-\delta T]\Phi\left(d_1\right) - Fe^{-rT}\Phi\left(d_2\right) + (1-\exp[-\delta T])V \tag{1.9}$$

where the additional $\exp[-\delta T]$ in the first term accounts for the reduction in asset value due to dividends distributed before maturity T. The last expression $(1-\exp[-\delta T])V$ does not appear in the traditional equation for the call option on a dividend-paying stock, since dividends do not accrue to option holders. Equation (1.9) is derived under the risk-neutral measure; therefore, the risk-free rate is taken to be the expected rate of return on the firm's value. This rate, however, is lowered by the dividend rate, and hence the terms d_1 and d_2 have to be modified to

$$d_1 = \frac{\ln(V_0 / F) + \left(r - \delta + 0,5\sigma_V^2\right)T}{\sigma_V\sqrt{T}},\ d_2 = d_1 - \sigma_V\sqrt{T}$$

where all parameters were defined above.

4. Implementation of the Model

The empirical use of any structural model is based on variables which are not directly observable. Similarly, in our case, the market value of assets V and asset volatility σ_V must be estimated in order to compute the expected LGD.[11] The procedure for estimating these variables was first proposed by Jones et al. (1984) for publicly listed companies, exploiting the prices of their shares. Their approach is based on simultaneously solving two equations which match the value of equity E and its volatility σ_E with two unknown variables V and σ_V. Jones et al. (1984) used relation (1.3) as the first equation. However, this equation does not consider dividend payouts and we will hence utilize the modified equation (1.9). The second equation linking the observable and unknown values results from the relation for the option delta

$$\sigma_E E = \sigma_V \exp[-\delta T]V\Phi(d_1) \tag{2.1}$$

and its derivation is presented, for example, in Seidler (2008). This system of two equations has to be solved to arrive at the unobservable market value of the firm's assets and their volatility. Due to the non-linearity of those equations it is necessary to solve the system iteratively.[12]

The accuracy of the expected LGD estimate is therefore dependent on the estimates of the parameters in equation (1.8). Although some of them, such as the debt's face value[13] and its maturity, are observable, some assumptions about them must be made before Merton's simplifying approach can be implemented. For example, the model requires the firm's capital structure to be reduced into a single liability. Since a large share of the firm's debt is not traded very often, we have to use the book values as a proxy. As a result, the book value of the total liabilities reported in firms' balance sheets is used as the notional face value of the zero coupon bond.

[11] The market value of the firm is the sum of the market value of its equity and debt. However, the market value of the debt is not usually available, since companies are not financed entirely by traded debt.

[12] To solve two non-linear equations of the form $F(x,y)=0$ and $G(x,y)=0$, the function $[F(x,y)]^2 + [G(x,y)]^2$ can be minimized (see Kulkarni et al., 2005).

[13] This holds only if the debt is traded.

To determine the maturity time of the zero coupon bond representing all the firm's liabilities, we could compute the weighted maturity of the individual claims' maturities.[14] However, our intention is to provide LGDs comparable across the sample of analyzed companies, which would hardly be practicable in the case of different maturities. Therefore, we will assume 5-year debt maturity for all companies, an assumption which should take into account the length of both short-term and long-term debt maturity.[15]

From our previous discussion it is obvious that the V and σ_V estimates are highly dependent through the system of two equations on the value and volatility of equity. While the market value of equity E is obtained simply as the closing price of the firm's shares at the end of the fiscal year multiplied by the outstanding number of stocks, the equity volatility depends on the chosen method of estimation. For that reason, it is desirable to use different types of estimation techniques for comparison.

The standard approaches to estimating σ_E can be based on historical stock price data or can exploit bond prices to get the so-called implied volatility. Nevertheless, since the latter volatility estimate incorporates all possible errors of the model used, and also considering our discussion about the illiquid and insufficient bond market, we will use only the historical approach based on the development of stock returns.

Since we set the maturity time to five years, we should also use long-term volatility for our predictions. For that reason, we used the volatility of five trading years (see Hull, 2002).[16] In addition, to take into account possible changes in volatility in the shorter run, we also estimate the last 250 trading days' volatility, similarly, for example, to Kulkarni et al. (2005).

