EXTRACTING INFORMATION FROM OPTIONS PREMIA: THE CASE OF THE RETURN OF THE ITALIAN LIRA TO THE ERM OF THE EMS*

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1. INTRODUCTION

The prices of financial derivatives (forward contracts, futures and options) reflect, at each moment, the expectations of economic agents regarding the future path of the prices of the underlying assets. On the other hand, the price of the underlying assets (e.g., Treasury bills, Treasury bonds, stocks and commodities) reflect market expectations of the future path of their economic determinants. While forward and futures contracts provide information on the expected value of the prices of the underlying assets, options premia allow the estimation of the risk-neutral probability density function (PDF) of the prices of the underlying assets.

In this context, the prices of financial derivatives contain potentially useful information for monetary authorities, namely in building indicators for monetary conditions, in assessing the impact of monetary policy actions, and to help detect anomalies in the functioning of the financial markets. These issues have been studied by several authors and central banks (see for instance Abken (1995), Bahra (1996), Deutsche Bundesbank (1995) and Söderlind and Svenson (1996)). Naturally, this information is also relevant from the viewpoint of portfolio and risk management by the private sector, and by financial institutions in particular.

The comparison between the PDF of the price of a financial asset, estimated at different instants for a given maturity, provides a measure for the path of market expectations, and for its dispersion. For instance, Campa, Chang and Reider (1997) analyse the reaction of foreign exchange markets to the re-entrance of the Italian lira in the Exchange Rate Mechanism of the European Monetary System (ERM-EMS) on 25 November 1996, and find that the implicit exchange rate volatility of the Italian lira decreased with its re-entrance in the ERM-EMS. This finding is consistent with the idea that the change in the exchange rate regime aimed at stabilising the exchange rate.

This study analyses the same episode, but using the premia of futures options of the 3-month interest rates of the Italian euro-lira[(1)]. We use daily observations of the settlement prices for the call-options and put-options traded in the London International Financial Futures Exchange (LIFFE).

The remainder of the paper is structured as follows: section 2 presents the most essential aspects of the estimation methodology; all technical details are left for Appendix. The third section applies the methodology and the last one concludes.

* The opinions of this paper represent the views of the authors, they are not necessarily those of the Banco de Portugal.
The authors benefited from the generosity of LIFFE concerning the statistical data, and thank Vitor Gaspar and José Ferreira Machado for their comments. The authors are liable for remaining errors and omissions. This research also benefited from the visit of Charles Thomas to the Banco de Portugal.

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(1) The analysis of Campa, Chang and Reider (1997) is exclusively based on the premia of over-the-counter currency options, with no reference to interest rates.

2. MAIN CONCEPTS AND ESTIMATION METHODOLOGY

As a rule, financial institutions price European options with the Black-Scholes formula(3). The main assumption underlying to the Black-Scholes model is that the rates of return have normal distribution (hence the prices of the underlying assets are log-normally distributed) and are independent and identically distributed. The problem is that the assumption of log-normality of the prices of the underlying assets is frequently in contradiction with reality.

For instance, this model considers that the implied volatility is constant for all strike prices and for all expiration dates, which regularly does not apply. In fact, for the same expiration date and underlying asset, the volatility is a convex function of the strike price, assuming higher values for the options with strike prices more distant to the expected future price of the underlying asset. The relation between implied volatility and the strike price is currently defined as volatility smile, being usually considered as a signal of rejection of the log-normality assumption.

Consider the valuation of an European call-option with expiration date T. Let $S_{T\,i}$ be the price of the underlying asset in moment T, and X the strike price of the European option. The premium at t of a call-option with term to maturity equalling $\tau = T - t$, $C(X,\tau)$, is the expected return of the option, discounting that return if the premium is paid upfront at date t. This corresponds to:

$$C(X,\tau) = e^{-r_{t,\tau}\tau} \int_0^{\infty} \max\left[S_T - X, 0\right] q_t\left(S_T\right) dS_T \qquad (1)$$

where $r_{t,\tau}$ is the relevant interest rate at the moment t (for maturity τ) and $q_t\ (S_T)$ is the (risk-neutral) PDF of the asset price S_T, conditional on the current asset price S_t.

