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# Exchange Rate Management and Inflation Targeting:

## Modeling the Exchange Rate in Reduced-Form New Keynesian Models

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*Abstract*
*This paper introduces a strategy for modeling the exchange rate when the monetary authority targets inflation while also managing the exchange rate using interventions. It does so in the framework of a standard reduced-form New Keynesian model of monetary transmission used in many institutions for research, forecasting, and monetary policy analysis. We propose a micro-founded modification to the UIP condition which allows for modeling of informal exchange rate bands. Our modeling strategy is useful for most hybrid IT regimes, including those with imperfect control over market interest rates.*

## 1. Introduction

The modeling of transmission mechanisms when the monetary authority operates inflation targeting (IT) as well as managing the exchange rate is an important issue for many central banks. The mix of a fixed exchange rate and an IT regime calls for adequate forecasting and modeling capacities, similar to those required by full fledged IT.

Various central banks are in need of such forecasting capacities. For instance, there are countries with a fixed or strongly managed exchange rate which are contemplating a gradual transition towards a more flexible exchange rate regime. Examples include Botswana, Belarus, Egypt, and Ukraine, among others. Although they peg their currencies either to a single foreign currency or to a basket of international currencies, they are simultaneously attempting to implement elements of inflation targeting by controlling the deviation of inflation from the target through adjustments to short-term interest rates.

At the same time, there are IT (or close-to-IT) countries with a flexible exchange rate that have gradually approached an exchange rate peg within the ERM II framework (Slovakia, for instance) or will do so in the future, such as the Czech Republic. The Czech Republic's almost free float[1] will in future be constrained by the ERM II mechanism with a fluctuation band, only to be entirely removed on joining the EMU later on. The freedom of the central bank to choose targets and interest rates to achieve those targets will thus gradually be reduced.

* The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the institutions they represent or any other person associated with them.

[1] See (Geršl, Holub, 2006) for a discussion of exchange rate interventions undertaken recently by the Czech National Bank.

Such a modeling capacity is useful for other countries, too, as the phenomenon of the coexistence of IT with some kind of exchange rate management is present in many IT countries, at least informally. Many of them, at times, attempt to control excessive exchange rate fluctuations by interventions of various forms (e.g., sterilization of inflows). Some of them (e.g., Hungary until recently) even recognize two explicit targets: in terms of exchange rate and inflation bands.

Indeed, the two intermediate targets are not incompatible (e.g., Mishkin and Savastano (2001)). In theory, there is no difference between inflation and exchange rate targeting, as long as the targets are defined consistently with each other. It is the practice of the two regimes that marks the difference. In pegged regimes, central banks are punished for mistakes in assessing economic fundamentals (such as the equilibrium real exchange rate) by the financial markets, which act swiftly and whose force is hard to balance. On the other hand, mistakes in IT regimes are punished by consumers (and the public in general) through expectations, which evolve only gradually. In practice, monetary policy makers have many more opportunities to correct their mistakes when targeting inflation rather than the exchange rate. For IT, it is enough for the policy to be correct on average, whereas for a peg, a single mistake can have devastating consequences.

This paper introduces a strategy for modeling exchange rate behavior when the monetary authority attempts to control both the exchange rate and the inflation rate, each with a different instrument: inflation with interest rates and the exchange rate through interventions.[2] It does so in the framework of a simple reduced-form New Keynesian (NK) model of monetary transmission, as used in many institutions for research, forecasting, and monetary policy analysis.

Our strategy differs from other approaches that combine IT with partial control over the exchange rate, in that it uses the exchange rate as an operational rather than an intermediate target. The exchange rate thus complements the interest rate as a monetary policy instrument rather than inflation as an intermediate target (as in (Parrado, 2004a), for instance). This is a more relevant approach in cases where control over money market interest rates is not yet perfectly established, or where changing interest rates is for some reason insufficient to achieve the intermediate inflation target and the central bank resorts to interventions. In addition to pure IT, our strategy encompasses the hybrid IT regimes of informal exchange rate corridors, pegged or crawling exchange rates, and imperfect control over market interest rates.

We keep our analysis technical and avoid discussing its policy or country implications. Although we model a variety of regimes, we do so by modifying one equation (UIP) in an otherwise standard model with a given parameterization. In these experiments, we do not attempt to model a particular country, and many of our comparisons are subject to the Lucas critique, because we do not change the model parameterization for different exchange rate regimes. Nevertheless, we believe these comparisons will be useful for modelers seeking particular model properties.

Despite the absence of policy discussions in this text, our strategy is useful for modeling many hybrid IT regimes. It encompasses IT with informal (unannounced)

[2] Interventions are often used as a tool to control exchange rate volatility even in IT regimes; see, for example, (Geršl, Holub, 2006). For a summary of central bank interventions in emerging economies, as well as many country studies, see (BIS, 2005).

exchange rate bands, crawling bands, as well as regimes with imperfect control over both exchange and interest rates. The only regimes our analysis is not useful for tackling are those with explicit (announced) exchange rate bands, whose disciplining effect on exchange rate behavior cannot be captured in a simple linear New Keynesian model.

The paper first discusses why modeling of an exchange rate managed through interventions (as opposed to interest rate changes) is more relevant in the institutional setting of many economies. It then introduces the canonical New Keynesian model of monetary transmission and explains the problems created by pure UIP for the model properties. In the next section, a modification of the UIP is proposed, one that is consistent with empirical findings on UIP and that can be linked to a systematic intervention policy of a central bank. It is shown how the modified equation can be calibrated to achieve a particular degree of exchange rate flexibility (through a probabilistic exchange rate band). Finally, the model is extended to encompass other hybrid IT cases, including those involving both exchange and interest rate corridors. The last section presents conclusions. The appendix explains in more detail the extreme case of an IT regime based on interest rate policy under an exchange rate peg.

## 2. Institutional Setting

The coexistence of inflation targeting and exchange rate management can be achieved in two institutionally different ways, each with different modeling implications. In one, the monetary authority affects the exchange rate solely through manipulation of interest rates, and the exchange rate then responds to interest rate differentials according to the interest rate parity arbitrage condition. In the other, the authority conducts exchange interventions independently of interest rate management, violating the interest rate parity arbitrage condition if necessary.

In reality, both practices are common and are often used concurrently. For instance, interest rates in Hungary (which explicitly followed both exchange and inflation targets until recently) were set so that the exchange rate did not escape its band. Were the exchange to escape the band, however, the central bank woul have intervened to fulfill the exchange rate target. In other cases as well, interventions are supported by dramatic changes in interest rates to preserve exchange rate targets (whether explicit or implicit), as happened, for instance, during the speculative attacks on the ERM in 1992.

The two practices have different modeling requirements. When the exchange rate is managed through interest rates simultaneously with inflation, the monetary authority has to consider both inflation and exchange rates in its interest rate rule as intermediate targets. Less flexible exchange rate regimes are represented by a high weight on the deviation of the exchange rate from the desired level, as in (Parrado, 2004a) or (Natalucci, Ravenna, 2002). The exchange rate itself is then modeled via the conventional uncovered interest rate parity (UIP) arbitrage relationship, and the exchange rate fluctuations remain confined by the appropriate management of policy rates.

Modeling of the situation where both the exchange rate and the interest rate are managed independently is more challenging and not well explored in the literature. In such case, interest rates are set with respect to the inflation target, while the role of UIP in determining the exchange rate must be reduced (depending on the flexibility of the exchange rate regime). As a special case, both the interest rate and the exchange rate may be used as independent instruments in targeting inflation.

Although modeling of both practices is important, this paper focuses only on exchange rate management through interventions and not through interest rates. This is for three reasons. First, managing the exchange rate via interest rates does not allow for simultaneous functioning of fixed exchange rate and IT regimes, because interest rate arbitrage sets interest rates at the parity implied by foreign interest rates. Second, the case of targeting the exchange rate through interest rates is relatively well developed in the literature (see (Parrado, 2004a), or (Natalucci, Ravenna, 2002)). Finally, using interest rates in targeting the exchange rate is not an adequate tool for modeling the coexistence of IT and managed exchange rates. The latter requires handling of interventions and (at least periodic) violation of the interest rate parity condition. This is consistent with the literature (e.g. (Krugman, 1991), and (Sarno, Taylor, 2001)), which finds that the exchange rate bands act as a signal altering the sensitivity of the exchange rate to its fundamentals (i.e., in this case the interest rate differential).

## 3. The Canonical New Keynesian Model of Monetary Transmission

In this section we present the canonical reduced-form New Keynesian model that is widely used for forecasting and policy analysis in central banks and other institutions, such as the IMF.[3] A more detailed description, including the rationale behind the model equations, can be found, for instance, in (Bulíř, Hurník, 2006), who used a similar model for analysis of disinflation costs in several EU member countries, as well as Berg, Laxton, and Karam in (BKL, 2006a and 2006b).

### 3.1 The Standard Model with Pure UIP

To motivate our work, we start by presenting the canonical model in the form suitable for a full-fledged IT country that leaves the exchange rate to float freely. Accordingly, we model the exchange rate using a purely forward-looking uncovered interest rate parity condition (UIP). Later in this section we modify the exchange rate equation in a way allowing for certain persistence in exchange rate movement, as proposed by Beneš, Vávra, and Vlček (BVV, 2002). These serve as a background for the discussion of exchange rate management in the following sections.