Furthermore, we used the exponentially weighted moving average (EWMA), where more recent observations carry higher weights. This method, better capturing the volatility dynamics, is recommended in RiskMetrics™ (1996). For our sample of companies we used monthly observations over the five years with a decay factor equal to 0.97.[17]

As the last estimate we used the long-run average variance from the GARCH(1,1) model, which we estimated for daily data over the 5-year interval in the form

$$\sigma_t^2 = b + \alpha_1 r_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 \qquad \alpha_0 > 0, \alpha_1 \geq 0, \alpha_2 \geq 0 \tag{2.2}$$

where $b=\alpha_0 \sigma_{LR}^2$, σ_{LR}^2 represents the long-run unconditional variance of the daily returns r and $\alpha_0, \alpha_1, \alpha_2$ are the weights, whose sum is equal to 1. We computed

[14] Another method widely used among academics is to group the short-term and long-term obligations and find out the maturity by weighting the maturities of those two groups. For example, Delianedis and Geske (2001) made an assumption of 1-year maturity for short-term and 10-year maturity for long-term debt. The weights would be the book values of the claims.

[15] By setting a longer time horizon, we should also avoid inaccuracies deriving from the fact that we use a poor diffusion process without possible jumps for the firm's asset value dynamics.

[16] In the case of an insufficiently long time series, we use the longest available one. This also holds for the other 5-year estimates computed later in this section.

[17] The decay factor determines the relative weights for a particular observation. The value 0.97 is based on the analysis relating to optimal λ provided in RiskMetrics™ (1996).

the long-run volatility from the estimated parameters as

$$\sigma_E = \sqrt{\frac{b}{1-\alpha_1-\alpha_2}} \tag{2.3}$$

For most of the companies in our sample, we estimated four types of daily equity volatility by the aforementioned methods. Those still needed to be scaled to obtain the annualized volatility used in later computations.

All the estimates are given in *Table 4* available at the webpage of this journal. Since higher equity volatility results in higher volatility of the firm's value and higher default risk, the choice of estimated σ_E can significantly influence the subsequent results. However, we consider it more desirable as a rule of prudence to provide overstated rather than understated LGD values. Therefore, we use the average of the two highest σ_E estimates, σ_E^*, as the parameter entering the system of two equations.

As the firm's expected rate of return, the derived system for obtaining the unobservable values of V and σ_V exploits the risk-free rate r_f, for which we used the yield on the 5-year government bond. Therefore, the last parameter that must be estimated in order to solve the equations is the dividend rate δ. Nonetheless, to obtain δ one needs to get the market value of the firm V. Hence, we use the approximate market value V' as the sum of the market value of the equity E and the book value of the debt.[18] Since we are estimating the 5-year horizon, we will use in the computations the adjusted rate δ^* capturing the dividend stream in the last five years instead of the one-year dividend rate δ.[19] We solved the two equations simultaneously by the iterative Newton search algorithm. The approximate value V' and the equity volatility were used as the starting values for V and σ_V, respectively. In almost all cases, the process converges within ten iterations.

For the estimate of the expected LGD in the risk-neutral measure we already know all the necessary parameters. However, as the risk-free rate can differ significantly from the real rate of return, we also estimate the expected market return on assets, μ_V, as the return on assets during the previous year. We can easily utilize the estimated values of the firm's market value V and get the one-year return μ_V as

$$\mu_V(t) = \frac{V(t) + Div(t) - V(t-1)}{V(t-1)} \tag{2.4}$$

where $V(t)$ is the firm's market value at the end of year t and $Div(t)$ denotes the sum of the common and preferred dividends declared during this year. Since the 5-year expected return will not be based solely on a one-year observation, we use in our calculations the adjusted μ_V^* again as the 5-year weighted average, in which in recent years carries more weight to react faster to current information.

5. Estimation of LGDs in the Czech Republic

We will implement the aforementioned methods on a sample of firms that are listed on the Prague Stock Exchange (PSE) and present the dynamics of the 5-year

[18] This approach, as Wong and Li (2004) show, overestimates the true market value of the firm.

[19] We used an exponentially weighted average with decay factor λ=0.9.

expected LGD for each company between 2000 and 2008. We restrict our sample to non-financial firms so that the leverage ratios are comparable across them. In addition, we exclude enterprises that were listed after 2007 because of the short time series of the share prices needed to estimate the asset volatility. The list of 27 analyzed companies can be found in *Table 3* available at the webpage of this journal.

The income statements and the balance-sheet items for our set of PSE corporations were obtained from Magnus (2008) database, and some of them were completed from company annual reports. Share prices, dividend yields, and the number of shares outstanding are available on the PSE website.[20] We used the time series of share prices from the beginning of 1999 to the end of 2008 and the accounting information reported at the end of the fiscal year. The series of 5-year risk-free interest rates comes from the ARAD database of the Czech National Bank (CNB).

The non-existence of dividend payouts in the seminal Merton model was modified in the last section. Still, one should also incorporate the costs of bankruptcy, which result in debt holders receiving less than the total firm value in the case of default. While Betker (1997) estimated the direct administration costs relating to bankruptcy at around 5 % of the firm's value, the study by Andrade and Kaplan (1998) indicates higher costs of financial distress – in the range of 15–20 %. Based on those empirical studies we consider exogenous common bankruptcy costs $(1 - \varphi)$ equal to 10 %.