In principle, the PDF can be assessed directly through a discrete approximation from the option premia for the different strike prices observed, since theoretically the risk-neutral PDF corresponds to the second derivative of the call-option price function(4):

$$q_t\left(S_T\right) = e^{r_{t,\tau}\tau} \cdot \frac{\partial^2 C(X,\tau)}{\partial X^2} \qquad (2)$$

However, the results obtained through this method are in general unsatisfactory(5). Many alternative techniques for estimating the PDF from the option premia have been suggested(6). One of the most currently used consists of estimating the parameters of a combination of two log-normal distributions:

$$q_t\left(S_T\right) = \theta L\left(\alpha_1, \beta_1; S_T\right) + (1-\theta) L\left(\alpha_2, \beta_2; S_T\right) \quad (3)$$

where $L\left(\alpha_1, \beta_1; S_T\right)$ and $L\left(\alpha_2, \beta_2; S_T\right)$ are lognormal distributions, α_1 and α_2 are the means of the the respective normal distributions, β_1 and β_2 are the standard deviations of the respective normal distributions and θ is the weight of the first distribution(7).

3. THE RETURN OF THE ITALIAN LIRA TO THE ERM-EMS

The impact of the return of the Italian lira to the ERM-EMS on the expected exchange rate was analysed in Campa, Chang and Reider (1997). Here we analyse the impact of this episode on the expected value of short-term interest rates. We use a daily sample of settlement prices of futures options expiring on 18 December 1996, for the various strike prices, throughout the whole period these were traded (from 19 March up to 16 December 1996).

Estimating the parameters of equation (3) yields the PDF that characterise the daily path of the expected 3-month interest rate of the Italian lira for 18 December 1996. Since the expiration date is fixed, one may observe that the variance of the distribution naturally decreases with the approximation of the expiration date. For instance, if the price of the underlying asset follows a log-normal distribution, when t increases a fraction of time (e.g.) one day, with time expressed in years)

(3) The Black-Scholes pricing formula was developed in Black and Scholes (1973), for premia of options on non-dividend-paying stocks. Thereafter, this formula was adapted to options on financial assets with different features.

(4) See Appendix for a derivation of the result.

(5) See for instance Soderlind and Svenson (1996).

(6) See for instance Adão *et al.* (1997) and Bahra (1997) on the methods of estimating PDF from option premia.

(7) The estimates for the parameters are obtained solving the optimisation problem presented in Appendix.

the standard deviation of $ln\ S_t$ decreases by $\sigma\left(\sqrt{\tau}-\sqrt{\tau-1/365}\right)$. Therefore, if the comparison is made between quite close days, this correction is irrelevant. However, for comparisons between days reasonably far away, any conclusions on the expectations' dispersion in different moments require the correction of the standard-deviation of $ln\ S_T$ by the “time” effect. This same correction principle was used for the estimated distributions.

Chart 1 displays the PDF adjusted for the “time” effect estimated for three distinct dates: 19 March, 19 November and 25 November, respectively, the date of the beginning of trade on options expiring on 18 December 1996, one week before and the day the Italian lira returned to the ERM-EMS.

The means of the distributions equalled 8.8, 7.1 and 7.06 per cent, respectively. It should be noted that the mean of the distribution did not change significantly with the return of the lira to the ERM-EMS. Nevertheless, the probability associated to higher interest rates decreases — i.e., the skewness of the distribution decreases.

A significant reduction in the distribution's mean occurred after with the legislative election which took place on 21 April 1996, as shown in chart 2. Indeed, on 17 April the distribution mean amounted to 8.6 per cent, while on 22 April the mean stood at 7.5 per cent. Contrary to what occurred with the return of the Italian lira to the ERM-EMS, no significant changes in the skewness of the PDF were recorded.

Chart 1
PROBABILITY DENSITY FUNCTIONS OF THE THREE-MONTH ITALIAN LIRA INTEREST RATE

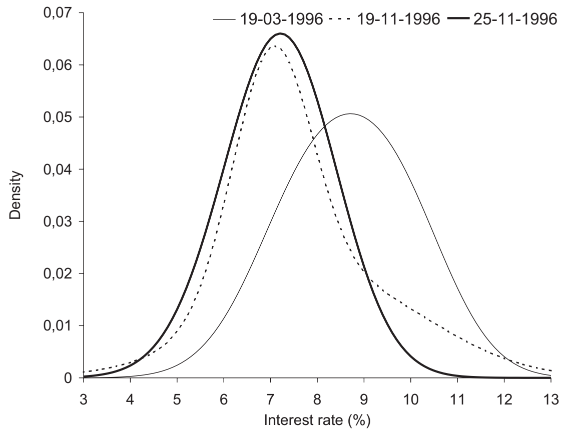
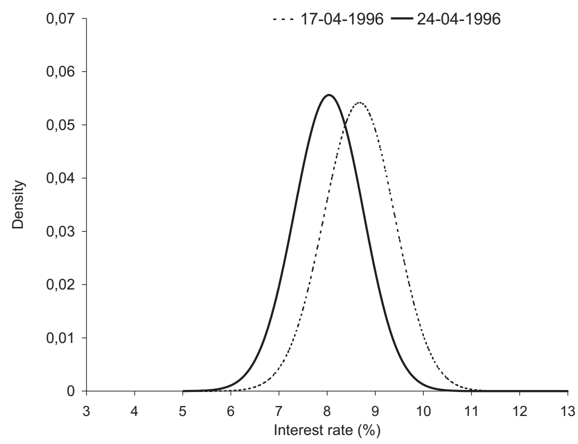