The canonical model consists of six behavioral equations that represent aggregate demand, aggregate supply, imported goods inflation, the uncovered interest rate parity condition, the term structure, and the policy-reaction function.[4] In addition, several identities are present. As it is not our intention to calibrate or estimate the model based on data available for any particular country, the model parameterization (i.e., the model coefficients) simply follows reasonable values that may be found in the relevant literature.

Aggregate demand, aggregate supply, and the equation for imported goods inflation take the following form:

[3] See (Berg, Laxton, Karam, 2006a) and (2006b) for the reference.

[4] Berg, Laxton, and Karam in (2006a) and (2006b) propose an even simpler version of the model consisting of four equations only. Unlike in our model here, the equations for imported goods inflation and the term structure are not used.

$$\hat{y}_t = a_1 \hat{y}_{t-1} - a_2 \hat{r}_t - a_3 \hat{z}_t + a_4 \hat{y}_t^* + \varepsilon_t^y \quad (1)$$

$$\pi_t = b_1 \pi_{t-1} + (1 - b_1 - b_2)\pi_{t+1}^e + \\ + b_2(\pi_{t-1}^M + \Delta\overline{z}) + b_3 \hat{y}_t + \varepsilon_t^\pi \quad (2)$$

$$\pi_t^M = c_1 \pi_{t-1}^M + (1 - c_1)(\pi_t^* - \Delta s_t) + \varepsilon_t^M \quad (3)$$

where $\hat{y}_t$, $\hat{r}_t$, $\hat{z}_t$, and $\hat{y}_t^*$ represent the deviations of actual output, the real interest rate, the real exchange rate, and foreign output from their respective non-inflationary (natural) levels, $\pi_t$, $\pi_{t+1}^e$, $\pi_t^*$, and $\pi_t^M$ stand for domestic, expected (model-consistent), foreign, and imported goods inflation, and $\Delta\overline{z}$ and $\Delta s_t$ represent changes in the trend real and nominal exchange rates. Structural shocks are denoted by $\varepsilon$. The variables are in logs, except for interest rates.

The policy rule and the term structure are represented as

$$i_t = d_1 i_{t-1} + (1 - d_1)(\overline{r} + \pi_{t+1}^e + d_2(\pi_{t+1}^e - \pi^T) + d_3 \hat{y}_t) + \varepsilon_t^i \quad (4)$$

$$I_t = (i + i_{t+1} + i_{t+2} + i_{t+3})/4 \quad (5)$$

where $i_t$ represents the policy (and market) short-term rate, $\overline{r}$ is the trend short-term real interest rate, $\pi^T$ is the inflation target, and $I_t$ is the long-term (one-year) nominal interest rate.[5]

Finally, the UIP equation is

$$s_t = s_{t+1} + (i_t - i_t^*)/4 - prem_t + \varepsilon_t^s \quad (6)$$

where $s_t$ is the nominal exchange rate at time $t$, $s_{t+1}$ is its model-consistent expectations, and $i_t^*$ is the foreign nominal (short-term) interest rate. The interest rate differential between domestic and foreign short (3M) interest rates $i_t - i_t^*$ is quoted in annual terms. *prem* is the premium required by investors for holding domestic securities.

The model further consists of several identities and transformations:

$$r_t = i_t - \pi_{t+1}^e \quad (7)$$

$$\hat{r}_t = r_t - \overline{r} \quad (8)$$

$$\overline{r} = \overline{r}^* - \Delta\overline{z} + prem \quad (9)$$

$$\Delta z_t = \Delta s_t + \pi_t - \pi_t^* \quad (10)$$

[5] The rule reacts to the deviation in the quarterly inflation rate from the targe one period ahead. More typical formulations involve reactions to a year-on-year inflation rate from the target several periods ahead. We chose this formulation because it is more parsimonious (introducing year-on-year inflation would add one more equation) and the Taylor rule specification is not central to our results.

TABLE 1 Parameters of the Canonical Model

| | In use | BKL – general suggestion | BKL – Canada |
|---|---|---|---|
| $a_1$ | 0.7 | 0.5–0.9 | 0.85 |
| $a_2$ | 0.1 | - | 0.1 |
| $a_3$ | 0.2 | - | 0.05 |
| $a_2 + a_3$ | 0.3 | 0.1–0.2 | 0.15 |
| $a_4$ | 0.3 | - | 0.25 |
| $b_1$ | 0.65 | > 0.5 | 0.8 |
| $b_2$ | 0.25 | - | 0.1 |
| $b_3$ | 0.5 | - | 0.3 |
| $c_1$ | 0.8 | - | - |
| $d_1$ | 0.8 | 0.5–1 | 0.5 |
| $d_2$ | 2.5 | - | 2.0 |
| $d_3$ | 0.5 | - | 0.5 |

$$\hat{z}_t = z_t - \overline{z}_t \tag{11}$$

$$i_t^* = \overline{r}^* + \pi_{t+1}^e \tag{12}$$

$$\overline{z}_t = \overline{z}_{t-1} + \Delta\overline{z}_t / 4$$

$$z_t = z_{t-1} + \Delta z_t / 4$$

where $r^t$ is the short-term real interest rate, $\overline{r}^*$ is the foreign trend real interest rate, and $\Delta z^t$ is the change in the real exchange rate. In our notation, bars denote (potentially exogenous) trend values of model variables with the property that $lim_{t\to\infty} \overline{x}_t = = lim_{t\to\infty} x_t, \forall x$ For instance, $\overline{z}_t$ is an exogenous trajectory of the real exchange rate trend. The model implies by construction that $\overline{z}_t = z_t$ in the steady state.

In calibrating the model we choose plausible parameter values that may correspond to an emerging market economy with strong nominal and real exchange rate transmission channels. *Table 1* gives a perspective of the chosen parameter values using BKL's general suggestions and their choice for Canada. Our parameter values differ from those for Canada to better reflect an emerging marking economy:

– First, the effect of the real exchange rate on the output gap (parameter $a_3$) is much stronger than that for Canada and exceeds the strength of the real interest rate effect on output ($a_2$). This reflects a much stronger real exchange rate effect on output in many emerging market economies, whose underdeveloped financial markets and high financial dollarization often reduce the relative strength of the real interest rate channel (see, for example, (Armas, Grippa, 2006), for the case of Peru). Moreover, in economies that peg their currencies, medium-term stability comes from fluctuations in relative prices (real exchange rates) and not real interest rates.

– Second, output in our model is less persistent and more dependent on the external economy than that of Canada (parameters $a_1$ and $a_4$).

– Third, the nominal exchange rate pass-through to inflation is much stronger and faster in our model than in the Canadian calibration. Parameter $b_2$, which measures the immediate pass-through, is more than twice as high as that for Canada. In the model simulation of an exchange rate shock (see later), almost 20 % of the shock is passed to prices within one quarter and the transmission is complete within about a year and a half.

– Fourth, the output gap effect on inflation ($b_3$) is much higher in our calibration to reflect the relatively low costs of disinflation in many emerging market economies. Our model was calibrated so that the sacrifice ratio (measuring the cumulative output loss needed for a sustained disinflation of 1 p.p.) is below 1, which is standard for many emerging market economies.

These parameter choices affect the model business cycle properties (transition dynamics), but are not central to our main derivations and conclusions. Although the impulse responses and other simulations performed later in the paper are specific to the parameter values chosen, the general lessons we draw are not affected by the particular parameter choice.

Crucial to our analysis are the long-term (steady state) properties of the canonical model, which are independent of the parameter choices. The model is constructed so that monetary neutrality holds, i.e., there is no long-term relationship between nominal and real variables, or their growth rates. In our notation, this implies the following steady state properties (no time subscripts indicate steady state values of the variables):

$$\pi = \overline{\pi} = \pi^T \quad (13)$$

$$\Delta z = \Delta \overline{z}$$

$$\Delta s = \Delta \overline{s}_t = -\pi_t + \Delta \overline{z}_t + \pi_t^* \quad (14)$$

$$\hat{r}, \hat{z}, \hat{y} = 0$$

On the other hand, the levels of the nominal exchange rate and prices in the steady state (or along a balanced growth path, BGP)[6] are not uniquely defined, because they follow stochastic trends, i.e., they can take any value depending on the shocks hitting the model economy and the initial conditions.

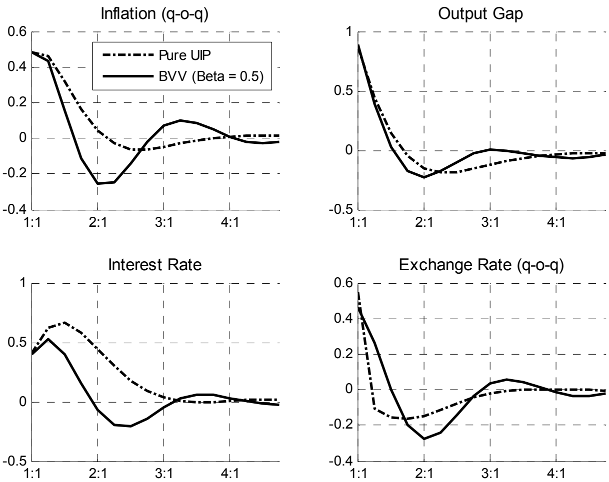
In the simulations below, $\pi^*$, $\hat{y}^*$, and $\overline{r}^*$ as well as $\Delta\overline{z}$, *prem*, and $\pi^T$ are assumed to equal zero for the sake of exposition (without affecting the generality of our conclusions).