The final formula for the 5-year expected LGD at the beginning of year t in the physical measure, including both dividend payouts and bankruptcy costs, is then

$$ELGD_t = 1 - \varphi \frac{V_t}{F_t} \exp[(\mu^*_{V,t} - \delta^*_t)T] \frac{\Phi(-d^*_1)}{\Phi(-d^*_2)} \tag{2.5}$$

$$d^*_1 = \frac{\ln(V_t / F_t) + \left((\mu^*_{V,t} - \delta^*_t) + 0,5\sigma^2_{V,t}\right)T}{\sigma_{V,t}\sqrt{T}}, \text{ and } d^*_2 = d^*_1 - \sigma_{V,t}\sqrt{T}$$

where the time subscripts represent the particular values at the beginning of year t, and $\mu_{V,t}^*$ and δ_t^* denote adjusted rates considering 5-year historical observations. One can get the expected LGD in the risk-neutral measure by replacing $\mu_{V,t}^*$ by r_f.

The results are given in *Table 1*, which presents the expected LGD for each company estimated at the beginning of every year during the period 2000–2008 in both the risk-neutral and physical measures.[21] All the parameters used for the computations are given in *Table 4* available at the webpage of this journal.

In the theoretical framework, the risk-neutral LGD is always the upper bound to its physical counterpart. Nevertheless, this holds only if the asset drift μ_V is greater than the risk-free rate. In the conventional analysis, the rate r_f is supposed to be always less than the drift μ_V. For example, Hillegeist et al. (2004) compute μ_V for PD estimates and use r_f as a minimum bound for μ_V, since they claim that expected growth rates lower than r_f are inconsistent with asset pricing theory. However, this

[20] This information is also available for Czech companies in the Magnus (2008) database.

[21] The estimates in the physical measure begin from the year 2001, since we lost one observation for acquiring the firm's growth rate.