Chart 2
PROBABILITY DENSITY FUNCTIONS OF THE THREE-MONTH ITALIAN LIRA INTEREST RATE



4. CONCLUSIONS

Financial derivatives provide relevant information of the expectations of economic agents regarding the futures path of prices of the underlying assets. While the prices of futures contracts deliver indications on the expected values, the prices of options allow for a more complete characterisation of the various future values.

Taking the 3-month Italian lira interest rate from futures options premia traded at LIFFE, we estimated the risk-neutra PDF of the 3-month Italian lira interest rate. We found that the return of the Italian lira to the ERM-EMS did not change significantly the expected short-term interest rates, despite reducing the skewness of the probability distribution. On the contrary, the results of the Italian legislative elections led to a significant shift of the distribution, thereafter characterised by a lower expected short-term interest rate. The electoral results resulted in a lessening of political uncertainty in Italy, and may have consolidated the expectations regarding the return of the Italian lira to the ERM-EMS. Therefore, when the return of the lira to the ERM took place, this uncertainty had already been discounted by the markets.

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APPENDIX

Deduction of equation (2)

Differentiating (1) in order to the strike price we obtain:

$$\frac{\partial C(X,\tau)}{\partial X} = -e^{-r_{t,\tau}\tau}\int_X^{\infty} q_t(S_T)dS_T = \\ = -e^{-r_{t,\tau}\tau}\left(1-\int_{-\infty}^{X} q_t(S_T)dS_T\right) \qquad (A1)$$

which yields,

$$1+\frac{\partial C(X,\tau)}{\partial X}e^{r_{t,r}\tau} = P_q\left[S_T \le X\right] \qquad (A2)$$

where P_q is the probability measure.

Differentiating (A2) in order to the strike price yields (2).

Estimation of the parameters of equation (3)

The parameters are estimated from the minimisation of a distance function between the sample premia and the premia obtained from the specified functional form. The following optimisation problem is solved:

$$\underset{\alpha_1,\alpha_2,\beta_1,\beta_2,\theta}{Min}\sum_{i=1}^{N}\left[C(X_i,\tau)-C_i^0\right]^2+\sum_{i=1}^{N}\left[P(X_i,\tau)-P_i^0\right]^2+ \\ \left[+\theta e^{\alpha_1+\frac{1}{2}\beta_1^2}+(1-\theta)e^{\alpha_2+\frac{1}{2}\beta_2^2}-e^{rt}S\right]^2 \qquad (A3)$$

subject to $\beta_1,\beta_2 > 0$ and $0 \le \theta \le 1$, and being:

$$C(X_i,\tau)=e^{-r\tau}\int_{X_i}^{\infty}\left[\theta L(\alpha_1,\beta_1;S_T)+ \\ +(1-\theta)L(\alpha_2,\beta_2;S_T)\right](S_T-X_i)dS_T \qquad (A4)$$

$$P(X_i,\tau)=e^{-r\tau}\int_{0}^{X_i}\left[\theta L(\alpha_1,\beta_1;S_T)+ \\ +(1-\theta)L(\alpha_2,\beta_2;S_T)\right](X_i-S_T)dS \qquad (A5)$$

where X_i *(i =1, …,N)* are the strike prices and $P(X,\tau)$ is the price of a put-option.

The first two terms in (A3) are the sum of square of the residuals between the estimated and the sample premia of call-options and put-options, respectively. The last term is an approximating factor of the estimated mean to the forward price.

This method has several advantages: firstly, it allows for flexible density functions, consequently permitting to characterise multimodal functions and functions exhibiting skewness or kurtosis; secondly, it allows to use simultaneously call-options and put-options premia, without any arithmetic transformation. However, it is a time-consuming procedure and the results are sensitive to the initial values.

To minimise these aspects, one may use a more comprehensive function, for instance, minimising only on call-options premia (and / or transforming the Put-options premia into call-options premia) and eliminating the approximating factor to the forward. The elimination of this factor allows to use the deviations between the estimated mean and the forward price as a measure of the degree of precision of the distribution obtained.