### 3.2 Problems with Pure UIP

This standard New Keynesian model with pure UIP as in (6) is difficult to calibrate to achieve reasonable properties in terms of impulse responses, because the exchange rate has little persistence. *Figures 1* to *3* show the exchange rate and interest rate impulse responses for three standard shocks: demand, supply, and exchange rate ($\varepsilon_t^y$, $\varepsilon_t^\pi$, $\varepsilon_t^s$).

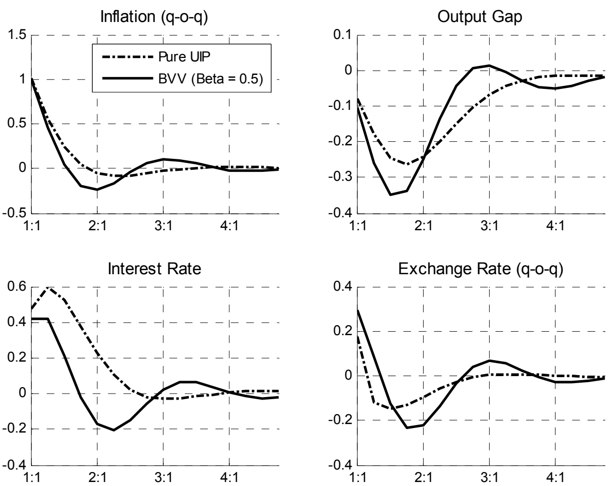
[6] We use steady state and BGP interchangeably.

FIGURE 1 Demand Shock




*Source*: own calculation

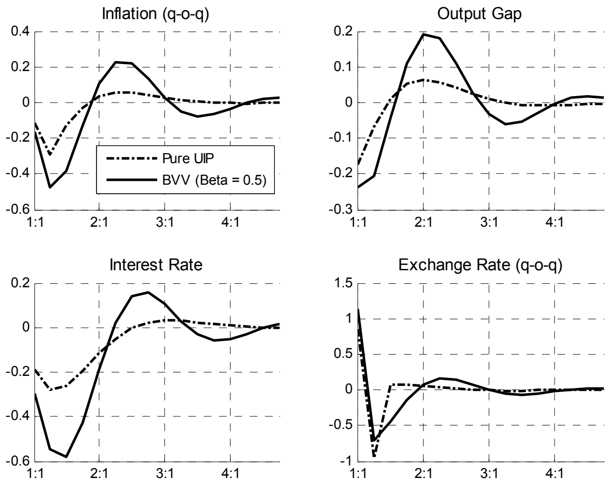
FIGURE 2 Inflation Shock




*Source*: own calculation

The pure forward-looking UIP makes the exchange rate jump from period to period. For instance, in *Figure 1* the exchange rate appreciates on impact in response to increased policy rates and then starts to depreciate to reach its new steady state level. The erratic exchange rate behavior is due to the forward-looking element in (6). It makes the current exchange rate level immediately adjust to the sum of all future interest rate differentials implied by the model behavior.

FIGURE 3 Exchange Rate Shock




*Source*: own calculation

Such exchange rate behavior is often inconsistent with the stylized facts. It also distorts monetary policy transmission, complicating the interpretation of the shock dynamics using the model mechanisms and thus reducing the usefulness of the model for policy analysis. This is a problem especially when modeling transmission in emerging market economies, where the direct exchange rate channel often has a dominant role and good calibration of that channel is essential for the successful use of any structural model.

The poor performance of pure UIP in structural models is related (but not equivalent) to the failure of empirical tests of the UIP. The so-called Fama regressions reject the UIP overwhelmingly, especially at short horizons (Fama, 1984), (Engels, 1996). The literature ascribes most of the deviations between the UIP and the observed ex post behavior of the exchange rate to expectation errors rather than a country risk premium (Campbell, 1997), (Froot, Frankel, 1989). Many hypotheses have been put forward to explain the serial correlation of expectation errors needed to reconcile the evidence with the theory (Engel, Mark, West, 2007). Among them, the so-called "peso problem" and a sudden shift in the monetary regime have been most prominent.[7]

Modelers have been trying to address the issue of low exchange rate persistence by modifying the pure UIP, mostly by reducing the extent of its forward-looking nature. This approach substitutes the model-consistent exchange rate expectations in (6) by a combination of backward and model-consistent expectations. This is the tactic used by BKL and BVV, among others.[8] The BVV case is more general than the BKL

[7] Note that calibration problems with pure UIP also arise in the full structural model that addresses the misspecification issue of reduced-form Fama regressions. Indeed, the structural model in this section does offer more options for describing exchange and interest rate behavior than the reduced-form Fama regressions, as seen in *Figures 1–3*. Nevertheless, pure UIP still causes large problems as regards calibrating the model to the stylized facts.

one, as it allows for non-zero growth of the exchange rate along the balanced growth path.[9] BVV postulate exchange rate expectations as a weighted average of model-consistent and myopic expectations that are consistent with the model behavior of the exchange rate in the long term:

$$s^e_{t+1} = \beta(s_{t-1} + 2\Delta\overline{s}_t) + (1-\beta)s_{t+1} \tag{15}$$

where

$$\Delta\overline{s}_t = -\overline{\pi}_t + \Delta\overline{z}_t + \overline{\pi}^*_t \tag{16}$$

or

$$\Delta\overline{s}_t = -\pi_t + \Delta\overline{z}_t + \pi^*_t \tag{17}$$

or

$$\Delta\overline{s}_t = -\pi_t + \Delta z_t + \pi^*_t \tag{18}$$

The first element in brackets in (15) is the myopic exchange rate expectations, which project the exchange rate in period $t + 1$ as an extrapolation of the past exchange rate using the trend growth rate of the real exchange rate and the inflation differential.

Such expectations are myopic (i.e., model-inconsistent) in the short term, but are consistent with rational model-based expectations in the long term, They, therefore, correspond to the empirical research that finds the UIP holding at long horizons. The term $\Delta\overline{s}_t$ is the change in the exchange rate consistent with long-term economic fundamentals represented by the inflation targets and the real exchange rate trend. By construction, $\Delta\overline{s}_t = \Delta s_t$ in the steady state, so the steady state properties in (13) are intact.

There are various interpretations of this myopic exchange rate expectation term. The most intuitive is that it reflects the exchange rate expectations of financial market analysts, who have a view of the long-term economic potential (represented by the trend real exchange rate appreciation of $\Delta\overline{z}_t$) and adjust it for inflation differentials, current, expected or long-term (represented by inflation targets). This needs to be done for two periods, as the analysts project the exchange rate from period $t - 1$ to period $t + 1$, hence the factor of 2. An equivalent interpretation is that it reflects the view of an analyst who has a simple monetarist model in mind, in which relative PPP holds when adjusted for the real exchange rate trend. In such a paradigm (which also holds in the steady state of our model), higher inflation inevitably leads to a weaker exchange rate, ceteris paribus. Finally, a technical interpretation is that such a formulation of myopic expectations is the only one with a backward-looking exchange rate ($s_{t-1}$) that is at the same time consistent with non-zero growth of the nominal exchange rate in the steady state.

Note that there are various possible definitions of $\Delta\overline{s}_t$ All are consistent with the long-term model properties, but each has different short-term implications.[10] Al-

[8] (Adolfson, Lasen, Linde, Villani, 2007) is another recent example of a UIP modification to introduce persistency into the exchange rate dynamics using an approach similar to, but less general than, BVV.

[9] In addition, BKL is more cumbersome, as it uses the UIP defined in real exchange and interest rates.

[10] There are other specifications belonging to the same class, such as those that rewrite each step in projecting the future exchange rate separately using the values for inflation and the real exchange rate trend of the particular period, e.g.: $s^e_{t+1} = \beta(s_{t-1} + \Delta\overline{s}_t + \Delta\overline{s}_{t+1}) + (1-\beta)s_{t+1}$.

though BVV use (17), all three definitions belong to the same category of UIP modifications. In the remaining text and simulations we will only use the first formulation in (16), because it is more convenient for modeling exchange rate corridors later on. However, our results can easily be re-interpreted using the other formulations.

The BVV UIP modification is useful, because it reduces the short-term jumpiness of the exchange rate and thus helps us to achieve more reasonable model properties in terms of impulse responses and the fit to the stylized facts. It is clear that the inclusion of the past exchange rate $s_{t-1}$ makes the exchange rate more persistent. *Figures 1* to *3* contrast the impulse responses of the pure UIP model with those of the BVV UIP model, showing more persistence in the exchange rate behavior of the latter. An additional advantage of the BVV approach is that it is very malleable, and researchers can experiment with any of the formulations of $\Delta\overline{s}_t$ in (15) which they believe best fit their requirements.