Table 1 5-year expected LGDs in the period 2000–2008

Company	Expected LGD (%) – risk neutral measure									Expected LGD (%) – physical measure							
	2000	2001	2002	2003	2004	2005	2006	2007	2008	2001	2002	2003	2004	2005	2006	2007	2008
CETV	-	-	-	-	-	-	18.0	22.5	21.4	-	-	-	-	-	-	23.1	19.1
Č. NÁM. PLAVBA	28.7	26.1	23.1	24.0	22.2	34.8	16.2	13.7	12.5	23.5	28.6	24.6	22.8	34.7	16.0	12.9	12.6
ČEZ	24.1	27.7	34.4	35.7	35.3	30.7	29.3	29.2	24.1	32.7	47.1	39.3	29.6	21.2	18.1	18.7	16.7
ECM	-	-	-	-	-	-	-	13.8	18.4	-	-	-	-	-	-	-	13.3
ENERGOAQUA	13.0	24.4	37.7	35.7	33.4	22.4	17.8	14.0	13.6	17.2	22.7	20.8	19.6	17.4	14.6	12.4	12.2
JČ PAPÍRNY VĚTŘNÍ	29.2	23.7	26.3	26.5	21.3	32.4	23.1	23.0	33.6	30.3	52.6	33.2	57.9	33.2	13.0	14.1	36.2
JM PLYNÁRENSKÁ	44.7	38.3	34.4	45.6	32.2	23.8	19.4	14.9	11.6	48.2	21.5	27.7	19.5	14.8	17.4	13.4	11.2
LÁZNĚ TEPLICE	17.7	16.4	16.9	15.5	17.0	17.2	16.9	15.9	14.7	49.5	12.6	11.5	12.9	13.7	14.1	14.7	14.9
LEČ. L. JÁCHYMOV	30.1	18.0	25.9	20.4	19.1	16.8	14.0	15.6	13.3	85.8	49.5	19.7	19.0	17.1	13.7	14.9	13.4
ORCO	-	-	-	-	-	-	21.3	22.5	29.5	-	-	-	-	-	-	13.2	16.7
PARAMO	30.4	17.6	16.2	20.5	19.5	23.8	25.0	21.4	22.5	78.4	65.4	44.3	16.5	20.6	19.1	18.7	19.6
PEGAS	-	-	-	-	-	-	-	28.4	19.0	-	-	-	-	-	-	-	20.4
PHILIP MORRIS	-	17.0	25.4	36.9	32.1	31.1	28.9	32.5	41.2	-	15.8	21.7	18.8	20.8	21.0	29.5	41.2
PR. ENERGETIKA	51.5	40.8	42.5	44.0	35.9	28.8	25.1	22.9	20.6	52.7	53.5	40.4	28.5	22.0	18.5	17.4	16.2
PR. PLYNÁREN.	30.2	33.0	34.6	40.3	38.5	36.8	30.9	29.1	26.4	66.7	33.2	36.3	36.4	26.6	19.4	19.2	18.9
PR. SLUŽBY	18.3	25.6	22.2	22.0	20.1	14.4	12.1	11.2	10.7	17.0	22.1	21.9	17.1	13.3	11.7	11.0	10.6
RM-S HOLDING	29.2	34.6	32.7	27.1	34.5	35.4	24.6	12.5	11.4	58.8	50.4	33.4	29.1	31.2	24.5	12.6	11.5
SETUZA	30.0	30.6	28.2	28.0	28.7	29.8	29.8	28.4	27.4	13.3	14.3	17.4	21.3	23.5	31.0	30.7	22.7
SLEZAN FM	26.4	34.4	34.4	32.4	29.4	30.2	25.9	23.1	18.5	88.3	70.4	27.8	23.1	27.9	25.3	23.2	19.8
SM PLYNÁREN.	31.5	25.1	40.6	29.9	33.7	33.7	23.9	21.7	19.2	25.2	36.5	21.1	29.6	33.5	21.5	19.6	17.4
SPOL. CH.H..VÝR.	20.0	16.2	23.0	23.4	24.9	22.4	25.5	22.0	20.0	70.1	37.8	28.1	23.9	15.8	14.5	13.7	13.2
SPOLANA	33.3	33.5	36.1	34.2	35.0	34.9	27.8	27.5	26.6	42.9	76.6	58.5	44.3	45.0	28.9	27.1	30.0
TELEFÓNICA	23.9	32.5	36.7	36.0	33.4	33.3	26.3	22.9	43.4	40.2	49.5	51.7	35.4	32.7	23.0	20.9	37.1
TOMA	29.9	29.1	23.0	23.5	21.0	19.7	23.5	21.4	18.7	67.5	24.2	29.6	18.4	15.6	16.5	15.8	13.4
UNIPETROL	36.1	30.1	26.5	24.8	26.4	29.8	35.0	36.3	31.3	24.0	25.3	23.4	22.1	27.0	18.8	22.3	21.6
VČ PLYNÁRENSKÁ	42.8	34.9	48.6	63.5	56.9	55.3	48.4	49.1	30.7	33.6	33.1	41.1	39.7	42.5	39.0	41.0	28.9
ZENTIVA	-	-	-	-	-	18.6	22.6	22.9	24.0	-	-	-	-	-	15.3	18.7	19.2
Mean (%)	29.6	27.7	30.4	31.4	29.6	28.5	24.5	22.9	22.4	46.0	38.3	30.6	26.6	25.0	19.8	19.2	19.5
Std. Dev. (%)	9.1	7.4	8.4	10.8	9.0	8.9	7.4	8.3	8.7	23.4	18.5	11.7	10.7	9.2	6.5	7.1	8.3

Source: computed from eq. (2.5)

approach can result in highly underestimated LGD values if the real growth rate is lower than r_f.[22] This can be demonstrated using selected results.

Paramo ended the year 2000 with a loss of more than CZK 430 million and an almost 24 % drop in its market value. This negative result has no impact on the expected risk-neutral LGD at the beginning of 2001 and its value is even below-average for the given year. However, the physical estimate captures the huge deterioration in the firm's asset value, which leads to a more than four times higher expected LGD. Also, Spolana recorded losses of about CZK 700 million as a result of negative developments in the plastics market in 2001. The subsequent year it was negatively affected by floods, which led to further losses. While the risk-neutral LGDs in these years do not incorporate any problem compared to the other years' estimates, the physical measure counterparts indicate the company's poor performance quite well. The same situation can be found in the case of Lázně Jáchymov in 2001, Slezan FM in 2001 and 2002, and Papírny Větřní in 2002 and 2004, for example. By contrast, when the growth rate of the firm's assets μ_V is higher than r_f, the risk-neutral estimates overstate the ELGD.

The relatively high ELGD in both measures for ČEZ in 2002 might seem contradictory, since ČEZ ended 2001 successfully with an increase in net profit of 26 % to more than CZK 9 billion. However, its share price dropped from an initial CZK 101 at the end of 2000 to CZK 77.5 at the end of 2001, which led to a more than 23 % decrease in the market value of its equity. This development, together with a high dividend rate, was reflected in an almost 14 % deterioration in its asset value and led to a significant increase in its ELGD. Similarly, a high decrease in the market value of equity caused a worsening of the predictions for Telefónica in 2002 and 2003. Nonetheless, the sharp rise in its ELGD in 2008 is solely due to a sharp increase in asset volatility.