The BVV approach is also handy in that it does not affect the fundamental long-term properties of the New Keynesian model in (13)–(14). In particular, there is no long-term relationship between nominal and real variables, and all real variables are on their predetermined trend trajectories. Most importantly for our analysis, the exchange rate level is not uniquely defined in the steady state, as it depends on the history of shocks and the initial condition. It is this property that is the object of our attention in the next section.

## 4. Inflation Targeting and Managed Floats

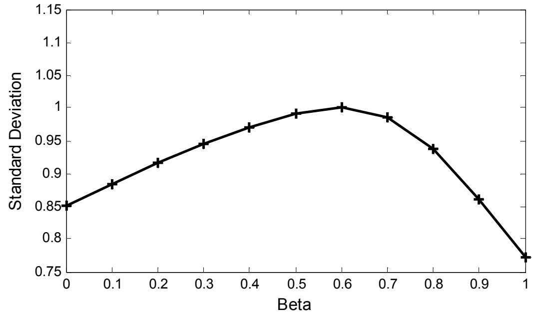
In this section we develop an alternative specification of the UIP that enables modeling of informal (i.e., stochastic) exchange rate bands or (crawling) pegs coexisting with IT. The specification introduces short-term deviations from the UIP, while preserving the UIP at long maturities. The short-term deviations are modeled as expectation errors that fluctuate according to the central bank's intervention policy (with the country's stock of reserves). We show how the modified UIP can be calibrated to keep the exchange rate within a stochastic band around a central parity and how it affects the short and long-term model properties.

### 4.1 Requirements

While instrumental in increasing exchange rate persistence, the BVV (or BKL) approach cannot be used for modeling exchange rate corridors or pegs. First, no value of $\beta$ in (15) gives a formulation consistent with a fixed exchange rate. *Figure 4* plots the unconditional standard deviation of the exchange rate implied by the model of Section 3 (and its calibration in terms of parameters and shock standard errors) for different values of $\beta$.[11] The power of $\beta$ to limit exchange rate volatility is small; for instance, the standard deviation of the exchange rate for $\beta = 1$ is still about 90 % of that implied by the model with pure UIP ($\beta = 0$). Moreover, the relationship between

[11] In this experiment the model economy is subjected to shocks drawn randomly from their covariance matrix implied by the data and the current model. This is done for several values of $\beta \in (0,1)$ The variance of the exchange rate error term has been set to zero, but the covariance matrix of the error terms does not change with different $\beta$. When calibrated to a particular country, each value of $\beta$ should be associated with a different covariance matrix of shocks.

FIGURE 4 Exchange Rate Volatility and $\beta$ (common model with BVV extension of the UIP)




*Source*: own calculation

exchange rate volatility and $\beta$ is not monotonic, making it difficult to calibrate $\beta$ according to some desired exchange rate volatility. Finally, it is difficult to motivate BVV by reference to central bank behavior limiting exchange rate fluctuations.

We therefore propose an alternative UIP modification with the following properties: (i) pure UIP holds at long horizons as well as in the steady state, as in BVV and as is consistent with empirical research on the UIP; (ii) it allows for modeling of an exchange rate (crawling) peg; (iii) it can be used to limit the exchange rate volatility around a central parity according to pre-defined stochastic bands; (iv) it can be linked to a systematic exchange rate management policy of a central bank.

A formulation satisfying these requirements could be used to model a variety of exchange rate regimes, ranging from pure floats, through informal exchange rate bands, to exchange rate (crawling) pegs. At the same time, it would be consistent with the empirical finding that the pure UIP discrepancies are due to expectation errors that disappear at long horizons, as well as having good behavioral foundations.

## 4.2 Exchange Rate Parity

As a first step in deriving the modified UIP, we introduce the concept of exchange rate parity. This is the level (or trajectory) of the exchange rate that is consistent with the inflation targets and real exchange rate in the long term. In other words, it is the exchange rate trajectory that the central bank needs to maintain on average if it wants to keep inflation on target in the long term:

$$s_t^P = s_{t-1} + \Delta\overline{s}_t \tag{19}$$

or

$$s_t^P = s_{t-1}^P + \Delta\overline{s}_t \tag{20}$$

In both definitions, the exchange rate parity is implied by extrapolating from the past exchange (parity) rate using the economy's fundamentals ($\Delta\overline{z}_t$) and long-term inflation differentials ($\overline{\pi}_t^* - \overline{\pi}_t$). We call these levels parity because they are implied by relative PPP (adjusted for the real exchange rate trend).

Both formulations of the parity level are consistent with achieving inflation targets, but they have different policy implications. In (19), the trajectory of the parity level is not predetermined and moves with the exchange rate level. Its slope, however, is determined at any point in time. Such a parity formulation has little direct practical use in terms of modeling central bank policies. It would require a central bank to let the exchange rate move within a band around the parity in one period, and then level shift the parity for the next period according to the actual exchange rate situation. While such patterns have been observed in countries with crawling band regimes in the past (e.g., Israel and Poland in the 1990s), level shifts in the parity have occurred very infrequently.

In (20), on the other hand, the whole trajectory of the parity level is predetermined, which is more appropriate for modeling the central parities of continuous exchange rate corridors. In the rest of the text and simulations we will use the parity definition in (20), but our results are robust to using (19) too.

### 4.3 Modified UIP

Our modified UIP is based on the following correction term, which is subtracted from the exchange rate expectations in the pure UIP equation:

$$corr_t = \frac{\beta}{1-\beta}\left[s_t - s_t^P\right],\ \beta \in \langle 0,1) \tag{21}$$

The correction reduces (increases) exchange rate expectations if the current level of the exchange rate is above (below) the exchange rate parity level. This correction introduces a direct effect of long-term fundamentals on future exchange rate expectations. For instance, if the exchange rate appreciates by more than what is implied by the long-term fundamentals (that is, $s_t > s_t^P$), the rational expectations of the future exchange rate are corrected downward, reflecting the unsustainable movement.[12]

When subtracting this term from the exchange rate expectation in pure UIP, we arrive at the following UIP modification:

$$s_t = s_{t+1} - corr_t + (i_t - i_t^*)/4 \tag{22}$$

$$= \beta s_t^P + (1-\beta)\left(s_{t+1} + (i_t - i_t^*)/4\right) \tag{23}$$

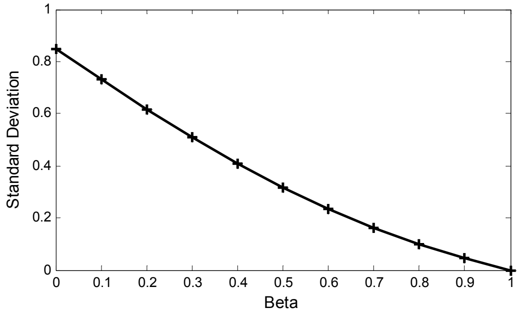
$$\beta \in \langle 0,1 \rangle \tag{24}$$

As seen from (23), the correction term reduces the sensitivity of the exchange rate to the interest rate differential, both current and future. This reduces the exchange rate jumpiness (increases its persistence), while also allowing for controlling of exchange rate volatility, as we will explain below. This behavior is consistent with the literature (e.g., (Krugman, 1991)) which finds that the exchange rate bands alter the sensitivity of the exchange rate to the interest rate differential at all points, not only in the immediate neighborhood of the bands.

The modified UIP (23) satisfies our requirements when used in the New Keynesian model of Section III in place of (6). First, the correction disappears at long horizons, because $s_t - s_t^P = \Delta s_t - \Delta \overline{s}_t$ and $\Delta s = \Delta \overline{s}$ along the balanced growth path by the construction of the model in Section 3. This is best seen from the stationary version of (25):

[12] Note that introducing this effect just reinforces the overall effect of long-term fundamentals on exchange rate movements: even in the model with pure UIP the exchange rate always grows at $\Delta \overline{s}_t$ along the balanced growth path (see the previous section).

FIGURE 5 Exchange Rate Volatility (deviation from the parity) and $\beta$ (new UIP)




*Source*: own calculation

$$\Delta s_t = \beta(\Delta \overline{s}_t) + (1-\beta)\left(\Delta s_{t+1} + \Delta s_t + (i_t - i_t^*)/4\right) \tag{25}$$

Second, (23) harbors a crawling peg as a special case of $\beta = 1$ Then, $s_t = s_t^P$ at all times, which can be used to model both hard and crawling pegs, depending on the trajectory of $s_t^P$. Because the parity $s_t^P$ is a trajectory given by assumptions about the real exchange rate trend and inflation targets, an appropriate choice of inflation targets can give it any shape, including that of a hard peg. The appendix deals with the hard peg case in more detail.

Third, the value of $\beta$ can be used to limit the exchange rate volatility within given (stochastic) bands around a central parity. For any $\beta > 0$ (23) and (21) guarantee that the exchange rate will always be at the parity level in the steady state. Also, outside the steady state, the exchange rate is more stable around the parity rate of $s_t^P$ with higher $\beta$. *Figure 5* demonstrates this using stochastic simulations of the full model: the unconditional volatility of the exchange rate falls with higher $\beta$. Although we cannot prove it formally, we checked that this relationship is monotonic also for a range of other model parameterizations. The parameter $\beta$ thus plays the role of a weight measuring the exchange rate flexibility in the particular exchange rate regime.