The downswing in economic activity due to global and domestic factors was not sufficiently incorporated into share prices at the end of 2007. Therefore, the average ELGD at the beginning of 2008 is relatively small, still capturing the good economic development in recent years. However, the financial turbulence during 2008 caused a considerable drop in market prices of equity and an increase in asset volatility for some of the companies analyzed. As the result, the rough average ELGD estimate[23] at the beginning of 2009 rose to almost 40 %, which indicates a significant increase in credit risk in the non-financial corporation sector. For some companies, the ELGD increased only modestly (Unipetrol, Telefónica, and Philip Morris) or was virtually unchanged from the previous year (Spolana, Toma, and Zentiva), while other companies experienced significant growth of their expected LGDs, which exceeded their historical values several fold (CETV – 74 %, Orco – 65 %, and Pegas – 70 %). This holds especially for companies that have been listed on the PSE for only a short period.

[22] The risk-neutral estimates are based on the same company structural values relating to credit risk as the physical estimates, except different assumptions are made about the expected growth of the company's assets. The more μ_V differs from r_f, the more inaccurate results they provide compared to the physical counterpart.

[23] These estimates are based on companies' share prices at the end of 2008, but still use accounting information from the previous year and therefore provide only illustrative estimates of the ELGD.

As mentioned earlier, the literature relating to the empirical evidence on realized LGDs focuses on different facilities in distinct countries and is based on diverse sample sizes across different periods. This makes comparison of our estimates with realized LGDs difficult. However, the average LGD based on 25 studies summarized by Grunert and Weber (2005) is about 30 %, which corresponds to our results. Direct comparison for the Czech Republic is even more problematic because of the insufficient database of actual LGDs. Moreover, the companies in our sample are among the better-rated companies, which generally have a relatively rare occurrence of default. Nevertheless, CNB (2008) gives LGDs for large enterprises of around 34 % for secured claims and 48 % for unsecured claims in 2008. Chalupka and Kopecsni (2008) used a set of micro-data on loans to corporations in the period 1995–2005 of an anonymous commercial bank and estimated an overall workout LGD of about 52 %. Considering that average the indebtedness in our sample is lower than the average indebtedness in the non-financial corporate sector,[24] the average ELGDs of the companies analyzed should be lower than the above-mentioned values.

6. Sensitivity Analysis

The sensitivity analysis relating to the initial Merton model discussed in the theoretical section assumed that all the necessary structural variables are known. However, as mentioned earlier, the value and volatility of the firm's assets are not directly observable and they have to be estimated through a system of two equations which hold only at a given time. Therefore, the following analysis concentrates on the sensitivity of the ELGD arising from potential changes in the company's structural variables influencing the estimates of σ_V and V. The main emphasis is put on leverage, defined as the ratio between total liabilities and the market value of all assets (F/V).

Before we present the ELGD sensitivity for the individual companies in the analyzed sample, we provide a general theoretical discussion based on different input parameter scenarios. The main difference between the current analysis and the previous one illustrated in *Figure 1* is that a change in leverage influences the estimate of the firm's asset volatility σ_V. Thus, as leverage increases, the weight of equity in the firm's value declines and the volatility decreases.[25] The rate of decline is presented for a given set of parameters in the first part of *Figure 2*.

This figure also illustrates the impact of an increase in the firm's leverage on the PD and ELGD. However, while the growth in leverage has an unambiguously positive effect on PD, the ELGD peaks at a particular leverage ratio and then starts to decrease.

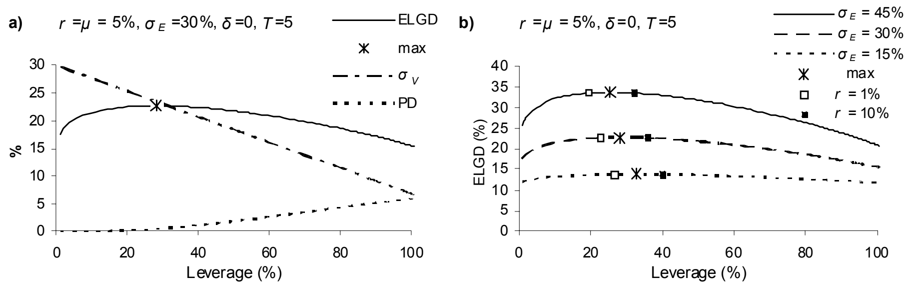
The negative relation between the ELGD and leverage may look counter-intuitive; nevertheless, it is caused by decreasing asset volatility σ_V.[26] Although the PD is

[24] This comparison is based on the economic results of non-financial enterprises with more than 100 employees provided by the Czech Statistical Office (CZSO, 2008).

[25] Note that σ_V is lower than σ_E. This is caused by the presence of leverage, since the debt is considered to be non-traded. With increasing leverage, equity occupies a lower share in the overall value of the firm and V is therefore less volatile than E.

[26] The previous analysis reported in *Figure 1* shows a strictly positive correlation between the ELGD and leverage. However, σ_V was taken as a constant and did not change with leverage.

Figure 2 ELGD Sensitivity Analysis



Source: computed from eq. (2.5) and system (1.9), (2.1)

increasing with leverage, the expected value of the firm's assets at maturity T, conditioned by default ($V_T<F$), has increased with respect to a given leverage. In other words, due to lower volatility σ_V it is less likely that the firm's expected value will be excessively below the default barrier F at time T and therefore the expected recovery ratio (V_T/F) in the case of default has increased.

The result is that by leaving the initial volatility of equity as a constant,[27] the increase in leverage causes a decline in asset volatility, which from a particular leverage ratio (L^* – the breakpoint) generates a negative correlation between the PD and the ELGD. Nevertheless, for all the presented scenarios the increase in the PD outweighs the decline in the LGD, and so the expected loss per unit of exposure (PD.ELGD) is strictly increasing with leverage.

Pursuing the issue further, we analyze the changes in breakpoints with respect to other parameters. The maximum ELGD points are presented for three different values of r_f and σ_E. As can be seen, the decline in the risk-free interest rate shifts the maximum ELGD points to the left, similarly to an increase in equity volatility (*Figure 2b*). It is evident that any increase in σ_E will lead (because of higher uncertainty) *ceteris paribus* to higher values of ELGD. However, the figure also presents the variability of the potential ELGDs along the whole range of leverages. While for σ_E=45 % the ELGDs vary from 22 to 33 %, the volatility for σ_E=30 % is only 7 percentage points, and in the case of σ_E=15 % the variability of the possible ELGDs is minimal. This further highlights the importance of volatility as a crucial variable for LGD predictions and indicates that companies with identical leverage ratios can have substantially different ELGD sensitivities. Further discussion regarding the sensitivity analysis can be found in Seidler (2008).

The empirical results for the analyzed sample are reported in the following table, which shows the leverage elasticity of the ELGD in both measures at the beginning of 2008.

As can be seen, most of the companies analyzed have inelastic ELGDs with respect to leverage. Only Spolek pro chem. a hut. výrobu has a negative elasticity, slightly exceeding 1. Based on our previous discussion we can analyze differences in

[27] The change in leverage will also affect the equity volatility. However, since for the computation we use the long-run volatility σ_E^*, on which sudden short-term changes have no effect, the assumption of constant σ_E in the sensitivity analysis is maintainable.

Table 2 Elasticity of ELGD with Respect to Leverage

Company	$\varepsilon^{ELGD^Q}_{Leverage}$	$\varepsilon^{ELGD}_{Leverage}$
CETV	0.071	0.022
Č. NÁM. PLAVBA	0.042	0.045
ČEZ	0.078	-0.034
ECM	-0.607	-0.643
ENERGOAQUA	0.188	0.080
JČ PAPÍRNY VĚTŘNÍ	0.116	0.129
JM PLYNÁRENSKÁ	0.198	0.092
LÁZNĚ TEPLICE	-0.055	-0.047
LEČ. L. JÁCHYMOV	0.026	0.028
ORCO	0.344	-0.128
PARAMO	-0.393	-0.498
PEGAS	0.341	0.405
PHILIP MORRIS	0.403	0.403
PR. ENERGETIKA	0.268	0.128
PR. PLYNÁREN	0.856	0.423
PR. SLUŽBY	0.011	0.004
RM-S HOLDING	0.022	0.024
SETUZA	-0.867	-0.890
SLEZAN FM	0.432	0.493
SM PLYNÁREN.	0.308	0.228
SPOL. CH.HUT.VÝR.	-1.072	-1.095
SPOLANA	-0.647	-0.477
TELEFÓNICA	0.175	0.150
TOMA	-0.093	-0.179
UNIPETROL	-0.025	-0.148
VČ PLYNÁRENSKÁ	0.271	0.244
ZENTIVA	0.012	-0.109

Note: $ELGD^Q$ denotes risk-neutral estimates of ELGD

the risk-neutral (ε^Q) and physical (ε^P) elasticity with respect to other parameters. For example, CET and Pr. Služby, with zero dividend rates and low leverage at the beginning of 2008, are located on the rising part of their ELGD sensitivity curves. However, because μ_V lowers the ELGD's rate of growth and the expected asset rate μ_V is higher than r_f for both companies, their "physical" elasticity is lower than ε^Q. On the contrary, Č. Nám. Plavba and JČ Papírny show an inverse inequality between ε^P and ε^Q since their $\mu_V < r_f$. [28]

The sensitivity analysis further illustrates the differences already pointed out between the risk-neutral and physical measure. However, a more important finding seems to be that the ELGD is quite inelastic with respect to leverage, and sudden changes in the ELGD do not generate significantly large changes in the expected LGD.