Fourth, the modified UIP can be derived from the systematic exchange rate management policy of a central bank. It is enough to assume that exchange rate expectations depend on the level of the country's foreign exchange reserves and to choose the appropriate functional form for this relationship corresponding to (21). Alternatively, we can assume that the rate at which the economy borrows on international financial markets depends on the level of foreign exchange reserves (say, as coverage for a current account deficit), e.g., $i_t^* + \phi(Fx_t), \phi'(.) < 0$ where $Fx$ refers to the central bank's foreign exchange rate reserves. Assuming further a particularly simple functional form of $\phi(Fx_t) = -\log(Fx_t/\overline{Fx}_t) = \overline{fx}_t - fx_t$, we get the following UIP condition dependent on the fluctuations of foreign exchange reserves around their long-term (steady-state) optimal level:

$$s_t = s_{t+1} + i_t - i_t^* - \left(\overline{fx}_t - fx_t\right) \tag{26}$$

Finally, assume that the central bank adjusts the reserves in order to limit the deviations of the exchange rate changes from the parity according to the rule:

$$\overline{fx}_t - fx_t = \lambda(s_t - s_t^P) \tag{27}$$

By substituting this rule into (26), we arrive at (23), with $\lambda = \beta/(1-\beta)$.[13]

Although a partial equilibrium, this derivation is very malleable and does not depend on many unrealistic assumptions. It can also easily be incorporated into a general equilibrium model.[14] However, because we only need the UIP in (23) in a reduced form, we are not tied to a particular behavioral motivation. For instance, we could derive a similar reduced-form UIP if we assumed that the central bank was concerned about sterilization costs when adjusting the level of reserves in response to exchange rate deviations, such as

$$\overline{fx}_t - fx_t = \lambda(\Delta s_t - \Delta\overline{s}_t) - \kappa\left(s_t - s_{t+1} + i_t^* - i_t\right)$$

In the special case of $\lambda = \kappa$ this rule also gives rise to (23).

### 4.4 Modeling of Exchange Rate Corridors

Using the modified UIP in the canonical NK model of Section 3 allows for modeling of hybrid IT regimes that use passive exchange rate management in addition to interest rate changes when targeting inflation. By passive exchange rate management we refer to situations where the central bank limits exchange rate volatility through some kind of band, but does not manipulate the band actively in order to target inflation (this is achieved by interest rates). In contrast, the case of active exchange rate management, which manipulates the exchange rate as an instrument in targeting inflation, is addressed in the next section.

The modified UIP can model informal exchange rate bands coexisting with IT, but cannot be used for explicit (pre-announced) bands. In our linear model the exchange rate flexibility and bands refer to stochastic behavior of the exchange rate (or its rate of change), so that the exchange rate fluctuations are contained within bands with a certain probability. Nothing prevents the exchange rate from escaping the bands should a relatively large shock hit the economy. When reached, the bands are not defended by any special operations. We call such bands informal (or implicit), as opposed to explicit or formally announced bands, which presume discreet ad hoc central bank operations to defend the bands when the exchange rate is in their vicinity. In the linear model we have no capacity to model explicit bands and their disciplining effects on exchange rate behavior (e.g., (Krugman, 1991)). Unlike explicit bands, implicit bands arise only as a result of systematic central bank behavior that is the same irrespective of the exchange rate position relative to the bands (such as that described in (27)).

[13] Note that when the central bank adjusts its reserves in response to exchange rate growth, such as $\overline{fx}_t - fx_t = \lambda(\Delta s_t - \Delta\overline{s}_t)$ one obtains the modified UIP consistent with the "jumping" definition of the exchange rate parity in (19).

[14] (Ho, 2004) is an example of a general equilibrium analysis of intervention, although in a flexible price equilibrium.

### 4.4.1 Calibration

Our approach is handy in that parameter $\beta$ serves as an index of exchange rate flexibility and can be calibrated to fit the width of an informal exchange rate corridor. Free floats and pegs arise as special cases of $\beta = 0$ and 1, respectively. For informal bands, $\beta$ can be chosen according to the relationship between the unconditional volatility of the exchange rate deviation from the parity and $\beta$, such as in *Figure 5*. For instance, given the calibration of the shock standard errors in our model (not reported), $\beta = 0.5$ implies that the exchange rate will, with 67% (95%) probability, be within a band around 0.3 % (0.6 %) wide.

In practice, the calibration of $\beta$ using the information in *Figure 5* can be complicated by the need to know the standard errors of the model structural shocks. Typically, the standard errors are not known in advance and are estimated (say, by maximum likelihood in a Kalman filter) knowing the full model structure, including the value of $\beta$. If the data allow, the standard errors and $\beta$ can be estimated jointly. When the data are not informative, an iterative procedure can be used in which an initial value of $\beta$ is used to estimate the standard errors of the model shocks. The resulting $\beta$-exchange rate volatility relationship (such as in *Figure 5*) is then used to modify $\beta$ and so forth.

### 4.4.2 Model Properties

Our UIP modification substantially alters both the steady state and transition model properties when compared to the models with either pure UIP or BVV formulations.

*Steady State Properties*

First, the exchange rate in our model does not follow a random walk in the steady state; it always ends up at the pre-determined parity level $s_t^P$ as the UIP correction (21) disappears. This is in contrast to the pure UIP and BVV approaches, where the steady state exchange rate level depends on the initial conditions and the history of shocks.

Second, the price level is also predetermined in the steady state, which means that inflation targeting becomes price level targeting. This follows from observing that when the nominal and real exchange rates are on their predetermined trajectories, the price level must be at the level implied by the initial condition and the sequence of inflation targets.

Third, the other steady state properties in (13) and (14) remain intact, except that the central bank loses its target autonomy. As long as it is committed to a particular exchange rate parity $s^P$, it can no longer choose its inflation target $\pi^T$ freely. In other words, its choice of parity and inflation target is constrained by the condition:

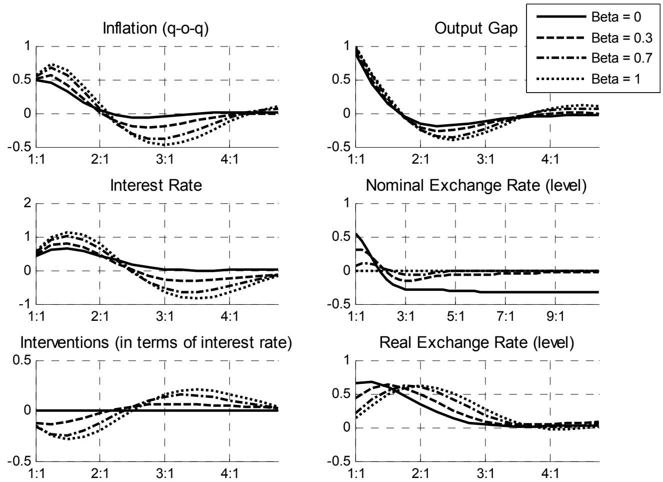
$$\pi^T = \Delta\overline{z} + \overline{\pi}^* - \Delta\overline{s}^P$$

This condition is especially binding if the country chooses a fixed exchange rate. Then, its inflation (and the target) in the long term is given by $\pi^T = \Delta\overline{z} + \overline{\pi}^*$.

*Transition Properties*

The exchange rate corridor also affects the impulse responses. We illustrate this using the output gap shock by varying $\beta$ and thus changing the width of the informal exchange rate band. Other shocks yield similar conclusions.[15]

FIGURE 6 Demand Shock: Corridors and Varying *β*




*Source*: own calculation

The response of output to a demand shock is not much affected by the different exchange rate corridors, unlike those of inflation and the policy variables, including interest and exchange rates, and interventions. *Figure 6* shows the impulse responses of key variables following the demand shock for several values of $\beta$ including the extreme cases of an exchange rate float and (crawling) peg.

Inflation and interest rates fluctuate much more with narrower exchange rate corridors. This is a result of the price-level-targeting property. Because the price level is predetermined in the steady state, a temporary inflation increase above the target has to be compensated by an inflation decrease below the target of an equal magnitude. As inflation fluctuates around the target, so do interest rates around their steady state level. In contrast, a pure float scenario allows for a smooth landing for both inflation and interest rates.

Inflation is contained faster with wider exchange rate bands (low values of $\beta$), as nominal and real exchange rates are allowed to appreciate more in response to an interest rate hike. As a consequence, the initial interest rate reaction is smaller with wider bands than with narrower ones. For narrower bands, inflation continues to build up for some time after the shock, before the stabilization forces of higher real interest (and exchange) rates are finally able to contain it.

Exchange rate volatility is clearly contained within the corridors and the exchange rate always returns to the baseline for any positive value of $\beta$. This is in contrast to a free float ($\beta$ = 0, i.e., pure UIP), when the exchange rate does not revert to the baseline (parity) level. It falls below the control trajectory for a given real exchange rate path, because the shock raises the price level in the long term.

Finally, the figure also displays the central bank interventions (expressed in interest rate equivalents) that are needed to sustain the corridor. The central bank has

[15] Note that the exchange rate shock only affects the intervention if the country practices a hard peg.

to violate the UIP at short horizons if it simultaneously wants to retain policy independence (in terms of interest rates targeting inflation) and limit the exchange rate fluctuations. In order to violate the UIP at short horizons the central bank has to engage in perpetual interventions. Thanks to the reduced-form formulation of the UIP in (23), interventions are a residual variable of our system. From (21) and (23) the interventions are proportional to the value of the correction term *corr* applied to the model-consistent expectations. This is understandable, because it is precisely this term that violates the pure UIP.

With the demand shock, the central bank has to intervene when a corridor is in place ($\beta > 0$). It intervenes at first by buying foreign exchange, as the interest rate reaction is larger than that required by a free float, and later by selling foreign exchange, when the interest rate response falls below that of the free float.

## 5. Hybrid Inflation Targeting: Exchange and Interest Rate Corridors

The approach of the previous section can be generalized to provide for cases where the central bank has imperfect control over both the exchange rate and the interest rate. In other words, both the exchange rate and the interest rate move in probabilistic corridors determined by the central bank's systematic behavior. Such generalization is most relevant for intermediate regimes that are experimenting with introducing inflation as an intermediate variable, while still engaged in exchange rate management, and replacing exchange rates with interest rates as instruments. Our strategy allows for imperfect control over market interest rates, which is a common situation in regimes whose monetary programs involve targeting monetary aggregates through either banking reserves or the monetary base as operational targets, and whose markets may not be developed enough to enable effective interest rate targeting.

The cornerstone of our generalized modeling strategy is symmetric treatment of both the exchange rate and the interest rate in a similar way as we treated the exchange rate in Section 4. Each is assumed to evolve around a policy-defined parity level that is made consistent with achieving the inflation target and the economy's long-term trends using Taylor-type rules. The following are the modified equations (which replace equations (4) and (6) of the original model in Section 3):

$$s_t = s_t^P + \hat{s}_t$$

$$s_t^P = s_{t-1}^P + \Delta\overline{s}_t + \gamma(\pi_{t+1}^e - \pi^{TAR}) \tag{28}$$

$$s_t = \beta s_t^P + (1-\beta)\left(s_{t+1} + (i_t - i_t^*)/4\right) \tag{29}$$

$$i_t = i_t^P + \hat{i}_t$$

$$i_t^P = \hat{i} + \alpha(\pi_{t+1}^e - \pi^{TAR}) \tag{30}$$

$$i_t = \chi i_t^P + (1-\chi)\left(4\left(s_t - s_{t+1}\right) + i_t^*\right) \tag{31}$$

$$\Upsilon_t = s_t - \left(s_{t+1} + (i_t - i_t^*)/4\right) \tag{32}$$

In the modified system, $s_t^P$ does not represent the parity level as before, but a policy level of the exchange rate that evolves around the parity in inflation targeting using a Taylor-type rule. We can afford this small inconsistency in our notation because conceptually these terms are the same, and our previous exposition arises as a special case of this general system. The equations introduce other new variables: $i^P$ denotes the policy level of interest rates (equivalent to that for exchange rates), and $\hat{s}$ and $\hat{i}$ are the measures of the actual deviation of the exchange rate and the interest rate from their policy levels.

Equations (28) and (29) represent the modified UIP equation introduced in the previous section – collapsing to (25) for $\gamma = 0$, while equations (30) and (31) modify the market interest rate behavior (replacing equation (4)) using the same principle.

The policy levels for both the exchange rate and the interest rate ($s_t^P$ $i_t^P$) move according to Taylor rules targeting inflation (the autoregressive smoothing terms of 0.8 are not displayed for the sake of exposition).[16] In the long term, they both return to the parity levels consistent with the inflation targets and long-term real economic trends. The parameters $\gamma$ and $\alpha$ (calibrated to 2 and 2.5, respectively) control the short--term sensitivity of the policy levels to the deviation of inflation from the target. Various other Taylor formulations are possible, including those with expected future deviations of inflation from the target or with output gaps.

The actual (market) values of the exchange rate and the interest rate converge to the policy levels (i.e., $\hat{s}, \hat{i} = 0$), but in the short term move around the policy levels to a degree parameterized between 0 and 1 using parameters $\beta$ and $\chi$. When the respective parameter reaches 1, the central bank controls the market rate perfectly at the desired policy level; in the other extreme, the rate is determined freely by market forces – by the uncovered interest rate parity condition. Note that the models of Sections 3 and 4 assumed the central bank had perfect control over market interest rates, hence $i_t$ was used in place of $i_t^P$

Finally, the variable $\Upsilon$ measures the extent to which the UIP condition is violated (expressed in %), and is an indicator of the intervention volumes necessary to maintain the interest rate and exchange rate simultaneously at their desired levels. Violating the arbitrage condition is only possible if there is an infinite supply of instruments – in this case the central bank engages in permanent interventions by buying or selling short-term instruments to roll the situation over (e.g., sterilized interventions in case of interest rate sensitive capital inflows in fixed exchange rate economies).

### 5.1 Interpreting $\beta$ and $\chi$

The parameters can be understood as putting informal corridors in place for both the exchange rate and the interest rate, whose width (in terms of volatility) is determined on the basis of the model parameterization and the shock standard errors.

The parameters represent various (unspecified) institutional factors that may prevent the central bank from enforcing the desired policy levels. For instance, the central bank may introduce an informal exchange rate corridor around the policy tra-

[16] A similar Taylor type rule for the exchange rate was used and estimated by Parrado (2004b) for Singapore.

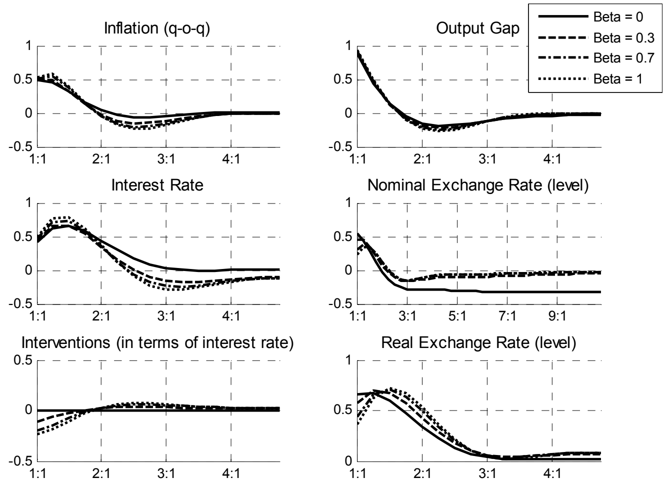
jectory in order to promote market development and facilitate greater exchange rate flexibility later on. On the other hand, central banks (especially those with nascent money markets) may not be able to limit the volatility of money market interest rates beyond a certain threshold, given by the institutional characteristics of the market.

The requirement of model stability puts some constraints on the possible combinations of the two parameter values. For instance, while it is possible that the exchange rate and/or the interest rate are forced to be on their policy level trajectories ($\beta = \chi = 1$), it is not possible for them both to be jointly set loose from these levels ($\beta = \chi = 0$), as there would be no mechanism stabilizing the model (and inflation) in the face of a shock. Note also that when the monetary authority has no control over the exchange (interest) rate, i.e., $\beta = 0$ $(\chi = 0)$, then its control over the interest (exchange) rate is necessarily perfect; in other words, the value of $\chi\,(\beta)$ becomes irrelevant.

The most empirically relevant cases are the following:

– $\beta = \chi = 1$; the central bank practices a hard peg exchange rate regime (at a level made consistent with the inflation target and economic fundamentals), but simultaneously manipulates market interest rates (either directly or through a reserve-based system) in order to achieve the targeted rate of inflation, as in Section 3. As a result of violating the market determination of both rates, the bank almost permanently intervenes. In the special case where $\gamma \neq 0$, the exchange rate peg moves in response to deviations of inflation from the target.
– $\beta = 0,\ \chi = 1$; the most standard inflation targeting case. The central bank exercises perfect control over money market interest rates as its only instrument for targeting inflation, and the exchange rate is freely floated.
– $\beta \in (0,1),\ \chi = 1$; the central bank exercises perfect control over money market interest rates as its main instrument for targeting inflation, but also imposes an informal corridor on its exchange rate, whose central parity is consistent with the target and economic fundamentals, as in Section 4. When $\gamma \neq 0$, the center of the corridor (i.e., the exchange rate parity) moves in response to deviations of inflation from the target. This situation prevails in several IT regimes (e.g., Hungary), but is also typical of the early stages of IT introduction.
– $\beta \in (0,1),\ \chi \in (0,1)$; the central bank does not have perfect control over money market interest rates, but at the same time allows some exchange rate flexibility. This situation prevails in some intermediate regimes transiting to greater exchange rate flexibility, but lacking the proper infrastructure to use other instruments – the exchange rate thus continues to play the role of instrument too.
– $\beta = 1,\ \chi \in (0,1)$ the central bank uses the exchange rate as its main instrument for targeting inflation, while market interest rates are allowed to move in a more or less loose corridor. This situation may arise in very small open economies with a dominant exchange rate channel (e.g., Singapore or New Zealand in the early 1990s).