7. Criticism and Limitations

The first implementation of Merton's model, applied by Jones et al. (1984), Ogden (1987), and Franks and Torous (1989), suggested that the model generates

[28] The values of leverage and expected asset growth are reported in *Table 4* available at the webpage of this journal.

lower credit spreads than those observed on the market. Similarly, more recent studies by Lyden and Saraniti (2001) and Helwege et al. (2004) showed that the basic Mertonian contingent claim model underpredicts the actual bond spread, especially for low-leveraged and low-volatility companies. Based on those findings, our ELGD estimates would be undervalued. However, considering that bond spreads also reflect market risk, tax, and liquidity effects, the aforementioned studies only confirm Merton's inability to capture other components of debt spreads, saying nothing about the model's ability to reveal default and recovery risk.[29]

Sarig and Warga (1989) did not compare the absolute values of theoretical corporate bond spreads, but only compared their rates of change with respect to change in the actual bond default riskiness and confirmed the good predictive power of Merton's model. Furthermore, Delianedis and Geske (2001) termed the difference between the observed and modeled spread the residual spread and empirically confirmed that the spreads estimated with the Merton approach correctly evaluate default risk and that the residual spread is driven by liquidity, tax, and other effects. These conclusions move towards the correctness of our LGD estimates, since the accuracy of the ELGDs is based on capturing the company's default risk, which should also be reflected in share prices. Nevertheless, the ELGD's stock market dependence can also embody excessive movements in share prices caused by market bubbles. Also, the stock market may not efficiently incorporate all publicly available information about the default probability and, especially in the case of a young market such as the Czech one, the limits of the information given by share prices – particularly for companies whose shares are not so frequently traded – should be considered.[30]

For the purposes of the Basel II framework, the ELGDs based on equity development are procyclical and, due to an increase in the minimum required capital, would lead to a credit crunch in a recession and contrariwise to overlending at a time of strong economic growth.[31] The definition of default used in the model corresponds more to a state of bankruptcy than to the obligor's 90 days past due obligation defined in Basel II. Thus, the model's definition of default leads to an overstated ELGD;[32] however, the companies analyzed should have high capabilities to raise funds. So, if a company is more than 90 days past due on its obligation, it has probably exhausted all means of raising funds and bankruptcy will follow. Different default definitions hence should not generate significant inaccuracies.

[29] In spite of these well known complications and imperfections, the majority of the literature empirically testing structural models has presumed that the credit spread is primarily due to default risk, since the other components are hardly tractable. This idea stems from the theoretical assumption that corporate bond markets are perfect and complete and trading takes place continuously (see Delianedis and Geske, 2001).

[30] Č. nám. plavba, Energoaqua, Jihomoravská plynárenská, Pražské služby, RM-S HOLDING, SLEZAN FM, and Východočeská plynárenská. Nonetheless, we estimate LGDs for these less liquid companies as well, because our estimates are based on 5-year volatility and we can still acquire some information even if the liquidity in one year is low.

[31] Implied market estimates should not substitute for estimated LGD values based on historical data as requested in Basel II. However, they may serve as an early warning signal and thereby improve the current credit management.

[32] A broad definition of default leads to a lower estimate of the PD but a higher estimate of the LGD because fewer exposures will be classified as "in default," but those will have relatively lower quality with a low recovery outlook. Conversely, higher default rates and also higher recoveries stem from a narrower and stricter definition of default.

The computations also do not consider any debt priority, therefore the ELGDs for secured and more senior claims should be lower than the presented estimates and conversely higher for subordinated debt. However, the distribution of the value of the bankrupted firm depends also on violation of the APR, which is difficult to predict for single cases. The bankruptcy costs were determined using other empirical studies. Nonetheless, bankruptcy laws and other procedures differ substantially by country and may therefore differ in the Czech Republic. A calibration on the empirical sample would be needed to obtain more accurate estimates, but the appropriate data sample is not available due to a low number of defaults of comparable companies. Also, our estimates cover only a relatively short time period, characterized (except last year) by relative stability and rapid financial growth.