FIGURE 7  Demand Shock: Changing Corridors and Varying $\beta$




*Source*: own calculation

## 5.2 Model Properties

We will now examine the effects of varying parameters $\beta$ and $\chi$ on the model properties. For brevity we illustrate them using the output gap shock, but the key findings are seen in other shocks too.

### 5.2.1 Varying $\beta$

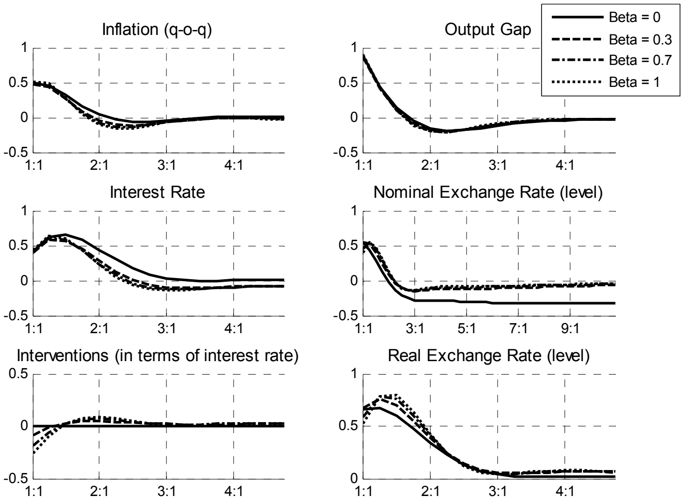
First, we run experiments changing the width of the exchange rate corridor by varying $\beta$ while keeping perfect interest rate control ( $\chi = 1$ ). The experiment is similar to the one in the previous section, except that the exchange rate parity (corridor) now moves actively in fighting inflation, thus aiding interest rate transmission (*Figure 7*).

Some of the results are qualitatively similar to those in Section 4. Specifically, the responses of output and real interest rates are not much affected by changing the corridor width. Inflation is still contained faster with wider exchange rate bands (very low values of $\beta$), despite the revaluation of the exchange rate corridor. With wider bands, nominal and real exchange rates are still allowed to appreciate more in response to an interest rate hike than is allowed by the appreciation of the band using the Taylor rule. As a consequence, the required interest rate reaction is still smaller with wider bands than with narrower ones.

However, these results depend on how aggressively the central bank moves the exchange rate corridor in response to higher inflation. *Figure 8*, for instance, shows the same simulation, but with $\gamma$ in the exchange rate rule raised to 4 instead of 2. Now, the exchange rate reacts by more than is implied by a pure float, allowing the interest rates in the corridor scenarios to be more relaxed on average compared to the pure float scenario.

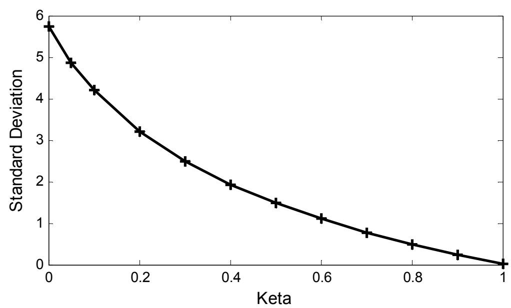
Finally, there are also quantitative differences compared to *Figure 6*. For instance, inflation and interest rates in *Figure 7* do not rise as much with narrower cor-

FIGURE 8 Demand Shock: Changing Corridors and Varying $\beta$ ($\gamma$ = 4)




*Source*: own calculation

FIGURE 9 Interest Rate Volatility (deviation from the policy level) and $\chi$ ($\beta$ = 0.5)




*Source*: own calculation

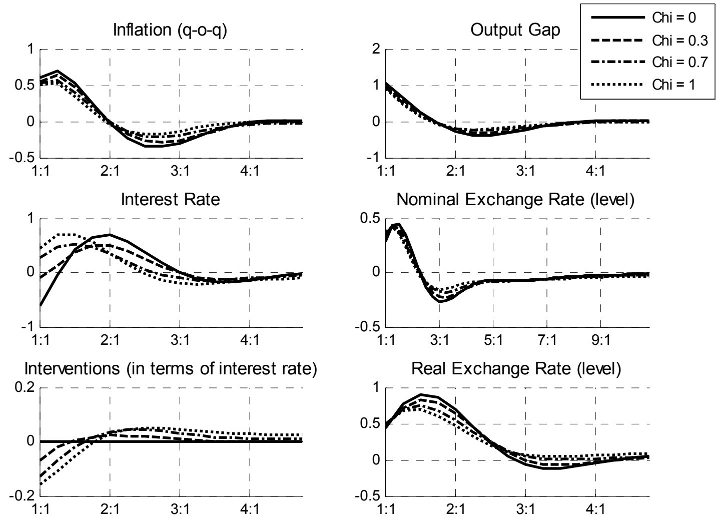
ridors as in *Figure 6*, because of the revaluation of the exchange rate corridor. Also the interventions, at least initially, are higher when revaluating the exchange rate corridor. This is more clearly seen in *Figure 8*, where the revaluation is higher.

### 5.2.2 Varying $\chi$

Second, we experiment with changing the width of the interest rate corridor by varying $\chi$ while keeping the exchange rate within the band implied by $\beta = 0.5$. *Figure 9* illustrates how varying $\chi$ limits the unconditional volatility of the interest rate deviation from the desired policy level.[17] As with $\beta$, this relationship is mono-

[17] Note that this experiment actually contains two effects. In addition to changing the interest rate corridor ($\chi$), the exchange rate corridor width also moves, despite $\beta$ being constant, because different $\chi$ imply different unconditional volatilities of the exchange rate. We would have to recalibrate $\beta$ for each value of $\chi$ to obtain the same exchange rate corridor width in these simulations.

FIGURE 10 Demand Shock: Corridors and Varying $\chi$ ($\beta$ = 0.5)




*Source*: own calculation

tonic for plausible parameterizations and can be used in calibrating $\chi$ to set the width of the interest rate corridor.

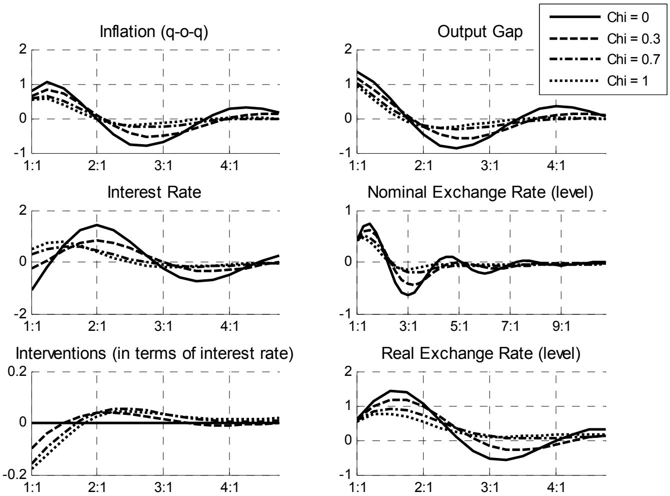
*Figure 10* shows the impulse responses after a demand shock for different values of $\chi$. In contrast to the exchange rate corridor experiments, the profiles for the most important variables, such as inflation, the exchange rate, and output, are not much affected by the interest rate corridor width.

Unlike most other variables, market interest rates are much affected by the width of the interest rate corridor. Interest rates even fall below the baseline when the central bank has little control over them. This is because in such situations market interest rates are determined purely by UIP arbitrage. Hence, few or no interventions are needed to support the corridor. And as the markets expect the exchange rate parity to tighten gradually (following the Taylor type rule), an inflow of capital brings market rates down. As a result of the lower interest rates, inflation is higher with wider interest rate corridors.

The experiment with an interest rate corridor points to the potential pitfalls of targeting inflation using the exchange rate as the only instrument. Inflation is higher with a wider corridor, as the real interest rate and exchange rate channels go against each other. The effect of this clash depends on the relative strength of the two channels in affecting output. Our baseline calibration (*Figure 10*) assigns a relatively low weight to the real interest rate channel (in line with the stylized emerging market calibration of Section 3). As a result, the increase in inflation resulting from falling market rates is relatively small in *Figure 10*. However, *Figure 11* shows the same experiment with the relative weights on the real interest rate and exchange rate in (1) reversed. The result is a marked increase in inflation and its volatility.

In summary, the width of the interest and exchange rate corridors matters more for inflation and interest rates than for output. Moving the exchange rate corridor in

FIGURE 11 Demand Shock: Corridors and Varying $\chi$ ($\beta$ = 0.5, $a_2$ = 0.2, $a_3$ = 0.1)




*Source*: own calculation

response to inflation helps to contain inflation faster and also allows for looser interest rates. Even then, though, a wider exchange rate corridor may be preferred (in terms of containing inflation) if the central bank is not able to move the corridor aggressively enough. Inflation is also contained faster when the control over market interest rates is tighter. Targeting inflation using the exchange rate as the only instrument risks counter-productive side effects, as the interest rate channel may work against the stabilization effects of the exchange rate.

## 6. Conclusions

We analyze the modeling implications of hybrid IT regimes that coexist with exchange rate management. We do so in the framework of a reduced-form New Keynesian model of monetary transmission. Our approach is new in modeling interest and exchange rates as independent instruments (operational variables) in targeting inflation, as opposed to using only interest rates to target both inflation and the exchange rate as intermediate objectives.

The modeling of such hybrid regimes requires an adjustment to the UIP condition which affects exchange rate expectations and reduces the sensitivity of the exchange rate to interest rate differentials. We introduce a UIP modification that can be linked to the foreign exchange interventions of a central bank. We show how it can be used to model (informal) exchange rate bands, i.e., stochastic bands that are not explicitly announced and defended by other means than a systematic intervention policy. We also extend the analysis to other hybrid regimes with imperfect control over interest rates.

The main contribution of our paper is a new approach to modeling informal exchange rate bands in the class of reduced-form small open economy models. This approach is thus suitable for modeling of IT countries that keep their exchange rates in informal bands as well as countries that are at an intermediate stage of transition from a fixed (or strongly managed) exchange rate regime to a more flexible one with elements of IT. With certain caveats, the approach can also be used for countries that are transiting from free float inflation targeting to an exchange rate based system, e.g. ERM II.

Although we stay away from policy or country lessons, our analysis points to several important properties of hybrid IT regimes:

– An exchange rate corridor introduces large fluctuations of interest and inflation rates around their steady state (target) values, because such a regime resembles price level targeting.
– Inflation is contained faster with a wider exchange rate corridor, as in that case the interest rate transmission is most effective by also working through the exchange rate channel. However, narrower corridors can result in lower inflation and interest rates if the central bank moves the corridor aggressively in response to future inflation.
– Targeting inflation using the exchange rate as the only instrument (letting market interest rates adjust) risks counter-productive side effects, as the interest rate channel may work against the stabilization effects of the exchange rate.
– Finally, the degree of control over interest and exchange rates matters more for inflation, interest rates, and interventions than for output.

## APPENDIX

### Inflation Targeting and Exchange Rate Peg

Modeling of the transmission mechanism when an exchange rate peg coexists with elements of inflation targeting has to satisfactorily address the following issues: (i) the ability of a central bank to set interest rates independently of interest rate parity, and (ii) the independence of the monetary authority to choose its inflation target.

The first issue is that of monetary policy instrument independence when the exchange rate is fixed, which under usual circumstances delegates monetary policy conduct to the outside world (a reference country). Under these normal circumstances, the sheer force of international arbitrage (under perfect capital mobility) would lead to convergence of nominal interest rates to those of the main trading partners (subject to a risk premium). The management of interest rates would thus lie outside the influence of the central bank. However, the monetary authority may achieve independence if it is willing to work against the forces of international arbitrage and provide sufficient liquidity to keep interest rates at any level it considers fit (to achieve the inflation target, for instance).

The issue essentially boils down to how to model the exchange rate-interest rate link in such cases. Although the central bank may in theory attempt to control the entire yield curve by controlling the supply of instruments with relevant maturities, in practice it controls (as an instrument) only short-run interest rates, while the long rates are determined by the market.

In this institutional setting, the key modeling challenge is how to make long interest rates follow foreign rates, when short-run rates are set independently and UIP arbitrage does not hold at short maturities.

The virtue of the UIP condition is that if the country credibly fixes its exchange rate, this alone is sufficient to guarantee that domestic yields follow foreign yields at long maturities. To see this, let us return to the uncovered interest rate parity equation (6). Because we are interested in the case of a credible peg, we assume the premium away from now on without any loss in generality.[18] Relationship (6) holds for long maturities as well, and by iterating the equation forward we obtain:

$$\begin{aligned} s_t &= s_T + \sum_{t}^{T} \left[ (i_s - i_s^*)/4 - prem_t \right], \ T \geq t \\ &= s_T + (I_t^T - I_t^{T*})(T - t + 1)/4 \end{aligned}$$

where $s_T$ is the expected exchange rate at some distant date $T$ and $I$ is the appropriate long-run interest rate between times $t$ and $T$ (valid for $T - t + 1$ quarterly periods and quoted in annual terms). If such relationships held with a credibly fixed exchange rate ($s_t = s_t$, $\forall T$, premium is zero), both short and long-run rates in the domestic economy would be determined by foreign rates:

$$i_t = i_t^*, \ I_t^T = I_t^{T*}$$

[18] A convenient assumption only. As long as forward exchange rates are unbiased predictors of future exchange rates, the risk premium is zero. In later sections we in fact derive a formula for modeling a risk premium stemming from exchange rate management by the monetary authority.

Hence, the UIP condition alone guarantees the equalization of yields on long securities when the monetary authority passively keeps the exchange rate peg.

When, on the other hand, the monetary authority controls its short-run nominal rates in violation of UIP, equalization of long yields has to be achieved by other means than a simple UIP condition. This is done by a particular combination of three elements: an interest rate rule, setting of the inflation target, and real interest rate arbitrage at long horizons.

The interest rate rule determines the interest rate at long horizons. To see this, consider a stylized interest rate rule, such as that in (4):

$$i_t = \bar{i}_t + \alpha(\pi_t - \pi^{TAR}) \qquad (33)$$
$$\bar{i}_t = \bar{r} + \pi_t$$

Interest rates react to a deviation of inflation $\pi_t$ from the target $\pi_t$. Such a rule implies that domestic long interest rates are:

$$I_t^T = 1/4 * (T - t + 1)\sum_t^T i_s$$
$$= 1/4 * (T - t + 1)\left[\alpha(P_t^T - P_t^{T,TAR}) + \sum_t^T \bar{i}_s\right]$$
$$P_t^T = \sum_t^T \pi_s, P_t^{T,TAR} = \sum_t^T \pi_s^{TAR}$$

where $P_t^T$ and $P_t^{T,TAR}$ are time $t$ expectations of the differences in the price levels between $T$ and $t$, based on the expectations of actual and target inflation rates, respectively.

To inspect how domestic long rates relate to foreign long interest rates, the setting of the target (influencing the expected price level $P$) and the determination of the long-run level of nominal (and hence real) interest rates have to be made more explicit.

The choice of the target is not independent, however, if the monetary authority wishes to control inflation and keep the exchange rate fixed; the long-run properties of the economy constrain it. The monetary authority cannot aspire to change real variables in the long run, hence the real exchange rate is exogenous to it. Assume a particular long-run trajectory of real exchange rates $\bar{z}_t$, such that

$$\lim_{t\to\infty} \bar{z}_t = s_t - p_t^* + p_t \qquad (34)$$
$$\lim_{t\to\infty} \Delta\bar{z}_t = \Delta s_t + \pi_t - \pi_t^*$$

Then, domestic inflation is exogenous to the monetary authority as long as it fixes $s_t\,(\Delta s_t = 0)$, and its stationary value is determined by the long-run depreciation of the real exchange rate and foreign inflation,

$$\pi^{TAR} = \bar{\pi}_t = \Delta\bar{z}_t + \pi_t^* \qquad (35)$$

The inflation target then has to be set in accordance with the stationary level of inflation, implying for the expected change in price between periods $T$ and $t$ based on the inflation target: $P_t^{T,TAR} = \bar{Z}_t^T + P_t^{T*}\ (\bar{Z}_t^T \equiv \bar{z}_T - \bar{z}_t)$.

Using (34), $P_t^{T,TAR}$ approaches the expected change in the actual price level: $lim_{T \to \infty} P_t^{T,TAR} = P_t^T$. The long enough interest rates are then given as:

$$\lim_{T \to \infty} I_t^T = 1/4 * (T - t + 1) \sum_t^T \bar{i}_s \tag{36}$$

Furthermore, the stationary values of nominal (and real) interest rates are also exogenous to monetary policy.[19] They are linked to foreign rates by assuming equalization of real returns in the long run.

The real interest rate parity for long-run trends has:

$$-\Delta \bar{z}_t = \bar{r}_t - \bar{r}_t^* \tag{37}$$

Observing (35), the stationary values of nominal rates are then equal:

$$\begin{aligned} \bar{i}_t &= \bar{i}_t^* - \pi^* - \Delta \bar{z}_t + \bar{\pi} \\ &= \bar{i}_t^* \end{aligned}$$

Going back to (36), the returns on long instruments are equalized at long enough horizons, as implied by international arbitrage:

$$\lim_{T \to \infty} I_t^T = \lim_{T \to \infty} I_t^{T*}$$

Hence, with a fixed exchange rate, the system of equations (35), (33), and (37), and the identities

$$\begin{aligned} \bar{z}_t &\equiv \Delta \bar{z}_t + \bar{z}_{t-1} \\ z_t &\equiv \Delta z_t + z_{t-1} \\ \Delta z_t &\equiv \Delta s_t + \pi_t - \pi_t^* \end{aligned}$$

and a provision guaranteeing the satisfaction of (34) provide both management of short-run interest rates as well as alignment of the implied long rates along their foreign benchmark. The provision guaranteeing the satisfaction of (34) comes from the rest of the model in Section 3.

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