The computed ELGDs also suffer from other shortcomings, such as the assumption of a constant interest rate, no tax shield, and other simplifications arising from the seminal Mertonian approach. On the other hand, more sophisticated models require a higher number of parameters, which have to be estimated. This increases the computational complexity and might therefore produce higher errors. Also, some of the changes introduced, relating, for example, to the stochastic interest rate, have unambiguous effects and sometimes have little impact on the results (Lyden and Saraniti 2001). Nevertheless, the empirical application of more complex models will be the goal of further research.

8. Conclusion

One of the topics currently being studied intensively in quantitative finance is the concept of Loss Given Default, which is rather unexplored territory in the credit risk area. Especially with the implementation of the New Capital Accord, LGD has received increased attention and become a frequent object of empirical and theoretical research. The goal of this paper was to present the basic knowledge concerning this key input parameter of credit risk analysis and above all to introduce a modeling technique which enables estimation of forward-looking LGDs from market observable data.

We exploited the information embedded in the stock market and implemented the Mertonian structural approach based on contingent claim analysis, which considers the residual value of a firm's assets to be the recovered amount in the case of default. We also pointed out the interdependence between the PD and the LGD, which implies that they should not be treated as independent in credit risk modeling.

We analyzed companies listed on the Prague Stock Exchange in the period 2000–2008 and computed the expected LGD for every single company in a given year. The average LGD of the sample across time was estimated to be in the range of 20–45 %. We also described estimation procedures exploiting equity prices and volatility and showed that the LGD is relatively inelastic with respect to the leverage of the company. We demonstrated that the LGD in the physical measure is a more reliable indicator than its risk-neutral counterpart.

The main value added of this work is that it demonstrates an estimation technique for implied market LGD utilizing the Mertonian approach. As a result, the paper presents unique estimates of LGDs for the Czech corporate sector. This should offer another perspective on LGD and provide a better understanding of the difficulties related to this credit risk parameter.

APPENDIX

Derivation of equation (1.7) – see Liu et al. (1997) and Resti and Sironi (2007)

Suppose that variable Y is log-normal and $\ln Y$ is normally distributed with mean μ and variance σ^2. Then variable $Z = (\ln Y - \mu)/\sigma$ has a standard normal distribution. The conditional mean of Y, given $Y<c$, can then be expressed as follows

$$E\left(Y \middle| Y<c\right) = E\left(\exp\left[\sigma Z+\mu\right] \middle\| \exp\left[\sigma Z+\mu\right]<c\right)$$

$$= E\left(\exp\left[\sigma Z+\mu\right] \middle\| Z<\left(\ln c-\mu\right)/\sigma\right) \tag{3.1}$$

To simplify the following expression, let's define

$$g = \left(\ln c-\mu\right)/\sigma \text{ and } h = \Phi(g) \tag{3.2}$$

where Φ (.) is the normal c.d.f. with these notations. Equation (3.1) becomes

$$E\left(Y \middle| Y<c\right) = h^{-1}\int_{-\infty}^{g} \exp[\sigma Z+\mu]\left(2\pi\right)^{-1/2}\exp[-z^2/2]dz$$

$$= \exp[\mu+\sigma^2/2]h^{-1}\int_{-\infty}^{g}\left(2\pi\right)^{-1/2}\exp[-(z-\sigma)^2/2]dz$$

$$= \exp[\mu+\sigma^2/2]\frac{\Phi\left(\left(\ln c-\mu\right)/\sigma-\sigma\right)}{\Phi\left(\left(\ln c-\mu\right)/\sigma\right)} \tag{3.3}$$

Considering the parameters of the normal distribution of $\ln V$ stated in (1.4), we can write the conditional mean of V_T, given $V_T<F$, as

$$E\left(V_T \middle| V_T<F\right) = \exp[\mu_v^* + \sigma_V^{*2}/2]\frac{\Phi\left(\left(\ln F-\mu_v^*\right)/\sigma_V^*-\sigma_V^*\right)}{\Phi\left(\left(\ln F-\mu_v^*\right)/\sigma_V^*\right)} \tag{3.4}$$

where $\mu_v^* = \ln V_0 + \left(\mu_V - 0,5\sigma_V^2\right)T$ and $\sigma_V^{*2} = \sigma_V^2 T$. After substituting and rearranging we get

$$E\left(V_T \middle| V_T<F\right) = \exp\left[\ln V_0 + \mu_V T\right]\frac{\Phi\left(-\dfrac{\ln\left(V_0/F\right)+\left(\mu_V+0,5\sigma_V^2\right)T}{\sigma_V\sqrt{T}}\right)}{\Phi\left(-\dfrac{\ln\left(V_0/F\right)+\left(\mu_V-0,5\sigma_V^2\right)T}{\sigma_V\sqrt{T}}\right)}$$

$$= V_0\exp[\mu_V T]\frac{\Phi(-d_1^*)}{\Phi(-d_2^*)} \tag{3.5}$$

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